Beth-Uhlenbeck approach to a hadron resonance gas with Mott effect: thermodynamics and chemical freezeout[1]

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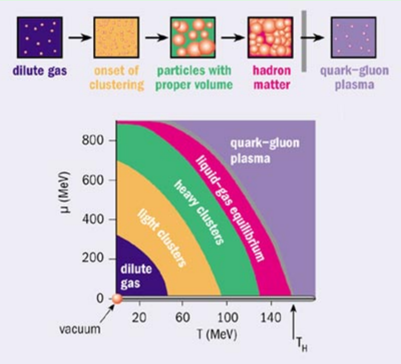
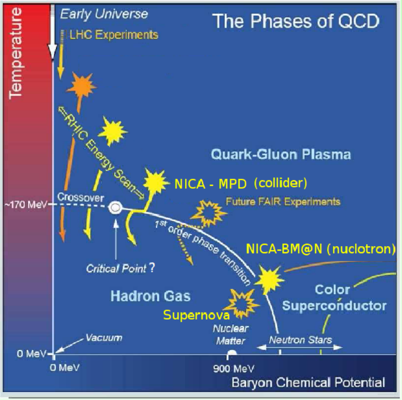
Hersonissos, Crete, Greece, June 12, 2014

[1] Collab.: J. Berdermann, M. Buballa, J. Cleymans, A. Dubinin, A. Radzhabov, K. Redlich, G. Röpke, L. Turko, A. Wergieluk, D. Zablocki ...

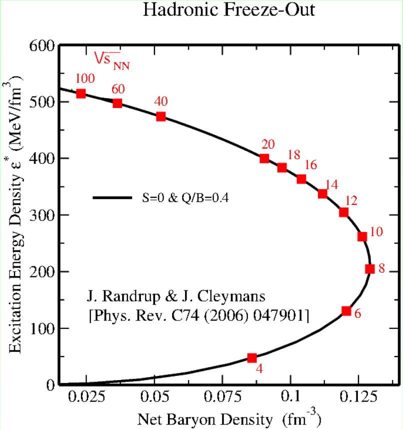
Rolf Hagedorn - Statistical model of particle production



QCD Phase Diagram & Heavy-Ion Collisions



Beam energy scan (BES) programs in the QCD phase diagram



Energy density vs. baryon density at freeze-out for different $\sqrt{s_{NN}}$(GeV)

Highest baryon densities at freeze-out shall be reached for $\sqrt{s_{NN}} \sim 8$ GeV $\longrightarrow$ QGP phase transition ?

Pion dissociation and Levinson's theorem
(A PNJL model case study)

- Gap eqn. & Bethe-Salpeter eqn. in PNJL quark matter
- Mott-Anderson dissociation/delocalization of pions
- Generalized Beth-Uhlenbeck EoS for quark-meson matter
- Levinson theorem & quark-meson thermodynamics

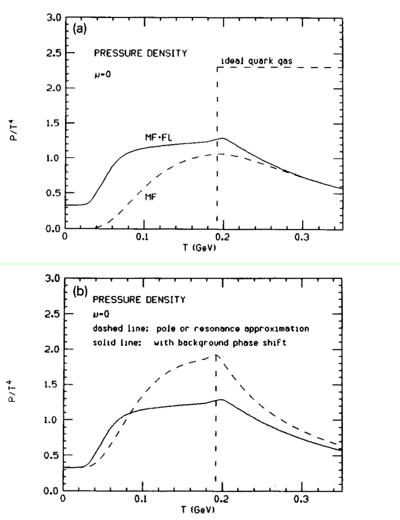
A. Wergieluk, D. Blaschke, Yu. Kalinovsky, A. Friesen, arxiv:1212.5245;
Dubna Report E2-2013-19; Phys. Part. Nucl. Lett. **7** (2013) 660.
D.B., M. Buballa, A. Dubinin, G. Röpke, D. Zablocki,
arxiv:1305.3907.v3; Annals Phys. in press (2014)
A. Dubinin, D. Blaschke, Yu. Kalinovsky; arxiv:1312.0559

The state of the art in January 1994

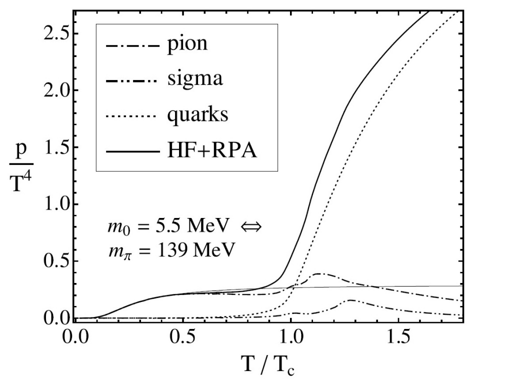
1) The NJL model:

(P. Zhuang, J. Hüfner and
S. P. Klevansky,
Nucl. Phys. A **576** (1994) 525)

Hüfner, Klevansky, Witzler, D.B.,
Dossenheim (2007)

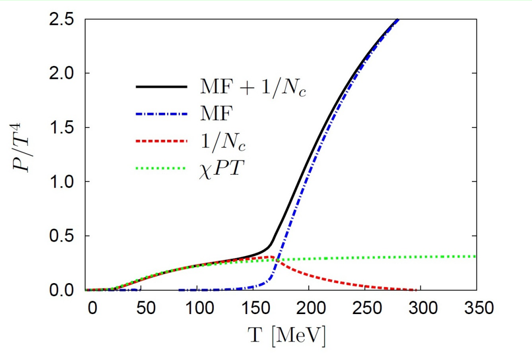


2) The PNJL model:



S. Roessner, T. Hell, C. Ratti and W. Weise,
Nucl. Phys. A **814** (2008) 118; [arXiv:0712.3152]

3) The nonlocal PNJL model:



(A. E. Radzhabov, D. Blaschke, M. Buballa and M. K. Volkov, Phys. Rev. D **83** (2011) 116004 [arXiv:1012.0664 [hep-ph]])

The PNJL model

Everything begins with a Lagrangian:

$$\mathcal{L}_{PNJL} = \bar{q}\left(i\gamma^\mu D_\mu - m_0 - \gamma^0\mu\right)q + \sum_{M=\sigma',\vec{\pi}'} G_M\left(\bar{q}\Gamma_M q\right)^2 - U(\Phi[A];T),$$

where $D_\mu = \partial_\mu - iA_\mu$,

$$U(\Phi;T) = T^4\left[-\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{3}\Phi^3 + \frac{b_4}{4}\Phi^4\right],$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3,$$

a_0	a_1	a_2	b_3	b_4	T_0 [MeV]
6.75	-1.95	-7.44	0.75	7.5	208

C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019,

B.-J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D **76** (2007) 074023.

The PNJL model

The partition function in the PNJL model:

$$\mathcal{Z}_{PNJL}[T,V,\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{ \int_0^\beta d\tau \int_V d^3x \left[\bar{q}\left(i\gamma^\mu(\partial_\mu - iA_\mu) - m_0 - \gamma^0\mu\right) q + \right.\right.$$

$$\left.\left. + G_S \left(\bar{q}\Gamma_{\sigma'} q\right)^2 + G_S \left(\bar{q}\vec{\Gamma}_{\pi'} q\right)^2 - U(\Phi[A];T) \right] \right\}$$

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$$\mathcal{Z}_{PNJL}[T,V,\mu] = \int \mathcal{D}\sigma'\mathcal{D}\vec{\pi}' \exp\left\{ -\left[\int_0^\beta d\tau \int_V d^3x \left(\frac{\sigma'^2 + \vec{\pi}'^2}{4G_S} + U(\Phi[A];T) \right) \right] + \right.$$

$$\left. + \mathrm{Tr}\ln\left[\beta S^{-1}[\sigma', \vec{\pi}']\right] \right\}$$

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$$\left. + \operatorname{Tr}\ln\left[\beta S^{-1}[\sigma', \vec{\pi}']\right]\right\}$$

$$\Omega_{FL}^{(2)}[T,V,\mu] = \frac{T}{V}\ln\left[\det\left(\frac{1}{2G_S} - \Pi_\sigma(q_0,\vec{q})\right)\right]^{-\frac{1}{2}} + \frac{T}{V}\ln\left[\det\left(\frac{1}{2G_S} - \Pi_{\vec{\pi}}(q_0,\vec{q})\right)\right]^{-\frac{3}{2}}$$

Thermodynamic potential - propagators - phase shifts

Thermodynamic potential for bosonic degree of freedom (mode) X

$$\Omega_{\rm X}(T,\mu) = \frac{1}{2}\frac{T}{V}\,{\rm Tr}\ln S_{\rm X}^{-1}(iz_n,\mathbf{q}) = \frac{1}{2}d_{\rm X}T\sum_n\int\frac{{\rm d}^3q}{(2\pi)^3}\ln S_{\rm X}^{-1}(iz_n,\mathbf{q})~,$$

$$= -d_{\rm X}T\sum_n\int\frac{{\rm d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}\,\frac{1}{iz_n-\omega}{\rm Im}\ln S_{\rm X}^{-1}(\omega+i\eta,\mathbf{q})~,$$

Propagator = complex function → polar representation

$$S_{\rm X}^{-1}(iz_n,\mathbf{q}) = G_{\rm X}^{-1}-\Pi_{\rm X}(iz_n,\mathbf{q}) = |S_{\rm X}|{\rm e}^{i\Phi_{\rm X}}~,~~\Phi_{\rm X}(\omega,\mathbf{q}) = -{\rm Im}\ln S_{\rm X}^{-1}(\omega-\mu_X+i\eta,\mathbf{q})$$

Beth-Uhlenbeck formula

$$\Omega_{\rm X}(T,\mu) = d_{\rm X}\int\frac{{\rm d}^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{{\rm d}\omega}{2\pi}\,n_X^-(\omega)\Phi_X(\omega,\mathbf{q})$$

$$= -d_{\rm X}\int\frac{{\rm d}^3q}{(2\pi)^3}\int_0^{\infty}\frac{{\rm d}\omega}{2\pi}\,[1+n_X^-(\omega)+n_X^+(\omega)]\Phi_{\rm X}(\omega,\mathbf{q})$$

$$= d_{\rm X}\int\frac{{\rm d}^3q}{(2\pi)^3}\int_0^{\infty}\frac{{\rm d}\omega}{2\pi}\left\{\omega+T\ln\left(1-{\rm e}^{-(\omega-\mu_{\rm X})/T}\right)\right.$$

$$\left.+T\ln\left(1-{\rm e}^{-(\omega+\mu_{\rm X})/T}\right)\right\}\frac{d\Phi_{\rm X}(\omega,\mathbf{q})}{d\omega}~.$$

The Generalized Beth-Uhlenbeck approach

$$P_M^{(2)} = -\Omega_M^{(2)}(T,\mu) = \frac{N_M}{2}\int \frac{d^3q}{(2\pi)^3}\left(\int_0^{+\infty}\frac{d\omega}{\pi}\left[\omega + 2T\ln\left(1-e^{-\beta\omega}\right)\right]\frac{d\Phi_M(\omega,\vec{q})}{d\omega}\right)$$

Mott-effect: bound state (delta function)→ resonance (spectral broadening)

$$\frac{d\Phi_M(\omega,\vec{q})}{d\omega} \quad \rightarrow \quad \begin{cases} \pi\ \delta(\omega - E_M), & T < T_{\rm Mott} \\ \frac{d\phi_R(\omega,\vec{q})}{d\omega}, & T > T_{\rm Mott} \end{cases}$$

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The Breit-Wigner ansatz for the phase shift derivative:

$$\frac{d\phi_R(s,T)}{ds} = A_R(s,T) = \frac{\pi}{\frac{\pi}{2} + \arctan\left(\frac{\vec{q}^2 + M_M^2}{M_M\Gamma_M}\right)}\ \frac{M_M\Gamma_M}{\left(s - M_M^2\right)^2 + (M_M\Gamma_M)^2}$$

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and the corresponding meson pressure ($\omega = \sqrt{\vec{q}^2 + s}$)

$$P_M(T) = \frac{N_M}{2}\int \frac{d^3q}{(2\pi)^3}\int_{4m^2}^{+\infty} ds\left(\omega + 2T\ln\left(1 - e^{-\beta\omega}\right)\right)D_M(s,T)$$

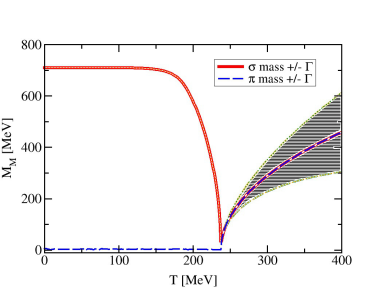
Meson masses with spectral broadening

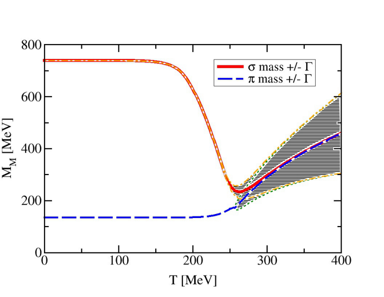
Separating the real and imaginary part of $\Pi_M\left(q_0,\vec{0}\right) = 4N_cN_f\ I_1 - 2N_cN_fP_M\ I_2$ results in coupled Bethe-Salpeter equations:

$$M_M^2 - \frac{1}{4}\Gamma_M^2 \ - \left(\begin{array}{c} 4m^2 \\ 0 \end{array}\right) = \frac{\frac{1}{4N_cN_fG_S} - 2I_1}{|I_2(q_0 = M_M - i\frac{1}{2}\Gamma_M)|^2} \cdot \mathrm{Re}\ I_2(q_0),$$

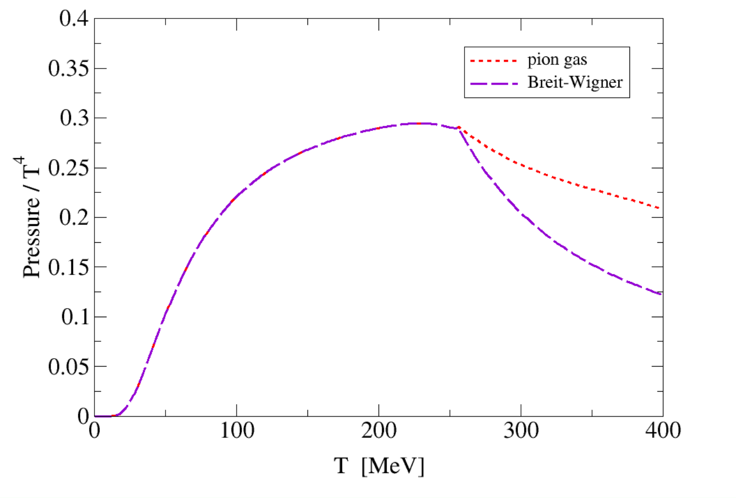
$$M_M\Gamma_M = \frac{\frac{1}{4N_cN_fG_S} - 2I_1}{|I_2(q_0 = M_M - i\frac{1}{2}\Gamma_M)|^2} \cdot \mathrm{Im}\ I_2(q_0).$$

See, e.g., D. Blaschke, M. Jaminon, Yu.L. Kalinovsky, *et al.*, NPA **592** (1995) 561





Pion pressure: massive pion gas and Breit-Wigner ansatz



Levinson's theorem

Breit-Wigner ansatz $\rightarrow$ ϕ_R is

$$\phi_R(s) = \frac{\pi}{\frac{\pi}{2} - \arctan\left(\frac{4m^2 - M_M^2}{M_M \Gamma_M}\right)} \left(\arctan\left[\frac{s - M_M^2}{M_M \Gamma_M}\right] - \arctan\left[\frac{4m^2 - M_M^2}{M_M \Gamma_M}\right] \right)$$

[it fulfills $\phi_R(s \rightarrow 4m^2) = 0$ and $\phi_R(s \rightarrow \infty) = \pi$]

violates Levinson's theorem which would require

$$\phi(s_{\mathrm{threshold}} = 4m^2) - \phi(\infty) = n\pi = 0 \ ,$$

since the number of bound states below threshold vanishes (n=0) for $T > T_{\mathrm{Mott}}$

$\rightarrow$ Solution: phase shift corresponding to scattering states missing!

Two contributions to the scattering phase shift: $\Phi_M = \phi_R + \phi_{sc}$

$$\Phi_M = -\arctan\left(\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2}\right) + \arctan\left(\frac{1 - 2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2} \cdot \frac{\mathrm{Im}\widetilde{I}_2}{P_M + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}\right) .$$

(P. Zhuang, J. Hufner, S. P. Klevansky, Nucl. Phys. A576, 525-552 (1994).)

Zhuang formula - derivation

$$\Phi_M = \phi_R + \phi_{sc} \ .$$

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We can represent the total scattering phase shift Φ_M as

$$\Phi_M = \frac{i}{2} \ln \frac{1 - 2G_S \Pi_M(\omega + i\eta, \vec{q})}{1 - 2G_S \Pi_M(\omega - i\eta, \vec{q})} \ .$$

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Using

$$\Pi_M \left(q_0, \vec{0}\right) = 4N_c N_f \ I_1 - 2N_c N_f P_M \ I_2 = \widetilde{I}_1 - P_M \widetilde{I}_2,$$

and

$$\frac{i}{2} \ln \left(\frac{1 - ix}{1 + ix}\right) = \arctan x$$

we show that

$$\Phi_M = -\arctan\left[\frac{2G_S P_M \mathrm{Im}\widetilde{I}_2}{1 - 2G_S \widetilde{I}_1 + 2G_S P_M \mathrm{Re}\,\widetilde{I}_2}\right].$$

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(several steps more)

$$\Phi_M = -\arctan\left[\frac{\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2} - \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\mathrm{Im}\widetilde{I}_2}{P_M + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}}{1 + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\mathrm{Im}\widetilde{I}_2^2}{P_M\mathrm{Re}\,\widetilde{I}_2 + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2^2}}\right].$$

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Using

$$-(\arctan\alpha \pm \arctan\beta) = -\arctan\left[\frac{\alpha\pm\beta}{1\mp\alpha\beta}\right]$$

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Using

$$-(\arctan\alpha \pm \arctan\beta) = -\arctan\left[\frac{\alpha\pm\beta}{1\mp\alpha\beta}\right]$$

we get

$$\Phi_M = -\arctan\left(\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2}\right) + \arctan\left(\frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\cdot\frac{\mathrm{Im}\widetilde{I}_2}{P_M + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}\right).$$

Our approach

Now then

$$\Phi_M = \phi_{sc} + \phi_R = -\arctan\left(\frac{\mathrm{Im}\widetilde{I}_2}{\mathrm{Re}\widetilde{I}_2}\right) + \arctan\left(\frac{1 - 2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2} \cdot \frac{\mathrm{Im}\widetilde{I}_2}{P_M + \frac{1-2G_S\widetilde{I}_1}{2G_S|\widetilde{I}_2|^2}\mathrm{Re}\widetilde{I}_2}\right)$$

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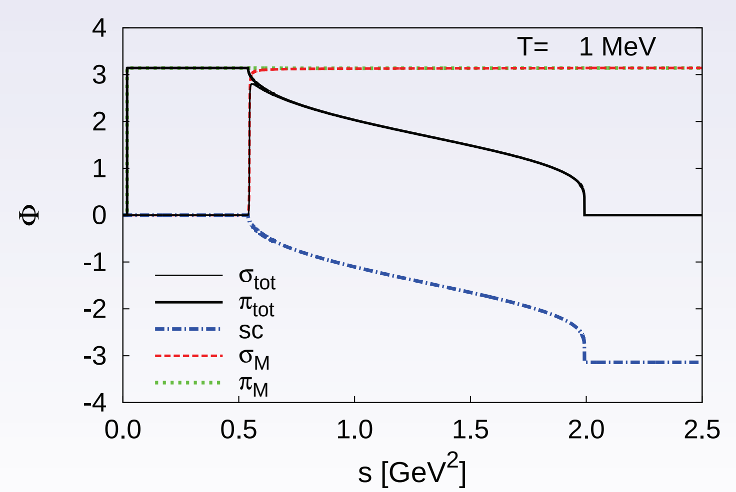
Our analysis is a combined approach:

$$D_M(s) = \frac{1}{\pi}\frac{d\phi_M(s)}{ds} = \begin{cases} \delta(s-M_M^2) + \frac{1}{\pi}\frac{d}{ds}\phi_{sc}(s)\ , & T < T_{\mathrm{Mott}}\ , \\ \frac{a_R}{\pi}\frac{\Gamma_M M_M}{(s-M_M^2)^2+\Gamma_M^2 M_M^2} + \frac{1}{\pi}\frac{d}{ds}\phi_{sc}(s)\ , & T > T_{\mathrm{Mott}}\ . \end{cases}$$

Phase shifts



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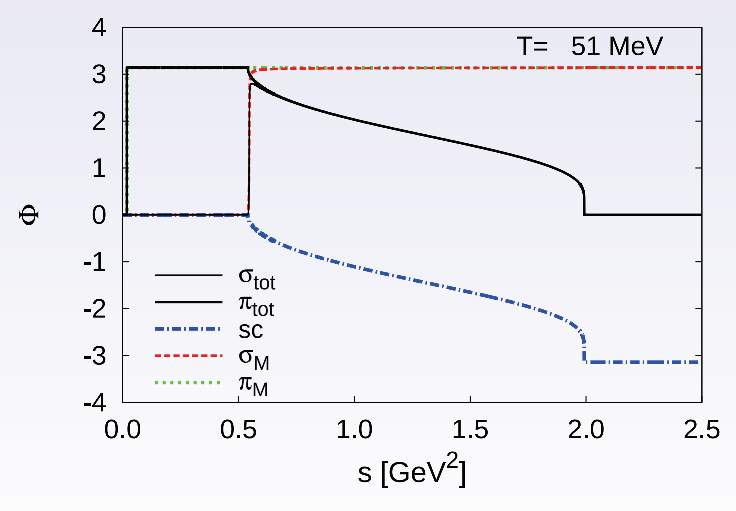

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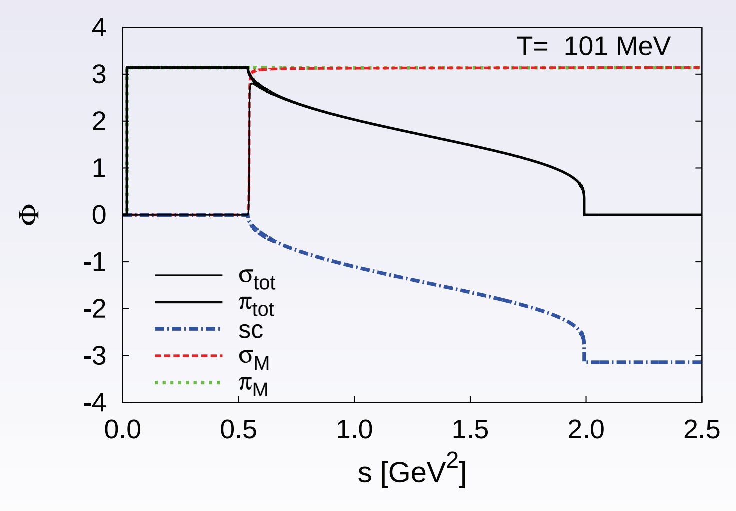

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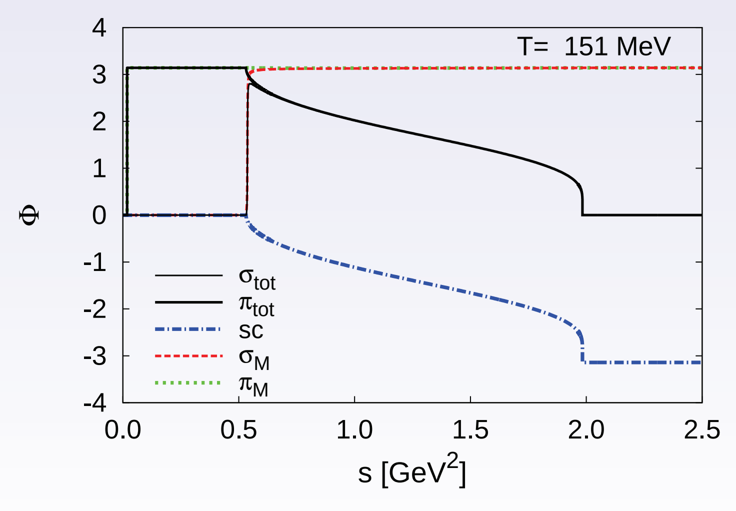

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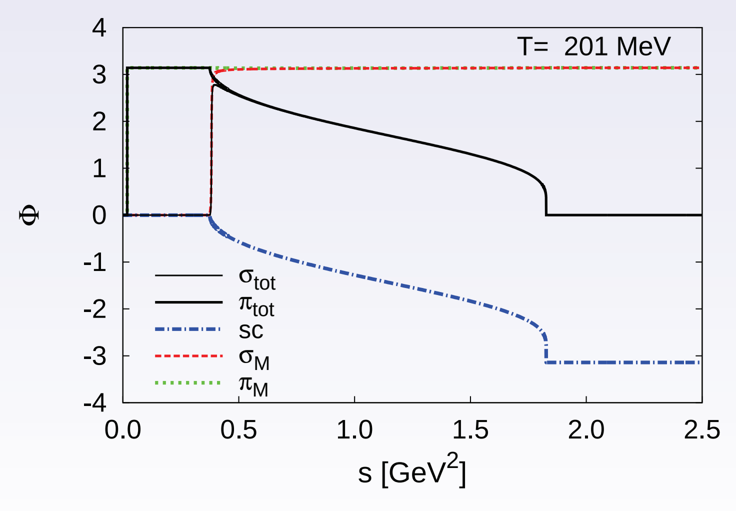

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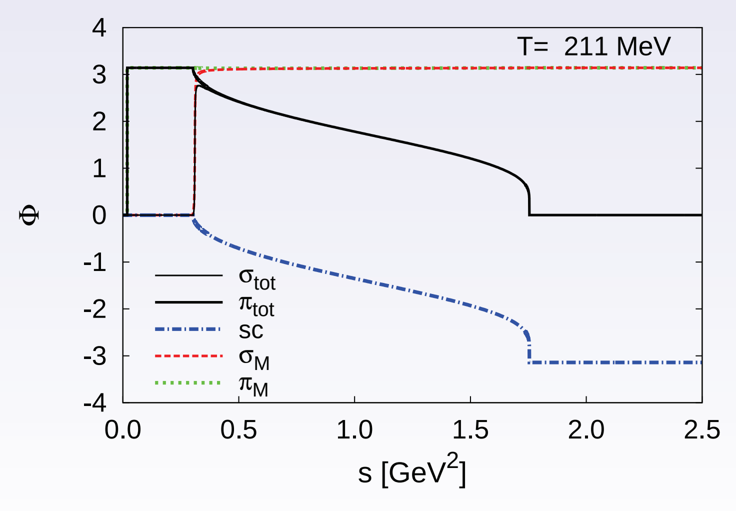
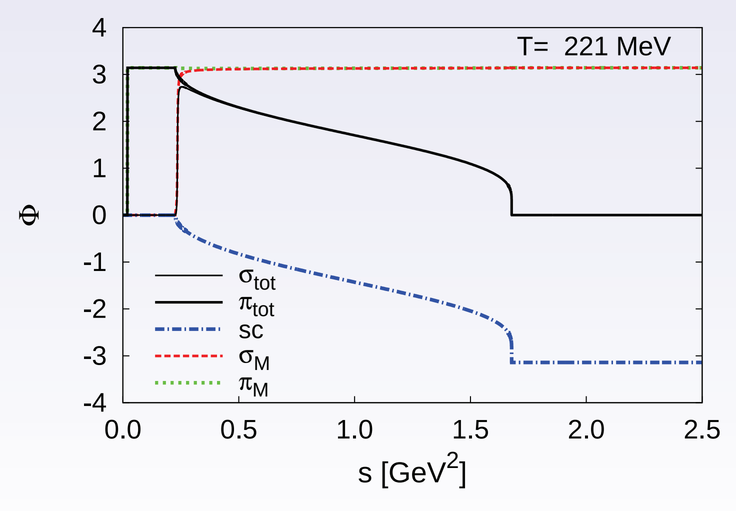
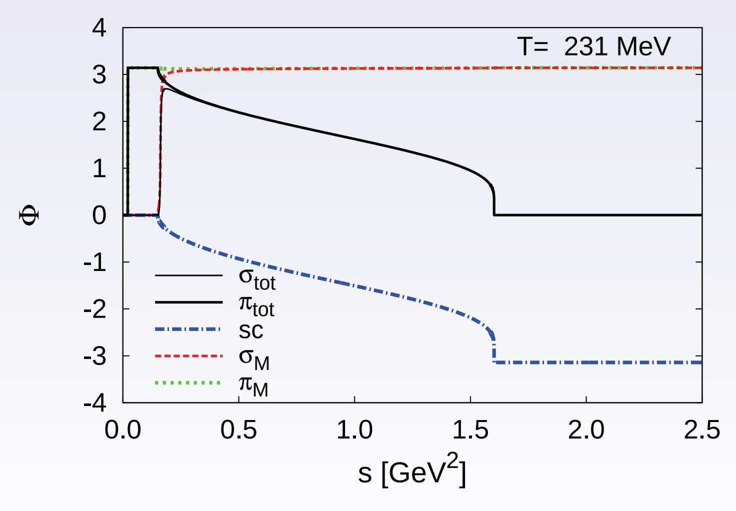


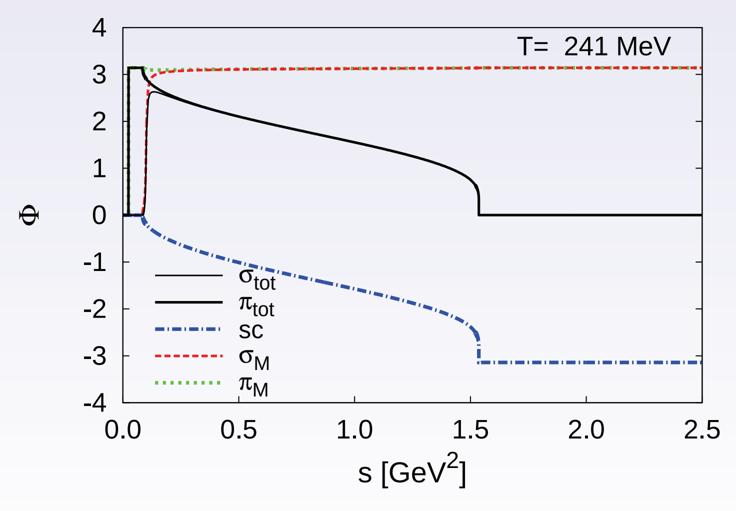
Phase shifts



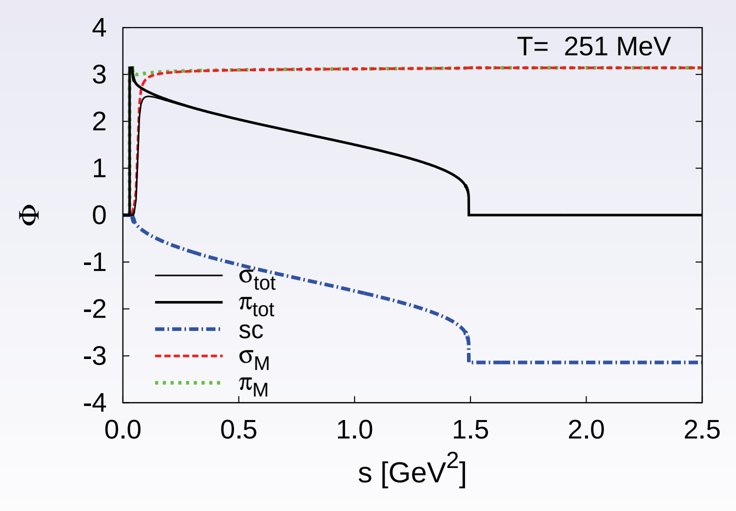
Phase shifts



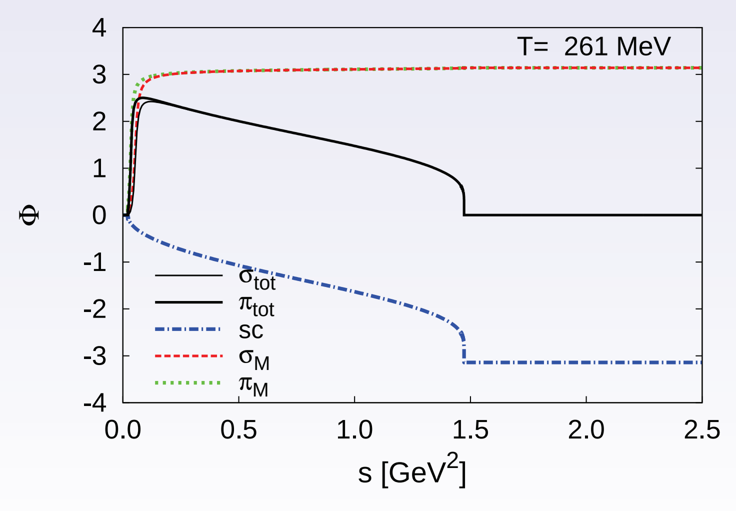
Phase shifts



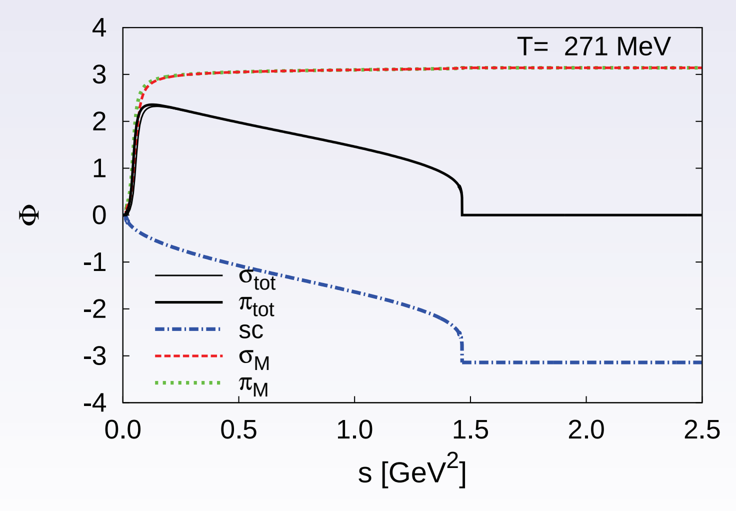
Phase shifts



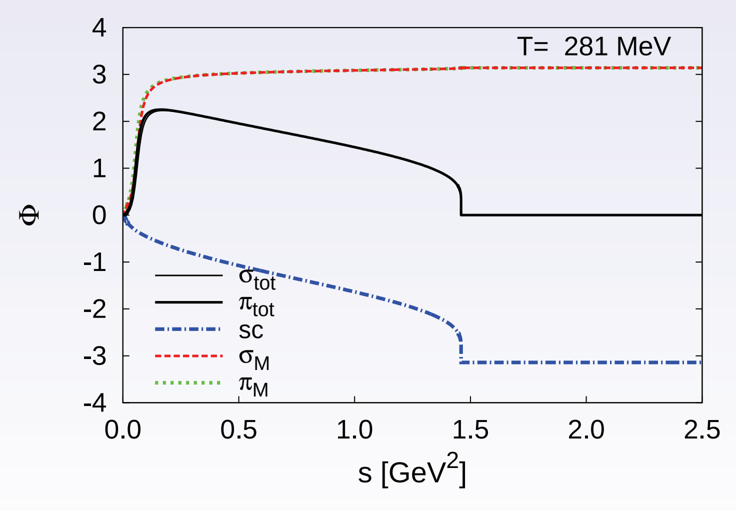
Phase shifts



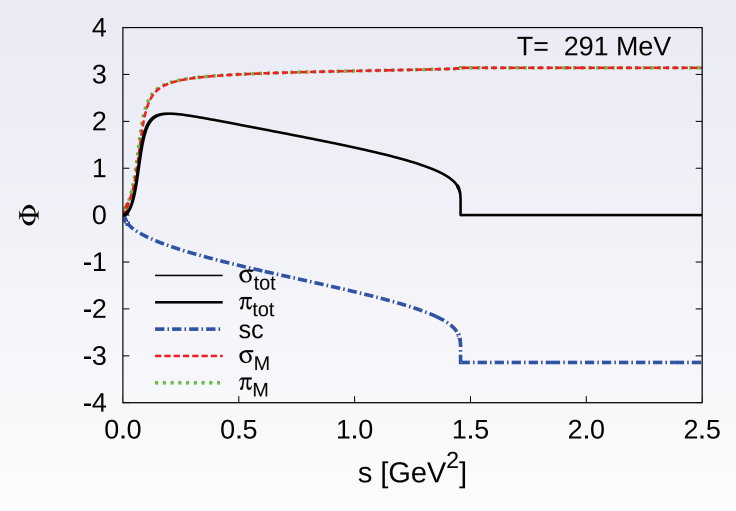
Phase shifts



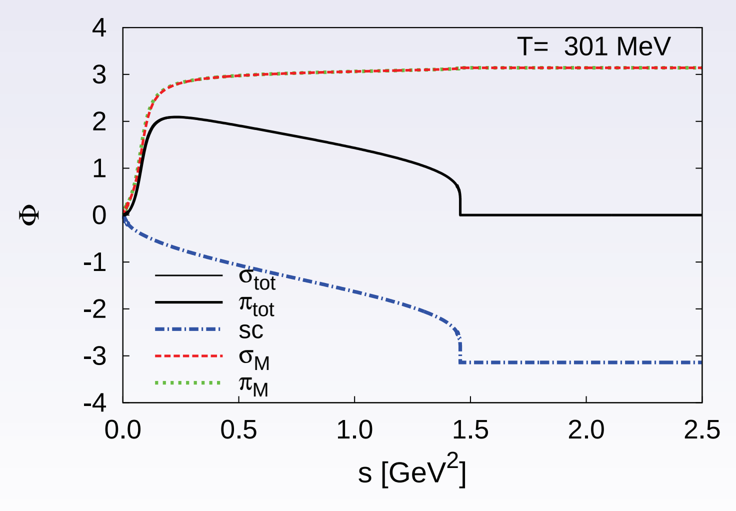
Phase shifts



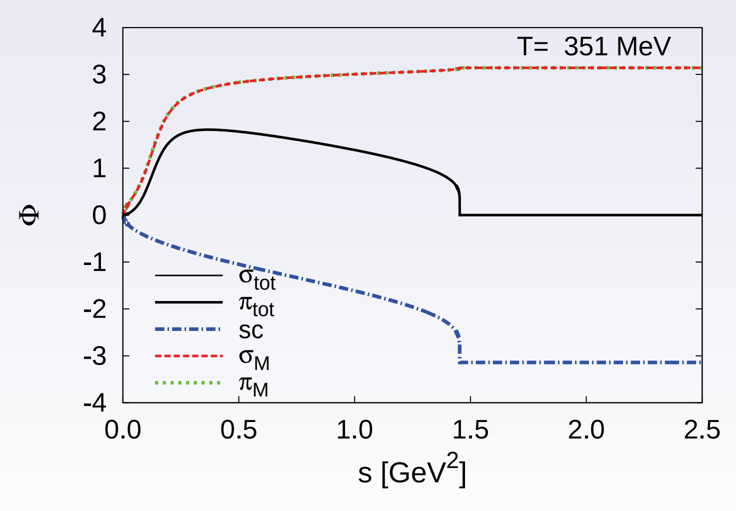
Phase shifts



Phase shifts



Phase shifts



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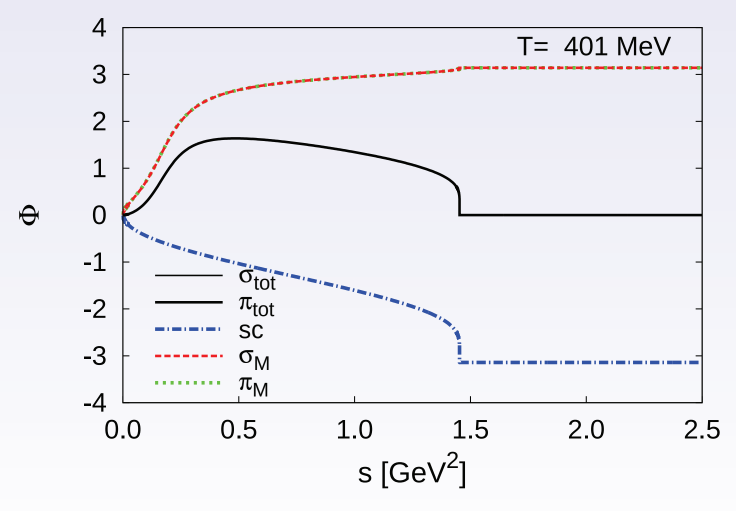
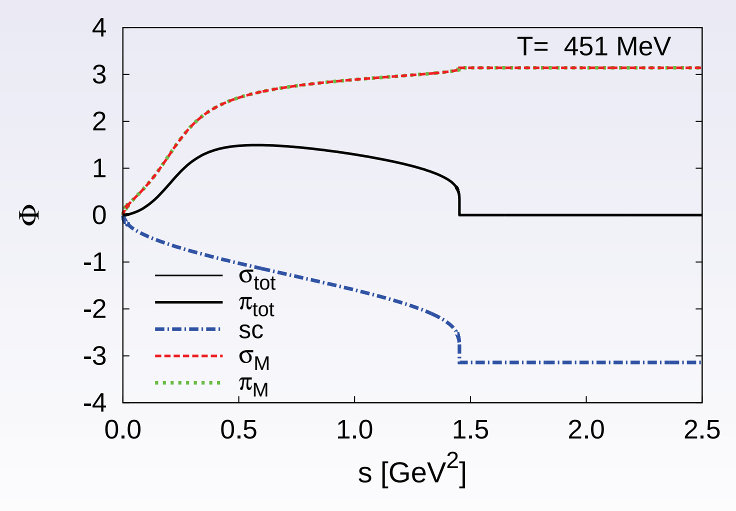
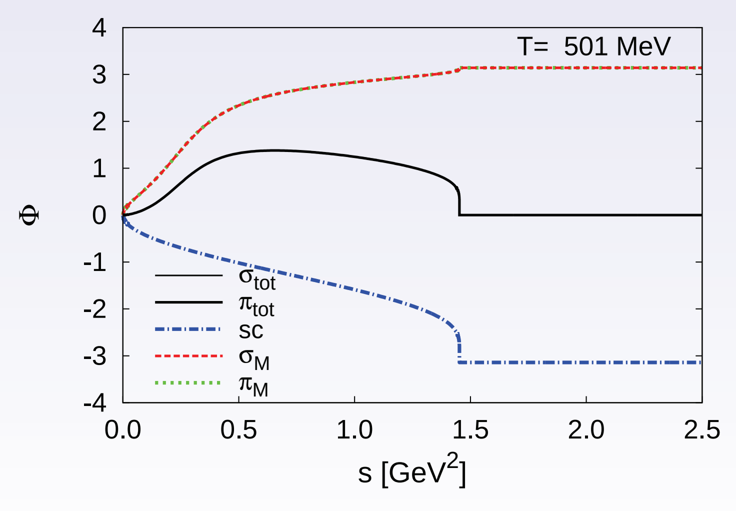


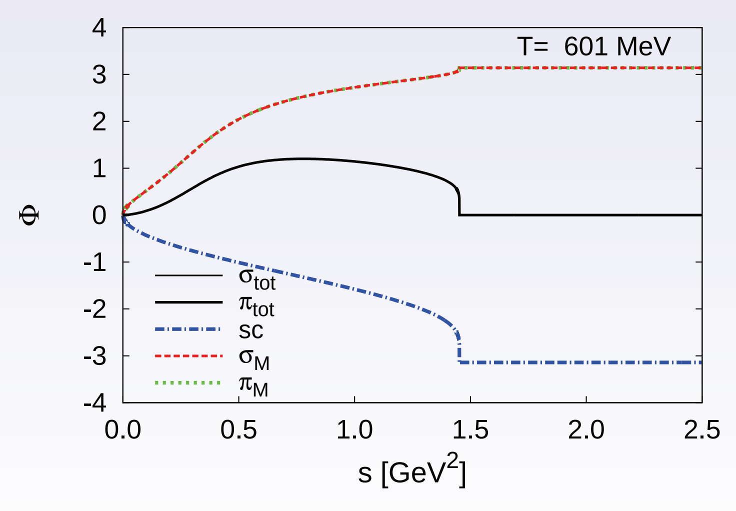
Phase shifts



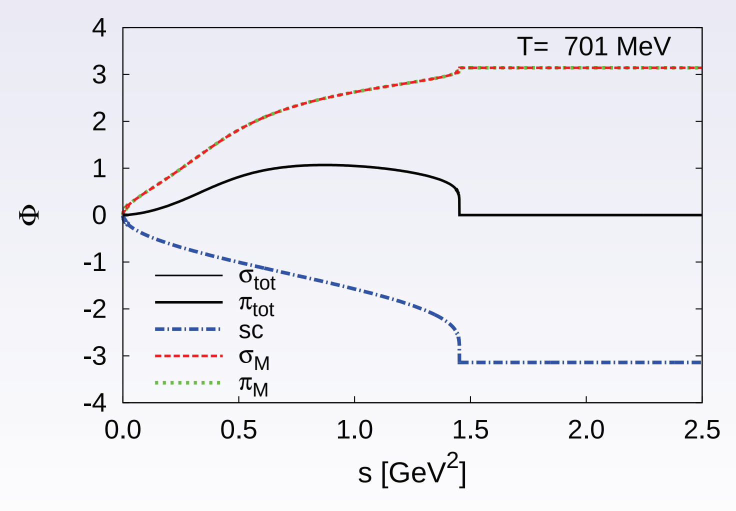
Phase shifts



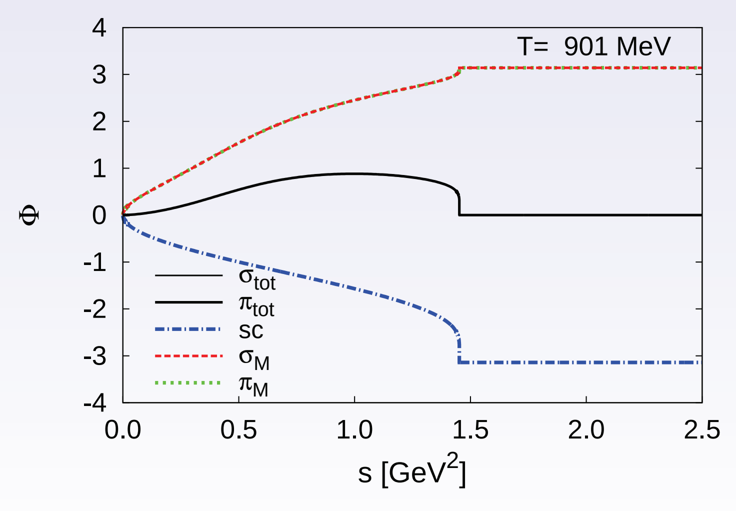
Phase shifts



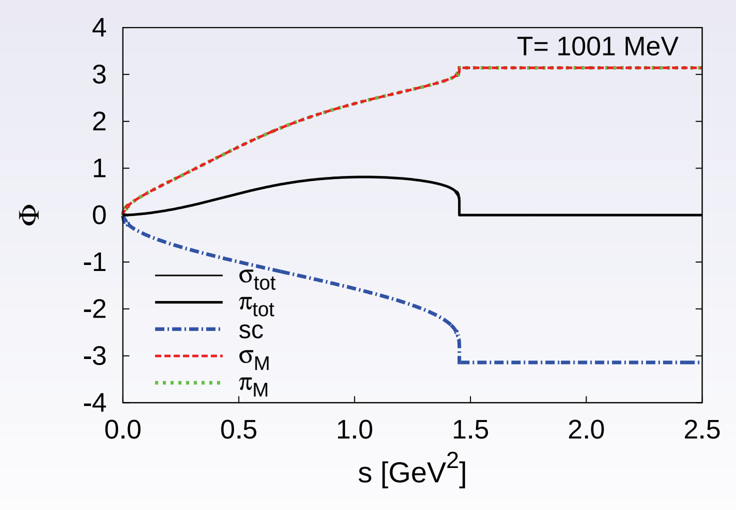
Phase shifts



Phase shifts



Phase shifts



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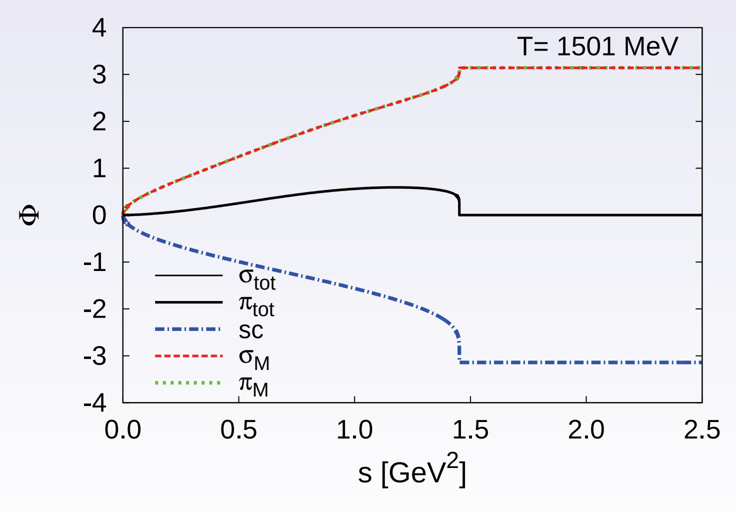

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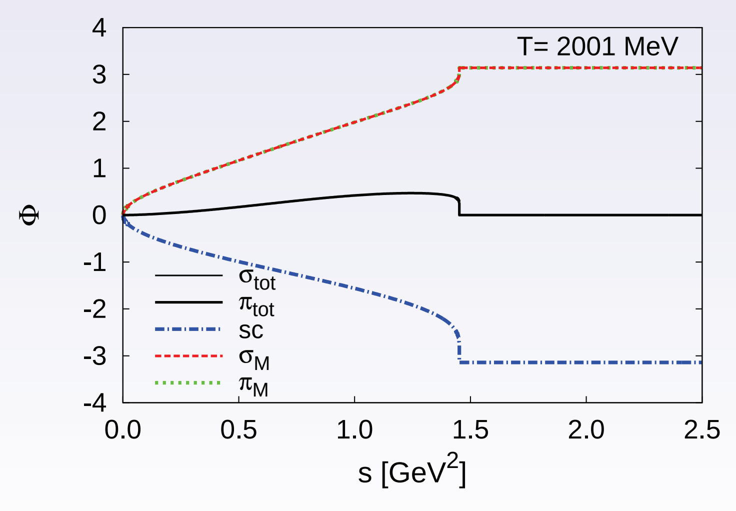
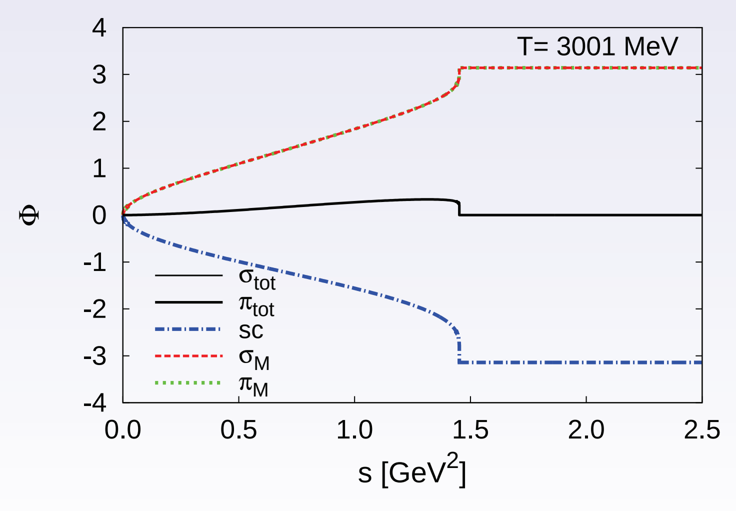
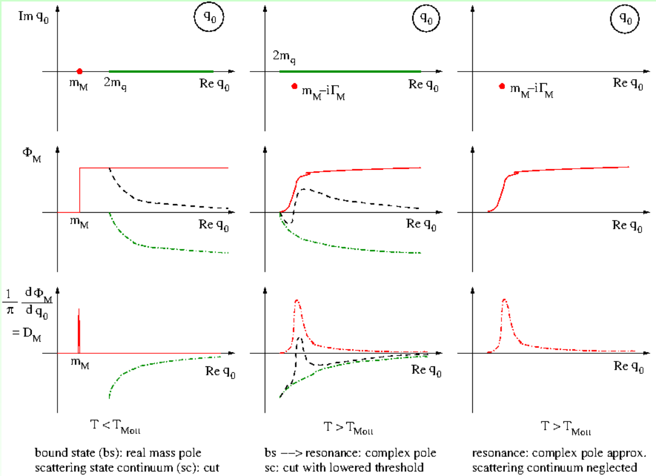


Phase shifts

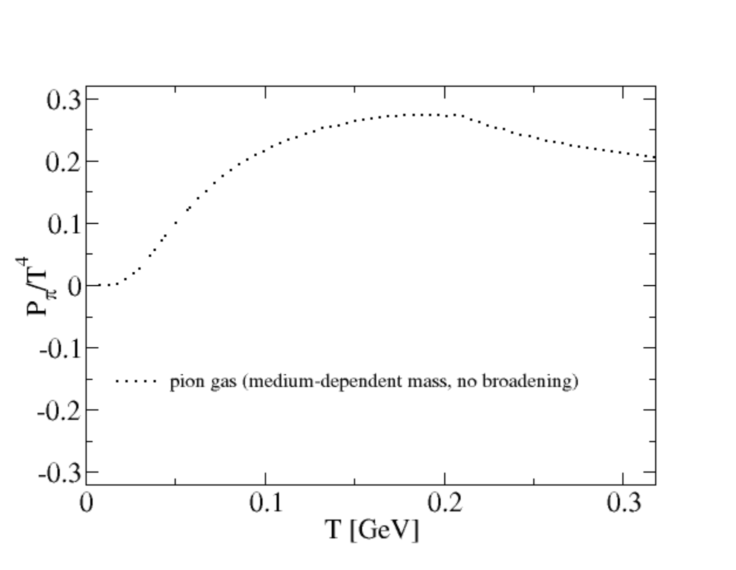


Summary: Levinson's Theorem & analytical properties



Pion pressure

Role of scattering continuum (Levinson theorem!) for pressure:



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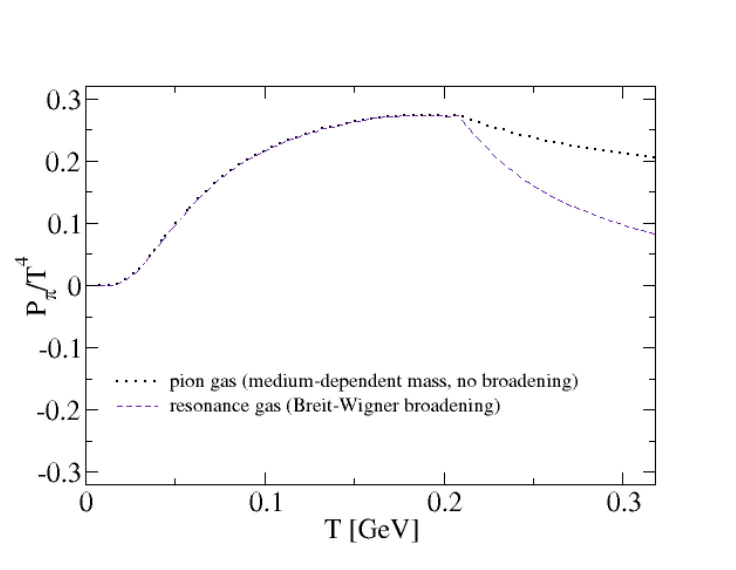

Pion pressure

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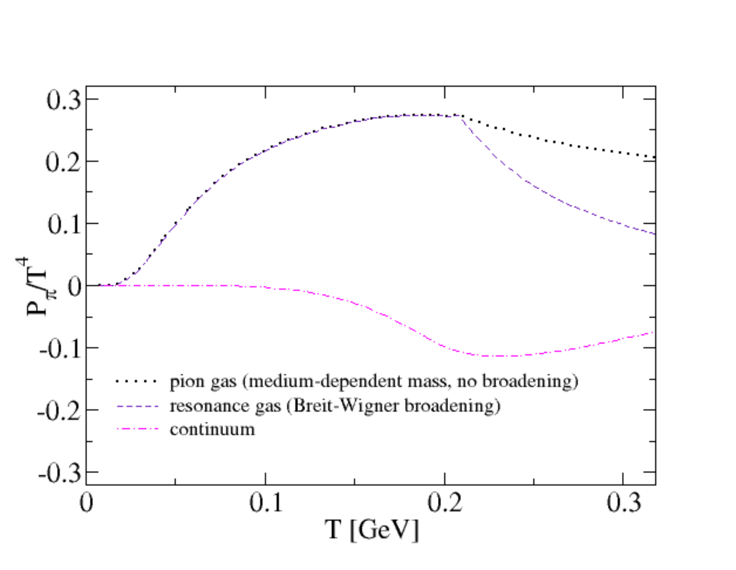

Pion pressure

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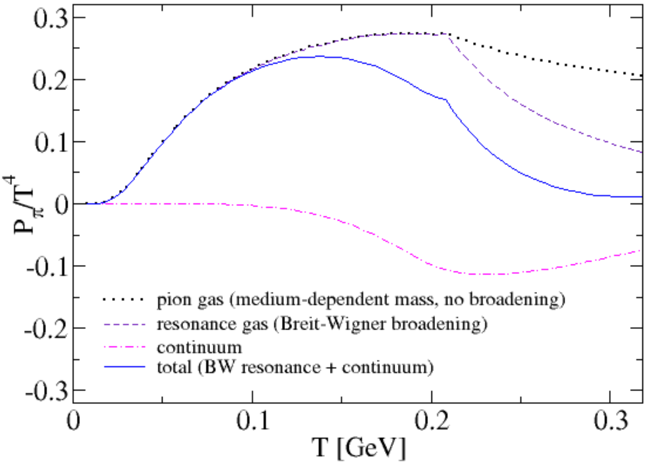
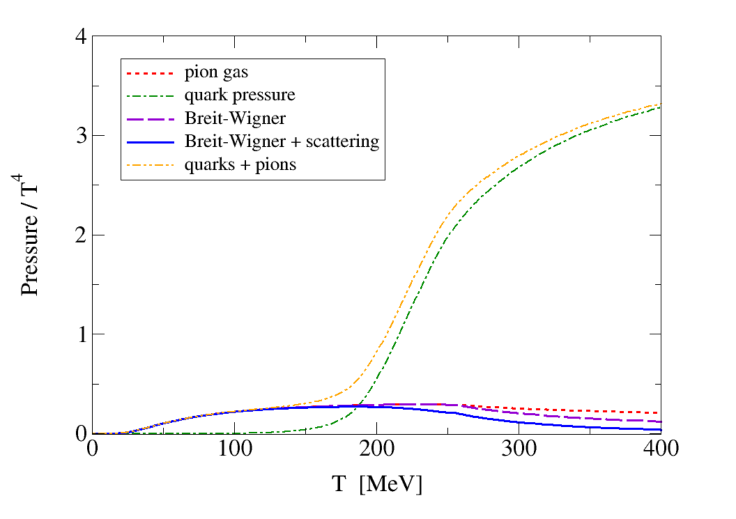


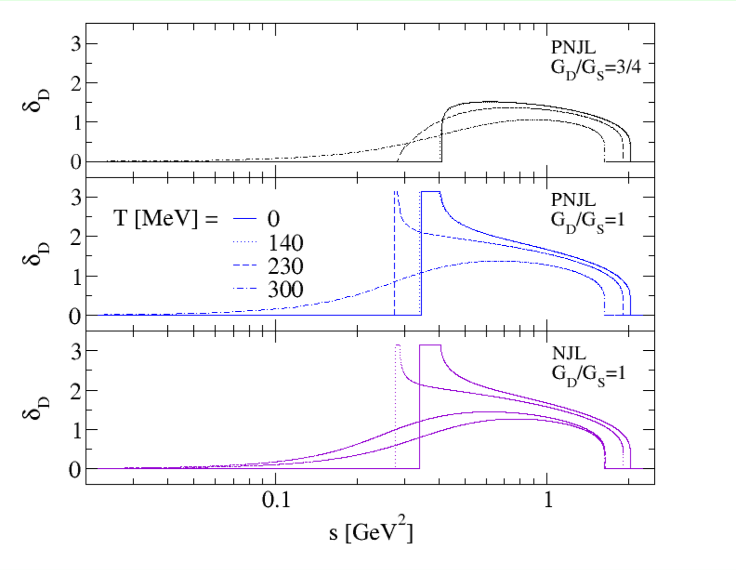
Quark + pion pressure



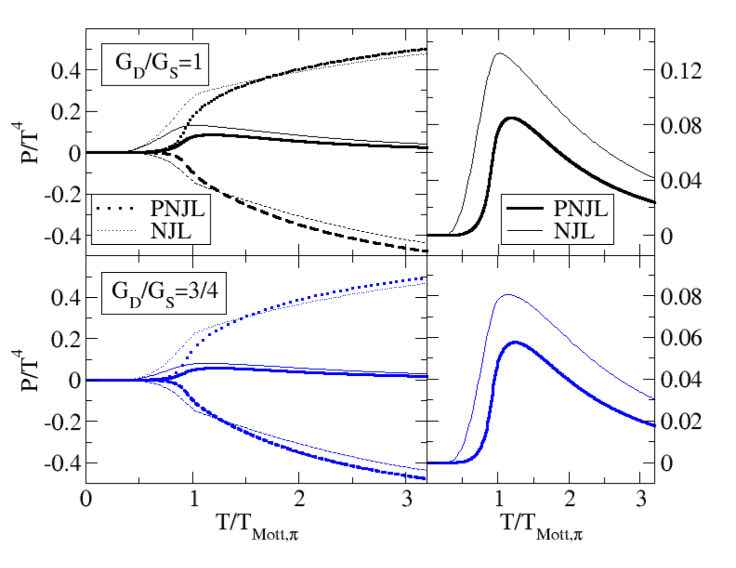
A fantastic result !!

A fantastic result !!

Diquark phase shifts at finite temperature

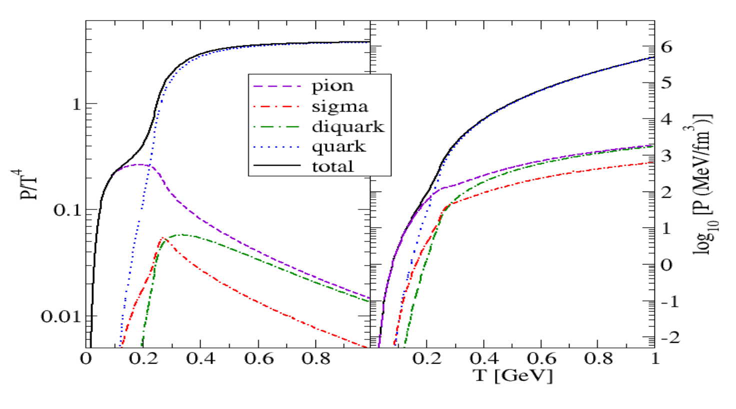


Polyakov-loop suppression of diquark pressure



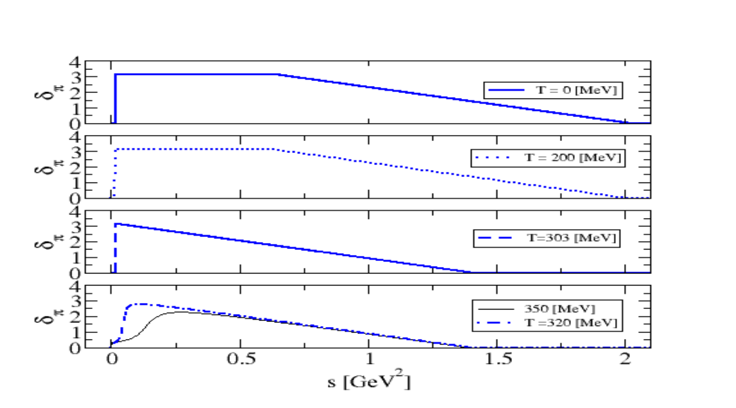
D.B., M. Buballa and A. Dubinin, in preparation (2014)

Partial pressures in a quark-meson-diquark system



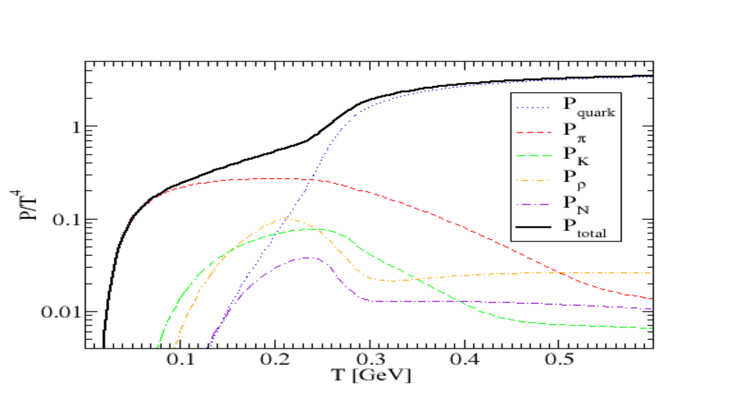
D.B., M. Buballa and A. Dubinin, in preparation (2014)

Generic model for hadronic phase shifts in medium



D.B., M. Buballa and A. Dubinin, in preparation (2014)

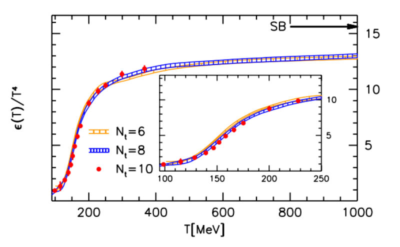
Schematic hadron resonance gas with Mott effect



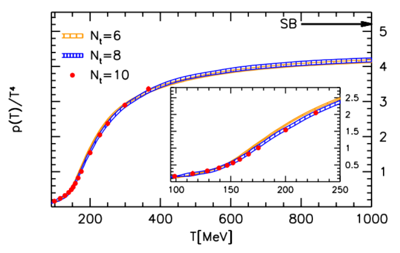
D.B., M. Buballa and A. Dubinin, in preparation (2014)

Conclusions - part I

- PNJL model: suitable for describing χSB and restoration at finite temperature, it describes pions as $q\bar{q}$ bound states and pseudo-Goldstone bosons $\quad \rightarrow \quad m(T)$, $M_M(T)$, $\Gamma_M(T)$
- pressure $P(T)$ for quark mean-field: suppression of quarks for $T < T_c$, correct SB limit
- Gaussian fluctuations in σ, $\vec{\pi}$: Generalized Beth-Uhlenbeck
- resonance approximation for pionic mode above $T_{\rm Mott}$ **violates** Levinson's theorem!
- an analytic formula for the continuum states' contribution to the scattering phase shift together with the Breit-Wigner ansatz for the resonance
- resulting phase shift obeys the Levinson theorem
  $\rightarrow \quad$ pressure reduction (ideally to zero) for $T > T_{\rm Mott}$
- **outlook**: semi-microscopic approach to implement Mott effect for hadrons (here: only pions) consistent with Levinson's theorem into hadron resonance gas (HRG) models



The energy density normalized by T^4
as a function of the temperature
on N_t =6,8 and 10 lattices.



The pressure normalized by T^4
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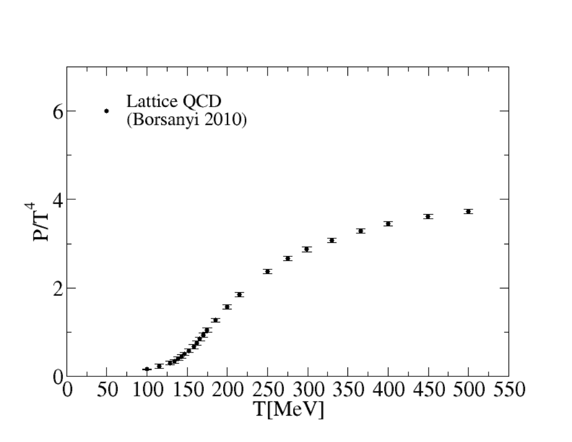
S. Borsanyi *et al.* *"The QCD equation of state with dynamical quarks,"* JHEP **1011**, 077 (2010)

Hagedorn resonance gas: comparison with Lattice QCD



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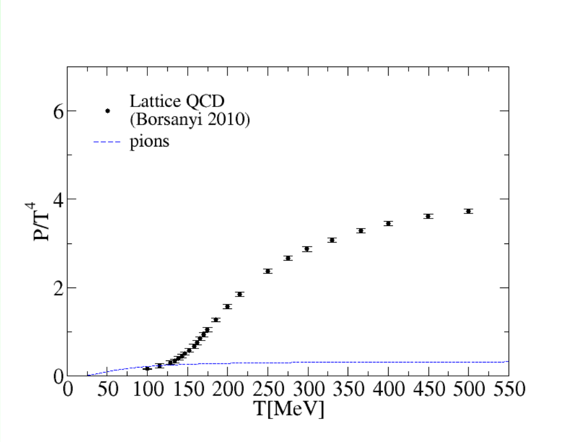
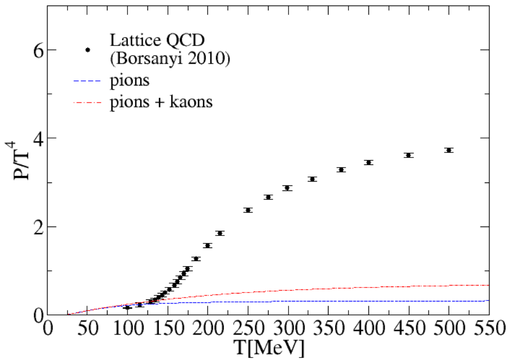
Hagedorn resonance gas: comparison with Lattice QCD



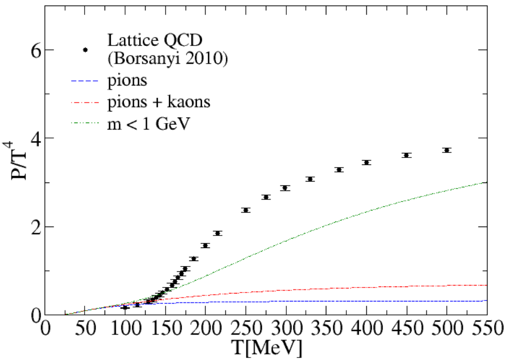
$$P_\pi(T) = 3T \int \frac{d^3p}{(2\pi)^3} \ln\left[1 - \mathrm{e}^{-\sqrt{p^2+m_\pi^2}/T}\right]$$

Hagedorn resonance gas: comparison with Lattice QCD



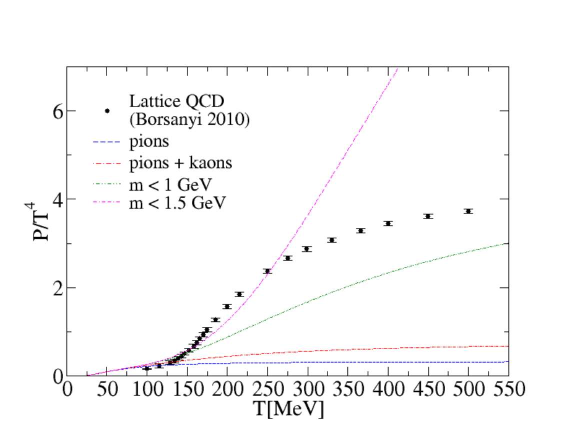
$$P_{\pi+K}(T) = \sum_{i=\pi,K} d_i T \int \frac{d^3p}{(2\pi)^3} \ln\left[1 - \mathrm{e}^{-\sqrt{p^2+m_i^2}/T}\right]$$

Hagedorn resonance gas: comparison with Lattice QCD



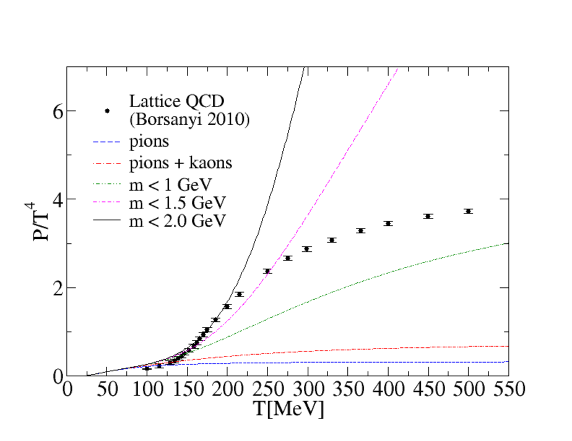
$$P_{\mathrm{HRG}}(T) = \sum_{i, m_i < 1\mathrm{GeV}} \delta_i d_i T \int \frac{d^3p}{(2\pi)^3} \ln\left\{1 + \delta_i \mathrm{e}^{-\sqrt{p^2+m_i^2}/T}\right\}$$

Hagedorn resonance gas: comparison with Lattice QCD



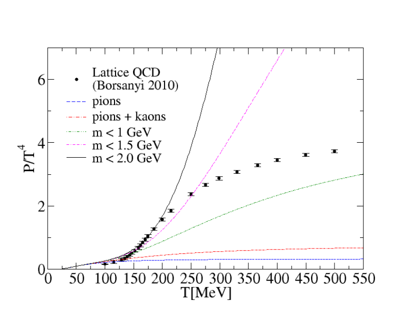
$$P_{\mathrm{HRG}}(T) = \sum_{i, m_i < 1.5\mathrm{GeV}} \delta_i d_i T \int \frac{d^3p}{(2\pi)^3} \ln \left\{ 1 + \delta_i \mathrm{e}^{-\sqrt{p^2+m_i^2}/T} \right\}$$

Hagedorn resonance gas: comparison with Lattice QCD



$$P_{\rm HRG}(T) = \sum_{i,m_i<2{\rm GeV}} \delta_i d_i T \int \frac{d^3p}{(2\pi)^3} \ln\left\{1+\delta_i {\rm e}^{-\sqrt{p^2+m_i^2}/T}\right\}$$

Hagedorn resonance gas: comparison with Lattice QCD



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Courtesy: M. Naskręt (UWr)

Hagedorn resonance gas: hadrons with finite widths

The energy density per degree of freedom with the mass M

$$\varepsilon(T,\mu_B,\mu_S) = \sum_{i:\ m_i<m_0} g_i\ \varepsilon_i(T,\mu_i;m_i)$$

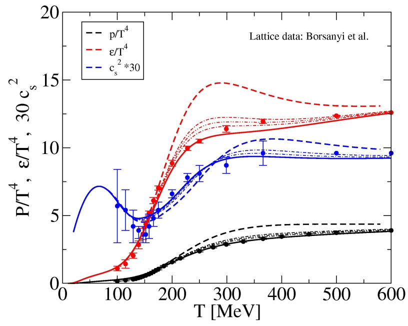
$$+ \sum_{i:\ m_i\geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2)\ A(M,m_i)\ \varepsilon_i(T,\mu_i;M),$$

Spectral function

$$A(M,m) = N_M \frac{\Gamma\cdot m}{(M^2-m^2)^2+\Gamma^2\cdot m^2}\ ,$$

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)$$

Hagedorn resonance gas: hadrons with finite widths



$$P(T) = T \int_0^T dT' \, \frac{\varepsilon(T')}{T'^2} \; .$$

N_m in the range from $N_m = 2.5$ (dashed) to $N_m = 3.0$ (solid).
$C_\Gamma = 10^{-4}$
$N_T = 6.5$
$T_H = 165$ MeV

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)$$

D.B. & K. Bugaev, Fizika B **13**, 491 (2004); PPNP **53**, 197 (2004)

State-dependent hadron resonance width

$$A_i(M, m_i) = N_M \frac{\Gamma_i \cdot m_i}{(M^2 - m_i^2)^2 + \Gamma_i^2 \cdot m_i^2} ,$$

$$\Gamma_i(T) = \tau^{-1}_{\rm coll,i}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T \; n_j(T)$$

D. B., J. Berdermann, J. Cleymans, K. Redlich, PPN 8, 811 (2011) [arXiv:1102.2908]

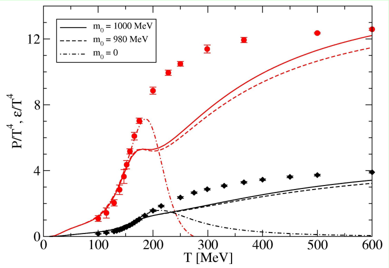
For pions (mesons)

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1} \; ; \; \langle \bar{q}q \rangle_T \Longrightarrow \text{Talk by J. Jankowski}$$

For nucleons (baryons)

$$r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu); \qquad r_0 = 0.45\text{fm} \;\; \text{pion cloud.}$$

Mott-Hagedorn resonance gas



Quarks and gluons are missing!

Mott-Hagedorn resonance gas: Pressure and energy density for three values of the mass threshold

$m_0 = 1.0$ GeV (solid lines)

$m_0 = 0.98$ GeV (dashed lines)

and

$m_0 = 0$ (dash-dotted lines)

Quarks and gluons in the PNJL model

Systematic expansion of the pressure as the thermodynamical potential in the grand canonical ensemble for a chiral quark model of the PNJL type beyond its mean field description $P_{\rm PNJL,MF}(T)$ by including perturbative corrections

$$P(T) = P_{\rm MHRG}(T) + P_{\rm PNJL,MF}(T) + P_2(T) \ ,$$

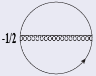
$$P_{\rm MHRG}(T) = \sum_i \delta_i d_i T \int \frac{d^3p}{(2\pi)^3} \int dM A_i(M, m_i) T \ln \left\{ 1 + \delta_i {\rm e}^{-[\sqrt{p^2+M^2}-\mu_i]/T} \right\} \ ,$$

Quark and gluon contributions

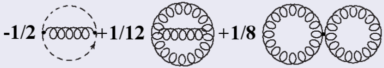
$$P_2(T) = P_2^{\rm quark}(T) + P_2^{\rm gluon}(T)$$

Quark and gluon contributions

$P_2^{\text{quark}}(T)$



$P_2^{\text{gluon}}(T)$



Total perturbative QCD correction

$$P_2 = -\frac{8}{\pi}\alpha_s T^4 (I_\Lambda^+ +$$

$$\frac{3}{\pi^2}((I_\Lambda^+)^2 + (I_\Lambda^-)^2))$$

$$\xrightarrow[\Lambda/T\to 0]{} -\frac{3\pi}{2}\alpha_s T^4$$

where

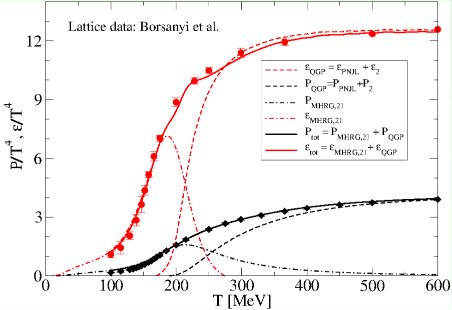
$$I_\Lambda^\pm = \int_{\Lambda/T}^\infty \frac{\mathrm{d}x \; x}{\mathrm{e}^x \pm 1}$$

- Energy corrections

$$\varepsilon_2(T) = T\frac{dP_2(T)}{dT} - P_2(T)\ .$$

Quarks, gluons and hadron resonances

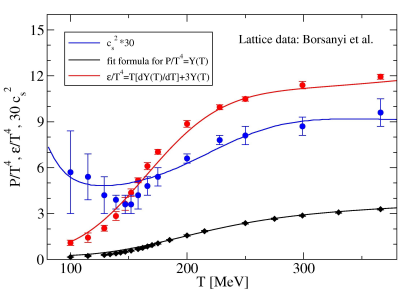
$$P_{\mathrm{MHRG}}(T) = \sum_i \delta_i d_i \int \frac{d^3p}{(2\pi)^3} \int dM\, A_i(M, m_i) T \ln\left\{1 + \delta_i \mathrm{e}^{-[\sqrt{p^2+M^2}-\mu_i]/T}\right\} ,$$



- Quark-gluon plasma contributions are described within the improved PNJL model with α_s corrections .
- Heavy hadrons are described within the resonance gas with finite width exhibiting a Mott effect at the coincident chiral and deconfinement transitions.

L. Turko, D. Blaschke, D. Prorok, J. Berdermann, J. Phys. Conf. Ser. **455**, 012056 (2013)

Quarks, gluons and hadron resonances II

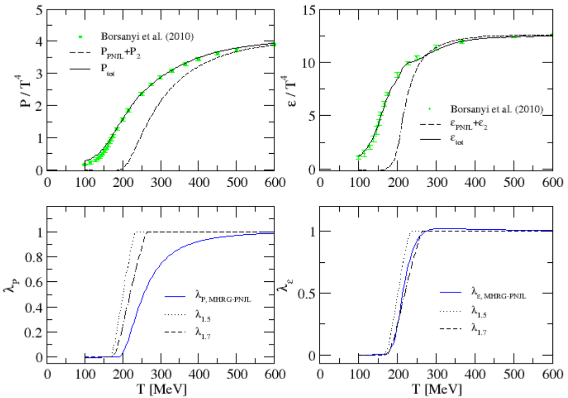


- Contribution restricted to the region around the chiral/deconfinement transition 170-250 MeV
- Fit formula for the pressure

$$P = aT^4 + bT^{4.4} \tanh(cT - d),$$

$$a = 1.0724,\ b = 0.2254,$$
$$c = 0.00943,\ d = 1.6287$$

Application: Parton fraction in the EoS → HIC Simulations



L. Turko et al., [arxiv:1402.xxxx] (07.02.2014)

Compare: M. Nahrgang et al., *Influence of hadronic bound states above* T_c ..., PRC 89, 014004 (2014), [arxiv:1305.6544]

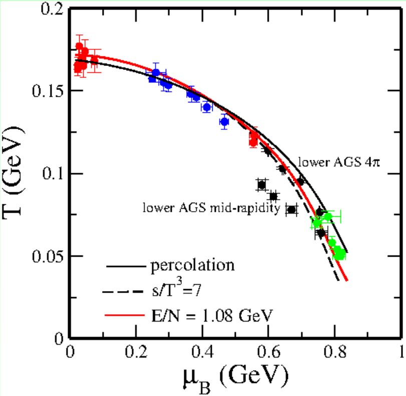
Conclusions - part II

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances.
- A generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium

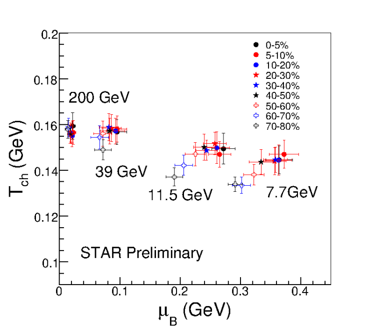
To do

- Join hadron resonance gas with quark-gluon model.
- Calculate kurtosis and compare with lattice QCD.
- Spectral function for all low-lying hadrons from microphysics (PNJL model ...).

Part III: Chemical Freeze-out in the QCD Phase Diagram



"Old" freeze-out data from RHIC (red), SPS (blue), AG (black), SIS (green).



"New" freeze-out data from STAR BES @ RHIC.
Centrality dependence!

F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C73 (2006) 044905
Lokesh Kumar (STAR Collab.), arxiv:1201.4203 [nucl-ex]

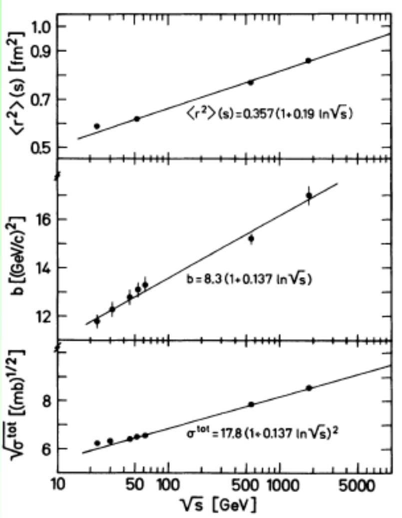
Chemical freeze-out condition

$$\tau_{\rm exp}(T,\mu) = \tau_{\rm coll}(T,\mu)$$

$$\tau_{\rm coll}^{-1}(T,\mu) = \sum_{i,j} \sigma_{ij} n_j$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

D.B. et al., Few Body Systems (2011) arxiv:1109.5391 [hep-ph]



B. Povh, J. Hüfner, PRD 46 (1992) 990

Hadronic radii and chiral condensate

$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} F_\pi^{-2}(T,\mu) \ .$$

$$F_\pi^2(T,\mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / m_\pi^2 .$$

$$r_\pi^2(T,\mu) = \frac{3m_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1} \ .$$

$$r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu) \ ,$$

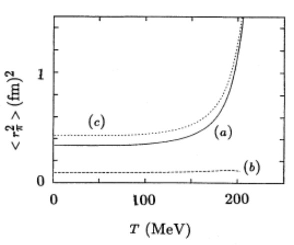
Expansion time from entropy conservation

$S = s(T,\mu)\ V(\tau_{\rm exp}) = {\rm const}$

$$\tau_{\rm exp}(T,\mu) = a\ s^{-1/3}(T,\mu) \ ,$$

D.B., J. Berdermann, J. Cleymans, K. Redlich,

Few Body Systems (2011) [arxiv:1109.5391]



H.-J. Hippe and S. Klevansky, PRC 52 (1995) 2172

Ladenburg (1992)

Clue to the effectiveness: (De)localization !

$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} F_\pi^{-2}(T,\mu) \ .$$

$$F_\pi^2(T,\mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / m_\pi^2 .$$

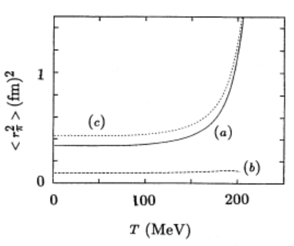
$$r_\pi^2(T,\mu) = \frac{3m_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1} \ .$$

$$r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu) \ ,$$

Effective hadron (de)localization at the chiral restoration transition, a la *Mott-Anderson (de-)localization* of electron wave functions in the insulator-metal transition [Nobel prize (1977)].

D.B., J. Berdermann, J. Cleymans, K. Redlich,

Few Body Systems (2011) [arxiv:1109.5391]



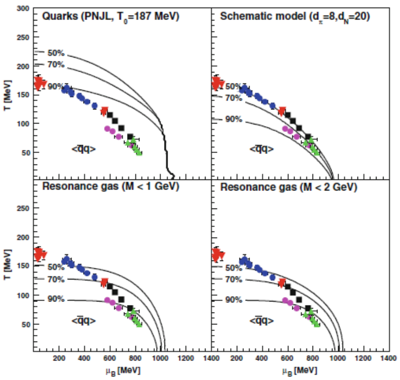
H.-J. Hippe and S. Klevansky, PRC 52 (1995) 2172

Sir N.F. Mott P.W. Anderson

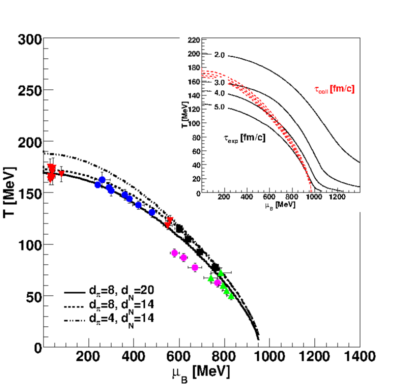
Chiral Condensate in a Hadron Resonance Gas

$$
\begin{aligned}
\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\rm vac}} &= 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \Bigg\{ 4N_c \int \frac{dp\, p^2}{2\pi^2} \frac{m}{\varepsilon_p} \left[f_\Phi^+ + f_\Phi^- \right] \\
&+ \sum_{M=f_0,\omega,\dots} d_M (2 - N_s) \int \frac{dp\, p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) \\
&+ \sum_{B=N,\Lambda,\dots} d_B (3 - N_s) \int \frac{dp\, p^2}{2\pi^2} \frac{m_B}{E_B(p)} \left[f_B^+(E_B(p)) + f_B^-(E_B(p)) \right] \Bigg\} \\
&- \sum_{G=\pi,K,\eta,\eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp\, \frac{p^2}{E_G(p)} f_G(E_G(p))
\end{aligned}
$$

S. Leupold, J. Phys. G (2006)
D.B., J. Berdermann, J. Cleymans,
K. Redlich, Few Body Systems
(2011)



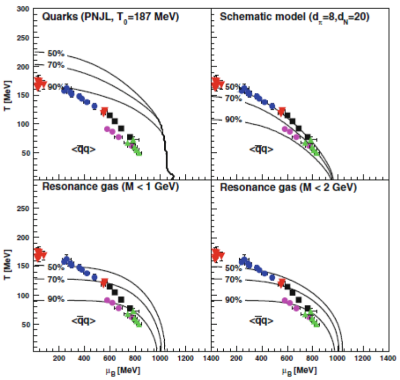
Chemical freeze-out vs. Condensate



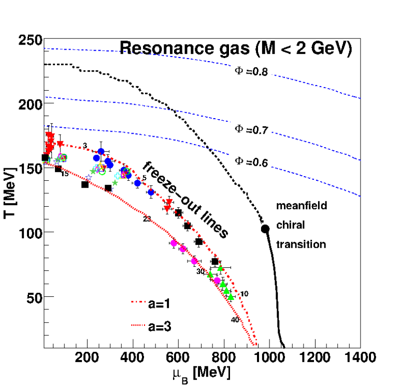
Chemical freeze-out from kinetic condition, schematic model

D.B., J. Berdermann, J. Cleymans, K. Redlich, Few Body Systems (2011)

Chemical Freeze-out and Chiral Condensate



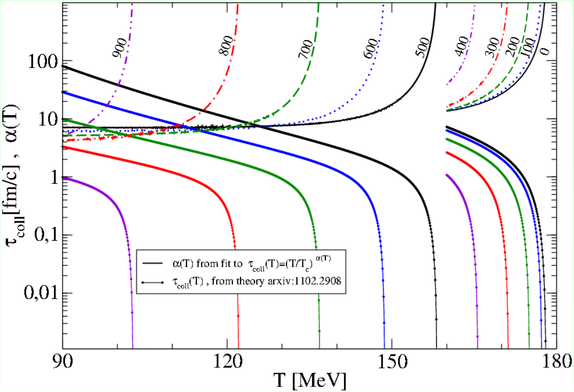
Chemical freeze-out vs. Condensate



Chemical freeze-out from kinetic condition, $a \sim$ inverse system size

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2014)

Strong T-Dependence of (inelastic) Collision Time



Klabucar, Berdermann (2006)

See: C. Blume in: NICA White Paper (2012)

C. Wetterich, P. Braun-Munzinger, J. Stachel, PLB (2004)

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2014)

Conclusions - part III

- The model works unreasonably well!
- Improvements are plenty:
  - Hadron mass formulae, e.g. from holographic QCD ...
  - Spectral functions - generalized Beth-Uhlenbeck
  - Thermodynamics ... hydrodynamics .
- Beyond freeze-out towards the deconfined phase: Mott-Hagedorn model

Visit (the University of) Wroclaw !

Thank you for collaboration !