QCD-like theories at finite density

4^{th} TURIC Network Workshop

& Non-Equilibrium Dynamics Symposium

Hersonissos, Crete, 9 June 2014

Lorenz von Smekal

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- QCD with Isospin Chemical Potential ↔ Two-Color QCD

  N. Strodthoff, B.-J. Schaefer & L.v.S., Phys. Rev. D85 (2012) 074007

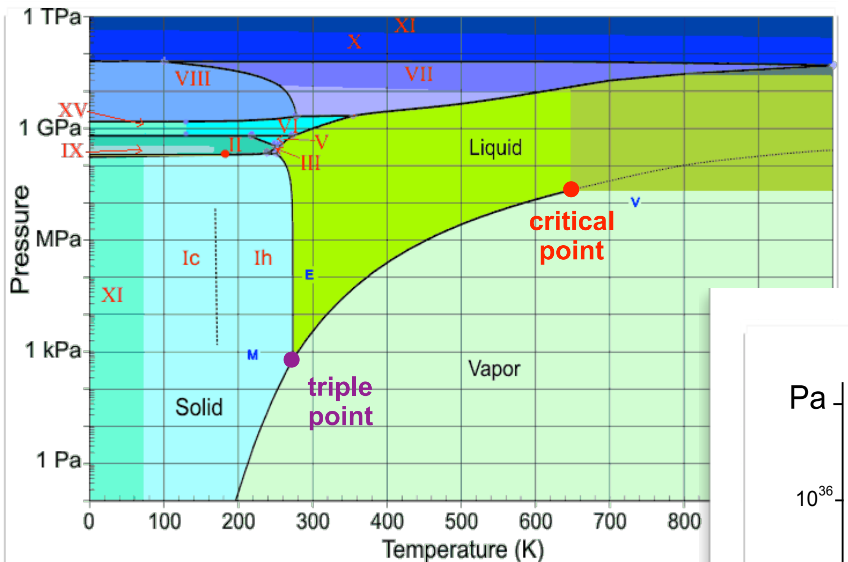
  K. Kamikado, N. Strodthoff, L.v.S. & J. Wambach, Phys. Lett. B 718 (2013) 1044

  N. Strodthoff & L.v.S., PLB 731 (2014) 350
- Isospin & Baryon Chemical Potential ↔ Polarised Fermi Gas
- G_2 Gauge Theory at Finite Baryon Density

  A. Maas, L.v.S., B. Wellegehausen & A. Wipf, Phys. Rev. D 86 (2012) 111901(R)

  B. Wellegehausen, A. Maas, A. Wipf & L.v.S., Phys. Rev. D 89 (2014) 056007
- Summary and outlook

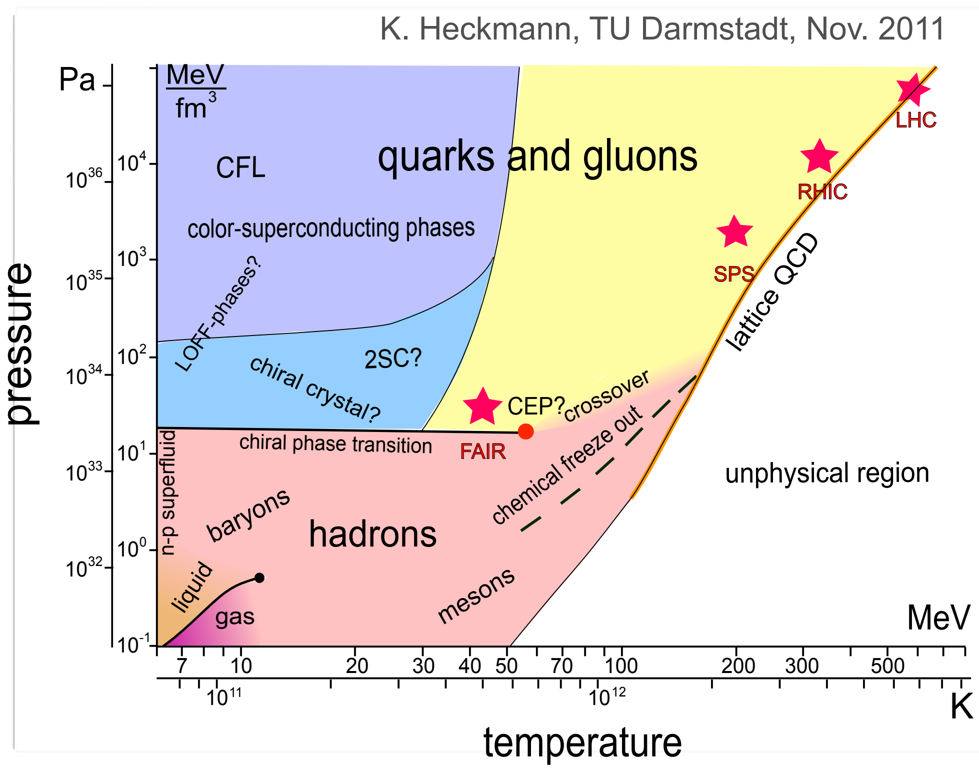
Phase Diagram



http://www.lsbu.ac.uk/water/phase.html

Water

QCD

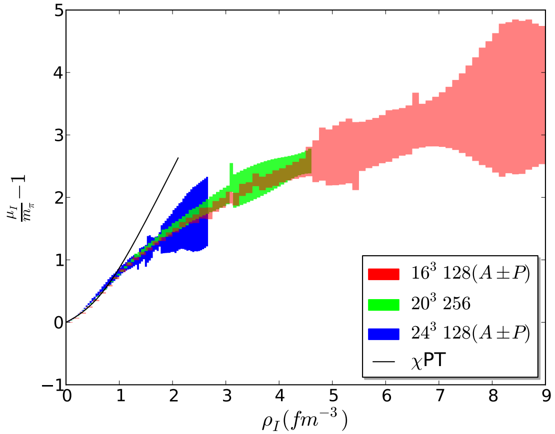


QCD-like Theories

Functional methods and effective models:

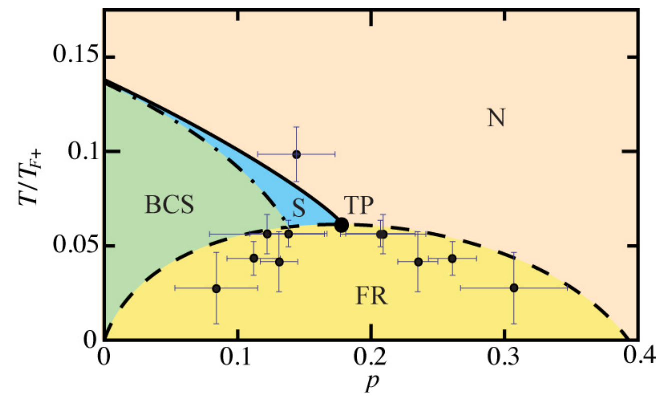
- **compare with lattice simulations**
  where there's no sign problem

QCD at finite isospin density

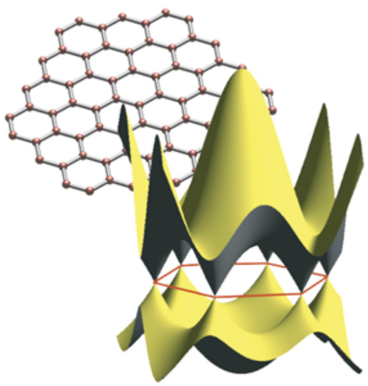


- **apply to ultracold fermi gases**
  exploit analogies and
  more experimental data

polarised fermi gas at unitarity



graphene



- **strongly correlated fermions in 2+1 dimensions**

  QED$_3$ (semimetal-insulator transition, $N_f < 4$),

  electronic properties of Graphene (half-filling, $N_f = 2$) – SFB 634

Fermion-Sign Problem

sign problem:

$$\big(\mathrm{Det}\, D(\mu_f)\big)^* = \mathrm{Det}\, D(-\mu_f)$$

- **in general, except if:**

(a) anti-unitary symmetry $\quad T D(\mu) T^{-1} = D(\mu)^* \qquad T^2 = \pm 1$

fermion color representation: | **Dyson index:**

(i) pseudo-real $\quad T^2 = 1$ **two-color QCD** $\quad \beta = 1$

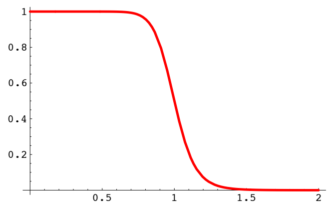
(ii) real $\quad T^2 = -1$ **adjoint QCD, or G_2-QCD** $\quad \beta = 4$

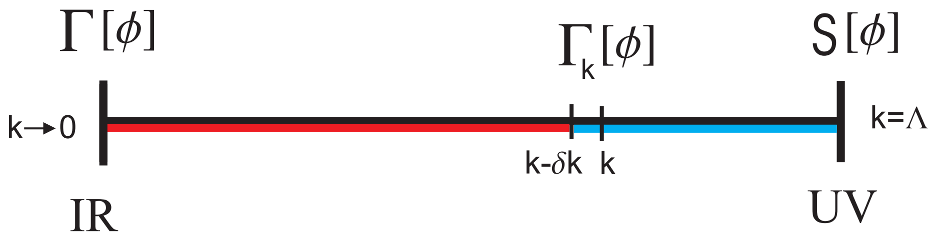
(b) two degenerate flavors with isospin chemical potential

fermion determinant $\quad \rightsquigarrow \ \mathrm{Det}\big(D(\mu_I) D(-\mu_I)\big) \qquad \beta = 2$

QCD at finite isospin density

Functional RG (Flow) Equations





Effective action: $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j]$

Legendre transform

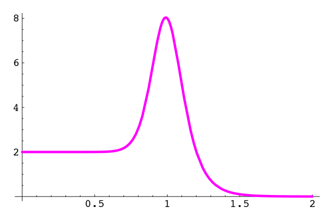
→ **1PI vertex functions**

$\Gamma^{(n)}(x_1, \ldots x_n)$

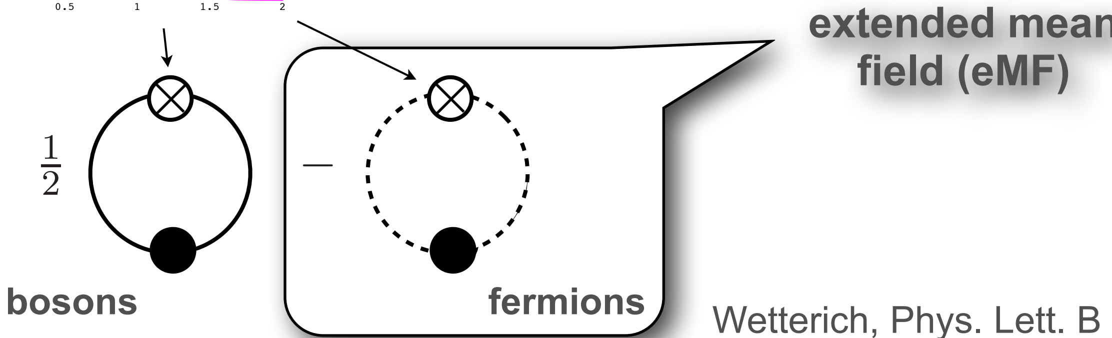
→ **grand potential**

$\Omega(T, \mu)$, at

$\phi_{\min} = \langle \phi \rangle_{T,\mu}$



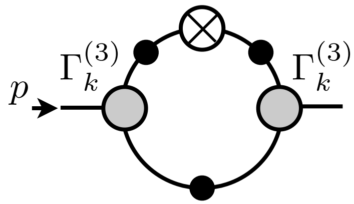
$k\partial_k \Gamma_k[\phi] = \frac{1}{2}$ (boson loop) $-$ (fermion loop)

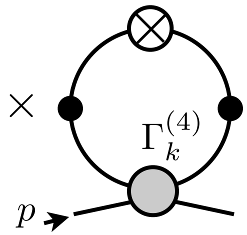


Wetterich, Phys. Lett. B 301 (1993) 90

Flow Equations for Correlation Functions

- **e.g. O(4) linear sigma model:**

$$k\partial_k \Gamma_k^{(2)}(p_0, \vec{p}) =$$ 

$-\frac{1}{2}\times$ 

- **continue to real time:**

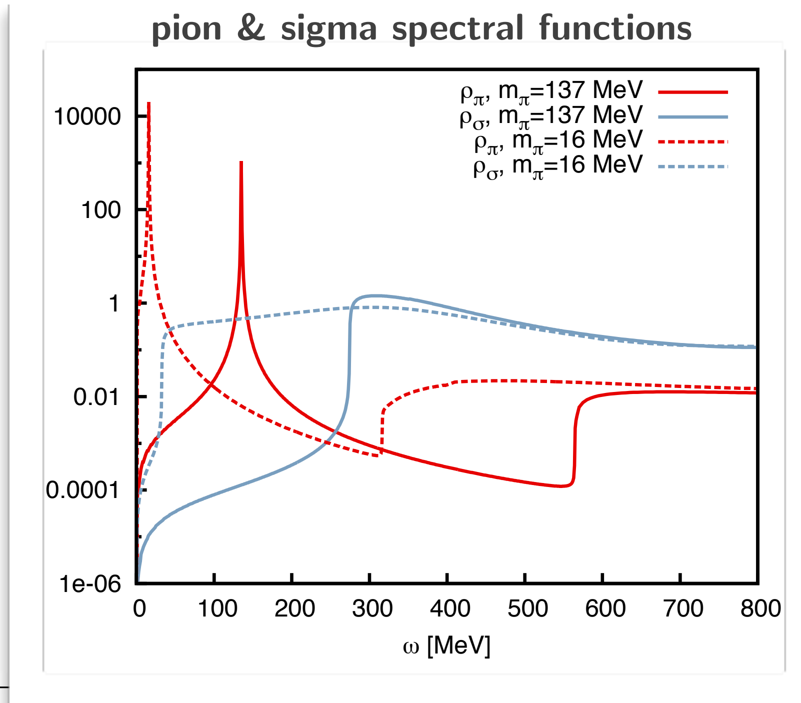
$$p_0 = -i(\omega + i\varepsilon) \quad \textbf{(retarded)}$$

$T = \mu = 0$:

Kamikado, Strodthoff, LvS & Wambach, EPJC 74 (2014) 2806

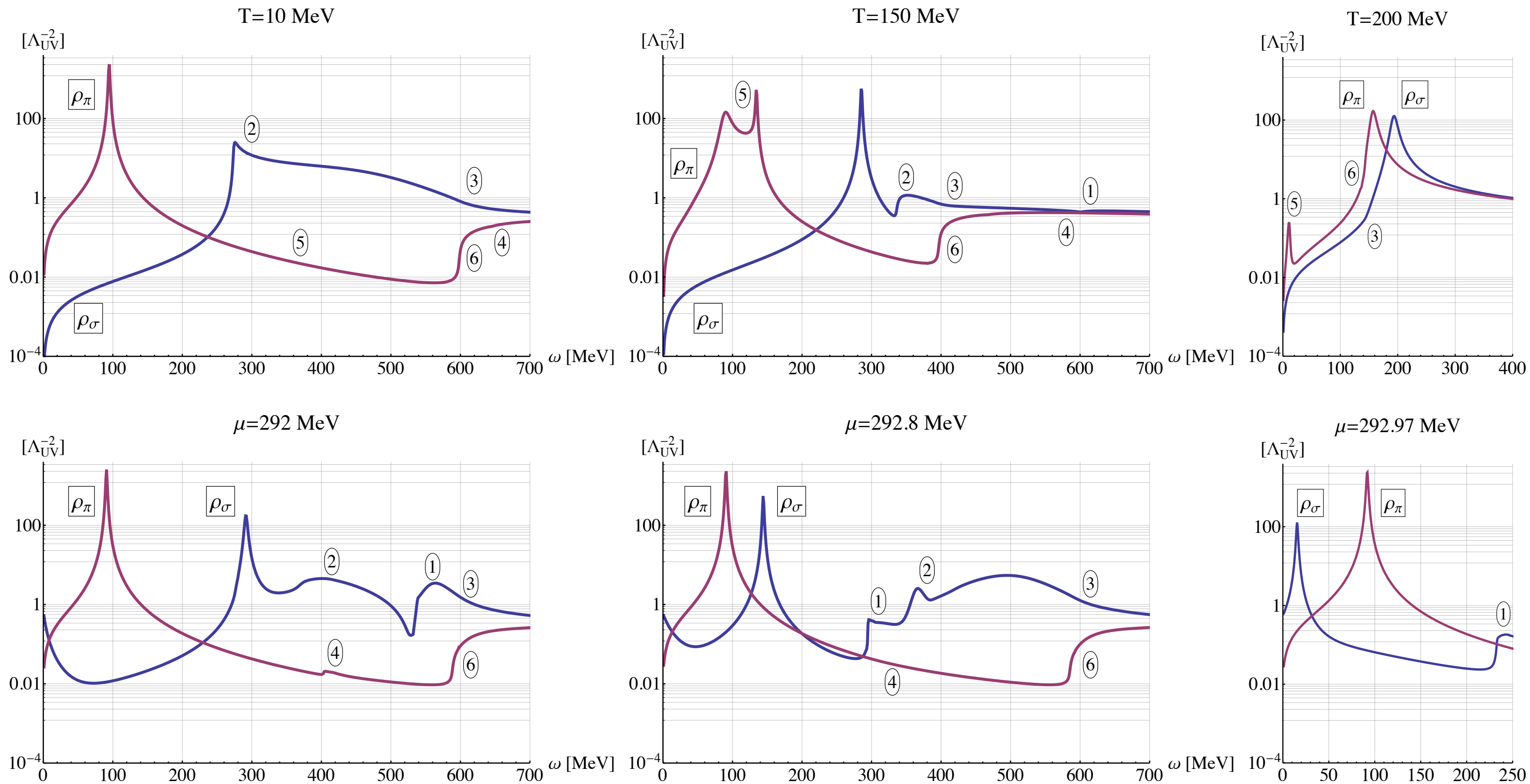
finite T and μ:

Tripolt, Strodthoff, LvS & Wambach, PRD 89 (2014) 034010



Spectral functions

- **QM model** **analytically continued FRG** **finite *T* and *μ***



1: $\sigma^* \to \sigma\sigma$, 2: $\sigma^* \to \pi\pi$, 3: $\sigma^* \to \bar{\psi}\psi$, 4: $\pi^* \to \sigma\pi$, 5: $\pi^*\pi \to \sigma$, 6: $\pi^* \to \bar{\psi}\psi$

[Tripolt, Strodthoff, LvS, Wambach, PRD 89 (2014) 034010]

see Arno Tripolt's talk tonight

QM Model with Isospin Chemical Potential

- **N_f = 2 quarks & mesons with Yukawa coupling:**

$$\begin{aligned}\mathcal{L} =& \bar{\psi}(\partial\!\!\!/ + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \mu\gamma^0 - \mu_I\tau_3\gamma^0)\psi \\ &+ \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi_0)^2 + U(\rho^2, d^2) - c\sigma \\ &+ \frac{1}{2}\left((\partial_\mu + 2\mu_I\delta_\mu^0)\pi_+(\partial_\mu - 2\mu_I\delta_\mu^0)\pi_-\right)\end{aligned}$$

- **chemical potentials:**

$$\mu_u = \mu + \mu_I \qquad \mu_d = \mu - \mu_I$$

$\mu \gg \mu_I$: $\mu_I \rightsquigarrow$ imbalance between up and down

$\mu_I \gg \mu$: $\mu \rightsquigarrow$ imbalance between up and anti-down

- **$\mu = 0$, map to QMD model for QC_2D:**

$$N_c\colon\ 3 \to 2 \qquad (\psi_u, \psi_d) \to (\psi_r, \tau_2 C\bar{\psi}_g) \qquad \mu_I \to \mu$$

$$\pi_+, \pi_- \to \Delta, \Delta^* \qquad \pi_0 \to \vec{\pi}$$

Two-Color QCD

- **extended flavor symmetry (Pauli-Gürsey), at $\mu = 0$**

  $$SU(N_f) \times SU(N_f) \times U(1) \text{ becomes } SU(2N_f)$$

  $N_f = 2$: connects pions and σ-meson with scalar (anti)diquarks.

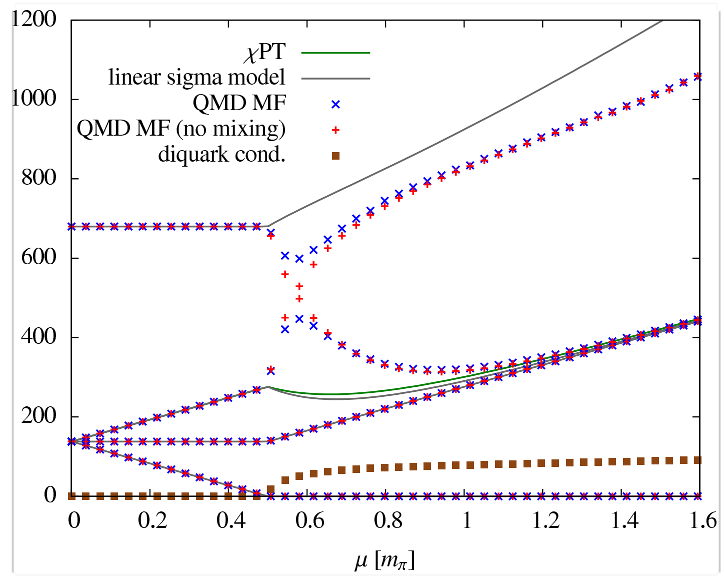
- **Dirac mass (quark condensate)**

  $$SU(4) \to Sp(2)$$

  or $$SO(6) \to SO(5)$$

  Coset: S^5 5 Goldstone bosons: pions and scalar (anti)diquarks

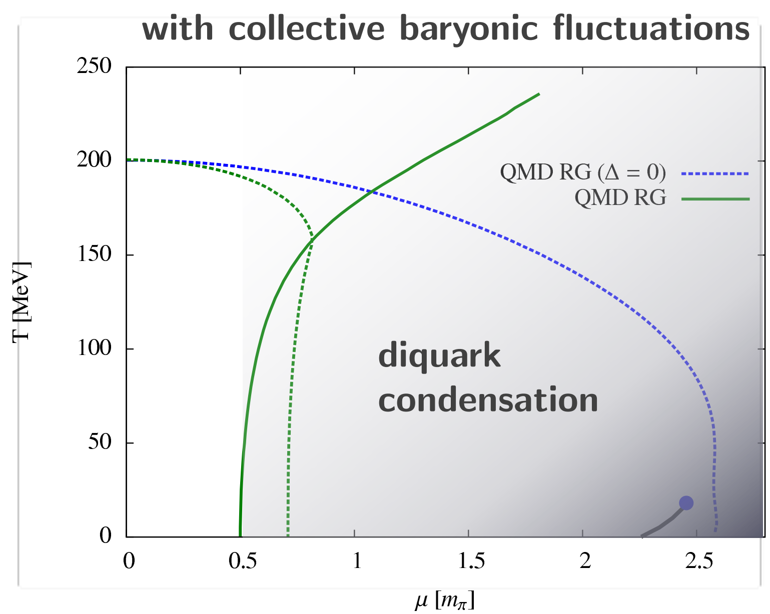
- **color-singlet diquarks (bosonic baryons)**

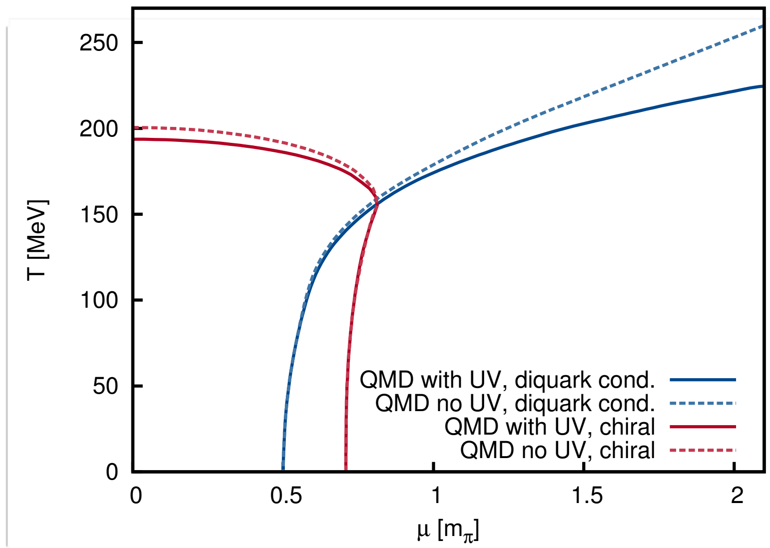


Strodthoff, Schaefer & LvS, Phys. Rev. D 85 (2012) 074007

Two-Color QCD

- **QMD model phase diagram**





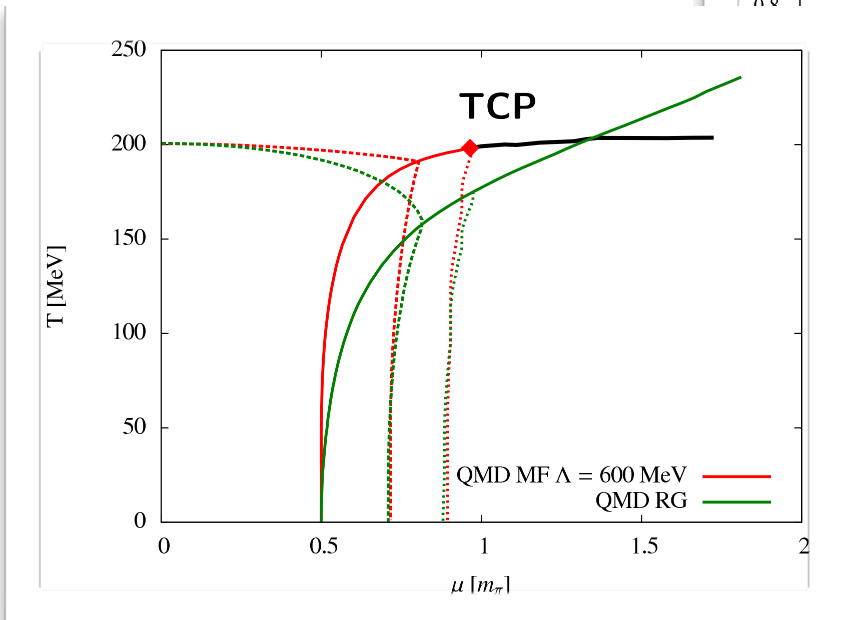
Strodthoff & L.v.S., PLB 731 (2014) 350

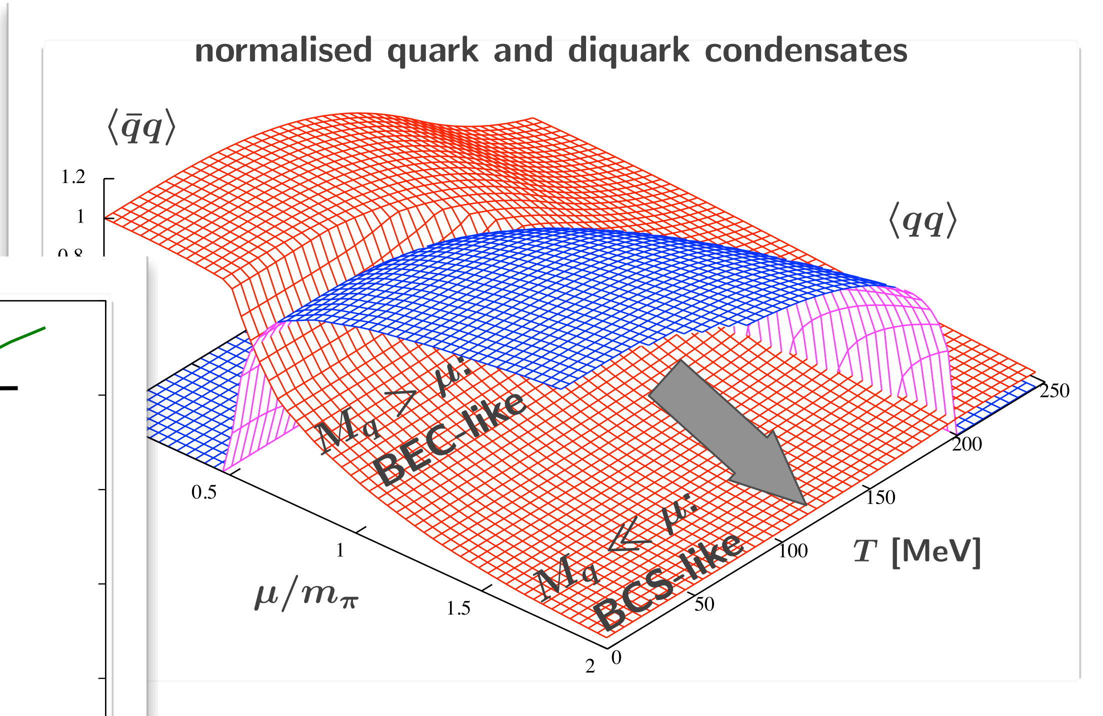
- no low-T 1^{st} order transition,
  no CEP at $\mu \sim 2.5\ m_\pi$!

Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007

Two-Color QCD

- **QMD model phase diagram**





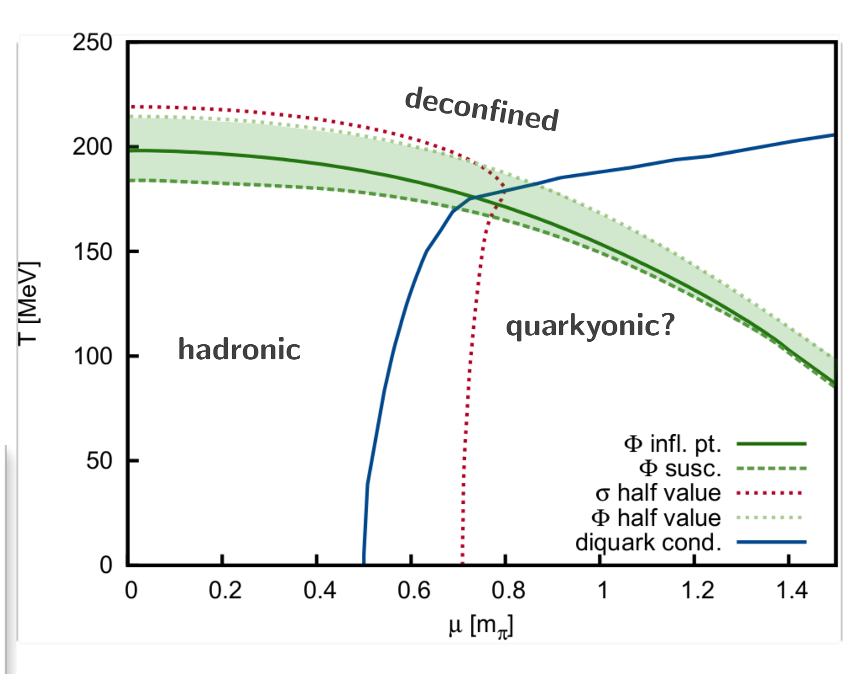
- Tricritical point predicted in:

  Splittorff, Toublan & Verbaarschot, Nucl. Phys. B 620 (2002) 290

Strodthoff, Schaefer & LvS, Phys. Rev. D85 (2012) 074007

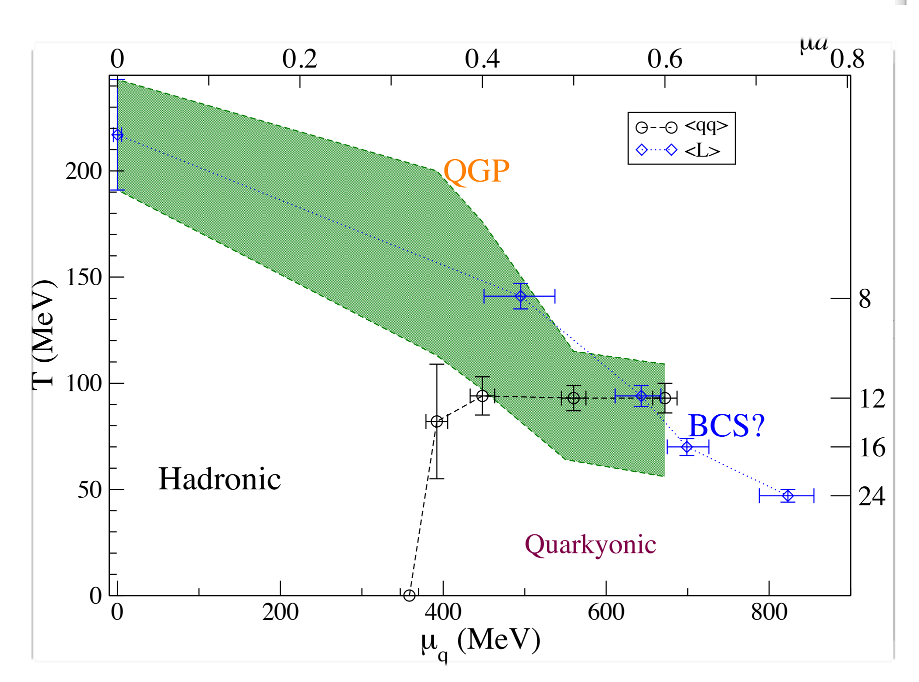
Two-Color QCD

- **Polyakov-Quark-Meson-Diquark model phase diagram:**



Strodthoff & L.v.S., PLB 731 (2014) 350

- **Lattice simulations:**

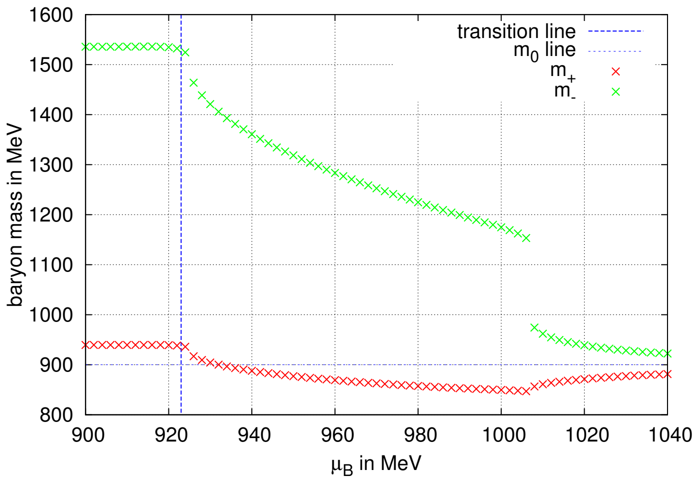


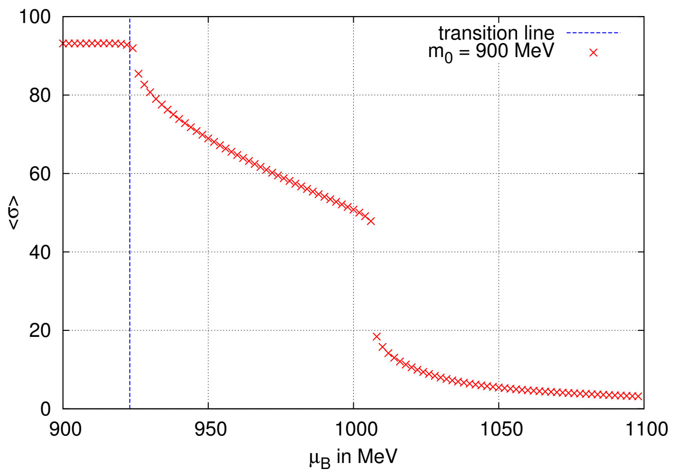
Can we describe the two-color world with the 3d effective lattice theory for heavy quarks?
[see Philipp Scior's talk this afternoon]

Cotter, Giudice, Hands & Skullerud, PRD 87 (2013) 034507

Nuclear Matter and Chiral Transition

- **Parity-Doublet Model with mesonic and baryonic fluctuations**

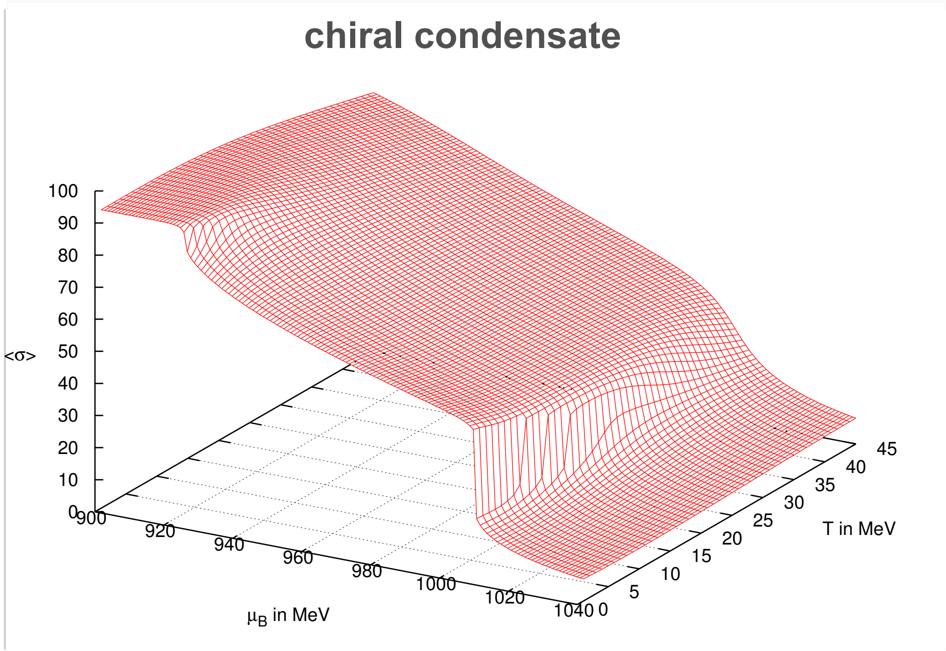




mean-field:

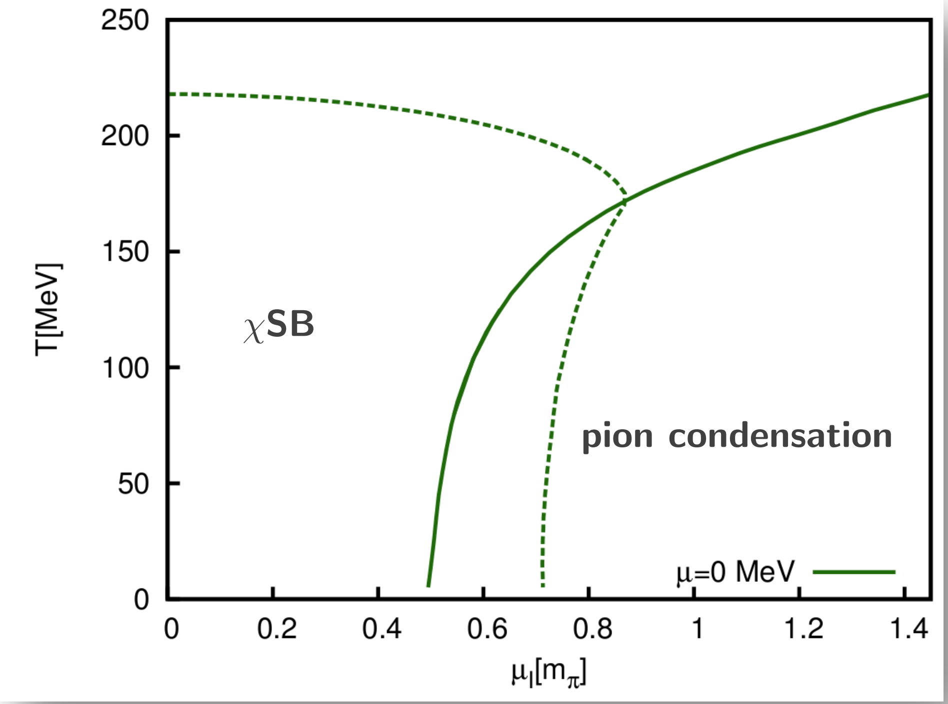
Zschiesche, Tolos, Schaffner-Bielich, Pisarski, PRC 75 (2007) 055202

FRG: MSc Thesis Johannes Weyrich, May 2014



QCD with Isospin Chemical Potential

- **QM Model with fluctuating chiral & pion condensates**



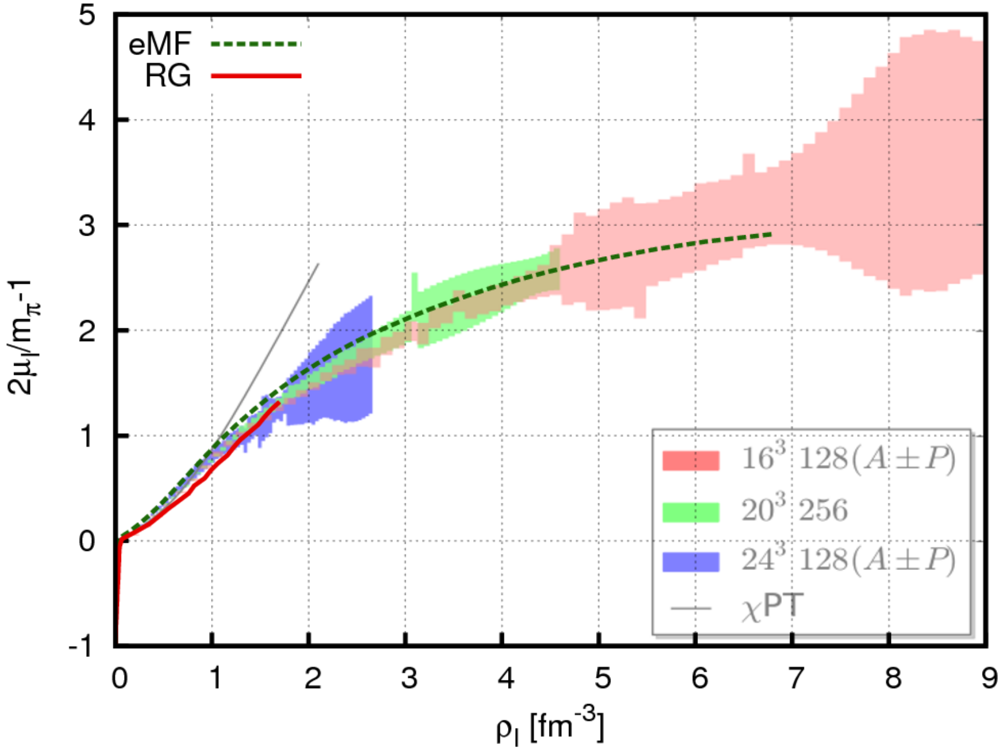
- **need 2 fields in effective potential**

$U = U(\rho^2, d^2)$, but replace $\rho^2 = \sigma^2 + \vec{\pi}^2$ and $d^2 = |\Delta|^2$
by $\rho^2 = \sigma^2 + \pi_0^2$ and $d^2 = \pi_1^2 + \pi_2^2 = \pi_+\pi_-$

Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

QCD with Isospin Chemical Potential

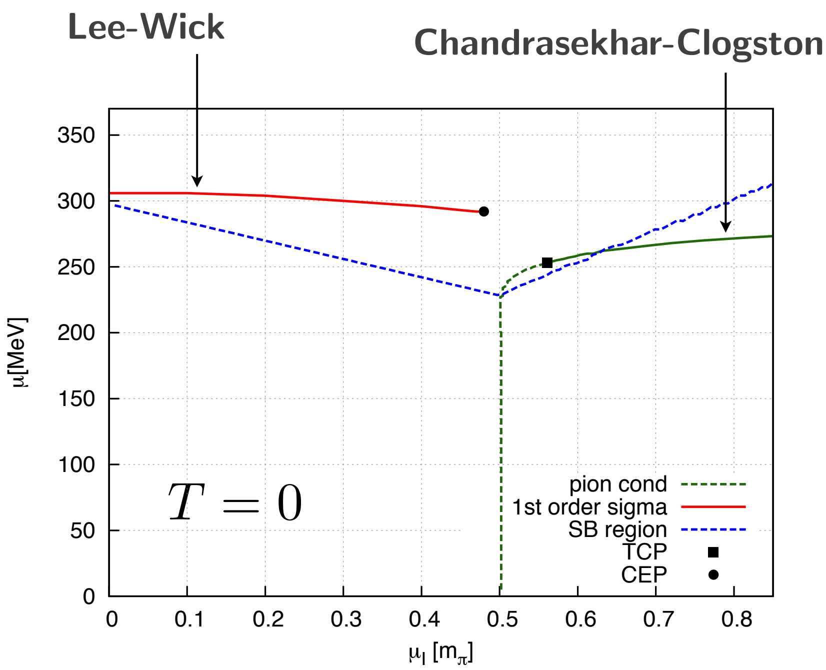
- *T* = 0 isospin density - FRG vs. lattice QCD:

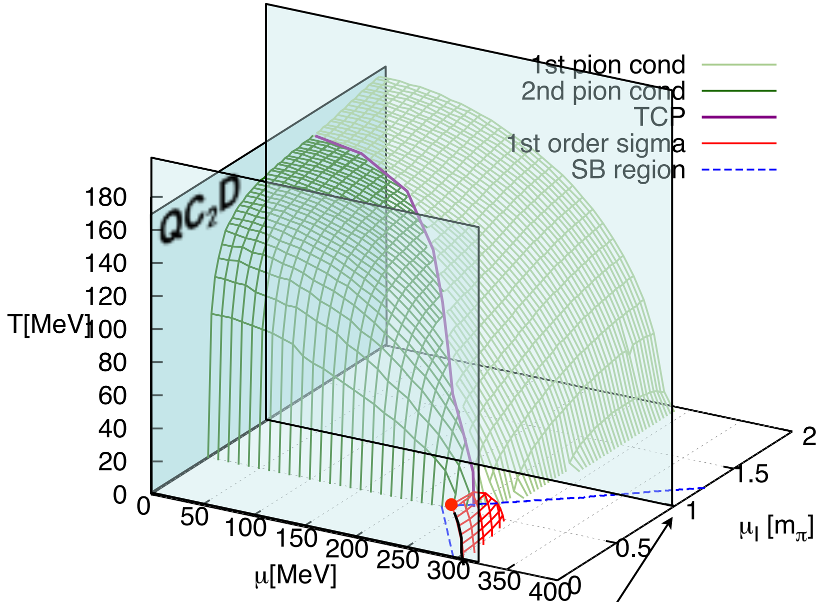


Kamikado, Strodthoff, LvS, PLB 718 (2013) 1044
Detmold, Orginos & Shi, Phys. Rev. D86 (2012) 054507

Baryon & Isospin Chemical Potential

- **Fermionic flow (extended mean-field):**





fixed $\mu_I = m_\pi$

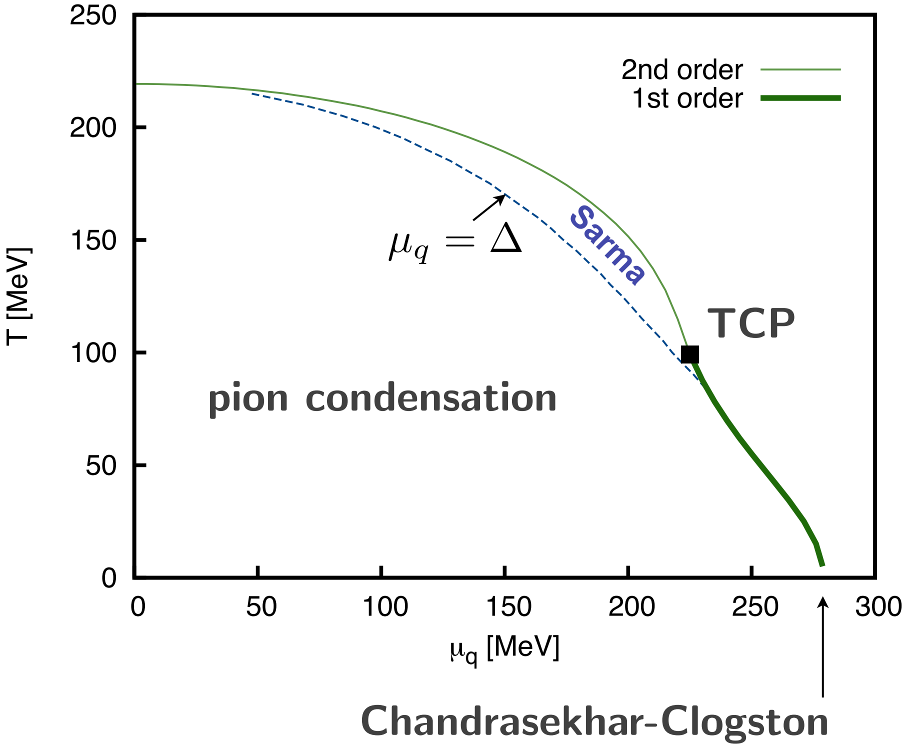
μ for (up/anti-down) imbalance

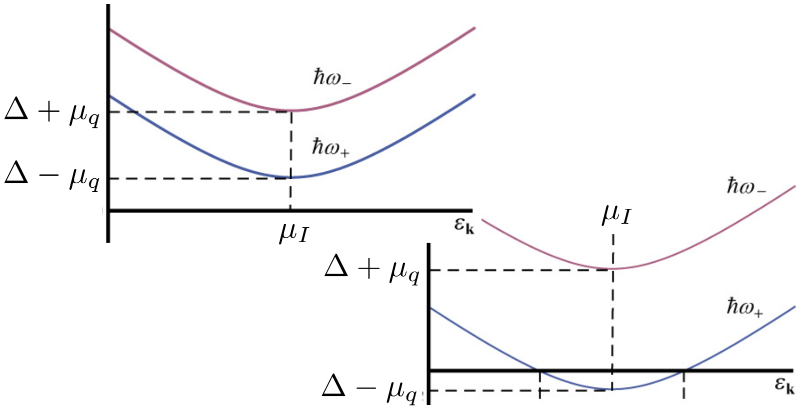
Kamikado, Strodthoff, LvS & Wambach, PLB 718 (2013) 1044

Up-Antidown Population Imbalance

- **Fermionic flow (extended mean-field):**

$\mu_I = m_\pi$





Up-Antidown Population Imbalance

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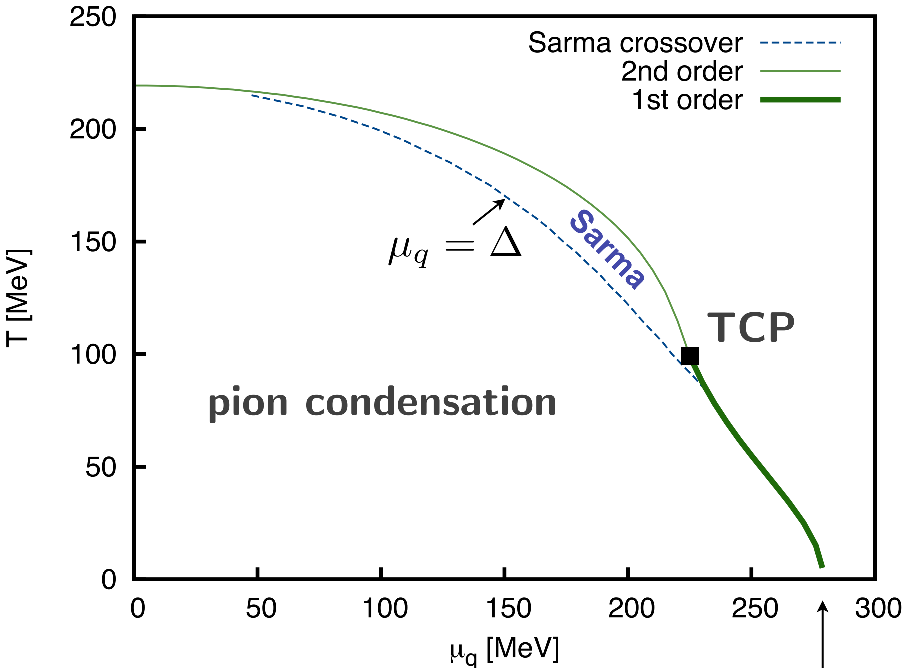
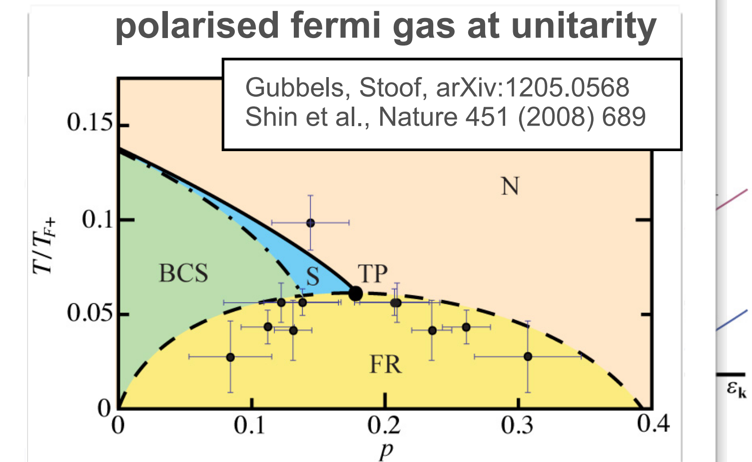
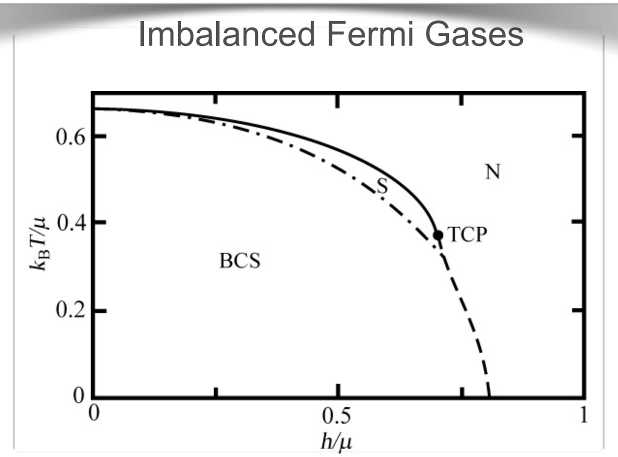
$\mu_I = m_\pi$



Chandrasekhar-Clogston





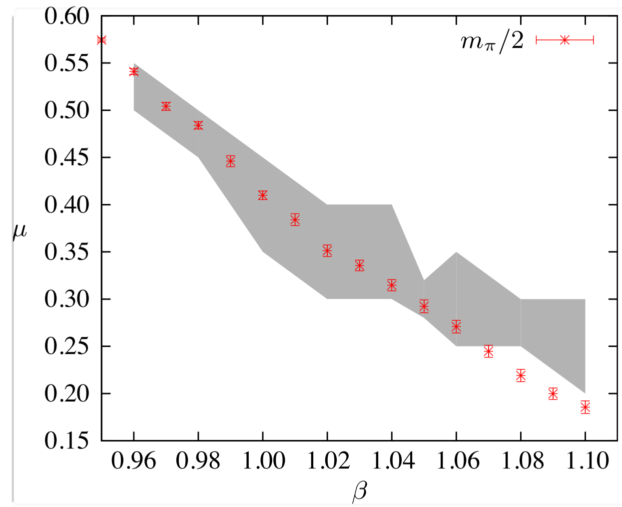
Gubbels, Stoof, arXiv:1205.0568

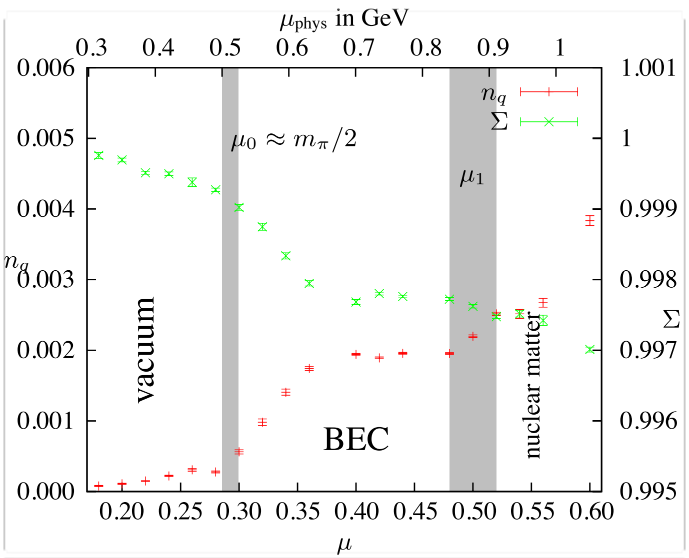
partially pol. superfluid near interface in fermi gases?

G_2 Gauge Theory at Finite Density

- real (positive), no sign problem (as adjoint QCD).
- rank 2, quenched 1st order deconfinement (as SU(3)).
- 7 colors, 14 gluons.
- diquark condensation (as two-color QCD).
- but has fermionic baryons also.
- breaks down to QCD:

$$G_2 \xrightarrow{\text{Higgs}} SU(3)$$





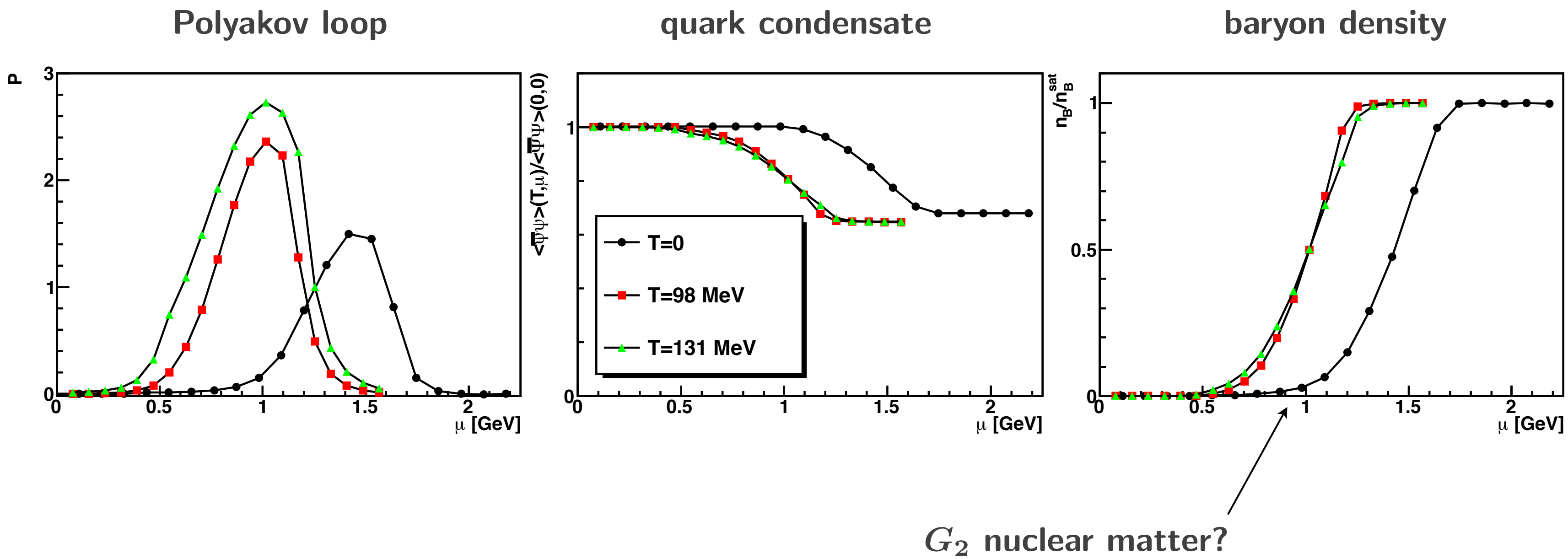
Holland, Minkowski, Pepe & Wiese, Nucl. Phys. B 668 (2003) 207

Wellegehausen, Wipf & Wozar, Phys. Rev. D 83 (2011) 114502

Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D 86 (2012) 111901R

G_2 Gauge Theory at Finite Density

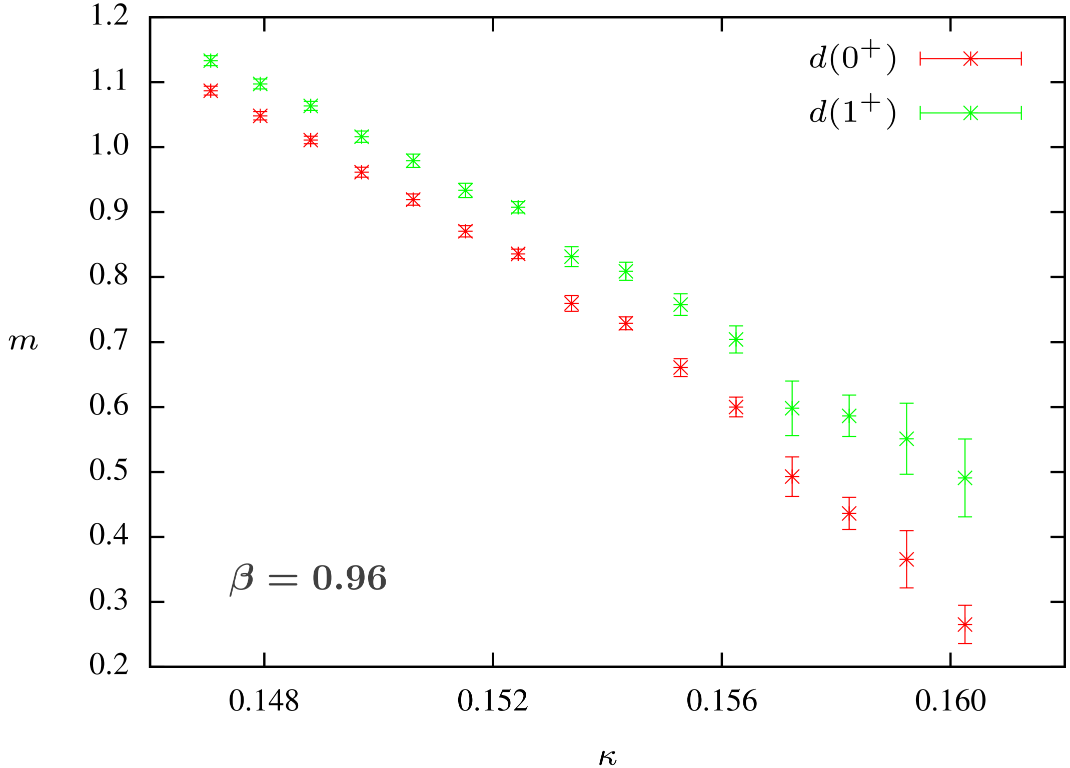
- but have fermionic baryons also
- finite baryon density (bosonic and fermionic)



Maas, LvS, Wellegehausen & Wipf, Phys. Rev. D 86 (2012) 111901R

G_2 Spectroscopy

- ***N_f = 1:*** real and positive for single flavor: $SU(2) \to U_B(1)$
  2 Goldstone bosons: scalar (anti)diquarks



- ***N_f = 2:***
  exact mass relations

$$m_{d(0+)} = m_{\pi(0-)}$$

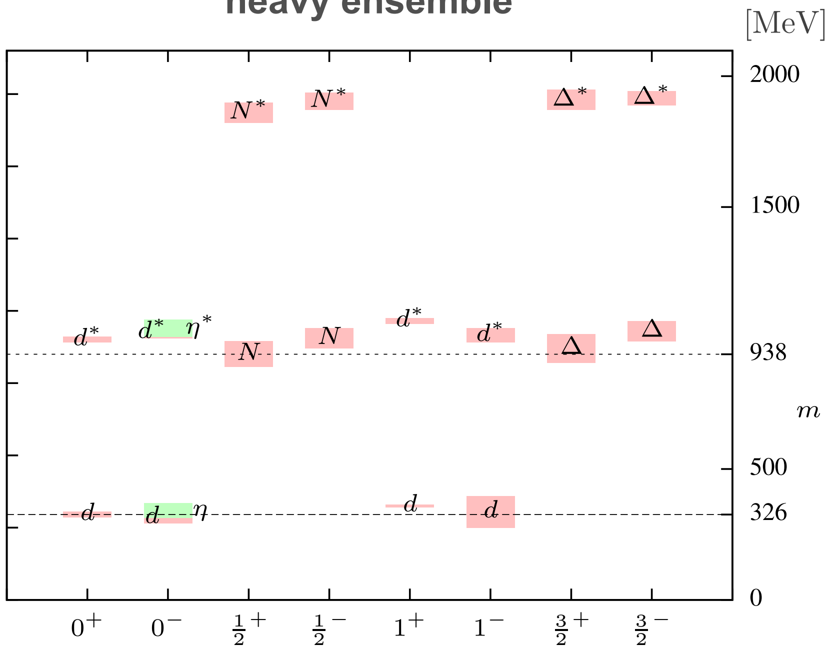
$$m_{d(1+)} = m_{\rho(1-)}$$

Name	β	κ	$m_{d(0+)}$
Heavy ensemble	1.05	0.147	326 MeV
Light ensemble	0.96	0.15924	247 MeV

Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

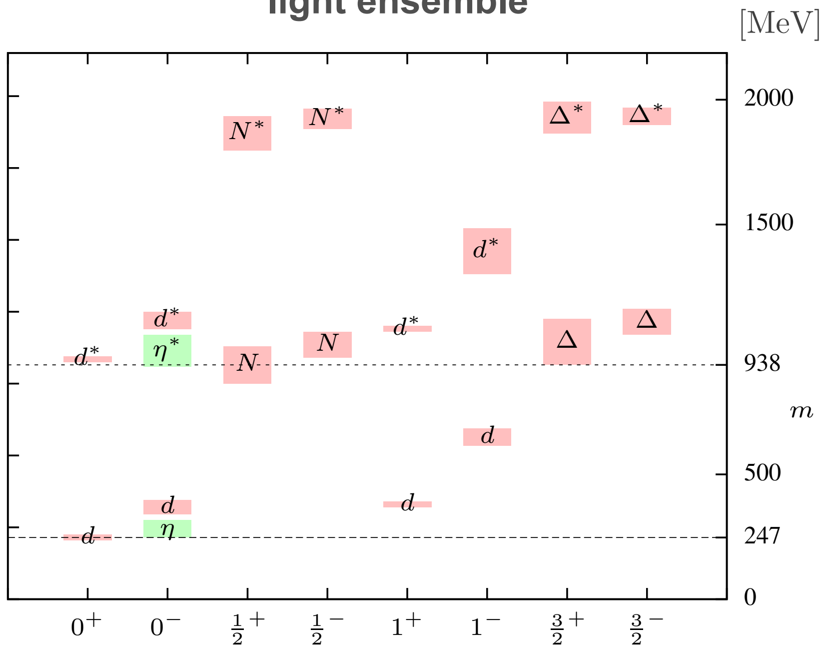
G_2 Spectroscopy

heavy ensemble



$\beta = 1.05, \ \kappa = 0.147$

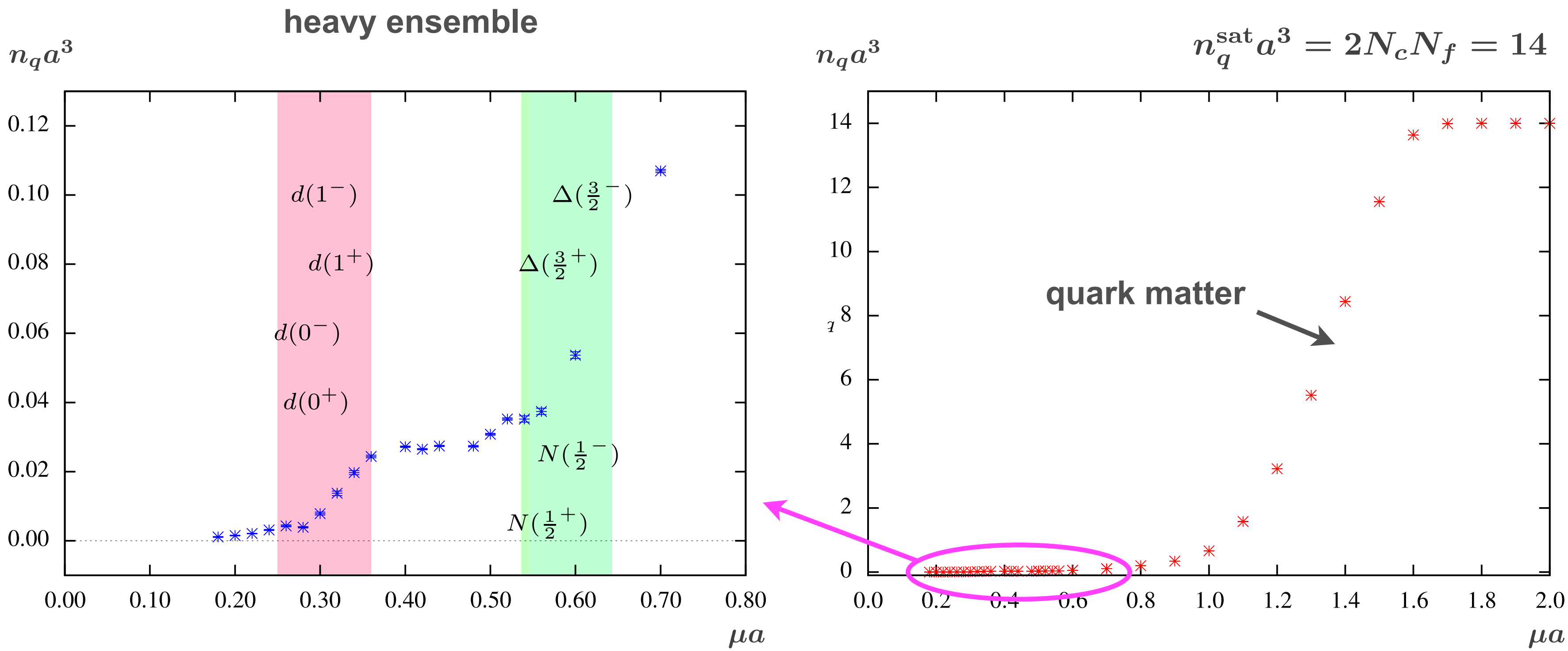
light ensemble



$\beta = 0.96, \ \kappa = 0.15924$

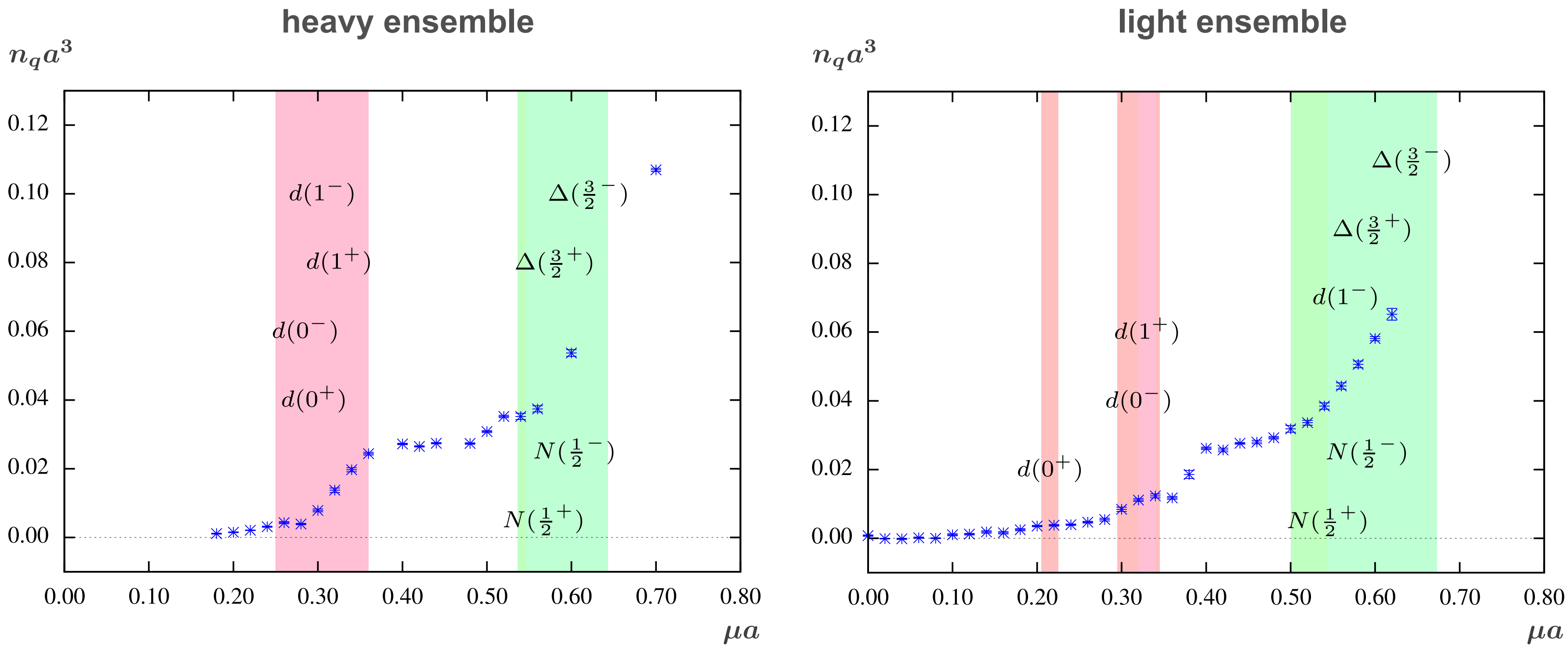
Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

Finite Baryon Density



Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

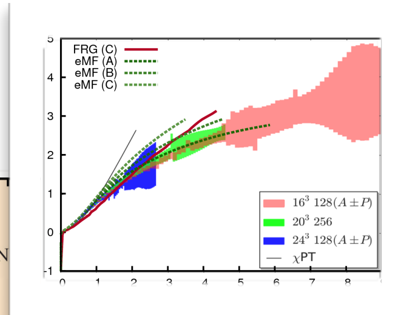
Finite Baryon Density

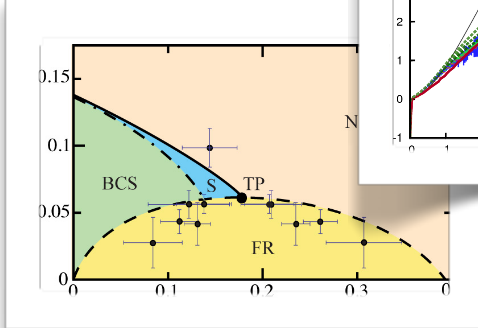


Wellegehausen, Maas, Wipf & LvS, PRD 89 (2014) 056007

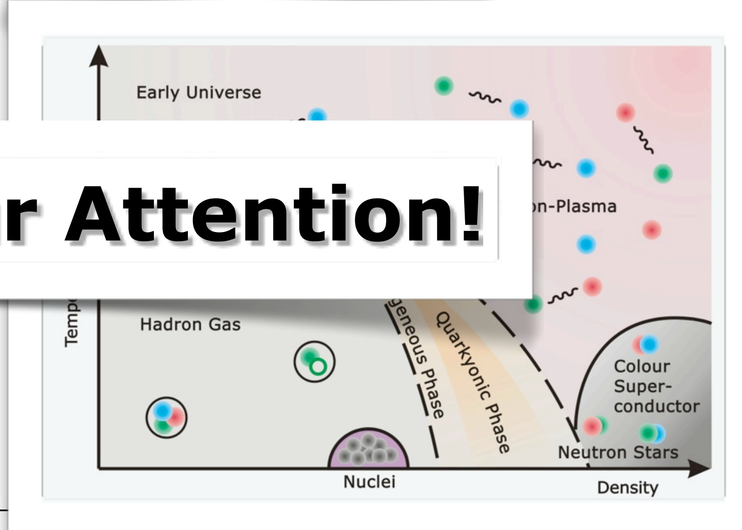
Summary & Outlook

- **Finite Isospin Density in QCD and Baryon Density in Two-Color QCD**
  - detailed understanding of phase diagram
  - functional methods and models vs. lattice MC
  - analogies with ultracold fermi gases BEC-BCS crossover, population imbalance with universal phase diagram...





- **Phase Diagram of G_2 Gauge Theory**
  - no sign problem – fermionic baryons
- **QCD Phase Diagram**
  - refined functional methods & models, baryonic dofs, finite volume...



Thank You for Your Attention!

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Helmholtz International Center