Hadronic interactions of heavy mesons

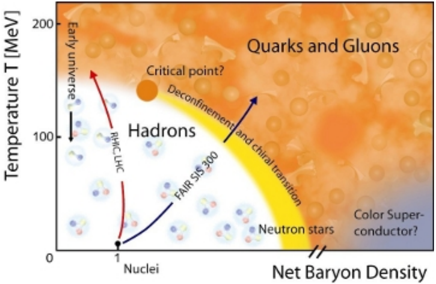
or how to implement realistic interactions
to study the heavy-meson diffusion

Juan M. Torres-Rincon

Subatech (CNRS-IN2P3)

Talk at NeD/TURIC 2014,
Crete. June 12, 2014

Introduction



- Focus on the hadronic side of the phase diagram
- Use effective field theories to compute hadronic interactions
- Ensure that the scattering amplitudes respect unitarity
- Compute some transport coefficients with these amplitudes

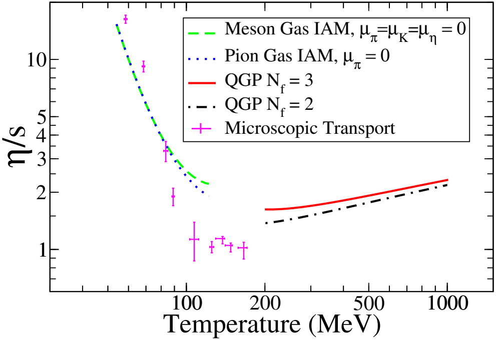
Hadronic Effective Theory + Unitarization Programme $\rightarrow$ Transport Coefficients

Hadronic Effective Theory + Unitarization Programme
$\rightarrow$ Transport Coefficients

Chiral Perturbation Theory + Inverse Amplitude Method
$\rightarrow$ Shear Viscosity
(Dobado, Llanes-Estrada and JMTR, 2009)

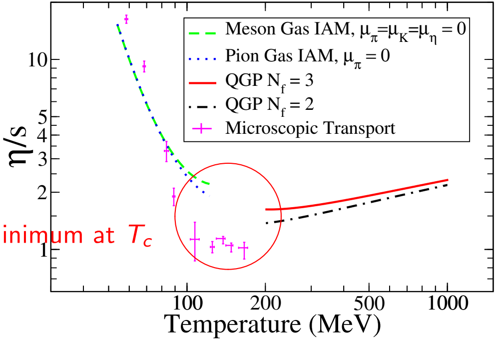
Hadronic Effective Theory + Unitarization Programme → Transport Coefficients

Chiral Perturbation Theory + Inverse Amplitude Method
→ Shear Viscosity
(Dobado, Llanes-Estrada and JMTR, 2009)



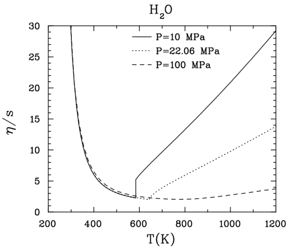
Hadronic Effective Theory + Unitarization Programme → Transport Coefficients

Chiral Perturbation Theory + Inverse Amplitude Method
→ Shear Viscosity
(Dobado, Llanes-Estrada and JMTR, 2009)

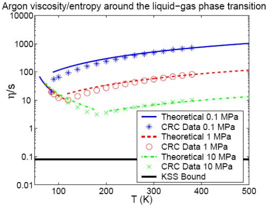


Notice the minimum at T_c

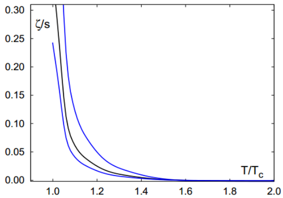
Extrema of transport coefficients



Cserani, Kapusta, McLerran, 2006



Dobado, Llanes-Estrada, JMTR, 2009



Karsch, Kharzeev, Tuchin, 2007

Other works in the same lines:
*Bulk viscosity in the linear σ model (Dobado and JMTR, 2012)
*Shear viscosity in a Fermi gas in the unitary limit (Chafin, Schaefer, 2012)
*Electrical conductivity in QCD (Cassing, Linnyk, Steinert and Ozvenchuk, 2013)

In this talk I will particularize on D mesons:

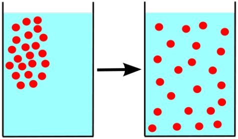
Heavy Meson Effective Theory + On-Shell Unitarization
$\rightarrow$ Heavy Meson Diffusion Coefficient

(Abreu, Cabrera, Llanes-Estrada, Tolos, Romanets, JMTR, 2011-2014)

In this talk I will particularize on D mesons:

Heavy Meson Effective Theory + On-Shell Unitarization
$\rightarrow$ Heavy Meson Diffusion Coefficient

(Abreu, Cabrera, Llanes-Estrada, Tolos, Romanets, JMTR, 2011-2014)

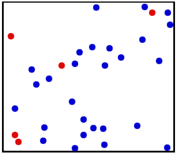


I will try to answer this question...

Does the diffusion coefficient of heavy mesons present an extremum at the QCD phase transition?

Diffusion coefficient

The physical picture is that of a Brownian particle travelling through a thermal bath and colliding with the bath's particles.



$$\langle (r(t) - r_0)^2 \rangle = 6 D_x t$$

The typical size probed by the Brownian particle is proportional to $\sqrt{t}$. The associated "speed" is the diffusion coefficient D_x.

Fokker-Planck equation

Fokker-Planck equation

$$\frac{\partial f(t,p)}{\partial t} = \frac{\partial}{\partial p_i} \left\{ F_i(p) f(t,p) + \frac{\partial}{\partial p_j} \left[\Gamma_{ij}(p) f(t,p) \right] \right\}$$

where $i = 1, 2, 3$ denote the spatial direction.

$F_i(p)$ and $\Gamma_{ij}(p)$ are the drag force and the momentum-space diffusion matrix.

Fokker-Planck equation

Fokker-Planck equation

$$\frac{\partial f(t,p)}{\partial t} = \frac{\partial}{\partial p_i} \left\{ F_i(p) f(t,p) + \frac{\partial}{\partial p_j} \left[\Gamma_{ij}(p) f(t,p) \right] \right\}$$

where $i = 1, 2, 3$ denote the spatial direction.

$F_i(p)$ and $\Gamma_{ij}(p)$ are the drag force and the momentum-space diffusion matrix.

In an isotropic gas, close to equilibrium and in the limit $p \to 0$ there is only one independent coefficient, say the drag force:

$$F(p) = \int d\mathbf{k} \; w(\mathbf{p},\mathbf{k}) \frac{\mathbf{k}\cdot\mathbf{p}}{p^2}$$

Fokker-Planck equation

Fokker-Planck equation

$$\frac{\partial f(t,p)}{\partial t} = \frac{\partial}{\partial p_i} \left\{ F_i(p) f(t,p) + \frac{\partial}{\partial p_j} \left[\Gamma_{ij}(p) f(t,p) \right] \right\}$$

where $i = 1, 2, 3$ denote the spatial direction.

$F_i(p)$ and $\Gamma_{ij}(p)$ are the drag force and the momentum-space diffusion matrix.

In an isotropic gas, close to equilibrium and in the limit $p \to 0$ there is only one independent coefficient, say the drag force:

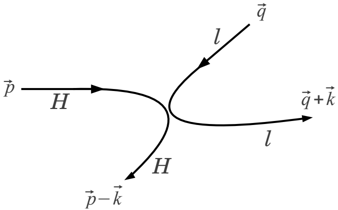
$$F(p) = \int d\mathbf{k} \; w(\mathbf{p},\mathbf{k}) \frac{\mathbf{k}\cdot\mathbf{p}}{p^2} \quad \to \quad D_x = \lim_{p\to 0} \frac{T}{MF(p)}$$

Fokker-Planck equation

Spatial diffusion coefficient

$$D_x = \frac{T}{M} \lim_{p \to 0} \left[\int d\mathbf{k} \; w(\mathbf{p}, \mathbf{k}) \frac{\mathbf{k} \cdot \mathbf{p}}{p^2} \right]^{-1}$$

where $w(\mathbf{p}, \mathbf{k})$ represents the probability of the Brownian particle with momentum $\mathbf{p}$ of having a collision and loosing a momentum $\mathbf{k}$.



Fokker-Planck equation

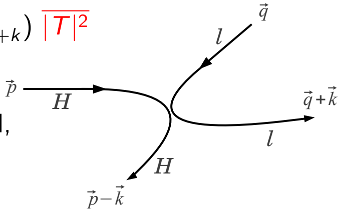
Spatial diffusion coefficient

$$D_x = \frac{T}{M} \lim_{p\to 0} \left[\int d\mathbf{k}\ w(\mathbf{p},\mathbf{k}) \frac{\mathbf{k}\cdot\mathbf{p}}{p^2} \right]^{-1}$$

where $w(\mathbf{p},\mathbf{k})$ represents the probability of the Brownian particle with momentum $\mathbf{p}$ of having a collision and loosing a momentum $\mathbf{k}$.

$$w(\mathbf{p},\mathbf{k}) = g_l \int \frac{d\mathbf{q}}{(2\pi)^9} f_l(\mathbf{q}) \frac{1}{2E^l_q} \frac{1}{2E^H_p} \frac{1}{2E^l_{q+k}} \frac{1}{2E^H_{p-k}} \times (2\pi)^4 \delta(E^H_p + E^l_q - E^H_{p-k} - E^l_{q+k}) \overline{|T|^2}$$

where $\overline{|T|^2}$ is the scattering amplitude squared, for which the effective theory is needed.



Meson-meson interaction

- The effective theory shares the symmetries of the underlying theory (QCD), in particular the chiral and heavy-quark symmetries
- These two symmetries are broken, but one can perturbatively construct an effective action in a systematic way
- Double expansion in $(m_{light}, p)/\Lambda_\chi$ and $1/m_{Heavy}$

Geng, Kaiser, Martin-Camalich and Weise *Phys.Rev.D82,05422 (2010)*

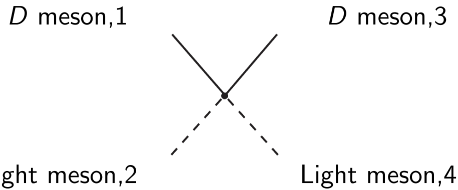
Meson-meson interaction

- The effective theory shares the symmetries of the underlying theory (QCD), in particular the chiral and heavy-quark symmetries
- These two symmetries are broken, but one can perturbatively construct an effective action in a systematic way
- Double expansion in $(m_{light}, p)/\Lambda_\chi$ and $1/m_{Heavy}$

Geng, Kaiser, Martin-Camalich and Weise *Phys.Rev.D82,05422 (2010)*

- SU(3) chiral symmetry is broken due to the light meson masses (π, K, $\bar{K}$, η) and the Lagrangian is taken at NLO.
- The heavy-quark symmetry expansion is kept at LO (we explicitly break flavor symmetry but not heavy-quark spin symmetry).

Meson-meson interaction



Tree level amplitude

$$V = \frac{C_0}{4F^2}(s-u) + \frac{2C_1}{F^2} h_1 + \frac{2C_2}{F^2} h_3 (p_2 \cdot p_4) + \frac{2C_3}{F^2} h_5 [(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

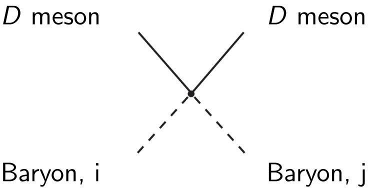
Isospin coefficients (known)

Low-energy constants (to be fixed by matching)

Meson-baryon interaction (in collaboration with L. Tolos and O. Romanets)

- Effective action for the interaction between D mesons and baryons.
- Also respects heavy-quark spin symmetry
- Uses a Weinberg-Tomozawa interaction (t-channel vector-meson exchange) and takes the static limit $t \to 0$ to produce a contact vertex
- DN and $D\Delta$ effective description:
  Mizutani, Ramos *Phys.Rev.C74, 065201 (2006)*
  Garcia-Recio et al. *Phys.Rev.D79, 054004 (2012)*
  Romanets et al. *Phys.Rev.D85 114032 (2012)*

Meson-baryon interaction



Tree level amplitude

$$V_{ij} = D_{ij} \frac{2\sqrt{s} - M_i - M_j}{4 f_i f_j} \sqrt{\frac{E_i + M_i}{2M_i}} \sqrt{\frac{E_j + M_j}{2M_j}}$$

D_{ij} coefficients depending on the channel $(IJSC)$ similar to the C_i for the light-meson sector.

Unitarization

Scattering matrix

$$S^\dagger S = 1$$

Unitarization

Scattering matrix

$$S^\dagger S = 1$$

Translated to the s-wave scattering amplitude

Unitarity condition

$$\mathrm{Im}\ V(s) = -\frac{\rho_{12}}{16\pi^2} |V(s)|^2$$

(two-body phase space)

$$\rho_{12} = \sqrt{\left(1 + \frac{(m_1 + m_2)^2}{s}\right)\left(1 - \frac{(m_1 - m_2)^2}{s}\right)}$$

Unitarization

Scattering matrix

$$S^\dagger S = 1$$

Translated to the *s*-wave scattering amplitude it reads

Unitarity condition

$$\text{Im } V(s) \neq -\frac{\rho_{12}}{16\pi^2} |V(s)|^2$$

(two-body phase space)

$$\rho_{12} = \sqrt{\left(1 + \frac{(m_1 + m_2)^2}{s}\right)\left(1 - \frac{(m_1 - m_2)^2}{s}\right)}$$

Unitarization allows to construct a new amplitude $V \to T$ that satisfies the unitary condition exactly.

On-Shell Unitarization

- Oller and Oset, *Nucl.Phys.A620 (1997) 438*
  Roca, Oset and Singh *Phys.Rev.D72 (2005) 014002*
- Based on two-body scattering resummation to construct a Bethe-Salpeter equation
- The method provides an algebraic solution to the integral equation

$$T = V + VGV + VGVGV + \cdots = V + VGT$$

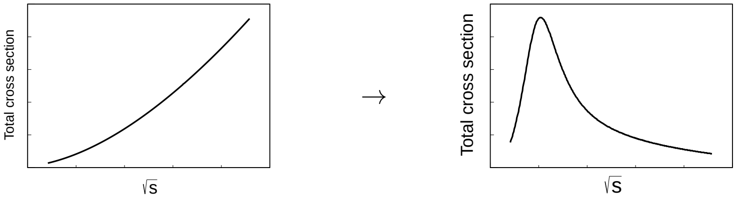
Unitarization

Unitarized amplitude

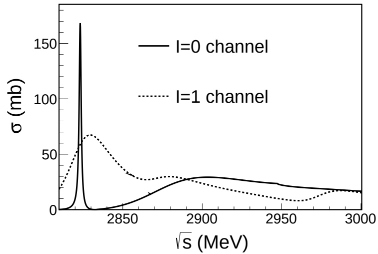
$$T = \frac{V}{1 - GV}$$

Benefits of unitarization:

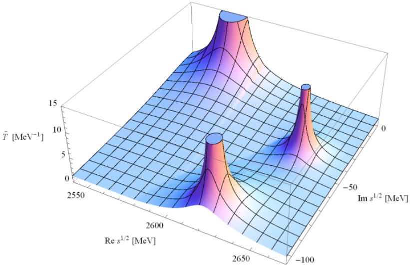
- S-matrix unitarity is now exactly satisfied
- Physical behavior of cross sections: saturation
- Potential presence of resonances and/or bound states



Unitarization

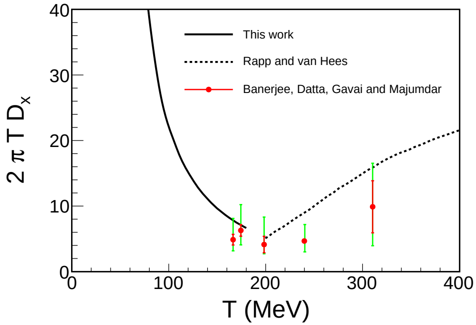


E.g. $DN \to DN$ total cross section



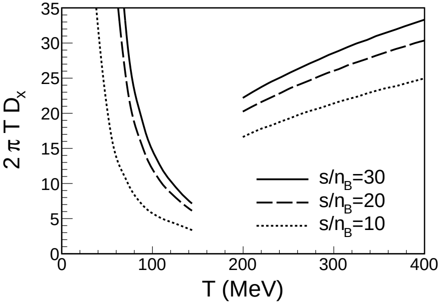
"Σ_c channel", Romanets et al. 2012

Diffusion coefficient at $\mu_B = 0$



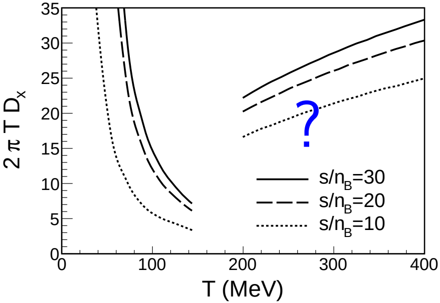
- Our result (Tolos and JMTR, 2013) contains interactions of D meson with π, K, $\bar{K}$, η, N, Δ
- The points are lattice-QCD data taken with $T_c = 160$ MeV
- In the QGP, a lattice-based potential is used in a $T-$matrix equation to compute the quark-quark scattering

Finite chemical potential



- For the QGP we adapt the result of Moore and Teaney, 2004 to the finite μ_B case with running coupling constant (as the simplest guess)
- In the $\mu_q = 0$ case, this result is known to overpredict (3-4 times) the lattice-QCD result

Finite chemical potential

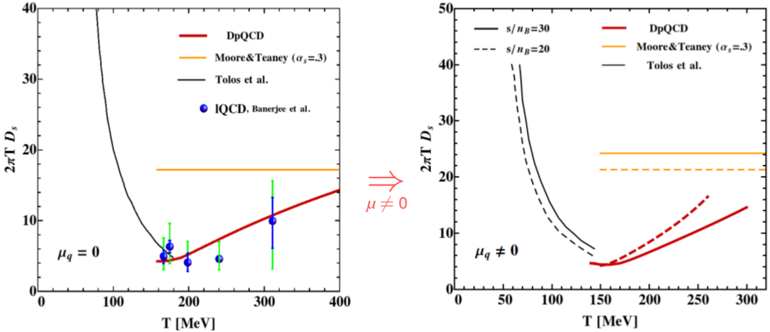


- For the QGP we adapt the result of Moore and Teaney, 2004 to the finite μ_B case with running coupling constant (as the simplest guess)
- In the $\mu_q = 0$ case, this result is known to overpredict (3-4 times) the lattice-QCD result

For recent updates on the QGP phase see talks of P.B. Gossiaux, V. Ozvenchuk and H. Berrehrah

Dressed-pQCD result

HQ spatial diffusion coefficient in HOT and DENSE medium



- $D_s = T/(M_Q \eta_D) = 2T^2/\kappa;\ \ \kappa = d < (\vec{p} - \vec{p}')^2 > /3d\tau,$ Nice agreement around the cross over T_c at $\mu = 0$
- Minimum at T_c for FAIR energies ??
- Adiabatic FAIR trajectories (with $S/N_B = 20 - 30$) following . . .
- Continuous transition $\rightarrow$ No 1[st] order transition $\rightarrow$ Consistent with our model assumptions
- HQ physics : Both partonic and hadronic worlds contribute ? $\rightarrow$ quantitative studies

References

For more information:

Charm

D-meson propagation in hot dense matter
Laura Tolos, Juan M. Torres-Rincon
Phys.Rev. D88 (2013) 074019
e-Print: arXiv:1306.5426 [hep-ph]

Bottom

Open bottom states and the anti-B meson propagation in hadronic matter
Juan M. Torres-Rincon, Laura Tolos, Olena Romanets
Phys.Rev. D89 (2014) 074042
e-Print: arXiv:1403.1371 [hep-ph]

Additional slides

Langevin equation

It is an alternative (but equivalent) description to the Fokker-Planck equation.

$$\frac{dx^i}{dt} = \frac{p^i}{m_H}$$
$$\frac{dp^i}{dt} = -F^i(p) + \xi^i(t)$$

with ξ^i a stochastic Gaussian force

$$\langle \xi^i(t) \rangle = 0$$
$$\langle \xi^i(t) \xi^j(t') \rangle = \Gamma^{ij}(p)\ \delta(t - t')$$

Relaxation time

Consider Newton's law (with $F^i = Fp^i$)

$$\frac{dp_i}{dt} = -F\ p^i$$

Assuming constant F one can solve the equation for $p(t)$

$$p(t) = p(0)\ e^{-t/F}$$

The inverse of F plays the role of a relaxation time τ_R

$$\tau_R\ =\ 1/F$$

Fluctuation-Dissipation Theorem

$F(p)$ is a deterministic drag force that causes energy loss (dissipation) whereas the diffusion coefficients are related to the strength of a stochastic (or fluctuating) force.

The fluctuation-dissipation theorem relates the 3 coefficients:

Fluctuation-Dissipation Theorem

$$F(p) + \frac{1}{p}\frac{\partial \Gamma_1(p)}{\partial p} + \frac{2}{p^2}\left[\Gamma_1(p) - \Gamma_0(p)\right] = \frac{\Gamma_1(p)}{m_H T}$$

Fluctuation-Dissipation Theorem

$F(p)$ is a deterministic drag force that causes energy loss (dissipation) whereas the diffusion coefficients are related to the strength of a stochastic (or fluctuating) force.
The fluctuation-dissipation theorem relates the 3 coefficients:

Fluctuation-Dissipation Theorem

$$F(p) + \frac{1}{p}\frac{\partial \Gamma_1(p)}{\partial p} + \frac{2}{p^2}\left[\Gamma_1(p) - \Gamma_0(p)\right] = \frac{\Gamma_1(p)}{m_H T}$$

In the static limit, i.e. when $p \to 0$ the two diffusion coefficients become degenerate and the Einstein relation is recovered

Einstein Relation

$$F = \frac{\Gamma}{m_H T}$$

$D-\pi$, $D^*-\pi$ interaction

Effective Lagrangian: L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

Chiral symmetry (NLO) + Heavy Quark symmetry (LO)

$$D=(D^0,D^+,D_s^+)\ ,\quad D^*_\mu=(D^{*0},D^{*+},D_s^{*+})_\mu$$

$$\mathcal{L}^{(1)}=Tr[\nabla^\mu D\nabla_\mu D^\dagger]-M_D^2Tr[DD^\dagger]-Tr[\nabla^\mu D^{*\nu}\nabla_\mu D^{*\dagger}_\nu]+M^2_{D^*}Tr[D^{*\mu}D^{*\dagger}_\mu]$$

$$+igTr\left[\left(D^{*\mu}u_\mu D^\dagger-Du^\mu D^{*\dagger}_\mu\right)\right]+\frac{g}{2M_D}Tr\left[\left(D^*_\mu u_\alpha\nabla_\beta D^{*\dagger}_\nu-\nabla_\beta D^*_\mu u_\alpha D^{*\dagger}_\nu\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

$$\nabla_\mu=\partial_\mu-\Gamma_\mu\ ;\quad \Gamma_\mu=\frac{1}{2}\left(u^\dagger\partial_\mu u+u\partial_\mu u^\dagger\right)\ ;\quad u_\mu=i\left(u^\dagger\partial_\mu u-u\partial_\mu u^\dagger\right)$$

g from experimental $D^{*+}\to D^*\pi^+$ decay width

$D-\pi$, $D^*-\pi$ interaction

$$\mathcal{L}^{(2)} = -h_0 Tr[DD^\dagger] Tr[\chi_+] + h_1 Tr[D\chi_+ D^\dagger] + h_2 Tr[DD^\dagger] Tr[u^\mu u_\mu] + h_3 Tr[Du^\mu u_\mu D^\dagger]$$

$$+h_4 Tr[\nabla_\mu D \nabla_\nu D^\dagger] Tr[u^\mu u^\nu] + h_5 Tr[\nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger] + \{D \to D^\mu\}$$

$$u = \sqrt{U} = \exp\left(\frac{i\Phi}{\sqrt{2}F}\right); \quad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

h_{even} subleading in the large-N_c limit
h_1 determined by the mass splitting between the D and D_S
h_3 and h_5 fixed by the pole mass and width of the $D_0(2400)$ resonance

D spectrum

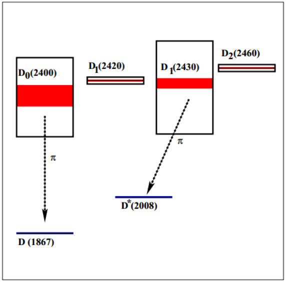


Fig. 1. The currently known low-lying D-meson system. The negative parity state D and D^* are represented as the blue lines. The four positive parity states have the mass measurement spread throughout the red boxes, while the hollow black boxes represent current estimates of their width. s-wave pion decays are depicted.

Table 1. Charged-average masses and experimental estimates [14] for the strong widths of the D-meson resonances. Units are MeV. Errors not quoted are about 1 MeV or less.

Meson	J^P	M (MeV)	Γ (MeV)
D	0^-	1867	-
D^*	1^-	2008	1
D_0	0^+	2360(40)	270(50)
D_1	1^+	2422	22(5)
D_1	1^+	2427(40)	380(150)
D_2	2^+	2460	30