Nonequilibrium dynamics and transport in a quark-meson model

by Alex Meistrenko
in collaboration with C. Wesp, H. van Hees and C. Greiner

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

H-QM Helmholtz Research School Quark Matter Studies

HIC for FAIR Helmholtz International Center

Outline

Motivation:

phase diagram, critical phenomena:
order parameter, critical slowing down

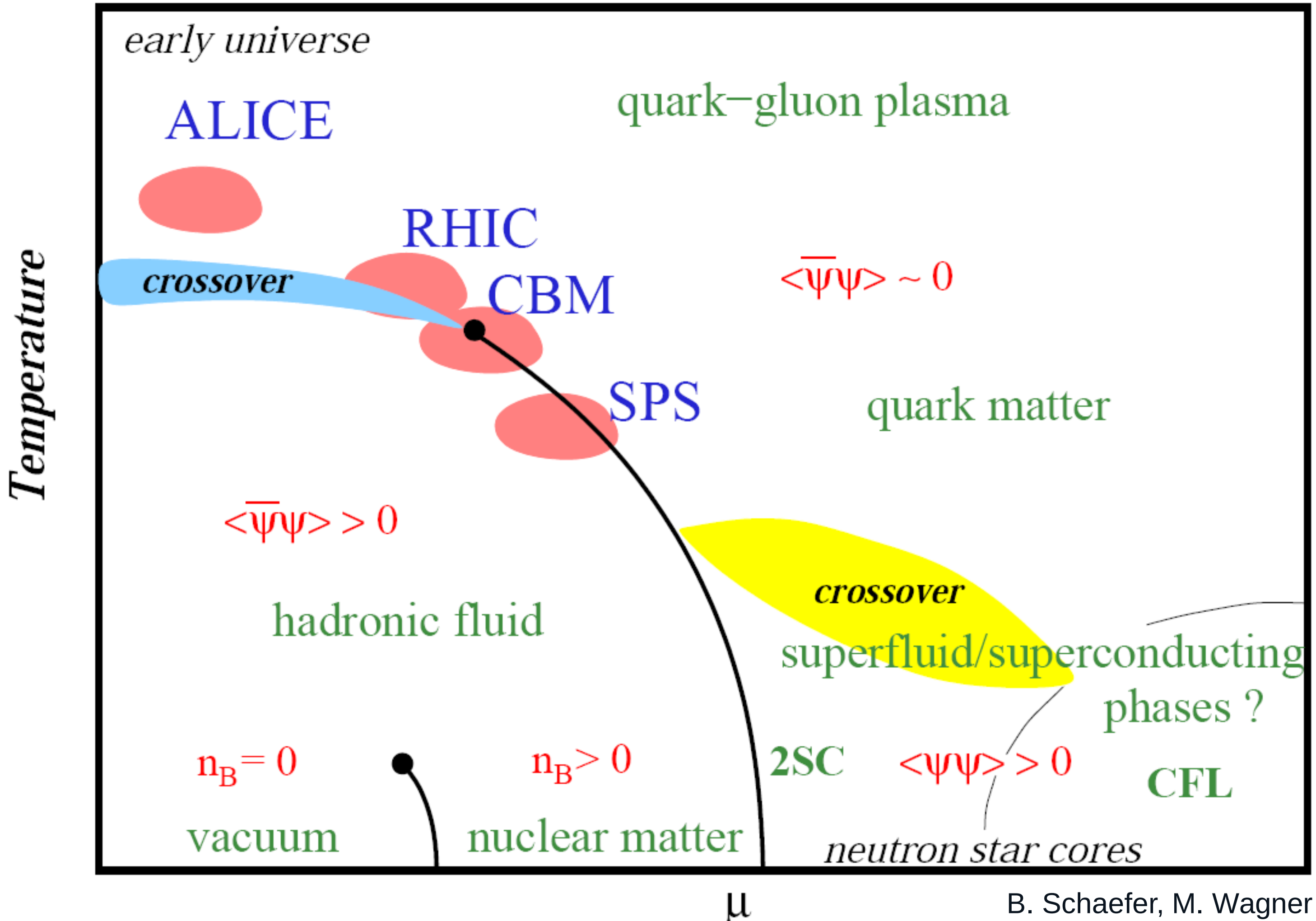
Model:

Lagrangian
spontaneously / explicitly broken chiral symmetry

1. mean-field and quarks with stochastic interaction
2. mean-field and Vlasov-Boltzmann approach:
   with selfconsistent mass equations,
   (collision terms and dissipation kernel)

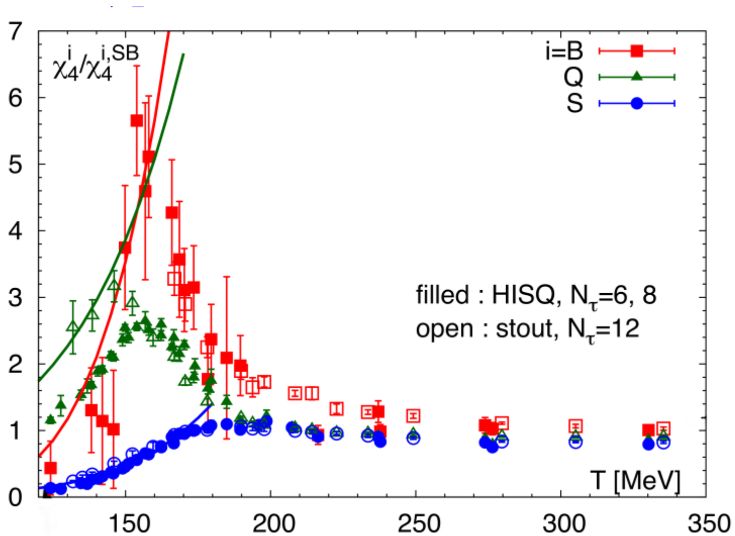
Summary and Outlook

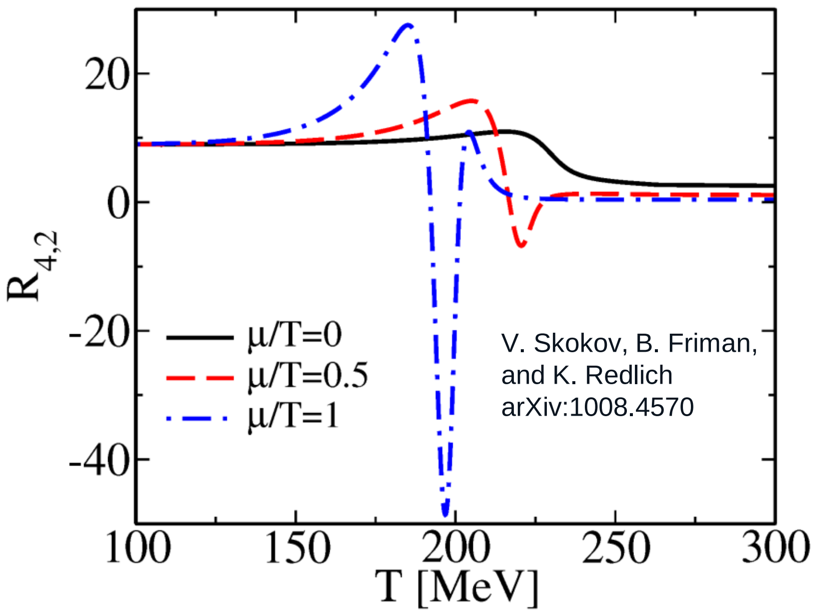
Motivation



B. Schaefer, M. Wagner
arXiv:0812.2855

Motivation





$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}\left(p/T^4\right)}{\partial\left(\mu_B/T\right)^l \partial\left(\mu_s/T\right)^m \partial\left(\mu_Q/T\right)^n}$$

$$R_{4,2} = \frac{\chi_4^B}{\chi_2^B} = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle$$

Model: Lagrangian

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - g\left(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5\right)\right]\psi - \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$

Yukawa coupling

O(N) theory / Φ^4 coupling

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - f_\pi m_\pi^2\sigma + U_0$$

parameters:

λ	$= 20$	self coupling
g	$= 3...6$	quark-sigma coupling
ν^2	$= f_\pi^2 - m_\pi^2/\lambda$	field shift term
f_π	$= 93\,MeV$	pion decay constant
m_π	$= 138\,MeV$	pion mass
U_0	$= m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$	ground state

Model: Lagrangian

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - g\left(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5\right)\right]\psi - \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$

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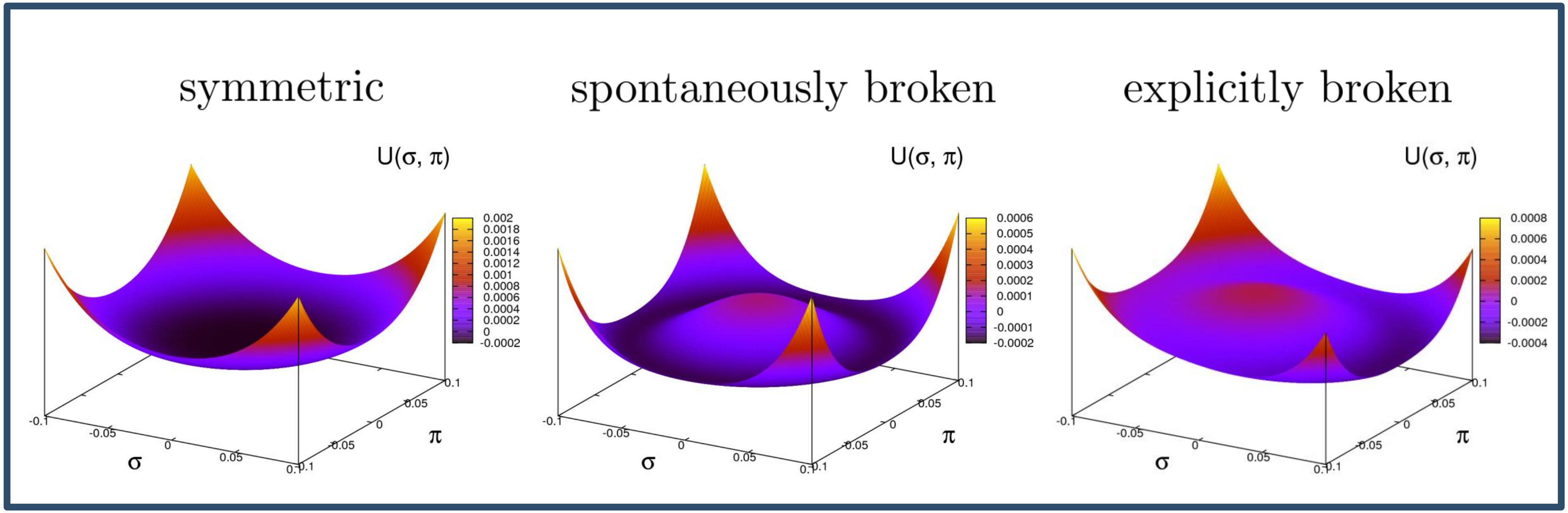


Mean-field + Vlasov + Interaction

by Christian Wesp

3D + 1 grid for the σ-field (similar for $\vec{\pi}$):

$$\partial_\mu \partial^\mu \sigma + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2\right) \sigma - f_\pi m_\pi^2 + g \boxed{\langle \bar{\psi}\psi \rangle} = I\left(\sigma \leftrightarrow \psi\psi\right)$$

Quarks as test particles:

$$\left[\partial_t + \frac{p}{E_\psi\left(t, \vec{x}, \vec{p}\right)} \cdot \nabla_{\vec{x}} - \boxed{\nabla_{\vec{x}} E_\psi\left(t, \vec{x}, \vec{p}\right)} \nabla_{\vec{p}}\right] f_\psi\left(t, \vec{x}, \vec{p}\right) = C\left(\psi\psi \to \psi\psi\,, \sigma \leftrightarrow \psi\psi\right)$$

$$\langle \bar{\psi}\psi \rangle \left(t, \vec{x}\right) = g d_\psi \sigma\left(t, \vec{x}\right) \int \frac{d^3p}{\left(2\pi\right)^3} \frac{f_\psi\left(t, \vec{x}, \vec{p}\right) + f_{\bar{\psi}}\left(t, \vec{x}, \vec{p}\right)}{\sqrt{\vec{p}^2 + M_\psi^2\left(t, \vec{x}, \vec{p}\right)}}$$

$$M_\psi^2\left(t, \vec{x}\right) = g^2\left[\sigma^2\left(t, \vec{x}\right) + \vec{\pi}^2\left(t, \vec{x}\right)\right]$$

stochastic model for interaction rates between field and particles:

$$P\left(\Delta E\,, \Delta\vec{p}\,, \psi\psi \leftrightarrow \sigma\right)$$

Damped Harmonic Oscillator

by Christian Wesp

classical damping:

$$\frac{\partial^2}{\partial t^2}\phi(t) + \boxed{\gamma\dot{\phi}(t)} + \phi(t) = 0 \quad \leftrightarrow$$

stochastic damping:

$$\frac{\partial^2}{\partial t^2}\phi(t) + \phi(t) = \boxed{\delta\phi(t)}$$

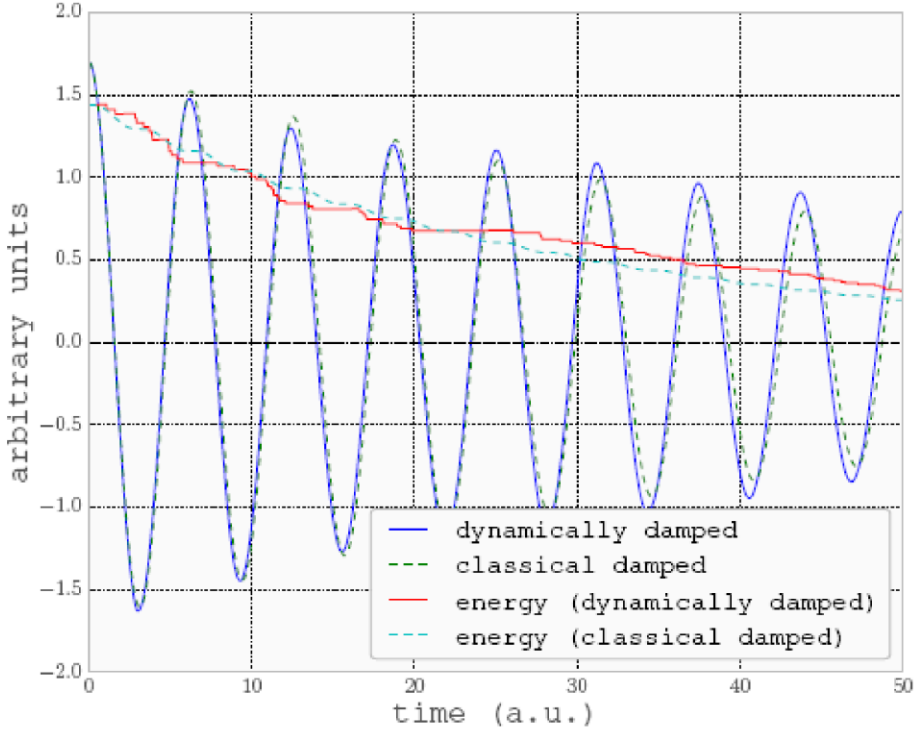


Figure: $\gamma = 0.35$, $N = 50$

$$P(\Delta E) = \gamma \cdot dt \cdot N$$

$$\text{with} \quad \Delta E = \frac{E_0}{N}$$

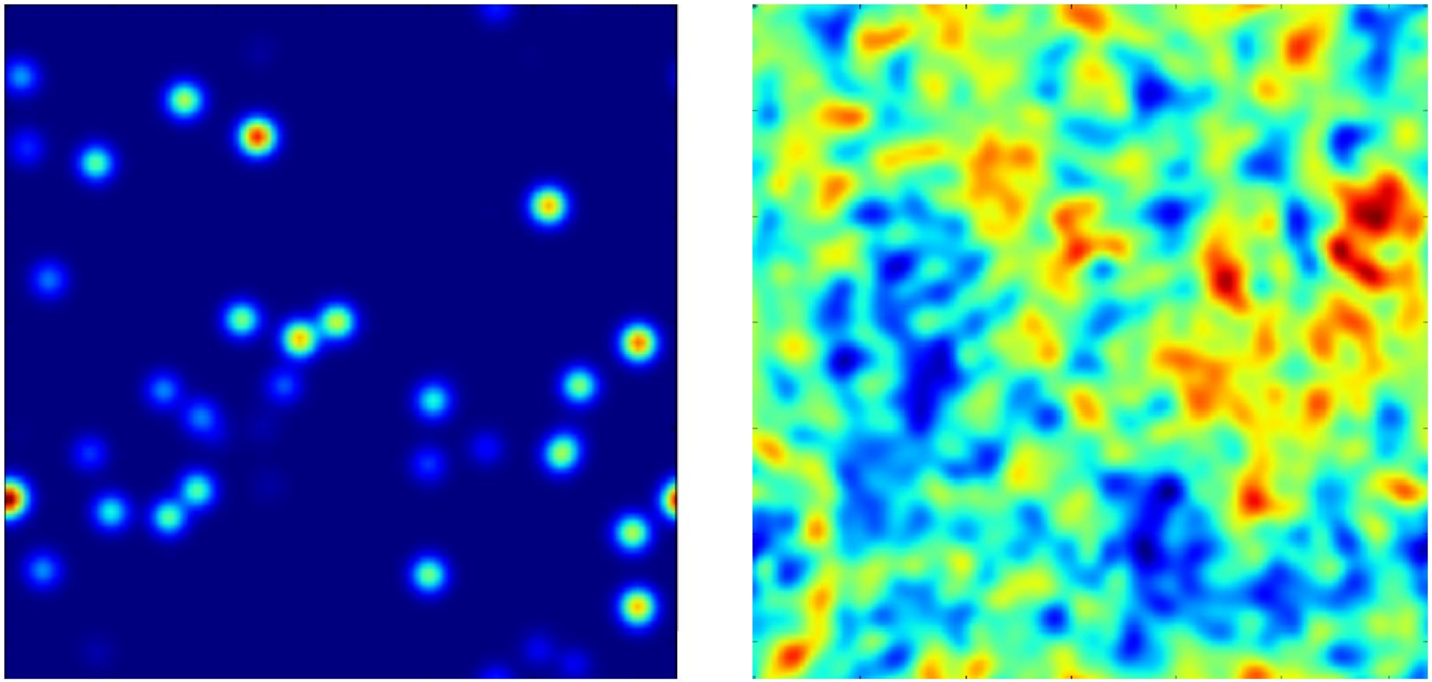
works also for a Langevin force:

$$\delta\phi(t) \sim -\gamma\dot{\phi}(t) + \kappa\xi(t)$$

H. van Hees, C. Wesp, A. Meistrenko and C. Greiner
arXiv:1311.6825

3D Field Dynamics

by Christian Wesp



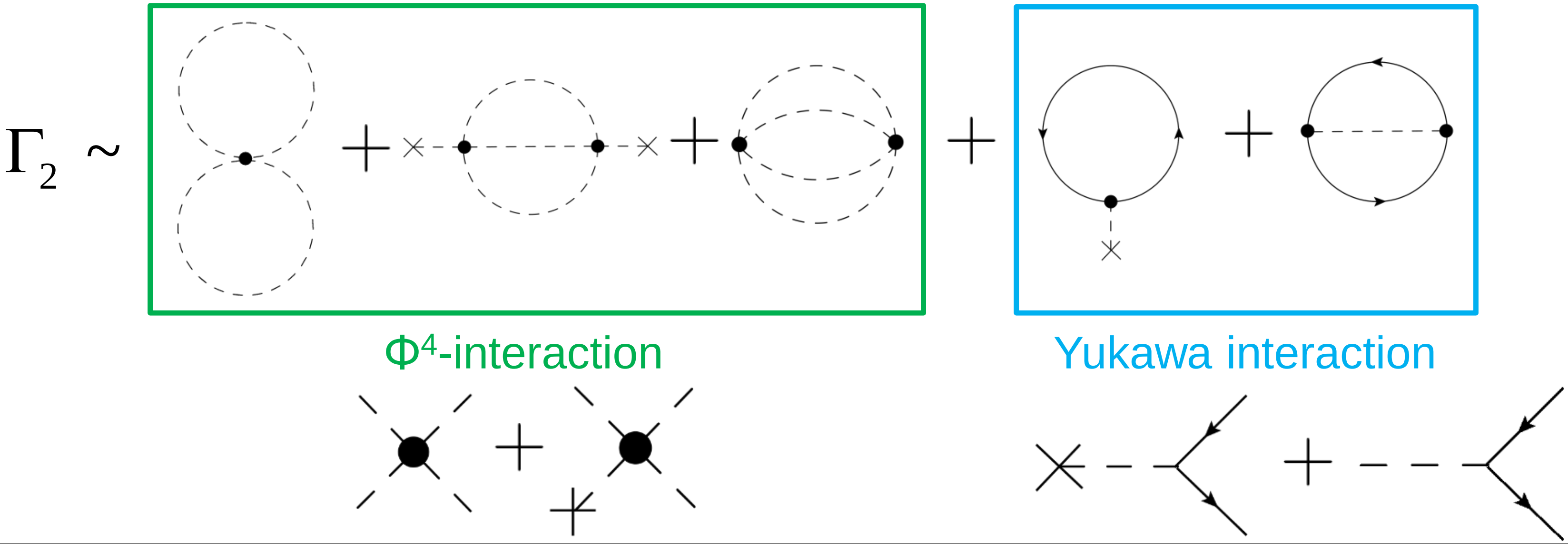
dynamic $\bar{\psi}\psi \leftrightarrow \sigma$ interaction

Noise is generated by particle-interactions, not by a noise-term!

2PI effective action

$$\Gamma[\phi,\psi,\bar{\psi},G,D] = S_{cl}[\phi] + \frac{i}{2}\cdot Tr\log G^{-1} + \frac{i}{2}\cdot TrG_0^{-1}G$$
$$- i\cdot Tr\log D^{-1} - i\cdot TrD_0^{-1}D + \Gamma_2[\phi,\psi,\bar{\psi},G,D]$$

relevant 2PI-diagrams (3 loops):



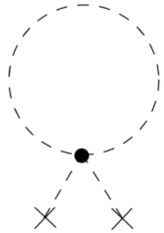
Mean-field EoM

$3D+1$ simulation:

$$\partial_\mu\partial^\mu\sigma+\lambda\left(\sigma^2+\vec{\pi}^2-\nu^2+3G_{\sigma\sigma}+3G_{\pi\pi}\right)\sigma-f_\pi m_\pi^2+g\left\langle\bar{\psi}\psi\right\rangle=0$$

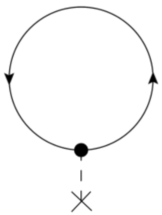
$$\partial_\mu\partial^\mu\vec{\pi}+\lambda\left(\sigma^2+\vec{\pi}^2-\nu^2+G_{\sigma\sigma}+5G_{\pi\pi}\right)\vec{\pi}+g\left\langle\bar{\psi}i\gamma_5\vec{\tau}\psi\right\rangle=0$$

$$G_{\phi\phi}\left(t,\vec{x}\right)=\frac{1}{2}\int\frac{d^3p}{(2\pi)^2}\frac{1+2N_\phi\left(t,\vec{x},\vec{p}\right)}{\sqrt{\vec{p}^2+M_\phi^2\left(t,\vec{x}\right)}}\quad\text{with}\quad\phi\in\{\sigma,\pi\}$$



$$\left\langle\bar{\psi}\psi\right\rangle\left(t,\vec{x}\right)=gd_\psi\sigma\left(t,\vec{x}\right)\int\frac{d^3p}{(2\pi)^3}\frac{f_\psi\left(t,\vec{x},\vec{p}\right)+f_{\bar{\psi}}\left(t,\vec{x},\vec{p}\right)}{\sqrt{\vec{p}^2+M_\psi^2\left(t,\vec{x},\vec{p}\right)}},$$

$$\left\langle\bar{\psi}i\gamma_5\vec{\tau}\psi\right\rangle\left(t,\vec{x}\right)=gd_\psi\vec{\pi}\left(t,\vec{x}\right)\int\frac{d^3p}{(2\pi)^3}\frac{f_\psi\left(t,\vec{x},\vec{p}\right)+f_{\bar{\psi}}\left(t,\vec{x},\vec{p}\right)}{\sqrt{\vec{p}^2+M_\psi^2\left(t,\vec{x},\vec{p}\right))}}$$



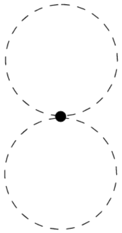
Vlasov-Boltzmann approach

self-consistent mass in the Vlasov equations of σ, $\vec{\pi}$ and ψ:

$$\left[\partial_t + \frac{p}{E_\phi\left(t,\vec{x},\vec{p}\right)}\cdot\nabla_{\vec{x}} - \nabla_{\vec{x}}E_\phi\left(t,\vec{x},\vec{p}\right)\nabla_{\vec{p}}\right] f_\phi\left(t,\vec{x},\vec{p}\right) = 0$$

$$M_\sigma^2\left(x\right) = \lambda\left(3\sigma^2 + 3\pi^2 - \nu^2\right) + 3\lambda G_{\sigma\sigma} + 3\lambda G_{\pi\pi}$$

$$M_\pi^2\left(x\right) = \lambda\left(\sigma^2 + 5\pi^2 - \nu^2\right) + \lambda G_{\sigma\sigma} + 5\lambda G_{\pi\pi}$$



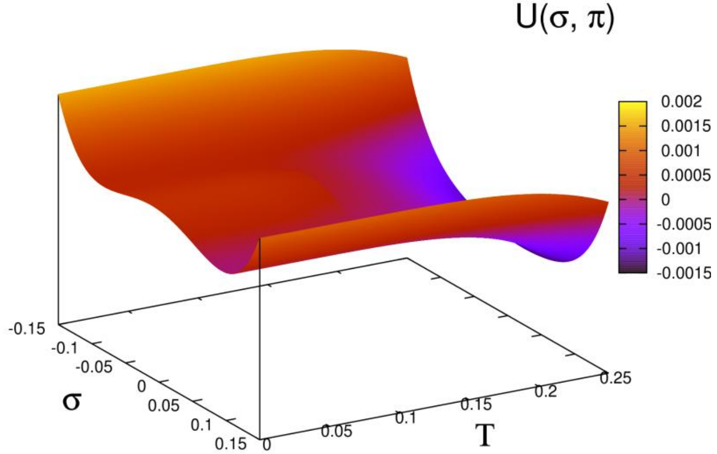
$$\left[\partial_t + \frac{p}{E_\psi\left(t,\vec{x},\vec{p}\right)}\cdot\nabla_{\vec{x}} - \nabla_{\vec{x}}E_\psi\left(t,\vec{x},\vec{p}\right)\nabla_{\vec{p}}\right] f_\psi\left(t,\vec{x},\vec{p}\right) = 0$$

$$M_\psi^2\left(t,\vec{x}\right) = g^2\left[\sigma^2\left(t,\vec{x}\right) + \vec{\pi}^2\left(t,\vec{x}\right)\right]$$

Effective potential

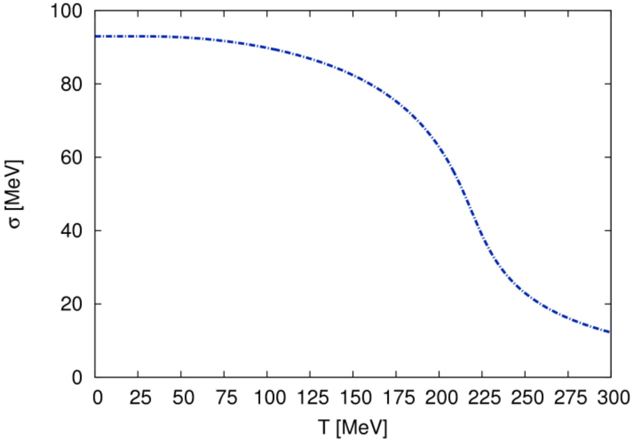
effective potential in Hartree approximation (without quarks):

$$\Omega = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - f_\pi m_\pi^2 - U_0$$
$$+ F_1\left(M_\sigma\right) + 3F_1\left(M_\pi\right)$$
$$- \frac{1}{2}\left(M_\sigma^2 - \lambda\left(3\sigma^2 + 3\pi^2 - \nu^2\right)\right) F_2\left(M_\sigma\right)$$
$$- \frac{3}{2}\left(M_\pi^2 - \lambda\left(\sigma^2 + 5\pi^2 - \nu^2\right)\right) F_2\left(M_\pi\right)$$
$$+ 3\frac{\lambda}{4}\left(F_2\left(M_\sigma\right)\right)^2 + 6\frac{\lambda}{4} F_2\left(M_\sigma\right) F_2\left(M_\pi\right) + 15\frac{\lambda}{4}\left(F_2\left(M_\pi\right)\right)^2$$



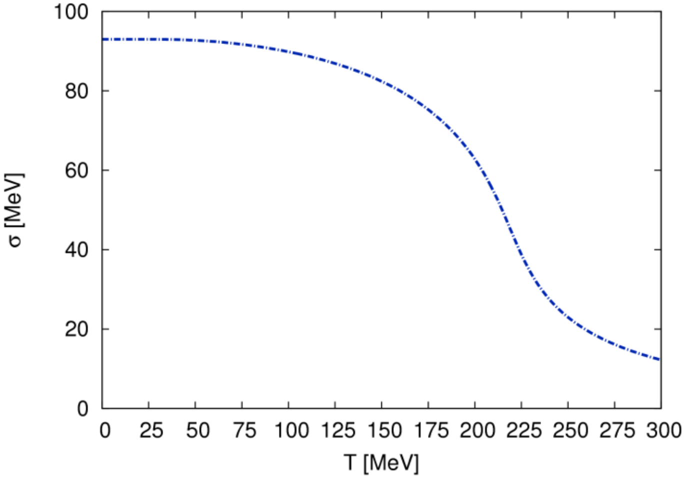
$$F_1\left(M_\phi\right) = T \int \frac{d^3k}{(2\pi)^3}\left[\frac{E_\phi}{2T} + \ln\left(1 - e^{-\beta E_\phi}\right)\right],$$
$$F_2\left(M_\phi\right) = \int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2E_\phi} + \frac{1}{E_\phi}\frac{1}{e^{\beta E_\phi} - 1}\right]$$

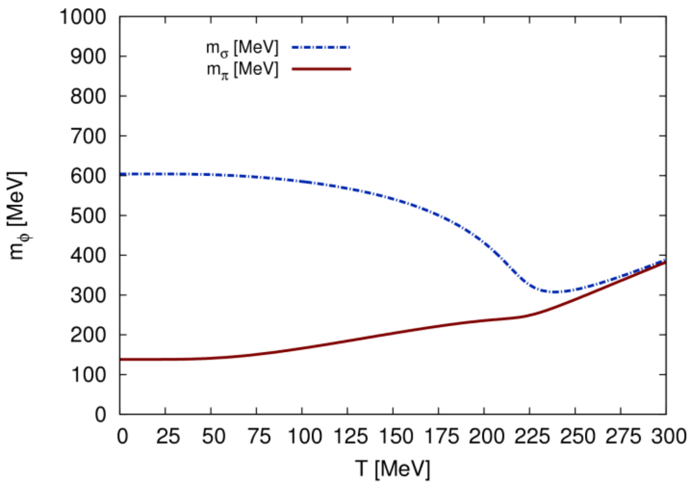
for renormalization:
J. Lenaghan, D. Rischke arXiv:nucl-th/9901049



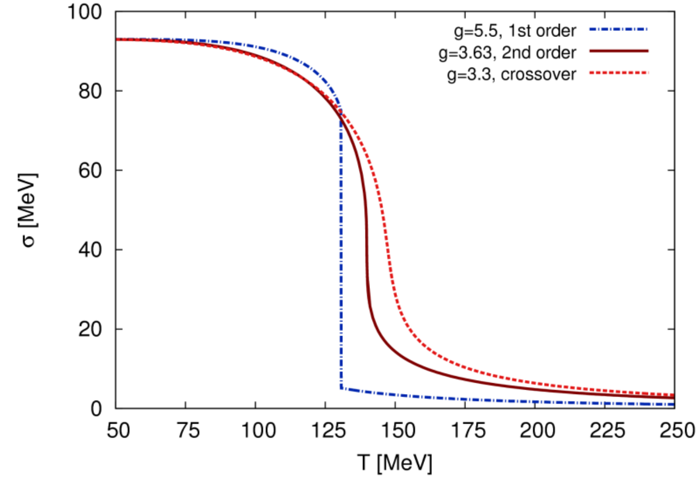
Equilibrium properties

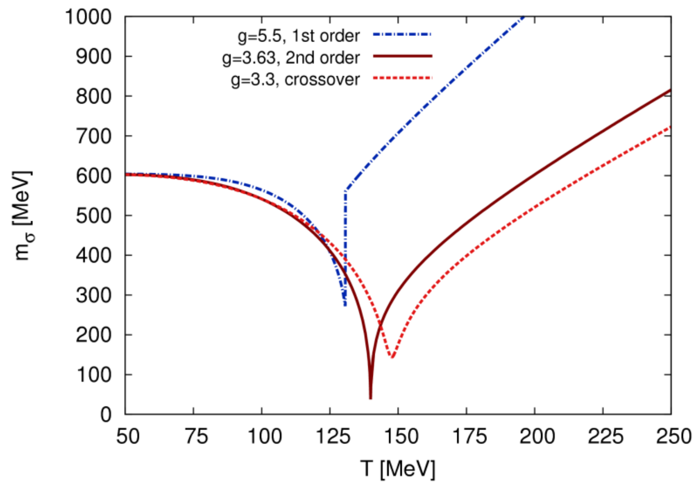
mean field + mesons





mean field + quarks

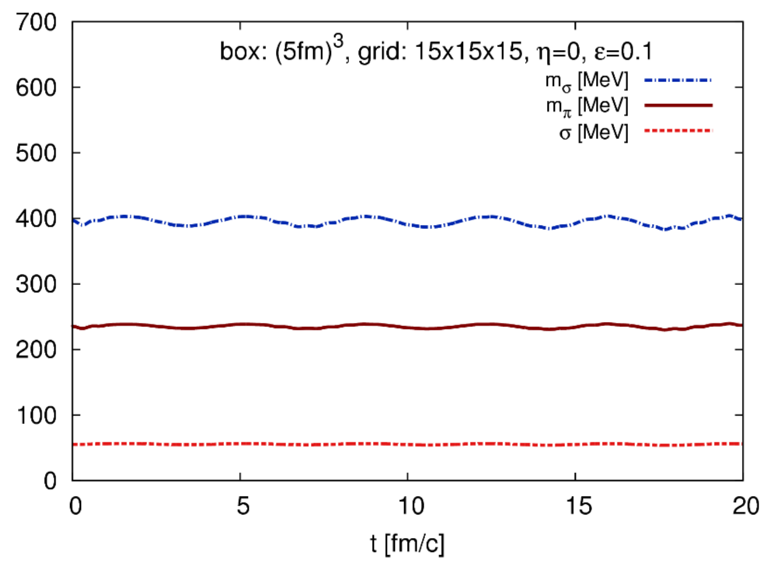


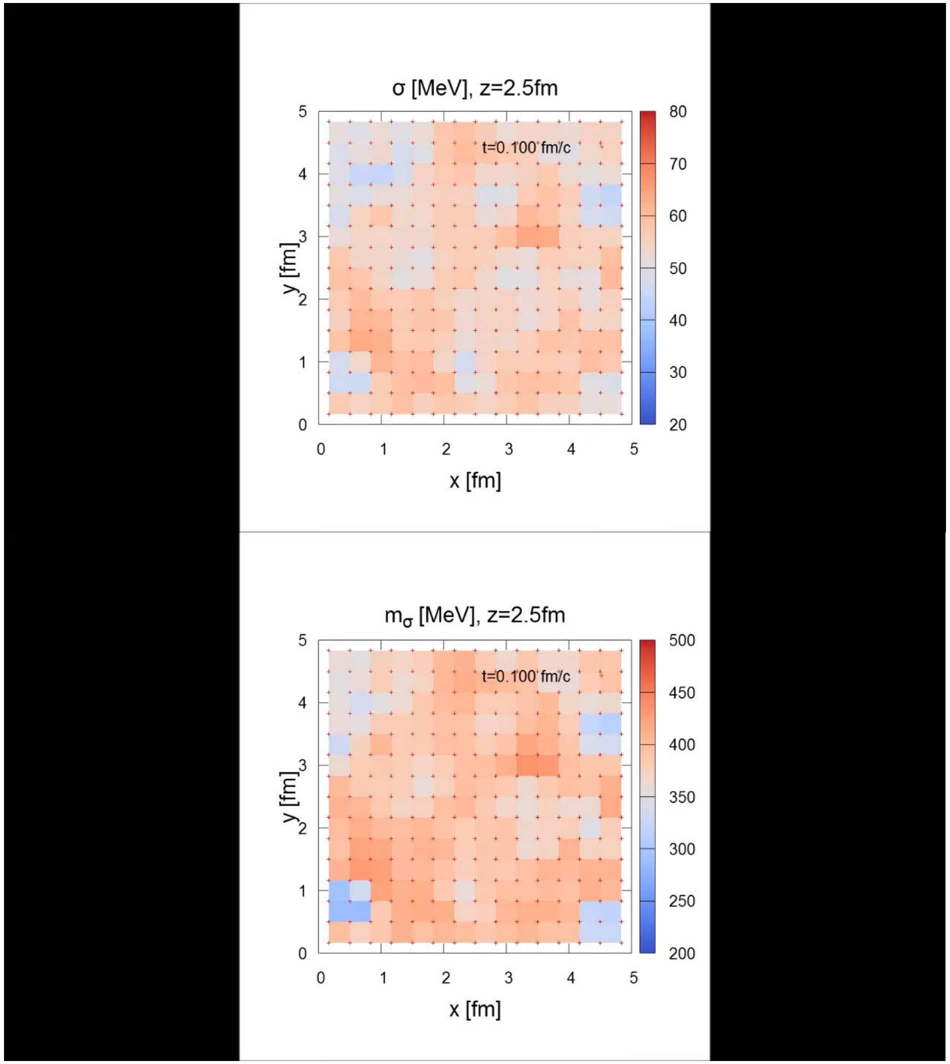


Dynamical simulations

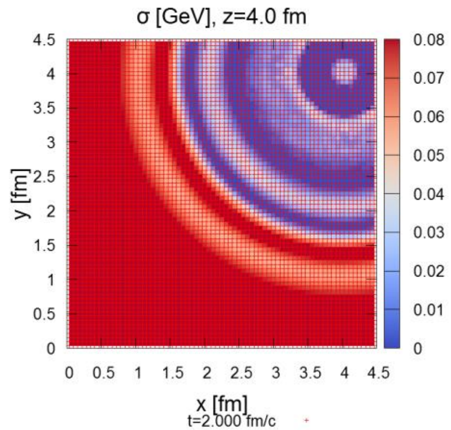
mean field + mesons

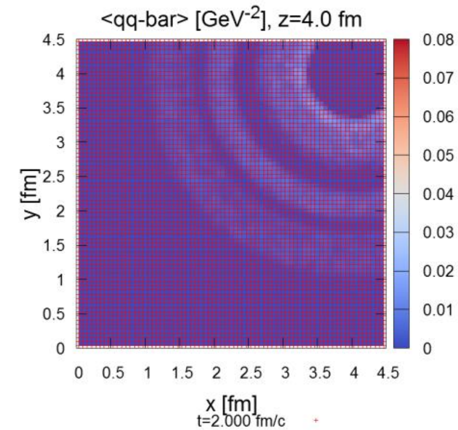
$T = 200\,MeV,\ \sigma = 10\,MeV$ (Gaussian fluctuations)

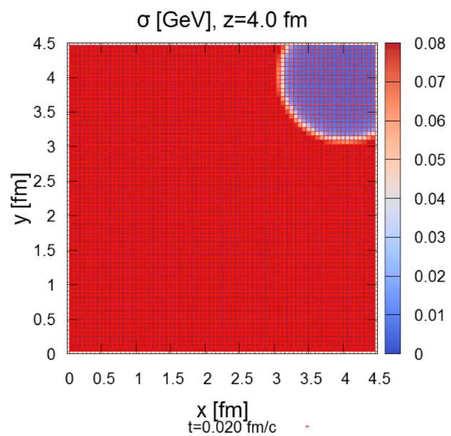


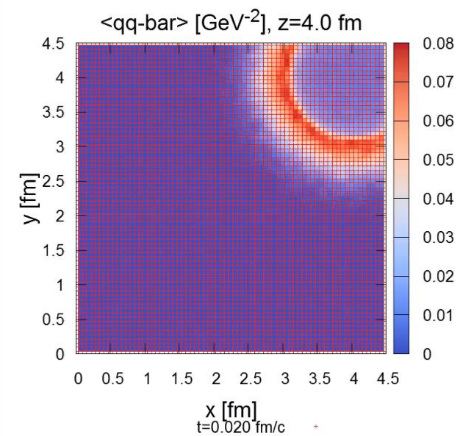


Dynamical simulations









Vlasov-Boltzmann approach

Mean-field equation with a dissipation kernel:

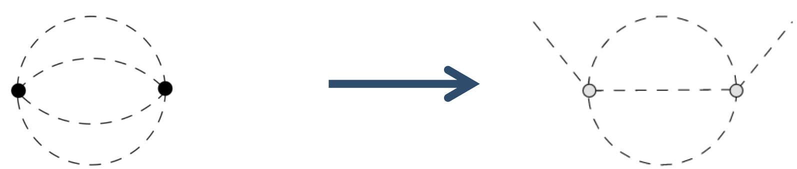
$\Gamma_2 \sim$

$$D\left(t,\vec{x}\right) \sim \int \frac{d^3k}{\left(2\pi\right)^3} e^{i\vec{k}\cdot\vec{x}} \int \frac{dk^0}{2\pi} \frac{\mathcal{M}\left(t,\vec{x},\vec{k}\right)}{E_k} \partial_t \sigma\left(t,\vec{k}\right) \pi\delta\left(E_k - k^0\right)$$

C. Greiner, B. Müller arXiv:hep-th/9605048

Vlasov-Boltzmann equation with $2 \rightarrow 2$ collision integral:

$\Gamma_2 \sim$



complexity of collision integrals: $\mathcal{O}\left(N^9\right) \rightarrow \mathcal{O}\left(N^6 - N^7\right)$ (with MC and symmetry)

Summary and outlook

- linear sigma model with Yukawa coupling to quarks
- 3D+1 non-equilibrium transport model:
  1. mean field + quarks with stochastic interaction
  2. mean field + mesons + quarks from 2PI

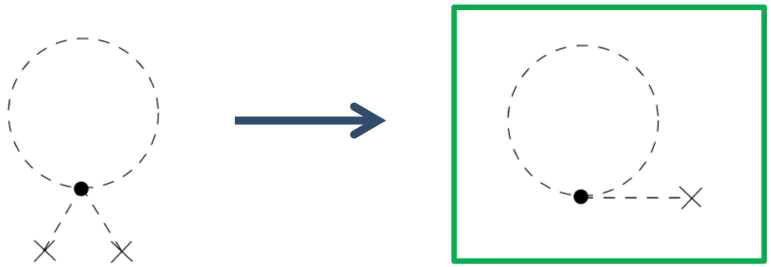
Outlook:

- dynamical simulation with collision terms
  - inelastic processes important
- study of net-quark number fluctuations

Thank you
for your attention!

2PI effective action – mean field

$$\Gamma \sim \frac{i}{2} \cdot Tr G_0^{-1} G \sim$$



$$\partial_\mu \partial^\mu \sigma + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + 3G_{\sigma\sigma} + 3G_{\pi\pi} \right) \sigma - f_\pi m_\pi^2 + g \left\langle \bar{\psi} \psi \right\rangle = 0$$

$$\partial_\mu \partial^\mu \vec{\pi} + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + G_{\sigma\sigma} + 5G_{\pi\pi} \right) \vec{\pi} + g \left\langle \bar{\psi} i \gamma_5 \vec{\tau} \psi \right\rangle = 0$$

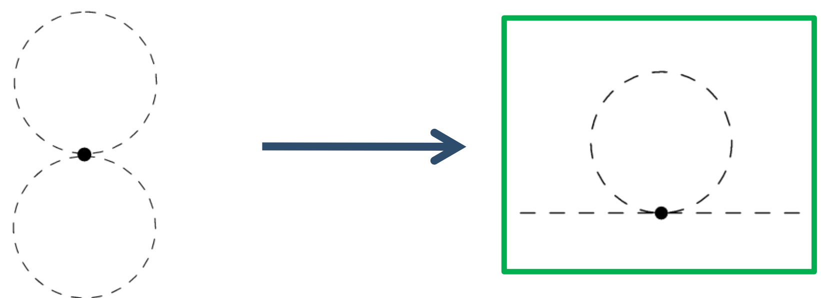
$$G_{\phi\phi}(t, \vec{x}) = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^2} \frac{1 + 2N_\phi (t, \vec{x}, \vec{p})}{\sqrt{\vec{p}^2 + M_\phi^2 (t, \vec{x})}} \quad \text{with} \quad \phi \in \{\sigma, \pi\}$$

2PI effective action – effective mass

local part of the self-energy from Hartree approximation:

$$iG^{-1}_{\phi\phi} = iG^{-1}_{0,\phi\phi} - i\Pi_{\phi\phi}\,, \qquad \Pi_{\phi\phi} = 2i\frac{\delta\Gamma_2}{\delta G}$$

$$\Gamma \sim \int_{\mathcal{C}} d^4x\, G_{\phi_i\phi_i} G_{\phi_j\phi_j} \sim$$



effective mass from the gap equation of the propagator: $iG^{-1}_{\phi\phi} = k^2 - M^2_\phi$

$$M^2_\sigma(x) = \lambda\left(3\sigma^2 + 3\pi^2 - \nu^2\right) + 3\lambda G_{\sigma\sigma} + 3\lambda G_{\pi\pi}$$
$$M^2_\pi(x) = \lambda\left(\sigma^2 + 5\pi^2 - \nu^2\right) + \lambda G_{\sigma\sigma} + 5\lambda G_{\pi\pi}$$

Dissipation kernel

Langevin equation (1D classical case):

$$\boxed{m\ddot{x}(t)} + \boxed{2\int_0^t dt\,\Gamma(t-t')\,\dot{x}(t')} - \boxed{F(x)} = \xi(t)$$

Mean-field equation with a dissipation kernel:

$\Gamma_2 \sim$

$$\boxed{\partial_\mu \partial^\mu \sigma} + \boxed{D(t,\vec{x})} + \boxed{\lambda\left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \frac{3}{2}G_{\sigma\sigma} + \frac{3}{2}G_{\pi\pi}\right)\sigma - f_\pi m_\pi^2 + g\left\langle \bar{\psi}\psi \right\rangle} = 0$$

$$D(t,\vec{x}) \sim \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \int \frac{dk^0}{2\pi} \frac{\mathcal{M}\left(t,\vec{x},\vec{k}\right)}{E_k} \partial_t \sigma\left(t,\vec{k}\right) \pi\delta\left(E_k - k^0\right)$$