Separation of Global Collective and Fluctuating Flow Patterns

3rd International Symposium on Non-equilibrium Dynamics

and 3rd TURIC Network Workshop,

9 - 14 June 2014, Hersonissos,

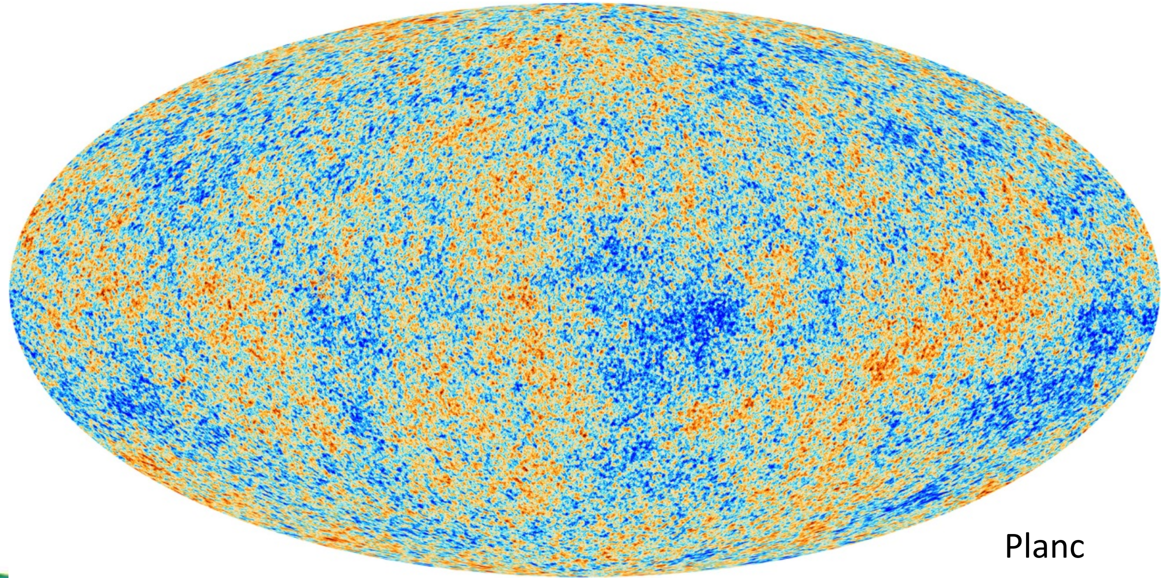
Crete, Greece,

L.P. Csernai and H. Stöcker

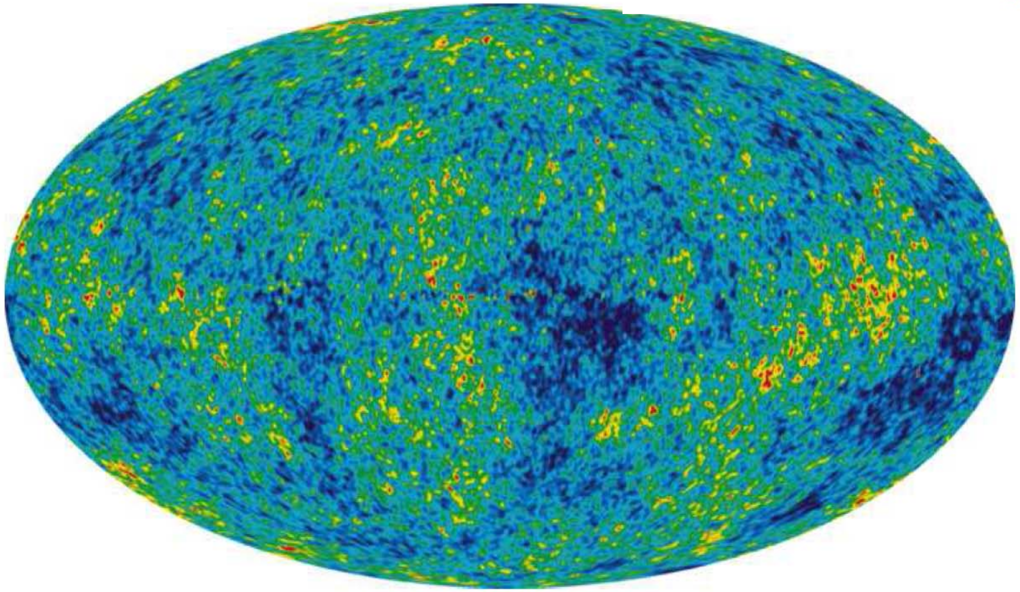
arXiv: 1406.1153 [nucl-th] → J. Phys. G

Outline

- Initial state: Central / Peripheral collision
- Symmetries: Initial State → Collective Flow
- How to split Collective flow & Fluctuations
- When Collective Flow identified: ***New patterns***
- Small viscosity (→ fluctuations & instabilities)
- Rotation
- Kelvin-Helmholtz Instability (KHI) ~ turbulence
- Observation of these



Planc



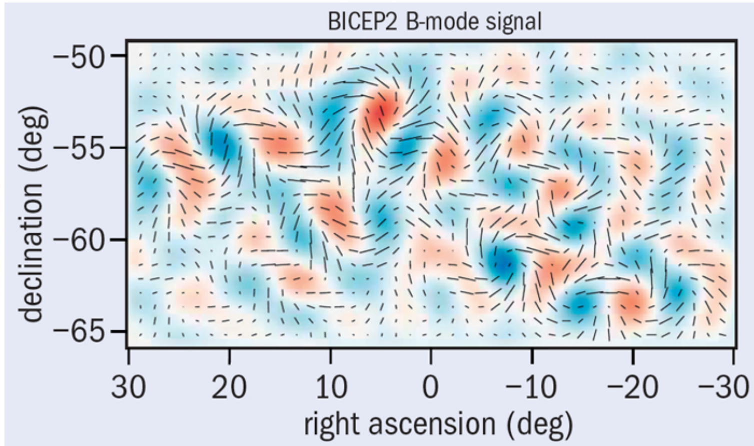
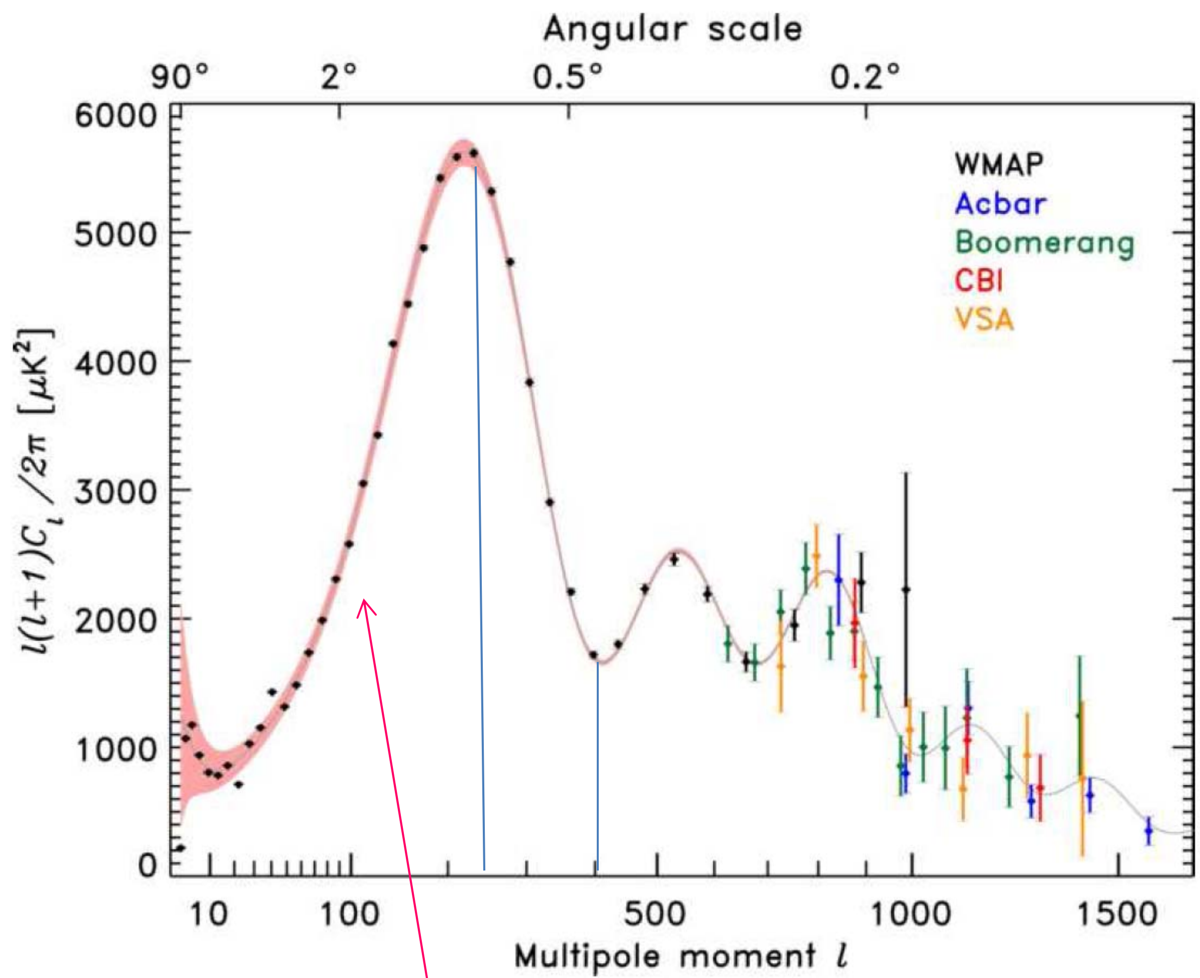


Figure 32: The CMB radiation temperature fluctuations from the 5-year WMAP data seen over the full sky. The average temperature is 2.725K, and the colors represents small temperature fluctuations. Red regions are warmer, and blue colder by about 0.0002 K.

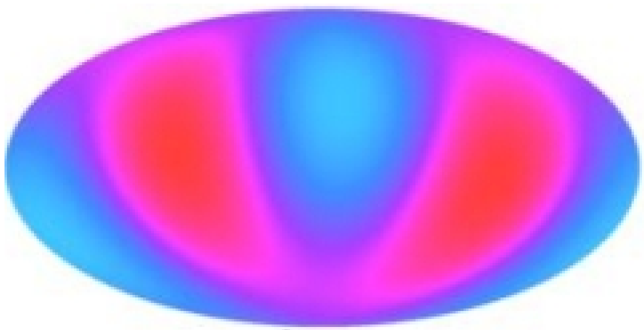


Longer tail on the negative (low ℓ) side ! (see discussion of "Skewness" later)

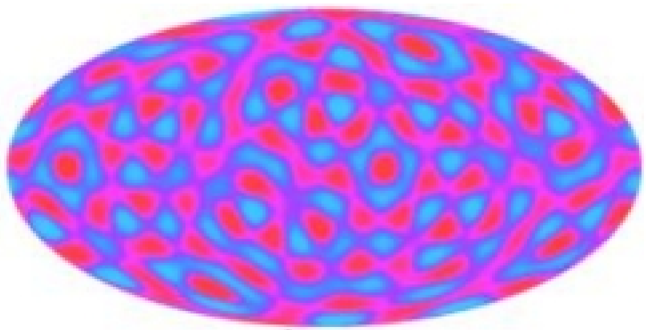
In Central Heavy Ion Collisions

~ like Elliptic flow, v_2

~ spherical with many (16) nearly equal perturbations



$\ell = 2$



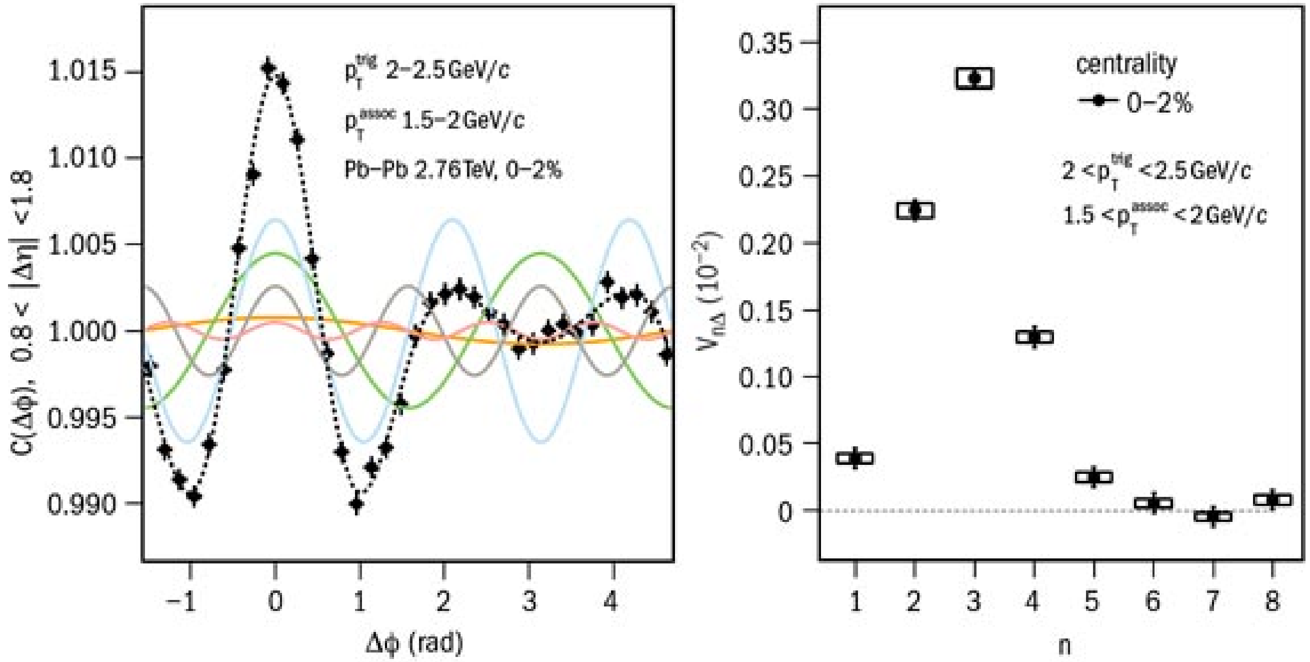
$\ell = 16$

CERN COURIER

Sep 23, 2011

ALICE measures the shape of head-on lead-lead collisions

Oct. 2011, p. 6



Flow originating from initial state fluctuations is significant and dominant in central and semi-central collisions (where from global symmetry no azimuthal asymmetry could occur, all Collective $v_n = 0$) !

Critical Fluctuations →

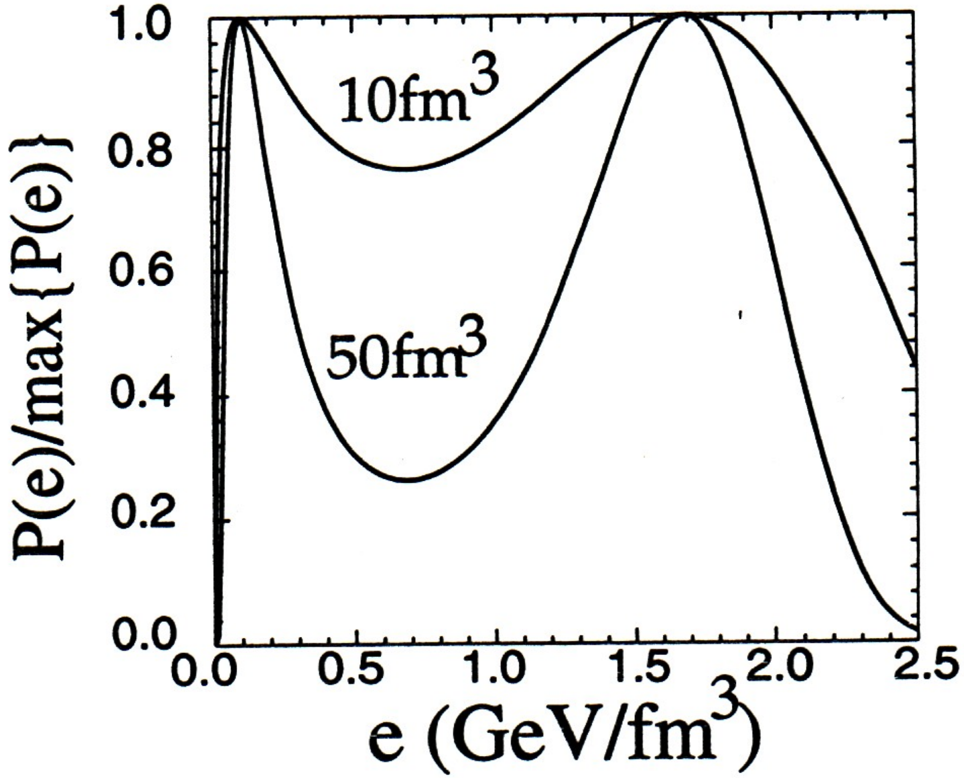


Fig. 2. *The relative probability of finding a state of a given energy density, e, in a system of given volume,* $\Omega = 10, 50\ fm^3$, *at a constant temperature,* $T = T_c$.

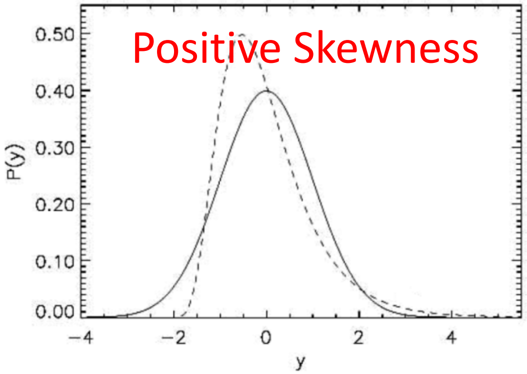
L.P. Csernai: Inroduction to Relativistic Heavy IonCollisions (Wiley, 1994)

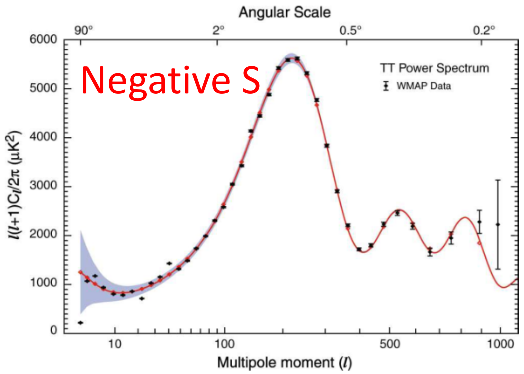
Fluctuations in Hadronizing QGP

L.P. Csernai[1,2], G. Mocanu[3] and Z. Néda[3]

PHYSICAL REVIEW C **85**, 068201 (2012)

Higher order moments can be obtained from fluctuations around the critical point. → Skewness and Kurtosis are calculated for the QGP → HM phase transition





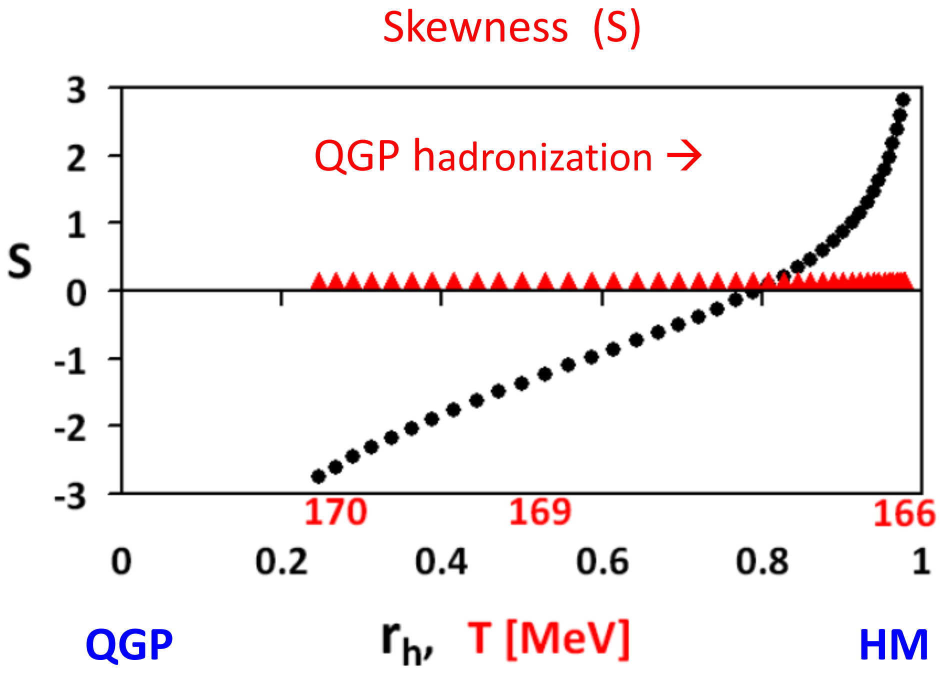
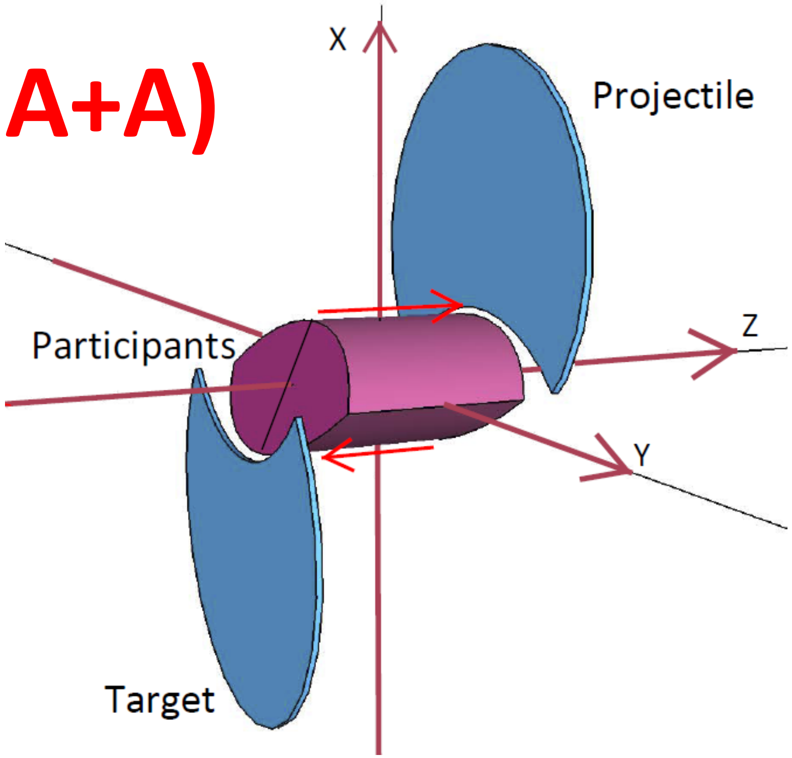


FIG. 4: (color online) Skewness as a function of the volume abundance of the hadronic matter (denoted as r_h, where 1 represents complete hadronization). The temperature scale is also indicated for clarity, the identifiers represent increments of 0.1 MeV in T. Results for $\Omega = 500 fm^3$.

Negative Skewness indicates Freeze-out mainly still on the QGP side.

Peripheral Collisions (A+A)



- Global Symmetries
- Symmetry axes in the global CM-frame:
  - (y ←→ -y)
  - (x,z ←→ -x,-z)
  - Azimuthal symmetry: ϕ-even (cos nϕ)
  - Longitudinal z-odd, (rap.-odd) for $v_{\_odd}$
  - Spherical or ellipsoidal flow, expansion

$$\frac{d^3N}{dy\,dp_t\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi) + 2v_2(y,p_t)\cos(2\phi) + \cdots\right]$$

$$\frac{d^3N}{dy\,dp_t\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,dp_t}\left[1 + 2v_1(y-y_{CM},p_t)\cos(\phi-\Psi_{RP}) + 2v_2(y-y_{CM},p_t)\cos(2(\phi-\Psi_{RP})) + \cdots\right]$$

- Fluctuations
- Global flow and Fluctuations are simultaneously present → ∃ interference
  - Azimuth - Global: even harmonics - Fluctuations : odd & even harmonics
  - Longitudinal – Global: v1, v3 y-odd - Fluctuations : odd & even harmonics
  - The separation of Global & Fluctuating flow is a must !! (not done yet)

Method to compensate for C.M. rapidity fluctuations

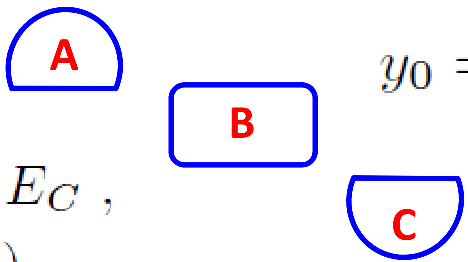
1. Determining experimentally EbE the C.M. rapidity
2. Shifting each event to its own C.M. and evaluate flow-harmonics there

L.P. Csernai[1,2], G. Eyyubova[3] and V.K. Magas[4]

PHYSICAL REVIEW C 86, 024912 (2012)

Determining the C.M. rapidity

The rapidity acceptance of a central TPC is usually constrained (e.g for ALICE $|\eta| < \eta_{lim} = 0.8$, and so: $|\eta_{C.M.}| << \eta_{lim}$, so it is not adequate for determining the C.M. rapidity of participants.



$$y_0 = 7.986$$

Participant rapidity from spectators

$$E_B = A_B\ m_{B\perp}\ \cosh(y^B) = E_{tot} - E_A - E_C\ ,$$
$$M_B = A_B\ m_{B\perp}\ \sinh(y^B) = -(M_A + M_C)$$

$$E_{tot} = 2A_{Pb}\, m_N\ \cosh(y_0)$$

$$E_A = A_P\, m_N\ \cosh(y_0),$$
$$E_C = A_T\, m_N\ \cosh(-y_0),$$

give the spectator numbers, A_P and A_T,

$$M_A = A_P\, m_N\ \sinh(y_0),$$
$$M_C = A_T\, m_N\ \sinh(-y_0),$$

$$y_E^{CM} \approx y^B = \text{artanh}\left(\frac{-(M_A + M_C)}{E_{tot} - E_A - E_C}\right)$$

Azimuthal Flow analysis with Fluctuations today

In contrast to the above formulation

$$\frac{d^3N}{dy\,dp_t\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi - \Psi_1^{EP}) + 2v_2(y,p_t)\cos(2(\phi - \Psi_2^{EP})) + \cdots\right],$$

Here Ψ_n^{EP} maximizes $v_n(y,p_t)$ in a rapidity range

Is this a complete ortho-normal series? Yes, if the Ψ_n^{EP} values are defined …..
We can see this by using: $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$, →

terms of the harmonic expansion

$$v_n\cos[n(\phi - \Psi_n^{EP})] = \underline{v_n\cos(n\Psi_n^{EP})}\;\cos(n\phi) + \underline{v_n\sin(n\Psi_n^{EP})}\;\sin(n\phi)$$

$$\Phi_n^{EP} \equiv \Psi_n^{EP} - \Psi_{RP}$$
$$\phi' \equiv \phi - \Psi_{RP}$$

Reaction Plane (EbE)

And the two coefficients:

$$^{c}v'_n \equiv v_n\cos(n(\Psi_n^{EP})) \qquad {}^{c}v'_n = {}^{c}v'_n(y - y_{CM}, p_t)$$
$$^{s}v'_n \equiv v_n\sin(n(\Psi_n^{EP})) \qquad {}^{s}v'_n = {}^{s}v'_n(y - y_{CM}, p_t)$$

→ terms of the harmonic expansion

$$v_n\cos[n(\phi-\Psi_n^{EP})] = v_n\cos[n(\phi'-\Phi_n^{EP})] = {}^{c}v'_n\cos(n\phi') + {}^{s}v'_n\sin(n\phi').$$

In Collider In EbE: CM,RP In EbE: CM,RP

Now: Separating Global Collective Flow & Fluctuations

the Global Collective flow in the configuration space has to be $\pm y$ symmetric

the coefficients of the $\sin(n\phi')$ terms should vanish: ${}^{s}v'_n = 0$

${}^{c}v'_n$ for odd harmonics have to be odd functions of $(y - y_{CM})$

for even harmonics have to be even functions of $(y - y_{CM})$

${}^{s}v'_n$ can be due to fluctuations only

Let us now introduce the rapidity variable $\mathbf{y} \equiv y - y_{CM}$

and let us construct even and odd combinations from the data:

$$v_{n\frac{even}{odd}}^{Coll.} \cos[n(\phi - \Psi_n^{EP})] = \frac{1}{2}\left[{}^{c}v'_n(\mathbf{y}, p_t) \pm {}^{c}v'_n(-\mathbf{y}, p_t)\right] \cos(n\phi')$$

$$v_{n\frac{even}{odd}}^{Fluct.} \cos[n(\phi - \Psi_n^{EP})] = \underbrace{\frac{1}{2}\left[{}^{c}v'_n(\mathbf{y}, p_t) \mp {}^{c}v'_n(-\mathbf{y}, p_t)\right]} \cos(n\phi') + \underbrace{{}^{s}v'_n(\mathbf{y}, p_t)} \sin(n\phi')$$

fluctuations must have the same magnitude for sine and cosine components

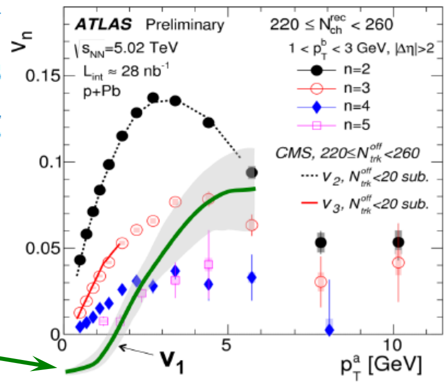
& for odd and even rapidity components.

Negative directed flow at low p_t [$v_1(p_t)$]

For Collective flow:
Due to softening of EoS at the QGP threshold $v_1(\mathbf{y})$ may become negative at low $\mathbf{y} > 0$.
Due to momentum conservation, and for $v_1(\mathbf{y})$ is odd , $\int dy\ v_1(\mathbf{y},p_t) = 0$ or $\langle v_1(p_t) \rangle = 0$
The Symmetrized $v_1^S(p_t)$ is usually still positive [Cs., Magas, Stöcker, Strottman, PRC84 (2011)]

In recent experiments:
Due to softening of EoS at the QGP threshold $v_1(\mathbf{y})$ may become negative at low $\mathbf{y} > 0$.
Due to momentum conservation, and for $v_1(\mathbf{y})$ is odd , $\int dy\ v_1(\mathbf{y},p_t) = 0$ or
The Symmetrized $v_1^S(p_t)$ is usually still positive [Cs., Magas, Stöcker, Strottman, PRC84 (2011)]

On the other hand, recent measurements yield negative $v_1^S(p_t)$ values at low rapidities, $p_t < 1.2 - 1.5$Gev/c [45, 46, 23]. The same is observed in model calculations both in fluid dynamics [47] and in molecular dynamics [43] with random fluctuating initial conditions. This is not unexpected.



See [Gyulassy et al., arXiv: 1405.7825]

There is a problem. In these works the participant C.M. was not identified. In this case adding up contributions with different C.M. points may lead to negative $v_1^S(p_t)$. See eqs. (2) & (3) of ref. [Cs., Magas, Stöcker, Strottman, PRC84 (2011)].

→ The Collective and Fluctuating flow effects interfere → **Identifying C.M. EbE**

Development of $v_1(y)$ at increasing beam energies

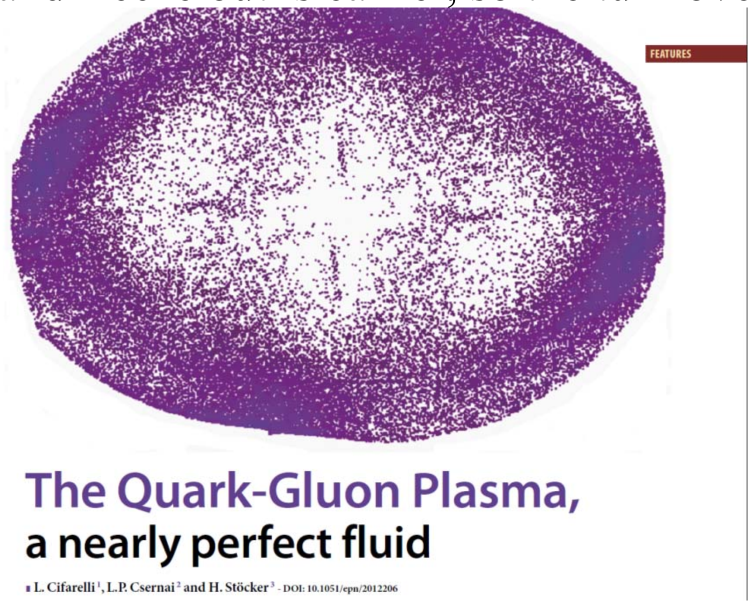
$v_1(y)$ observations show a central antiflow slope, $\partial v_1(y)/\partial y$, which is gradually decreasing with increasing beam energy [23]:

$$\frac{\partial v_1(y)_{odd}}{\partial y} = \begin{cases} -1.25\% & \text{for} \quad 62.4 \text{ GeV} \quad \text{(STAR)} \\ -0.41\% & \text{for} \quad 200.0 \text{ GeV} \quad \text{(STAR)} \\ -0.15\% & \text{for} \quad 2760.0 \text{ GeV} \quad \text{(ALICE)} \end{cases}$$

This can be attributed to smaller increase of p_t and the pressure, and the shorter interaction time, and **also to increasing rotation**.

In [Cs., Magas, Stöcker, Strottman, PRC84 (2011)] we predicted this rotation, but the turnover depends on the balance between rotation, expansion and freeze out. Apparently expansion is still faster and freeze out is earlier, so the turn over to the Positive side is not reached yet.

Interesting collective flow phenomena in low viscosity QGP →



PRL **97,** 152303 (2006) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2006

Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

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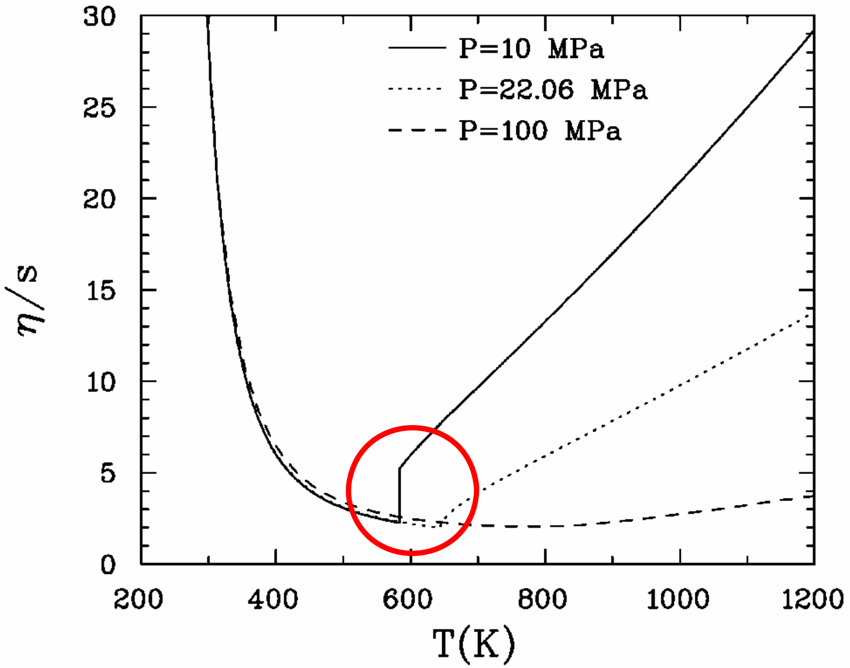
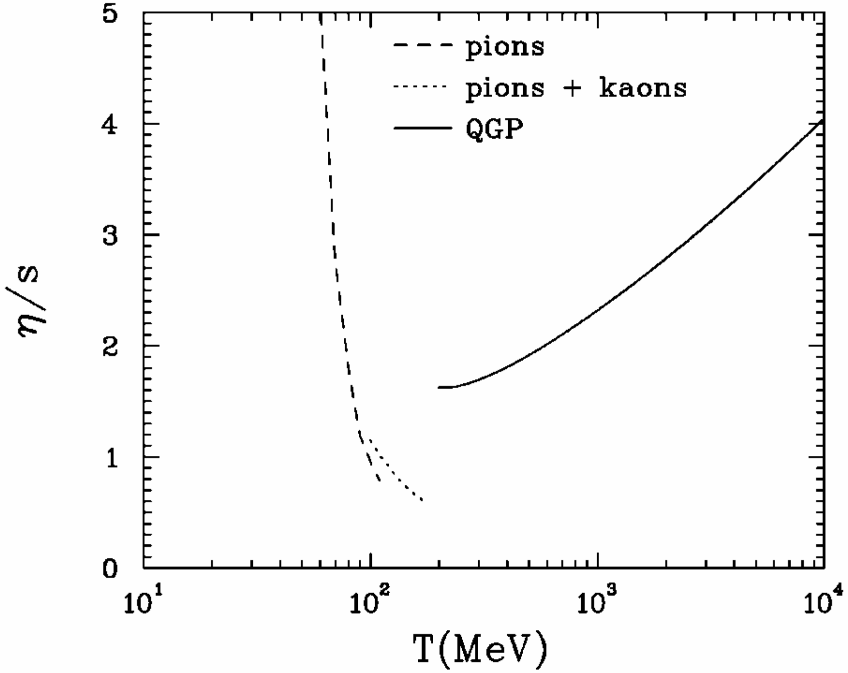
Viscosity vs. T has a minimum at the 1st order phase transition. This might signal the phase transition if viscosity is measured. At lower energies this was done.

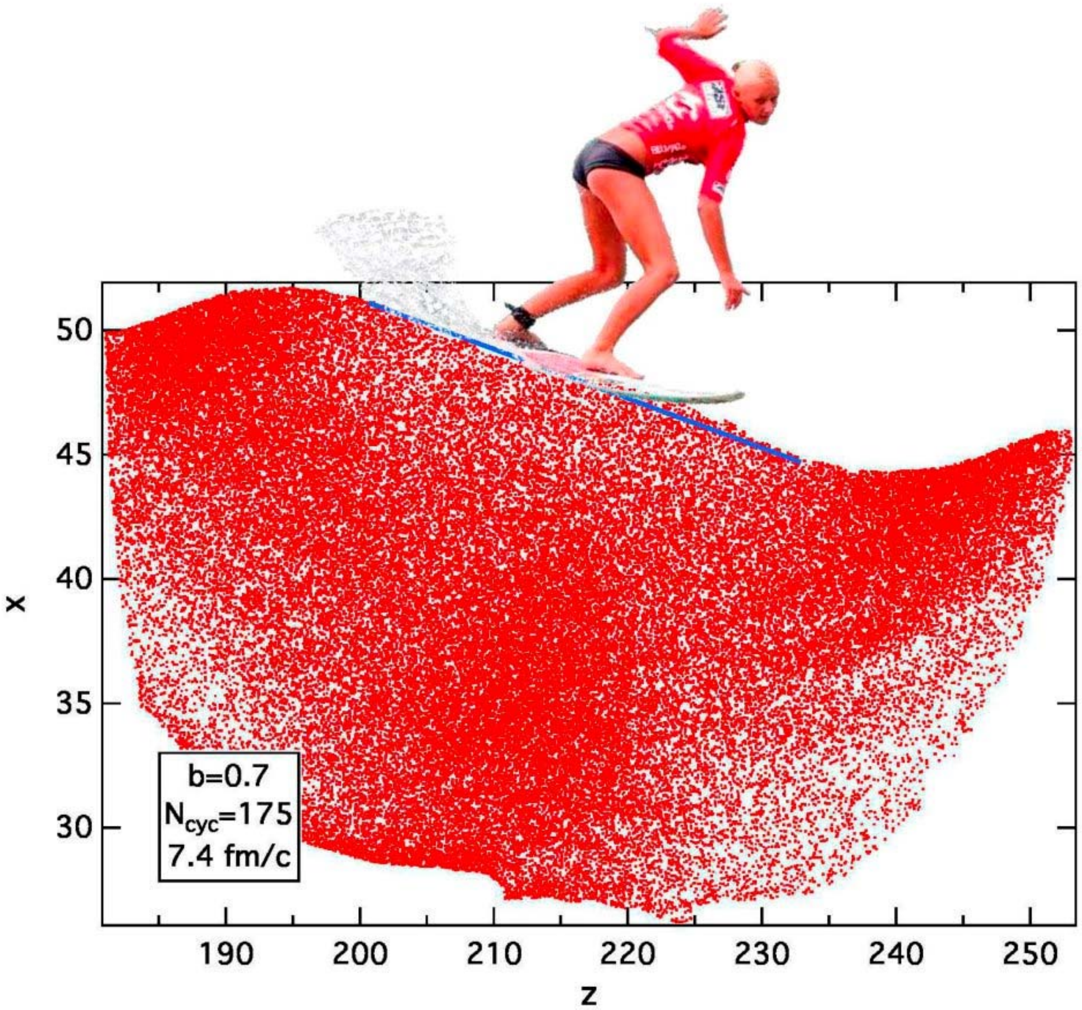
Water



QGP



Surfing on breaking waves of Quark-gluon Plasma



Kelvin-Helmholtz instability in high-energy heavy-ion collisions

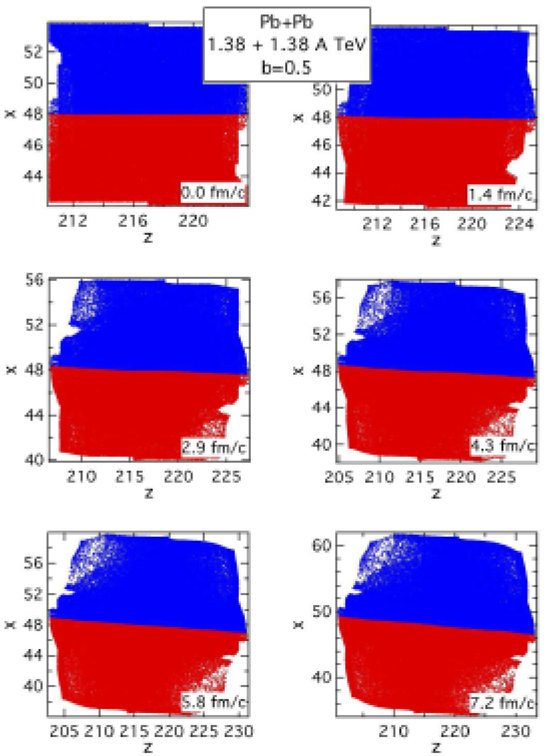
L.P. Csernai[1,2,3], D.D. Strottman[2,3], and Cs. Anderlik[4]

PHYSICAL REVIEW C **85**, 054901 (2012)

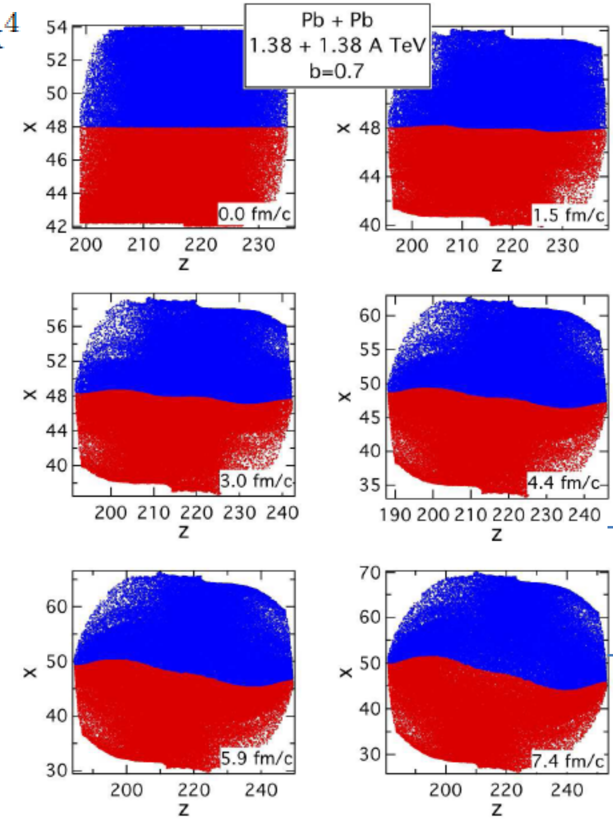
arXiv:1112.4287v3 [nucl-th]

KHI →

ROTATION



KHI



2.4 fm

FIG. 1: (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a $1.38A + 1.38A$ TeV peripheral, $b = 0.7 b_{\max}$, Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are $dx = dy = dz = 0.4375$fm and the time-step is 0.04233 fm/c The number of randomly placed marker particles in each fluid cell is 8^3. The axis-labels indicate the cell numbers in the x and z (beam) direction. The initial development of a KH type instability is visible from $t = 1.5$ up to $t = 7.41$ fm/c corresponding from 35 to 175 calculation time steps).

Classical

Relativistic

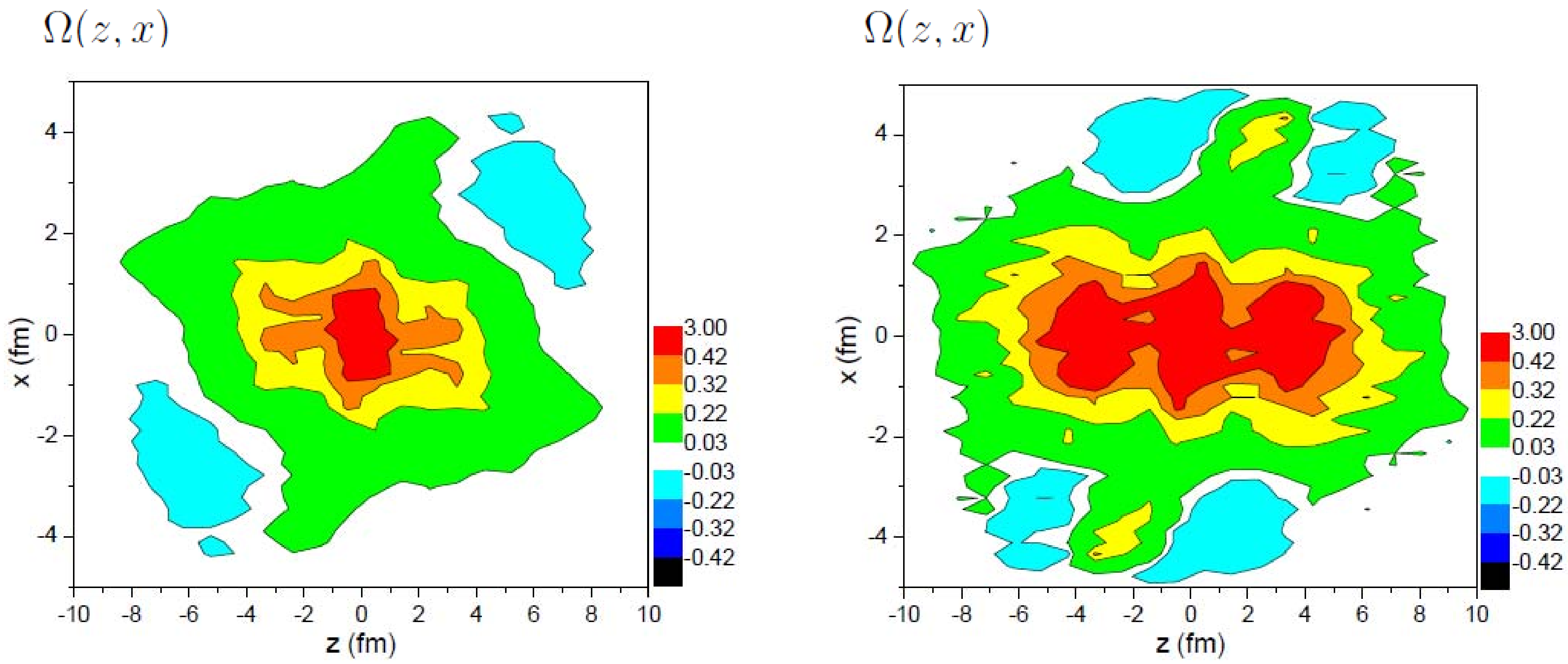
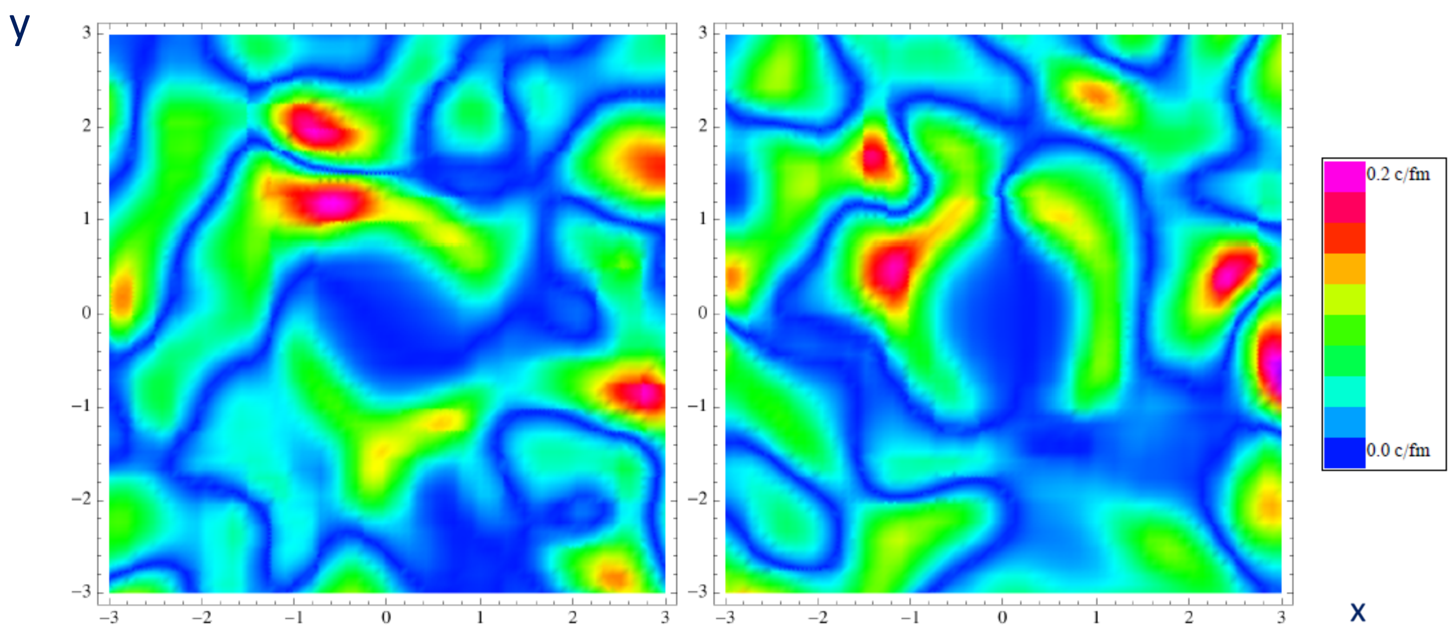


FIG. 5: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at t=3.56 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$. The average vorticity in the reaction plane is 0.0538 / 0.10685 for the classical / relativistic weighted vorticity respectively.

Onset of turbulence around the Bjorken flow

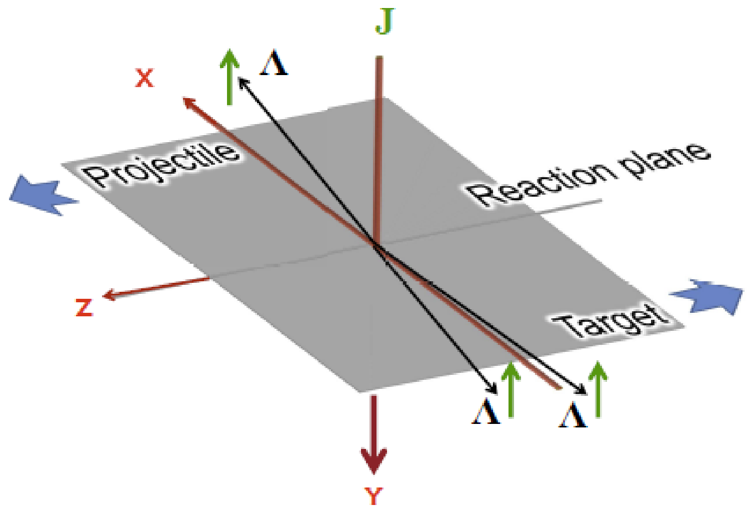
S. Floerchinger & U. A. Wiedemann,   JHEP 1111:100, 2011; arXiv: 1108.5535v1



absolute value of vorticity $|\partial_1 u^2 - \partial_2 u^1|$ divergence $|\partial_1 u^1 + \partial_2 u^2|$

- Initial state Event by Event vorticity and divergence fluctuations.
- Amplitude of random vorticity and divergence fluctuations are the same
- In dynamical development viscous corrections are negligible   (→ no damping)
- Initial transverse expansion in the middle (±3fm) is neglected (→ no damping)
- High frequency, high wave number fluctuations **may feed** lower wave numbers

Detecting rotation: Lambda polarization

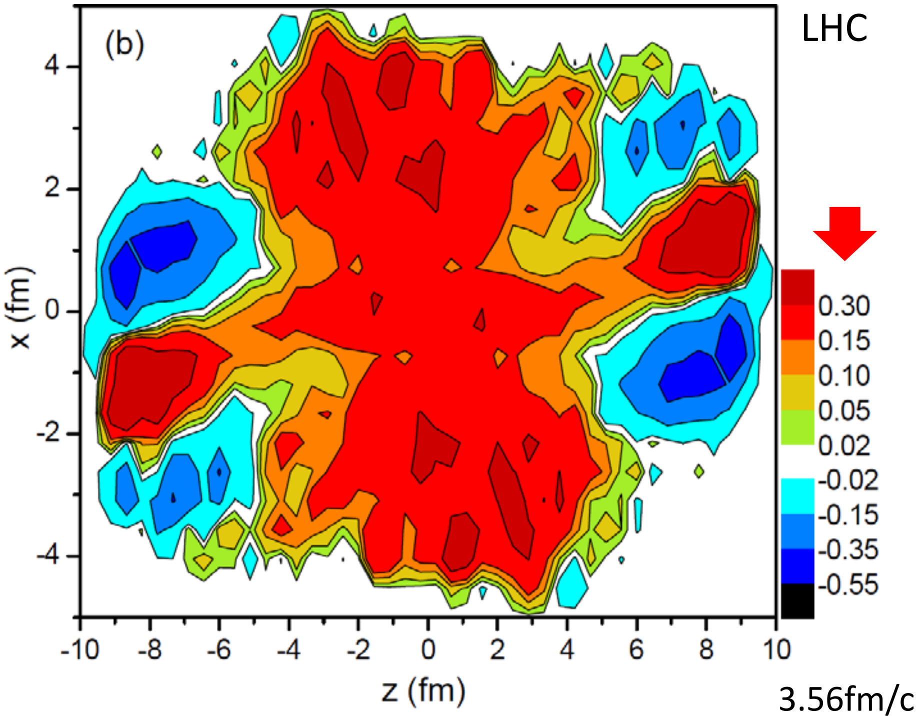


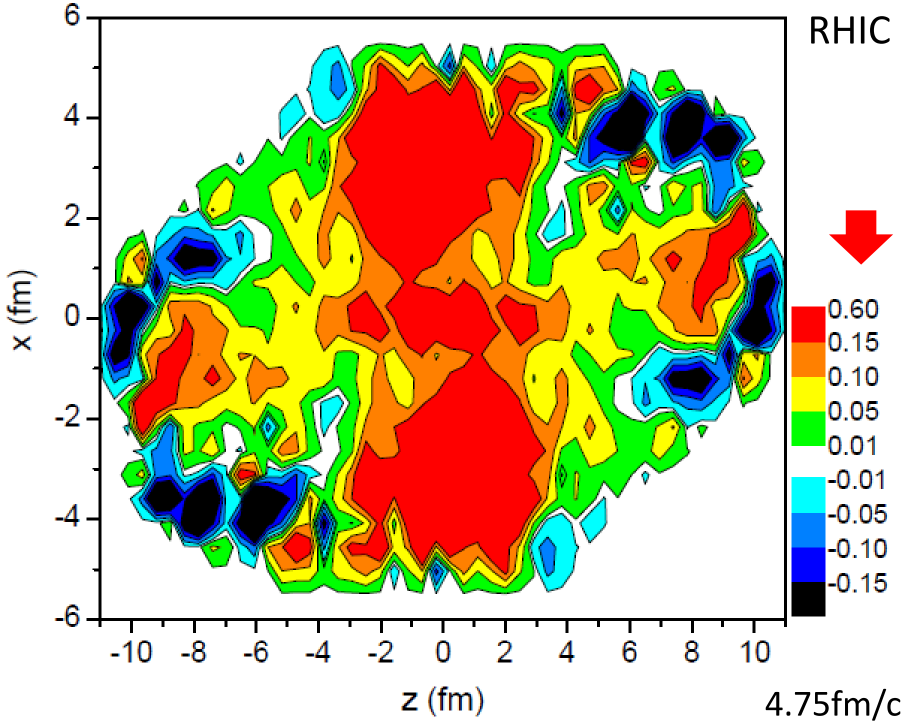
$$\mathbf{\Pi}(p) = \frac{\hbar\varepsilon}{8m}\frac{\int \mathrm{d}V\, n_F\, (\nabla \times \boldsymbol{\beta})}{\int \mathrm{d}V\, n_F}$$

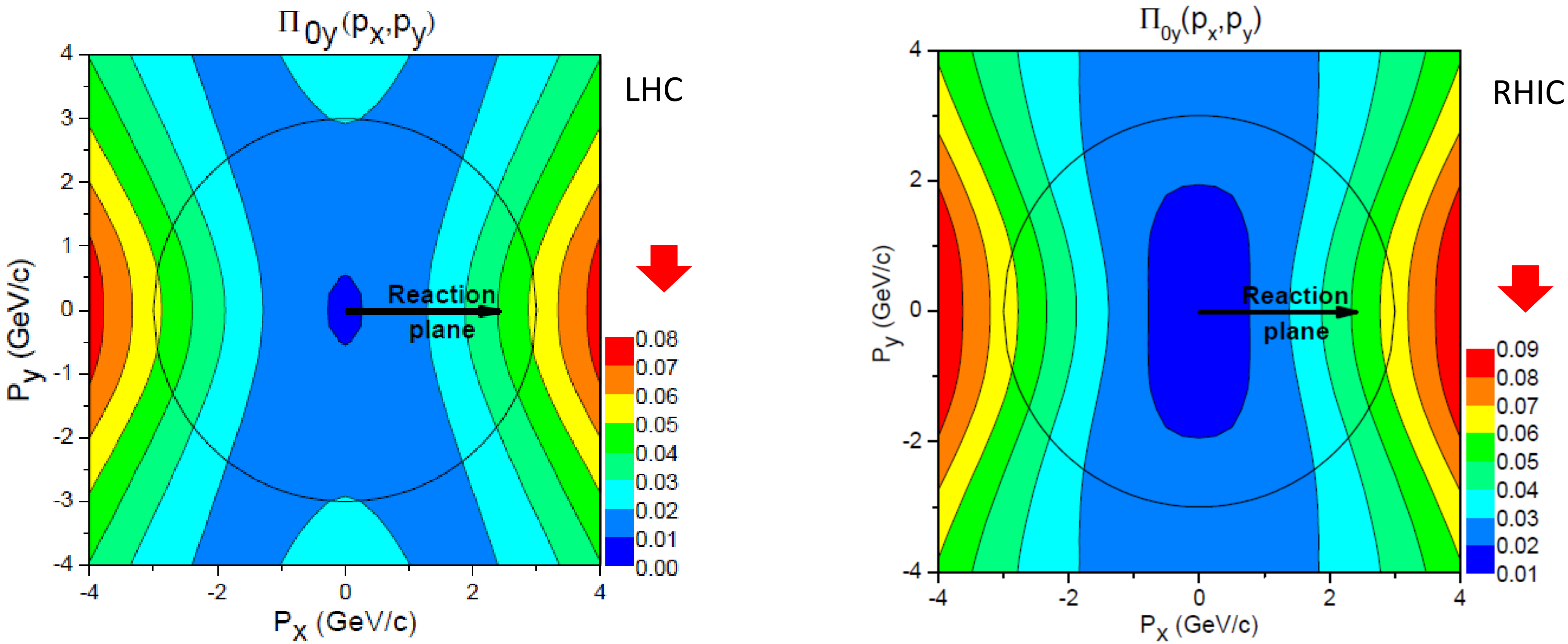
$$\beta^\mu(x) = (1/T(x))u^\mu(x)$$ ← From hydro

$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$

[F. Becattini, L.P. Csernai, D.J. Wang, Phys. Rev. C **88**, 034905 (2013)]





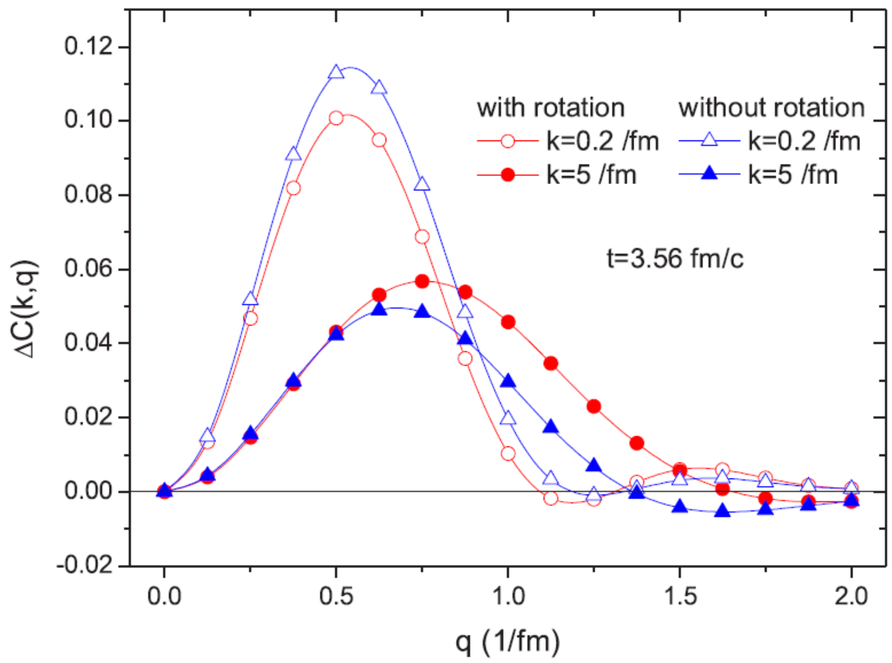


- **The POLARIZATION of Λ and $\overline{\Lambda}$ due to thermal equipartition with local vorticity is slightly stronger at RHIC than at LHC due to the much higher temperatures at LHC.**
- **Although early measurements at RHIC were negative, these were averaged over azimuth! We propose selective measurement in the reaction plane (in the +/- x direction) in the EbE c.m. frame. Statistical error is much reduced now, so significant effect is expected at $p_x \geq 3$ GeV/c.**

Differential HBT method

PHYSICAL REVIEW C **89**, 034916 (2014)

FIG. 2. (Color online) Differential correlation function, $\Delta C(k,q)$, at the final time with and without rotation.



We can rotate the frame of reference:

$$k'(\alpha) = \begin{Bmatrix} k_{x'} \\ k_{z'} \end{Bmatrix} = \begin{Bmatrix} k_x \cos\alpha - k_z \sin\alpha \\ k_z \cos\alpha + k_x \sin\alpha \end{Bmatrix}.$$

$$\rightarrow \quad \Delta C_\alpha(k', q'),$$

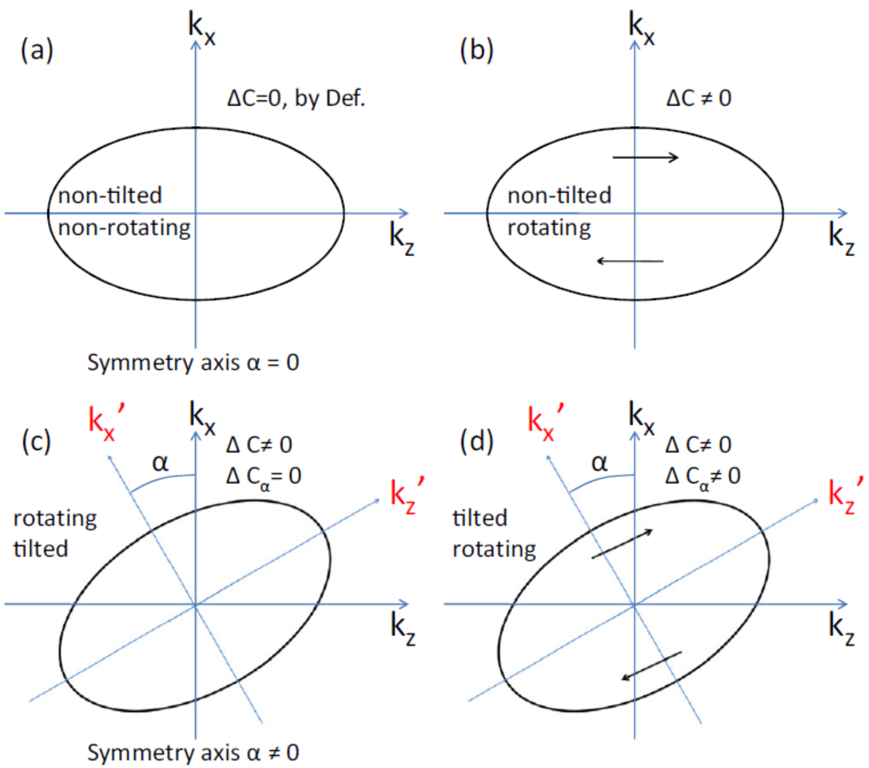


FIG. 3. (Color online) Sketch of the configuration in different reference frames, with and without rotation of the flow. The nonrotating configurations may have radial flow velocity components only. The DCF, $\Delta C_\alpha(k,q)$, is evaluated in a K' reference frame rotated by an angle α in the x,z, reaction plane. We search for the angle α, where the nonrotating configuration is "symmetric," so that it has a "minimal" DCF as shown in Fig. 4.

Signs of rotation

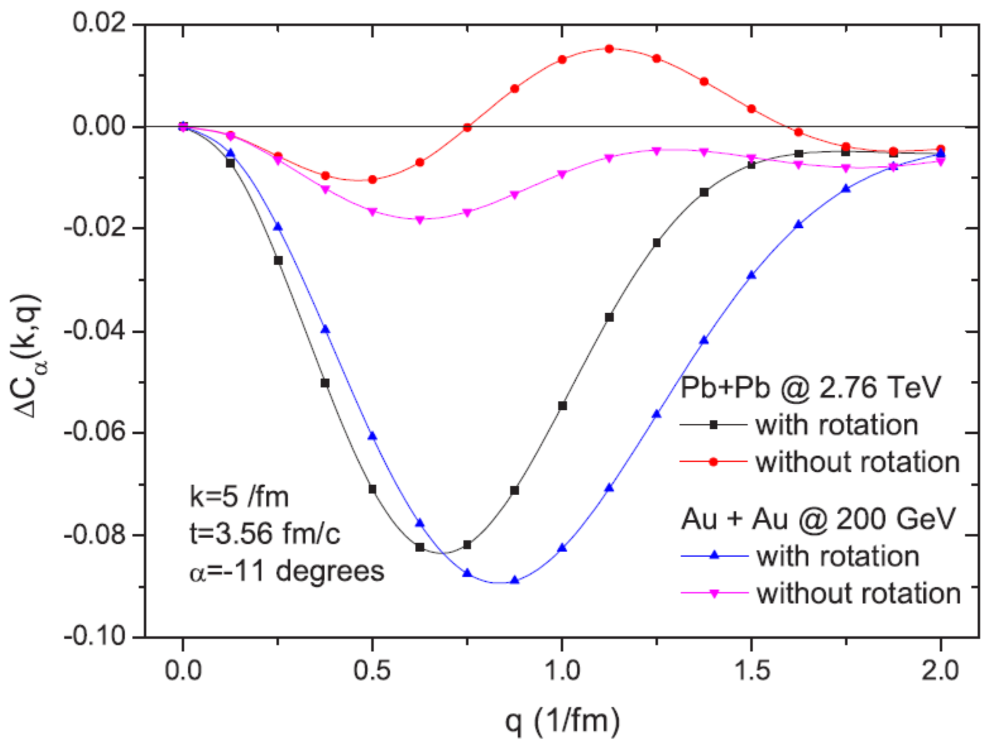


FIG. 5. (Color online) The DCF with and without rotation in the reference frames, deflected by the angle α, where the rotationless DCF is vanishing or minimal. In this frame the DCF of the original, rotating configuration indicates the effect of the rotation only. The amplitude of the DCF of the original rotating configuration doubles for the higher energy (higher angular momentum) collision.

To perform the analysis in the rotationless symmetry frame one can find the symmetry axis the best with the azimuthal HBT method, which provides even the transverse momentum dependence of this axis [20]. It is also important to determine the precise event-by-event c.m. position of the participants [21] and minimize the effect of fluctuations to be able to measure the emission angles accurately, which is crucial in the present $\Delta C(k,q)$ studies.

Summary

- We have shown how to split
  Collective flow & Fluctuations
- When Collective Flow is identified: ***New patterns***
- Small viscosity (→ fluctuations & instabilities)
- Rotation
- Kelvin-Helmholtz Instability (KHI) ~ turbulence
- These are observable in polarizations and in HBT