Mach cones in viscous heavy-ion collisions

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in collaboration with B. Betz, Z. Xu and C. Greiner

I. Bouras et al., Phys. Rev. Lett. 103:032301 (2009)

I. Bouras et al., PRC 82, 024910 (2010)

I. Bouras et al., Phys.Lett. B710 (2012)

I. Bouras et al., arXiv:1401.3019 (2014)

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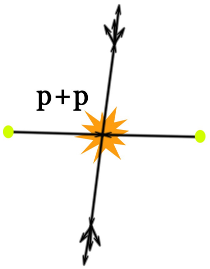
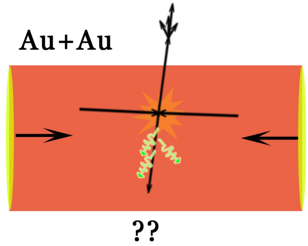
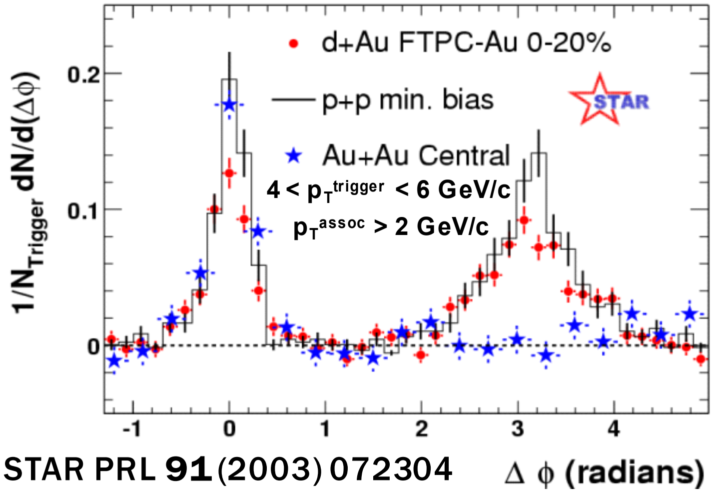
Jet-quenching and two-particle correlations in HIC

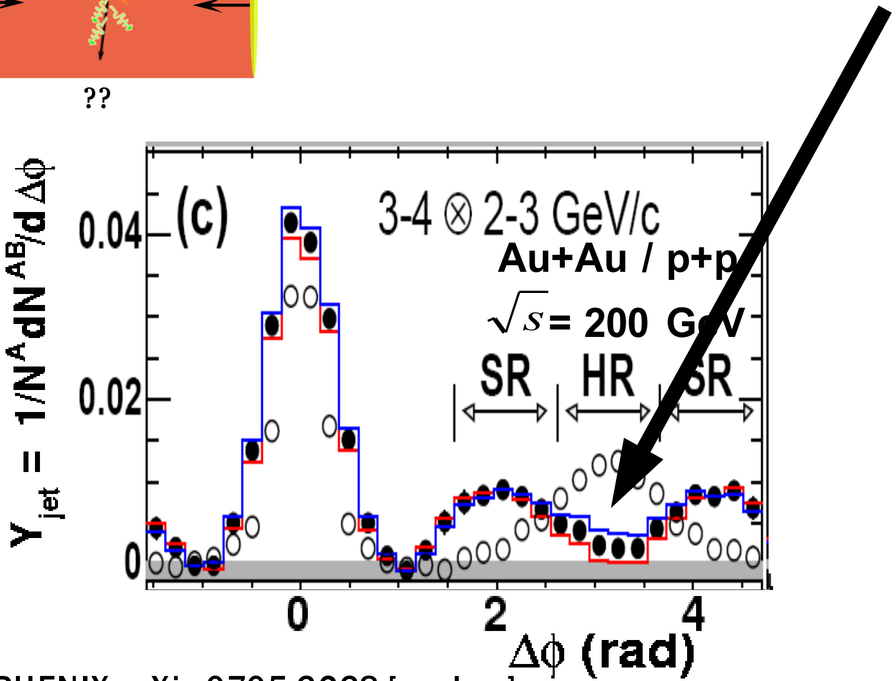




Jet-medium interaction?



STAR PRL **91** (2003) 072304



PHENIX arXiv:0705.3238 [nucl-ex]

Phys.Rev.C77:011901,2008

Possible contributions to this double-peak structure:

- **Deflected jets**
- **Jet-medium interaction resulting into Shock waves in form of Mach cones**
- **Triangular flow originated from initial state fluctuations** → Phys.Rev.Lett. 110 (2013) 012302
  Phys. Rev. Lett. 103, 242301 (2009)

Do Mach Cones have something to do with double peaks?
Do they contribute to the double-peak structure observed in experiments?

The Parton Cascade BAMPS

- Transport algorithm solving the **Boltzmann equation** using Monte Carlo techniques

$$p^{\mu}\partial_{\mu} f(x,p)=C_{22}+C_{23}+\ldots$$

Boltzmann
Approach for
Multi-
Parton
Scatterings

- Stochastic interpretation of collision rates

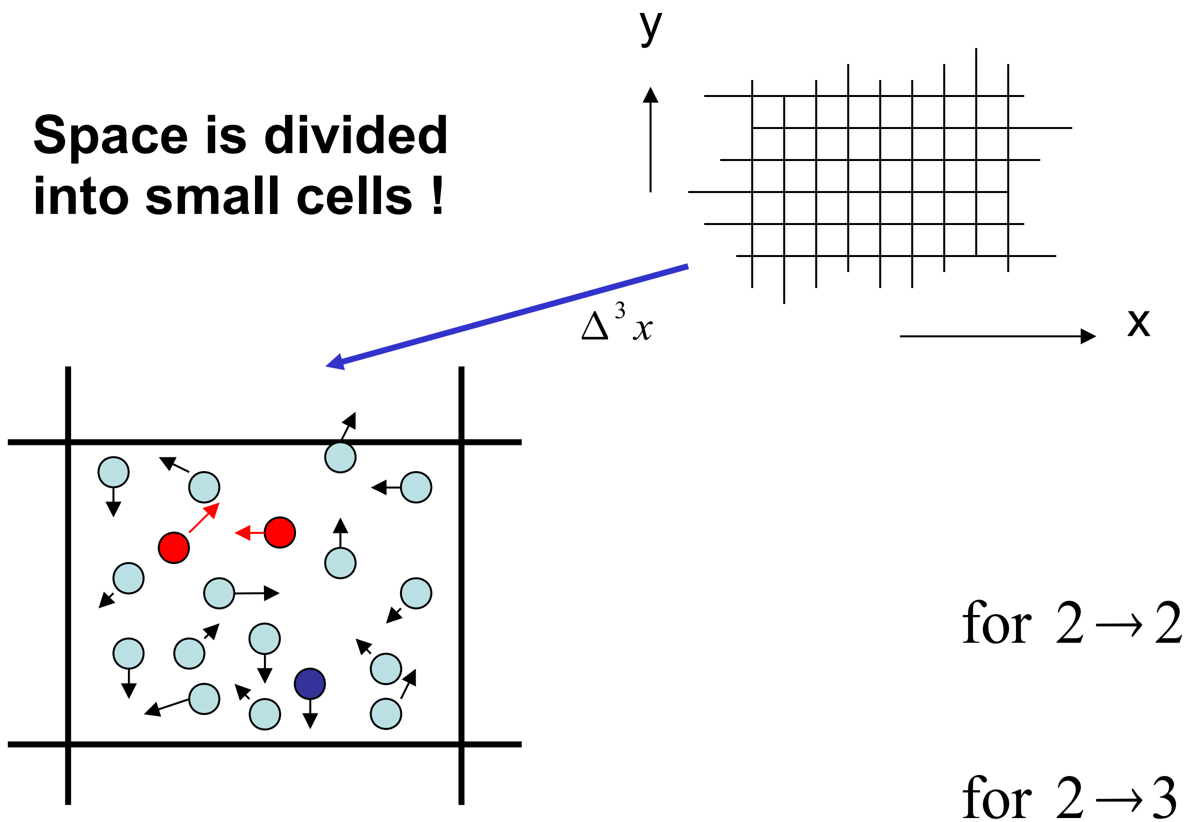
$$P_{2i}=v_{rel}\frac{\sigma_{2i}}{N_{test}}\frac{\Delta t}{\Delta^3 x}$$

Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

- In general:
  pQCD interactions, 2 ↔ 3 processes,
  quarks and gluons

The Parton Cascade BAMPS

Space is divided into small cells !



Boltzmann
Approach for
Multi-
Parton
Scatterings

$$\text{for } 2\rightarrow 2 \quad P_{22} = v_{rel} \frac{\sigma_{22}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

$$\text{for } 2\rightarrow 3 \quad P_{23} = v_{rel} \frac{\sigma_{23}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

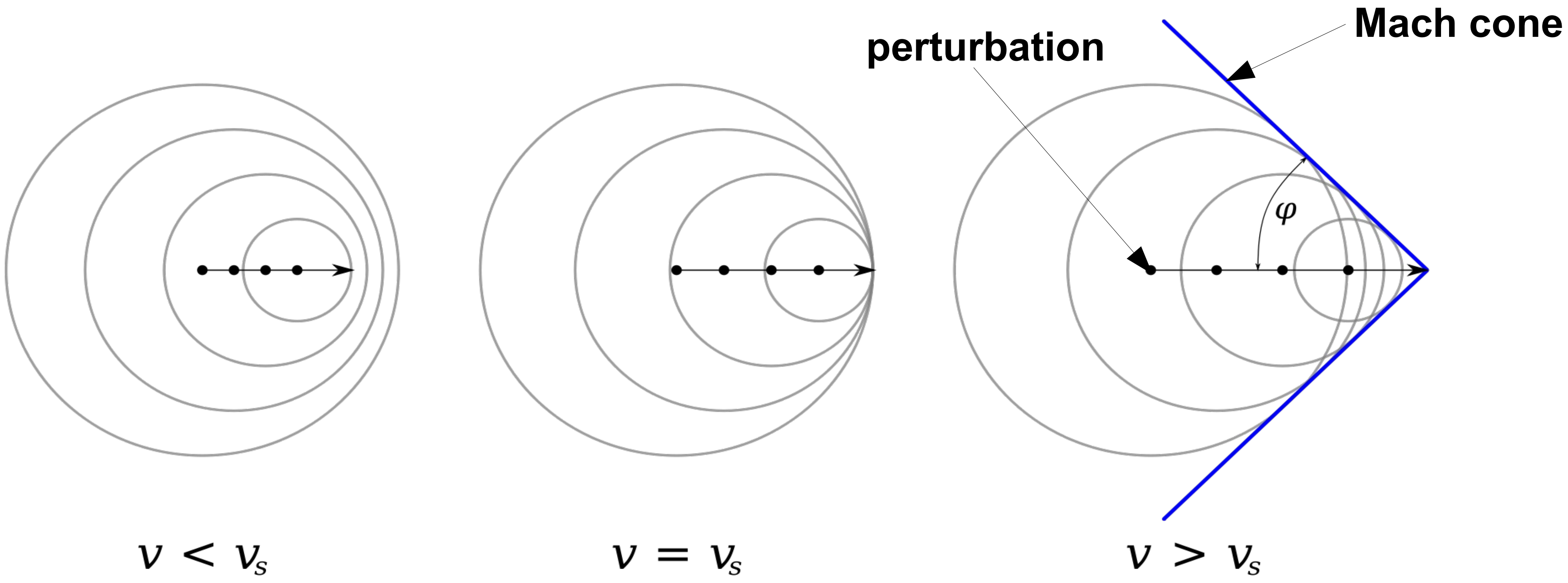
$$\text{for } 3\rightarrow 2 \quad P_{32} = \frac{1}{8E_1E_2E_3} \frac{I_{32}}{N_{test}^2} \frac{\Delta t}{(\Delta^3 x)^2}$$

Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

$$I_{32} = \frac{1}{2} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} |M_{123\rightarrow 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p'_1 - p'_2)$$

Mach Cones

- **If a source (perturbation) is propagating faster than the speed of sound, then a Mach Cone structure is observed**



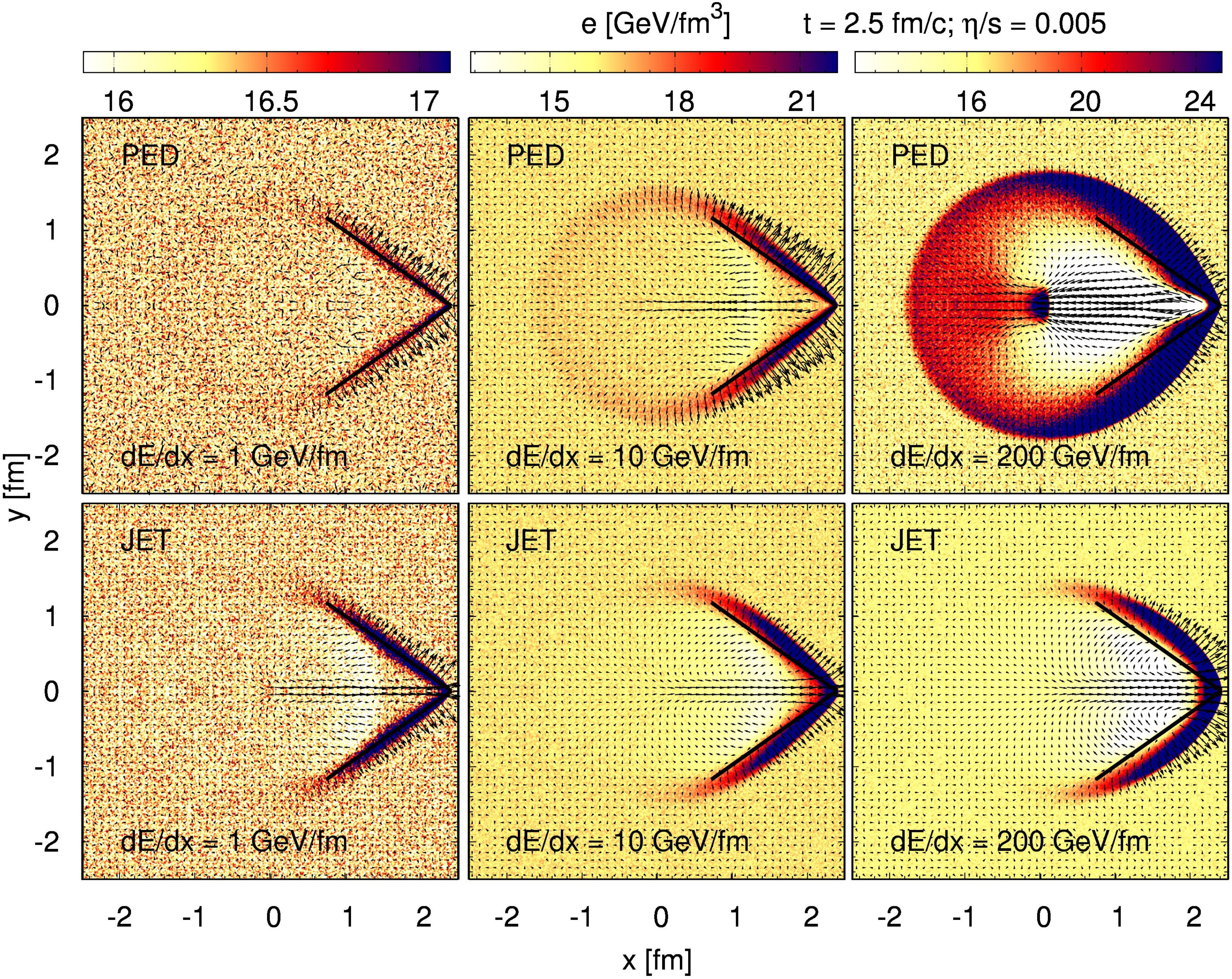
Investigation of Mach Cones In a static system

- Static System, no expansion

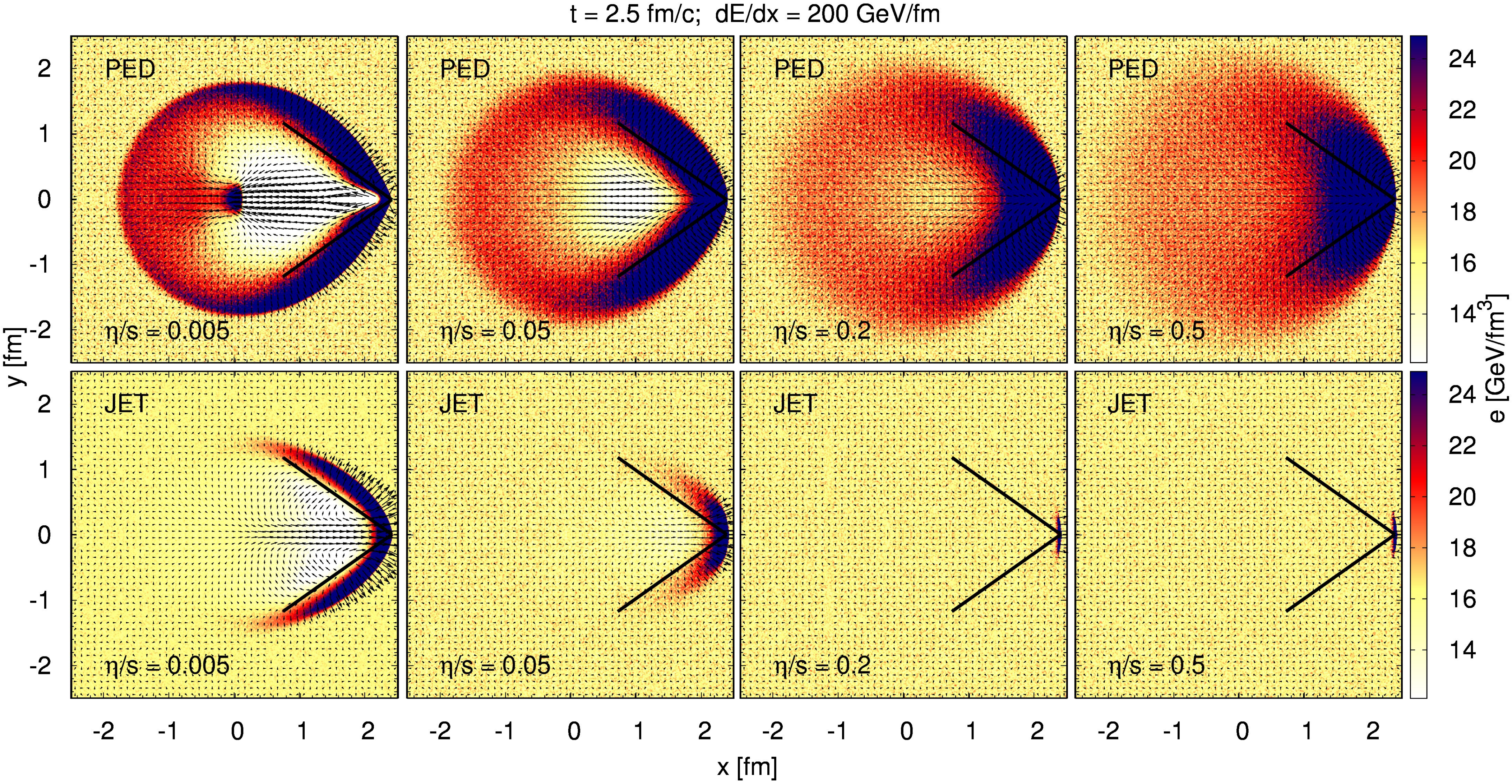
- Jet energy is fixed and cannot be deflected → two different scenarios of the energy deposition:

- PED: pure energy deposition
- JET: energy and momentum deposition

Ideal Solutions of Mach Cones



Viscous Solutions of Mach Cones



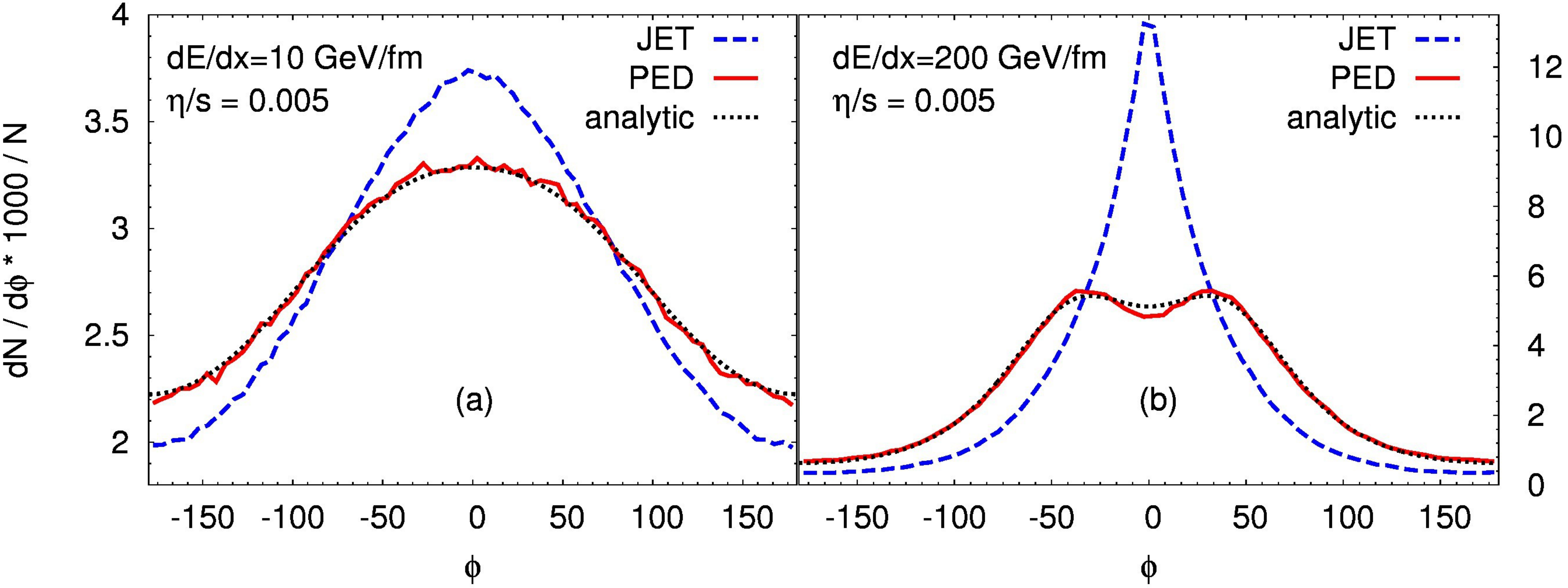
Mach Cone structure still visible for $\eta/s = 0.1 - 0.15$

Mach Cones in BAMPS

Two Particle Correlations for ideal solution

Numerical Results

10 GeV/fm **IDEAL** **200 GeV/fm**



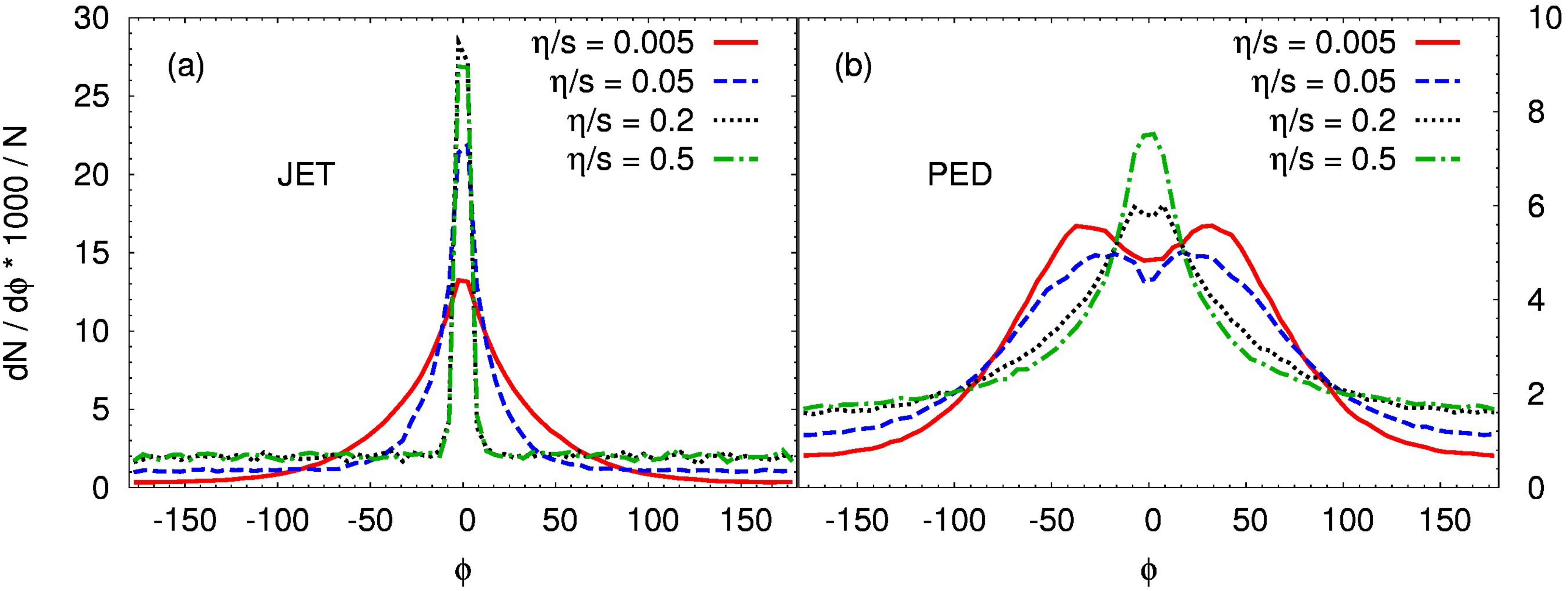
The source term plays a big role for observation a double peak structure

Mach Cones in BAMPS

Two Particle Correlations for viscous solution

Numerical Results

VISCOUS



Viscosity does not help for the development fo the double peak structure

Investigation of Mach Cones In Full relativistic HIC

- Initial conditions as given at RHIC using a parametrization for the distribution function

- The main difference to the static is the (longitudinal and transverse) expansion of the medium

- Jet energy is not fixed. Jet looses energy and can be deflected

- For simplicity we investigate only full central collisions, b=0 fm, and focus only at midrapidity

Initial conditions of the bulk medium

- **Glauber initial conditions in transverse direction**
- **Parametrization for the non-thermal single-distribution function**

$$f(\vec{x},\vec{p}) = K\frac{1}{E}\left(\frac{Q^n}{Q^n+p_T^n}\right)^m \exp\left(-\frac{y_{\mathrm{rap}}^2}{\sigma_y^2}\right)\exp\left(-\frac{z^2}{\sigma_z^2}\right)T_{\mathrm{A}}\left(x+\frac{b}{2},y\right)T_{\mathrm{B}}\left(x-\frac{b}{2},y\right)$$

Nuclear Thickness function

$$T_{\mathrm{A}}(x,y) = \int_{-\infty}^{+\infty} dz \rho_{\mathrm{A}}(x,y,z)$$

Wood-Saxon distribution

$$\rho_{\mathrm{A}}(\vec{x}) = \frac{\rho_0}{1+\exp\left(\frac{|\vec{x}|-R_{\mathrm{A}}}{D}\right)}$$

with

$D = 0.54\ \mathrm{fm}$

$Q = 1.3\,\mathrm{GeV}$

$m = 1.5$

$n = 4$

$\sigma_y = 1$

$\sigma_z = 0.13\,\mathrm{fm}$

$K = 0.0135$

$A = B = 197$

$b = 0\ \mathrm{fm}$

$\rho_0 = 0.17\,\mathrm{fm}^{-3}$

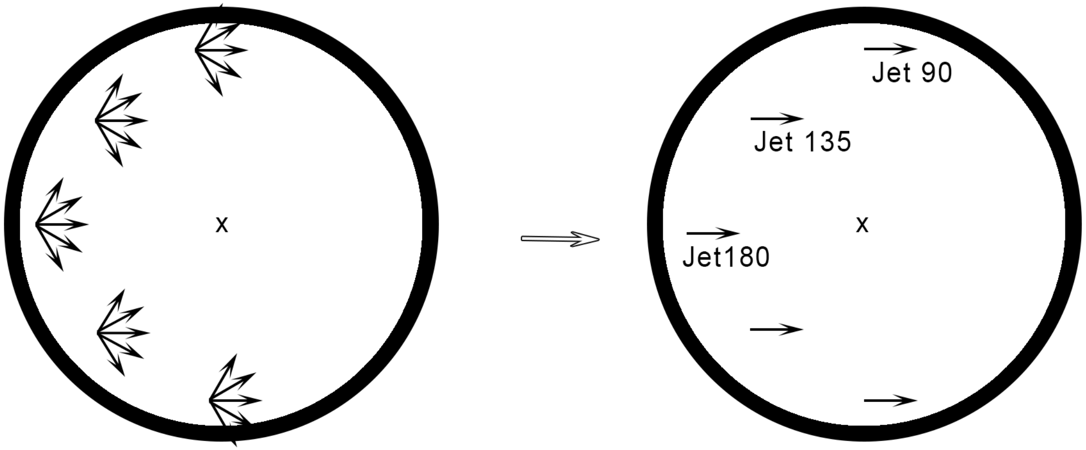
$R_{\mathrm{A}} = 1.12A^{1/3} - 0.86A^{-1/3}$

Jet initialisation on top

→ Surface Emission

- Jet is initialised on a semicircle in the midrapidity, while we have to consider several jet paths
  → Due to symmetry reasons we can can neglect several possible jet paths

- We neglect the near-side jet and consider only the jet traversing the medium



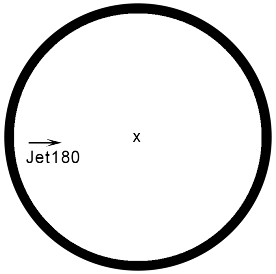
Betz et al. Phys.Rev.Lett. 105 (2010) 222301

$$\vec{x}_{\text{jet}} = r \begin{pmatrix} \cos\phi_{\text{jet}} \\ \sin\phi_{\text{jet}} \\ 0 \end{pmatrix}$$

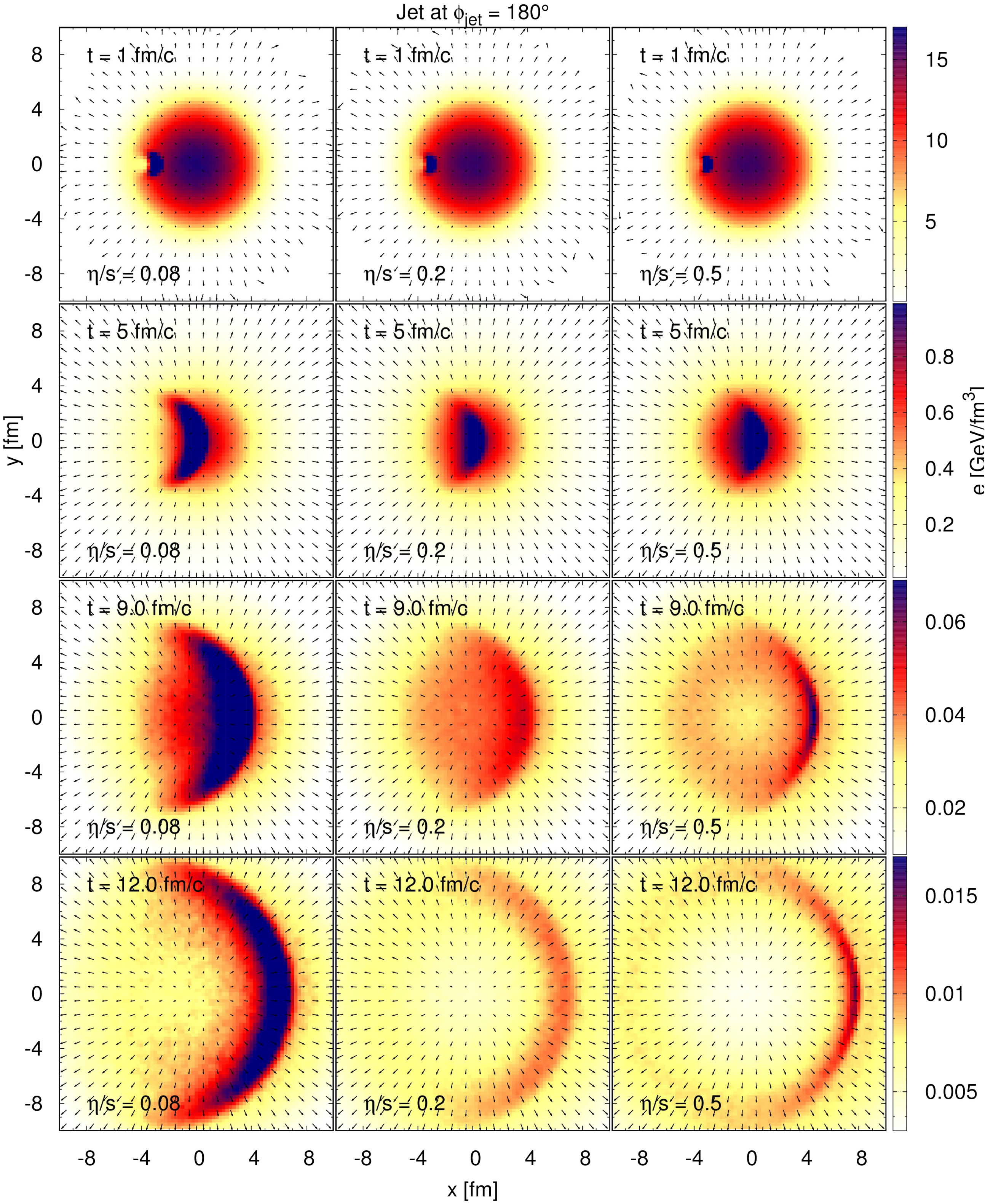
$$r = 4 \text{ fm}$$

$$p_x = E_{\text{jet}} = 20 \text{ GeV}$$

Scenario I

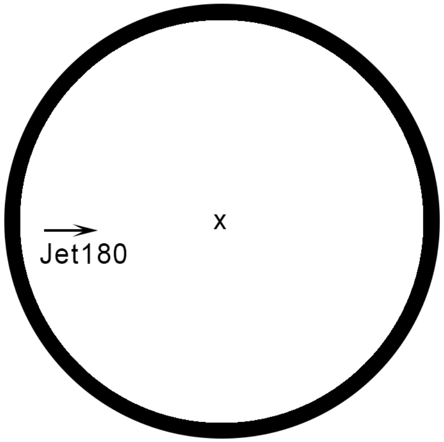


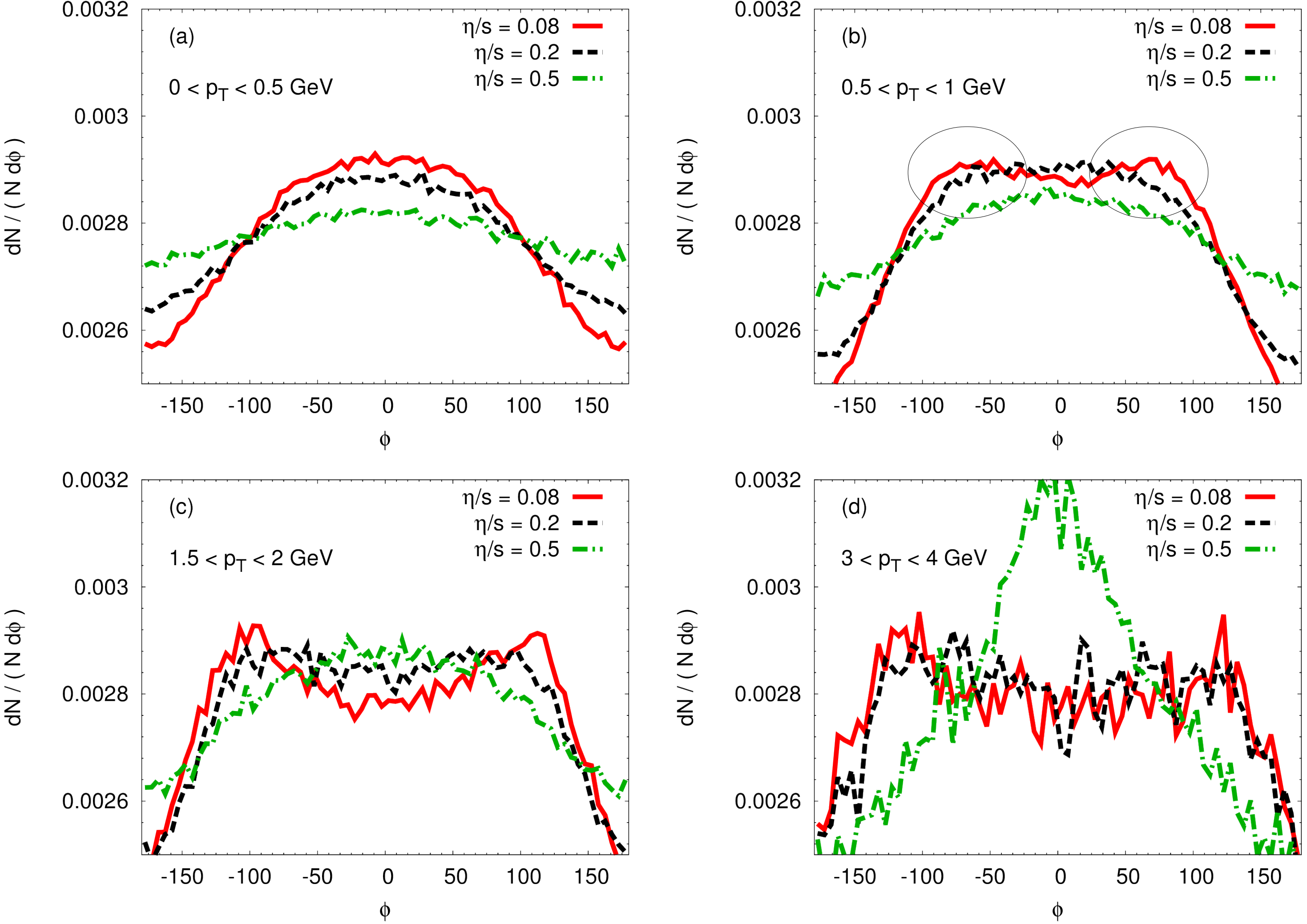
- Single jet event on the semi circle
- Results are shown at midrapidity for several values of viscosities and time steps
- Jet propagates in opposite direction to radial flow
- Small viscosity means strong shock wave development - Large viscosity smooths out the characteristic structure
- Shock front region of Mach cone is strongly curved due to jet quenching



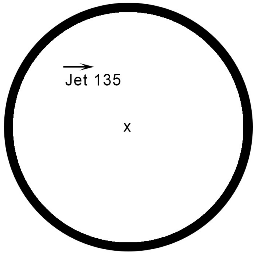
Scenario I

- Single jet event on the semi circle is able to generate a double-peak structure
- Head shock and diffusion wake is superimposed by the radial flow, contribution of Mach cone wings can show up
- Double-peak structure shows up only for higher-momentum particles
- Viscosity tends to destroy the double-peak structure





Scenario II

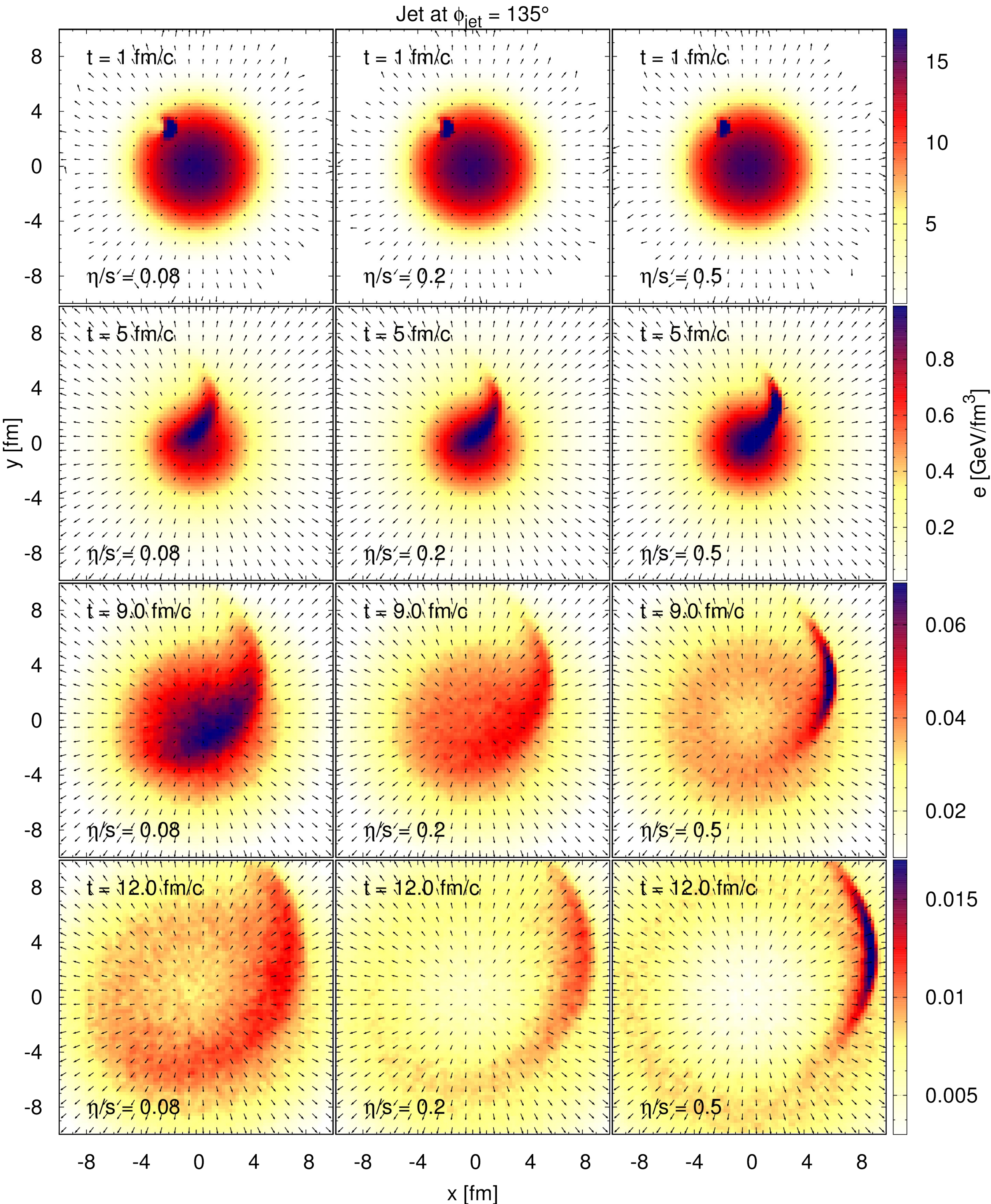


- Single jet event on the semi circle

- Results are shown at midrapidity for several values of viscosities and time steps

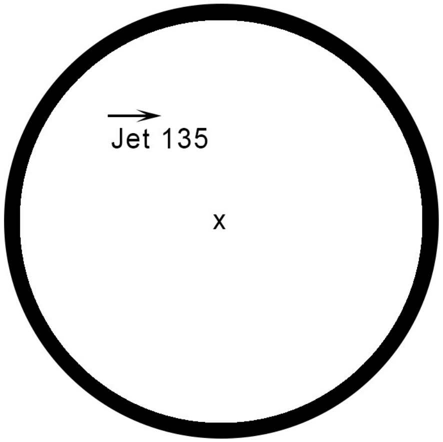
- Jet-induced Mach cone is strongly distorted due to radial flow

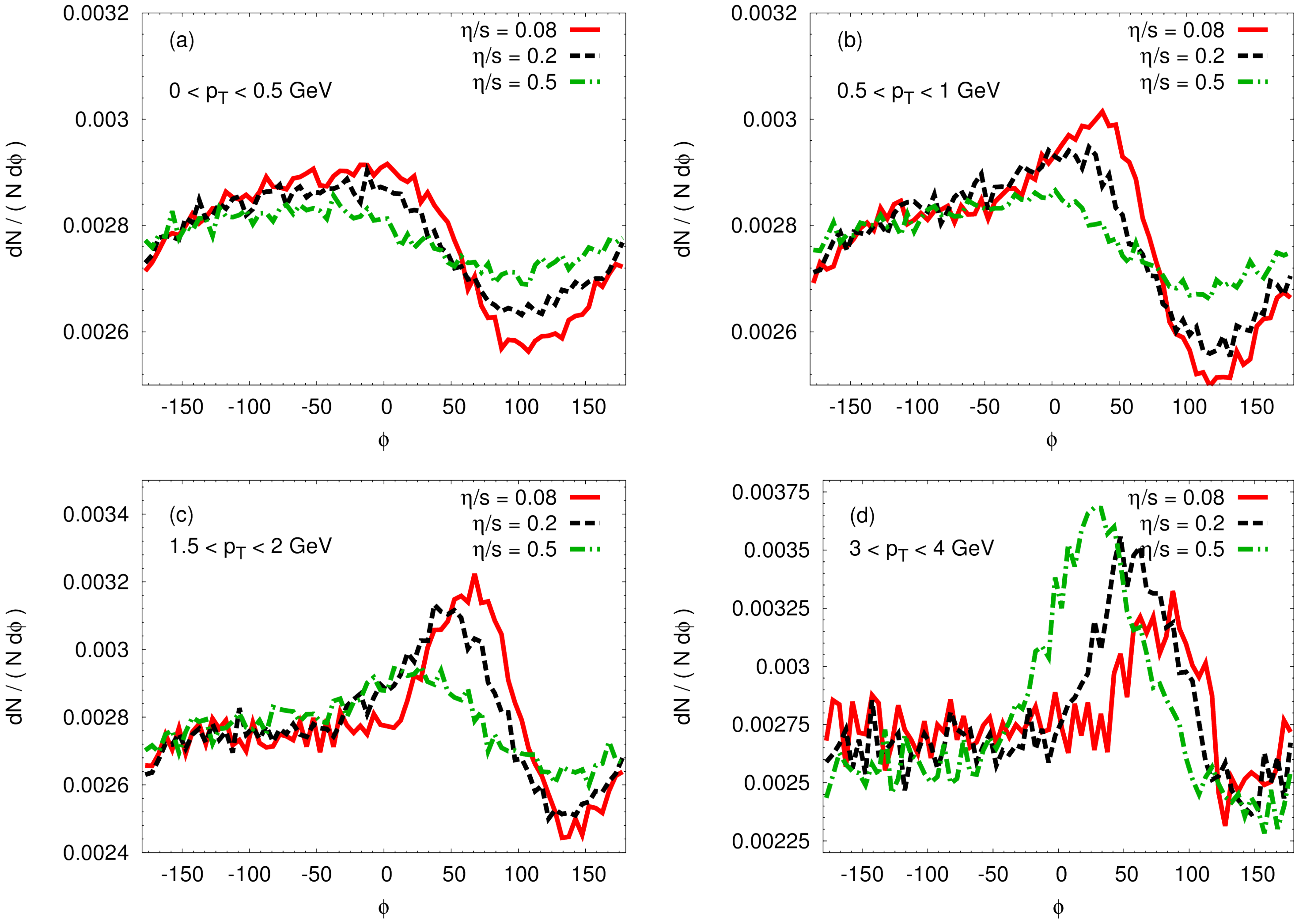
- Small viscosity means strong shock wave development - Large viscosity smooths out the characteristic structure



Scenario II

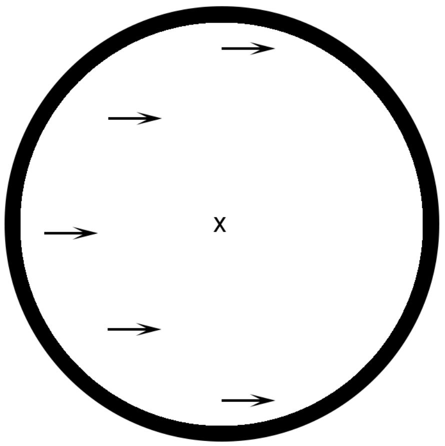
- Single jet event on the semi circle generates only one peak
- Head shock and diffusion wake is only deflected, which generates the one peak
- Viscosity tends to turn the peak into the the initial propagation direction

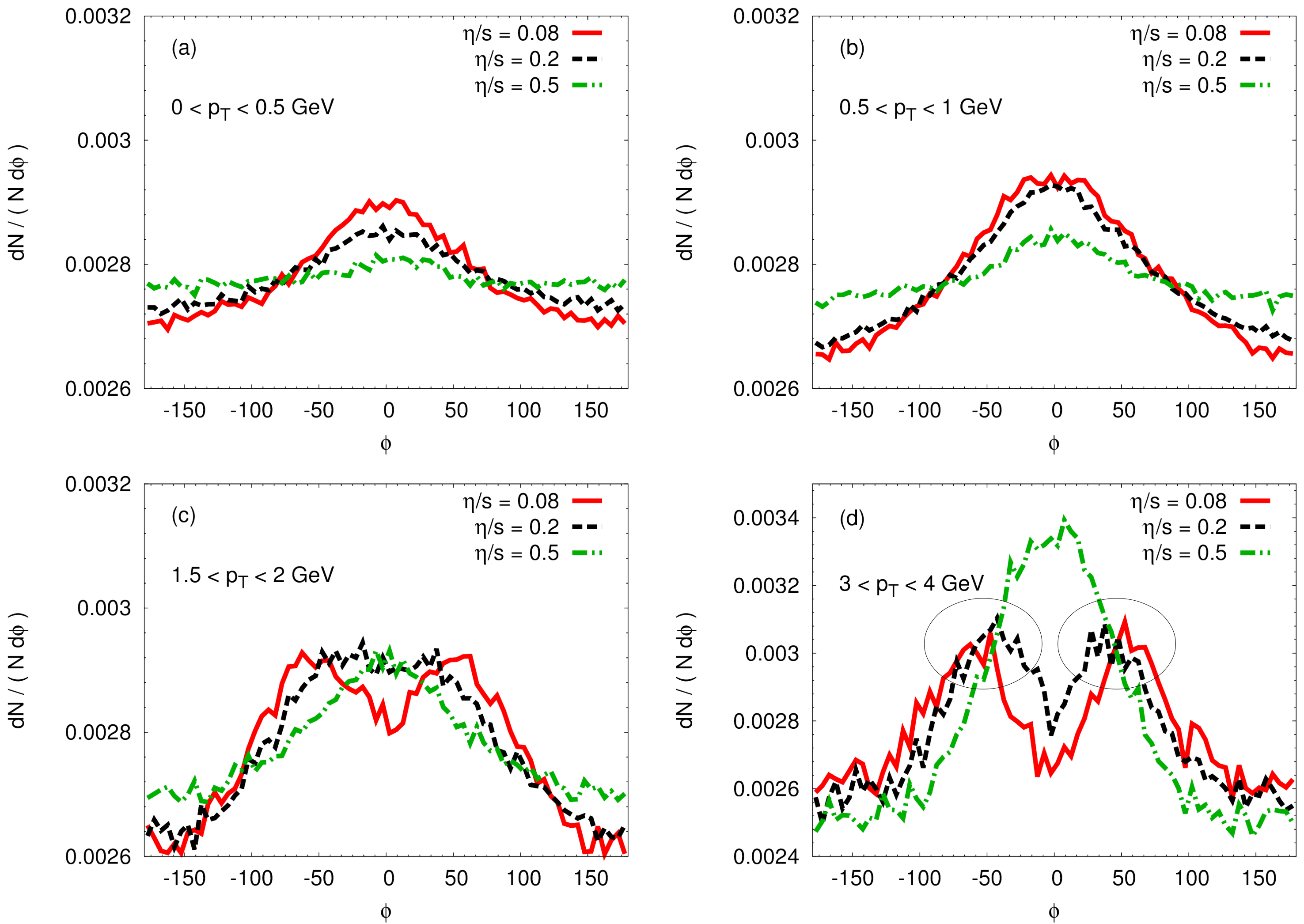




Scenario III

- We take all paths of the jet on the semi-circle
- A double-peak structure appears due to the contribution of the Mach cone wings (scenario I) and the superposition of the deflected and distorted jet-induced Mach cones (scenario II)
- Viscosity tends to destroy this double-peak structure





Conclusion ....

We considered the contribution to the double-peak structure originating from jet-medium interaction inducing Mach cones in a simplified setup...

- In a static system the double-peak structure is overshadowed by the head shock and diffusion wake

- In a scenario where the interplay with the medium plays a role, our studies show that a double-peak structure can be generated by the Mach cone wings in a single jet event, but its contribution seems to be very small

- The largest contribution comes from distorted jet-induced Mach cones

- In case viscosity is too large, any signal of Mach cones or double-peak structure ist destroyed

….Outlook

Implementation of initial stage fluctuations in BAMPS

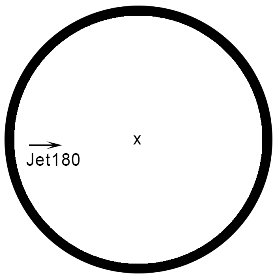
in collaboration with K. Gallmeister

- Monte Carlo Glauber sampling instead of smooth initial sampling

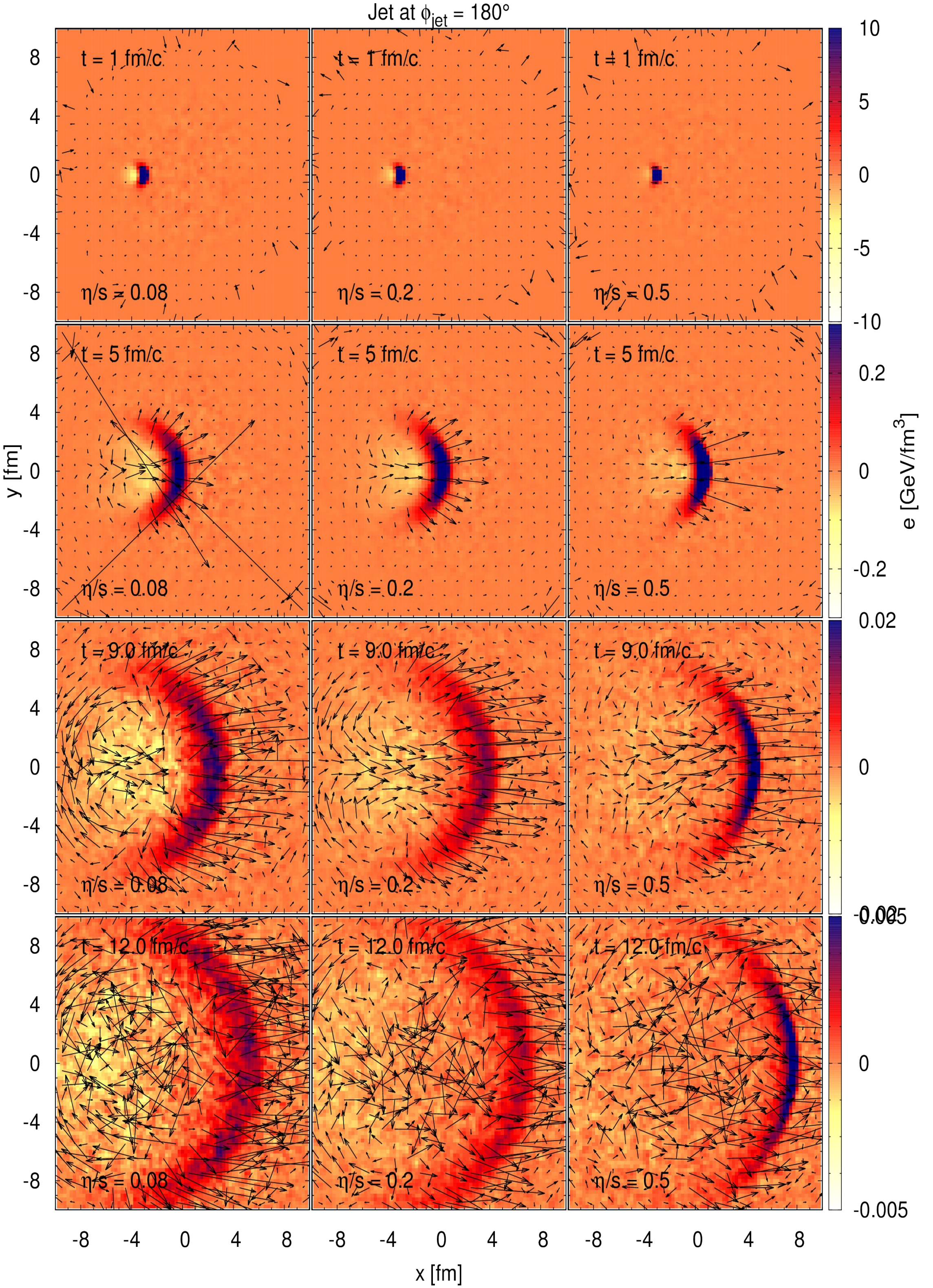
Focus:

- A + A and p + A collisions
- Extracting flow observables v2, v3 and compare them with initial excentricities
- Extracting two-particle correlations

Scenario I



- Mach cone evolution after subtracting the background



The Parton Cascade BAMPS

For this setup :

- **Boltzmann gas, isotropic cross sections, elastic processes only**
- **Implementing a constant η/s, we locally get the cross section σ_{22}:**

$$\eta = \frac{4}{15}\frac{\epsilon}{R^{tr}}$$

Transport collision rate R^{tr}

For isotropic elastic collisions:

$$R^{tr}_{22} = n\frac{2}{3}\sigma_{22}$$

$$\epsilon = 3\mathrm{n}\mathrm{T}$$

$$s = 4\mathrm{n} - n\ \ln(\lambda_{fug})$$

$$\lambda_{fug} = \frac{n}{n_{eq}} \qquad n_{eq} = \frac{g}{\pi^2}T^3$$

g = 16 for gluons

Z. Xu & C. Greiner,
Phys.Rev.Lett.100:172301,2008

$$\sigma_{22} = \frac{6}{5}\frac{T}{s}\left(\frac{\eta}{s}\right)^{-1}$$

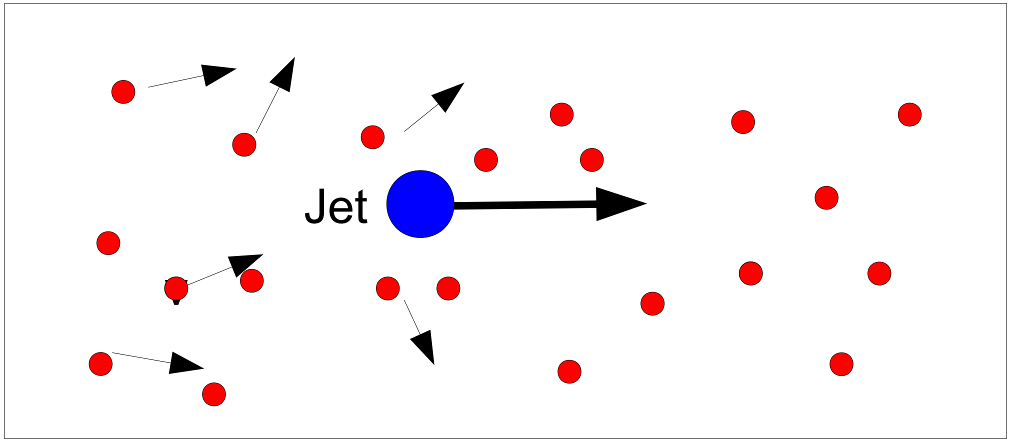
Static Box in BAMPS

- Static Box with a constant temperature. Medium is initially in thermal equilibrium
→ no expansion of the medium

- Two different source terms are applied for this study

Punch Through Scenario

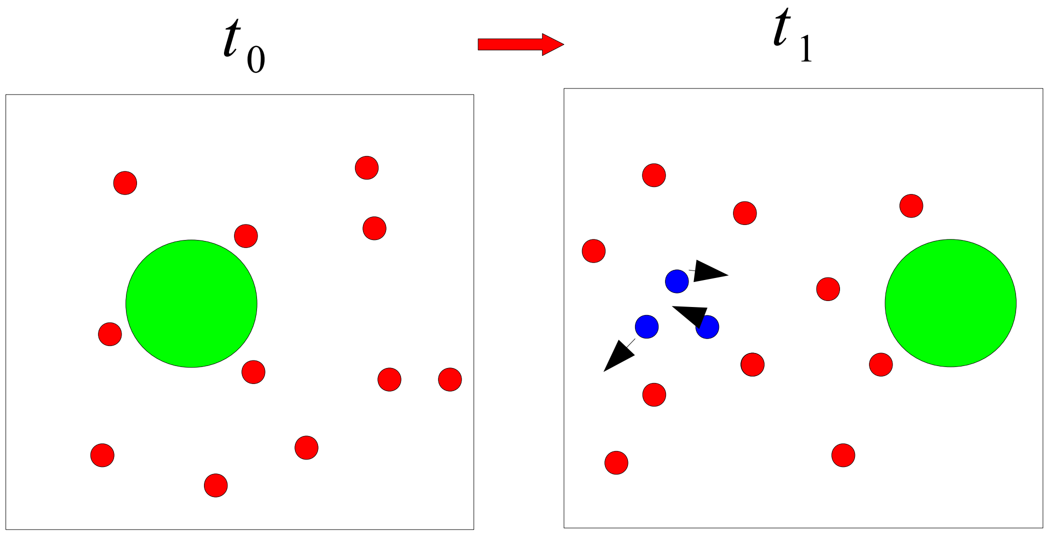
A scenario usefull to investigate the shape and development of ideal Mach Cones



- **Jet has finite initial energy and momentum E = pz and is massless; no transverse momentum → px = py = 0**
- **The Jet deposits energy to the medium due to binary collisions with particles**
- **After every collision with a thermal particle of the medium the energy of the jet gets recharged to its inital value**

Pure energy deposition Scenario

Energy deposition via the creation of thermal distributed particles



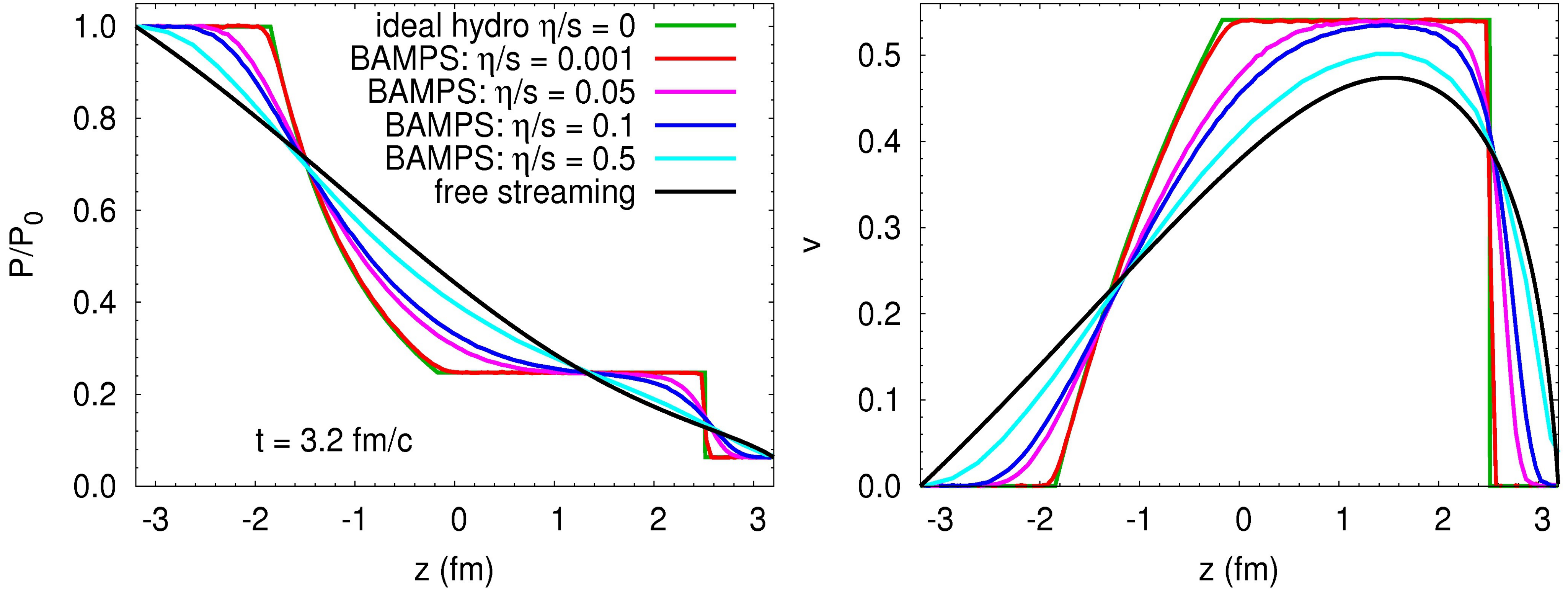
- The source (green) propagates with the speed of light and generates new particles (blue) at different timesteps
- The advantage of that method: a constant energy deposition but no momentum deposition, because new particles are thermal distributed

$$\longrightarrow f_{ped}(x,p)=e^{-E/T}$$

The Relativistic Riemann Problem

Investigation of Shock Waves in one dimension

Boltzmann solution of the relativistic Riemann problem ->what effects have viscosity?



Transition from ideal hydro to free streaming

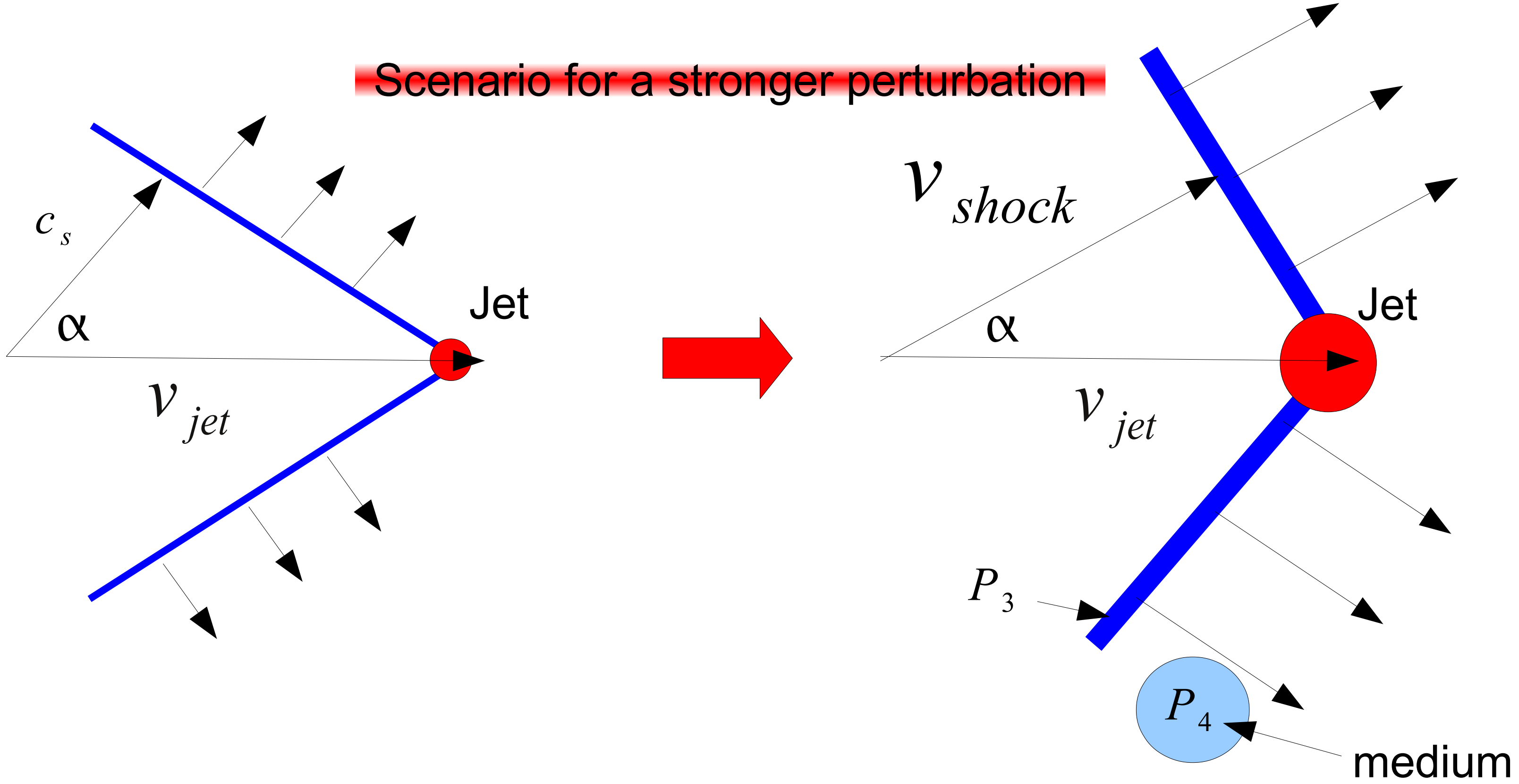
I. Bouras et al., Phys. Rev. Lett. 103:032301 (2009)

I. Bouras et al., PRC 82, 024910 (2010)

Mach Cones

Mach angle dependence

Scenario for a stronger perturbation



- In the case of a stronger perturbation the energy deposition is larger and therefore shock waves develop which exceed the speed of sound. Therefore the angle is approximately given by

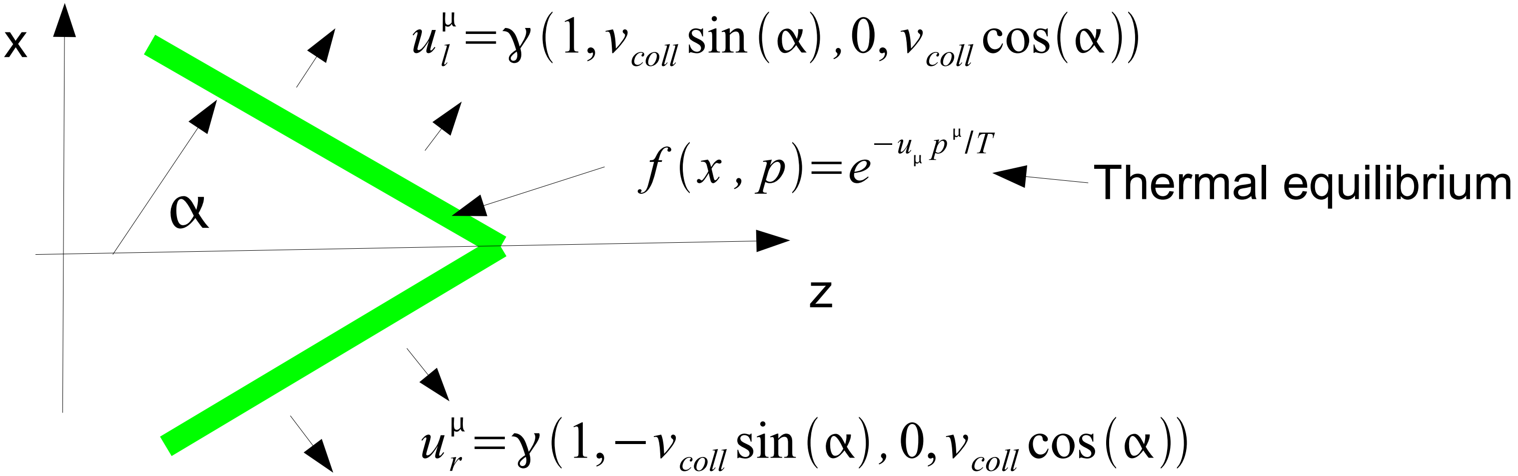
$$\alpha = \arccos \frac{v_{shock}}{v_{jet}} \qquad v_{shock} = \left[\frac{(P_4 - P_3)(e_3 + P_4)}{(e_4 - e_3)(e_4 + P_3)} \right]^{\frac{1}{2}}$$

- The emission angle α changes to smaller values than in the weak perturbation case

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution

Assume two wings in thermal equilibrium



$$u_l^{\mu} = \gamma \left(1, v_{coll} \sin(\alpha), 0, v_{coll} \cos(\alpha)\right)$$

$$f(x, p) = e^{-u_{\mu} p^{\mu} / T}$$

Thermal equilibrium

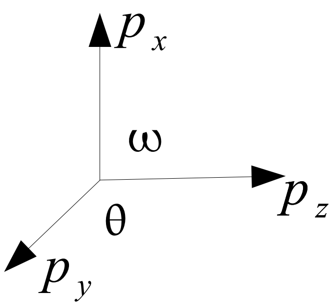
$$u_r^{\mu} = \gamma \left(1, -v_{coll} \sin(\alpha), 0, v_{coll} \cos(\alpha)\right)$$

alpha is a const and corresponds to the Mach angle, where v_coll is the collective velocity of matter velocity in the wings

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution

- We are looking for the angle ω, which is the angle in the p_x and p_z plane

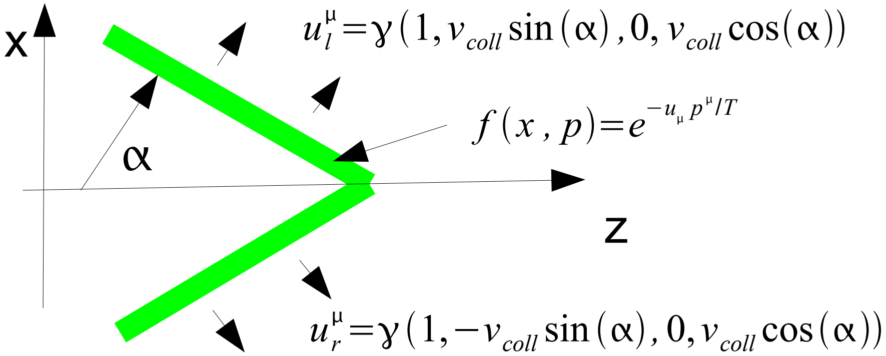


$$p_z = p\cos(\omega)\sin(\theta)$$
$$p_x = p\sin(\omega)\sin(\theta)$$
$$p_y = p\cos(\theta)$$

One calculate for each wing the particle distribution

$$\frac{dN}{d\omega} = \frac{V}{(2\pi)^3}\iint p^2 \sin(\theta) e^{-u_\mu p^\mu/T}\, dp\, d\theta$$

In the end one has to add both contributions!



$$u_l^\mu = \gamma(1, v_{coll}\sin(\alpha), 0, v_{coll}\cos(\alpha))$$

$$f(x,p) = e^{-u_\mu p^\mu/T}$$

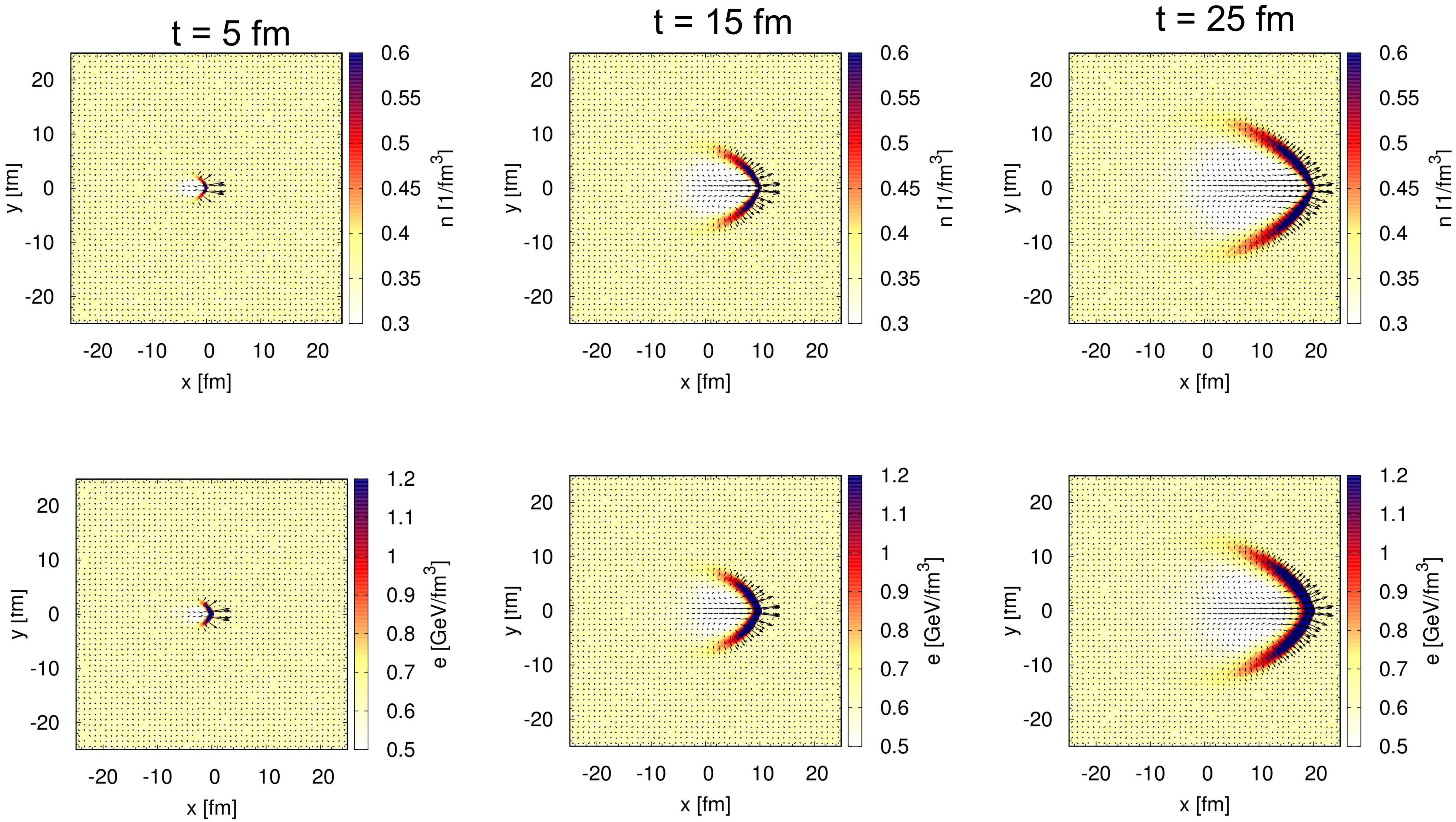
$$u_r^\mu = \gamma(1, -v_{coll}\sin(\alpha), 0, v_{coll}\cos(\alpha))$$

Mach cones at Bevalac?

Proton gas

$$\sigma = 40\,mb$$
$$m = 938\,MeV$$
$$n = 2\mathrm{n}_0$$
$$dE/dx = 20\,GeV/fm$$

W. Scheid, H. Muller & W. Greiner, Phys.Rev.Lett., 1974
D. Rischke, H. Stöcker & W. Greiner, Phys. Rev. D, 1990



t = 5 fm

t = 15 fm

t = 25 fm

Mach cones in static medium

- Curved structure due to jet quenching
- Viscosity destroys the structure