Inhomogeneous chiral symmetry-breaking phases in isospin-asymmetric matter

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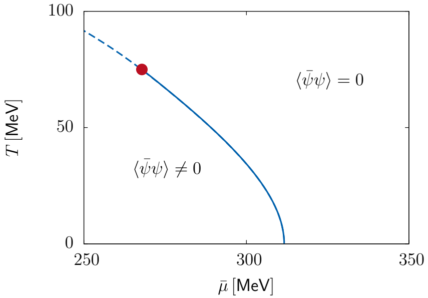
HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

EMMI

HIC for FAIR
Helmholtz International Center

Motivation:
QCD phase diagram



- order-parameters mostly assumed to be spatially constant, e. g.

$$\left\langle \bar{\psi}\psi \right\rangle (\vec{x}) = \left\langle \bar{\psi}\psi \right\rangle$$

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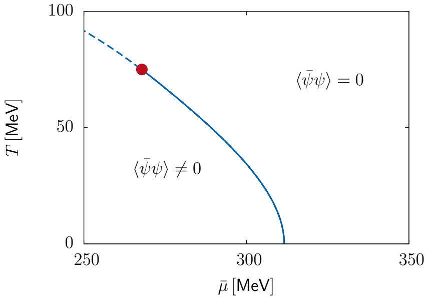

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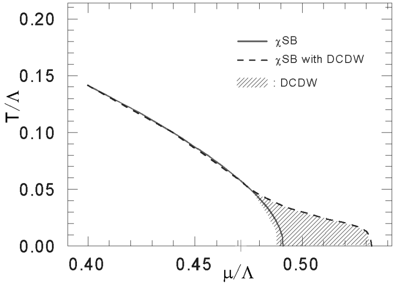

Nakano, Tatsumi, Phys. Rev. D 71:114006 (2005)

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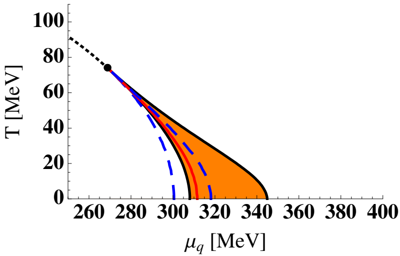

Nickel, Phys. Rev. D 80:074025 (2009)

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  - first-order phase-transition covered by inhomogeneous region

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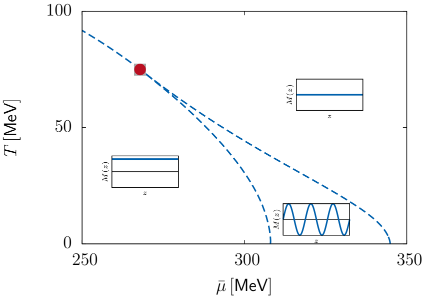

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QCD phase diagram

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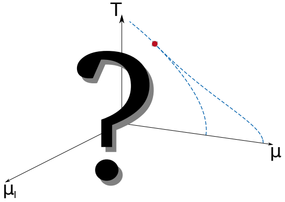
- isospin-asymmetric matter $\mu_u \neq \mu_d$
- relevance in nature?

Examples:

1. heavy ion collisions
   - excess of neutrons in heavy nuclei
2. neutron stars
   - requirement of electrical neutrality



Our model

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- $N_f = 2$ Nambu–Jona-Lasinio (NJL) model

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi + G\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau^a\psi\right)^2\right), \ \psi = (u, d)^T$$

- mean-field approximation
- condensates

$$S_u(\vec{x}) = \left\langle \overline{u}u \right\rangle, \qquad P_u(\vec{x}) = \left\langle \overline{u}i\gamma_5 u \right\rangle$$
$$S_d(\vec{x}) = \left\langle \overline{d}d \right\rangle, \qquad P_d(\vec{x}) = \left\langle \overline{d}i\gamma_5 d \right\rangle$$

- retain spatial dependence of the condensates in z-direction (1d modulation)

$$S_f(\vec{x}) \to S_f(z), \qquad P_f(\vec{x}) \to P_f(z)$$

Thermodynamic potential

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- thermodynamic potential

$$\Omega(T,\mu;M(z)) = -\frac{TN_c}{V}\sum_{E_n}\ln\left(2\cosh\left(\frac{E_n-\hat{\mu}}{2T}\right)\right) + \Omega_{\text{cond}} + \text{const.}$$

where

$$\Omega_{\text{cond}} = \frac{G}{L}\int_0^L dz\left(\left(S_u(z)+S_d(z)\right)^2 + \left(P_u(z)-P_d(z)\right)^2\right)$$

- $\hat{\mu}$: isospin-asymmetric matter

$$\hat{\mu} = \begin{pmatrix} \mu_u & \\ & \mu_d \end{pmatrix} = \begin{pmatrix} \bar{\mu}+\delta\mu & \\ & \bar{\mu}-\delta\mu \end{pmatrix}$$

where $\delta\mu = (\mu_u - \mu_d)/2 = \mu_I/2, \quad \bar{\mu} = (\mu_u+\mu_d)/2 = \mu_B/3$

Thermodynamic potential

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- E_n: eigenvalues of mean-field Hamiltonian $\tilde{H}_{\text{MF}} = \text{diag}(\tilde{H}^u_{\text{MF}}, \tilde{H}^d_{\text{MF}})$

$$\tilde{H}^f_{\text{MF}} = \begin{pmatrix} i\sigma^i\partial_i & M_f(z) \\ M_f^*(z) & -i\sigma^i\partial_i \end{pmatrix}, \; f \in \{u, d\}$$

- effective mass $M_f(z)$ (chiral limit $m = 0$):

$$M_f(z) = -2G\,(S_f(z) + S_h(z) + i\,(P_f(z) - P_h(z)))$$

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- Problem: Inhomogeneous chiral condensates couple different momenta
  $\rightarrow$ diagonalization of $\tilde{H}_{\mathrm{MF}}$ *very* difficult

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- Problem: Inhomogeneous chiral condensates couple different momenta
  $\rightarrow$ diagonalization of $\tilde{H}_{\text{MF}}$ *very* difficult
- Solution: Assume periodicity of the condensates to expand as Fourier series

$$S_f(z) = \sum_{q_k} S_{f,\vec{q}_k} \exp\left(iq_k z\right), \qquad P_f(z) = \sum_{q_k} P_{f,\vec{q}_k} \exp\left(iq_k z\right)$$

and exploit lattice structure for brute-force approach in momentum space

One-dimensional modulations of $S_f(z)$ and $P_f(z)$: Solitonic modulation

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- fully self-consistent analytical solution of the order-parameter known from 1+1 dimensional Gross-Neveu model (Nickel, Phys. Rev. D 80:074025, 2009)

$$M_u(z) = M_d(z) = \nu\Delta \,\mathrm{sn}(\Delta z|\nu)$$

- but: also simplified shapes possible $\Rightarrow$ brute force diagonalization of the Hamiltonian; e. g. for

$$S_f(z) = \Delta_f \cos(q_f z) \qquad P_f(z) = \Delta_f \sin(q_f z)$$

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- minimization of the thermodynamic potential $\Omega(T, \bar{\mu}, \mu_I)$ w. r. t. to (Δ, ν) resp. $(\Delta_u, \Delta_d, q_u, q_d)$

One-dimensional modulations of $S_f(z)$ and $P_f(z)$: Solitonic modulation

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- simple picture: quarks favor values for $q \sim \mu$ in the inhomogeneous phase
- question: what happens if $\mu_u \neq \mu_d$; non-degenerate flavors prefer different q's?

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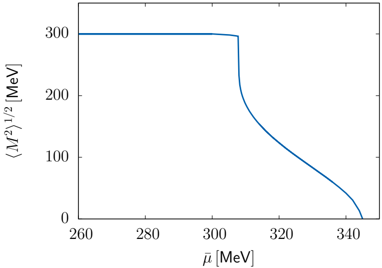
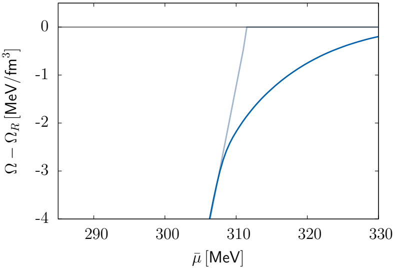


$T = 0, \mu_I = \quad 0$



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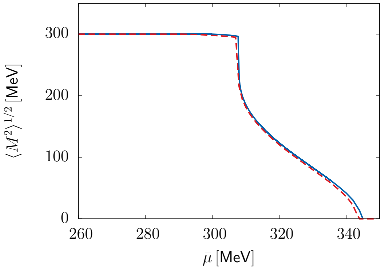
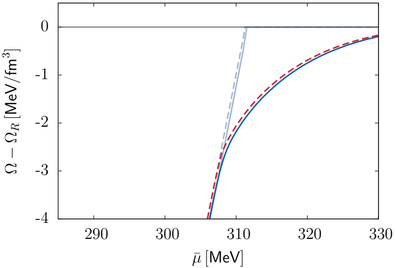


$T = 0, \mu_I = 20\,\text{MeV}$



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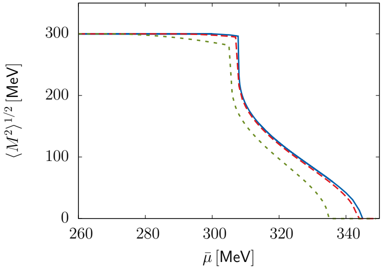
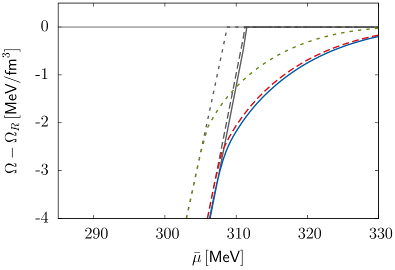


$T = 0, \mu_I = 60\,\text{MeV}$



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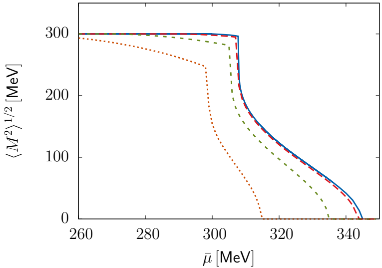
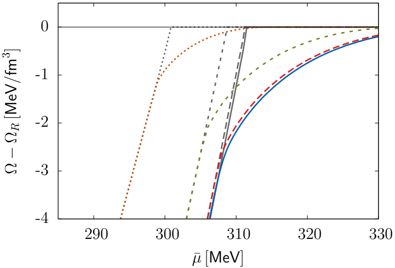


$T = 0$, $\mu_I = 120$ MeV



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One-dimensional solitonic modulations: Equal quark periodicities

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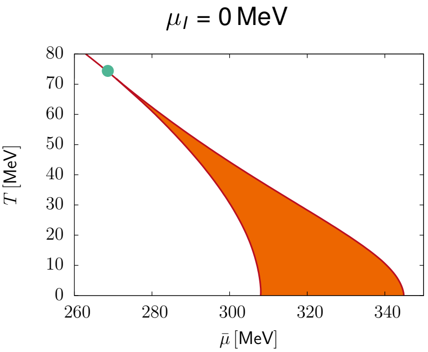
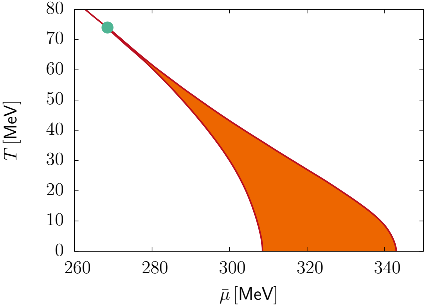


- inhomogeneous region bordered by second-order phase transition lines

One-dimensional solitonic modulations: Equal quark periodicities

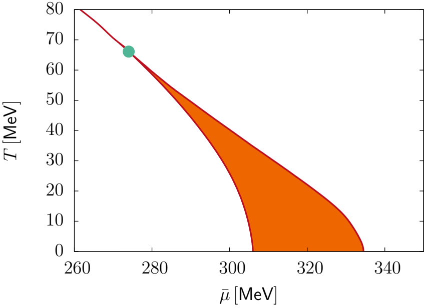
μ_I = 20 MeV



- size of inhomogeneous region gets reduced for non-zero μ_I

One-dimensional solitonic modulations: Equal quark periodicities

$\mu_I = 60\,\text{MeV}$



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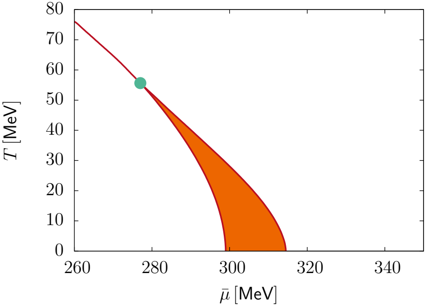
μ_I = 120 MeV



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One-dimensional modulations: Unequal quark periodicities

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- idea: allow different periodicities for the two quark flavors
- here: resort to simpler shapes

$$S_f(z) = \Delta_f \cos(q_f z), \quad P_f(z) = \Delta_f \sin(q_f z)$$

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- Solution: consider rational ratio instead

$$q_u = mq, \quad q_d = nq, \quad m/n \in \mathbb{Q}$$

then minimize Ω w. r. t. $(m/n, q, \Delta_u, \Delta_d)$

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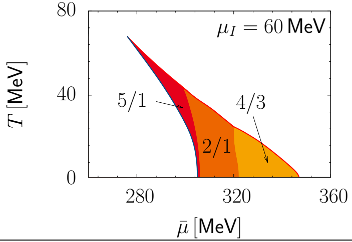
- inhomogeneous chiral symmetry breaking phase can be stabilized



One-dimensional modulations: Flavor mixing

(e. g. Frank, Buballa, Oertel, Phys. Lett. B 562, 221 (2003))

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- idea: study inhomogeneous phases with varying degree of flavor-mixing
- generalize four-fermion interaction in the model

$$G\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau^a\psi\right)^2\right) \rightarrow$$

$$(1-\alpha)G\left((\bar{\psi}\psi)^2 + (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2\right)$$
$$+\,\alpha G\left((\bar{\psi}\psi)^2 - (\bar{\psi}\tau^a\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2\right)$$

- effective mass $M_f(z)$ (chiral limit $m = 0$):

$$M_f(z) = -4G\left((1-\alpha)S_f(z) + \alpha S_h(z) + i\left((1-\alpha)P_f(z) - \alpha P_h(z)\right)\right), \;\; f \neq h$$

- degree of flavor-mixing controlled by parameter $\alpha \in [0, 1]$ (at fixed G)

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- degree of flavor-mixing controlled by parameter $\alpha \in [0, 1]$ (at fixed G)

$\Rightarrow$ previously considered: $\boxed{\alpha = 0.5}$

$\Rightarrow$ here: $\boxed{\alpha = 0}$

No flavor mixing: $\alpha = 0$
Thermodynamic potential

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- thermodynamic potential formaly consist of two independent parts

$$\Omega(T, \mu) = \sum_{f=u,d} \Omega_f(T, \mu_f)$$

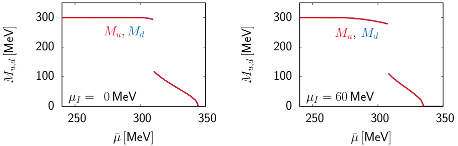
- no self-consistent analytical solution for space dependent order-parameter
- here: resort to simpler shapes

$$S_f(z) = \Delta_f \cos(q_f z), \quad P_f(z) = \Delta_f \sin(q_f z)$$

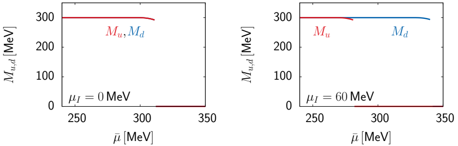
- minimize Ω w. r. t. $(\Delta_u, \Delta_d, q_u, q_d)$

No flavor mixing: α = 0
Phase structure for non-vanishing μ_I

- maximally coupled quark flavors: α = 0.5



- uncoupled quark flavors: α = 0



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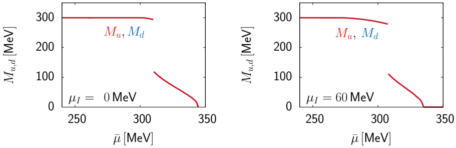

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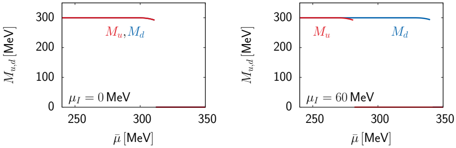

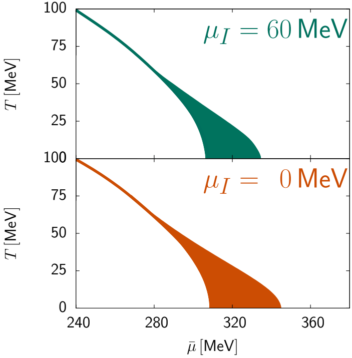

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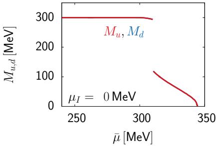

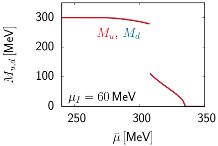

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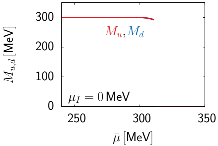
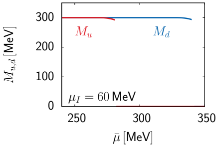
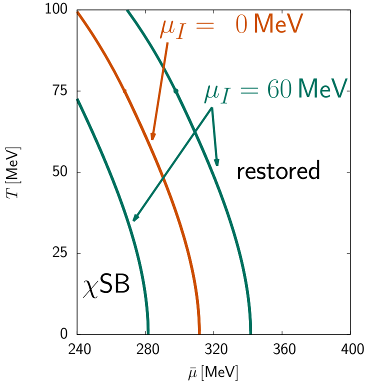
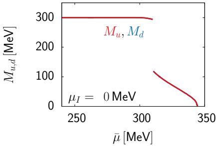


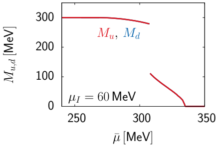




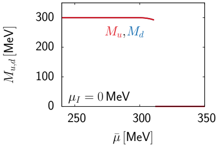
No flavor mixing: $\alpha = 0$
Phase structure for non-vanishing μ_I

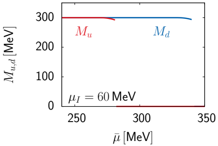
- maximally coupled quark flavors: $\alpha = 0.5$ ✓

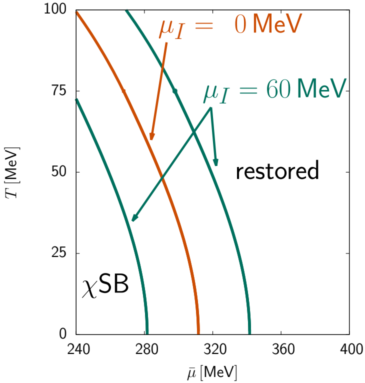




- uncoupled quark flavors: $\alpha = 0$







No flavor mixing: $\alpha = 0$
Inhomogeneous condensates

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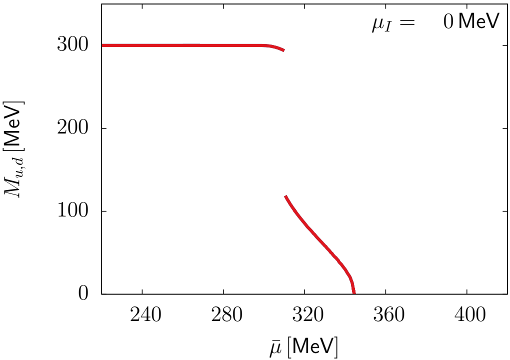
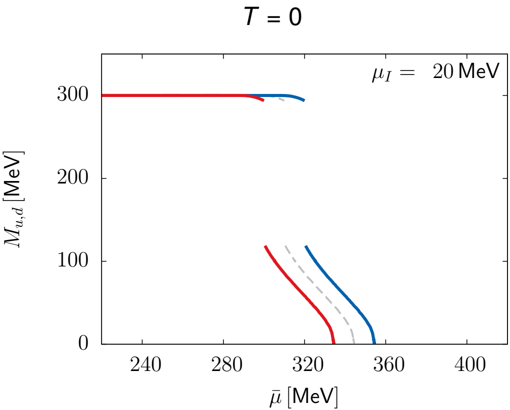
$T = 0$



No flavor mixing: $\alpha = 0$
Inhomogeneous condensates



No flavor mixing: $\alpha = 0$
Inhomogeneous condensates

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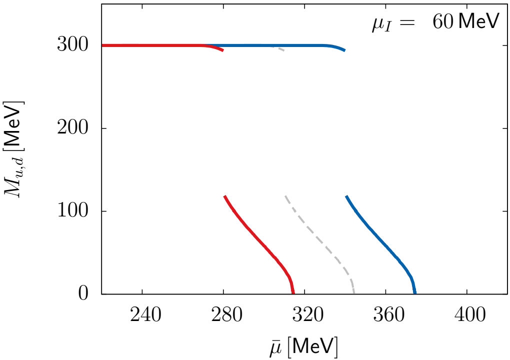

No flavor mixing: $\alpha = 0$
Inhomogeneous condensates

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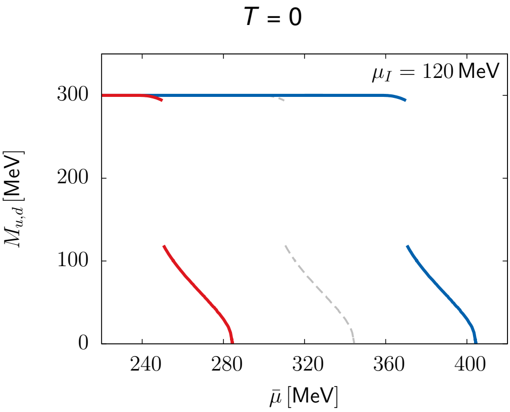
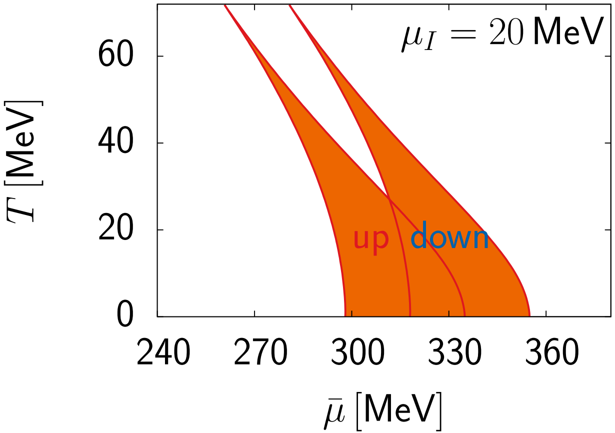


No flavor mixing: $\alpha = 0$
Inhomogeneous condensates



No flavor mixing: $\alpha = 0$
Inhomogeneous condensates

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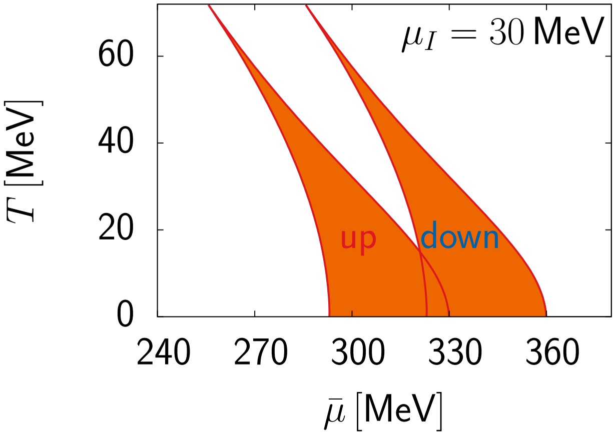

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Inhomogeneous condensates

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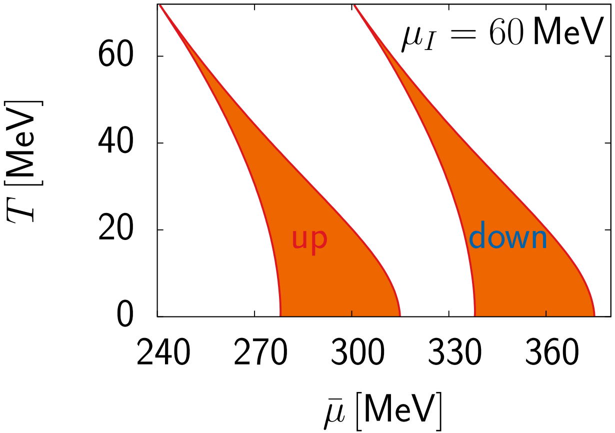

Summary & Outlook

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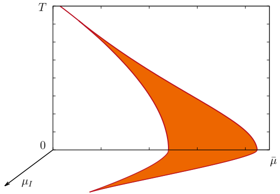
Maximally coupled quark flavors $\alpha = 0.5$

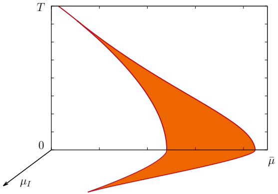


- inhomogeneous chiral symmetry breaking phases occur below $\mu_I < \mu_I^c$
- less sensitive to additional pairing stress if not limited to equal periodicities

Summary & Outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

Maximally coupled quark flavors α = 0.5



- inhomogeneous chiral symmetry breaking phases occur below $\mu_I < \mu_I^c$
- less sensitive to additional pairing stress if not limited to equal periodicities

Uncoupled quark flavors α = 0.0

- formalism for inhomogeneous phases extended to generalized four-fermion interaction
- inhomogeneous chiral symmetry breaking phases still occur

Outlook

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- neutron star matter $\rightsquigarrow$ electric charge neutrality and β-equilibrium
- two-dimensional lattice where up and down quark condensates vary independently in different directions?
- consider more interactions, e. g.
  - vector interaction $\rightsquigarrow$ talk by M. Schramm on monday
  - inhomogeneous charged pion condensation or color superconductivity