Non-equilibrium Dynamics, Hersonissos, Crete, 9-14 June, 2014

Non-equilibrium Dynamics of the Chiral Fluid

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with Christoph Herold, Marlene Nahrgang and Marcus Bleicher

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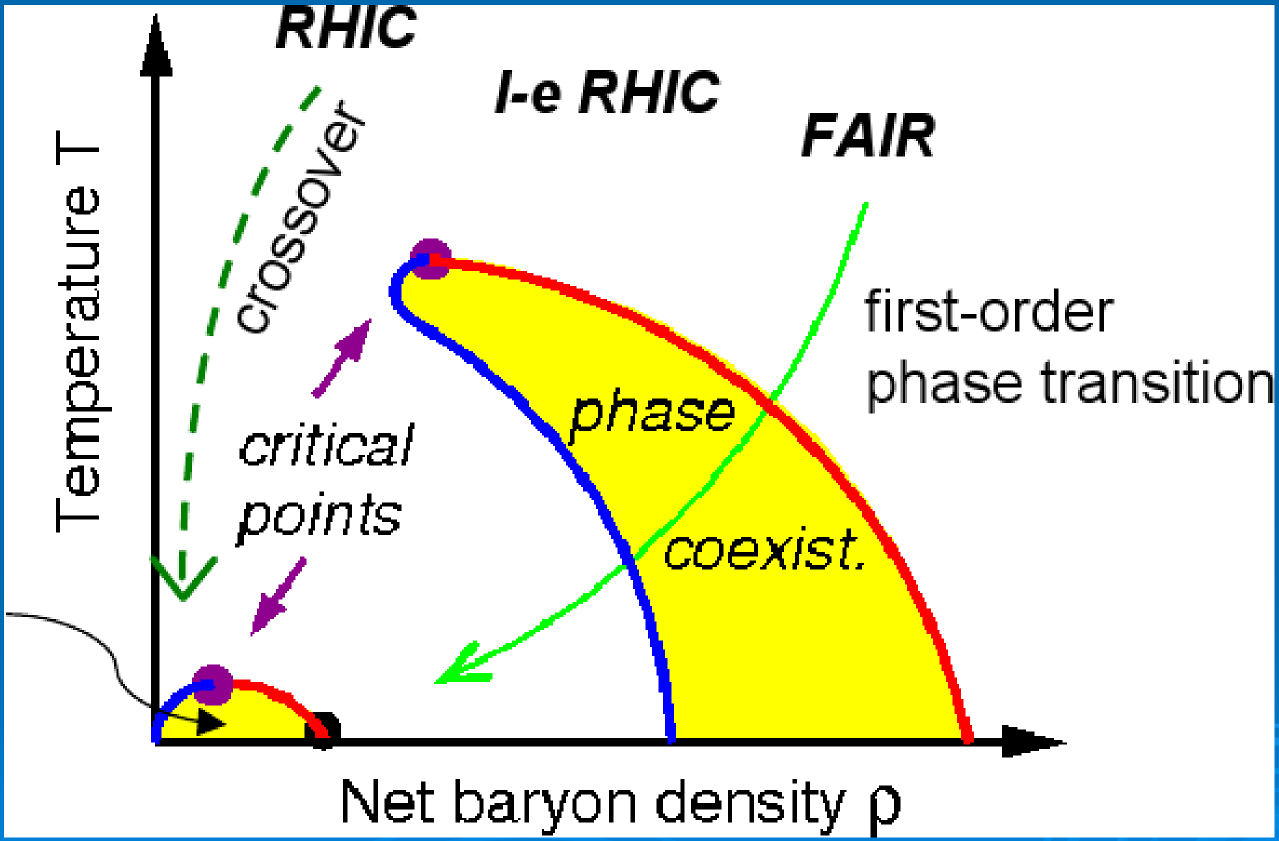
Moscow

FIAS Frankfurt Institute for Advanced Studies

HIC for FAIR Helmholtz International Center

NATIONAL RESEARCH CENTER KURCHATOV INSTITUTE

”QCD” phase diagram



Contents

- Introduction: Effects of fast dynamics
- Effective potential and fluctuations of order parameter
- Chiral fluid dynamics with damping and noise
- Extension to finite baryon densities
- Dynamical domain formation in 1st order tansition
- Conclusions

This talk is based on recent works:

M. Nahrgang, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, The impact of dissipationand noise on fluctuations in chiral fluid dynamics, J. Phys. G 40 (2013) 055108;

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Chiral fluid dynamics with explicit propagation of the Polyakov loop, Phys. Rev. C 87 (2013) 014907;

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Formation of droplets with high baryon density at the QCD phase transition in expanding matter, Nucl. Phys. A 925 (2014) 14;

I. Mishustin, T. Koide, G. Danicol, G. Torrieri, Dynamics and stability of chiral fluid, Phys. Atom. Nucl. (in press), arXiv:1401.4103.

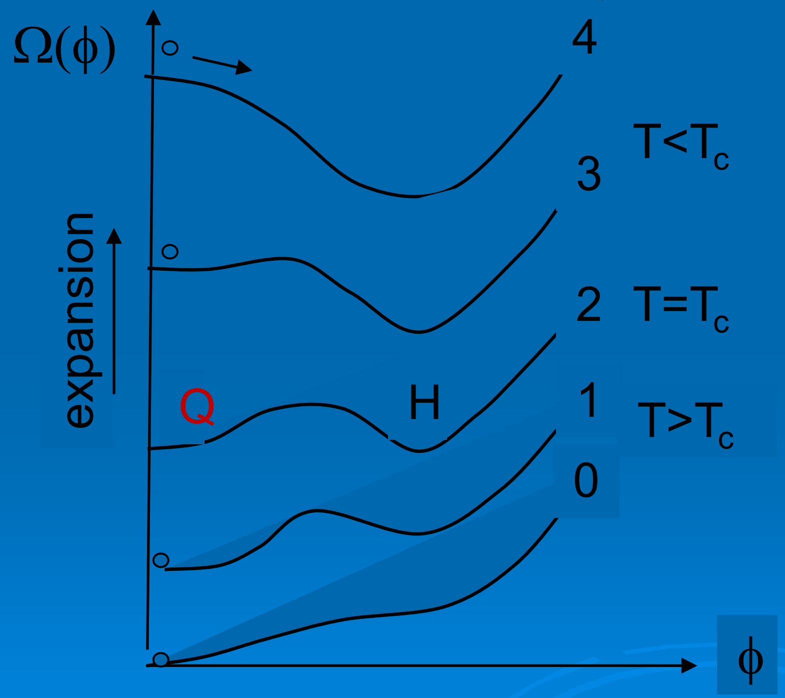
Effects of fast dynamics

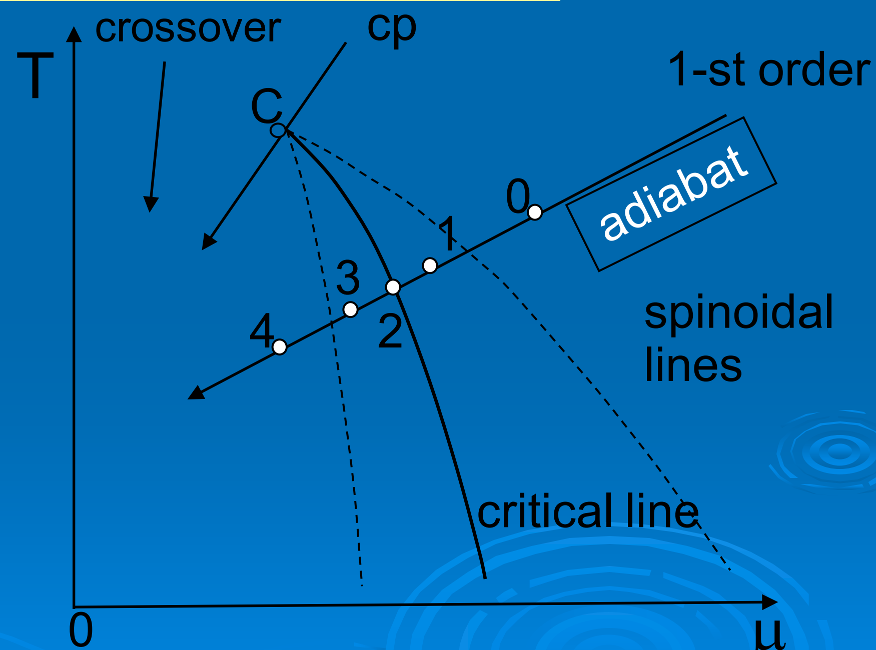
Effective thermodynamic potential for a 1[st] order transition

$$\Omega(\phi;T,\mu)=\Omega_0(T,\mu)+\frac{a}{2}\phi^2+\frac{b}{4}\phi^4+\frac{c}{6}\phi^6$$

a,b,c are functions of T and μ

Equilibrium ϕ is determined by $\frac{\partial\Omega}{\partial\phi}=0 \Rightarrow P=-\Omega(\phi_{eq})$

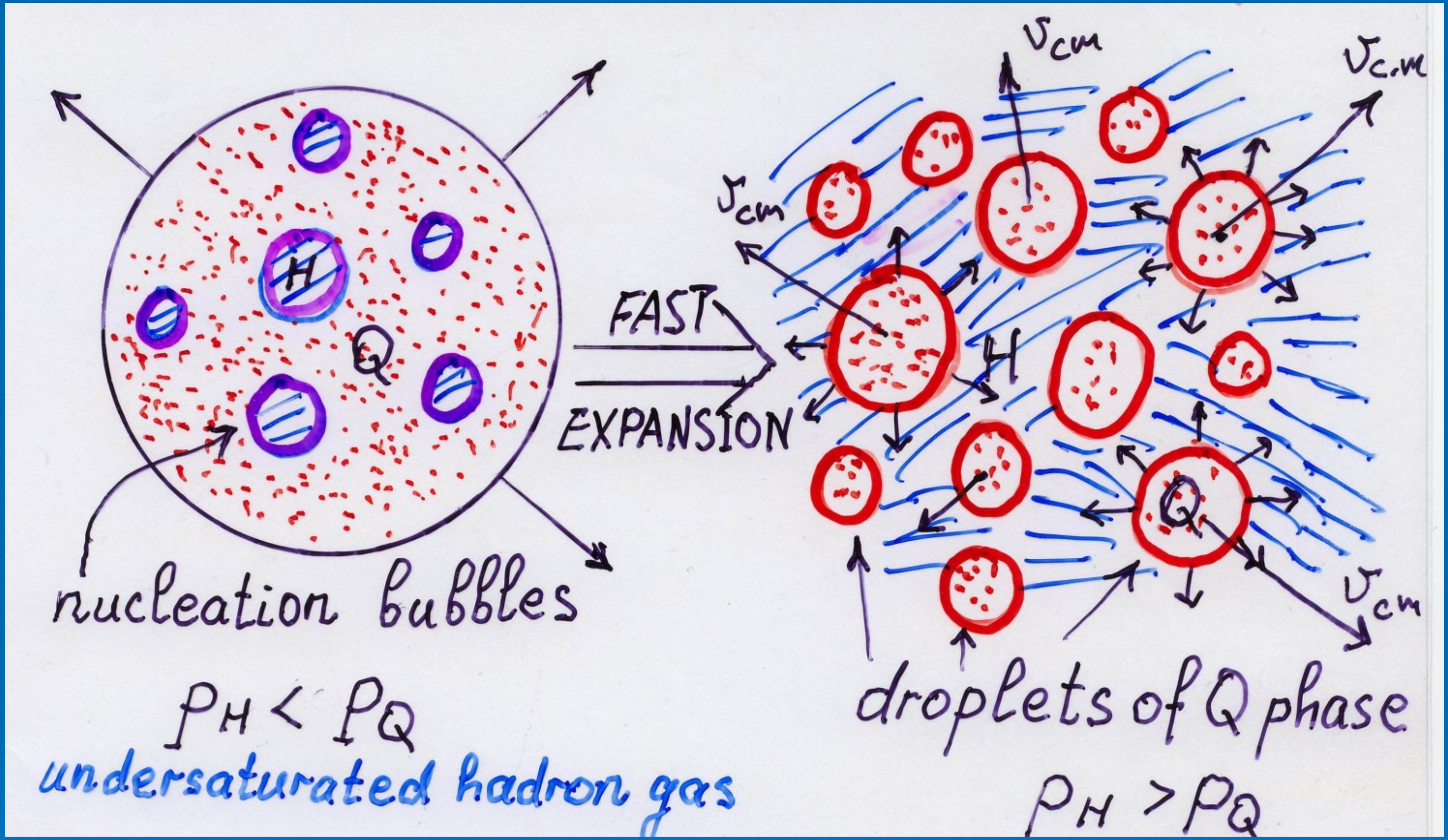




In rapidly expanding system 1-st order transition is delayed until the barrier between two competing phases disappears - spinodal decomposition

I. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. A681 (2001) 56

Rapid expansion through a 1st order phase transition



The system is trapped in a metastable state until it enters the spinodal instability region, when Q phase becomes unstable and splits into droplets

Csernai&Mishustin, 1995; Mishustin, 1999; Rafelski et al. 2000; Randrup, 2003; Steinheimer&Randrup 2013; …

Simple model for chiral phase transition

Scavenius, Mocsy, Mishustin&Rischke, Phys. Rev. C64 (2001) 045202

Linear sigma model (LσM) with constituent quarks

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi})]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi}),$$

$$U(\sigma, \vec{\pi}) = \frac{\ }{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \ \ <\sigma>_{\rm vac} = f_\pi \rightarrow H = f_\pi m_\pi^2$$

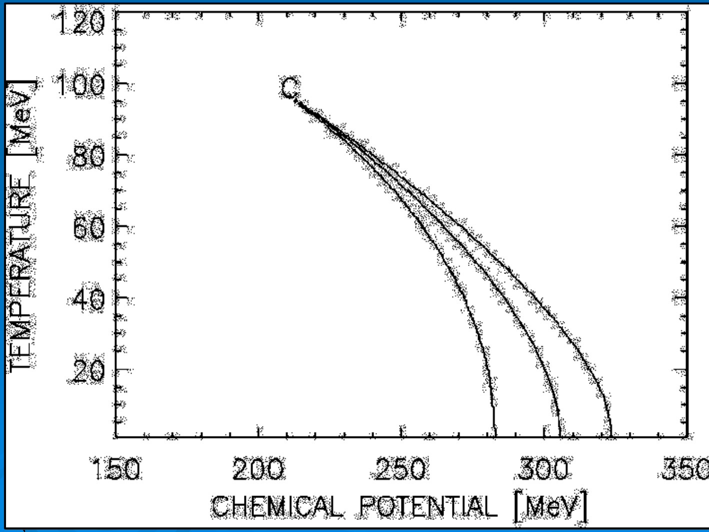
Effective thermodynamic potential contains contributions of mean field and quark fluid:

$$U_{\rm eff}(\sigma; T, \mu) = U(\sigma, \boldsymbol{\pi}) + \Omega_q(m; T, \mu)$$

$$m^2 = g^2(\sigma^2 + \boldsymbol{\pi}^2)$$

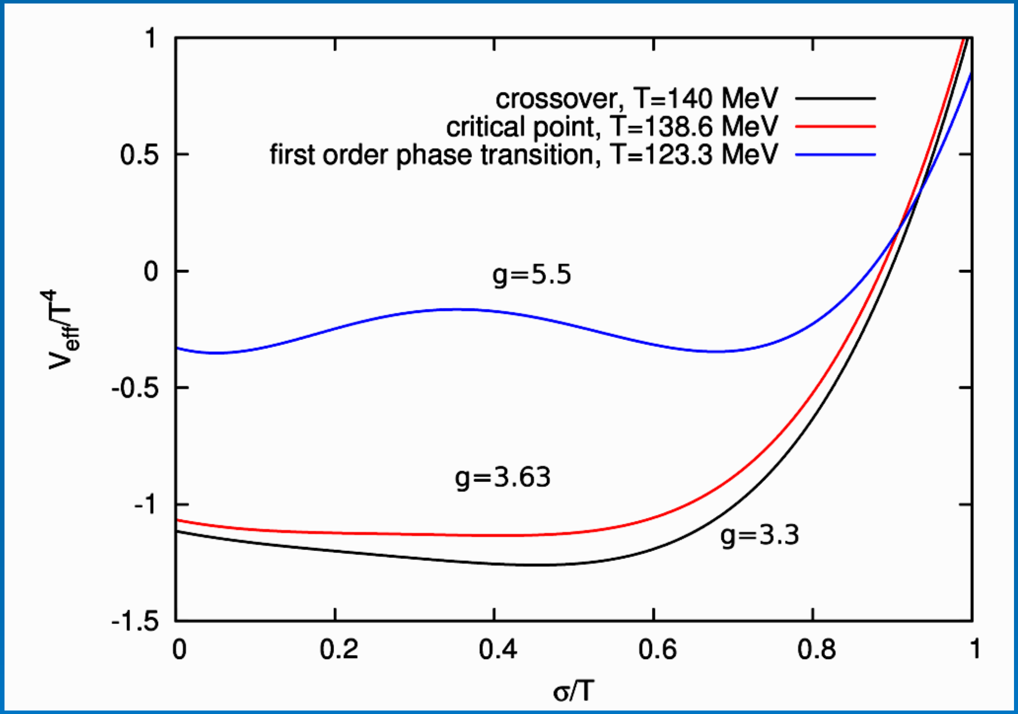
CO, 2nd and 1st order chiral transitions are obtained in T-μ plane.

Phase diagram



Effective thermodynamic potential

$$\Omega_q(m;T,\mu) = -\nu_q T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left\{ \ln\left[1+\exp\left(\frac{\mu-\sqrt{m^2+p^2}}{T}\right)\right] + (\mu \to -\mu) \right\}, \quad \nu = 2N_f N_c$$



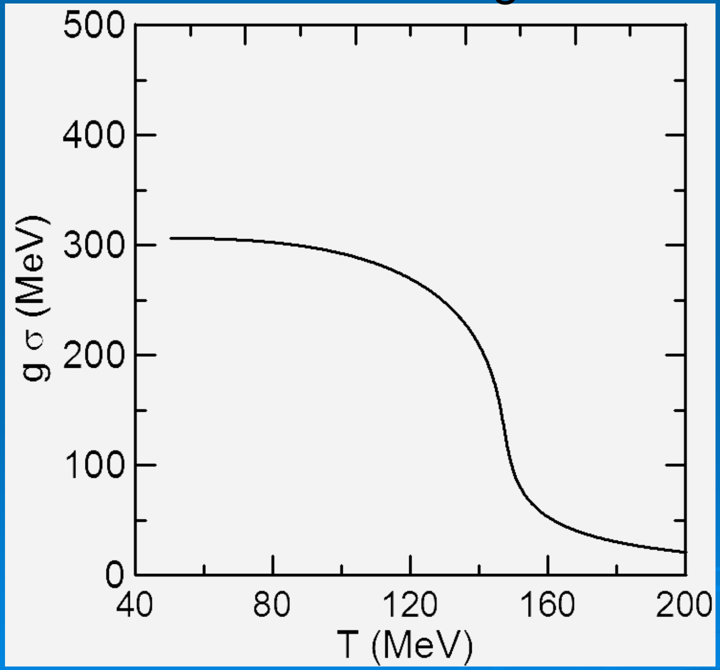
First we consider μ=0 system but tune the order of the chiral phase transition by changing the coupling g.

Equilibrium order parameter field

$$\lambda\left(\sigma^2-\nu^2\right)\sigma+g\rho_s(\sigma)=0,\ \rho_s(\sigma)=\frac{\partial\Omega_{\mathrm{q}}(m;T,\mu)}{\partial\sigma}$$

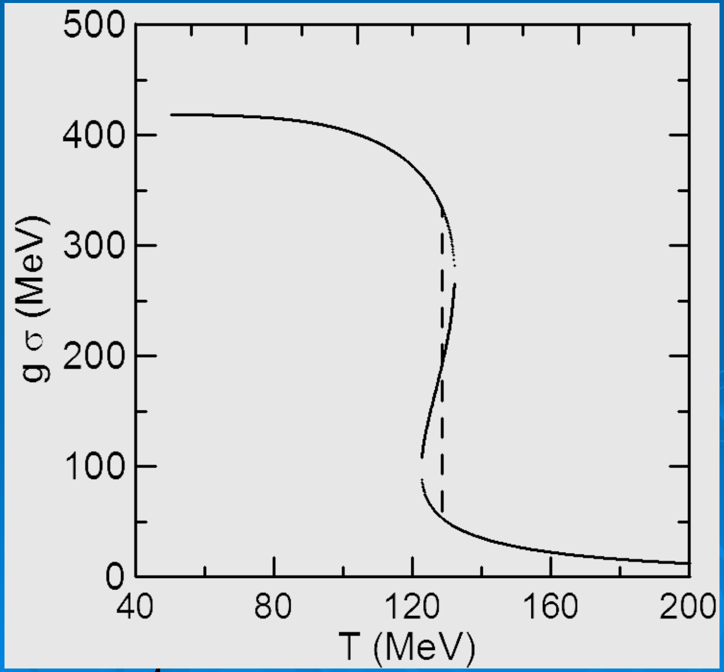
$$m_\sigma^2(\tau)=\frac{\partial^2U_{eff}}{\partial\sigma^2}=\lambda^2(3\sigma^2-\sigma^2{}_0)+g\frac{\partial\rho_s}{\partial\sigma}$$

crossover g=3.3



Only 1 equilibrium solution at each T

1-st order g=4.5



3 solutions at 122 MeV<T<132 MeV unstable states - spinodal instability

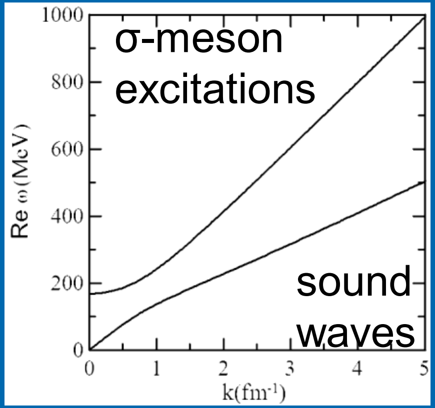
Spectrum of plane-wave fluctuations

I. Mishustin, T. Koide, G. Danicol, , G. Torrieri, Phys. Atom. Nucl.; arXiv:1401.4103.

$$\delta\sigma(x) = \delta\sigma(\omega,k)e^{i\omega t - ikx}$$

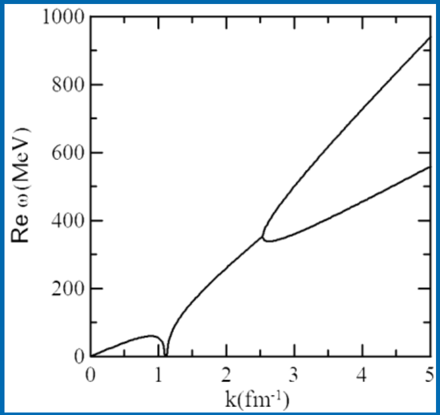
Generally two branches: 1) sound branch $\omega^2 \approx c_s^2 k^2$

2) sigma branch $\omega^2 \approx m_\sigma^2 + k^2$

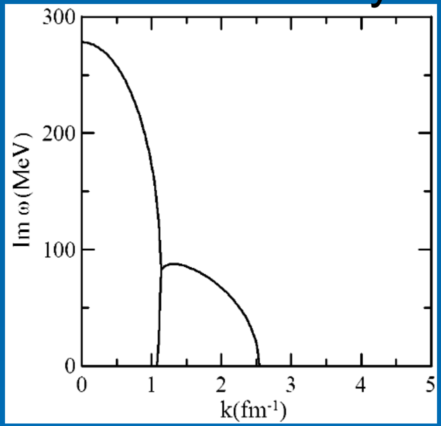


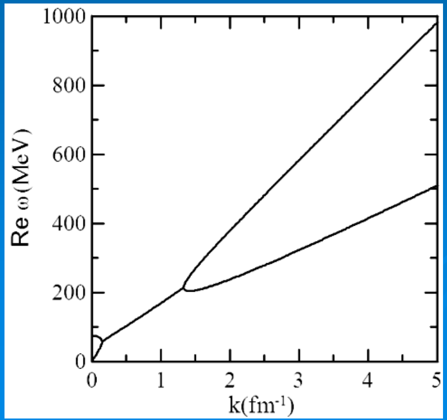
T=120 MeV

Solutions with $\omega^2<0$ indicate instability

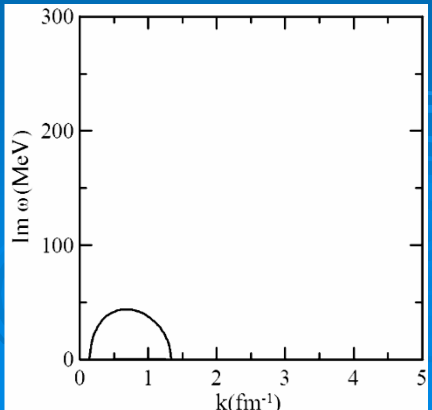


T=125 MeV





T=131 MeV



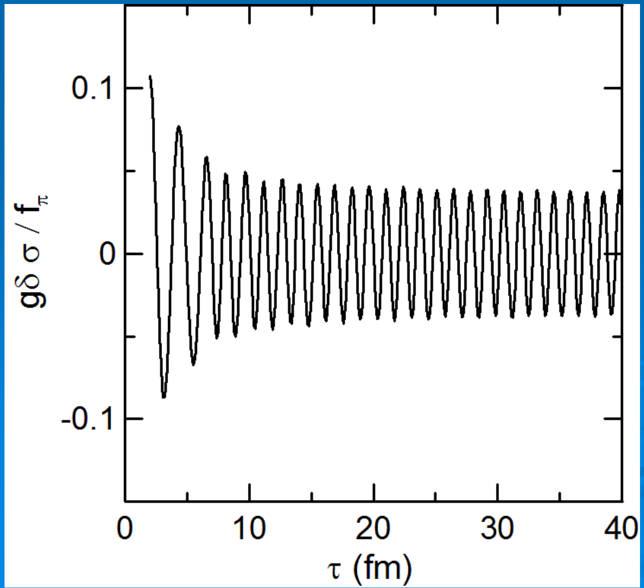
Fluctuations in Bjorken background

$$\delta\sigma(\eta,\tau) = \delta\sigma_k(\tau)e^{ik\eta},\ \tau = \sqrt{t^2 - z^2}$$

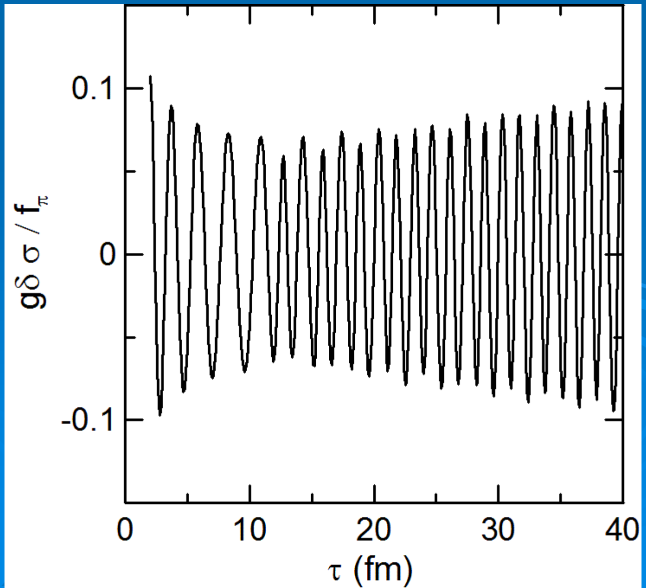
$$\left[\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) + \frac{k^2}{\tau^2} - \Gamma^2(\tau)\right]\delta\sigma_k(\tau) = f(T)$$

$$\Gamma^2(\tau) = -m_\sigma^2(\tau) + g\,\frac{\partial s/\partial\sigma}{\partial s/\partial T}\,\frac{\partial\rho_s}{\partial T}$$

Crossover transition (g=3.3)



1st order transition (g=4.5)



CFD with dissipation and noise

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134;
K. Paech, H. Stocker and A. Dumitru, Phys. Rev. C 68 (2003) 044907;
M. Nahrgang, C. Herold, S. Leupold, , C. Herold, M. Bleicher, Phys. Rev. C 84 (2011) 024912;
M. Nahrgang, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, J. Phys. G40 055108.

Fluid is formed by constituent quarks and antiquarks which interact with the chiral field via quark effective mass $m = g\sigma$

CFD equations are obtained from the energy momentum conservation for the coupled system fluid+field

$$\partial_\nu (T_{\text{fluid}}{}^{\mu\nu} + T_{\text{field}}{}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}{}^{\mu\nu} = -\partial_\mu T_{\text{field}}{}^{\mu\nu} \equiv S^\nu$$

$$S^\nu = -(\partial^2\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma})\partial^\nu\sigma = (g\rho_s + \eta\partial_t\sigma)\partial^\nu\sigma$$

We solve generalized e. o. m. with friction (η) and noise (ξ):

$$\partial_\mu\partial^\mu\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma} + g < \bar{q}q > + \eta\partial_t\sigma = \xi$$

Langevin equation for the order parameter

$$<\xi(t,\vec{r})>=0, \qquad <\xi(t,r)\xi(t',r')>=\frac{1}{V}m_\sigma\eta\delta(t-t')\delta(r-r')\coth\left(\frac{m_\sigma}{2T}\right)$$

Calculation of damping term

T.Biro and C. Greiner, PRL, 79. 3138 (1997)

M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, PRC 84, 024912 (2011)

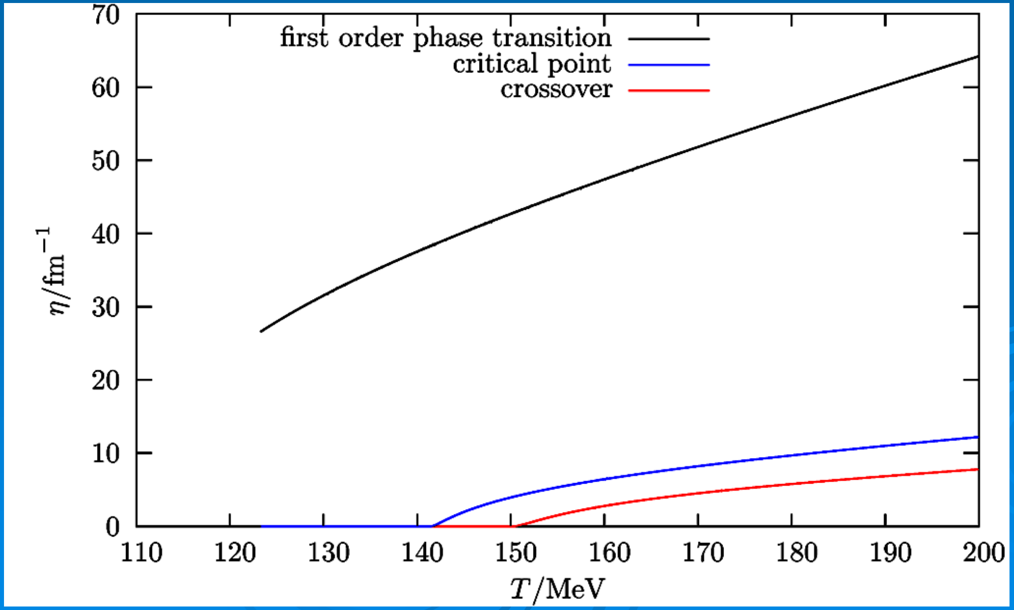
The damping is associated with the processes:

$$\sigma \to \bar{q}q, \ \ \sigma \to \pi\pi$$

It has been calculated using 2PI effective action

$$\eta = g^2 \frac{\nu_q}{\pi m_\sigma^2}\left[1 - 2n_F\left(\frac{m_\sigma}{2}\right)\right]\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{3/2}$$

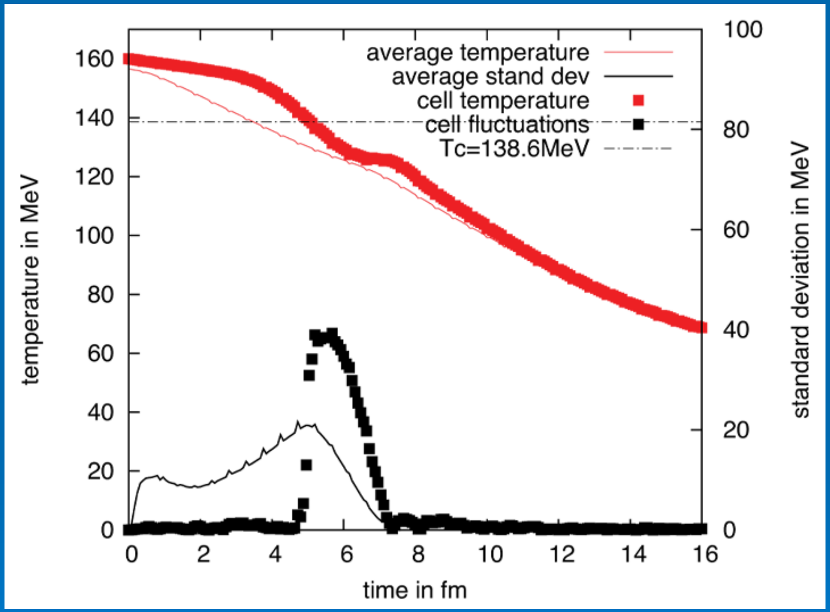
Around Tc the damping is due to the pion modes, η=2.2/fm

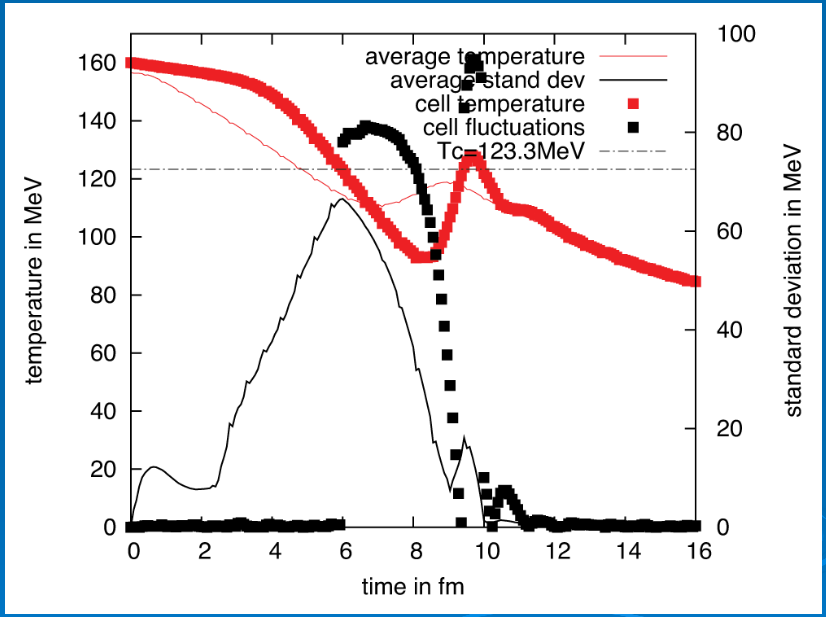


Dynamic simulations: Bjorken-like expansion

Initial state: cylinder of length L in z direction, with ellipsoidal cross section in x-y direction

$$\text{At } t=0: \; v(z)=\frac{2z}{L}0.2c, \;\; -\frac{L}{2}<z<\frac{L}{2}; \;\; v_x=v_y=0; \;\; T=160\ MeV$$





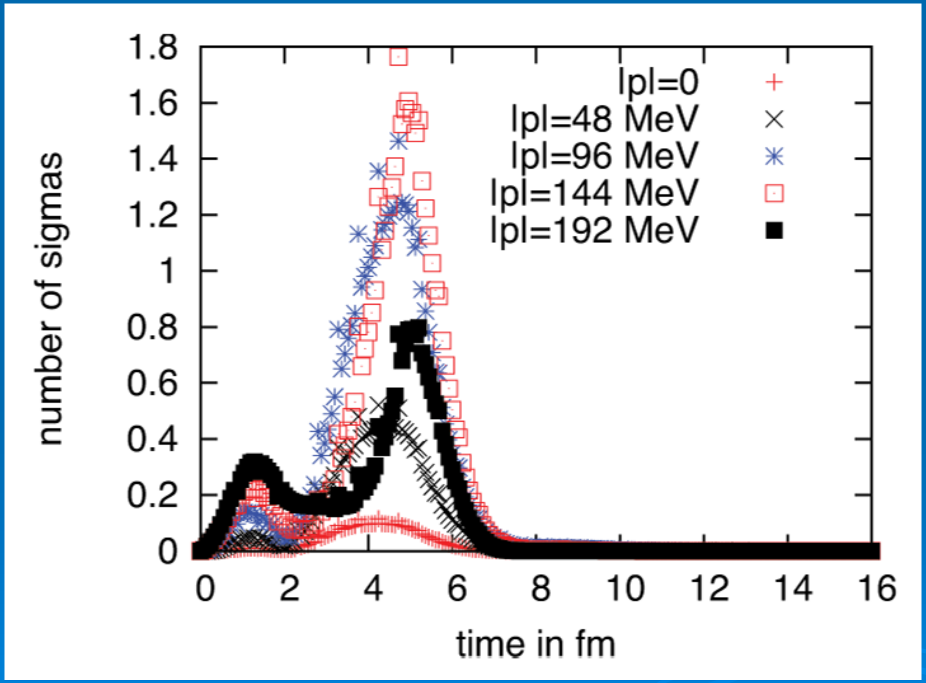
Mean values and standard deviation of T for the whole system and for a central cell (1 fm^3) are shown as a function of time
Supercooling and reheating effects are clearly seen in the 1-st orde transition, fluctuations become especially strong after 4 fm/c

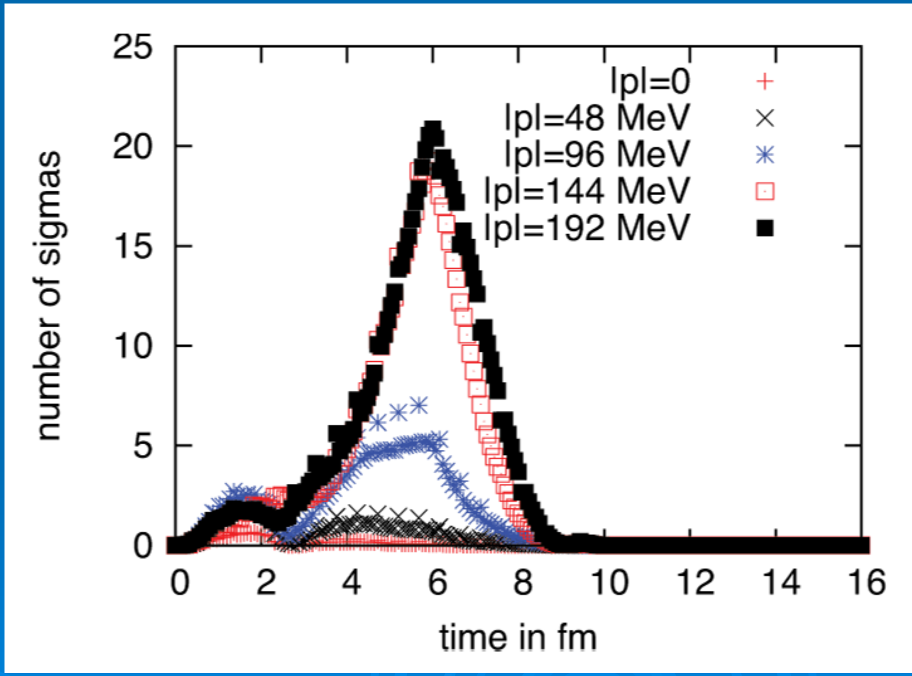
Sigma fluctuations in expanding fireball

$$\frac{dN_\sigma}{d^3k} = \frac{1}{(2\pi)^3}\frac{1}{2\omega_k}[\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2], \ \omega_k = \sqrt{m_\sigma^2 + k^2}, \ \ m_\sigma^2 = \frac{\partial^2 U_{\mathrm{eff}}}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\mathrm{eq}}}$$

Critical point (g=3.63)

First order (g=5.5)





Fluctuations are rather weak at critical point (left), but increase strongly at the 1st order transition (right) after 4 fm/c

Extension to finite baryon densities

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Nucl. Phys. A 925 (2014) 14;

- Include μ-dependence in Polyakov loop potential, (cf. Schäfer, Pawlowski, Wambach Fukushima)

$$\mathcal{U}(\ell, T, T_0)\ , \quad T_0 \to T_0(\mu)$$

- Calculate grand canonical potential for finite chemical potential

$$\Omega_{q\bar{q}} = -2N_f T \int \frac{d^3p}{(2\pi)^3} \left\{ (\ln \left[1 + 3\ell e^{-\beta(E-\mu)} + 3\ell e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}\right] + (\mu \to -\mu) \right\}$$

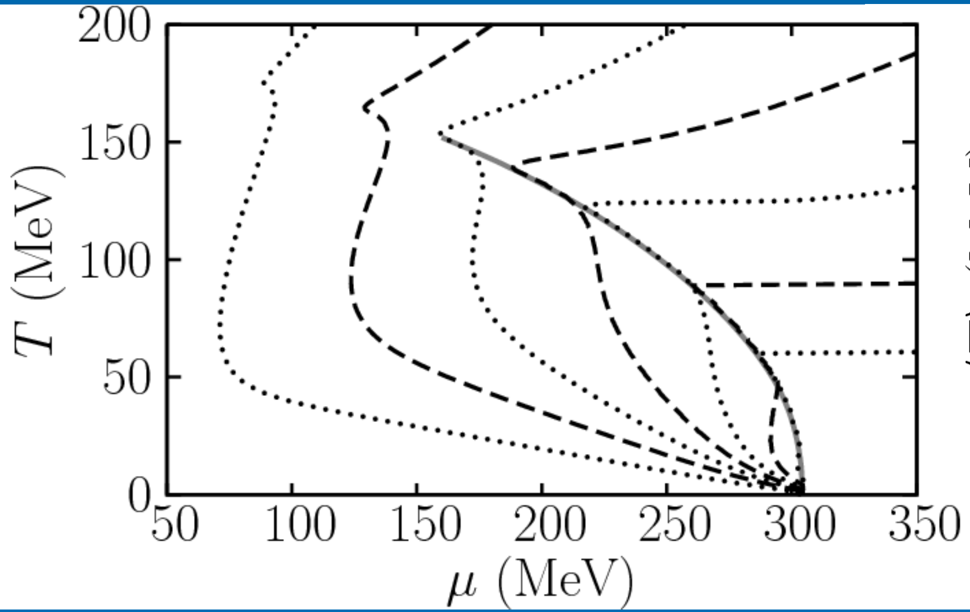
- Propagate (net) baryon density in the hydro sector

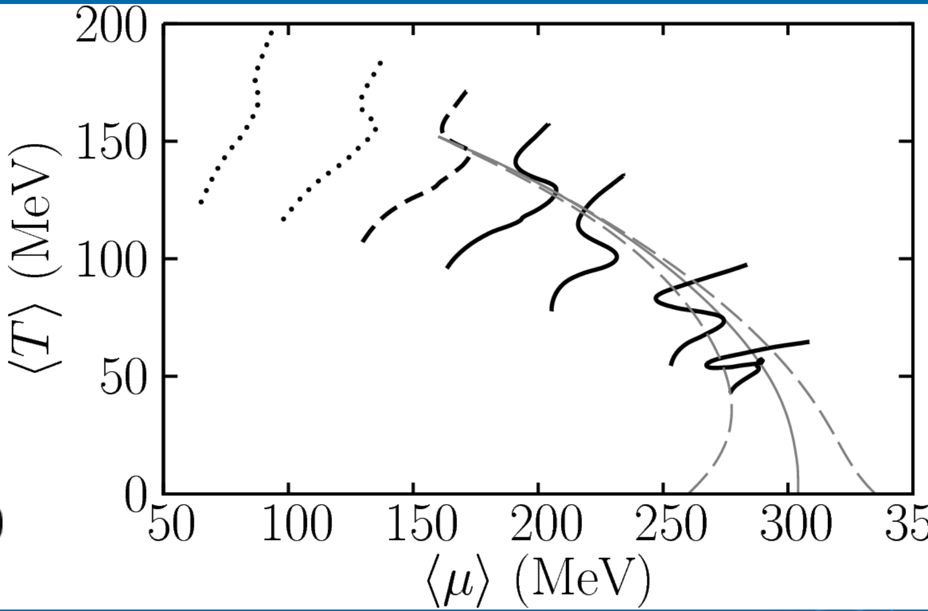
$$\partial_\mu n^\mu = 0\ , \quad n^\mu = \rho u^\mu$$

Trajectories on the T-μ plane

CFD alculations are done for spherical fireball of R=4 fm

Isentropic expansion | Hydrodynamic evolution

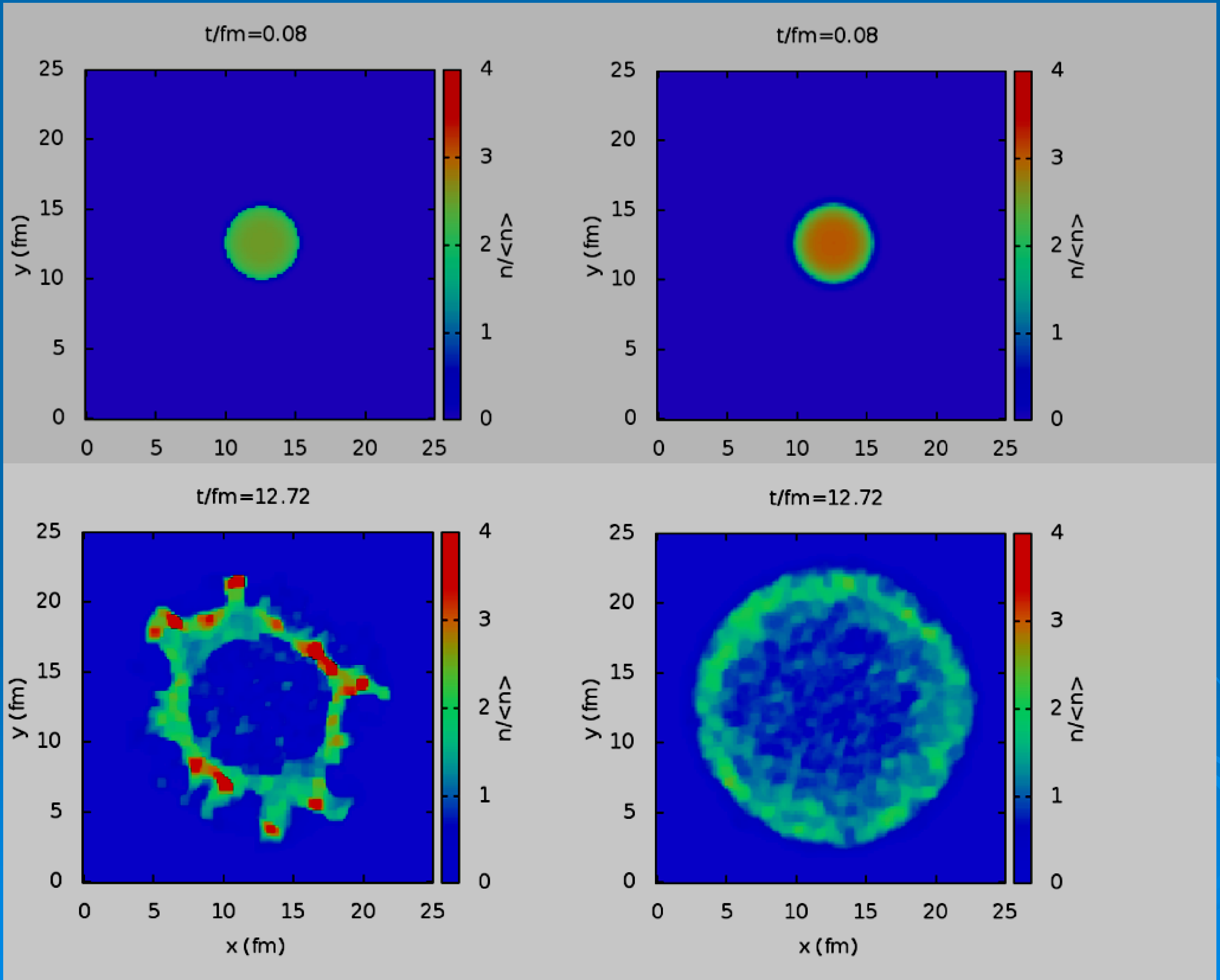




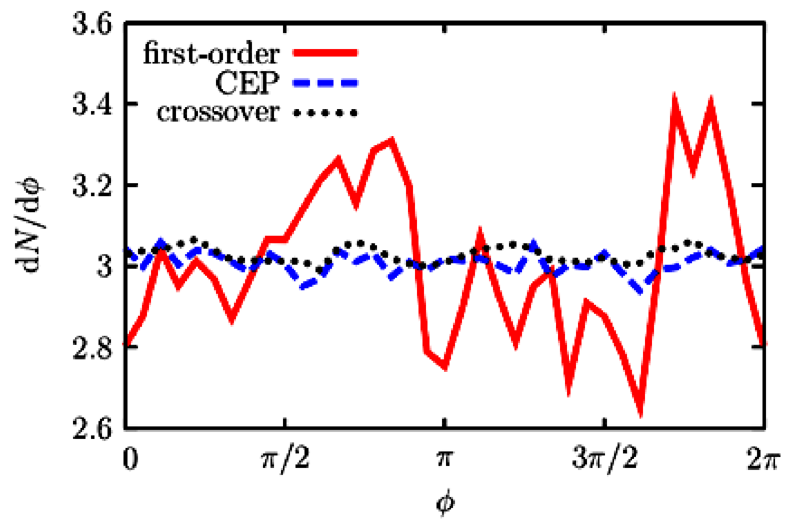
- Trajectories close to isentropes for crossover and CP
- Non-equilibrium "back-bending" is clearly seen in FO case
- At strong FO transition the system is trapped in spinodal region
- for a significant time

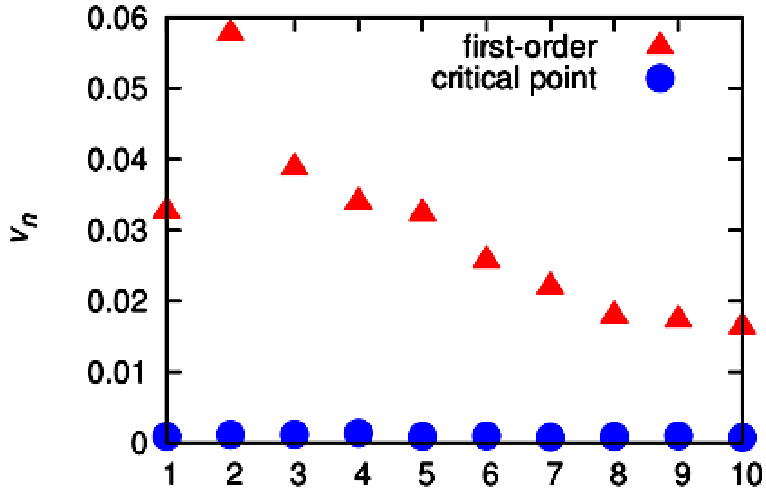
Dynamical fragmentation

First order **Critical point**



Observing high-density domains in expanding system





Azimuthal fluctuations of net-B In single events: strong enhancement at first order PT

High harmonics of baryonic flow (averaged over many events):

$v_n = \langle \cos[n(\varphi - \varphi_n)] \rangle$

New developments

- In the previous calculations the EOS had a P=0 point at a finite baryon density (like the MIT bag model), that makes possible stable quark droplets
- It is interesting to see what happens in a more realistic case when quark droplets are unstable at zero pressure (J. Steinheimer et al, PRC 89 (2014) 034901)
- there exist several models which have such a property, in particular so called Quark-Hadron Model developed at FIAS (S. Schramm et al.)

SU(3) chiral quark-hadron (QH) model

V. Dexheimer, S. Schramm, Phys, Rev. C 81 (2010) 045201

Includes: a) 3 quarks (u,d,s) plus baryon octet,

b) scalar mesons (σ, ς), vector meson (ω)

c) Polyakov loop (l)

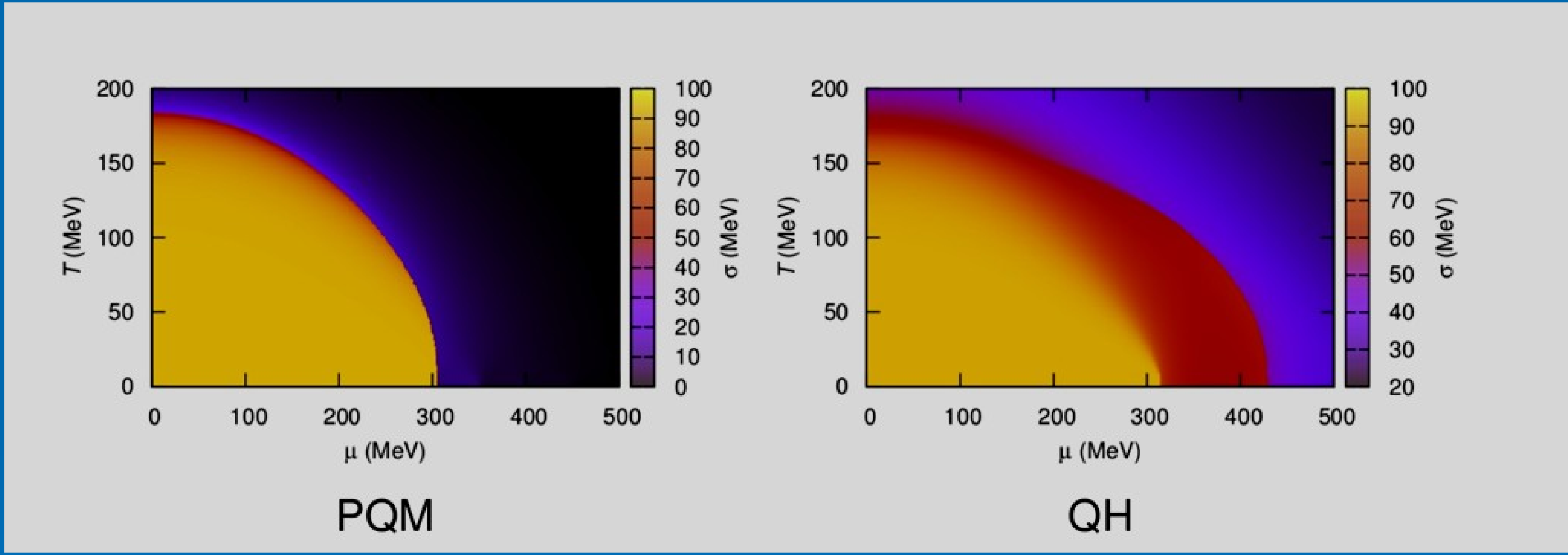
$$\mathcal{L} = \sum_i \overline{\psi}_i \left(i\gamma^\mu \partial_\mu - \gamma^0 g_{i\omega}\omega - M_i\right)\psi_i + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - U(\sigma,\zeta,\omega) - \mathcal{U}(\ell)$$

Effective masses:

$$\begin{aligned} M_q &= g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1-\ell) \\ M_B &= g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{B\ell}\ell^2 \end{aligned}$$

PQM vs. QH: phase diagram

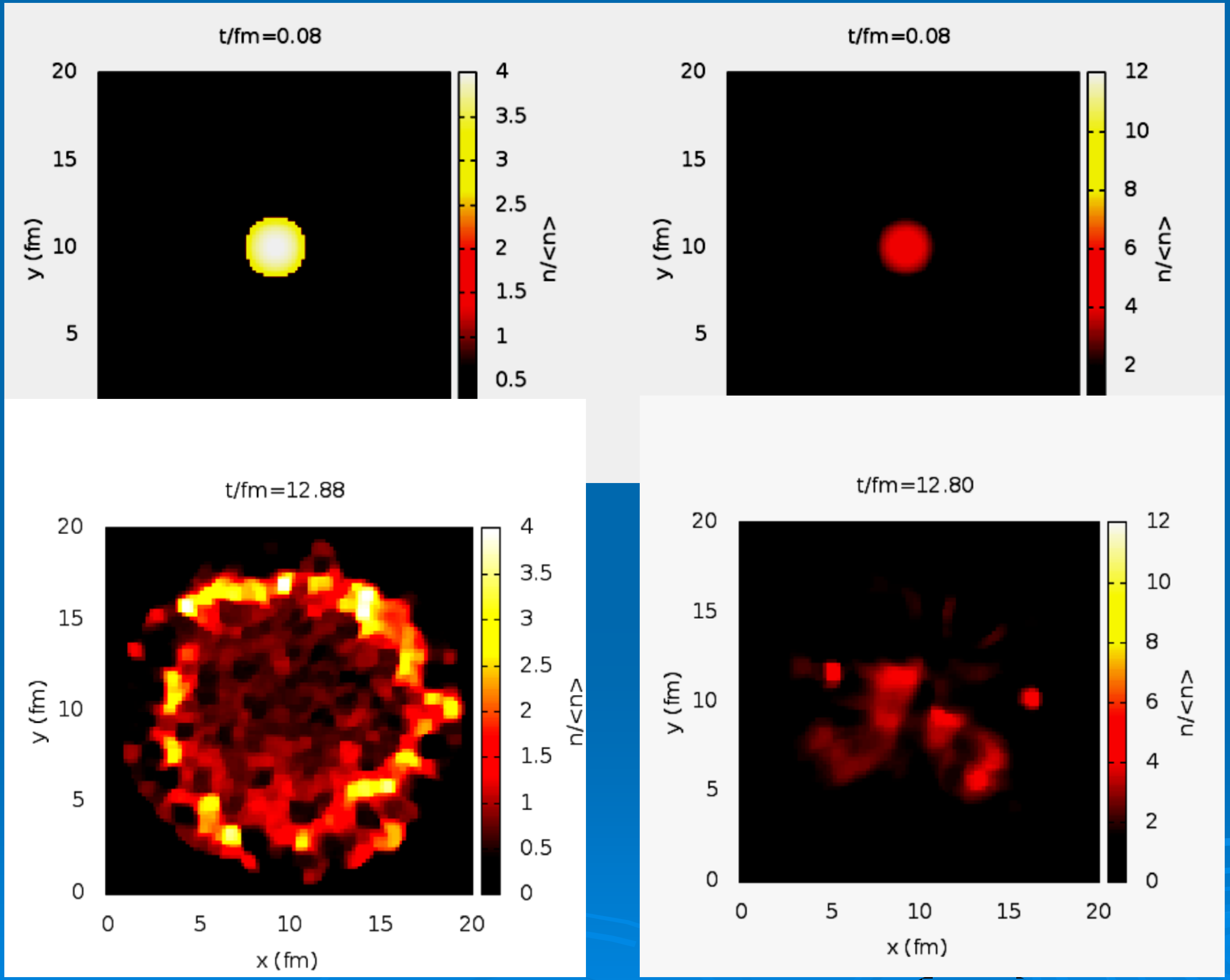
Herold, Limphirat, Kobodaj, Yan, Seam Pacific Conference 2014



Nuclear ground state at $\mu_N=3\mu\approx m_N$ is reproduced correctly
QH predicts two phase transitions:
1) **liquid-gas PT at μ≈300 MeV, and**
2) **deconfinement/chiral PT at higher μ≈450 MeV**

PQM vs. QH: domain formation

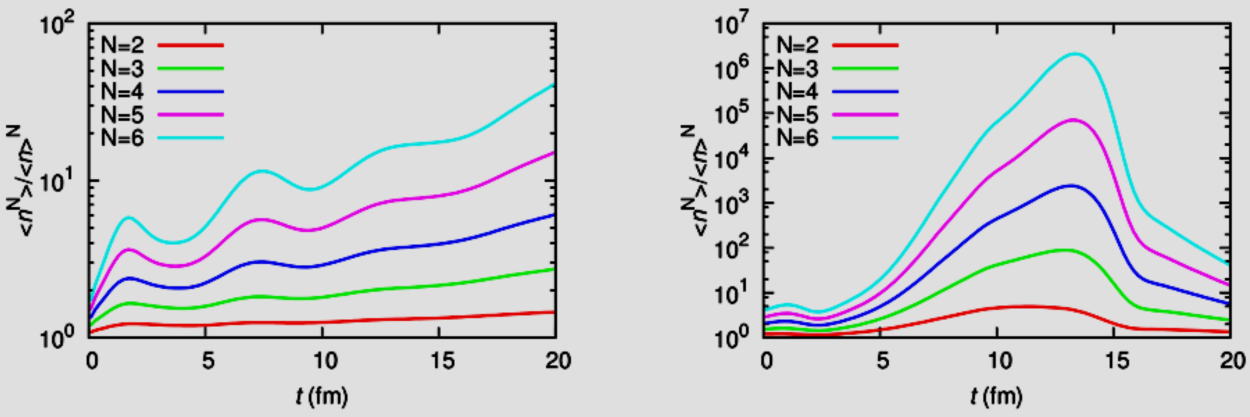
Herold, Limphirat, Kobodaj, Yan, Seam Pacific Conference 2014



QH predicts domains with much higher densities!

PQM vs. QH: density moments

$$\langle n^N \rangle = \int \mathrm{d}^3x \, n(x)^N P_n(x) \qquad \text{with} \quad P_n(x) = \frac{n(x)}{\int \mathrm{d}^3x \, n(x)}$$

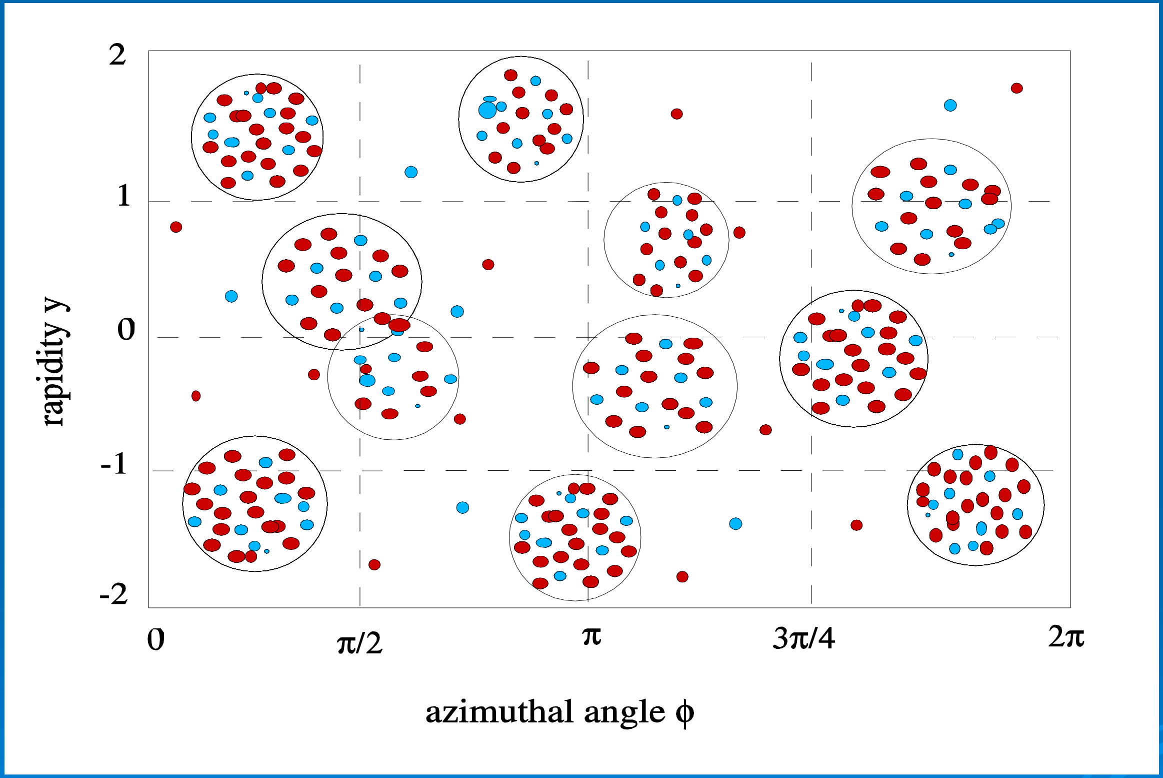


PQM eos QH eos

Strong clustering effect survives even at late times, t>15 fm/c

Experimental signal of droplets

Look for non-statistical fluctuations in kinematic observables of net baryons in individual events, i. e. , azimuthal angle , rapidity, transverse momentum



The bumps in distributions correspond to the emission from individual domains.

Conclusions

- Phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium

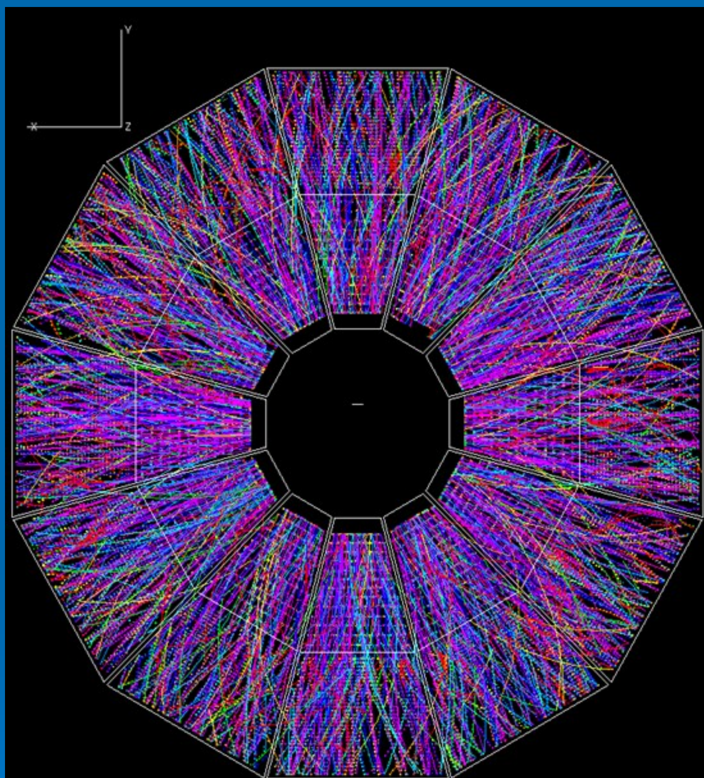
- 2nd order phase transitions (with CEP) are too weak to produce significant observable effects

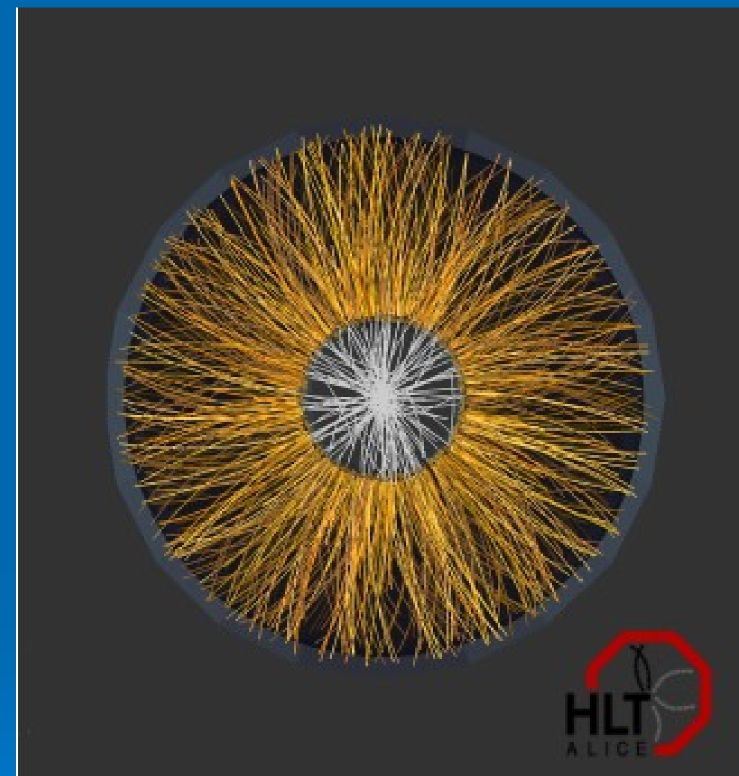
- Non-equilibrium effects in a1st order transition (spinodal decomposition, dynamical domain formation) may help to identify the phase transition

- If large QGP domains are produced in the 1-st order phase transition they will show up by large non-statistical fluctuations in single events

Ultrarelativistic A+A collisions

RHIC (STAR) Charged particles tracks LHC (ALICE)





Au+Au, $\sqrt{s} = 130\,\mathrm{AGeV}$

Pb+Pb, $\sqrt{s} = 2.76\,\mathrm{ATeV}$

Look for bumpiness in the net baryon distributions!