Fluctuations of conserved charges: lattice meets experiment

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S. Borsanyi, Z. Fodor, S. Katz, S. Krieg, C. R., K. Szabo, arXiv:1403.4576

Motivation

- We live in a very exciting era to understand the fundamental constituents of matter and the evolution of the Universe
- We can create the deconfined phase of QCD in the laboratory
- Lattice QCD simulations have reached unprecedented levels of accuracy
    - physical quark masses
    - several lattice spacings $\longrightarrow$ continuum limit
- The joint information between theory and experiment can help us to shed light on QCD

Susceptibilities of conserved charges

- The deconfined phase of QCD can be reached in the laboratory
- Need for unambiguous observables to identify the phase transition
  - susceptibilities of conserved charges (baryon number, electric charge, strangeness)

    S. Jeon and V. Koch (2000), M. Asakawa, U. Heinz, B. Müller (2000)
- A rapid change of these observables in the vicinity of T_c provides an unambiguous signal for deconfinement
- They can be calculated on the lattice as combinations of quark number susceptibilities
- They can be directly compared to experimental measurements

The observables under study

- The chemical potentials are related:

$$\begin{aligned}\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q;\\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q;\\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.\end{aligned}$$

- susceptibilities are defined as follows:

$$\chi^{BSQ}_{lmn} = \frac{\partial^{\,l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Quadratic susceptibilities:

$$\chi^X_2 = \frac{1}{VT^3}\langle N^2_X\rangle$$

- Correlators between different charges:

$$\chi^{XY}_{11} = \frac{1}{VT^3}\langle N_X N_Y\rangle.$$

Physical meaning

- Diagonal susceptibilities measure the response of quark densities to an infinitesimal change in the chemical potential

$$\chi_2^X = \frac{\partial^2 p/T^4}{\partial(\mu_X/T)^2} = \frac{\partial}{\partial(\mu_X/T)}\left(n_X/T^3\right)$$

  - ➠ A rapid increase of these observables in a certain temperature range signals a phase transition

- Non-diagonal susceptibilities measure the correlation between different quark flavors

$$\chi_{11}^{XY} = \frac{\partial^2 p/T^4}{\partial(\mu_X/T)\partial(\mu_Y/T)} = \frac{\partial}{\partial(\mu_Y/T)}\left(n_X/T^3\right)$$

  - ➠ They can provide information about bound-state survival above the phase transition

Relating lattice results to experimental measurement

- the first four cumulants are:

$$\chi_1 = \langle(\delta x)\rangle \qquad \chi_2 = \langle(\delta x)^2\rangle$$

$$\chi_3 = \langle(\delta x)^3\rangle \qquad \chi_4 = \langle(\delta x)^4\rangle - 3\langle(\delta x)^4\rangle^2$$

- we can relate them to higher moments of multiplicity distributions:

$$\text{mean}: \ M = \chi_1 \qquad \text{variance}: \ \sigma^2 = \chi_2$$

$$\text{skewness}: \ S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \ \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch (2012)

Caveats

- Effects due to volume variation because of finite centrality bin width V. Skokov, B. Friman, K. Redlich, PRC (2013)
- Finite reconstruction efficiency
- Spallation protons
- Canonical vs Gran Canonical ensemble
- Proton multiplicity distributions vs baryon number fluctuations
- Final-state interactions in the hadronic phase J.Steinheimer *et al.*, PRL (2013)

Caveats

- Effects due to volume variation because of finite centrality bin width V. Skokov, B. Friman, K. Redlich, PRC (2013)
  - Experimentally corrected by centrality-bin-width correction method
- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A. Bzdak, V. Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Numerically very similar once protons are properly treated M. Asakawa and M. Kitazawa, PRC (2012), M. Nahrgang *et al.*, 1402.1238 See talk by Paolo Alba this afternoon
- Final-state interactions in the hadronic phase J.Steinheimer *et al.*, PRL (2013)
  - Consistency between different charges = fundamental test

Relations between chemical potentials

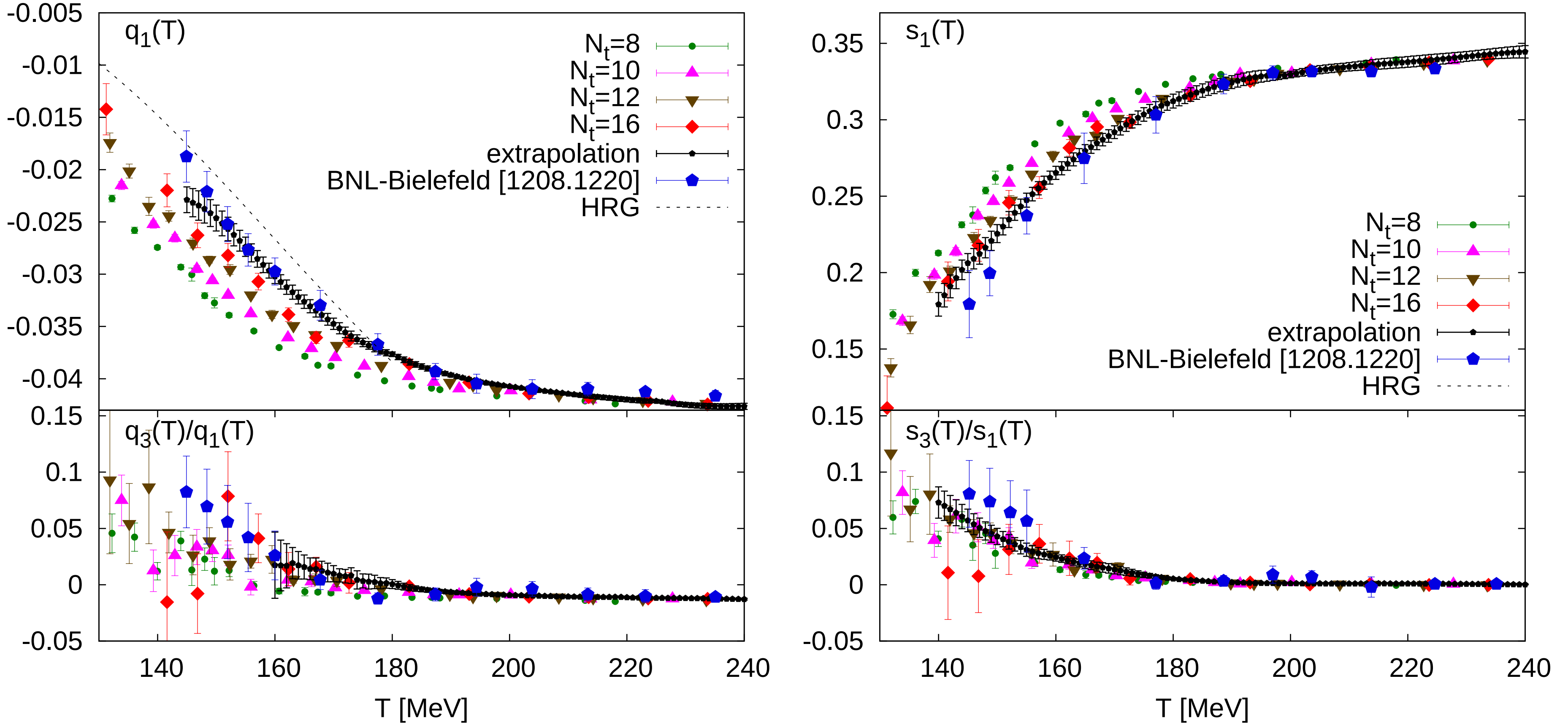
- μ_B, μ_S and μ_Q are NOT independent:

$$\langle n_S\rangle = 0 \qquad \langle n_Q\rangle = \frac{Z}{A}\langle n_B\rangle \qquad \Rightarrow \qquad \frac{Z}{A} = 0.4$$

- By expanding n_B, n_S and n_Q up to μ_B^3 we get:

$$\begin{aligned} \mu_Q(T,\mu_B) &= q_1(T)\mu_B + q_3(T)\mu_B^3 + ... \\ \mu_S(T,\mu_B) &= s_1(T)\mu_B + s_3(T)\mu_B^3 + ... \end{aligned}$$

Taylor coefficients: results



WB Collaboration: PRL (2013)

- μ_Q turns out to be very small
- Agreement between WB and BNL-Bielefeld collaborations

Thermometer and Baryometer

- R^B_{31}: thermometer

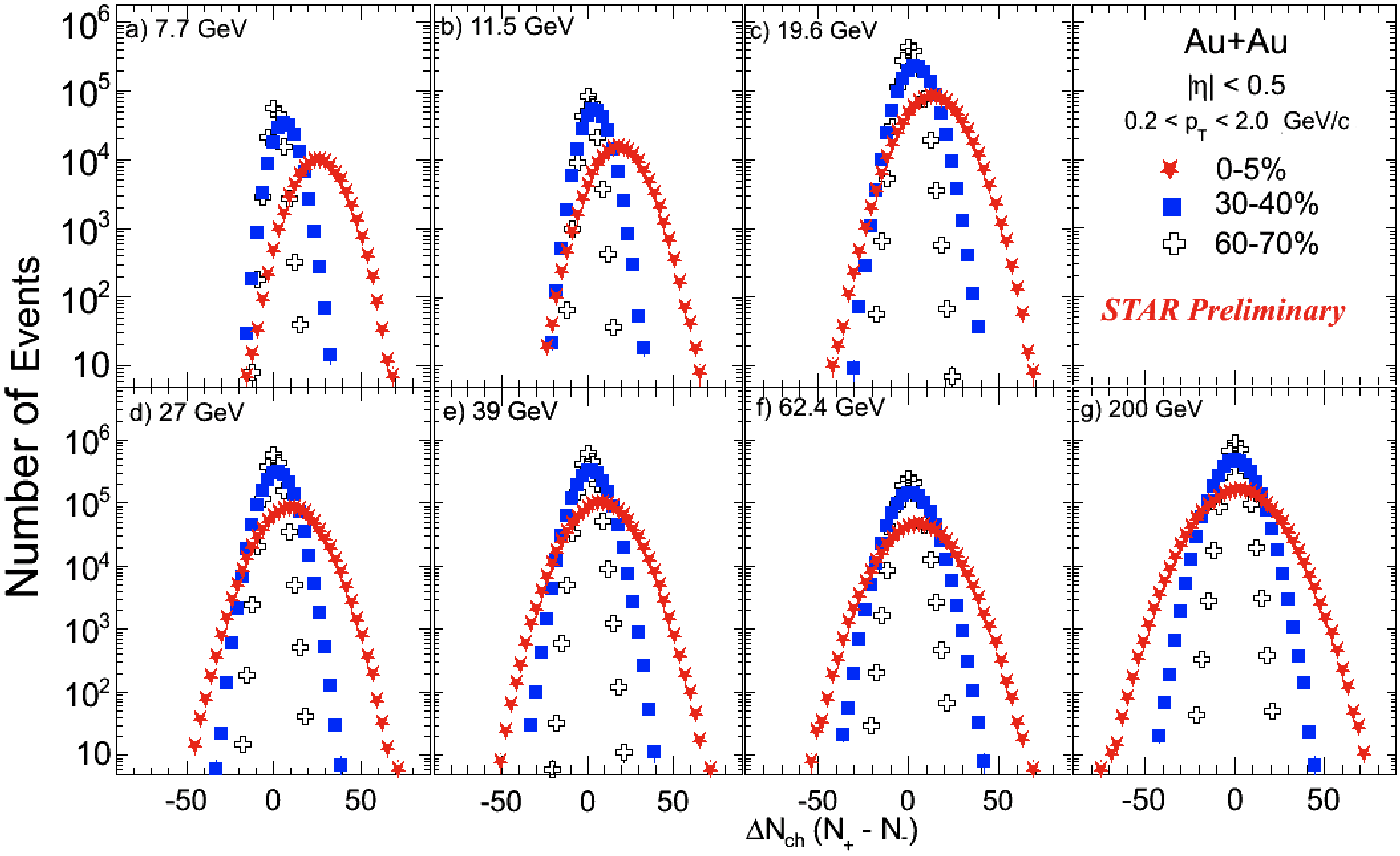
$$R^B_{31}(T,\mu_B)=\frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)}=\frac{\chi^B_4(T,0)+\chi^{BQ}_{31}(T,0)q_1(T)+\chi^{BS}_{31}(T,0)s_1(T)}{\chi^B_2(T,0)+\chi^{BQ}_{11}(T,0)q_1(T)+\chi^{BS}_{11}(T,0)s_1(T)}+\mathcal{O}(\mu_B^2)$$

- Expand numerator and denominator around $\mu_B=0$: ratio is independent of μ_B
- R^B_{12}: baryometer

$$R^B_{12}(T,\mu_B)=\frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)}=\frac{\chi^B_2(T,0)+\chi^{BQ}_{11}(T,0)q_1(T)+\chi^{BS}_{11}(T,0)s_1(T)}{\chi^B_2(T,0)}\frac{\mu_B}{T}+\mathcal{O}(\mu_B^3)$$

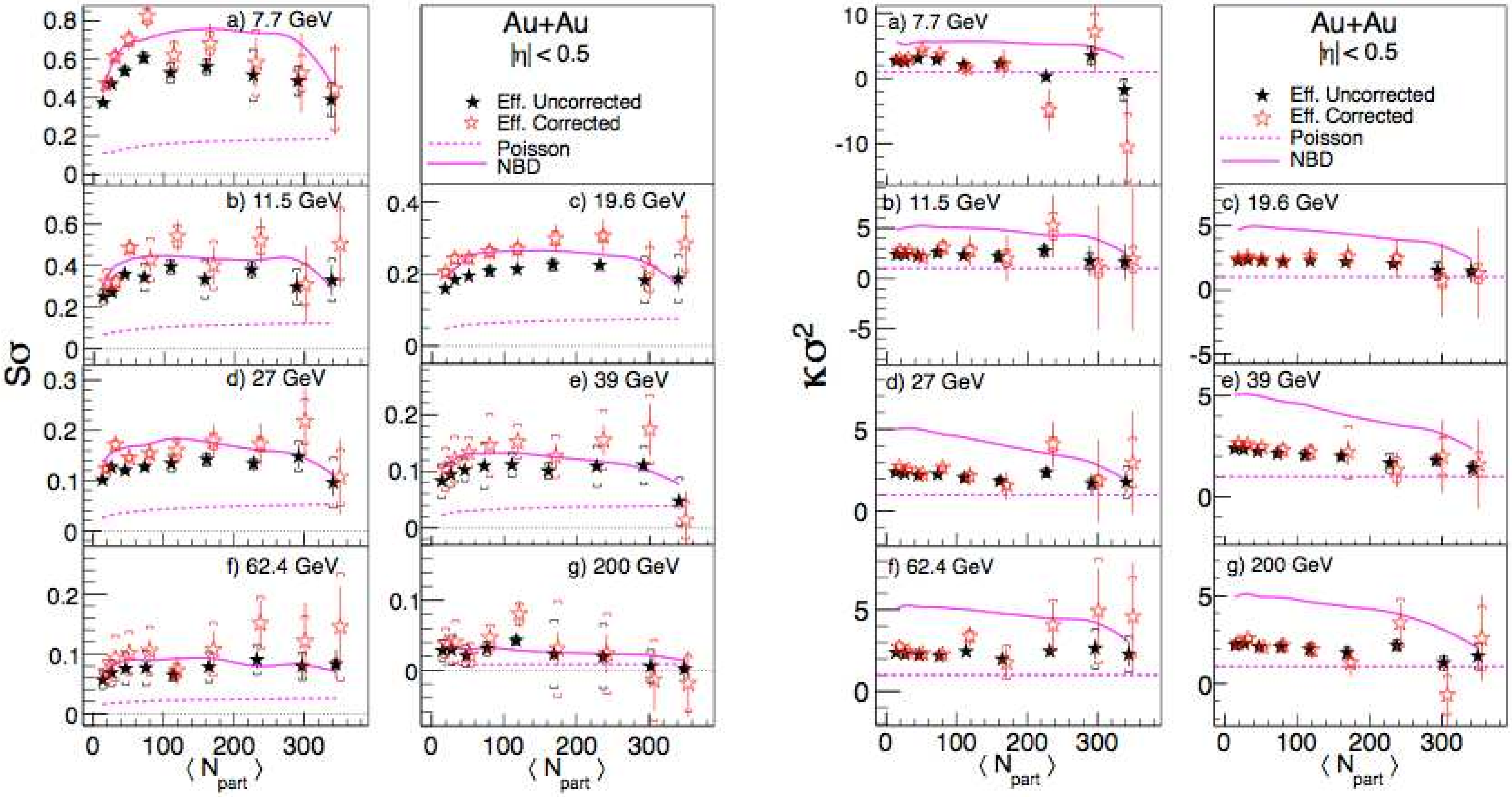
- Expand numerator and denominator around $\mu_B=0$: ratio is proportional to μ_B

Experimental measurement I



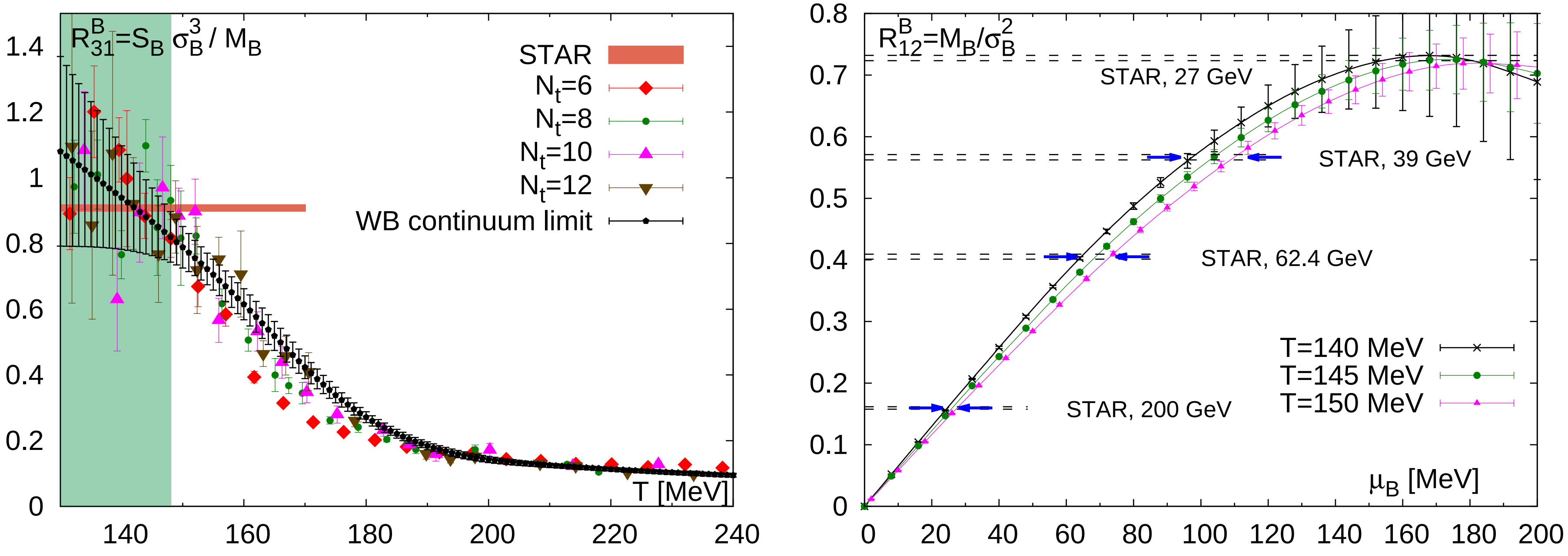
Star Collaboration: arXiv 1212.3892

Experimental measurement II



Star Collaboration: arXiv 1402.1558

Extracting freeze-out parameters from baryon number

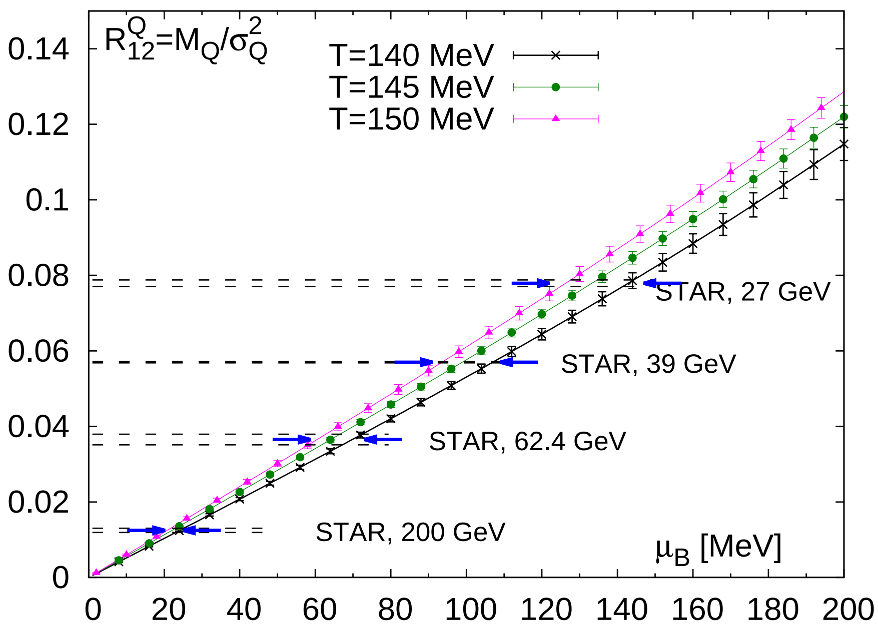


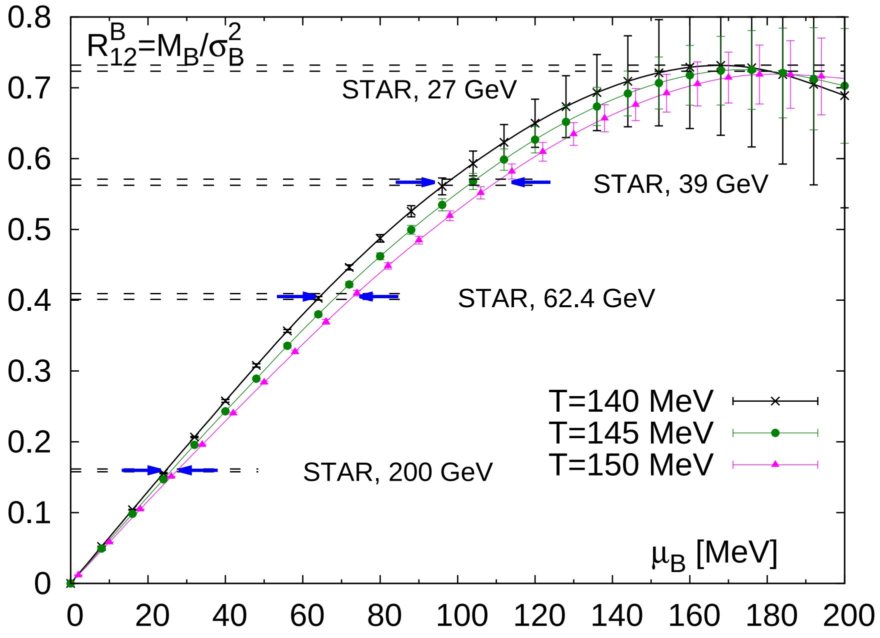
WB Collaboration: arXiv 1403.4576; STAR data from 1309.5681

- Upper limit: $T_f \leq 148 \pm 4$ MeV

$\sqrt{s}[GeV]$	μ_B^f [MeV]
200	25.6±2.4
62.4	69±5.7
39	104±10
27	-

Extracting freeze-out parameters from electric charge





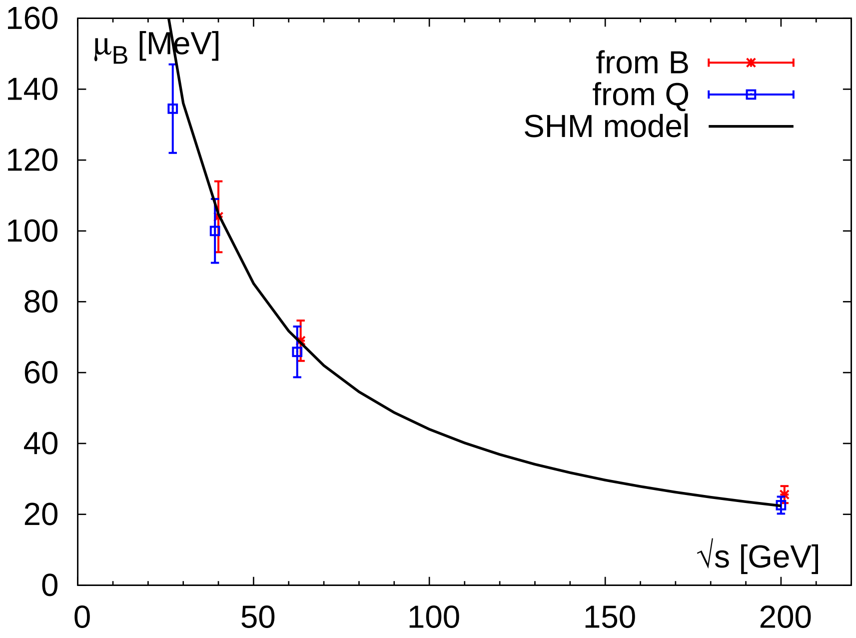
WB Collaboration: arXiv 1403.4576; STAR data from 1309.5681 and 1402.1558

- It is of fundamental importance to test the consistency between the freeze-out parameters obtained with different conserved charges

- This consistency check validates the method and shows equilibration of the medium

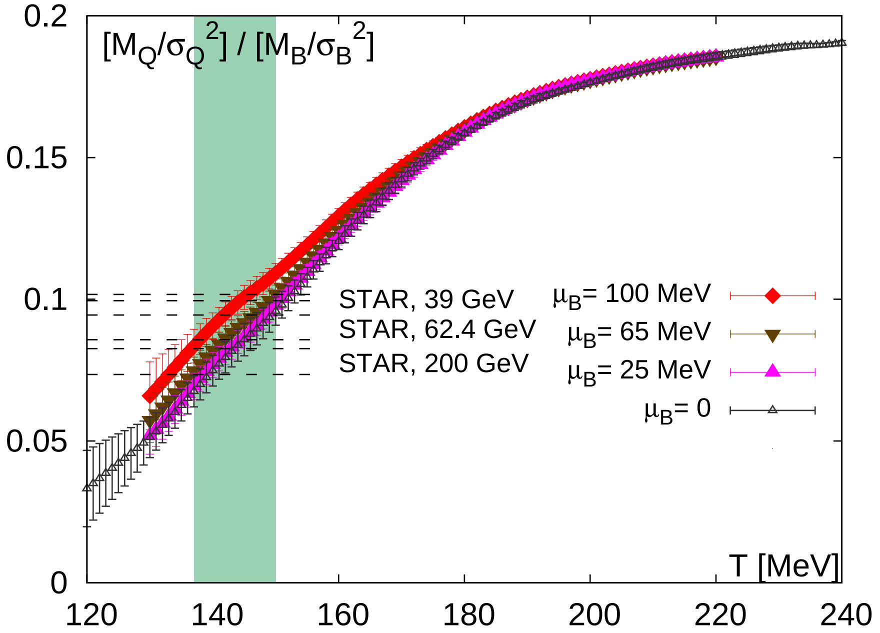
Consistency is found!

$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.6±2.4	22.6±2.4
62.4	69±5.7	65.9±7.2
39	104±10	100±9
27	-	134.5±12.5



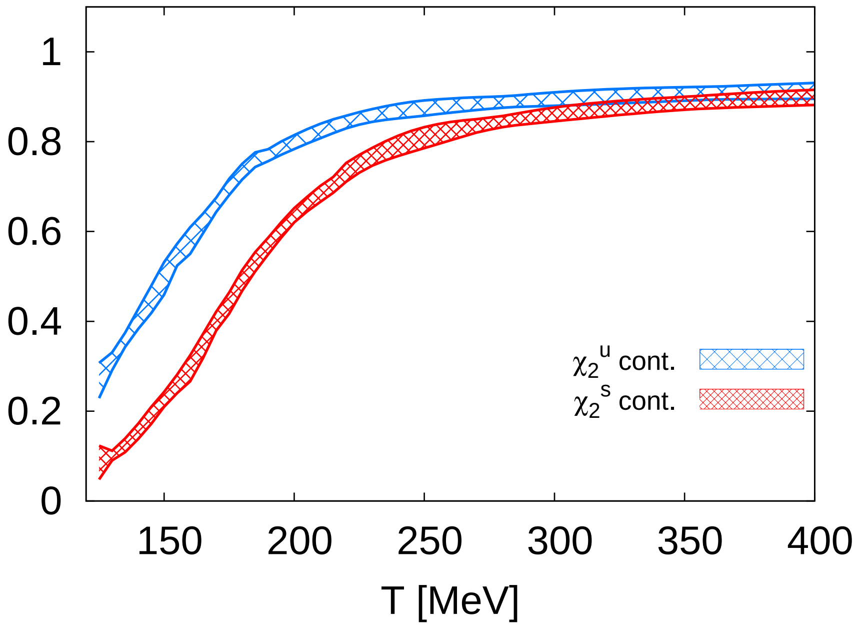
Lattice: WB Collaboration: arXiv 1403.4576; SHM: Andronic *et al.*, NPA (2006)

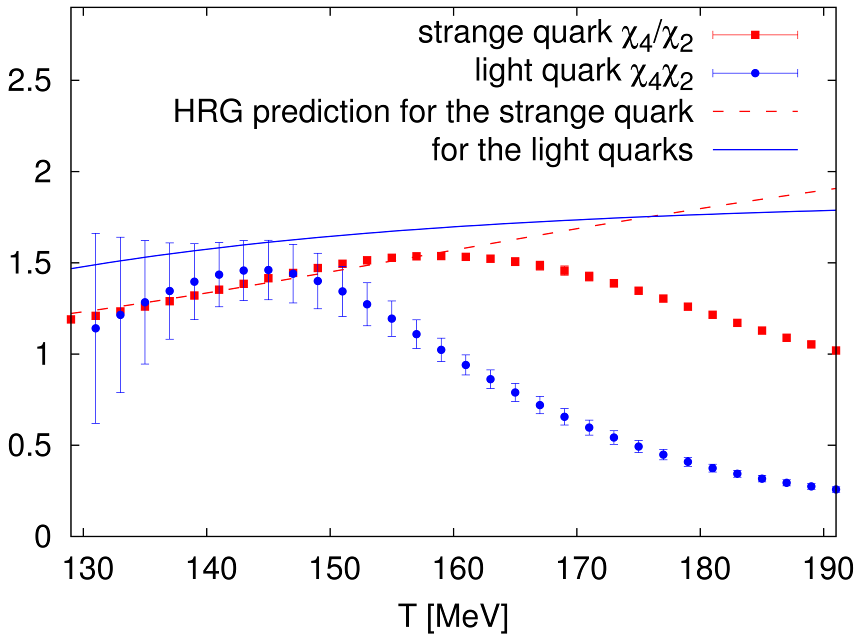
Ratio of ratios



$$R_{12}^Q/R_{12}^B = [\chi_1^Q/\chi_2^Q]/[\chi_1^B/\chi_2^B] = [M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$$

Strange vs light thermometer





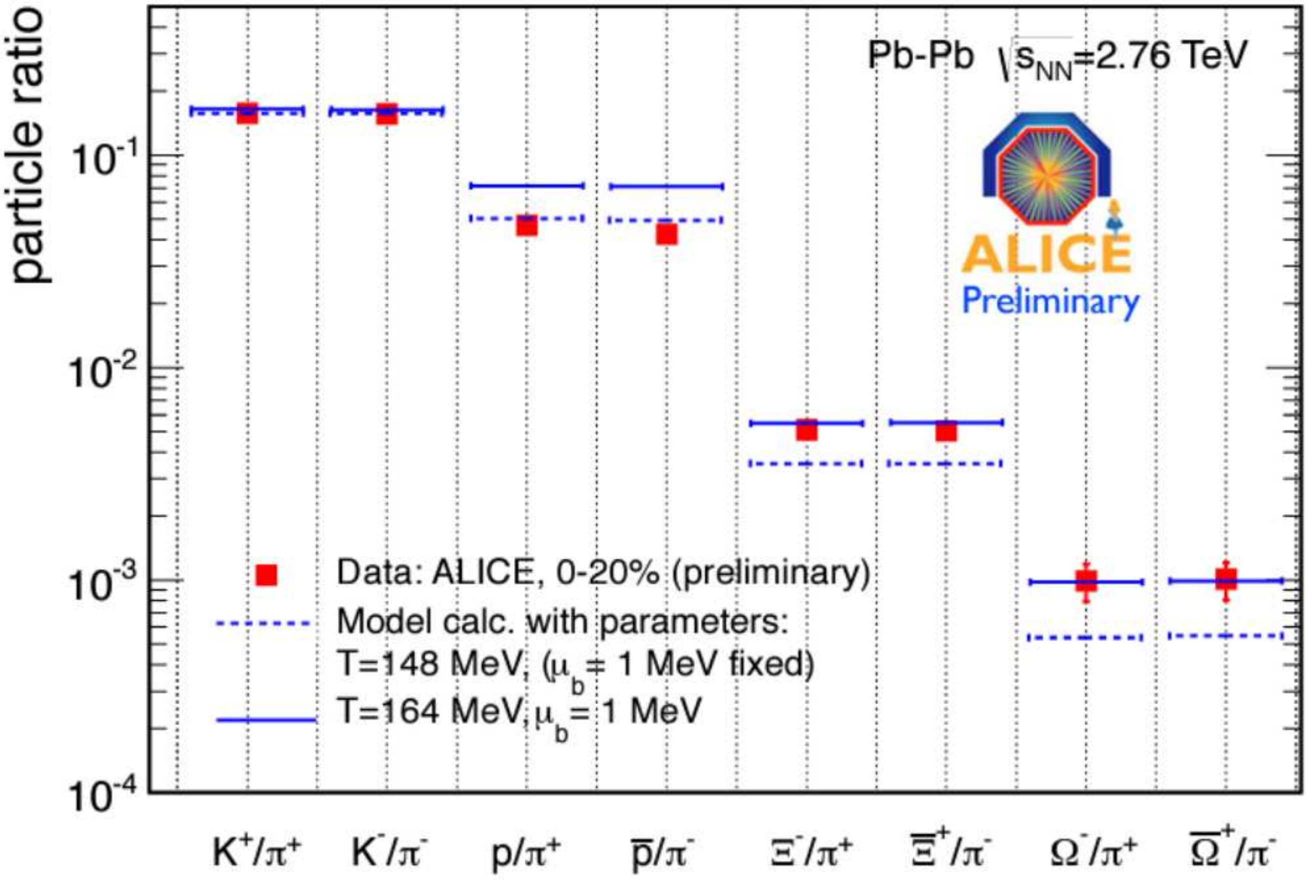
S. Borsanyi *et al.*: JHEP (2012); R. Bellwied *et al.*: PRL (2013)

- Flavor-specific fluctuations show separation between light and strange quarks
- Does it mean that light and strange quarks have different freeze-out temperatures?

See talk by Valentina Mantovani Sarti tomorrow afternoon

Freeze-out temperature from experiment

- Fit to yields of identified particles: Statistical Hadronization Model (SHM)
- Model-dependent. Parameters: freeze-out temperature and chemical potential



R. Preghenella
for ALICE
SQM 2012
arXiv:1111.7080
Acta Phys. Pol.

Conclusions

- It is possible to extract freeze-out parameters from first principles
- Higher order fluctuations of baryon number:
  - $R^B_{31}(T,\mu_B) = \frac{\chi^B_3(T,\mu_B)}{\chi^B_1(T,\mu_B)}$: Thermometer
  - $R^B_{12}(T,\mu_B) = \frac{\chi^B_1(T,\mu_B)}{\chi^B_2(T,\mu_B)}$: Baryometer
- Higher order fluctuations of electric charge:
  - independent measurement
  - $R^Q_{12}(T,\mu_B) = \frac{\chi^Q_1(T,\mu_B)}{\chi^Q_2(T,\mu_B)}$: Baryometer
- The freeze-out parameter sets obtained from B and Q are consistent with each other
- Looking forward to strangeness fluctuation data!