VNIVERSITÀ
DEGLI·STVDI
DI·FERRARA

University of Ferrara

INFN Istituto Nazionale di Fisica Nucleare

Heavy ions collision modeling with ECHO-QGP

Valentina Rolando

June, 11th 2014

ECHO-QGP Collaboration

The ECHO-QGP collaboration involves the Universities of Ferrara, Firenze and Torino.

ECHO-QGP

L. Del Zanna, V. Chandra, G. Inghirami, V. Rolando, A. Beraudo, A. De Pace, G. Pagliara, and A. Drago, and F. Becattini.
Relativistic viscous hydrodynamics for heavy-ion collisions with ECHO-QGP.
Eur.Phys.J.,C73:2524, 2013. arXiv(nucl-th):1305.7052

Overwiew on ECHO-QGP

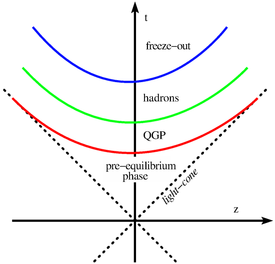
ECHO-QGP is a development of ECHO

ECHO

L. Del Zanna, O. Zanotti, N. Bucciantini, and P. Londrillo.
ECHO: an Eulerian Conservative High Order scheme for general relativistic magnetohydrodynamics and magnetodynamics
Astron.Astrophys., 473:11–30, 2007. arXiv(astro-ph):0704.3206

The original ECHO code can handle non-vanishing conserved-number currents as well as electromagnetic fields, which are essential for the astrophysical computations, in any (3+1)-D metric of General Relativity.

Setup



- Initial stage: Optical Glauber model (energy density/entropy density profile) or MC Glauber model
- Hydro stage:
  - the evolution can be purely ideal or viscous
  - can handle both Minkowski or Bjorken coordinates
  - it is designed to use any EoS, tabulated or Analytical
- Decoupling stage: Cooper-Frye prescription (mean spectrum and event generation)

ECHO-QGP Features

- ECHO-QGP has been originally conceived to be publicly released
  - user-friendly
  - exhaustive documentation and tutorials
- Designed to perform serial or parallel simulations
- Built-in standard tests initialization ($e.g.$ shock tube, Bjorken expansion, Gubser's solution . . .)
- Highly Customizable at runtime ($e.g.$ output, end criterion, grid, collision parameters, . . .)
- Several post-processing tools already included

The equations

Ideal hydro

$$g^{\mu\nu} = \begin{pmatrix} - & & & \\ & + & & \\ & & + & \\ & & & + \end{pmatrix}$$

Orthogonal projector

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$$

Covariant derivative

$$d_\mu = \underbrace{-u_\mu D}_{D \equiv u^\alpha d_\alpha} + \underbrace{\nabla_\mu}_{\nabla_\mu \equiv \Delta_\mu^\alpha d_\alpha}$$

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ EoS \end{cases}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

$$N^\mu = nu^\mu + V^\mu$$

$$T^{\mu\nu} = eu^\mu u^\nu + P\Delta^{\mu\nu} + w^\mu u^\nu + w^\nu u^\mu$$

The equations

viscous hydro

$$T^{\mu\nu} = eu^{\mu}u^{\nu} + (P+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$De + (e+P+\Pi)\theta + \pi^{\mu\nu}\sigma_{\mu\nu} = 0,$$

$$(e+P+\Pi)Du_{\nu} + \nabla_{\nu}(P+\Pi) + \Delta_{\nu}^{\beta}\nabla_{\alpha}\pi^{\alpha}_{\beta} + Du^{\mu}\,\pi_{\mu\nu} = 0,$$

$$D\Pi = -\frac{1}{\tau_{\Pi}}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta,$$

$$\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}D\pi^{\alpha\beta} = -\frac{1}{\tau_{\pi}}(\pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta - \lambda(\pi^{\mu\lambda}\omega^{\nu}_{\lambda} + \pi^{\nu\lambda}\omega^{\mu}_{\lambda})$$

Set of equations

$$\begin{cases} d_{\mu}N^{\mu} = 0, \\ d_{\mu}T^{\mu\nu} = 0 \\ EoS \\ \pi^{\mu\nu}\ \text{evolution} \\ \Pi\ \text{evolution} \end{cases}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

The equations

viscous hydro

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} N \equiv N^0 \\ S_i \equiv T_i^0 \\ E \equiv -T_0^0 \\ N\Pi \\ N\pi^{ij} \end{pmatrix}, \quad \mathbf{F}^k = |g|^{\frac{1}{2}} \begin{pmatrix} N^k \\ T_i^k \\ -T_0^k \\ N^k\Pi \\ N^k\pi^{ij} \end{pmatrix}$$

$$\mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2}T^{\mu\nu}\partial_i g_{\mu\nu} \\ -\frac{1}{2}T^{\mu\nu}\partial_0 g_{\mu\nu} \\ n[-\frac{1}{\tau_\pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta] \\ n[-\frac{1}{\tau_\pi}(\pi^{ij} + 2\eta\sigma^{ij}) - \frac{4}{3}\pi^{ij}\theta + \mathcal{I}_0^{ij} + \mathcal{I}_1^{ij} + \mathcal{I}_2^{ij}] \end{pmatrix}.$$

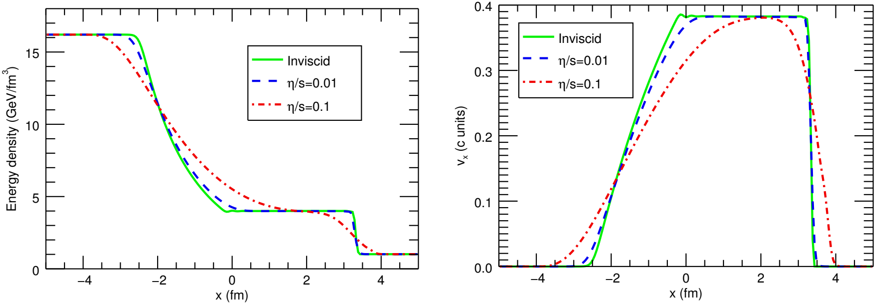
Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ EoS \\ \pi^{\mu\nu} \text{ evolution} \\ \Pi \text{ evolution} \end{cases}$$

Conservative form

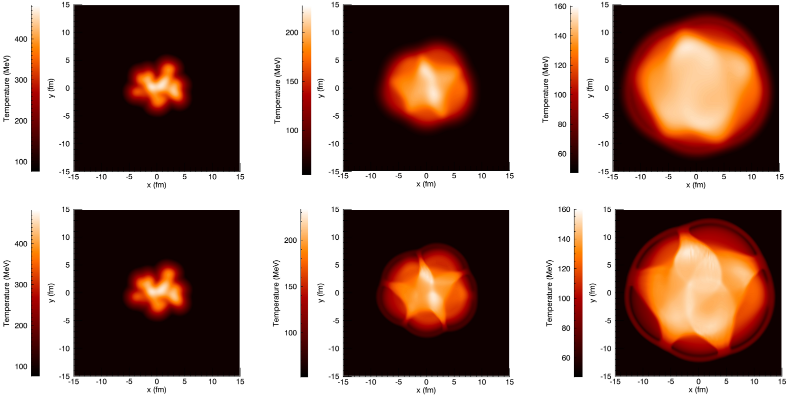
$$\partial_0 U + \partial_k F^k = S$$

Test: (2+1)-D shock tubes



$T^L = 0.4$ GeV ($P^L = 5.40$ GeV/fm^3) and $T^R = 0.2$ GeV ($P = 0.34$ GeV/fm^3)
$\eta/s = 0, 0.01, 0.1$ at $t = 4$ fm/c.

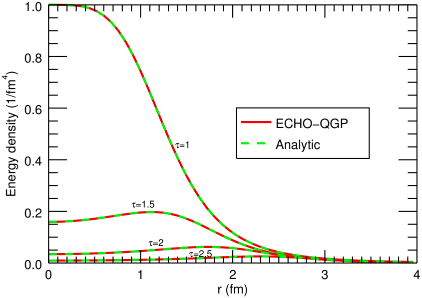
The effect of viscosity

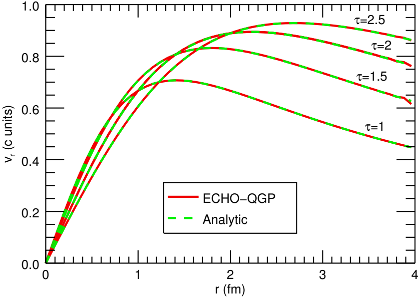


Test:(2+1)-D with azimuthal symmetry

Ideal Gubser Test

Analytic solution from symmetry consideration [1]:



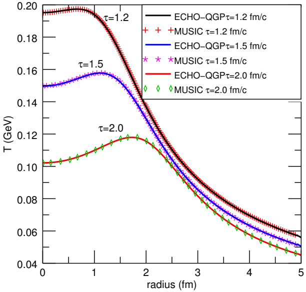


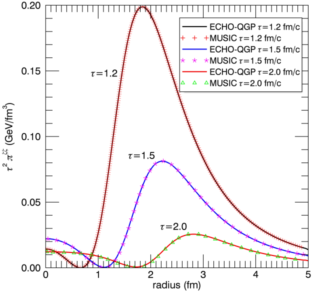
[1] S. S. Gubser. Symmetry constraints on generalizations of Bjorken flow. *PRD*82:085027, 2010.

Test: (2+1)-D with azimuthal symmetry

Viscous Gubser Test

Analytic solution from symmetry consideration in for the Israel-Stewart frame[2]:





[2] H. Marrochio, et al. Solutions of Conformal Israel-Stewart Relativistic Viscous Fluid Dynamics. 2013.arXiv(nucl-th):1307.6130

Decoupling fluid to particles

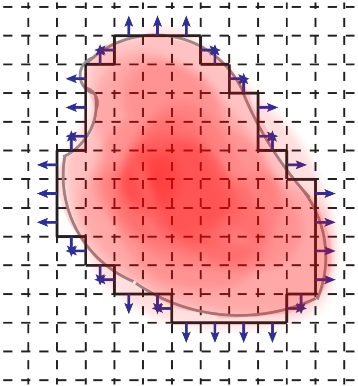
Isothermal hypersuface: our implementation

$$f_i(x,p) = \left[e^{-\frac{1}{T}(u^\nu p_\nu + \mu_i)} \pm 1 \right]^{-1}$$

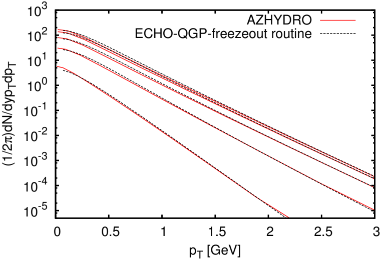
$$E\frac{\mathrm{d}^3 N_i}{\mathrm{d}p^3} = \frac{g_i}{(2\pi)^3} \int_\Sigma -f_i(x,p) p^\mu \mathrm{d}^3\Sigma_\mu$$

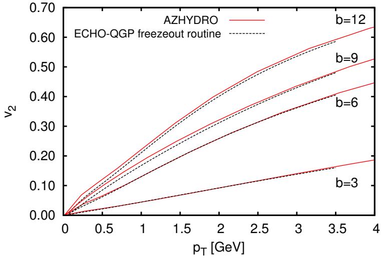
$$\mathrm{d}^3\Sigma_\mu = \begin{pmatrix} \mathrm{d}V^{\perp\tau} \\ \mathrm{d}V^{\perp x} \\ \mathrm{d}V^{\perp y} \\ \mathrm{d}V^{\perp\eta} \end{pmatrix} = \begin{pmatrix} \tau\,\Delta x \Delta y \Delta\eta_s\; s^\tau \\ \tau\,\Delta y \Delta\eta_s \Delta\tau\; s^x \\ \tau\,\Delta\eta_s \Delta\tau \Delta x\; s^y \\ \frac{1}{\tau}\,\Delta\tau \Delta x \Delta y\; s^\eta \end{pmatrix}$$

$$s^\mu = -\mathrm{sign}\left(\frac{\partial T}{\partial x^\mu}\right)$$



Freeze-out routine: tests with AZHYDRO[3]





σ_{NN} mb	τ_0 fm	e_0 Gev fm^{-3}	α	b fm	μ_π GeV	T_{freeze} GeV
40	0.6	24.5	1	0,3,6,9,12	0.0622	0.120

Table : The grid spacing here used is: $\Delta x = \Delta y = 0.4$ fm $\Delta\tau = 0.16$ fm.

[3] P. F. Kolb, J. Sollfrank and U. W. Heinz, transverse flow and the quark hadron phase transition *PRC* 62:054909, 2000

Decoupling fluid to particles

Update

$$E\frac{\mathrm{d}^3N_i}{\mathrm{d}p^3} = \frac{g_i}{(2\pi)^3}\int_\Sigma -f_i(x,p)p^\mu \mathrm{d}^3\Sigma_\mu$$

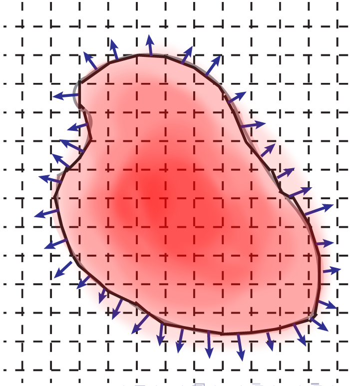
$$\mathrm{d}\Sigma_\mu = \sum_i \varepsilon_{\mu\alpha\beta\gamma}\frac{1}{6}s_i a_i^\alpha b_i^\beta c_i^\gamma$$

The devel version of ECHO-QGP embeds CORNELIUS

P. Huovinen and H. Petersen

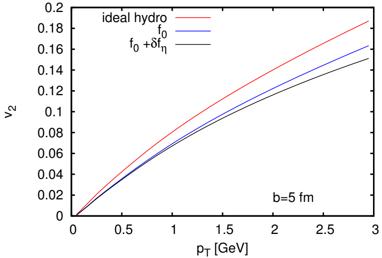
Particlization in hybrid models

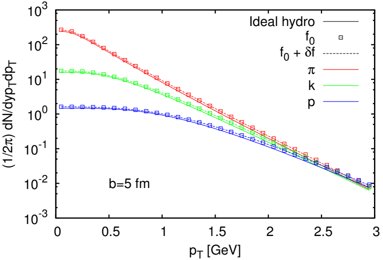
EPJ A 48 (2012) 171 arXiv:1206.3371 [nucl-th]



The effect of viscosity[4]

$$\delta f(x,p) = f_0(1 \pm f_0)\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2T^2(e+p)}$$

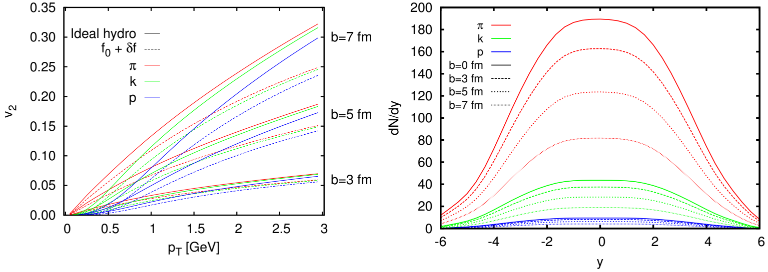




[4] D. Teaney. Effects of viscosity on spectra, elliptic flow, and HBT radii. *PRC* 68:034913, 2003.
R. Baier et al. Dissipative hydrodynamics and heavy ion collisions *PRC* 73:064903, 2006.

The effect of viscosity[4]

$$\delta f(x,p) = f_0(1 \pm f_0)\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2T^2(e+p)}$$



[4] D. Teaney. Effects of viscosity on spectra, elliptic flow, and HBT radii. *PRC* 68:034913, 2003.
R. Baier et al. Dissipative hydrodynamics and heavy ion collisions *PRC* 73:064903, 2006.

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

summary

- ECHO-QGP is a robust high-order shock-capturing code, solving either ideal or viscous (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP will be made available soon . . . *stay tuned!*
- More ongoing physics studies (vorticity, fluctuation propagations . . .)

Summary and Outlook

outlook

- Recover of the original ECHO feature of evolving EM fields
- Inclusion of conserved currents

Thank you!

backup slides

Decomposition of $d_\mu u_\nu$

The covariant derivative of the fluid velocity can be decomposed in its *irreducible tensorial parts* as

$$d_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu} - u_\mu D u_\nu + \tfrac{1}{3}\Delta_{\mu\nu}\theta$$

(transverse, traceless, and symmetric) *shear tensor*

$$\begin{aligned}\sigma_{\mu\nu} &= \tfrac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \tfrac{1}{3}\Delta_{\mu\nu}\theta,\\ &= \tfrac{1}{2}(d_\mu u_\nu + d_\nu u_\mu) + \tfrac{1}{2}(u_\mu D u_\nu + u_\nu D u_\mu) - \tfrac{1}{3}\Delta_{\mu\nu}\theta,\end{aligned}$$

(transverse, traceless, and antisymmetric) *vorticity tensor*

$$\begin{aligned}\omega_{\mu\nu} &= \tfrac{1}{2}(\nabla_\mu u_\nu - \nabla_\nu u_\mu)\\ &= \tfrac{1}{2}(d_\mu u_\nu - d_\nu u_\mu) + \tfrac{1}{2}(u_\mu D u_\nu - u_\nu D u_\mu),\end{aligned}$$

expansion scalar

$$\theta = \nabla_\mu u^\mu = d_\mu u^\mu.$$

energy, momentum and stress tensor equations

$$
\begin{aligned}
&De + (e + P + \Pi)\theta + \pi^{\mu\nu}\sigma_{\mu\nu} = 0, \\
&(e + P + \Pi)Du_\nu + \nabla_\nu(P + \Pi) + \Delta_\nu^{\beta}\nabla_\alpha\pi^{\alpha}_{\beta} + Du^\mu\,\pi_{\mu\nu} = 0, \\
&D\Pi = -\frac{1}{\tau_\Pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta, \\
&\Delta^\mu_\alpha\Delta^\nu_\beta D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta - \lambda(\pi^{\mu\lambda}\omega^\nu_\lambda + \pi^{\nu\lambda}\omega^\mu_\lambda)
\end{aligned}
$$

Exploit orthogonality and derive:

$$
\begin{aligned}
D\pi^{\mu\nu} &= -\frac{1}{\tau_\pi}(\pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta + \mathcal{I}_1^{\mu\nu} + \mathcal{I}_2^{\mu\nu}, \\
\mathcal{I}_1^{\mu\nu} &= (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)Du_\lambda, \\
\mathcal{I}_2^{\mu\nu} &= -\lambda(\pi^{\mu\lambda}\omega^\nu_\lambda + \pi^{\nu\lambda}\omega^\mu_\lambda).
\end{aligned}
$$

Conservative form of equations

$$\partial_0 \mathbf{U} + \partial_k \mathbf{F}^k = \mathbf{S},$$

where

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} N \equiv N^0 \\ S_i \equiv T^0_i \\ E \equiv -T^0_0 \\ N\Pi \\ N\pi^{ij} \end{pmatrix}, \quad \mathbf{F}^k = |g|^{\frac{1}{2}} \begin{pmatrix} N^k \\ T^k_i \\ -T^k_0 \\ N^k\Pi \\ N^k\pi^{ij} \end{pmatrix}$$

$$\mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2}T^{\mu\nu}\partial_i g_{\mu\nu} \\ -\frac{1}{2}T^{\mu\nu}\partial_0 g_{\mu\nu} \\ n[-\frac{1}{\tau_\pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta] \\ n[-\frac{1}{\tau_\pi}(\pi^{ij} + 2\eta\sigma^{ij}) - \frac{4}{3}\pi^{ij}\theta + \mathcal{I}^{ij}_0 + \mathcal{I}^{ij}_1 + \mathcal{I}^{ij}_2] \end{pmatrix}.$$

EoS

ECHO-QGP allows the use of any tabulated EOS of this kind, if provided by the user in the format $(T, e/T^4, P/T^4, c_s^2)$, with $c_s^2 \equiv dP/de$.

Transport coefficient setup

Following

- Huichao Song and Ulrich W Heinz, *Interplay of shear and bulk viscosity in generating flow in heavy-ion collisions*, PRC **81**, 2010, 024905.
- Piotr Bozek, *Flow and interferometry in 3+1 dimensional viscous hydrodynamics*, PRC **85** (2012), 034901.

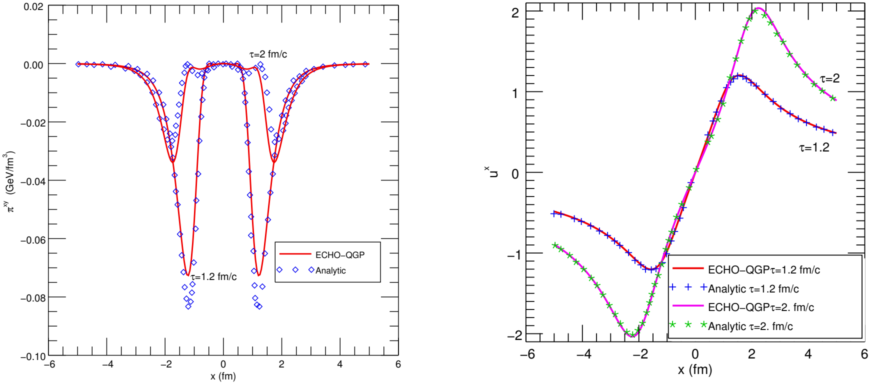
in ECHO-QGP the choice is

$$\tau_\pi = \tau_\Pi = \frac{3\eta}{sT}$$
$$\zeta = 2\eta\left(\tfrac{1}{3} - c_s^2\right)$$
$$\lambda = \frac{\lambda_2}{\eta}$$

Test: (2+1)-D with azimuthal symmetry

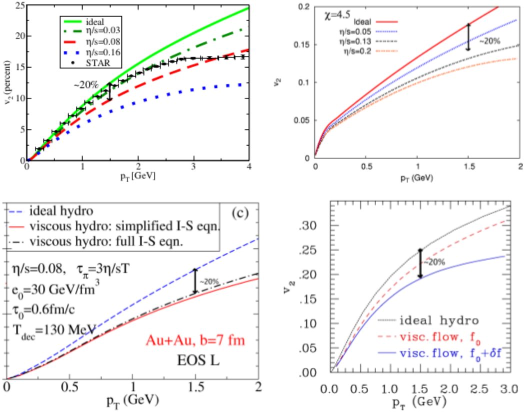
Viscous Gubser Test

Analytic solution from symmetry consideration in the Israel-Stewart frame[5]:



[5] H. Marrochio, et al. of Conformal Israel-Stewart Relativistic Viscous Fluid Dynamics. 2013.arXiv(nucl-th):1307.6130

The effect of viscosity



top left P. Romatschke and U. Romatschke. Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC? *PRL*, 99:172301, 2007.

top right K. Dusling and D. Teaney. Simulating elliptic flow with viscous hydrodynamics. *PRC* 77:034905, 2008.

bot left H. Song and U. W. Heinz. Multiplicity scaling in ideal and viscous hydrodynamics. *PRC* 78:024902, 2008.

bot right D. Molnar and P. Huovinen. Dissipative effects from transport and viscous hydrodynamics *JPG*, 35:104125, 2008.