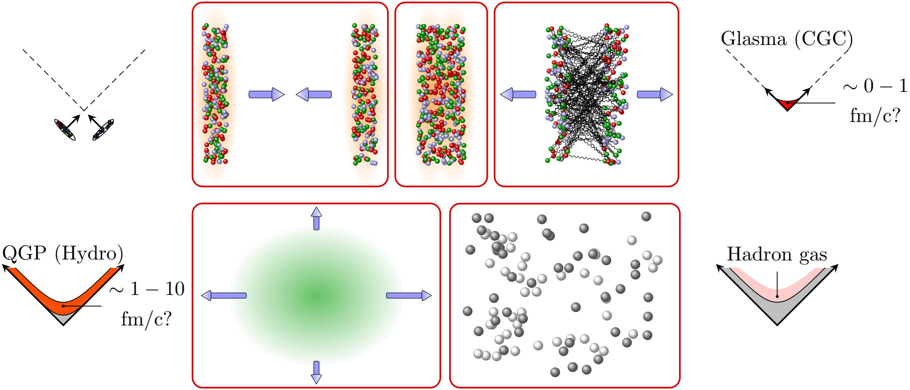
Towards the understanding of hydrodynamization in the quark gluon plasma

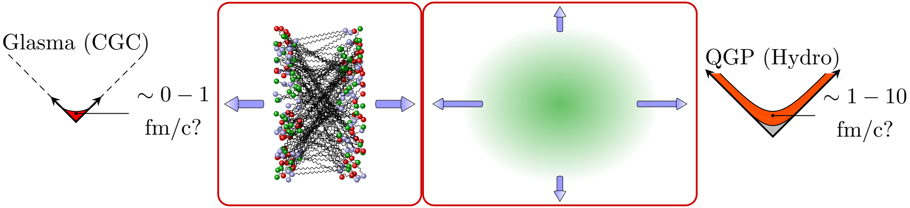
Hersonissos, 10th June 2014

Thomas EPELBAUM

IPhT

Heavy ion collision : the current picture





Large anisotropy

(negative P_L)

Small anisotropy

($P_L \sim P_T$)

Long time puzzle: Does (fast) hydrodynamization occur?

What is Hydrodynamics?

I) Macroscopic theory
II) Few field variables: P_L, P_T, ϵ, $\vec{u}$
III) Conservation law: $\partial_\mu T^{\mu\nu} = 0$
IV) Need input:

1) Equation of state $f(P_L, P_T) = \epsilon$
2) Small anisotropy
3) Short isotropization time
4) Initialization: $\epsilon(\tau_0)$, $P_L(\tau_0)$? ...
5) Viscous coefficients: shear viscosity η,...

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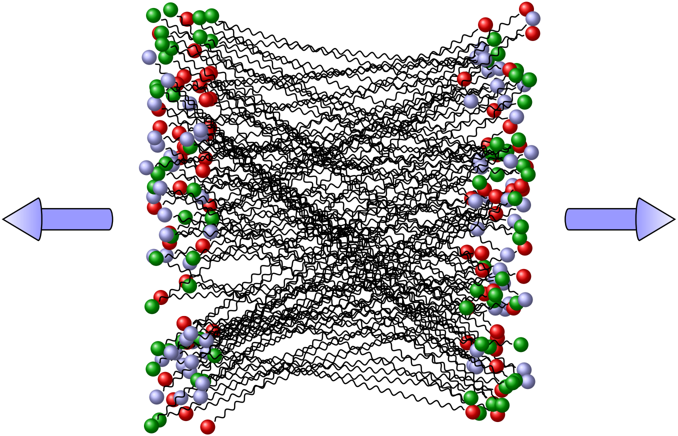
None of this is easy to get from QCD

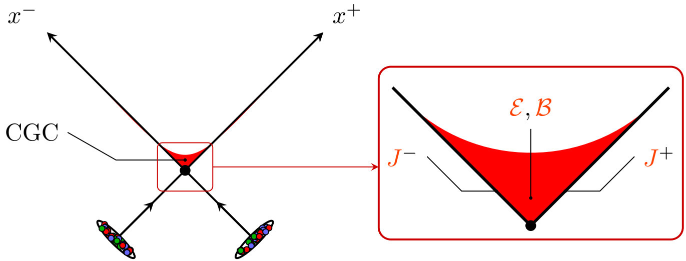
Weakly coupled method at dense regime:

$\alpha_s \ll 1$ **but** $f_{\text{gluon}} \sim \frac{1}{\alpha_s}$



THE COLOR GLASS CONDENSATE [MCLERRAN, VENUGOPALAN (1993)]

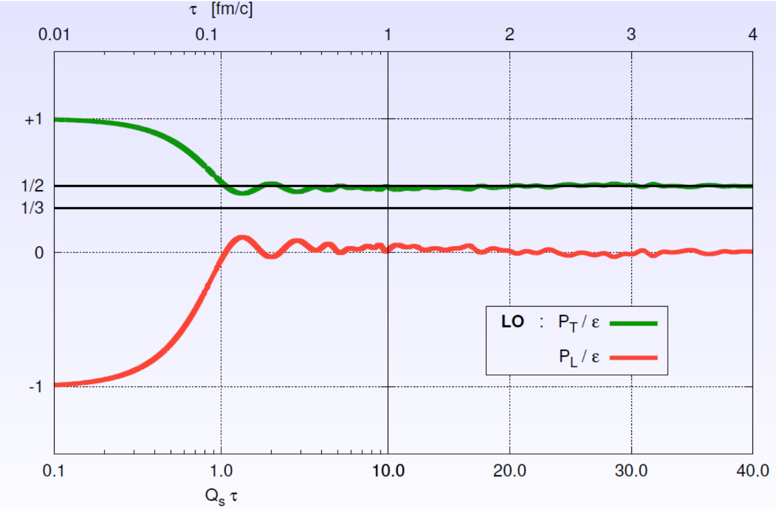
$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_\mu \mathcal{A}^\mu$$



LO: $\epsilon = \frac{1}{2}\underbrace{\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right)}_{\text{Classical color fields}}$ $\qquad$ $\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \underbrace{J^\nu}_{\text{Color sources on the light cone}}$

[KRASNITZ, VENUGOPALAN (1998)]

Strong anisotropy at early time



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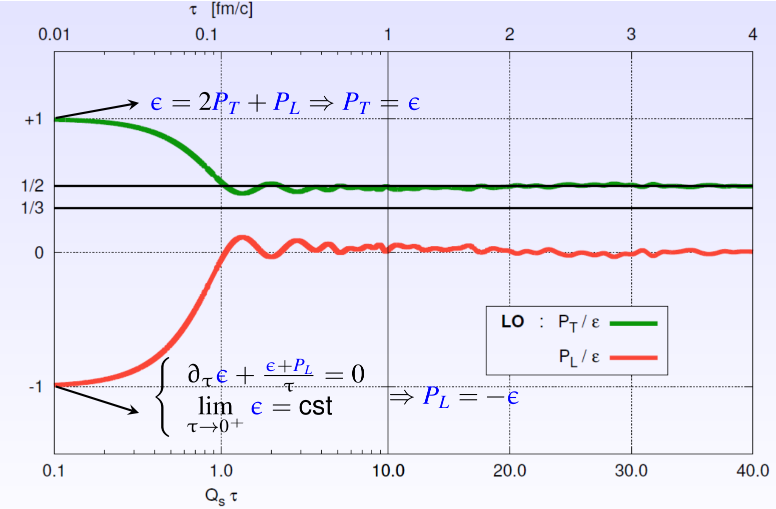
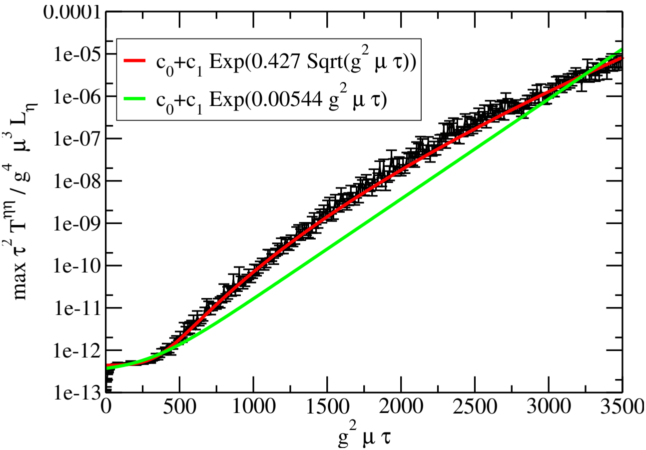


$$E(x) = \underbrace{\mathcal{E}(\boldsymbol{x}_\perp)}_{\text{LO}} + \underbrace{\int_{\vec{k}} e_{\vec{k}}(x)}_{\text{NLO}} + \cdots$$

$e_{\vec{k}}(x)$ perturbation to $\mathcal{E}(\boldsymbol{x}_\perp)$ created by a plane wave of momentum $\vec{k}$ in the remote past.

$e_{\vec{k}}(x)$ obeys to the linear Equation Of Motion



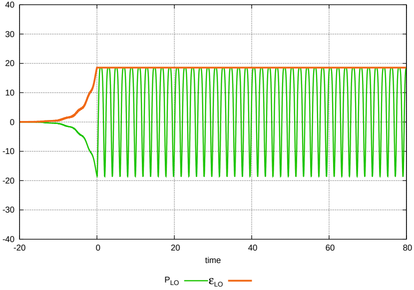
[ROMATSCHKE, VENUGOPALAN (2006)]

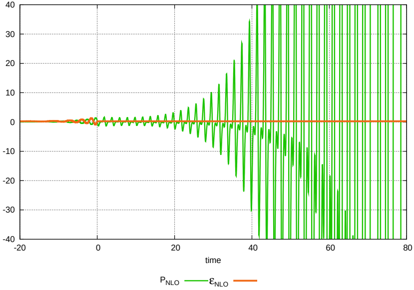
Small Fluctuations grow exponentially (Weibel instability)

[MROWCZYNSKI (1988)]

- Because of instabilities, the NLO correction eventually becomes as large as the LO ⇒ Important effect, should be included

- NLO alone will grow forever ⇒ unphysical effect, should be taken care of





- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

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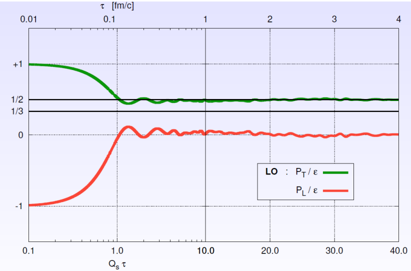
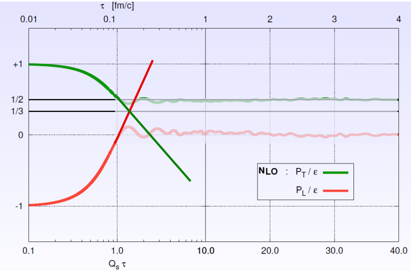
- NLO alone will grow forever ⇒ unphysical effect, should be taken care of





- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

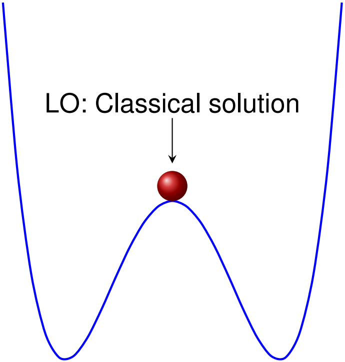
- At the initial time $\tau = \tau_0$, take:

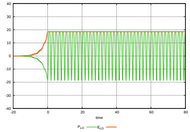
$$\vec{E}(\tau_0, \vec{x}) = \vec{\mathcal{E}}_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

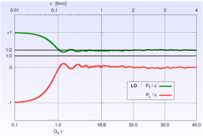
  where $c_{\vec{k}}$ are random coefficients: $\left\langle c_{\vec{k}} c_{\vec{k}'} \right\rangle \sim \delta_{\vec{k}\vec{k}'}$

- Solve the **Classical** equation of motion $D_\mu F^{\mu\nu} = J^\nu$
- Compute $\left\langle \vec{E}^2(\tau, \vec{x}) \right\rangle$, where $\langle\rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)
- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \to \infty$ [GELIS, LAPPI, VENUGOPALAN (2008)]

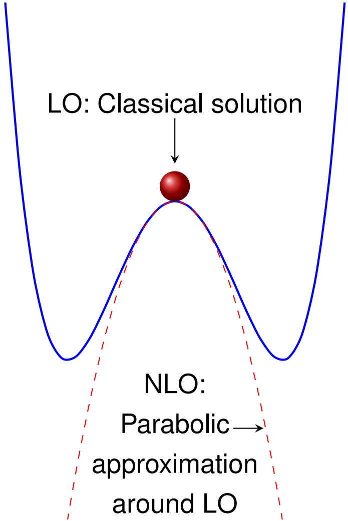
This gives: LO+NLO+Subset of higher orders

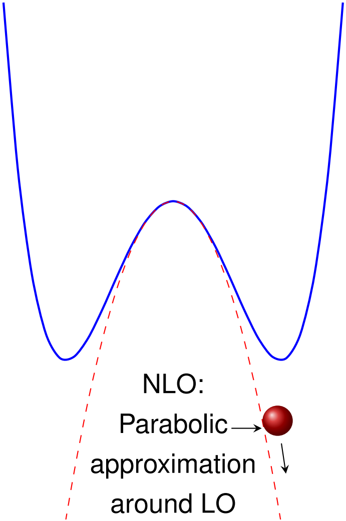

LO: Classical solution

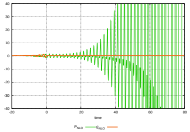

time
P_LO
ε_LO

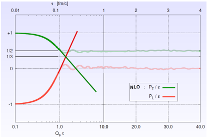

τ [fm/c]
Q_s τ
LO : P_T / ε
P_L / ε

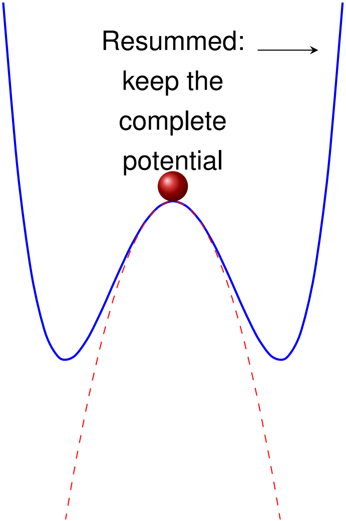
Implications of the classical-statistical method



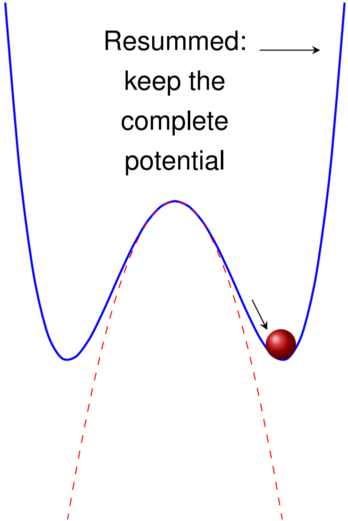

NLO:
Parabolic
approximation
around LO


time

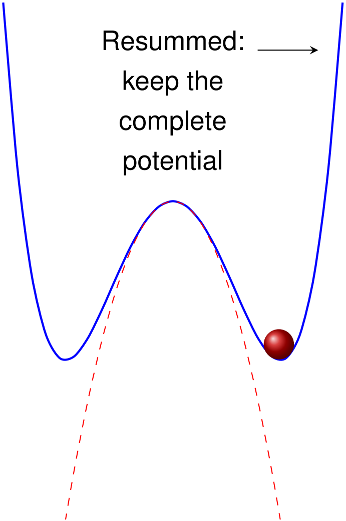

τ [fm/c]
NLO
P_T / ε
P_L / ε
$Q_s \tau$


Resummed:
keep the
complete
potential

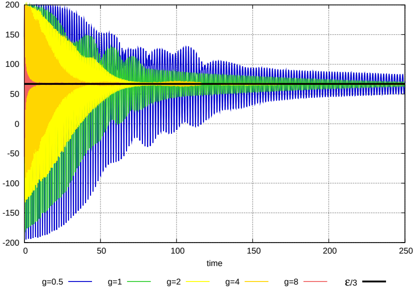
Implications of the classical-statistical method



IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD

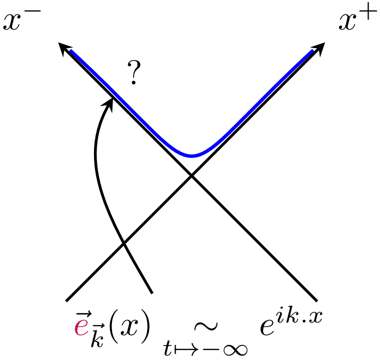


$$\phi(\tau_0, \vec{x}) = \varphi_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} a_{\vec{k}}(\tau_0, \vec{x})$$



$$\Box\phi + \frac{g^2}{6}\phi^3 = 0$$

- Need to know $\vec{e}_{\vec{k}}(\tau_0, \vec{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone $(\tau_0 > 0)$



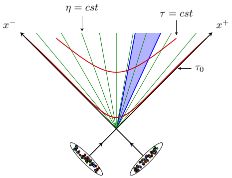
This can be done analytically [TE,GELIS 1307:1765]

Result at $\tau = 0^+$

$$e^{i}_{\nu\vec{k}_\perp}(\tau,\boldsymbol{x}_\perp,\eta) = i\nu\, e^{i\nu\eta}\left[F^{i,2}_{\nu\vec{k}_\perp}(\mathcal{U}_2,\tau,\boldsymbol{x}_\perp) - F^{i,1}_{\nu\vec{k}_\perp}(\mathcal{U}_1,\tau,\boldsymbol{x}_\perp)\right]$$

$$e^{\eta}_{\nu\vec{k}_\perp}(\tau,\boldsymbol{x}_\perp,\eta) = e^{i\nu\eta}\mathcal{D}^{i}\left[F^{i,2}_{\nu\vec{k}_\perp}(\mathcal{U}_2,\tau,\boldsymbol{x}_\perp) - F^{i,1}_{\nu\vec{k}_\perp}(\mathcal{U}_1,\tau,\boldsymbol{x}_\perp)\right]$$

- $\mathcal{U}_{1,2}$ depends on the color sources $J^{\pm}$ of the nuclei
- Analytical checks performed on the solution
  - Gauss's law
  - linearized Yang-Mills EOM
  - Orthonormality of the mode functions



Initial condition

$$E^\mu(x) = \mathcal{E}^\mu + \int_{\nu \boldsymbol{k}_\perp} c_{\nu \boldsymbol{k}_\perp} \, e^\mu_{\nu \boldsymbol{k}_\perp}(\tau_0, \boldsymbol{x}_\perp, \eta)$$

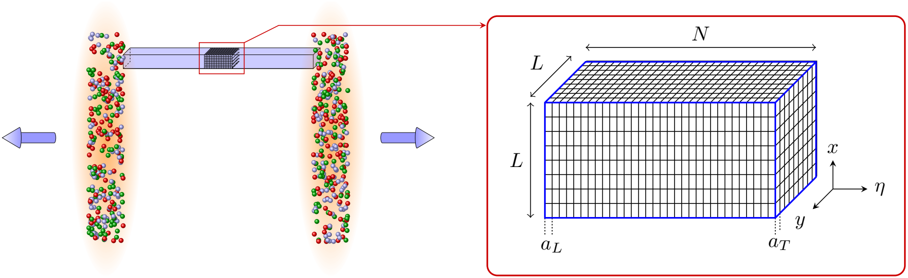
Time evolution ($I = x, y, \eta$) for each configuration

$$D_\mu F^{\mu I} = 0 \qquad \Rightarrow \qquad T^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} - F^{\mu\rho} F^{\nu}{}_{\rho}$$

Cross checks: Gauss's law

$$D_\mu E^\mu = 0$$

Gauge potential $A^\mu \to$ link variables (exact gauge invariance on the lattice)

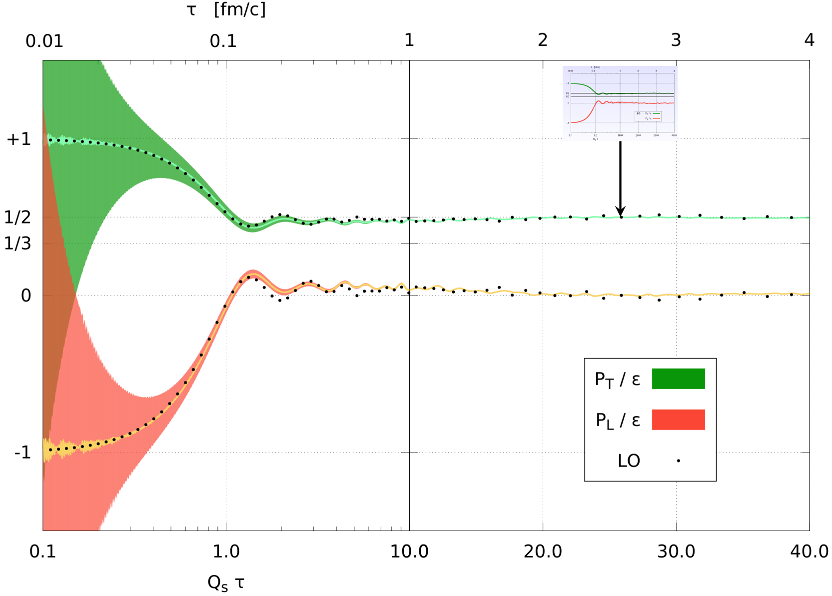


Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$

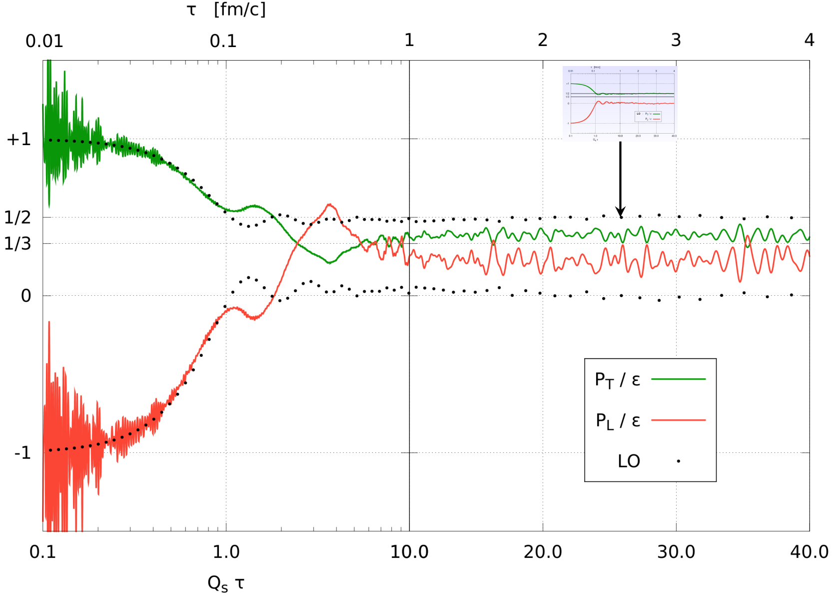
NUMERICAL RESULTS [TE,GELIS (2013)]

$\alpha_s = 8\,10^{-4}$ $(g = 0.1)$



NUMERICAL RESULTS [TE,GELIS (2013)]

$$\alpha_s = 2\,10^{-2}\ (g = 0.5)$$



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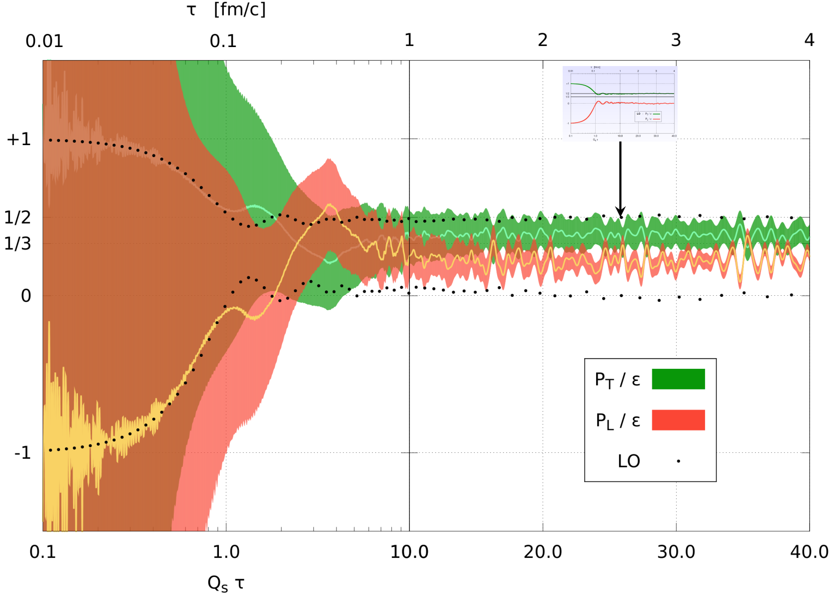


Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0 \tau^{-\frac{4}{3}}}_{\text{Ideal hydro}} - \underbrace{2\eta_0 \tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0 \tau^{-1}$)

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta \epsilon^{-\frac{3}{4}} \sim 300$.

If the system is nearly thermal

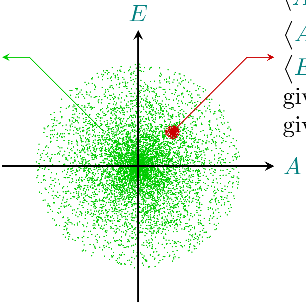
$$\epsilon^{\frac{3}{4}} \sim s \Longrightarrow \frac{\eta}{s} \text{ close to } \frac{1}{4\pi}$$

Does (fast) hydrodynamization occur?

- Correct NLO spectrum from first principles
- $\frac{P_L}{P_T} \approx 0.6$ for $g = 0.5$ at $\tau \sim 1 fm/c$
- No need for strong coupling to get isotropization
- Assuming simple first order viscous hydrodynamics

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

$\langle A \rangle \sim 0$, $\langle E \rangle \sim 0$
$\left\langle A^2 \right\rangle - \langle A \rangle^2 \sim \frac{Q_s^2}{g^2}$
$\left\langle E^2 \right\rangle - \langle E \rangle^2 \sim \frac{Q_s^4}{g^2}$
May give correct answer at LO
Not correct at NLO



$\langle A \rangle \sim \frac{Q_s}{g}$, $\langle E \rangle \sim \frac{Q_s^2}{g}$
$\left\langle A^2 \right\rangle - \langle A \rangle^2 \sim Q_s^2$
$\left\langle E^2 \right\rangle - \langle E \rangle^2 \sim Q_s^4$
give correct answer at LO
give correct answer at NLO

$$T^{\mu\nu}_{\text{resum}} \sim \frac{Q_s^4}{g^2} + c_0\Lambda^4 + c_2(Q_s)\Lambda^2 + \ldots$$

Quartic divergences can be subtracted with a simulation where

$$E^{\mu a}(x) = 0 + \sum_{\lambda,c}\int_{\boldsymbol{k}} c_{\boldsymbol{k}\lambda c}\, e^{\mu a}_{\boldsymbol{k}\lambda c}(\tau_0, \boldsymbol{x}_\perp, \eta)$$

Gives a $T^{\mu\nu}_{\text{part renor}} = T^{\mu\nu}_{\text{resum}} - T^{\mu\nu}_{\text{vac}}$

Anisotropic system $\Rightarrow \Lambda_T = k_{\perp,\max}$ and $\Lambda_L = k_{z,\max} = \frac{\nu_{\max}}{\tau}$

$$\epsilon_{\text{part renor}} \sim \frac{Q_s^4}{g^2} + \frac{Q_s^2 \nu_{\max}^2}{\tau^2} + \ldots$$

$$P_{L_{\text{part renor}}} \sim \frac{Q_s^4}{g^2} + \frac{Q_s^2 \nu_{\max}^2}{\tau^2} + \ldots$$

$$P_{T_{\text{part renor}}} \sim \frac{Q_s^4}{g^2} + Q_s^2 k_{\perp,\max}^2 + \ldots$$

How to deal with the $\frac{Q_s^2 \nu_{\max}^2}{\tau^2}$ terms $\rightarrow$ fitted for the time being.

Otherwise $\epsilon_{\text{part renor}}$ and $P_{L_{\text{part renor}}}$ behaves as τ^{-2} at early time.

$$\langle P_T \rangle_{\text{phys.}} = \langle P_T \rangle_{\substack{\text{backgd.} \\ +\ \text{fluct.}}} - \langle P_T \rangle_{\substack{\text{fluct.} \\ \text{only}}}$$

$$\langle \epsilon, P_L \rangle_{\text{phys.}} = \underbrace{\langle \epsilon, P_L \rangle_{\substack{\text{backgd.} \\ +\ \text{fluct.}}}}_{\text{computed}} - \underbrace{\langle \epsilon, P_L \rangle_{\substack{\text{fluct.} \\ \text{only}}}}_{\text{computed}} + \underbrace{A\,\tau^{-2}}_{\text{fitted}} .$$

The additional term is the only one that can satisfy Bjorken's law

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0$$

and the Equation Of State:

$$\epsilon = 2P_T + P_L$$