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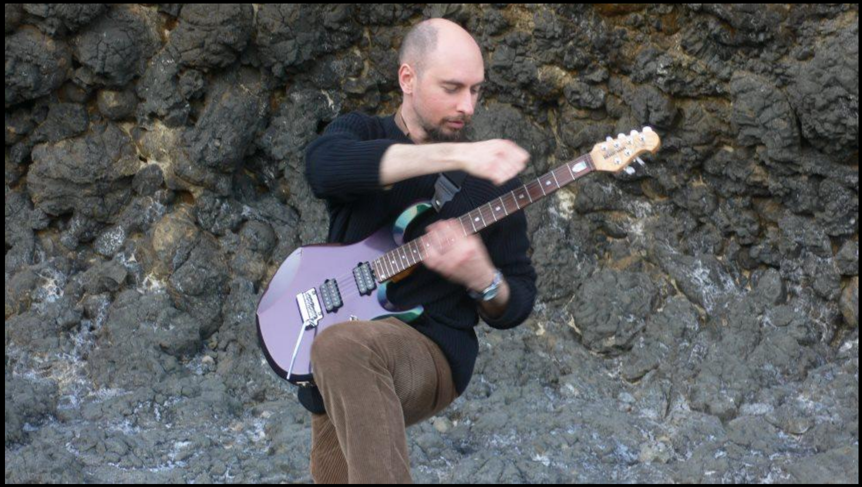
THERMALIZATION, ISOTROPIZATION AND FLOWS OF THE SHATTERED CGC

NeD/TURIC 2014

Collaborators:
- *Vincenzo Greco*
- *Salvatore Plumari*
- *Francesco Scardina*

In this talk:

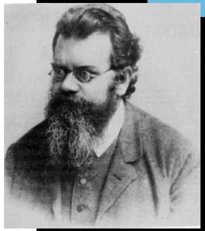
- Transport theory for heavy ion collisions
- Thermalization and Isotropization
- Flows
- Conclusions and Outlook



Boltzmann equation and QGP

In order to ***simulate*** the temporal evolution of the fireball we solve the ***Boltzmann equation*** for the parton distribution function ***f***:

$$\left\{p^{\mu}\partial_{\mu} + \left[p_{\nu}F^{\mu\nu} + m\partial^{\mu}m\right]\partial^{p}_{\mu}\right\}f(x,p) = C[f]$$



Field interaction (EoS)

Collision integral

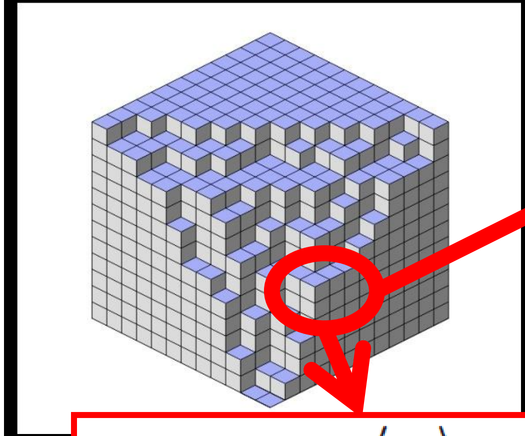
Collision integral: change of ***f*** due to collision processes in the phase space volume centered at (***x***,***p***).
Responsible for deviations from ideal hydro (non vanishing η/s).

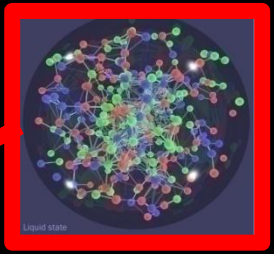
We map by C[f] the phase space evolution of a fluid which dissipates with a given value of η/s.

One can expand C[f] over microscopic details (2<->2,2<->3…), but in a hydro language this is irrelevant: ***only the global dissipative effect of C[f] is important.***

Transport *gauged* to hydro

We use ***Boltzmann equation*** to simulate a fluid at ***fixed eta/s*** rather than fixing a set of microscopic processes.
Total Cross section is ***computed*** in ***each configuration space cell*** according to ***Chapman-Enskog equation*** to give the ***wished value of eta/s***.





(.) Collision integral is gauged in each cell to assure that the fluid dissipates according to the desired value of eta/s.

(.) Microscopic details are not important: the specific microscopic process producing eta/s is not relevant, only macroscopic quantities are, in analogy with hydrodynamics.

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho}\frac{1}{\sigma}$$

Transport ⟵ ⟶ ***Hydro***

Description in terms of parton distribution function

Dynamical evolution governed by macroscopic quantities

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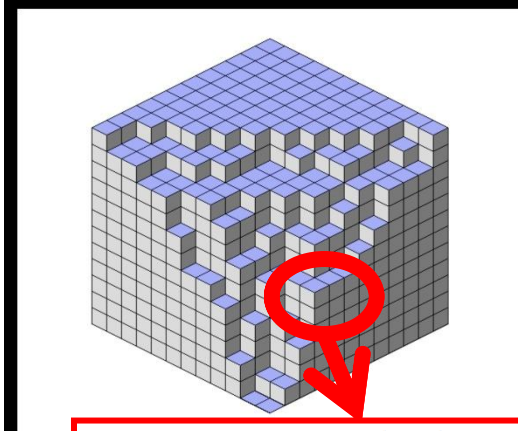
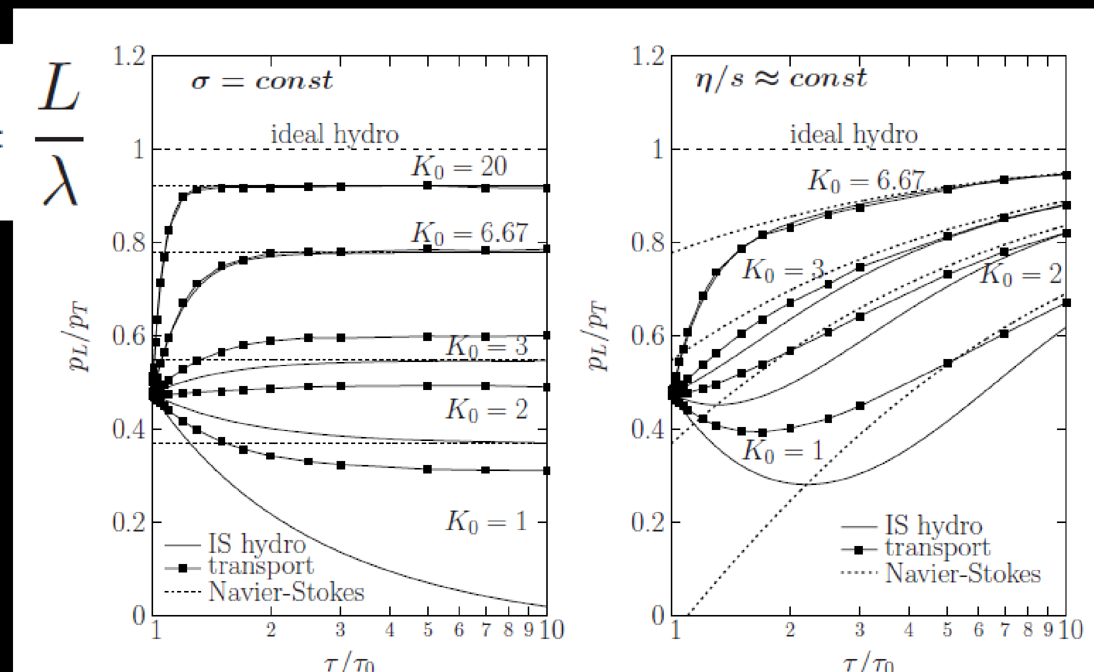


$$K_0 = \frac{L}{\lambda}$$

$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho} \frac{1}{\sigma}$$



Huovinen and Molnar, PRC79 (2009)

There is agreement of hydro with transport also in the non dilute limit

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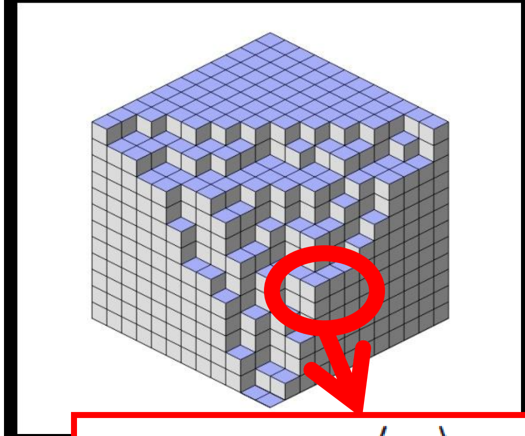

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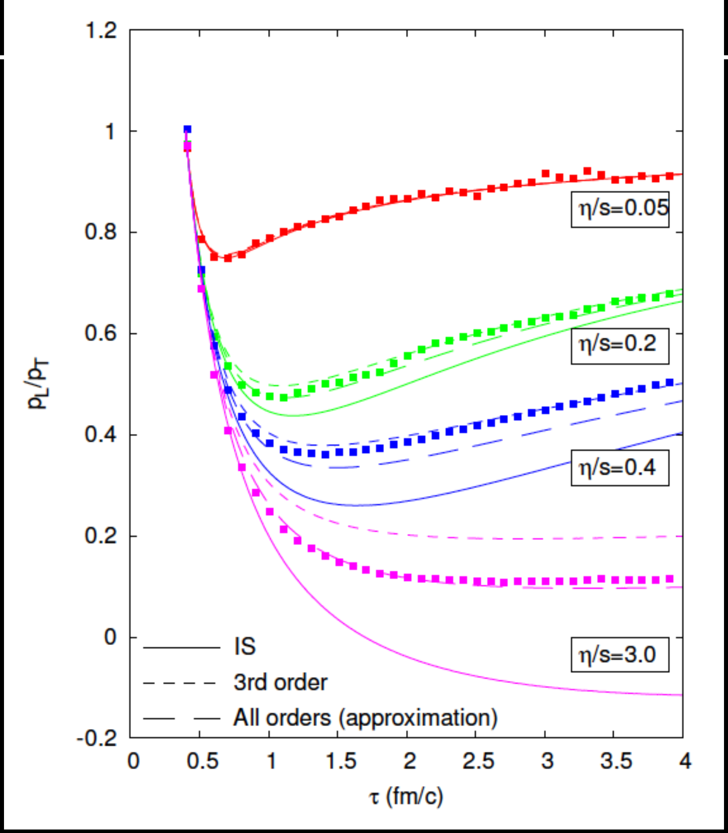

El, Xu, Greiner, Phys.Rev. C81 (2010) 041901

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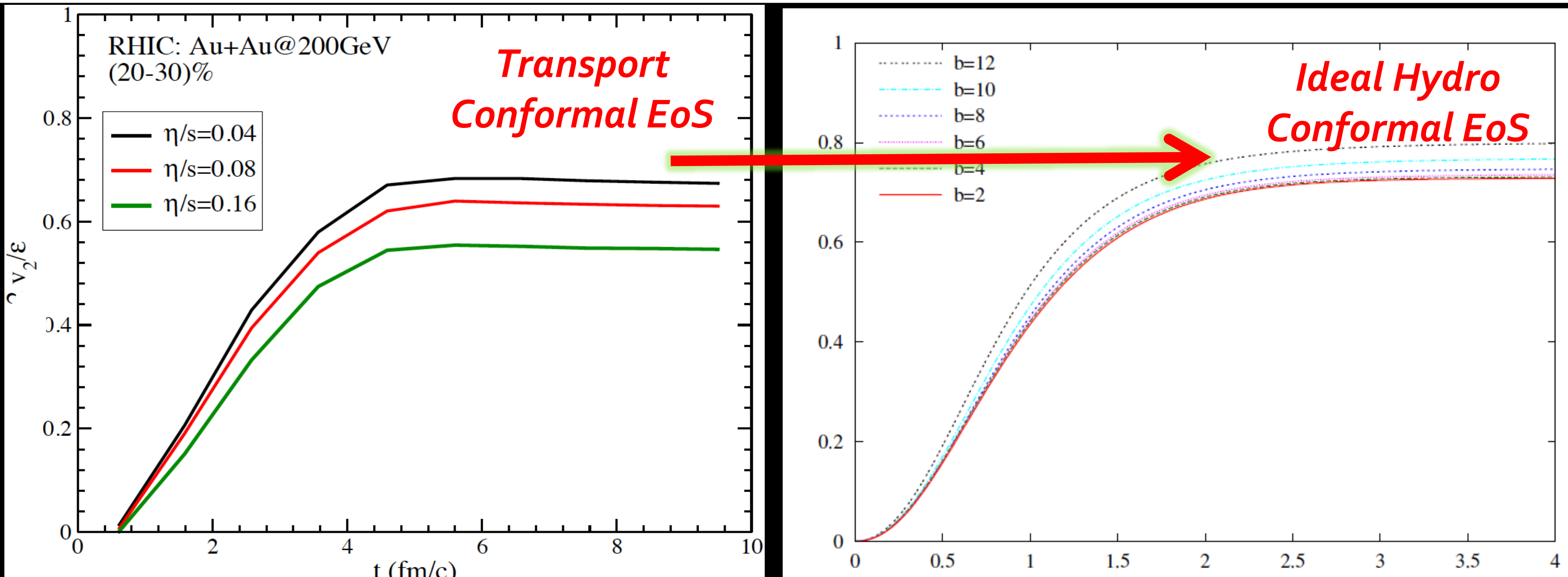

Bhalerao *et al.*, PLB627 (2005)

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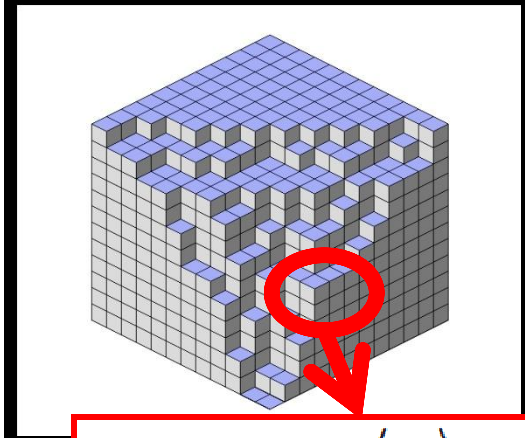
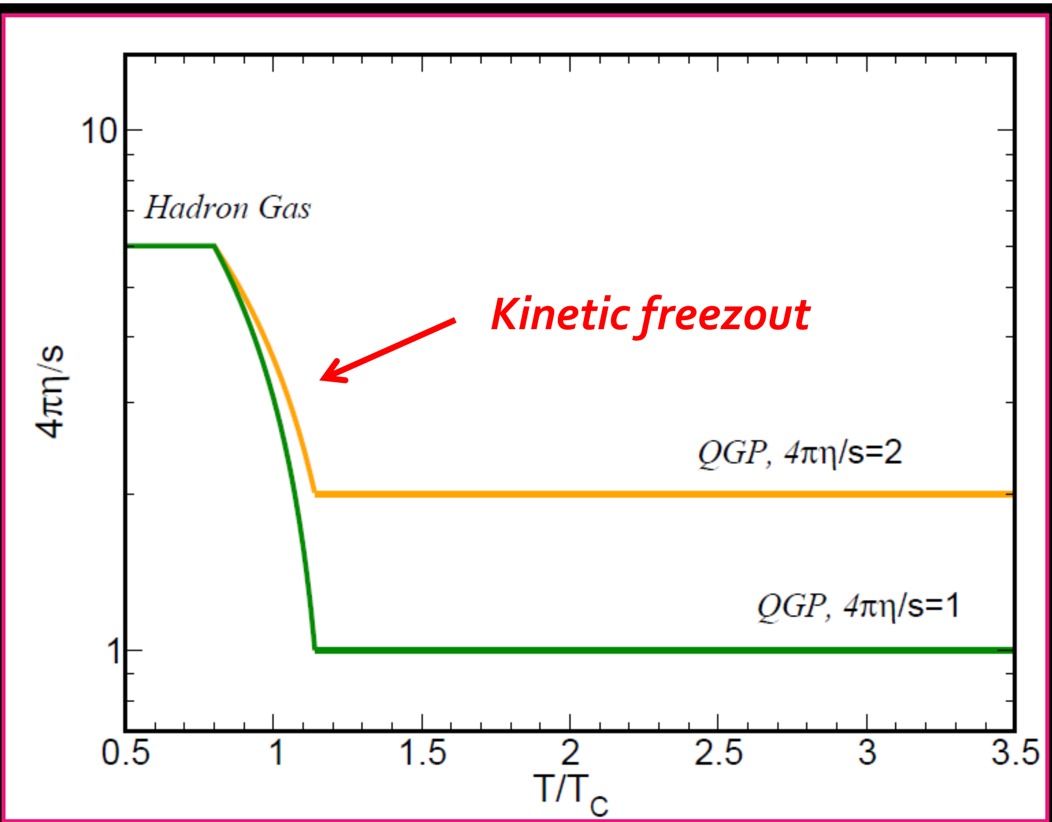


$$\frac{\eta}{s} = \frac{\langle p \rangle}{g(m_D)\rho}\frac{1}{\sigma}$$



A ***smooth kinetic freezout*** is implemented in order to gradually reduce the strength of the interactions as the temperature decreases below the critical temperature.

Why transport for uRHICs?

$$\left\{p^{\mu}\partial_{\mu}+\left[p_{\nu}F^{\mu\nu}+m\partial^{\mu}m\right]\partial_{\mu}^{p}\right\}f(x,p)=C[f]$$

- Starting from 1-body distribution function f(x,p) and not from $T_{\mu\nu}$:
  - Implement non-equilibrium implied by CGC-Qs scale (beyond ε_x)
  - Include off-equilibrium at high and intermediate p_T :

    Relevant at LHC due to large amount of minijet production
  - freeze-out self-consistently related with $\eta/s(T)$
- It's not an expansion in η/s:

  - valid also at high η/s -> LHC ($T>>T_c$)
- Appropriate for heavy quark dynamics *[Santosh's talk on Friday]*
- f(x,p) and kinetic equations are useful to grasp informations about early glasma evolution (McLerran's talk)

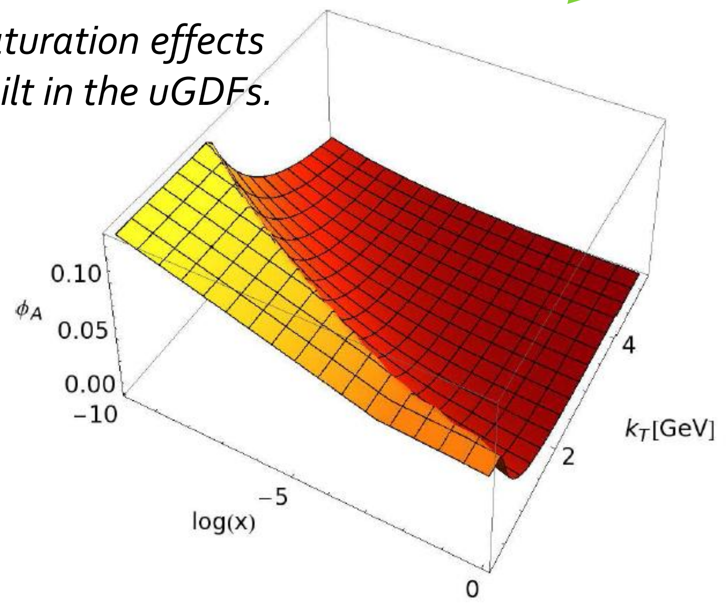
Initial condition: fKLN

Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
Kharzeev *et al.*, Phys. Lett. B**561**, 93 (2003)
Nardi *et al.*, Phys. Lett. B**507**, 121 (2001)
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)
Hirano and Nara, Nucl. Phys. A**743**, 305 (2004)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]

(f)KLN spectrum

$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2} \int_0^{p_T} d^2k_T \alpha_s(Q^2) \times\phi_A\left(x_A, \frac{(p_T+k_T)^2}{4}; \boldsymbol{x}_\perp\right) \times\phi_B\left(x_B, \frac{(p_T-k_T)^2}{4}; \boldsymbol{x}_\perp\right)$$

Saturation effects built in the uGDFs.



Gluon production is ***damped*** for momenta ***below the saturation scale***

This spectrum models ***gluons produced*** after the ***shattering of the color glass condensate***

Initial condition: fKLN

Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
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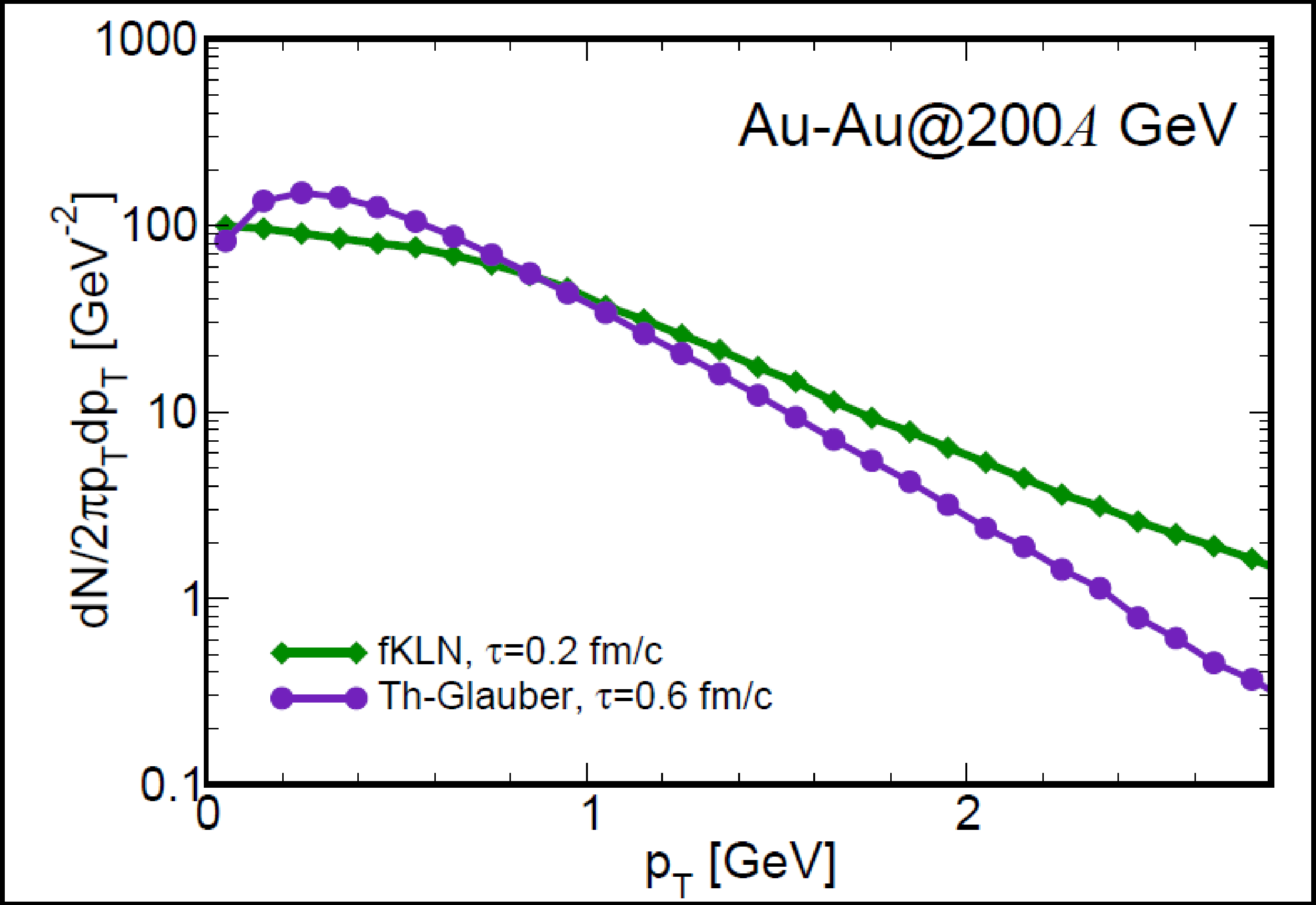
Our goal is computing:
() Thermalization
() Isotropization
() Flows, in particular v_2 and v_3
for this model of ***shattered color-glass condensate, whose initial spectrum is out of equilibrium.***

Nomenclature borrowed by Hirano and Monnai, 2011

M. R. *et al.*, PLB727 (2013)
M. R. *et al.*, PRC 89 (2014)

Initial spectra



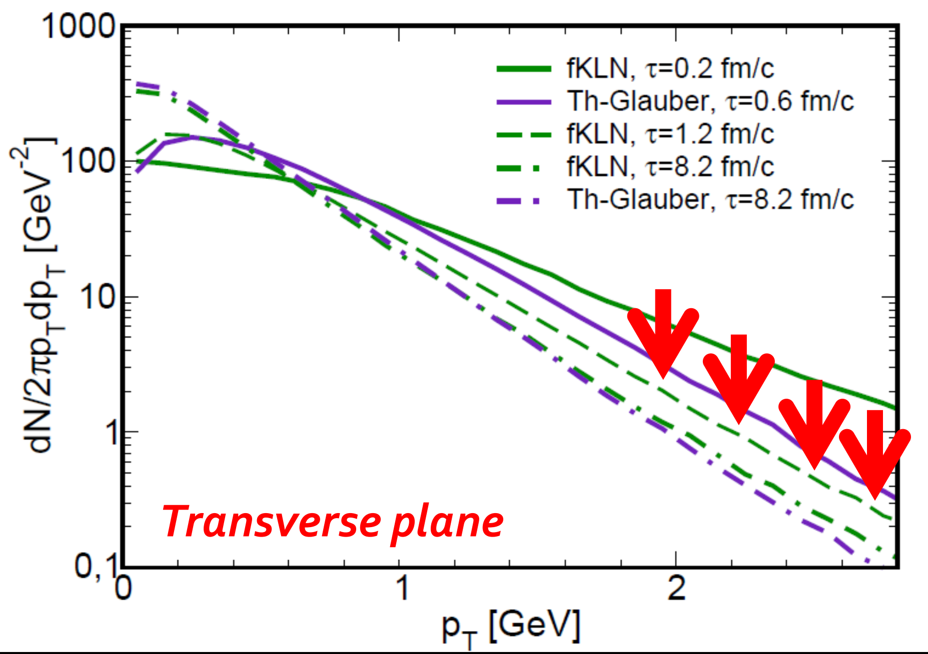
Our novelty:

For fKLN we consider the ***initial spectrum given by the theory at small transverse momenta.***

M. R. *et al.*, PLB727 (2013)
M. R. *et al.*, PRC 89 (2014)

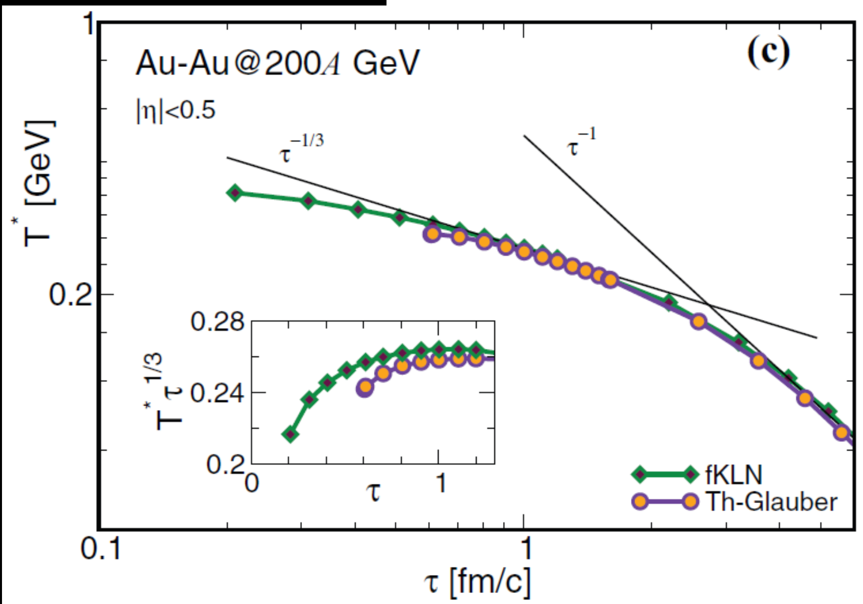
Thermalization

AuAu@200A GeV



Thermalization in less than 1 fm/c,
in agreement with:
Greiner *et al.*, Nucl. Phys. A**806**, 287 (2008).

Longitudinal direction



We have dynamics in the early stages of the simulation, which prepares the momentum distribution to build up the elliptic flow.

Similar results for Pb-Pb collisions

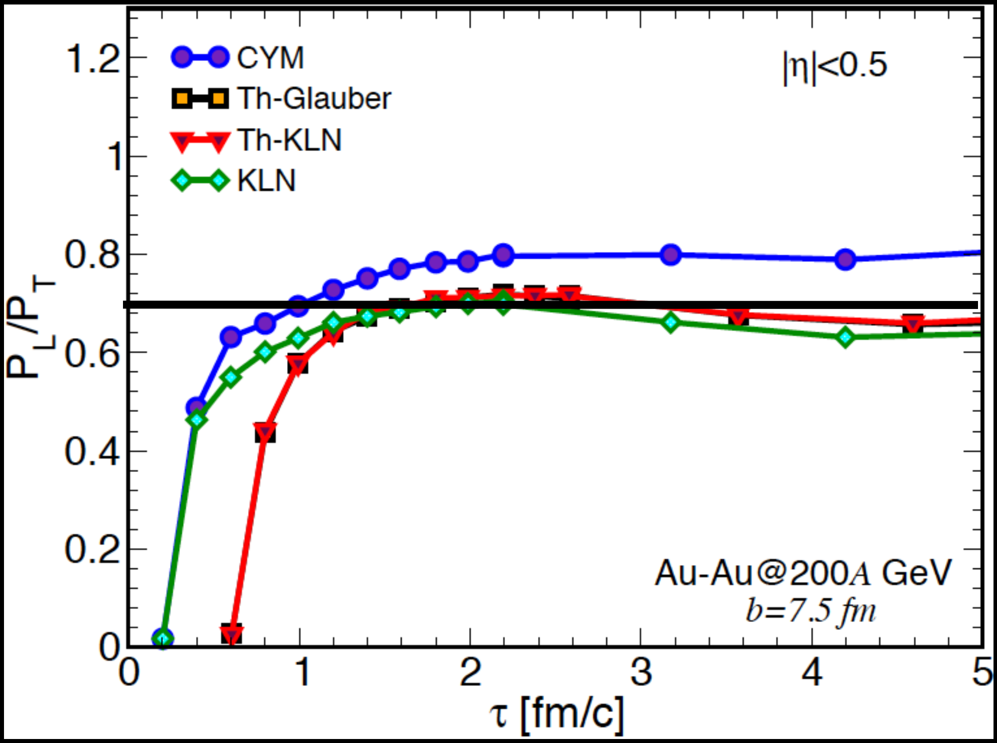
M. R. *et al.*, PRC 89 (2014)

Pressure isotropization

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(x,p)$$

$$P_T = \frac{1}{V}\int_\Omega d^2x_\perp d\eta \, \frac{T_{xx}+T_{yy}}{2} \, ,$$

$$P_L = \frac{1}{V}\int_\Omega d^2x_\perp d\eta \, T_{zz} \, ,$$



t=1/Qs≈0.1-0.2 fm/c -> P_L/P_T > 0
Gelis & Epelbaum
arXiV:1307.2214

CYM (IP-Glasma) spectrum:
Courtesy of B. Schenke
& R. Venugopalan

Fast isotropization in strong coupling, τ less then 1 fm/c

M. R. *et al.*, PRC 89 (2014)

Pressure isotropization

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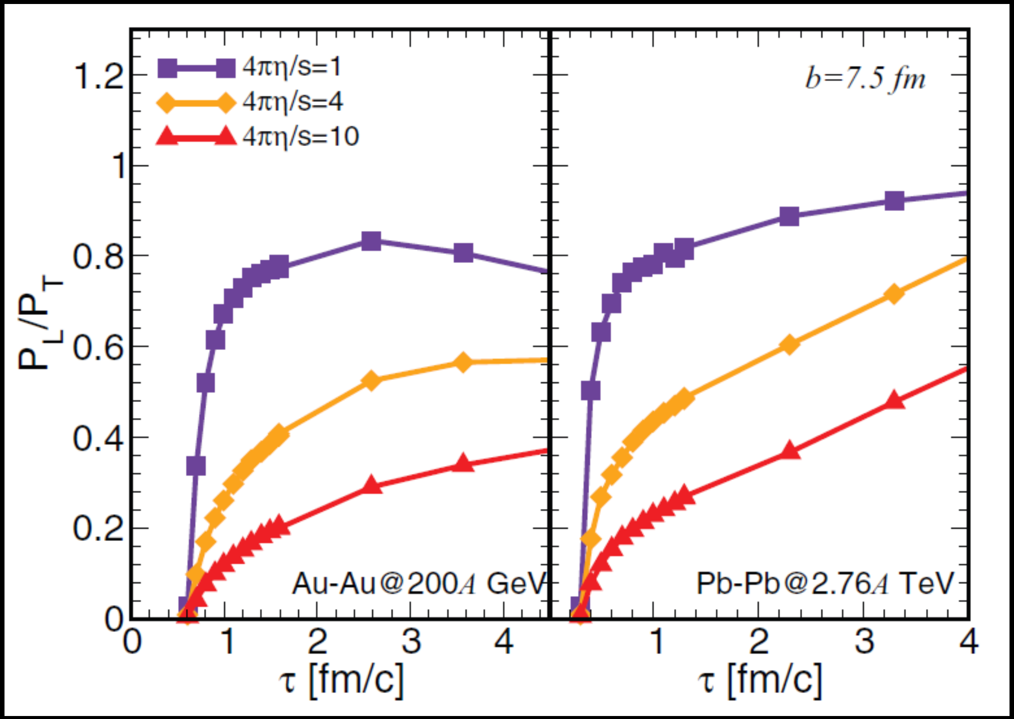

- For $\eta/s > 0.3$ one misses fast isotropization in P_L/P_T (τ about 2-3 fm/c)
- For $\eta/s \approx$ pQCD no isotropization

Pressure isotropization

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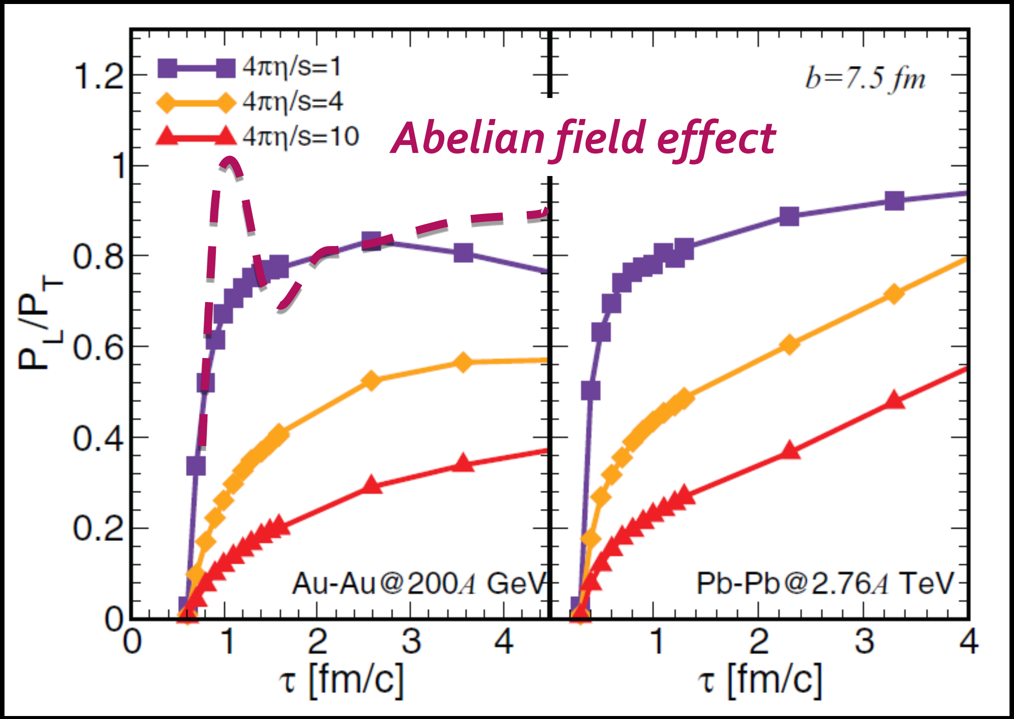


✧ For η/s > 0.3 one misses fast isotropization in P_L/P_T (τ about 2-3 fm/c)

✧ For η/s ≈ pQCD no isotropization

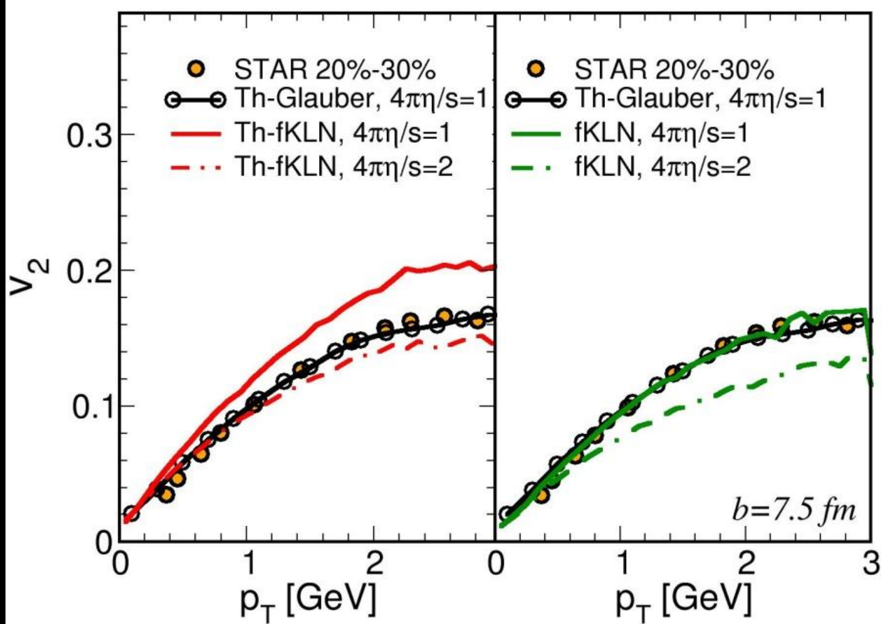
✧ Semi-quantitative agreement with Florkowski *et al.*, PRD88 (2013) 034028
ours is ***3+1D*** and ***full collision integral***; however ***no gauge fields***

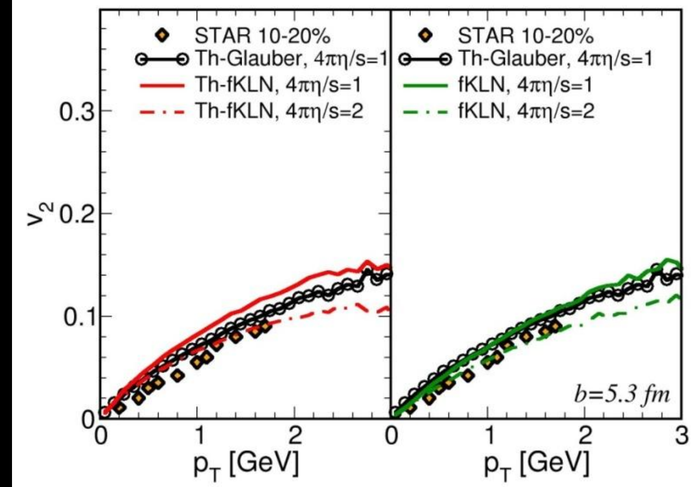
M. R. *et al.*, PLB727 (2013)
M. R. *et al.*, PRC 89 (2014)

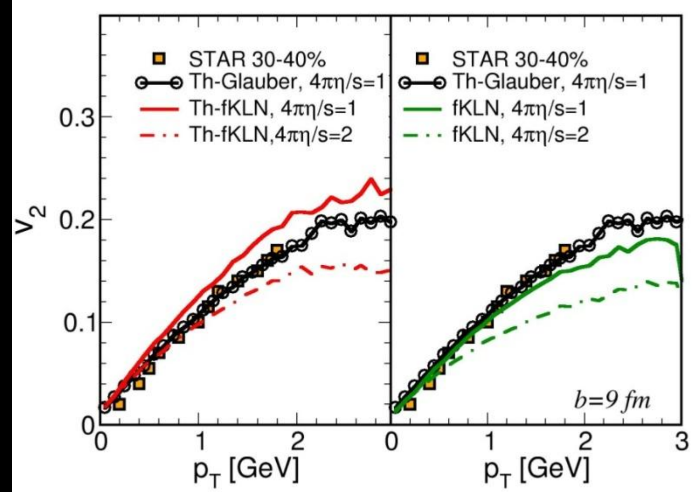
Elliptic flow from Transport

Au-Au collision
RHIC energy

Hydro-like ***KLN initial***







Larger eccentricity of KLN implies larger v_2

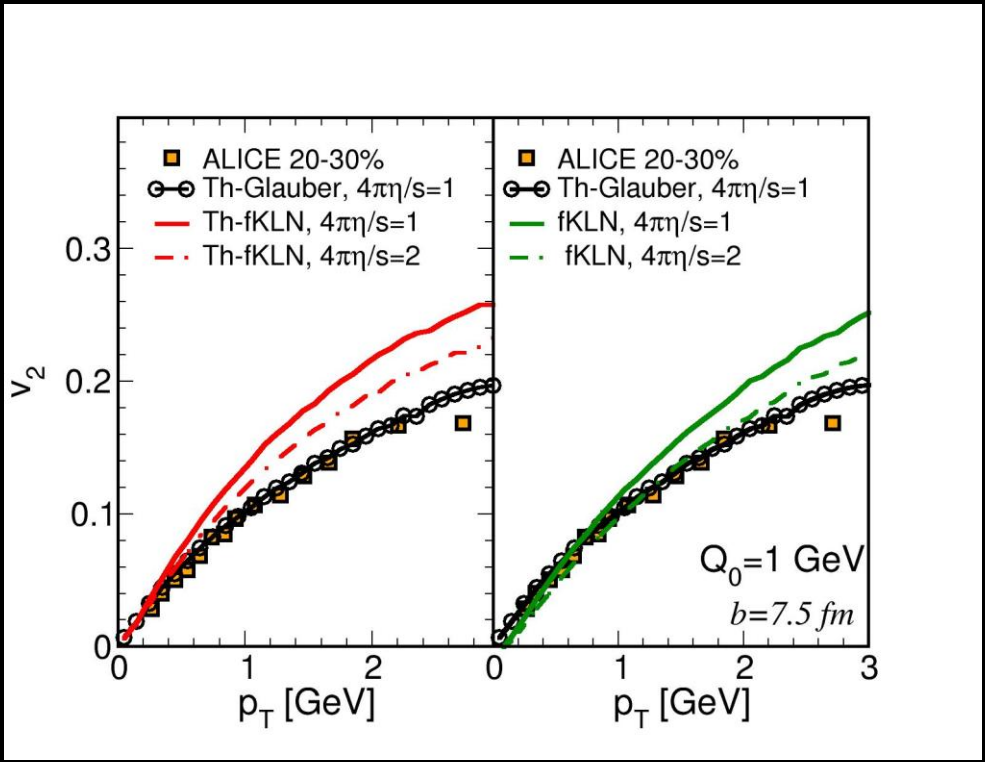
Results in fair agreement with hydro:
Song *et al.*, PRC83 (2011)

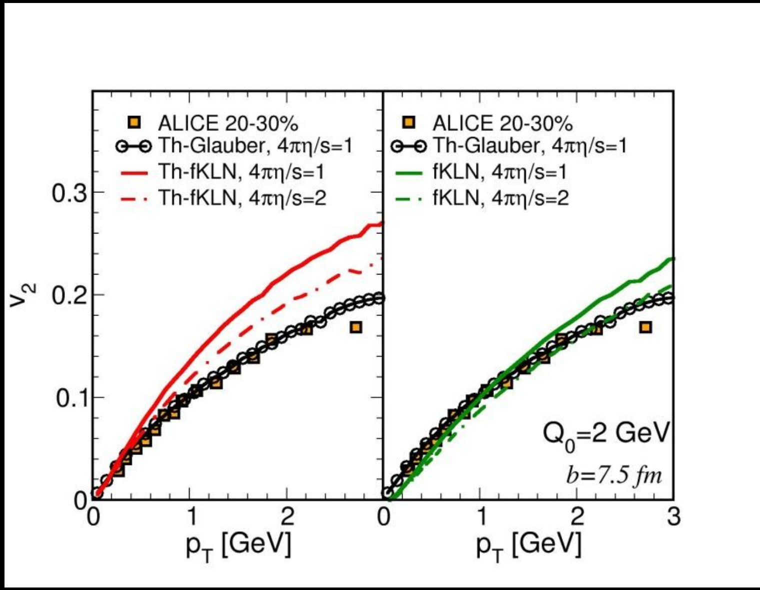
M. R. *et al.*, PRC 89 (2014)

Elliptic flow from Transport

Pb-Pb collision

LHC energy





*Elliptic flow computations show this quantity is **very sensitive** to the **initial conditions:***

.) Initial anisotropy (eccentricity)

.) Initial momentum distribution

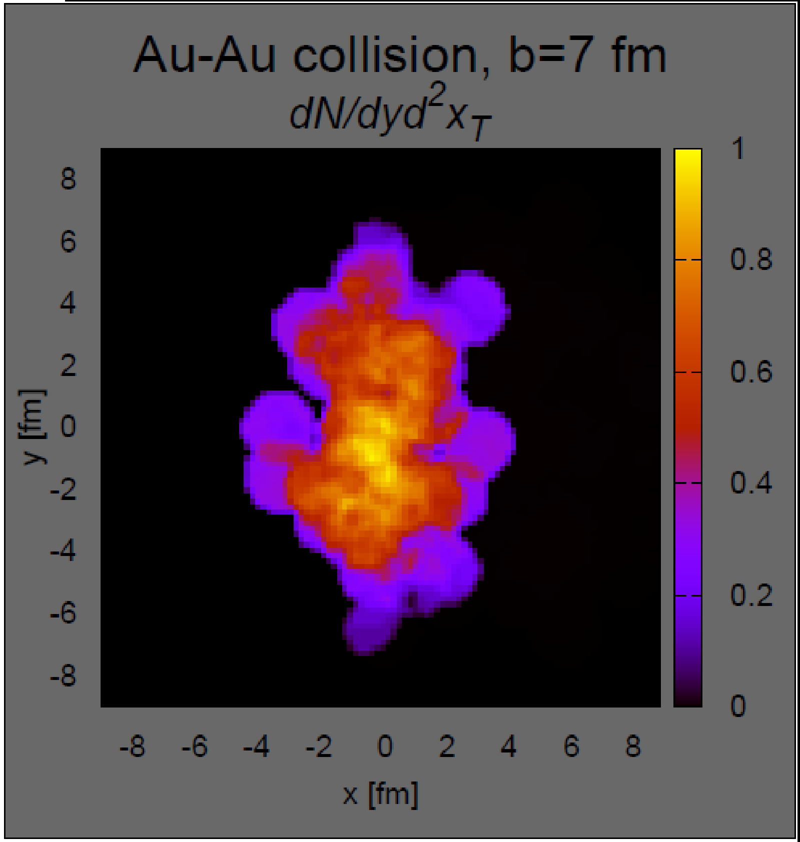
Measurements of elliptic flow in experiments might permit to identify the best theoretical initial conditions.

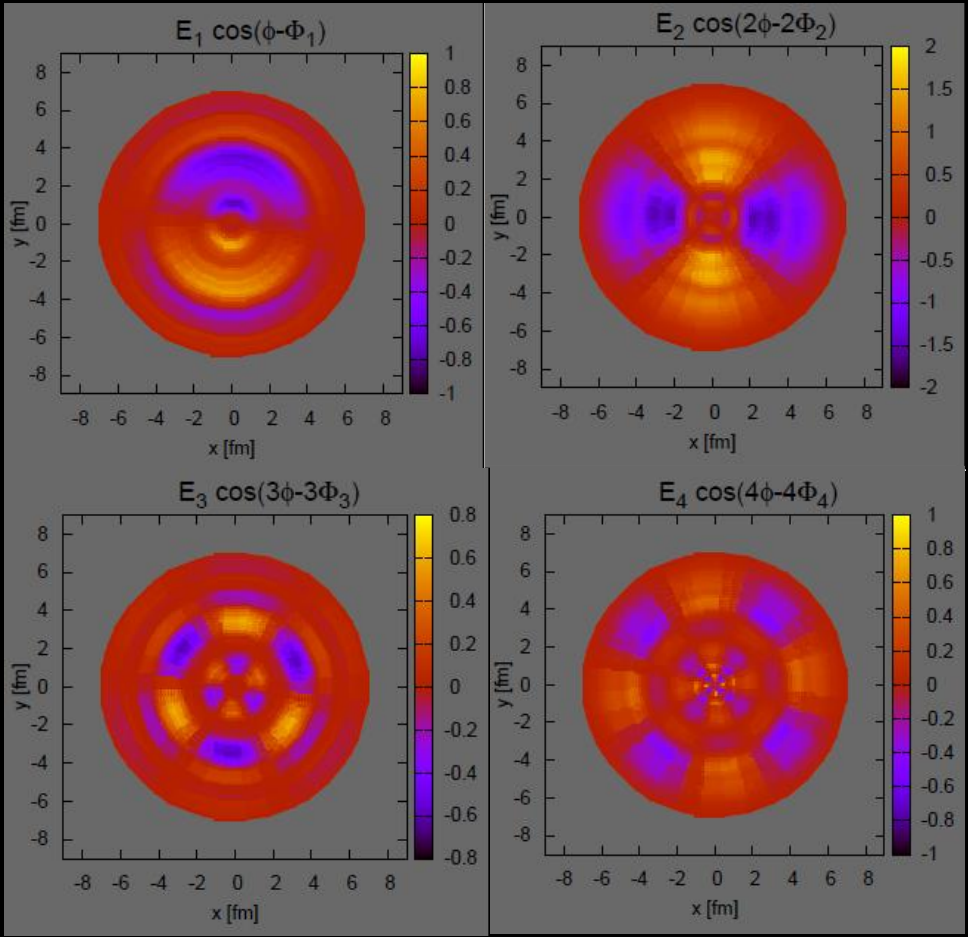
M. R. *et al.*, in preparation

Triangular flow from Transport

Initial state anisotropy parametrization

$$\frac{dN}{x_\perp dx_\perp d\phi} = \frac{dN}{x_\perp dx_\perp}\left[1 + 2\sum_{n=1}^{\infty} E_n(x_\perp)\cos\left(n(\phi - \Psi_n)\right)\right]$$

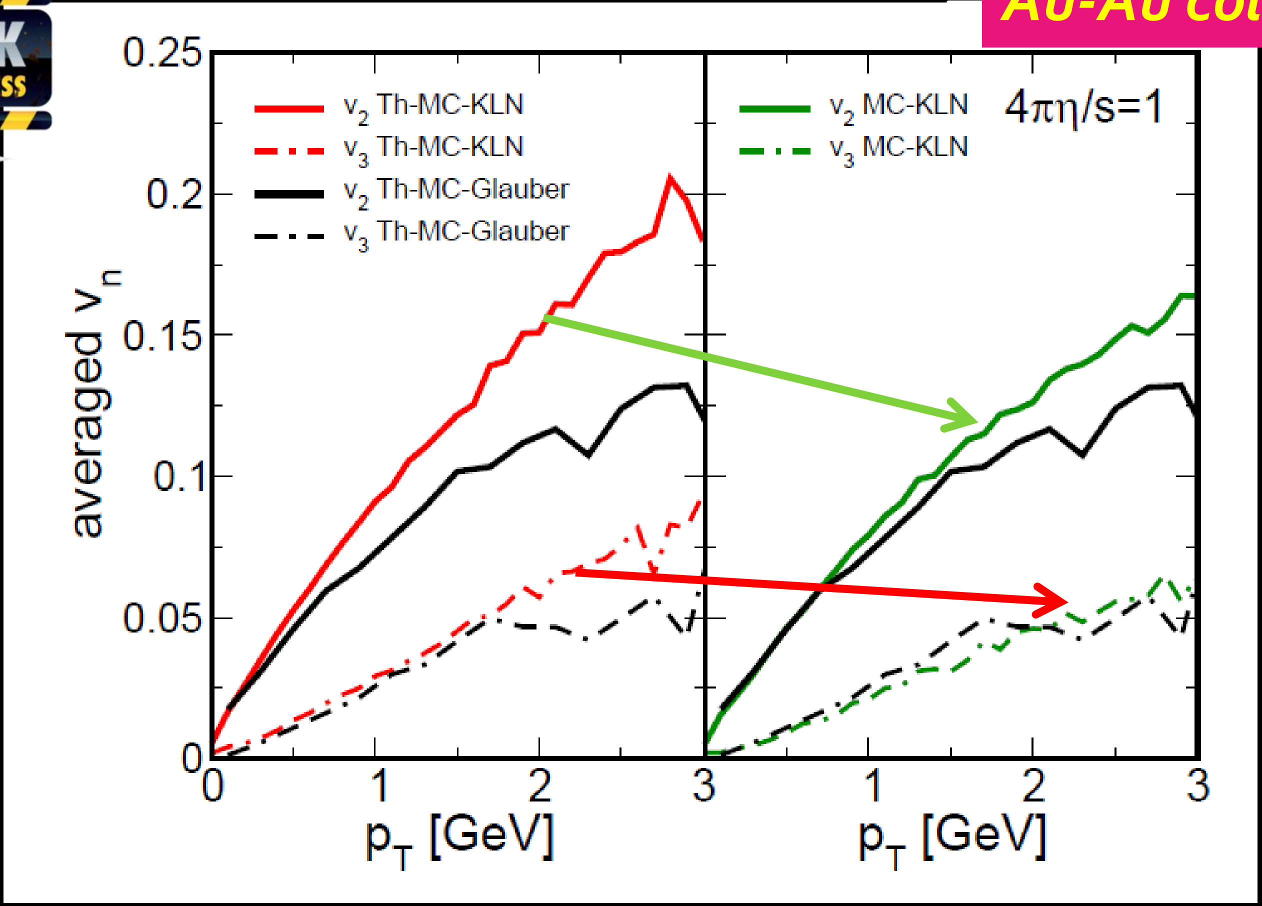




M. R. *et al.*, in preparation

Triangular flow from Transport

Au-Au collision



Elliptic and Triangular flows of MCKLN turns out to be in agreement with the MCGlauber ones, both with minimal viscosity.

Conclusions

- ***Kinetic Theory*** permits to compute ***elliptic flow*** of plasma**,** as well as its ***thermalization times*** and ***isotropization efficiency***.
- ***Initial distribution in momentum space affects the flow and the building up of momentum anisotropy.***
- ***Elliptic and Triangular flows of MCKLN turns out to be in agreement with the one of MCGlauber, both with the same viscosity.***

Outlook

WiP

(.)*Transient Bose-Einstein condensate*

BE condensation, in particular at LHC energy, expanding geometry

[Blaizot *et al.*, NPA920 (2013), NPA873 (2012)]

WiP

(.)Initial conditions from classical field dynamics

Implementation of initial color fields in abelian approximation

[Florkowski *et al.*, PRD 88 (2013)]

WiP

(.)Fluctuations in the initial condition

Systematic study of higher order harmonics

(.)Inelastic processes

Implementation of 2 to 3 and 3 to 2 processes in the collision integral

Thanks for your attention

Good wood does not grow in comfort:
the stronger the wind, the stronger the tree is.

Few remarks on fKLN

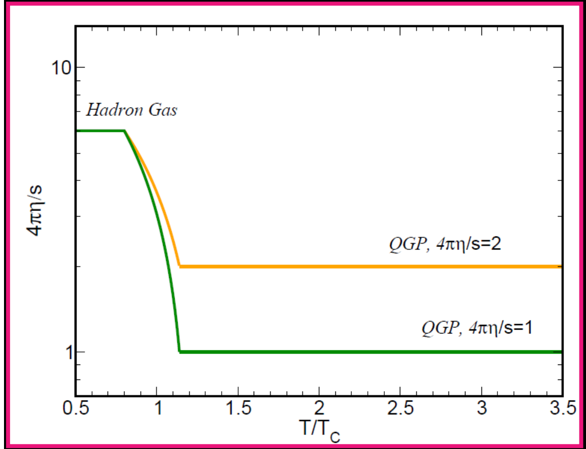
- fKLN is not glasma [Blaizot et al., NPA846 (2010)]
- We neglect initial field dynamics, which however ***should decay*** within $1/Q_s$
- It is not our purpose to insist on exact reproduction of experimental data [See instead IP-Glasma calculations, Gale *et al.*, PRL110 (2013)]

Rather, we want to solve another problem, namely compute the role of the initial nonequilibrium distribution in momentum space, often neglected in hydro and hybrid calculations

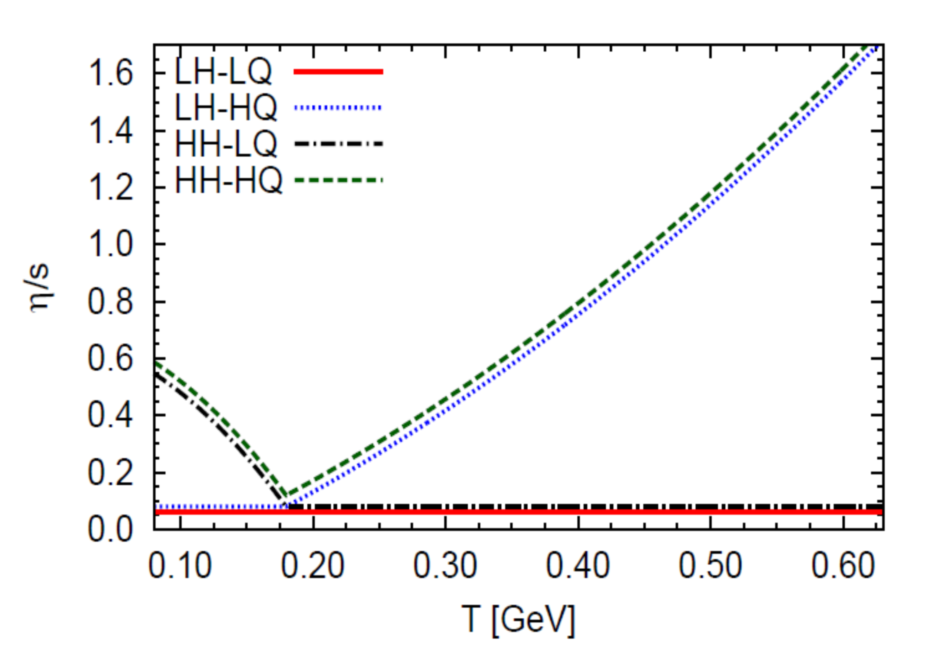
- Hydro widely uses KLN, and we are interested to compare the two approaches

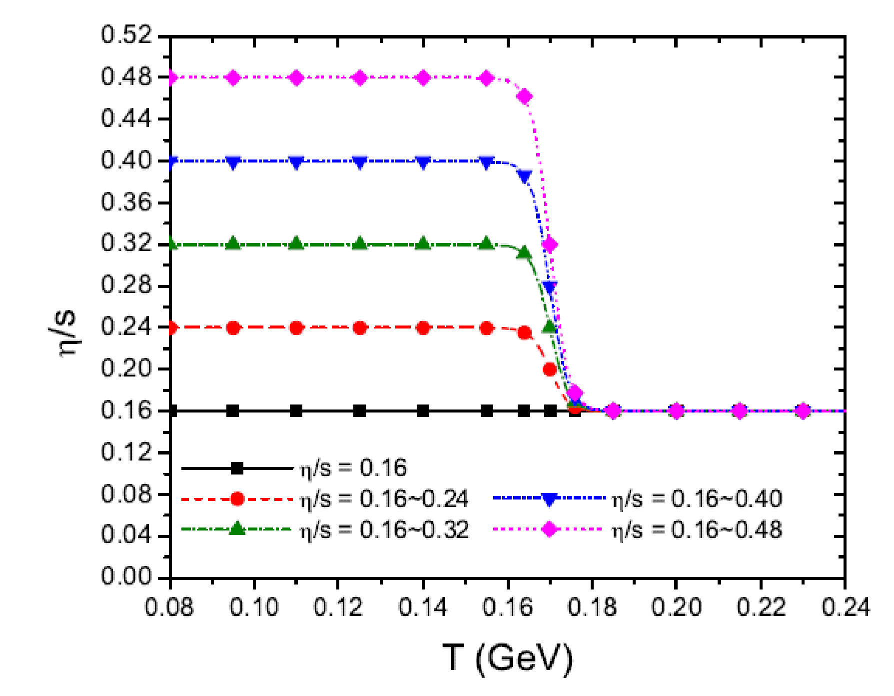
Viscometer: Schen et al., arXiv1308:2111
Thermometer: Schen et al., arXiv1308:2440
Flow computations: Hirano and Nara, PRC**79** (2009)
Hirano and Nara, NPA**743** (2004)

Various η/s used in hydro



Temperature dependence of eta/s already used in hydro simulations recently.

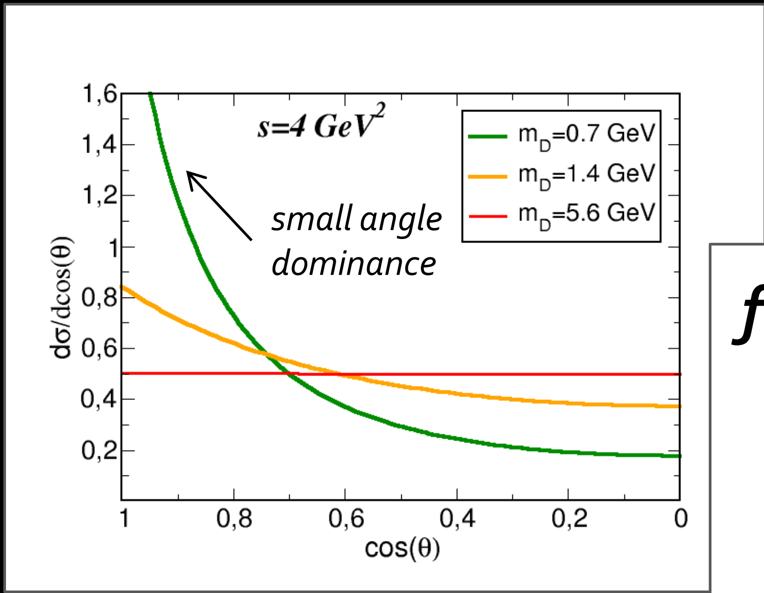




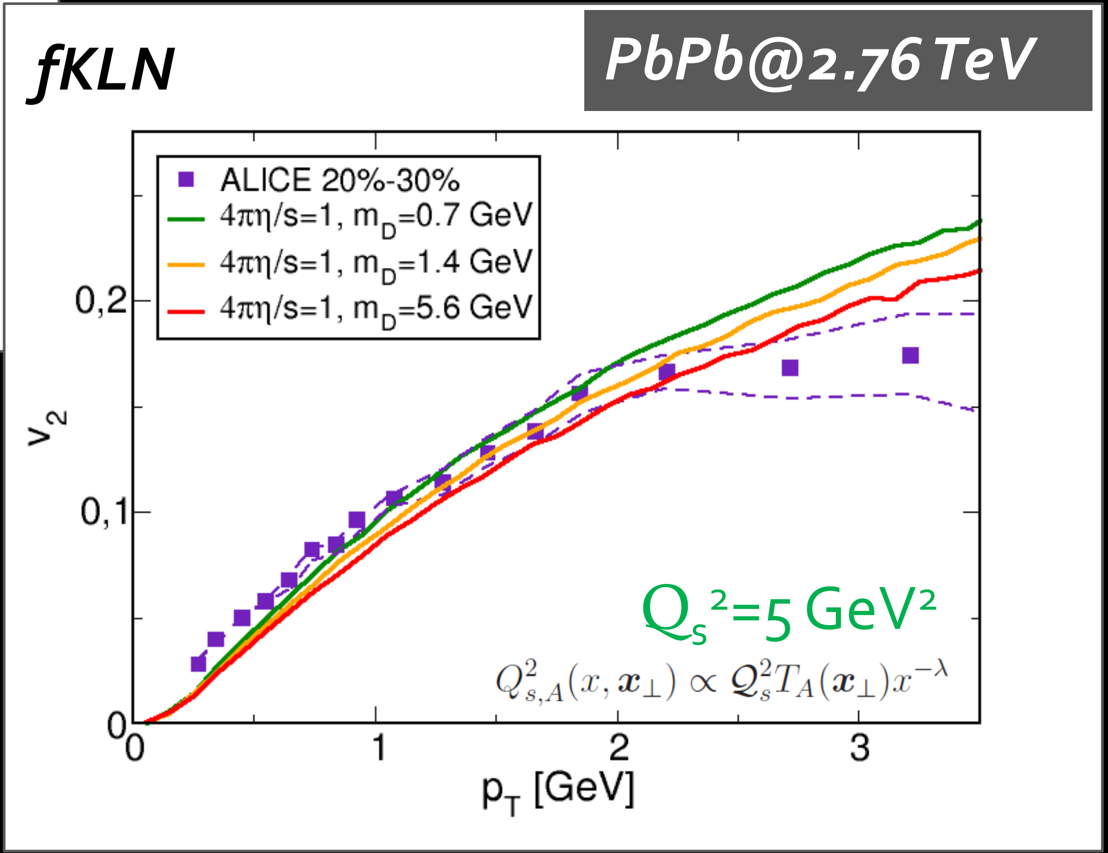
H. Niemi *et al.*, PRC86 (2012), PRL106 (2011)

Shen and Heinz, PRC83 (2011)

Are micro-details important?



$$\frac{d\sigma_{gg\to gg}}{dt} = \frac{9\pi^2\alpha_s^2}{2}\frac{1}{(t-m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$



Same cross section used in:

Zhang *et al.*, PLB 455 (1999)

Molnar and Gyulassy, NPA 697 (2002)

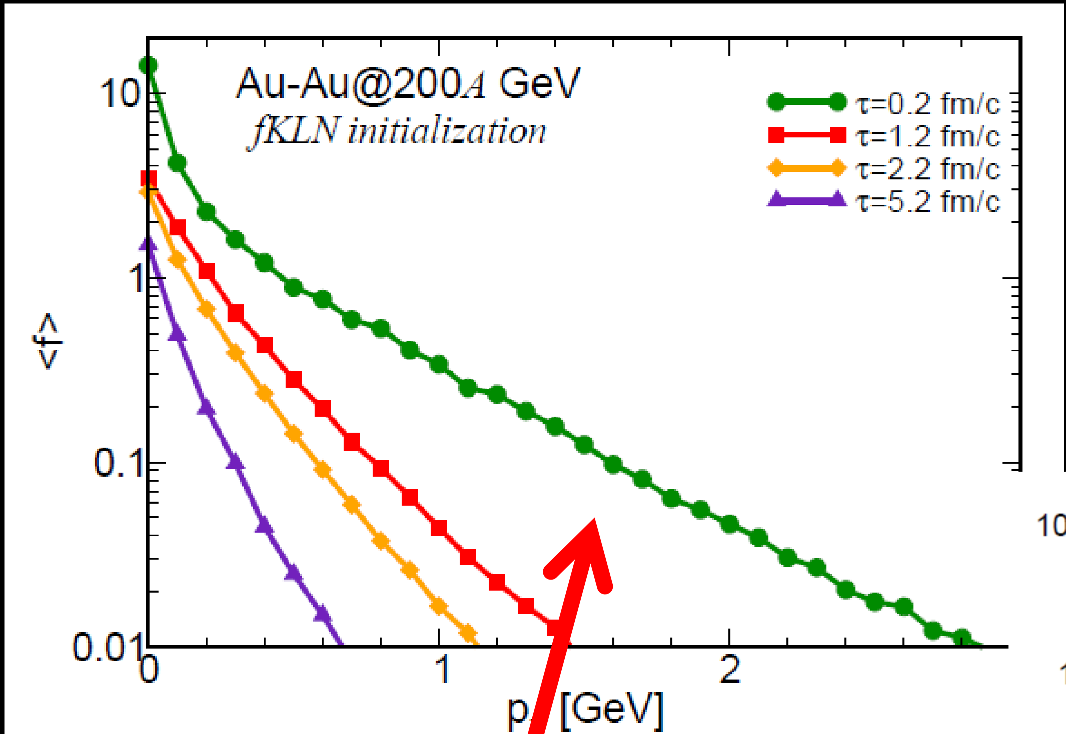
Greco et al., PLB 670 (2009)

Increasing m_D makes the cross section isotropic. However: ***Strong change of the cross section does not result in a strong change of the elliptic flow.***

M. R. *et al.*, in preparation

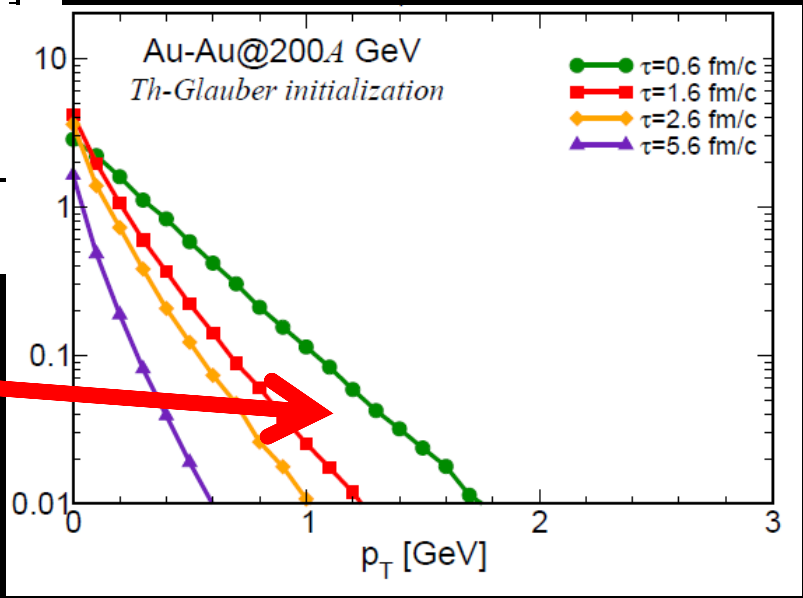
M. R. *et al.*, in preparation
M. R. *et al.*, work in progress

Invariant distributions



$$f = \frac{(2\pi)^3}{g} \frac{\Delta N}{\Delta^2 x_\perp \Delta^2 p_T} \frac{1}{\Delta z \Delta p_z}$$

Longitudinal and transverse expansions help to dilute the system, lowering the invariant distribution functions



1+f factors in the collision integral, arising from the bosonic nature of gluons, should not modify in a substantial way our results on elliptic flow

Preliminary result: no change due to 1+f (at RHIC energy).

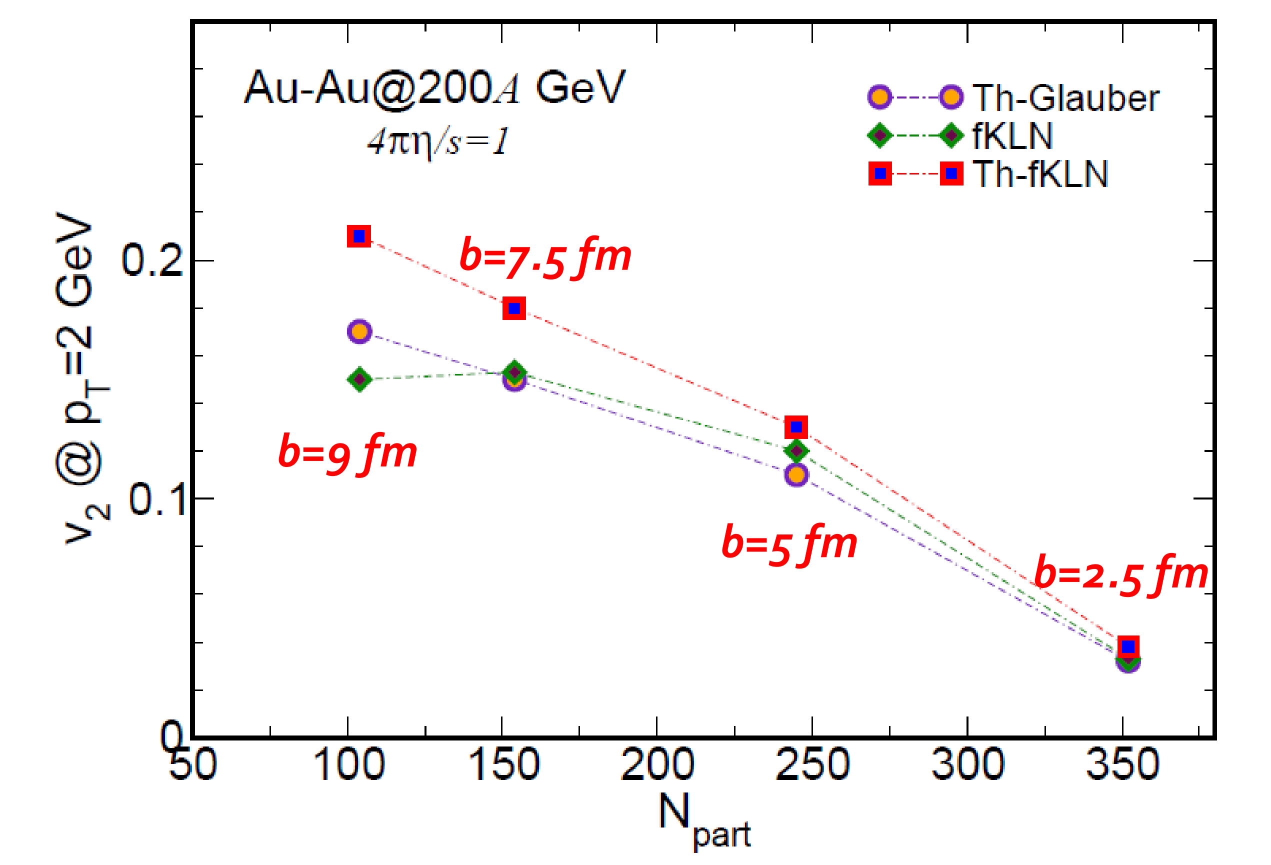
M. R. *et al.*, in preparation

Elliptic flow from Transport

Au-Au collision

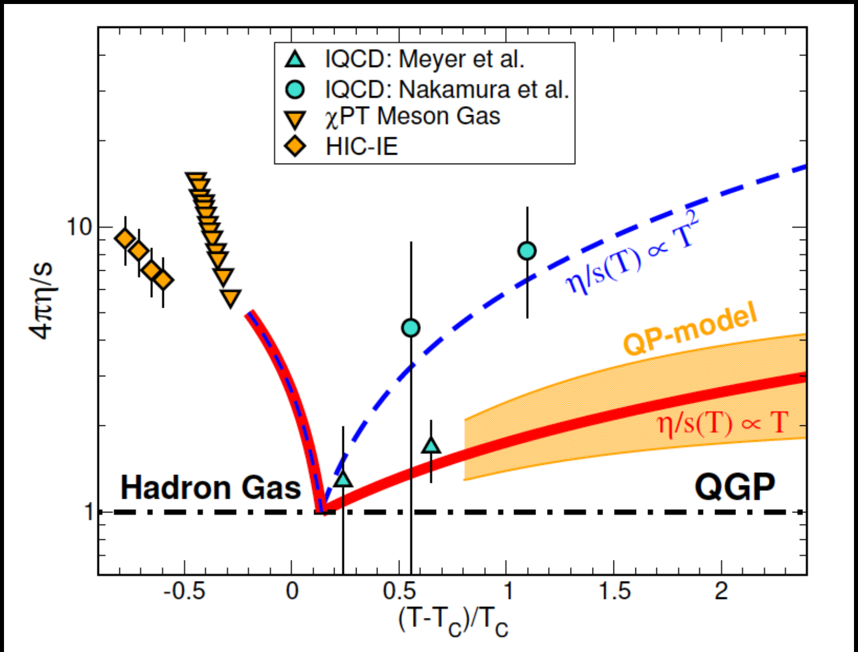
RHIC energy

Summary of the effect on differential v_2

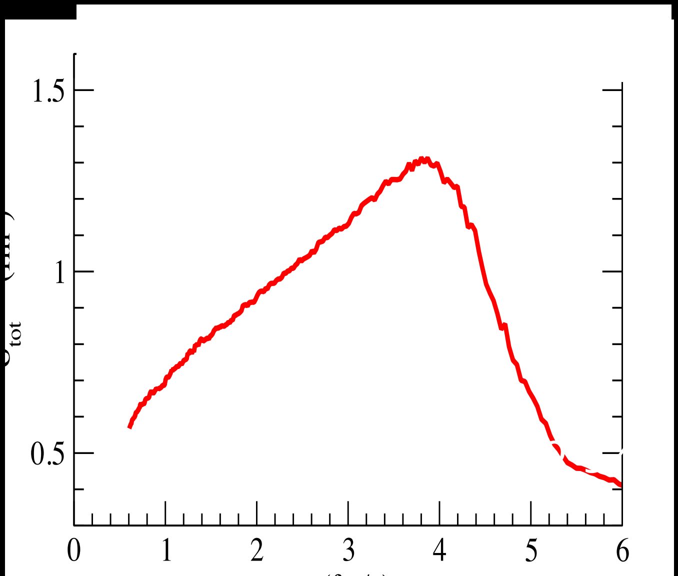


For more central collisions the effect on v2 becomes milder.

Cross section and freeze-out



- ✓ η/s increases in the cross-over region, realizing a smooth f.o. self-consistently dependent on h/s:
- ✓ Different from hydro that is a sudden cut of expansion at some $T_{f.o.}$



$$\sigma^* = g(a)\sigma_{tot} \approx \frac{1}{15}\frac{\bar{p}}{\rho}\frac{1}{\eta/s}$$

$\rho(\tau_o)$=23 fm^{-3}, η/s=0.08 → σ_{ToT}= 6 mb

$$\sigma_{pQCD} = \frac{9\pi\alpha_s^2}{m_D^2}\ , \quad \alpha_s = \frac{4\pi}{11\ln\left(\frac{2\pi T}{\Lambda}\right)}\ , \quad m_D^2 = 4\pi\alpha_s(T)T$$

T_o=340 MeV → σ_{pQCD}= 3.6 mb

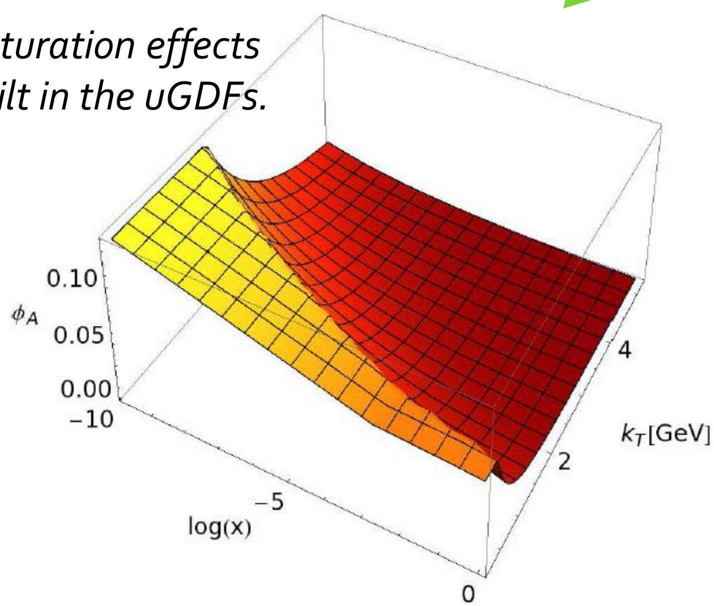
Initial condition: fKLN

Nardi *et al.*, Nucl. Phys. A**747**, 609 (2005)
Kharzeev *et al.*, Phys. Lett. B**561**, 93 (2003)
Nardi *et al.*, Phys. Lett. B**507**, 121 (2001)
Drescher and Nara, PRC**75**, 034905 (2007)
Hirano and Nara, PRC**79**, 064904 (2009)
Hirano and Nara, Nucl. Phys. A**743**, 305 (2004)
Albacete and Dumitru, arXiv:1011.5161[hep-ph]

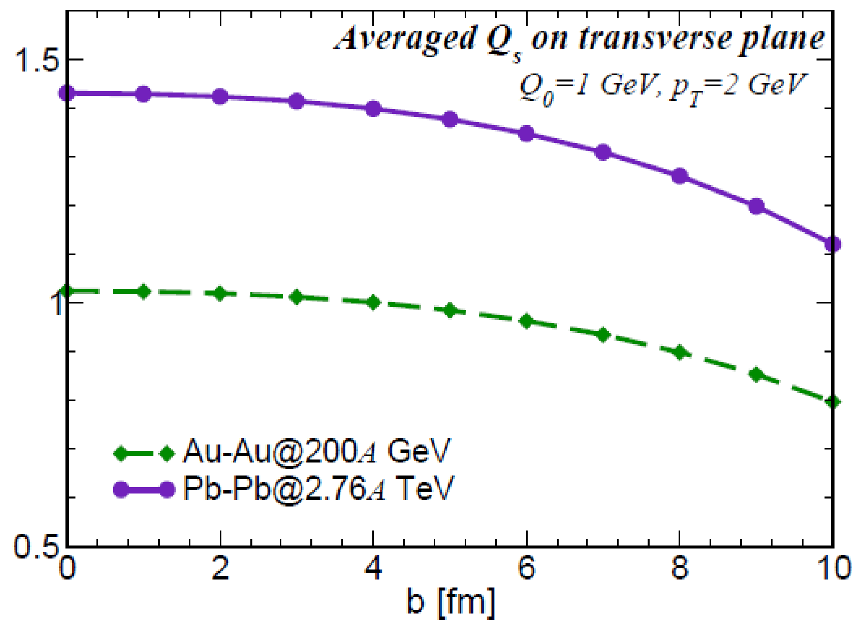
(f)KLN spectrum

$$\frac{dN_g}{d^2x_\perp dy} \propto \int \frac{d^2p_T}{p_T^2}\int_0^{p_T} d^2k_T \alpha_s(Q^2) \times \phi_A\left(x_A, \frac{(p_T+k_T)^2}{4}; \boldsymbol{x}_\perp\right) \times \phi_B\left(x_B, \frac{(p_T-k_T)^2}{4}; \boldsymbol{x}_\perp\right)$$

Saturation effects built in the uGDFs.

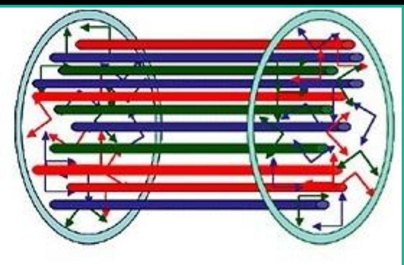


$$Q^2_{s,A}(x, \boldsymbol{x}_\perp) = Q_0^2\left(\frac{T_A(\boldsymbol{x}_\perp)}{1.53p_A(\boldsymbol{x}_\perp)}\right)\left(\frac{0.01}{x}\right)^\lambda$$



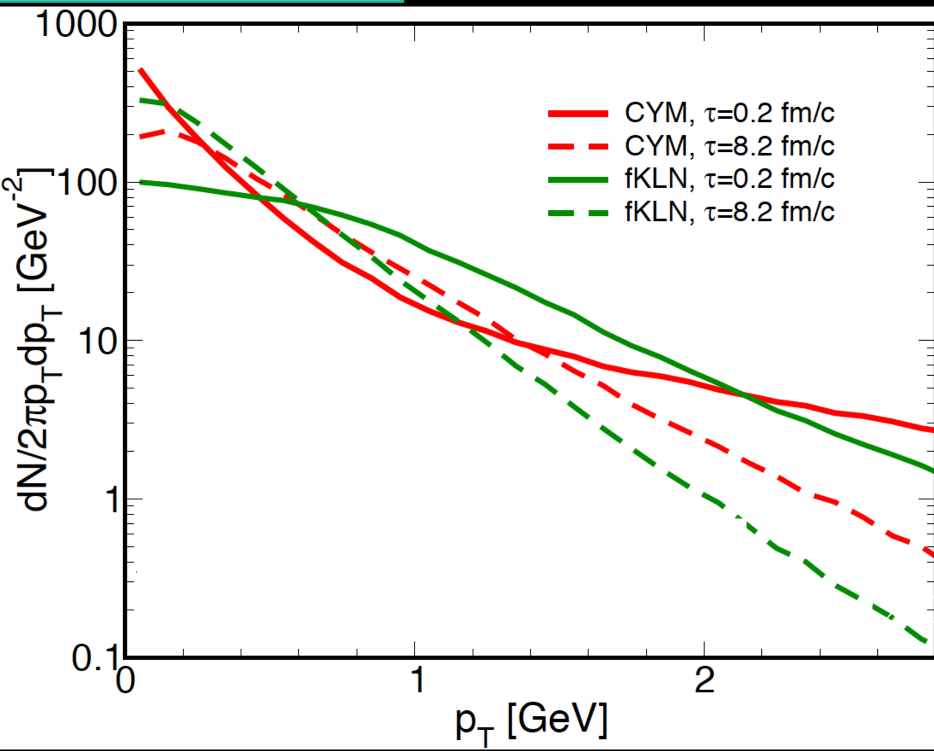
For Pb-Pb collision average Qs can be larger [Lappi, EPJC71 (2011)]

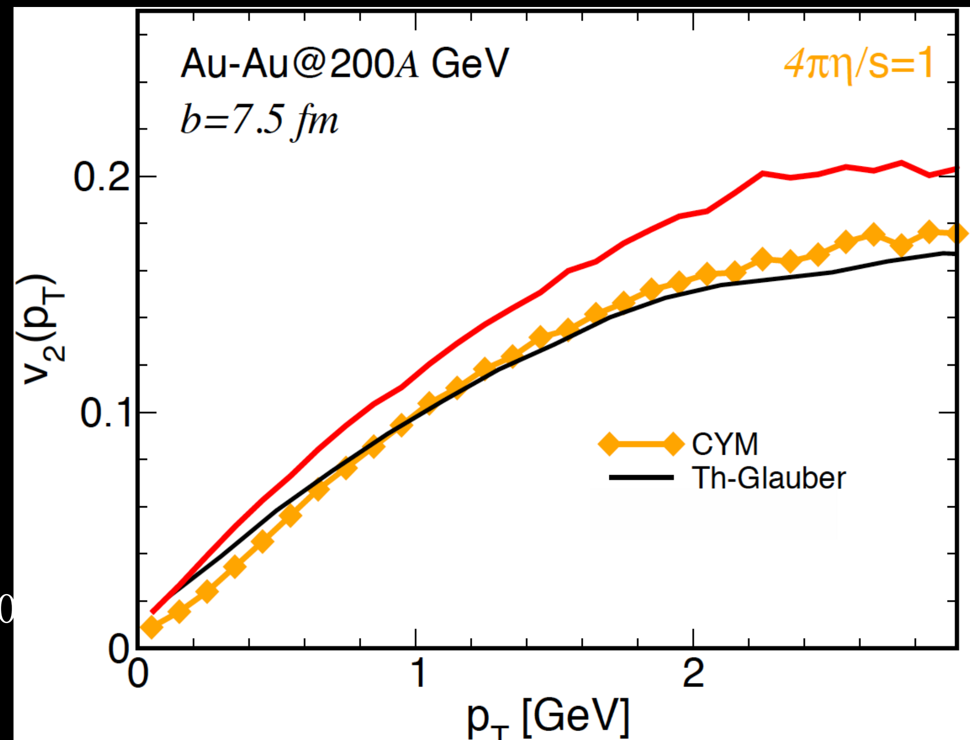
KLN vs Classic Yang-Mills



$$[D_\mu, F^{\mu\nu}] = J^\nu$$

Factorization of parton distr. funct
not valid in AA -> Classic field approach





The effect nearly disappears but indeed there is <u>nearly No saturation!</u>

The slope is the opposite of KLN

No real progress in the determination of h/s(T) w/o knowing initial spectra

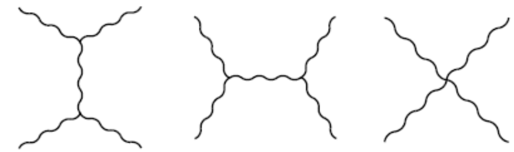
Onset of Bose –Einstein Condensation

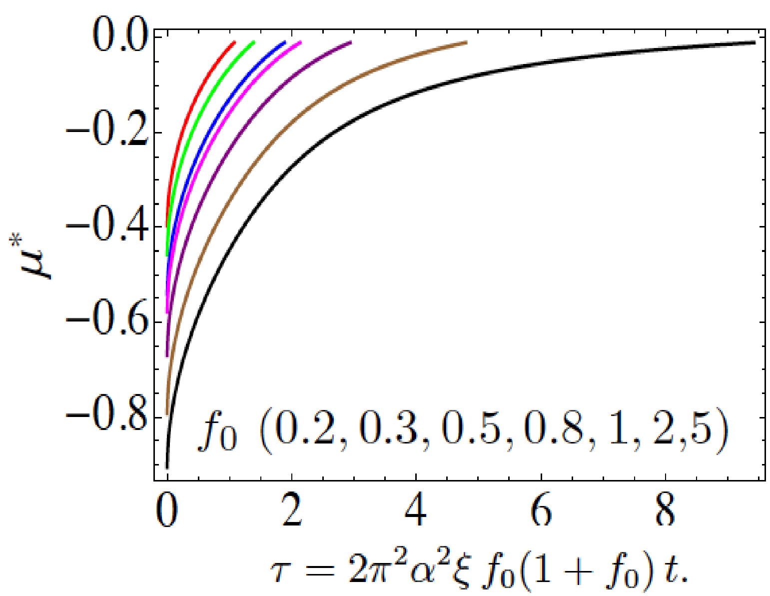
Boltzmann with Bose enhancement (1+f) -> Bose-Einstein distribution function

$$C_{22}=\frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3p'}{(2\pi)^3 2E'_p}\frac{d^3q'}{(2\pi)^3 2E'_q}\left|\mathrm{M}_{pq->p'q'}\right|^2\left[f(q')f(p')(1+f(p))(1+f(q))\right.$$
$$\left.-f(q)f(p)(1+f(p'))(1+f(q'))\right](2\pi)^4\delta^4(p+q-p'-q')$$

In Small-angle Approximation → Fokker-Planck

$$\mathcal{D}_\tau f(\tau,\boldsymbol{p})=\boldsymbol{\nabla}\cdot\left[I_a\boldsymbol{\nabla}f(\tau,\boldsymbol{p})+\frac{\boldsymbol{p}}{p}I_b\,f(\tau,\boldsymbol{p})[1+f(\tau,\boldsymbol{p})]\right]$$





$$f_{eq}(p)=\frac{1}{e^{(p-\mu)/T}-1},$$

$$m^*=T^*\,ln\left(1+\frac{1}{f(0)}\right)\xrightarrow{\text{equilibrium}} m$$

Onset of BEC for $f_0 > 0.156$

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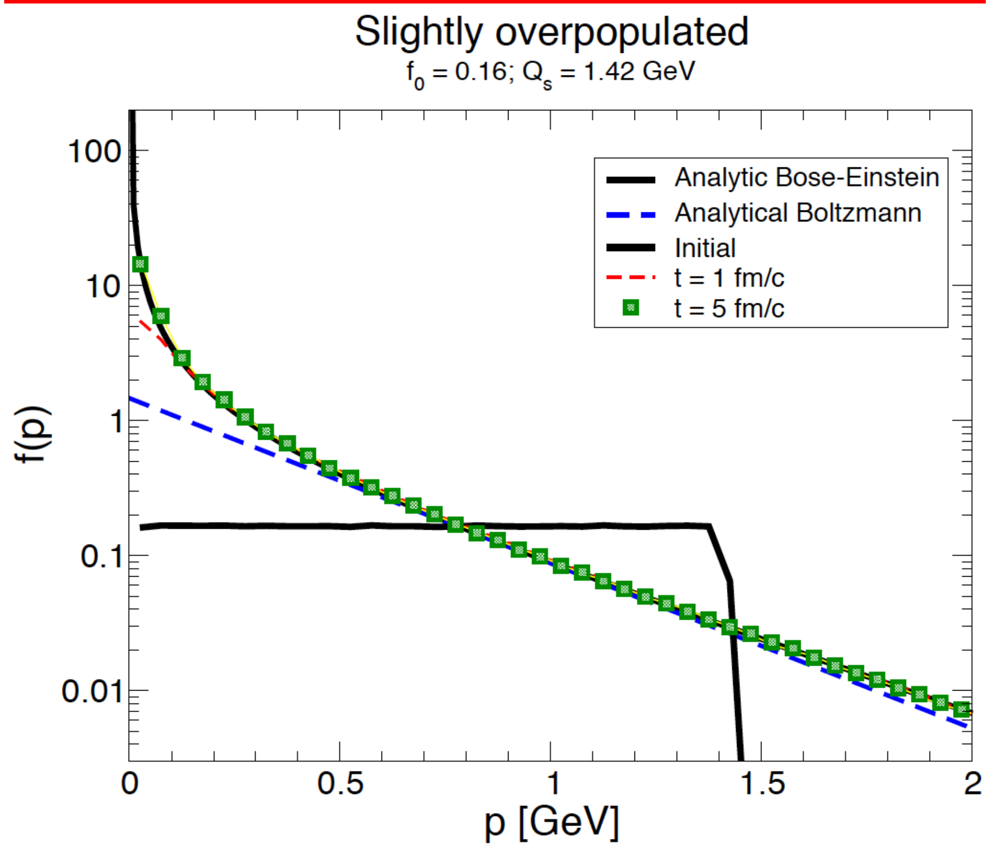
Solving Relativistic-Boltzmann with (1+f) factor

$$C_{22}=\frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3p'}{(2\pi)^3 2E'_p}\frac{d^3q'}{(2\pi)^3 2E'_q}\left|M_{pq->p'q'}\right|^2\left[f(q')f(p')(1+f(p))(1+f(q))\right.$$
$$\left.-f(q)f(p)(1+f(p'))(1+f(q'))\right](2\pi)^4\delta^4(p+q-p'-q')$$

$$P_{22}=\frac{\Delta N^{2\leftrightarrow 2}_{coll}}{\Delta N_1 \Delta N_2}=\frac{\Delta t}{\Delta^3 x}v_{rel}\sigma_{p,q\rightarrow p',q'}\left[1+f(p')\right]\left[1+f(q')\right]$$

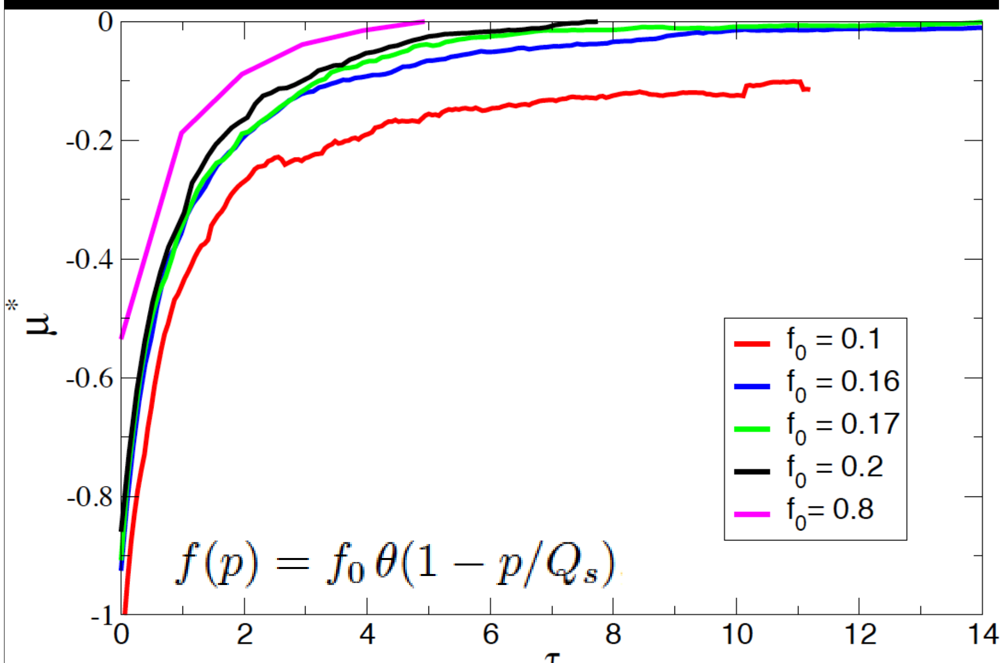


$$\frac{d\sigma^{gg\rightarrow gg}}{dt}=\frac{9\pi\alpha_s^2}{\left(m_D^2-t\right)^2}$$

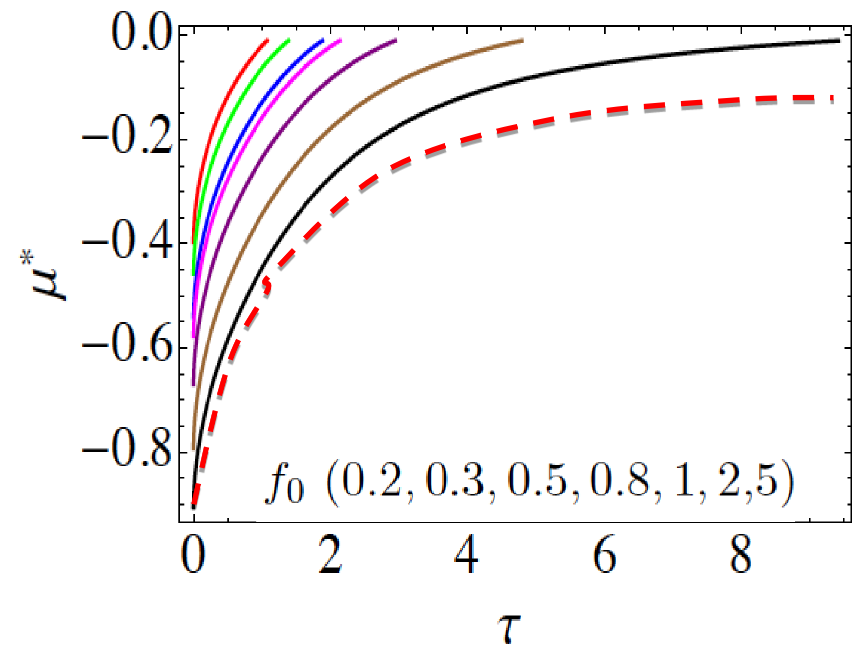
$$f(p)=f_0\,\theta(1-p/Q_s)$$

Relativistic-Boltzmann vs Fokker-Planck

$$m^* = T \ln\left(1 + \frac{1}{f(0)}\right) \xrightarrow{\text{equilibrium}} m$$



$$\tau = 2\pi^2\alpha^2 \xi f_0(1 + f_0)\, t.$$



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Quite good agreement, transition to BEC for $f_o > 0.16$ in both cases!

Generally similar behavior FP or BM, but m_D is small