Spectral Functions from the Functional Renormalization Group

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Ralf-Arno Tripolt[a], Nils Strodthoff[b], Lorenz von Smekal[a,c], Jochen Wambach[a,d]

[a] Theoriezentrum, Institut für Kernphysik, TU Darmstadt
[b] Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg
[c] Institut für Theoretische Physik, Justus-Liebig-Universität Giessen
[d] GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt

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Outline

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I) Introduction and motivation

II) Theoretical setup

- functional renormalization group (FRG)
- quark-meson model
- analytic continuation procedure

III) Results for finite temperature and chemical potential

- phase diagram
- spectral functions

IV) Summary and outlook

I) Introduction and Motivation

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[courtesy L. Holicki]

What is a Spectral Function?

A spectral function $\rho(\omega)$ can be defined as:

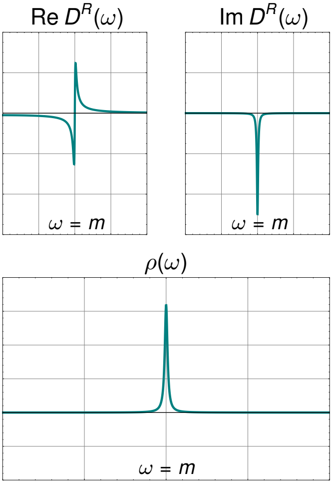
- $\rho(\omega) \equiv \frac{i}{2\pi}\left(D^R(\omega) - D^A(\omega)\right)$

For a free scalar field with mass m,
the retarded and advanced propagators are:

- $D^R(\omega) = \left((\omega + i\varepsilon)^2 - m^2\right)^{-1}$
- $D^A(\omega) = \left((\omega - i\varepsilon)^2 - m^2\right)^{-1}$

and the spectral function is, for $\varepsilon \to 0$:

- $\rho(\omega) = \mathrm{sgn}(\omega)\,\delta(\omega^2 - m^2)$



[J. Kapusta and C. Gale, *Finite-Temperature Field Theory*, Cambridge Univ. Press (2006)]

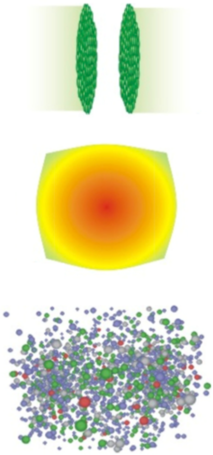
Why are Spectral Functions interesting?

Spectral functions give information on the particle spectrum and determine both real-time and imaginary-time propagators,

- $$D^R(\omega) = -\int d\omega' \, \frac{\rho(\omega')}{\omega' - \omega - i\varepsilon}$$
- $$D^A(\omega) = -\int d\omega' \, \frac{\rho(\omega')}{\omega' - \omega + i\varepsilon}$$
- $$D^E(\omega_n) = \int d\omega' \, \frac{\rho(\omega')}{\omega' + i\omega_n}$$

and thus allow access to many observables, e.g. transport coefficients like the shear viscosity:

- $$\eta = \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \left\langle \left[T_{ij}(x), T^{ij}(0) \right] \right\rangle$$



[B. Mueller, arXiv: 1309.7616]

The Analytic Continuation Problem

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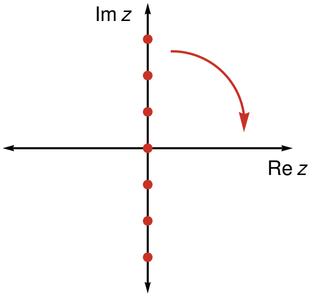
In Euclidean QFT, energies are discrete for $T > 0$ and there are infinitely many analytic continuations $D(z)$ from the imaginary to the real axis with:

- $D(z)|_{z=i\omega_n} = D^E(i\omega)$



Ambiguity is resolved by imposing physical (Baym-Mermin) boundary conditions:

- $D(z)$ goes to zero as $|z| \to \infty$
- $D(z)$ is analytic outside the real axis

When based on numeric results for the discrete frequencies ω_n, cf. lattice QCD, this analytic reconstruction is an ill-posed problem and very difficult.

[N. Landsman, C. v. Weert, Physics Reports 145, 3&4 (1987) 141]
[G. Baym, N. Mermin, J. Math. Phys. 2 (1961) 232]

II) Theoretical Setup

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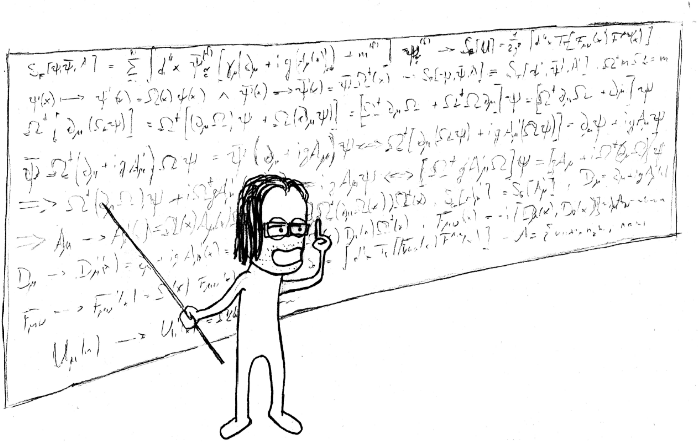

[courtesy L. Holicki]

Functional Renormalization Group

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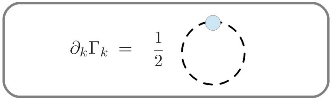
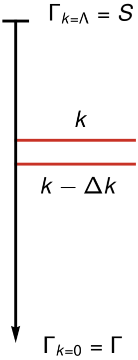
Flow equation for the effective average action Γ_k:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$





- Γ_k interpolates between bare action S at $k = \Lambda$ and full quantum effective action Γ at $k = 0$
- regulator R_k acts as a mass term and suppresses fluctuations with momenta smaller than k

[C. Wetterich, Phys. Lett. B301 (1993) 90]

Quark-Meson Model

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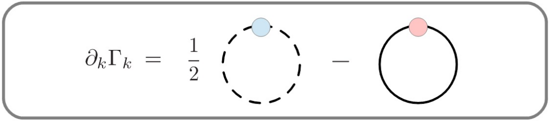
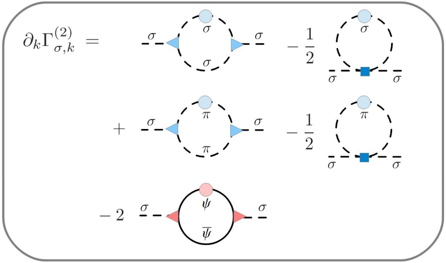
Ansatz for the scale-dependent effective average action:

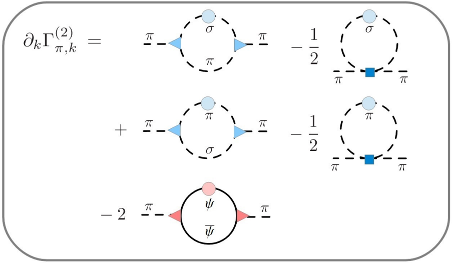
$$\Gamma_k[\bar\psi, \psi, \phi] = \int d^4x \left\{ \bar\psi \left(\partial\!\!\!/ + h(\sigma + i\vec\tau\vec\pi\gamma_5) - \mu\gamma_0 \right) \psi + \frac{1}{2}(\partial_\mu \phi)^2 + U_k(\phi^2) - c\sigma \right\}$$

- effective low-energy model for QCD with two flavors
- describes spontaneous and explicit chiral symmetry breaking
- flow equation for the effective average action:



Flow Equations for 2-Point Functions





Quark-meson vertices given by Yukawa interaction:

$$\Gamma^{(2,1)}_{\bar{\psi}\psi\sigma} = h, \qquad \Gamma^{(2,1)}_{\bar{\psi}\psi\vec{\pi}} = ih\gamma^5\vec{\tau}$$

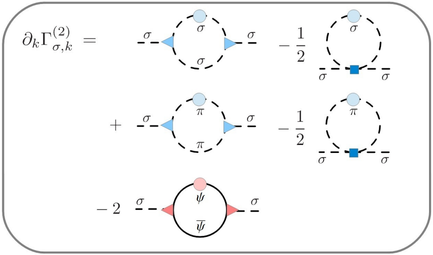
Mesonic vertices obtained from scale-dependent effective potential:

$$\Gamma^{(0,3)}_{k,\,\phi_i\phi_j\phi_m} = \frac{\delta^3 U_k}{\delta\phi_m\delta\phi_j\delta\phi_i}, \qquad \Gamma^{(0,4)}_{k,\,\phi_i\phi_j\phi_m\phi_n} = \frac{\delta^4 U_k}{\delta\phi_n\delta\phi_m\delta\phi_j\delta\phi_i}$$

Decay Channels for Sigma Meson

Possible processes affecting the sigma 2-point function $\Gamma^{(2)}_{\sigma,k}$:

- $\sigma^* \to \sigma\,\sigma, \quad \omega \geq 2\, m_\sigma$
- $\sigma^* \to \pi\,\pi, \quad \omega \geq 2\, m_\pi$
- $\sigma^* \to \bar{\psi}\,\psi, \quad \omega \geq 2\, m_\psi$

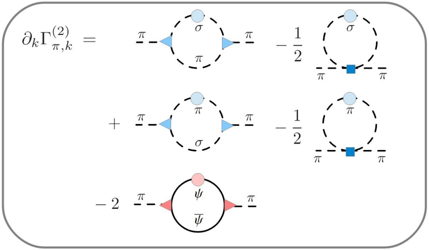


Diagrams involving 4-point vertices only give rise to ω-independent contributions.

Decay Channels for Pions

Possible processes affecting the pion 2-point function $\Gamma^{(2)}_{\pi,k}$:

- $\pi^* \to \sigma\,\pi, \quad \omega \geq m_\sigma + m_\pi$
- $\pi^*\,\pi \to \sigma, \quad \omega \leq m_\sigma - m_\pi$
- $\pi^* \to \bar{\psi}\,\psi, \quad \omega \geq 2\,m_\psi$



Diagrams involving 4-point vertices only give rise to ω-independent contributions.

Analytic Continuation Procedure

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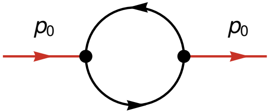
Two-step procedure on the level of flow equations to obtain $\Gamma^{(2),R}(\omega)$:

1) Use periodicity in external energy $p_0 = n\,2\pi T$:

- $n_{B,F}(E + ip_0) \rightarrow n_{B,F}(E)$

2) Substitute p_0 by continuous real frequency:

- $\Gamma^{(2),R}(\omega) = -\lim_{\epsilon \to 0} \Gamma^{(2),E}(p_0 = i\omega - \epsilon)$



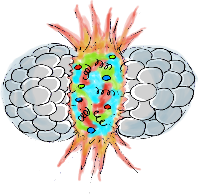
Spectral function is then given by:

- $$\rho(\omega) = \frac{1}{\pi} \frac{\mathrm{Im}\,\Gamma^{(2),R}(\omega)}{\left(\mathrm{Re}\,\Gamma^{(2),R}(\omega)\right)^2 + \left(\mathrm{Im}\,\Gamma^{(2),R}(\omega)\right)^2}$$

[N. Landsman, C. v. Weert, Physics Reports 145, 3&4 (1987) 141]

[A. Das, R. Francisco, J. Frenkel, Phys. Rev. D 86 (2012) 047702]

III) Results for finite T and μ



[courtesy L. Holicki]

Phase Diagram

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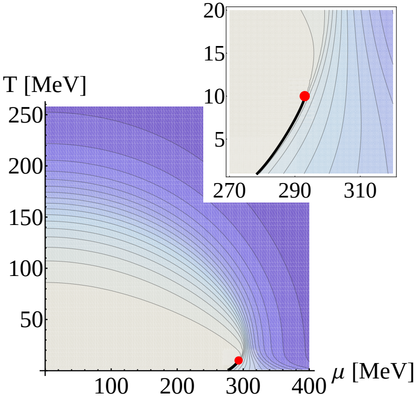
Phase diagram of the quark-meson model:

- chiral order parameter $\sigma_0 \equiv f_\pi$, decreases with darker color
- critical endpoint at μ = 293 MeV and T = 10 MeV
- we will study spectral functions along μ = 0 and T = 10 MeV line



[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Masses and Order Parameter

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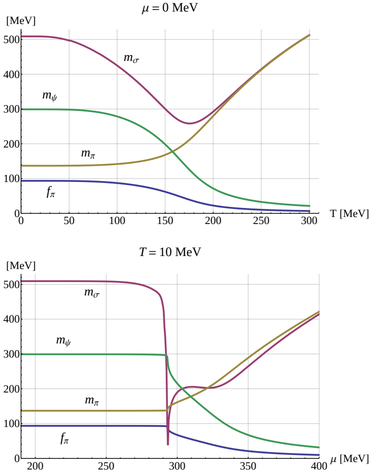
Screening masses and order parameter in vacuum, T = 0 and μ = 0:

- σ_0 = 93.5 MeV
- m_π = 138 MeV
- m_σ = 509 MeV
- m_ψ = 299 MeV

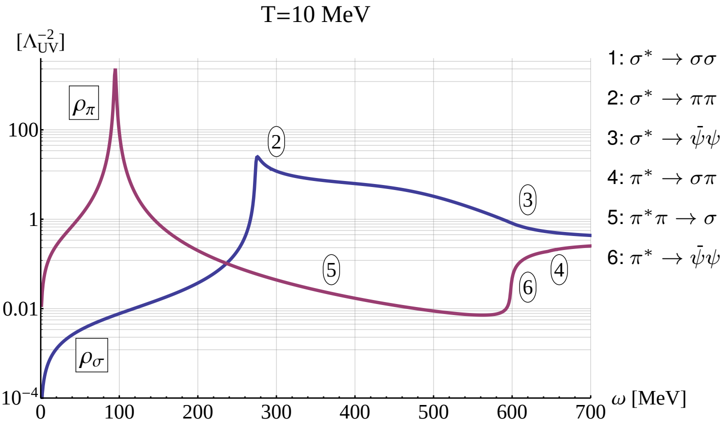
Screening masses determine thresholds in spectral functions, e.g. at T = 10 MeV, μ = 0:

- $\sigma^* \to \pi\,\pi, \quad \omega \geq 2\,m_\pi \approx 280$ MeV
- $\sigma^* \to \bar{\psi}\,\psi, \quad \omega \geq 2\,m_\psi \approx 600$ MeV



[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Spectral Functions along $\mu = 0$



[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

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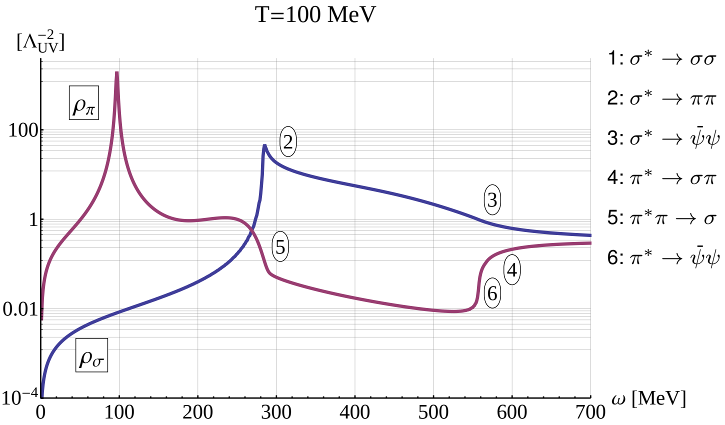

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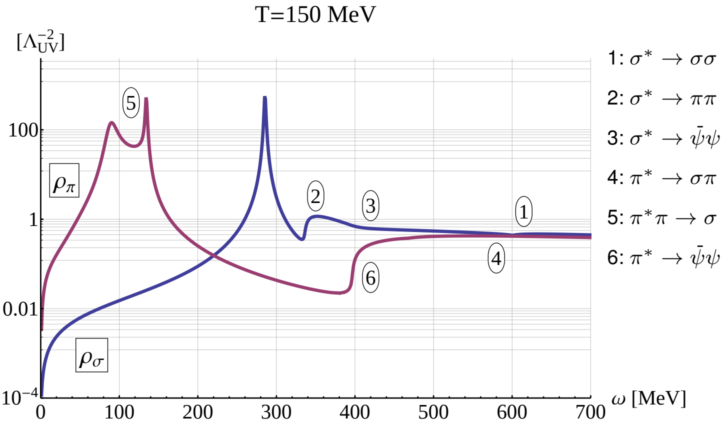

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Spectral Functions along μ = 0

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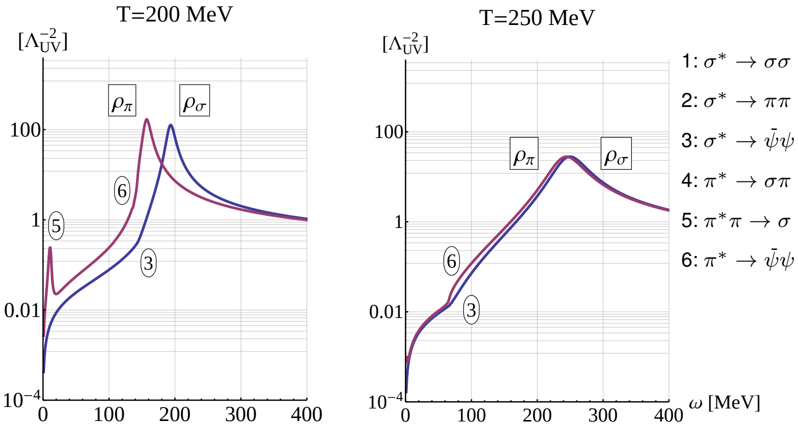

[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Spectral Functions along μ = 0 - Animation

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Spectral Functions along T = 10 MeV

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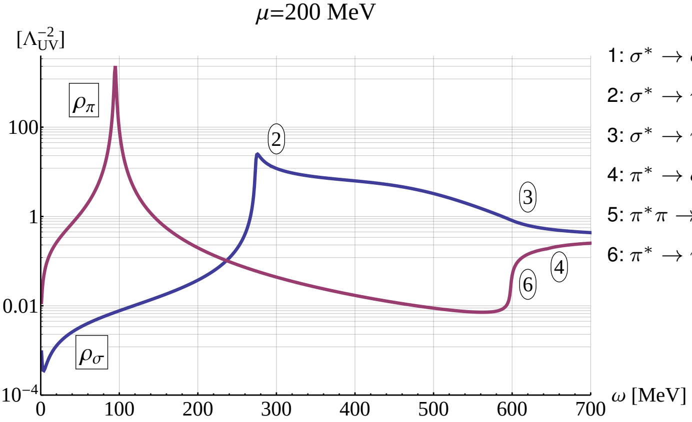

1: $\sigma^* \to \sigma\sigma$

2: $\sigma^* \to \pi\pi$

3: $\sigma^* \to \bar{\psi}\psi$

4: $\pi^* \to \sigma\pi$

5: $\pi^*\pi \to \sigma$

6: $\pi^* \to \bar{\psi}\psi$

[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Spectral Functions along T = 10 MeV

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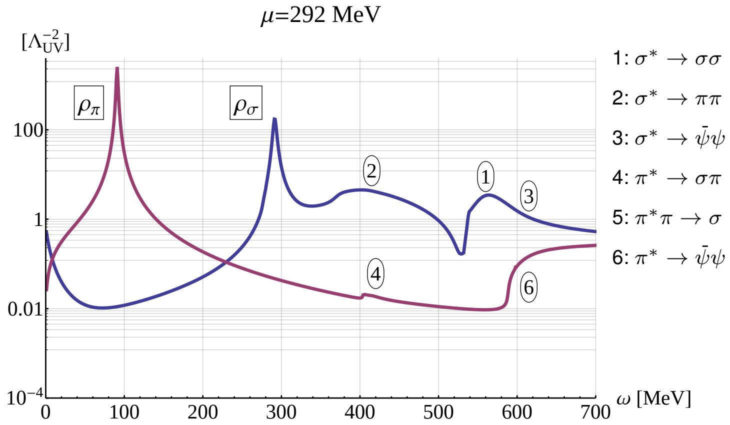

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Spectral Functions along T = 10 MeV

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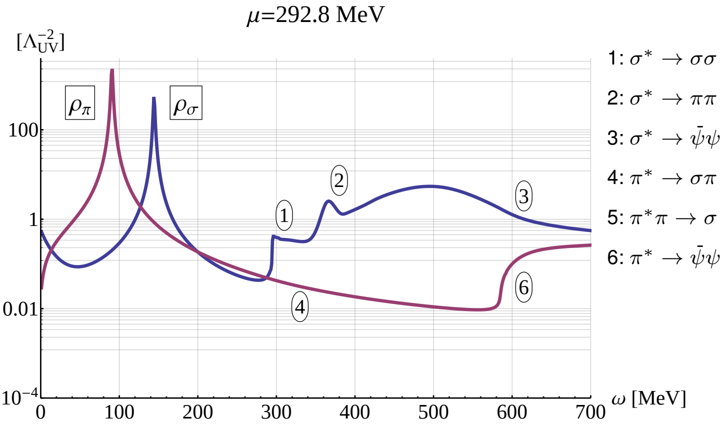

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Spectral Functions along T = 10 MeV

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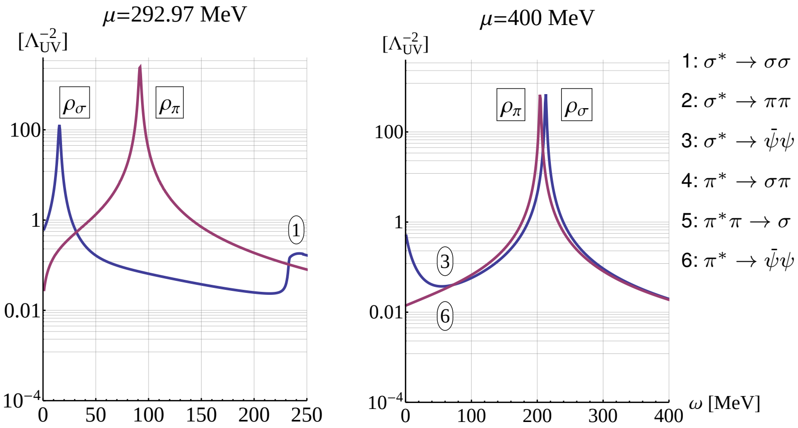

[R.-A. T., N. Strodthoff, L. v. Smekal, J. Wambach, Phys. Rev. D **89**, 034010 (2014)]

Summary and Outlook

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A new method to obtain spectral functions at finite T and μ from the FRG has been presented:

- involves analytic continuation from imaginary to real frequencies on the level of the flow equations
- feasibility of the method demonstrated by calculating mesonic spectral functions in the quark-meson model
- allows to iteratively take into account full momentum dependence of 2-point functions without assumptions on structure of propagator
- inclusion of finite external spatial momenta will allow for calculation of transport coefficients like the shear viscosity