

# ***HYDJET++ MODEL FOR RELATIVISTIC HEAVY ION COLLISIONS AT RHIC AND LHC***

***L. Bravina, H. Brusheim Johansson, J. Crkovska, G. Eyyubova,  
V. Korotkikh, I. Lokhtin, L. Malinina, S. Petrushanko,  
A. Snigirev and E. Zabrodin***

***University of Oslo and Moscow State University***

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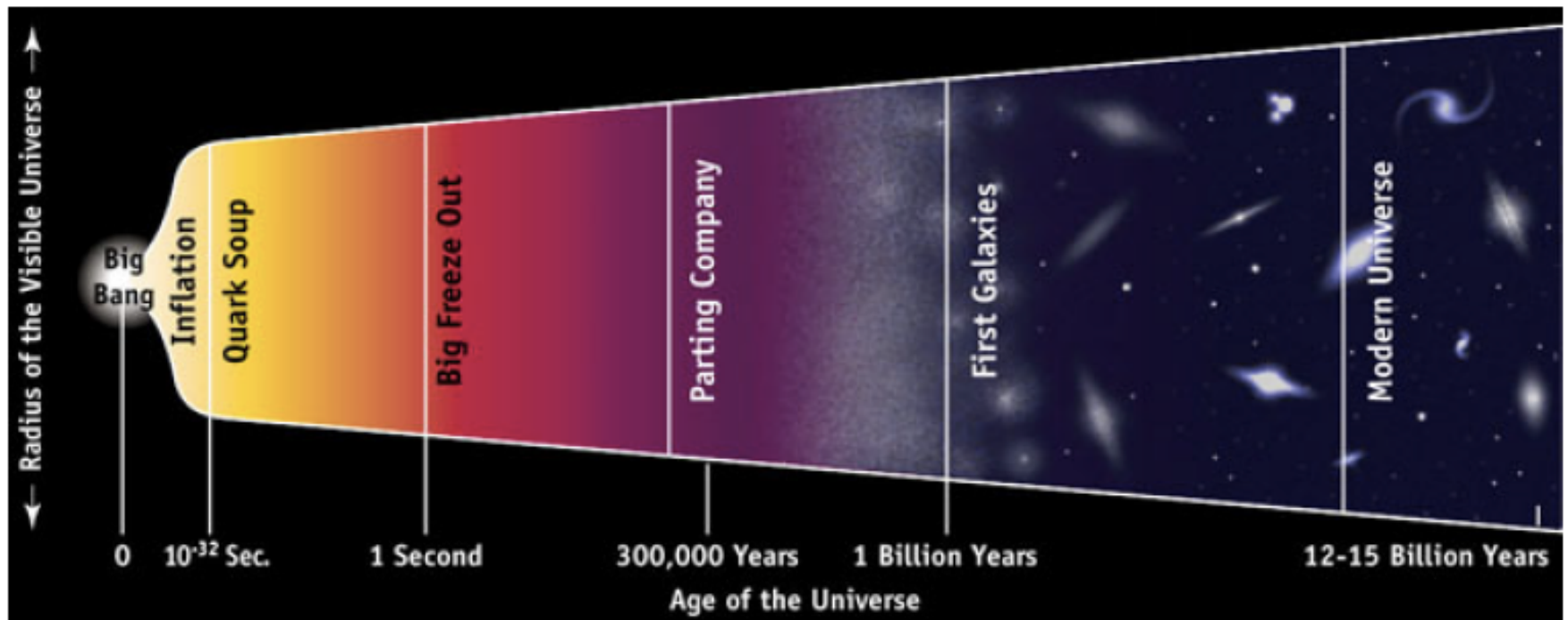
# OUTLINE

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- I. HYDJET++ model (hydro + jets)
- II. Description of elliptic flow in relativistic heavy ion collisions
- III. Influence of resonance decays
- IV. NCQ-scaling for  $v_2$  at RHIC and LHC
- V. Model results for the ratio  $v_4/(v_2)^2$  at RHIC and LHC
- VI. High harmonics and ridge
- VII. Conclusions

# Heavy-Ion Collisions

- aim to recreate conditions as in Universe shortly after Big Bang
- high energy densities and temperatures  $\Rightarrow$  new state of matter
- LHC (CERN), RHIC (BNL)



# Collective flow

- flow is a collective motion of particles
- pressure gradient inside fireball creates an outward motion of matter
- bulk properties of created medium
- can be described by thermodynamics
- the most thoroughly studied both in experiment and theory

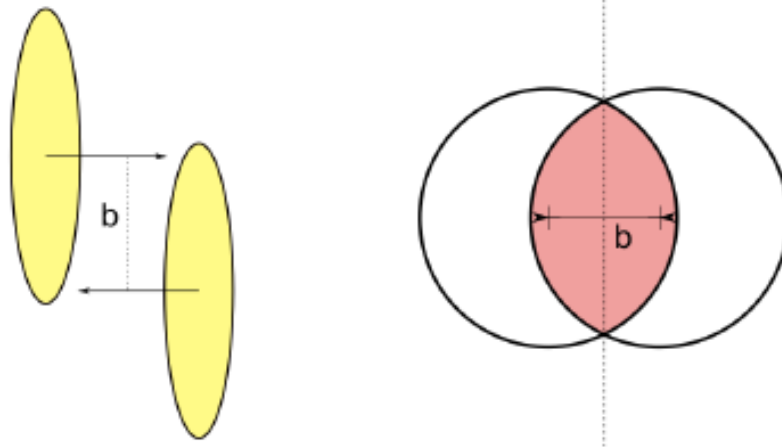
# Flow in HIC

## Isotropic flow

- additional momentum boost due to medium expansion

## Anisotropic flow

- initial asymmetry of overlap region function of  $b$
- restoration of spatial isotropy  $\Rightarrow$  momentum anisotropy
- information about early stages of collision



# Azimuthal distribution

## Fourier decomposition

- angular anisotropy can be developed into a Fourier series

$$\frac{dN}{d\varphi} = \frac{N}{2\pi} \left[ 1 + 2 \sum v_n \cos(n(\varphi - \Psi_R)) \right] \quad (1)$$

$$v_n = \langle \cos(n(\varphi - \Psi_R)) \rangle$$

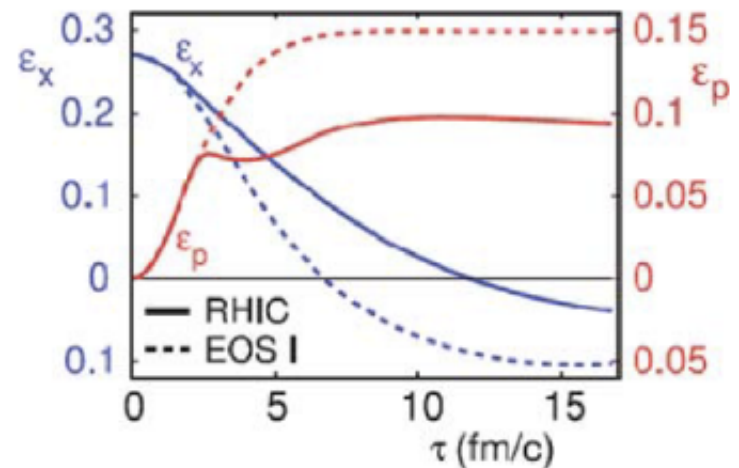
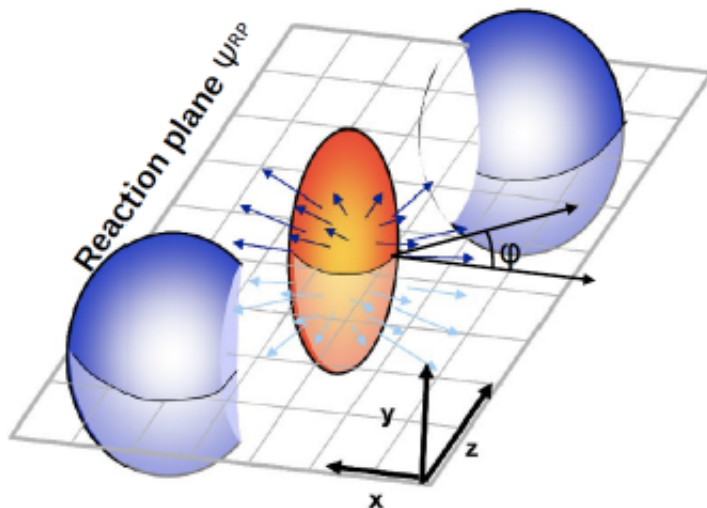
$$\varepsilon_n = \sqrt{\langle r^2 \cos(n\varphi) \rangle^2 + \langle r^2 \sin(n\varphi) \rangle^2} / \langle r^2 \rangle$$

- $\varphi$  is azimuthal angle in momentum
- $\Psi_R$  is reaction plane angle
- the first term represents isotropic flow
- $v_1$  directed flow,  $v_2$  elliptic flow,  $v_3$  triangular flow

# Elliptic flow

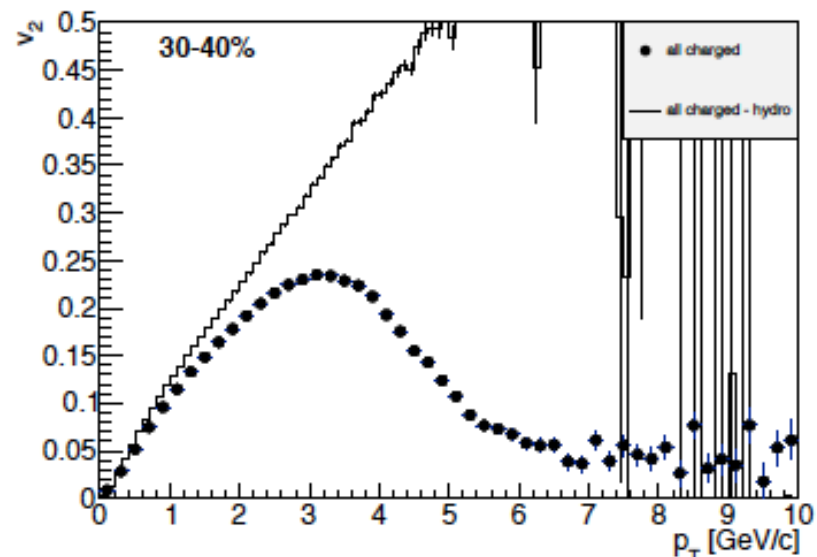
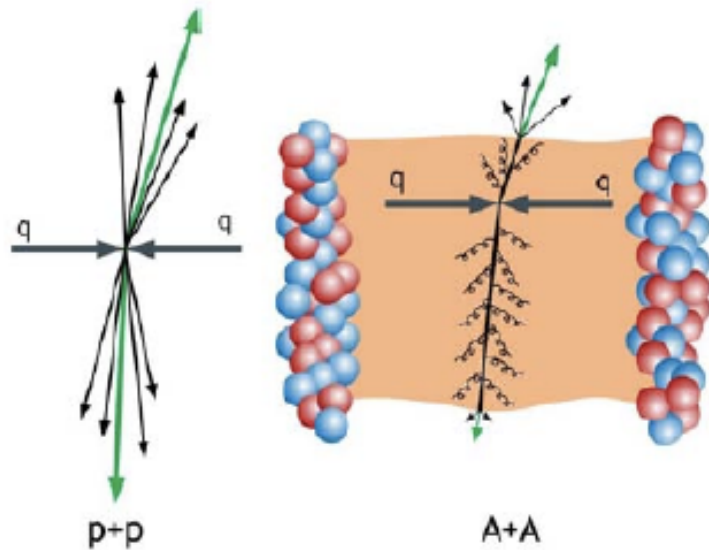
$$v_2 = \langle \cos(2(\varphi - \Psi_R)) \rangle$$

- dominant contribution to the anisotropic flow
- originates from the almond shape of overlap region
- evolution reduces spatial anisotropy  $\Rightarrow$  momentum anisotropy  $\Rightarrow$  self-quenching phenomenon



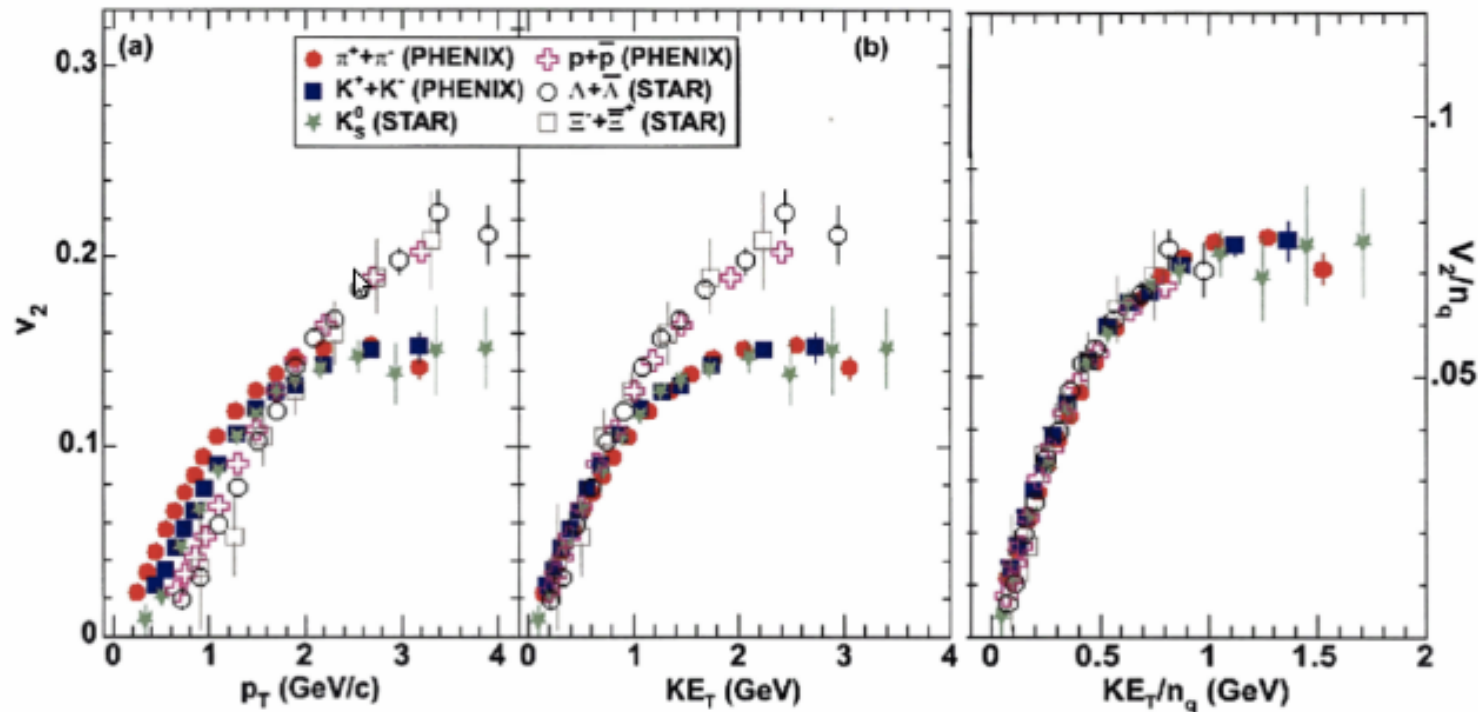
# Jet quenching

- high- $p_T$   $q\bar{q}$  pair moving in opposite direction  $\Rightarrow$  2 cones at  $\Delta\phi \approx \pi$
- pair created at the edge of the fireball  $\rightarrow$  one jet exits the region immediately, the other travels through the medium





# Number-of-constituent-quark scaling



Phys. Rev. Lett 98 (2007) 162301

- favours  $v_2$  formation on partonic level
- favours coalescence-production scenario

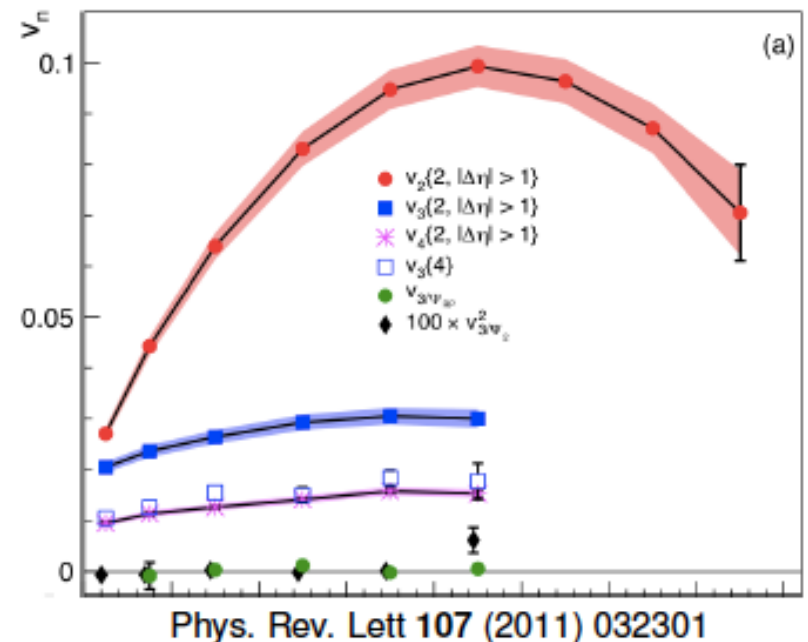
# Event-by-event fluctuations

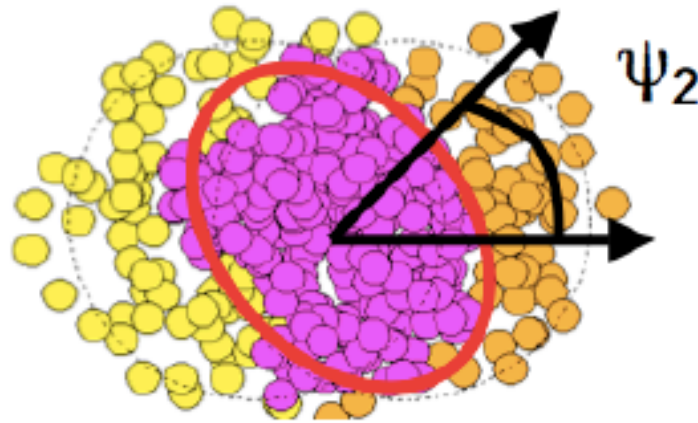
- largest pressure gradient along minor axis  $\Psi_n$

$$v_n(\Psi_n) = \langle \cos(n(\varphi - \Psi_n)) \rangle$$

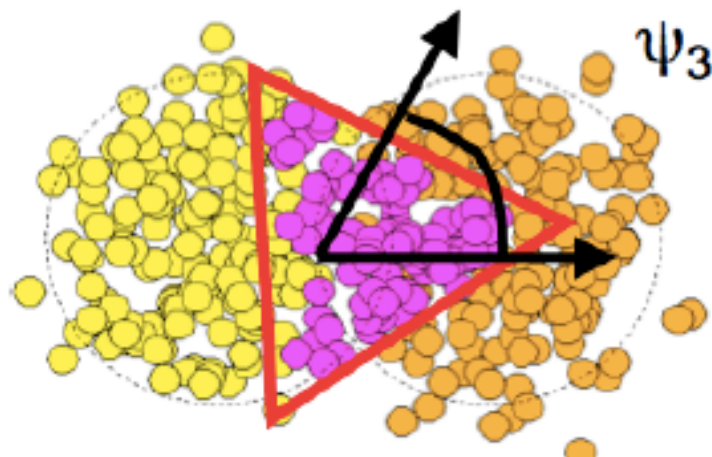
- $\Psi_2$  and  $\Psi_3$  uncorrelated
- $v_2$  and  $v_3 \Rightarrow$  higher coefficients

$\Rightarrow$  need at least  $\Psi_2$  and  $\Psi_3$  to describe the system





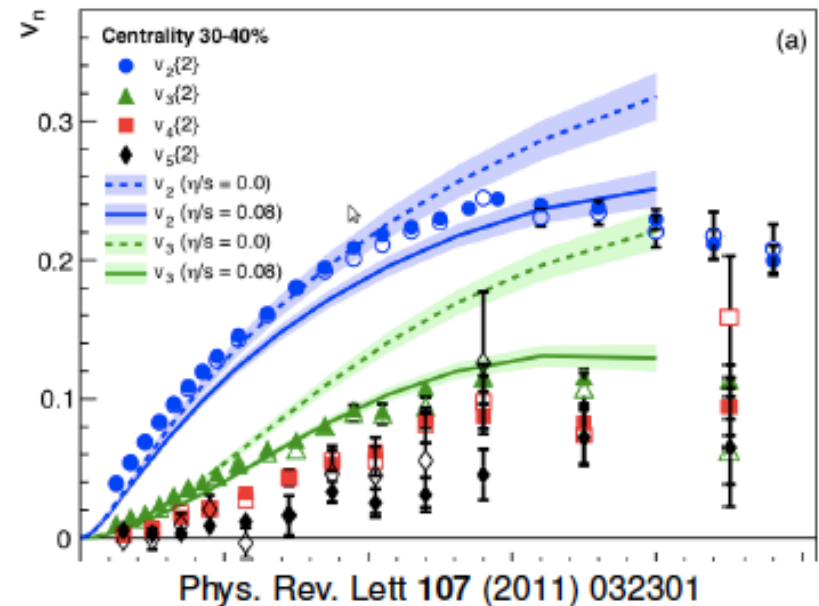
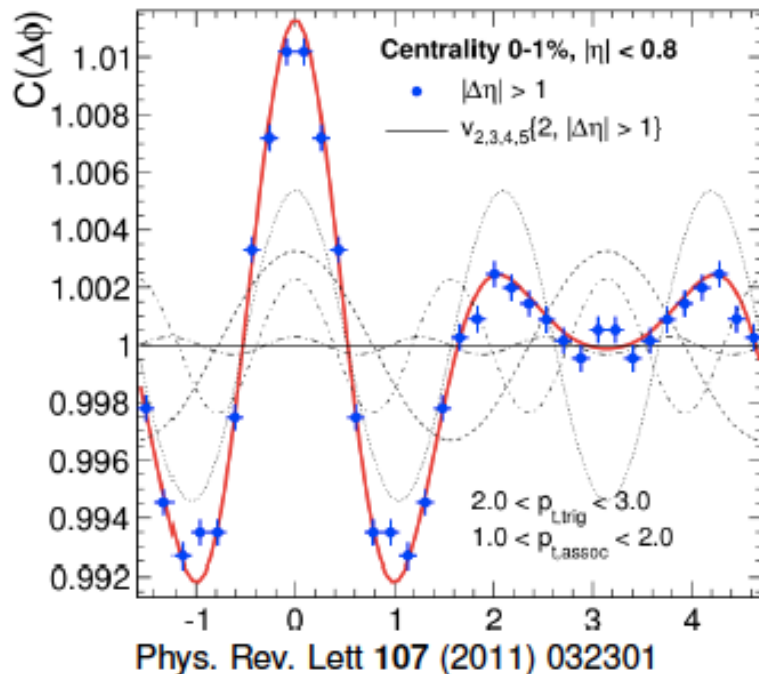
$$\epsilon = \frac{\sqrt{\langle (r^2 \cos(2\phi))^2 \rangle + \langle (r^2 \sin(2\phi))^2 \rangle}}{\langle r^2 \rangle}$$



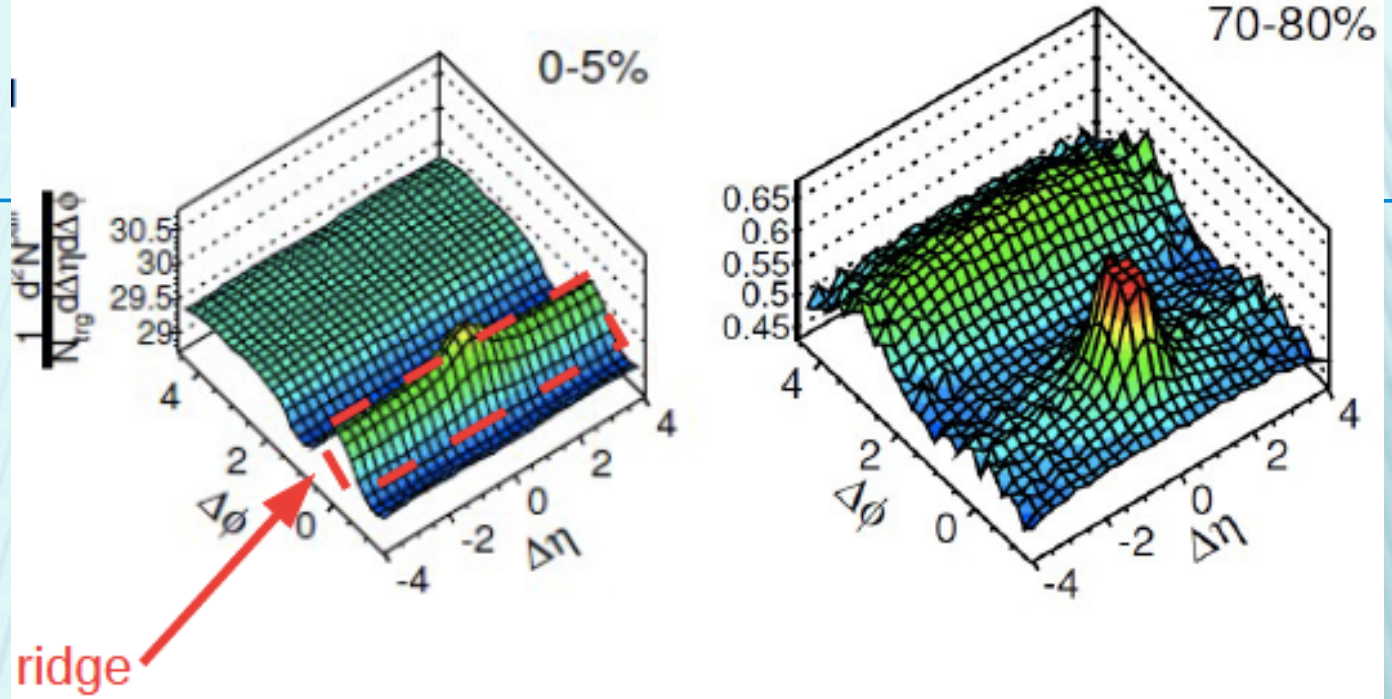
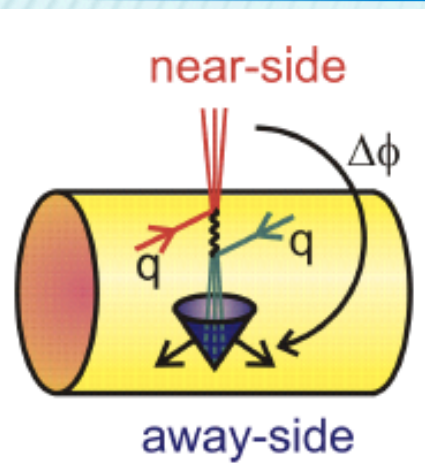
$$\epsilon_3 = \frac{\sqrt{\langle (r^2 \cos(3\phi))^2 \rangle + \langle (r^2 \sin(3\phi))^2 \rangle}}{\langle r^2 \rangle}$$

# Higher harmonics

- information on initial geometry
- more sensitive to viscosity
- explain shape of 2-particle correlation



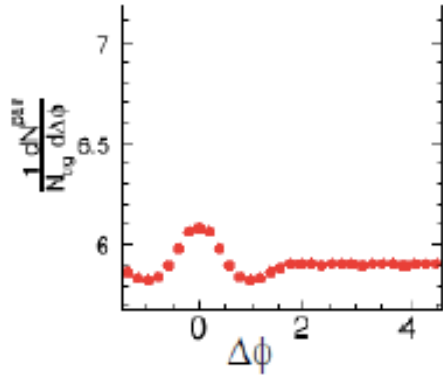
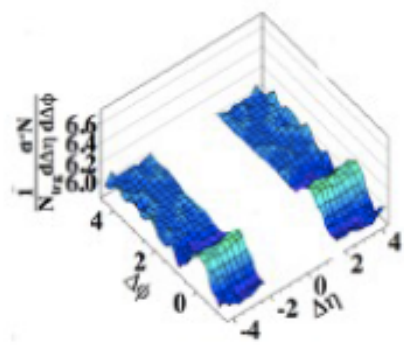
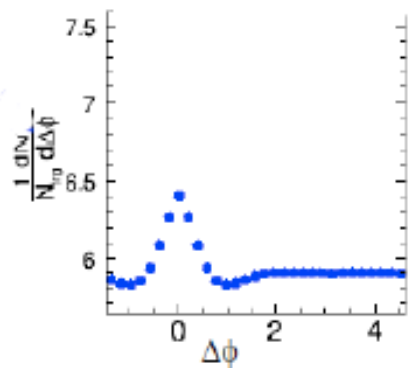
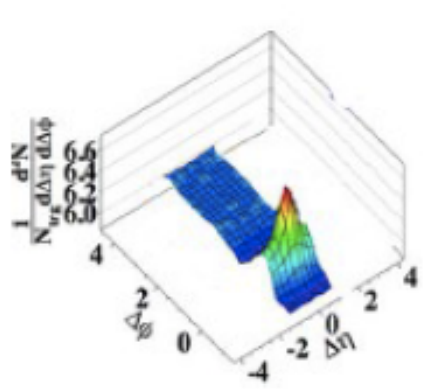
# RIDGE



CMS, PbPb 2.76 TeV, *Eur. Phys. C* 72 (2012) 10052,  $3 < p_{t}^{\text{trig}} < 3.5 \text{ GeV}/c$ ,  $1 < p_{t}^{\text{assoc}} < 1.5 \text{ GeV}/c$

## THE JET REGION

## RIDGE REGION



**I. HYDJET++ =  
FASTMS + HYDJET**

# HYDJET++ event generator

I.Lokhtin, L.Malinina, S.Petrushanko, A.Snigirev, I.Arsene, K.Tywoniuk,  
Comp. Phys. Commun.180 (2009) 779-799 (arXiv:0809.2708[hep-ph])

- The soft part of HYDJET++ event represents the "thermal" hadronic state.
  - ✓ multiplicities are determined assuming thermal equilibrium
  - ✓ hadrons are produced on the hypersurface represented by a parameterization of relativistic hydrodynamics with given freeze-out conditions
  - ✓ chemical and kinetic freeze-outs are separated
  - ✓ decays of hadronic resonances are taken into account (360 particles from **SHARE** data table) with "home-made" decayer

*the model reproduces soft hadroproduction features at RHIC  
(particle spectra, elliptic flow, HBT)*

- The hard, multi-partonic part of HYDJET++ event is identical to the hard part of Fortran written HYDJET (PYTHIA6.4xx + PYQUEN1.5) => **now PYTHIA Pro-Q20 tune !!** PYQUEN event generator is used for simulation of rescattering, radiative and collisional energy loss of hard partons in expanding quark-gluon plasma created in ultrarelativistic heavy ion AA collisions. HYDJET++ includes nuclear shadowing correction for parton distributions (important at LHC!) Impact-parameter dependent parameterization of *nuclear shadowing* (K.Tywoniuk, I.Arsene, L.Bravina, A.Kaidalov and E.Zabrodin, Phys. Lett. B 657 (2007) 170)

## Model parameters.

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1. Thermodynamic parameters at chemical freeze-out:  $T_{ch}$  ,  $\{\mu_B, \mu_s, \mu_Q\}$
2. If thermal freeze-out is considered:  $T_{th}$  ,  $\mu\pi$ -normalisation constant
3. Volume parameters:  $T, \Delta T, R$
1.  $\rho_{max}$  -maximal transverse flow rapidity for Bjorken-like parametrization
5.  $\eta_{max}$  -maximal space-time longitudinal rapidity which determines the rapidity interval  $[-\eta_{max}, \eta_{max}]$  in the collision center-of-mass system.
6. Impact parameter range: minimal  $b_{min}$  and maximal  $b_{max}$  impact parameters
7. Flow anisotropy parameters  $\delta(b), \epsilon(b)$

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## PYTHIA+PYQUEN obligatory parameters

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9. Beam and target nuclear atomic weight  $A$
10.  $\sqrt{s_{NN}}$  -c.m.s. energy per nucleon pair (PYTHIA initialization at given energy)
11. **ptmin** – minimal pt of parton-parton scattering in PYTHIA event (ckin(3) in /pysubs/)
12. **nhsel** flag to include jet production in hydro-type event:

- 0 - jet production off (pure FASTMC event),
- 1 - jet production on, jet quenching off (FASTMC+njet\*PYTHIA events),
- 2 - jet production & jet quenching on (FASTMC+njet\*PYQUEN events),
- 3 - jet production on, jet quenching off, FASTMC off (njet\*PYTHIA events),
- 4 - jet production & jet quenching on, FASTMC off (njet\*PYQUEN events);

13. **ishad** flag to switch on/off nuclear shadowing



# Parameters of energy loss model in PYQUEN

(default, but can be changed from the default values by the user)

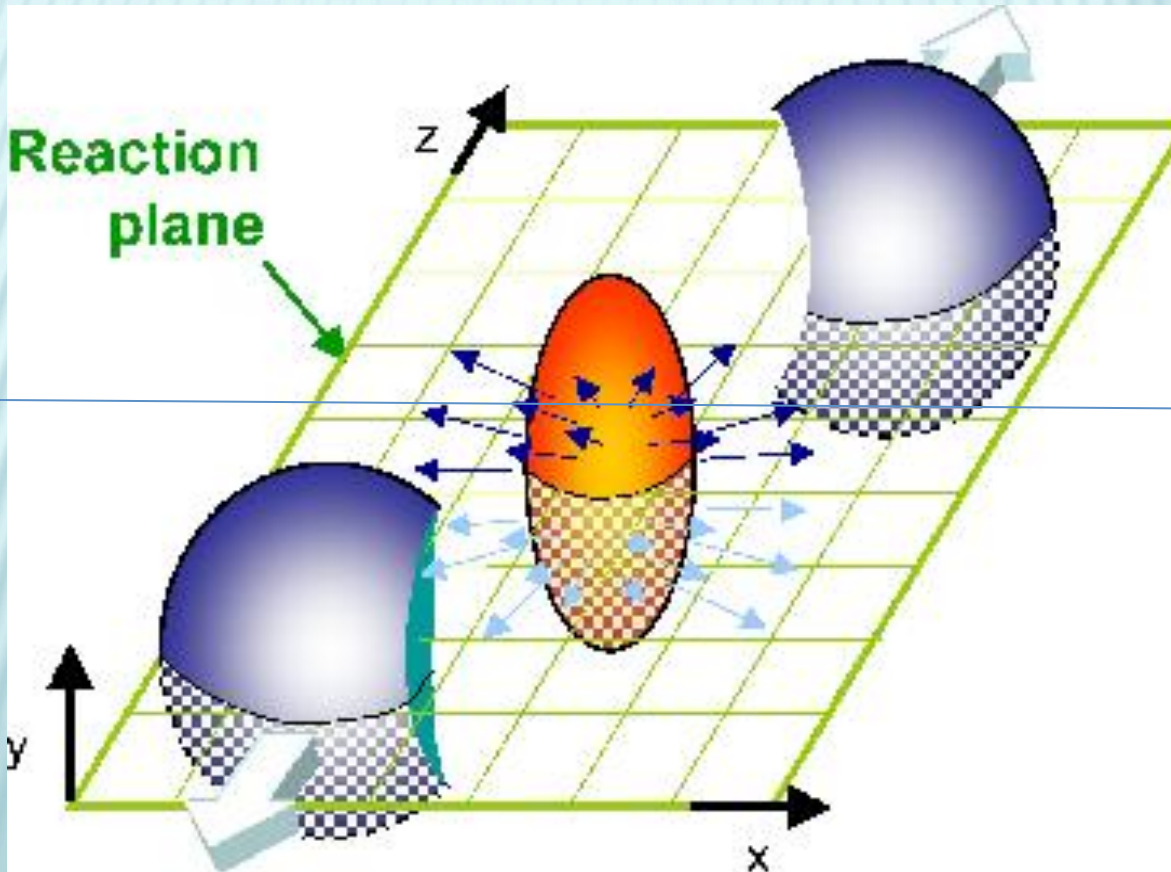
1. **T0** - initial temperature of quark-gluon plasma for central Pb+Pb collisions at mid-rapidity (initial temperature for other centralities and atomic numbers will be calculated automatically)  
at LHC: **T0=1 GeV**, at RHIC(200 AGeV) **T0=0.300 GeV**
2. **tau0** - proper time of quark-gluon plasma formation  
at LHC: **tau0=0.1 fm/c**, at RHIC(200 AGeV) **tau0=0.4 fm/c**
3. **nf** - number of active quark flavours in quark-gluon plasma (nf=0, 1, 2 or 3) at LHC: **nf=0**, at RHIC(200 AGeV) **nf=2**
4. **ienglu** - flag to fix type of medium-induced partonic energy loss (ienglu=0 - radiative and collisional loss, ienglu=1 - radiative loss only, ienglu=2 - collisional loss only, default value is ienglu=0);  
**ianglu** - flag to fix type of angular distribution of emitted gluons (ianglu=0 - small-angular, ianglu=1 - wide-angular, ianglu=2 - collinear, default value is ianglu=0).  
**ienglu=0**

**II. Elliptic flow in  
HYDJET++ : interplay  
of hydrodynamics and  
jets**

# Anisotropic flow

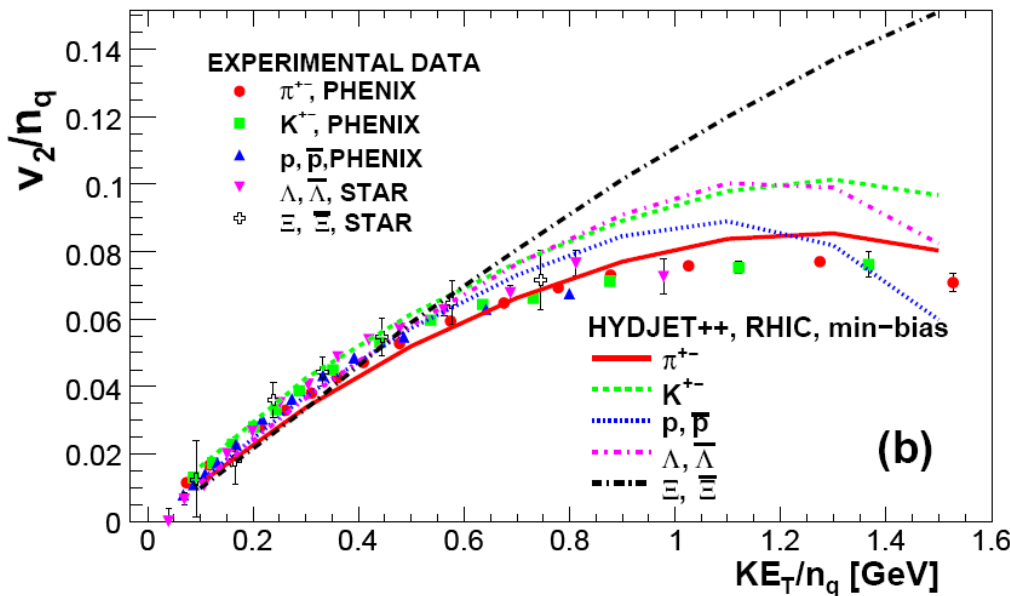
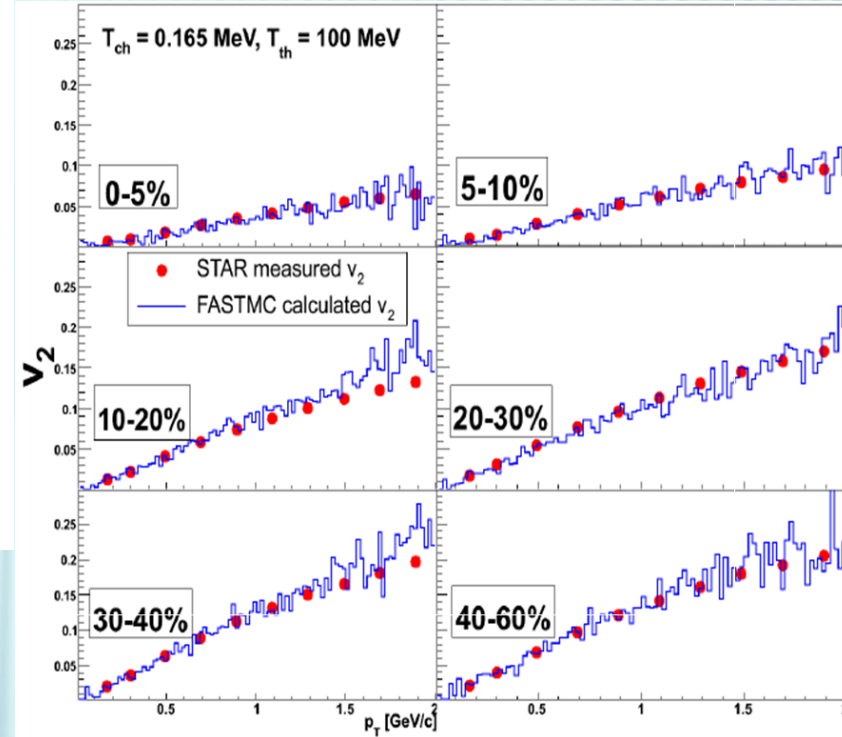
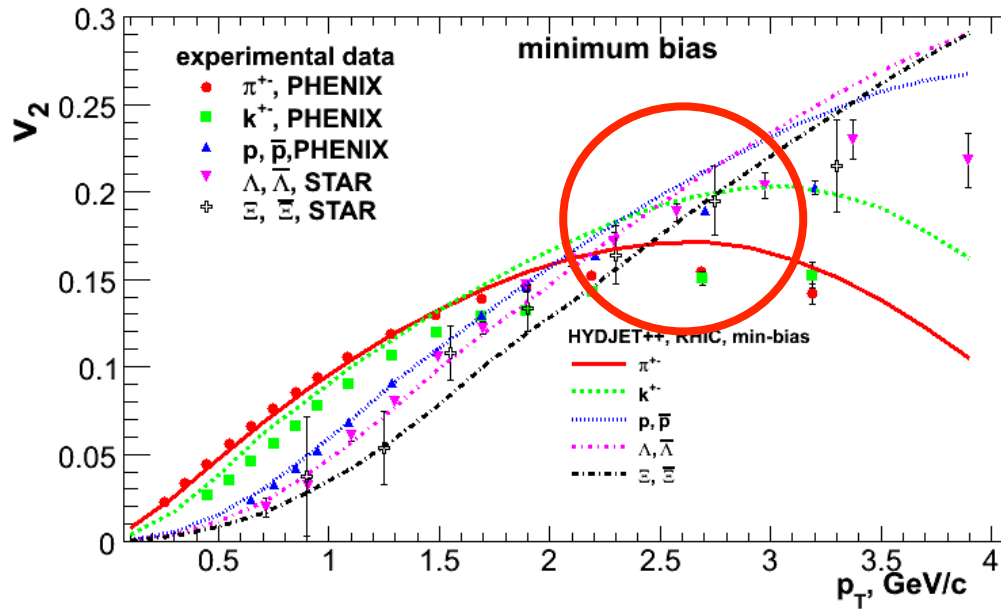
S.Voloshin and Y.Zhang, Z.Phys.C70 (1996) 665

$$\frac{dN}{d\varphi} = \frac{1}{2\pi} \left( 1 + \sum_{n=1}^{\infty} 2v_n(p_t) \cos[n(\varphi - \psi_r)] \right)$$



# RHIC DATA VS. HYDJET++ MODEL

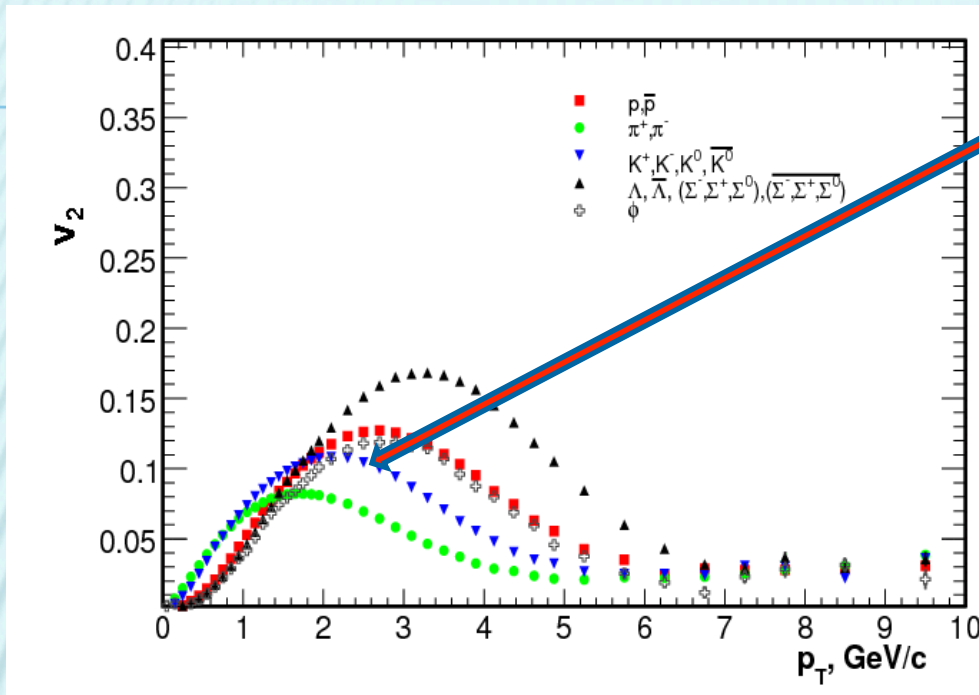
Au+Au @ 200 AGeV



Elliptic flow

G. Eyyubova et al., PRC 80 (2009) 064907;  
N.S. Amelin et al., PRC 77 (2008) 014903

# $V_2$ in HYDJET++ for different particles (centrality 30%)



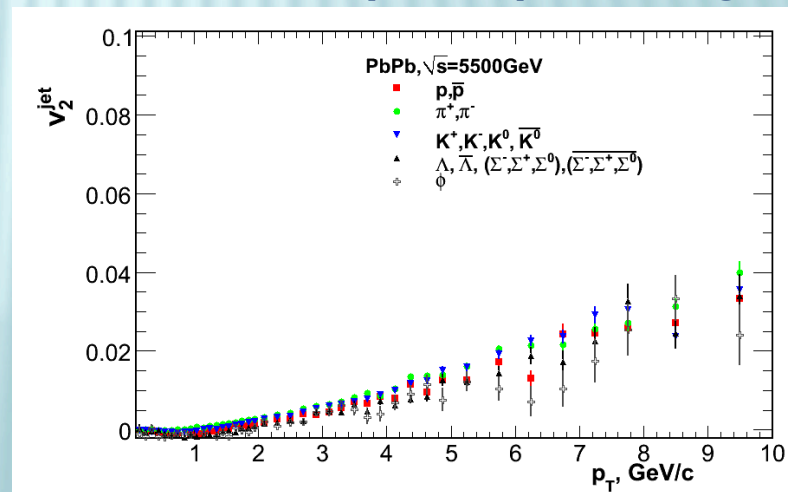
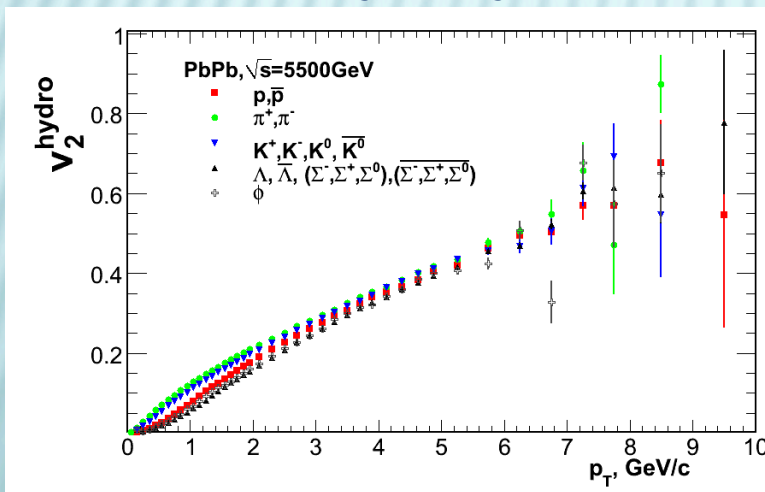
Mass ordering in soft  $p_T$  regions then breaks.

Why?

Hydrodynamics gives mass ordering of  $v_2$ .  
The model possesses crossing of baryon and meson branches.

Hydrodynamics

Jet part + quenching



Interplay between hydrodynamics and jets

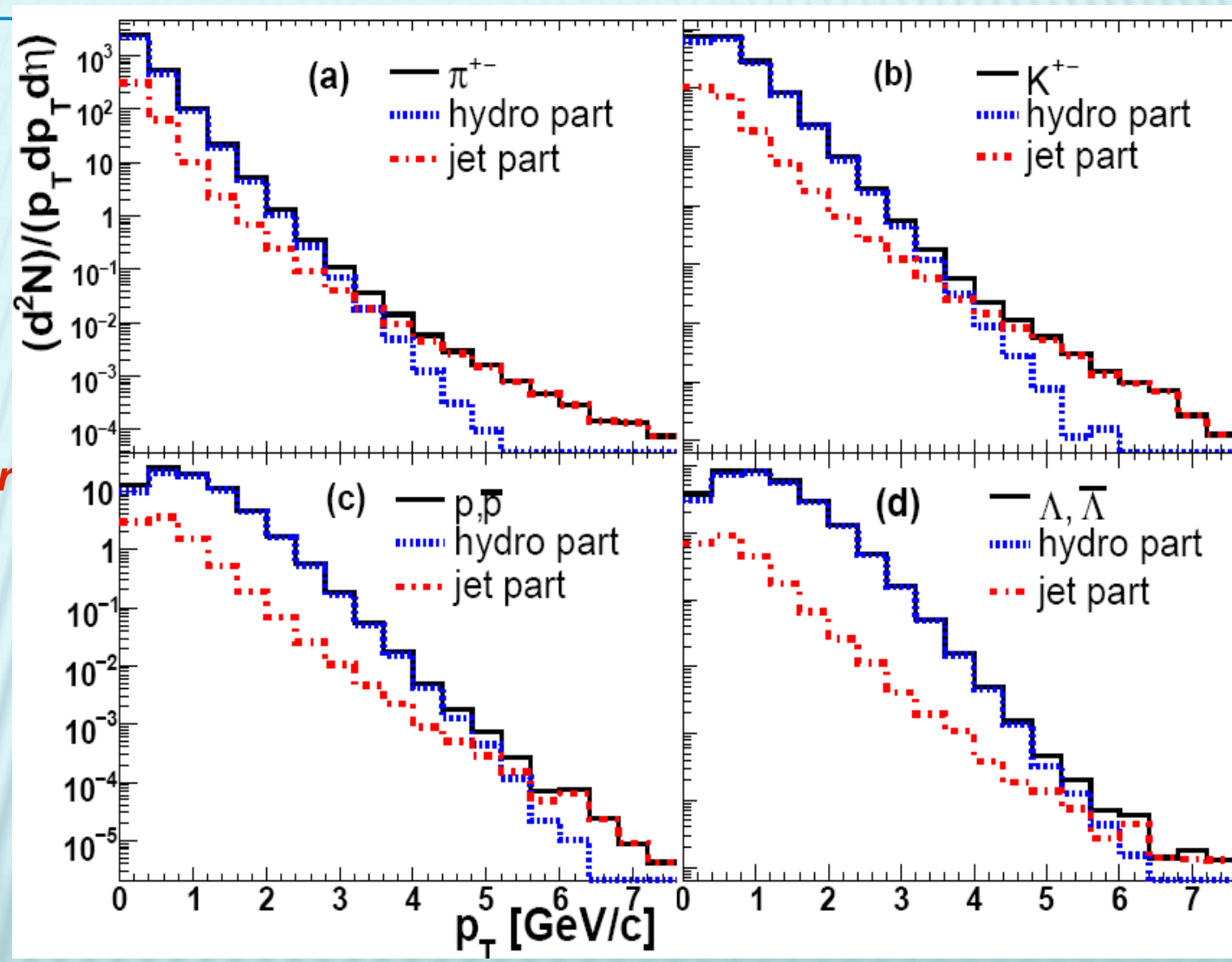
# The $p_T$ spectra of $\pi, K, p, \Lambda$ with HYDJET++ model, $\sqrt{s}=200\text{GeV}$

The slope for the hydro part depends strongly on mass:

- the heavier the particle -- the harder the spectrum



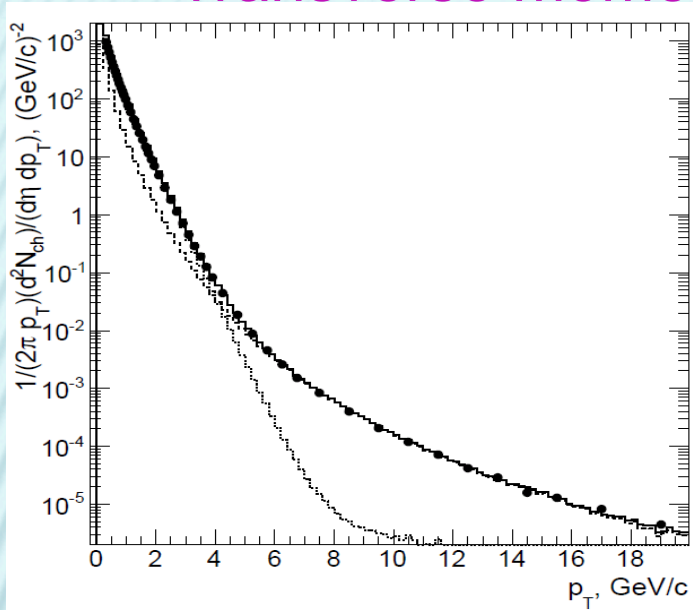
The hydro part dies out earlier for light particles than for heavy ones



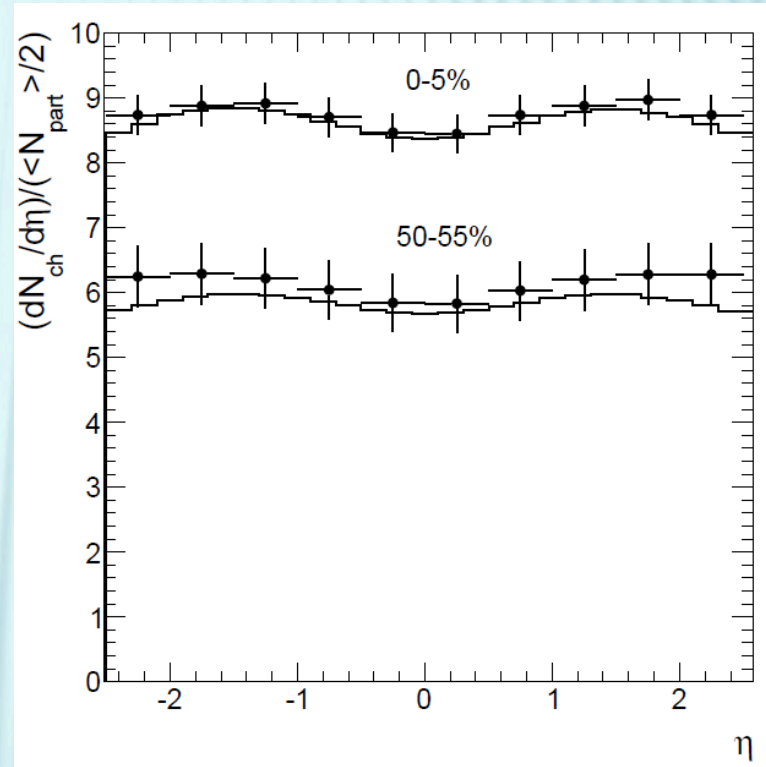
# LHC DATA VS. HYDJET++ MODEL

Transverse momentum

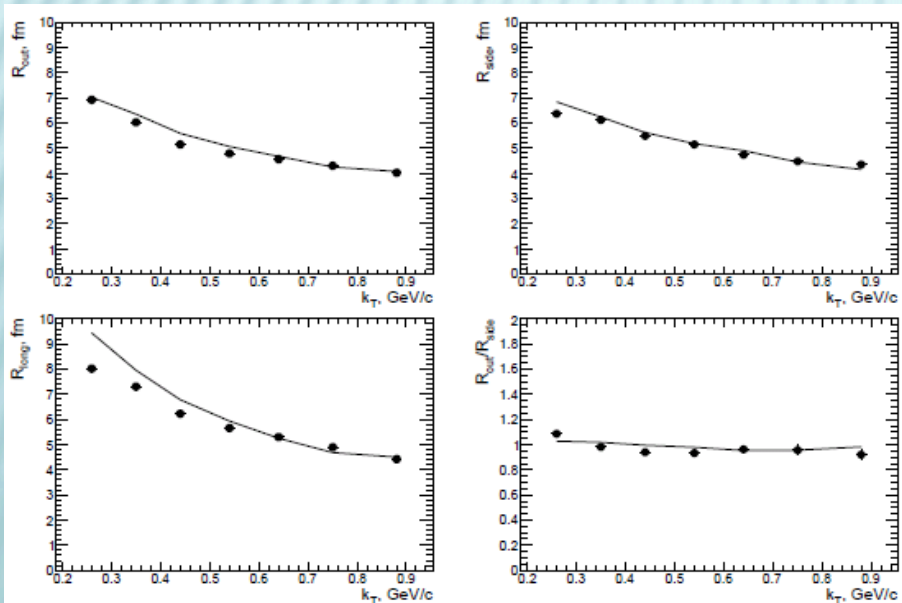
Pb+Pb @ 2.76 ATeV



Rapidity



I. Lokhtin et al., Eur. Phys. J C72 (2012) 2045

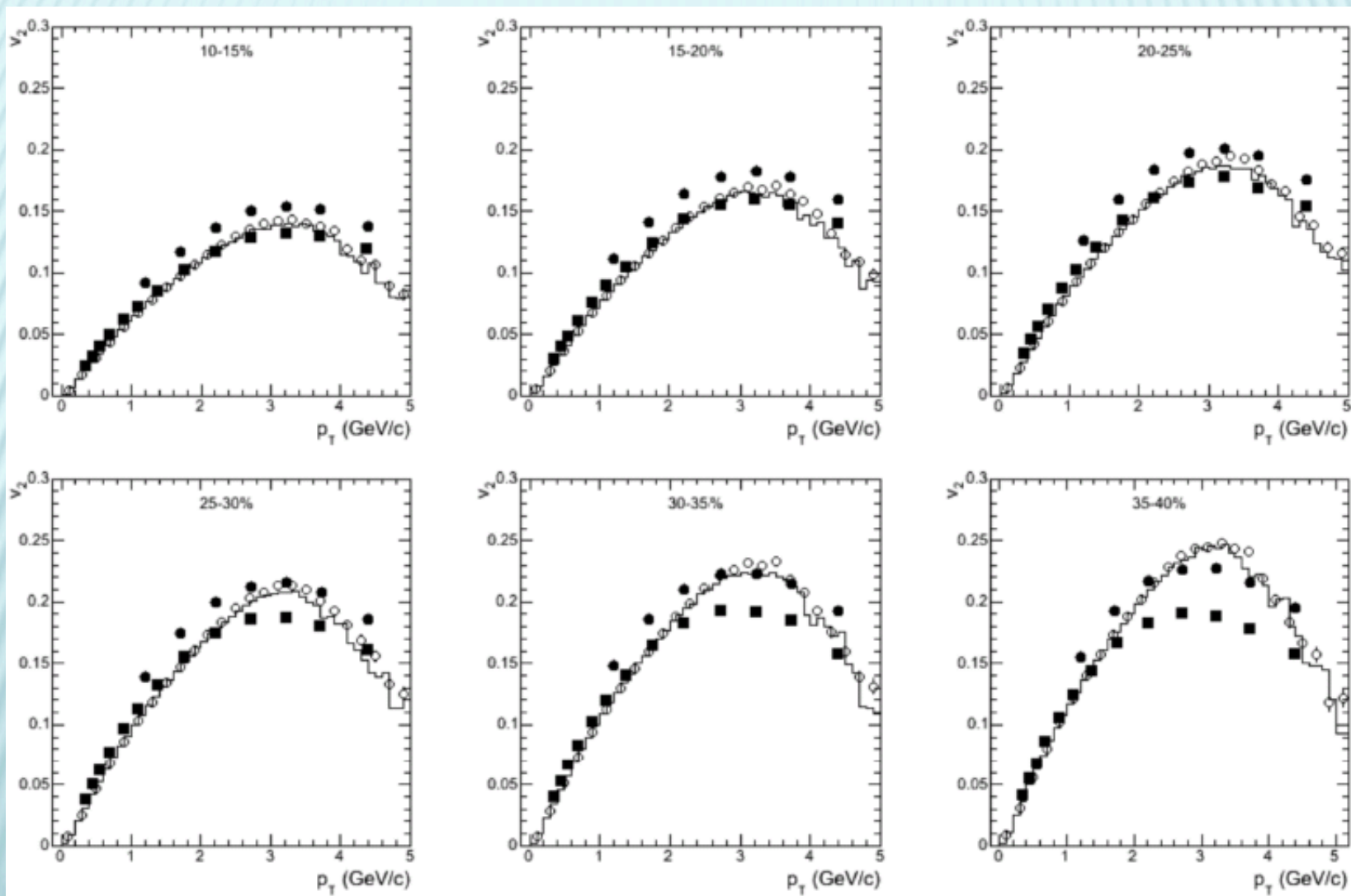


Correlation radii (femtoscropy)

# LHC DATA VS. HYDJET++ MODEL

## Elliptic flow

Pb+Pb @ 2.76 ATeV



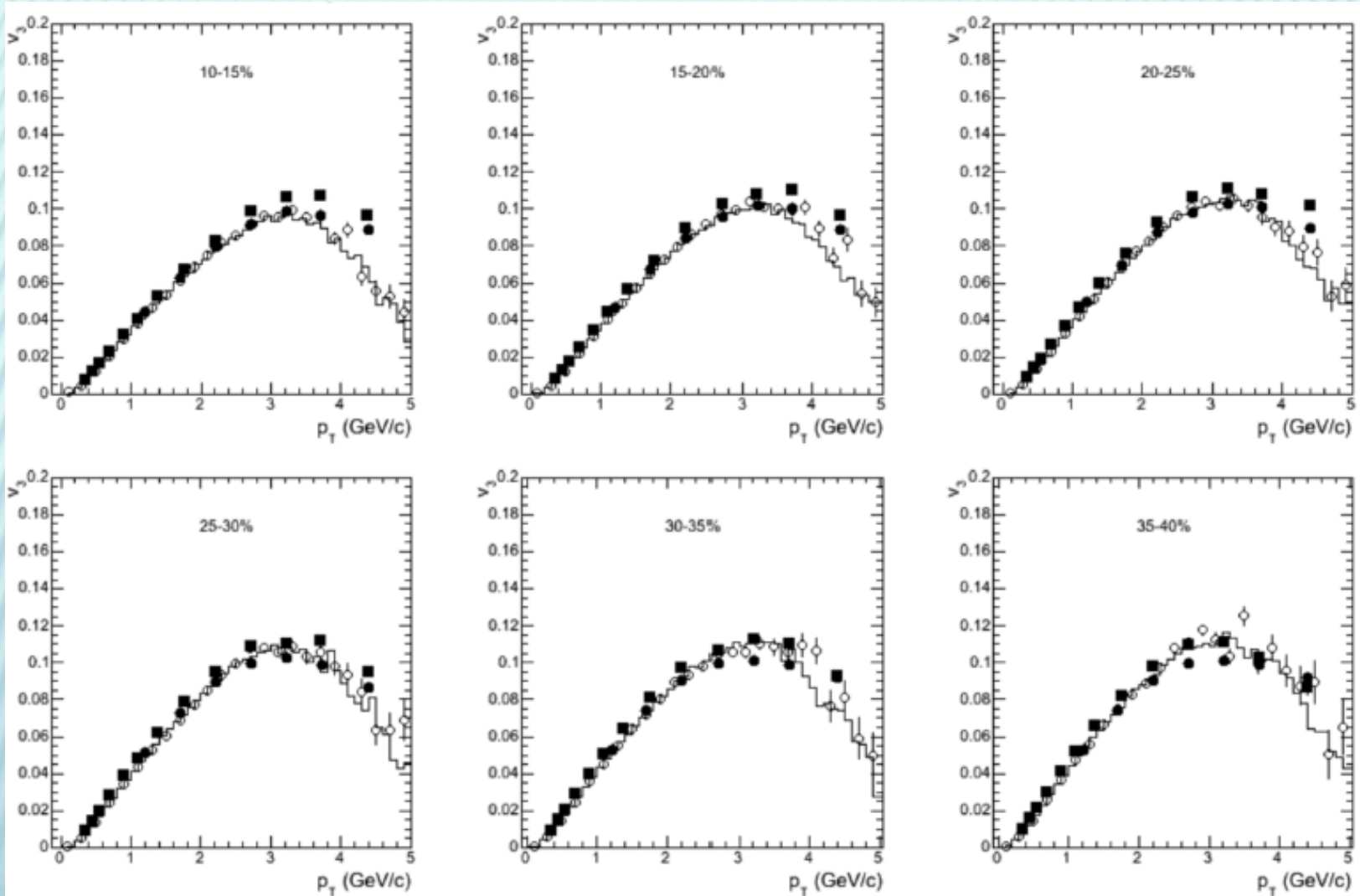
Closed points: CMS data  $v_2\{2\text{Part} \& \text{LYZ}\}$ ;  
Open points and histograms: HYDJET++  $v_2\{\text{EP} \& \text{Psi}^2\}$



# LHC DATA VS. HYDJET++ MODEL

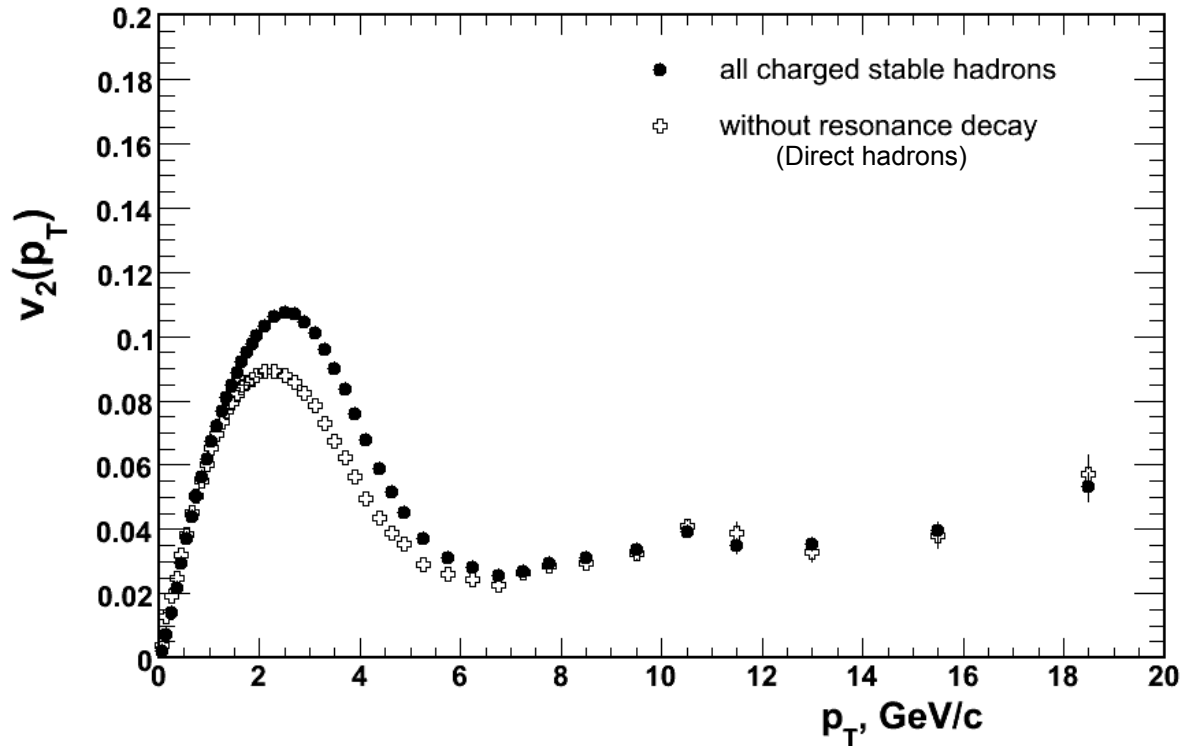
## Triangular flow

Pb+Pb @ 2.76 ATeV



# **III . Influence of resonance decays**

## Influence of resonance decay on $v_2$ value



PbPb collisions,  $c=30\%$

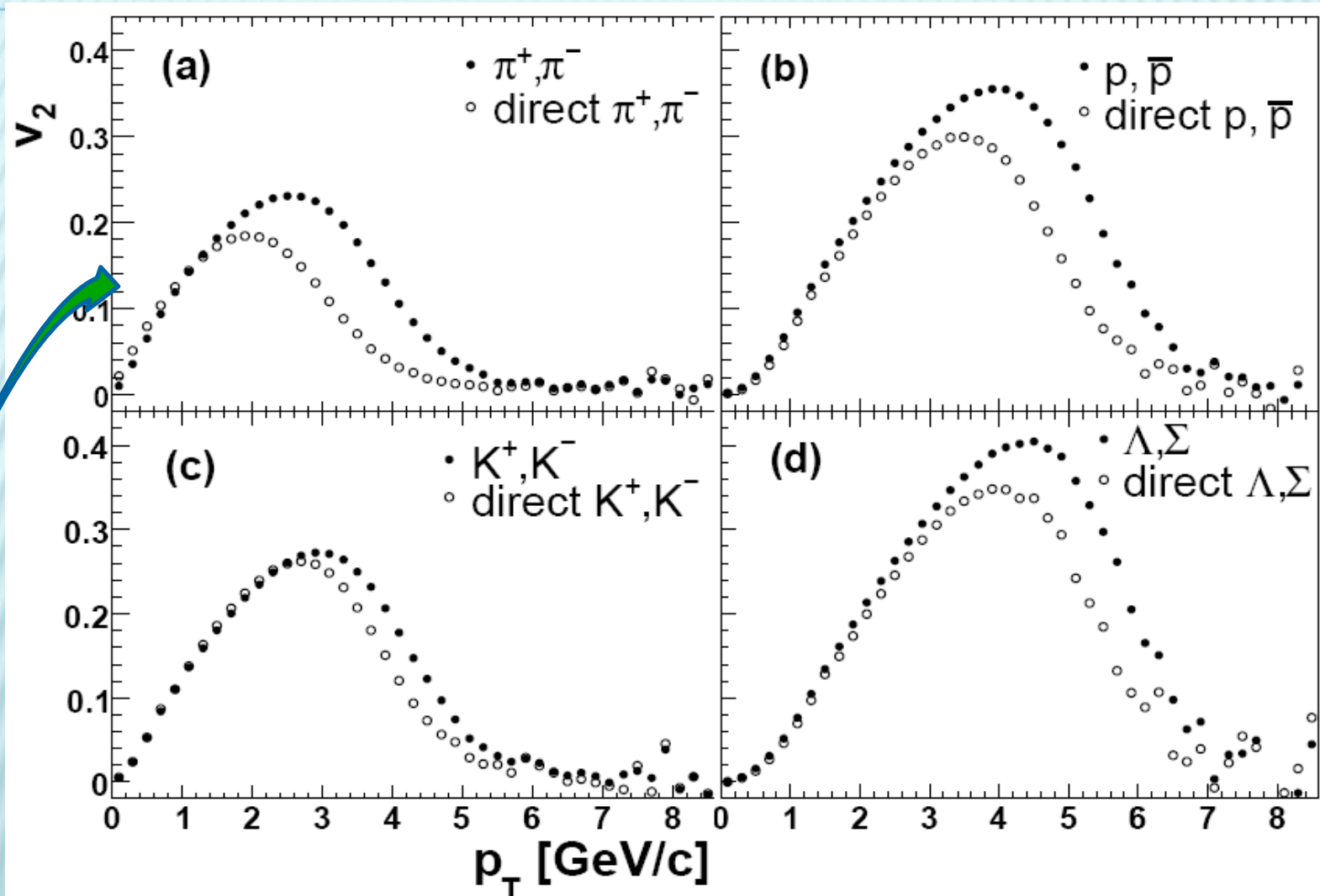
The elliptic flow of directly produced particles is smaller than that for all particles.

TABLE I: Yields of the particles produced directly and with resonance decays,  $5.6 \cdot 10^6$  events,  $c=42\%$ , midrapidity

|          | $\pi^\pm$ | $K + \bar{K}$ | $p + \bar{p}$ | $\Lambda + \bar{\Lambda} + \Sigma + \bar{\Sigma}$ | $\phi$ |
|----------|-----------|---------------|---------------|---|--------|
| all      | 860       | 185           | 63.8          | 42.3  | 6.55   |
| direct   | 169       | 81.4          | 18.6          | 14.2  | 6.5    |
| direct % | 20 %      | 44 %          | 30 %          | 39 %  | 99 %   |

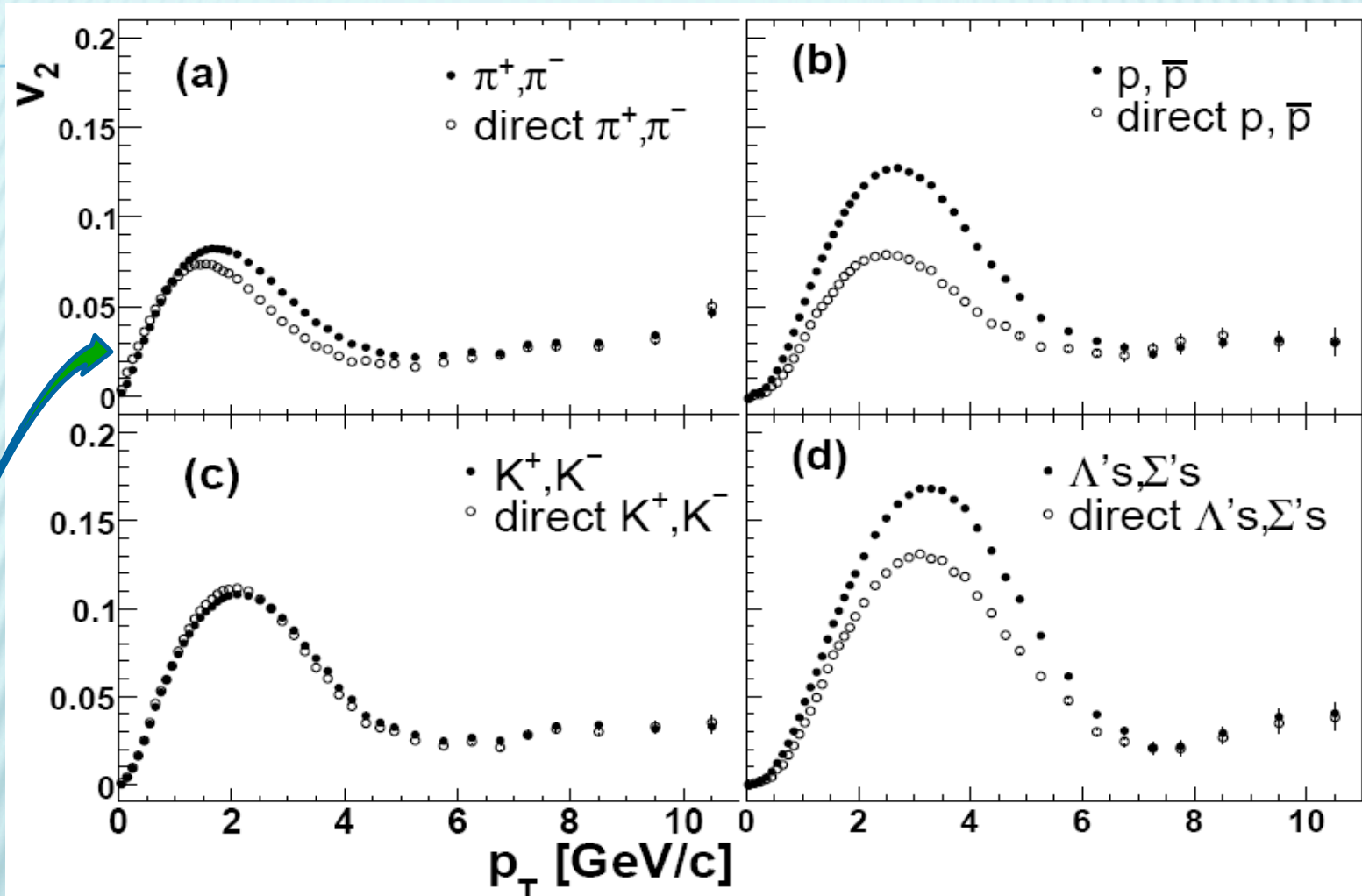
# Influence of resonance decays for different type of particles at RHIC

G. Eyyubova et al., PRC 80 (2009) 064907



**Pions and kaons:** the resulting flow is **weaker** at low- $p_T$  and **larger** at high- $p_T$   
**Baryons:** the resulting flow is **stronger** than the flow of direct particles

# Influence of resonance decays for different type of particles at LHC

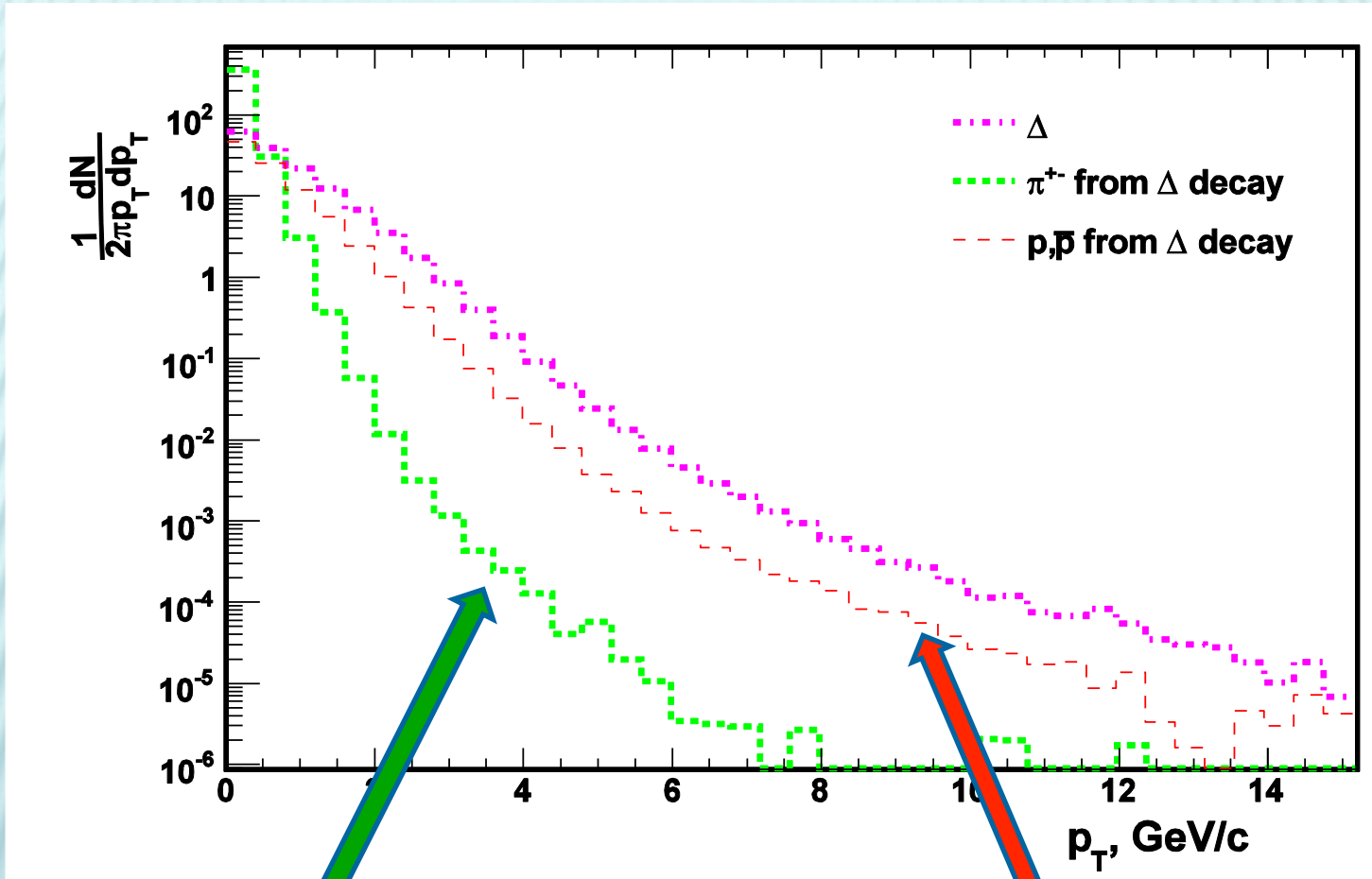


**Pions:** the resulting flow is weaker at low-pt and larger at high-pt

**Kaons:** both flows almost coincide

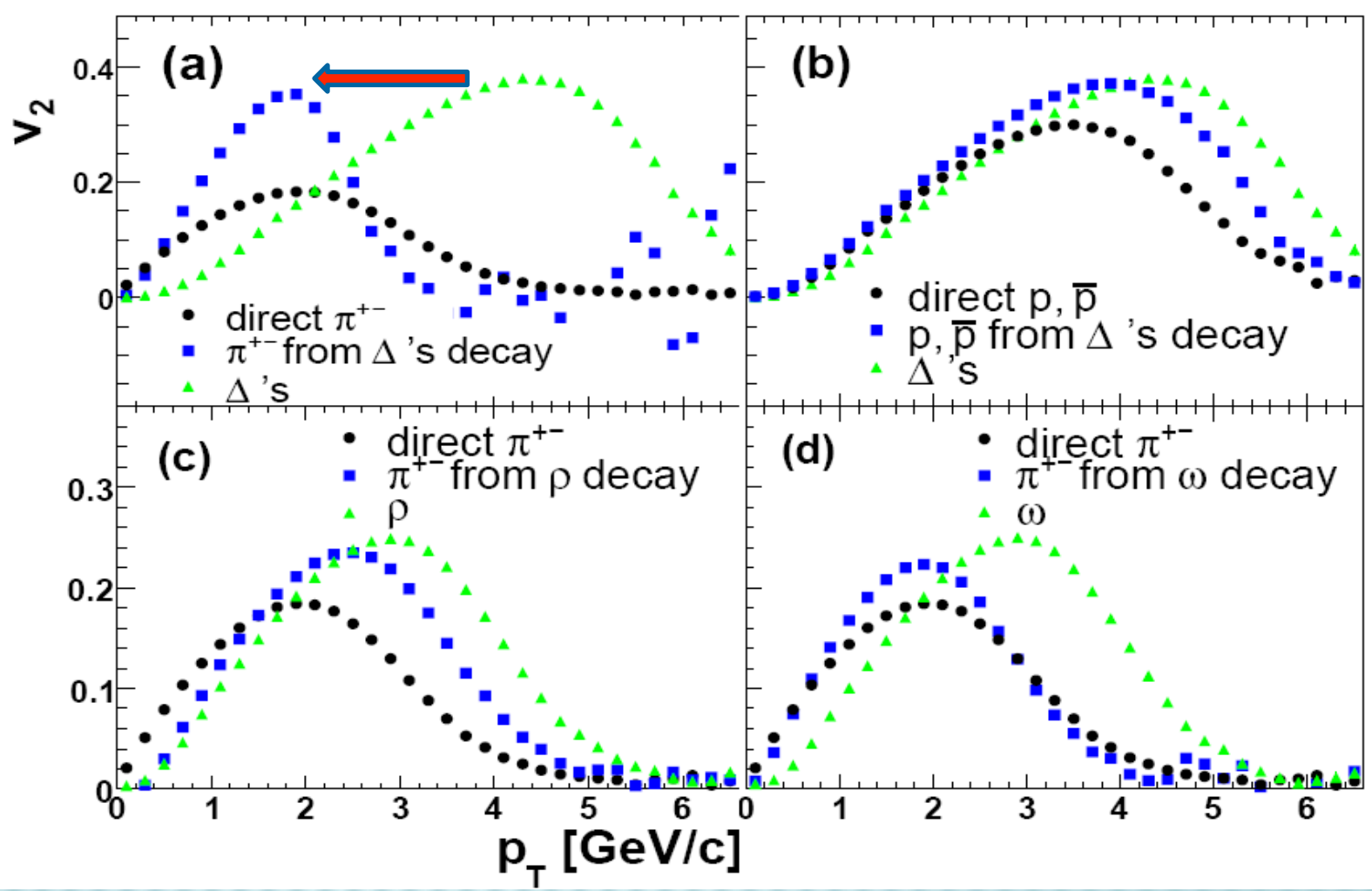
**Baryons:** the resulting flow is stronger than the flow of direct particles

# TRANSVERSE MOMENTUM OF SECONDARY PARTICLES



The secondary pion spectrum is much softer than proton spectrum

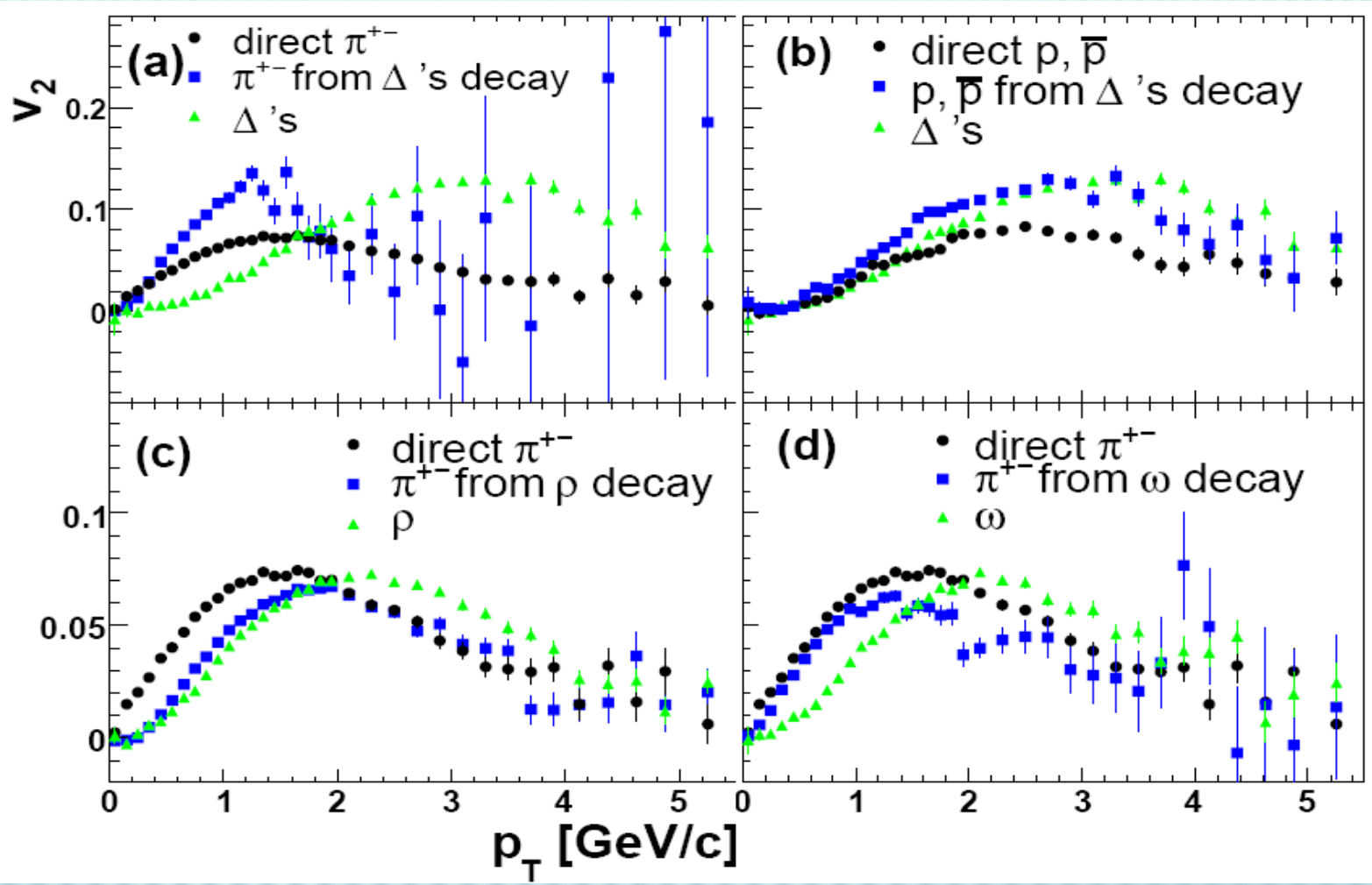
# ELLIPTIC FLOW OF DIRECT AND SECONDARY PARTICLES AT RHIC



G. Eyyubova et al., PRC 80 (2009) 064907

The heavier resonances have larger  $v_2$  at high transverse momenta  
 The decay kinematics keeps this high  $v_2$  for products of resonance decays

# ELLIPTIC FLOW OF DIRECT AND SECONDARY PARTICLES AT LHC

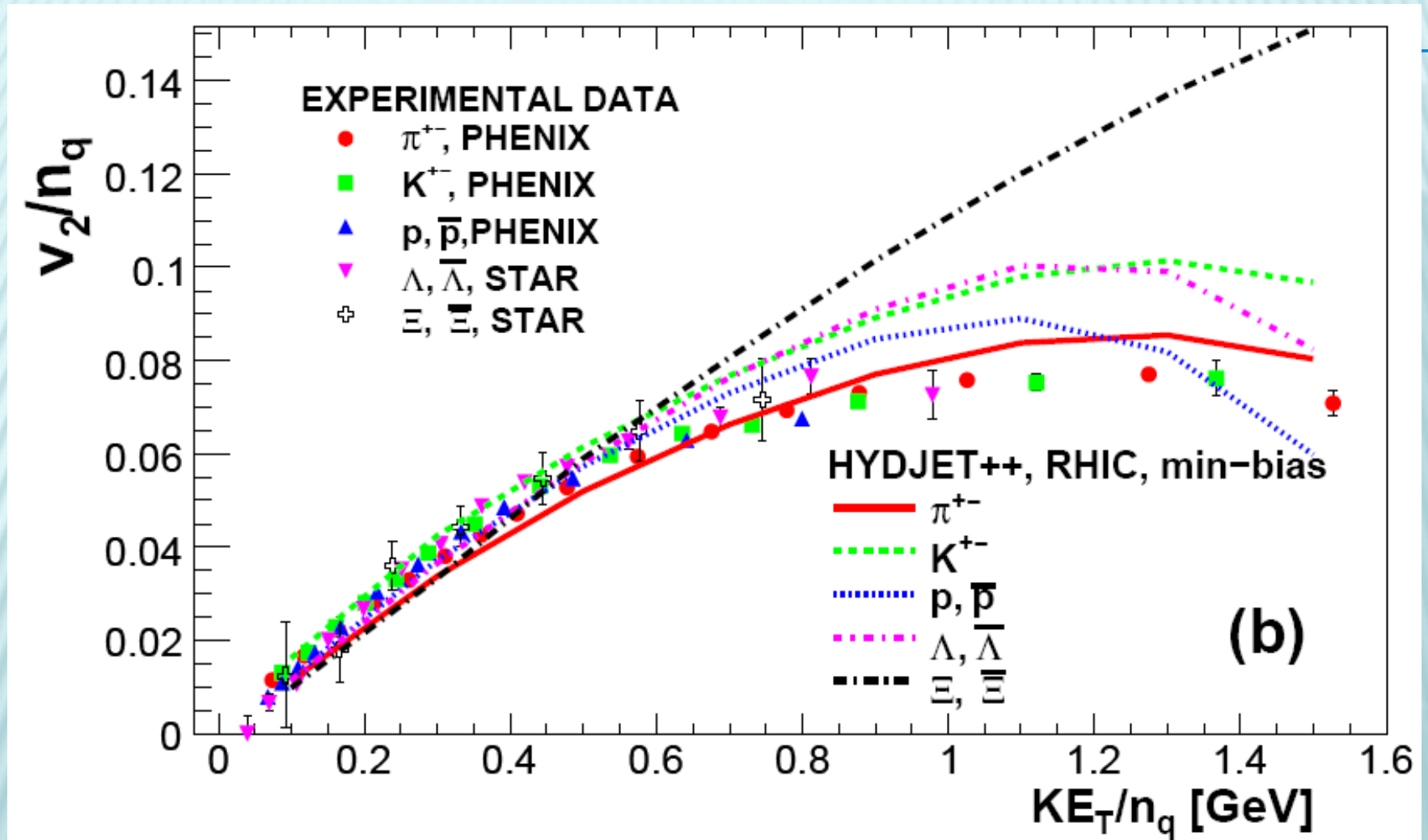


At low transverse momenta: pions from baryon resonances enhance the flow; pions from meson resonances reduce it



# **IV. Number-of- constituent- quark (NCQ) scaling**

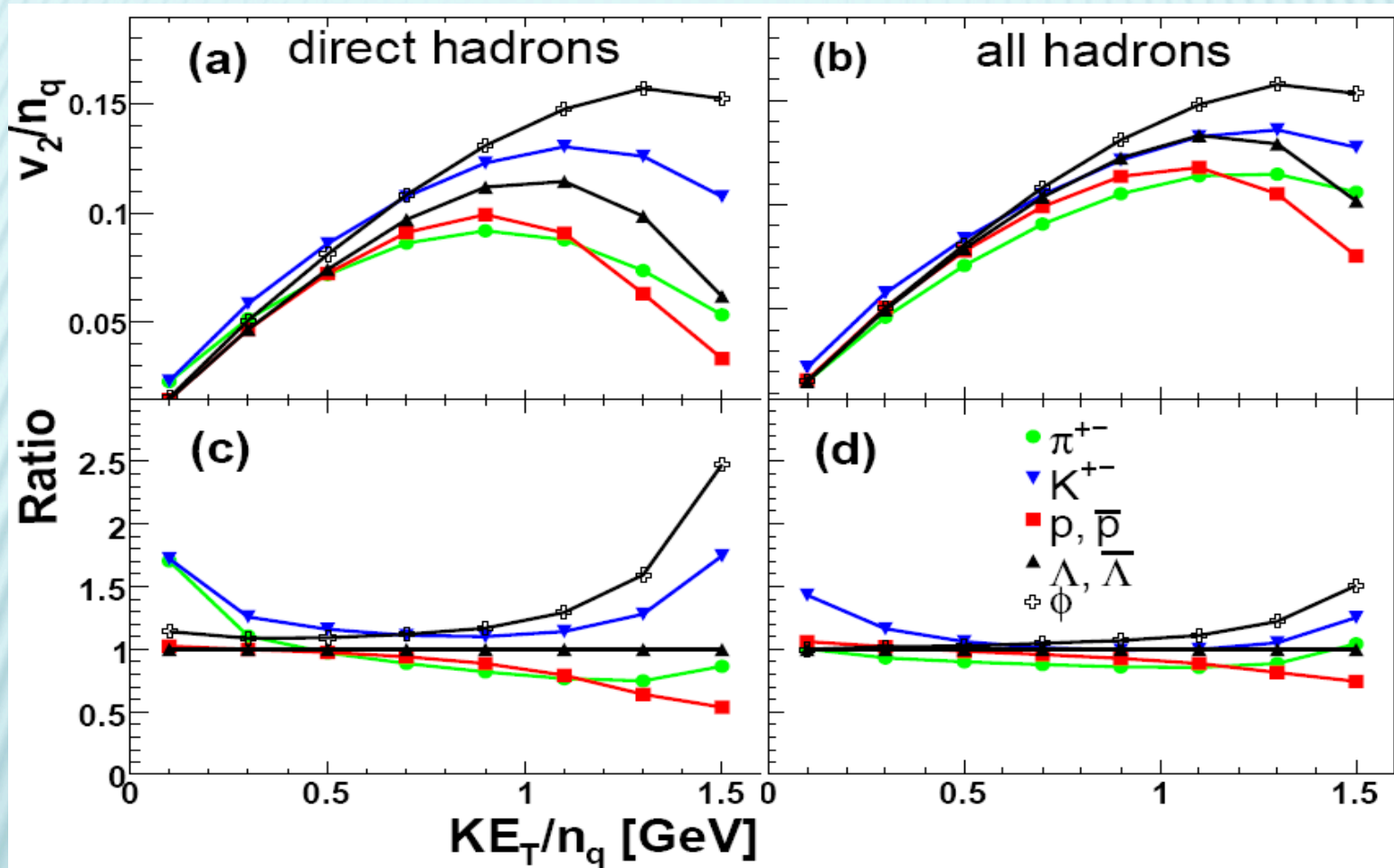
## COMPARISON WITH RHIC DATA



The agreement seems to be good at  $KE_T/n_q < 0.7$  GeV

# Number-of-constituent-quark scaling at RHIC

Direct particles: scaling is not good. All particles:  $KE_T/n_q$  scaling

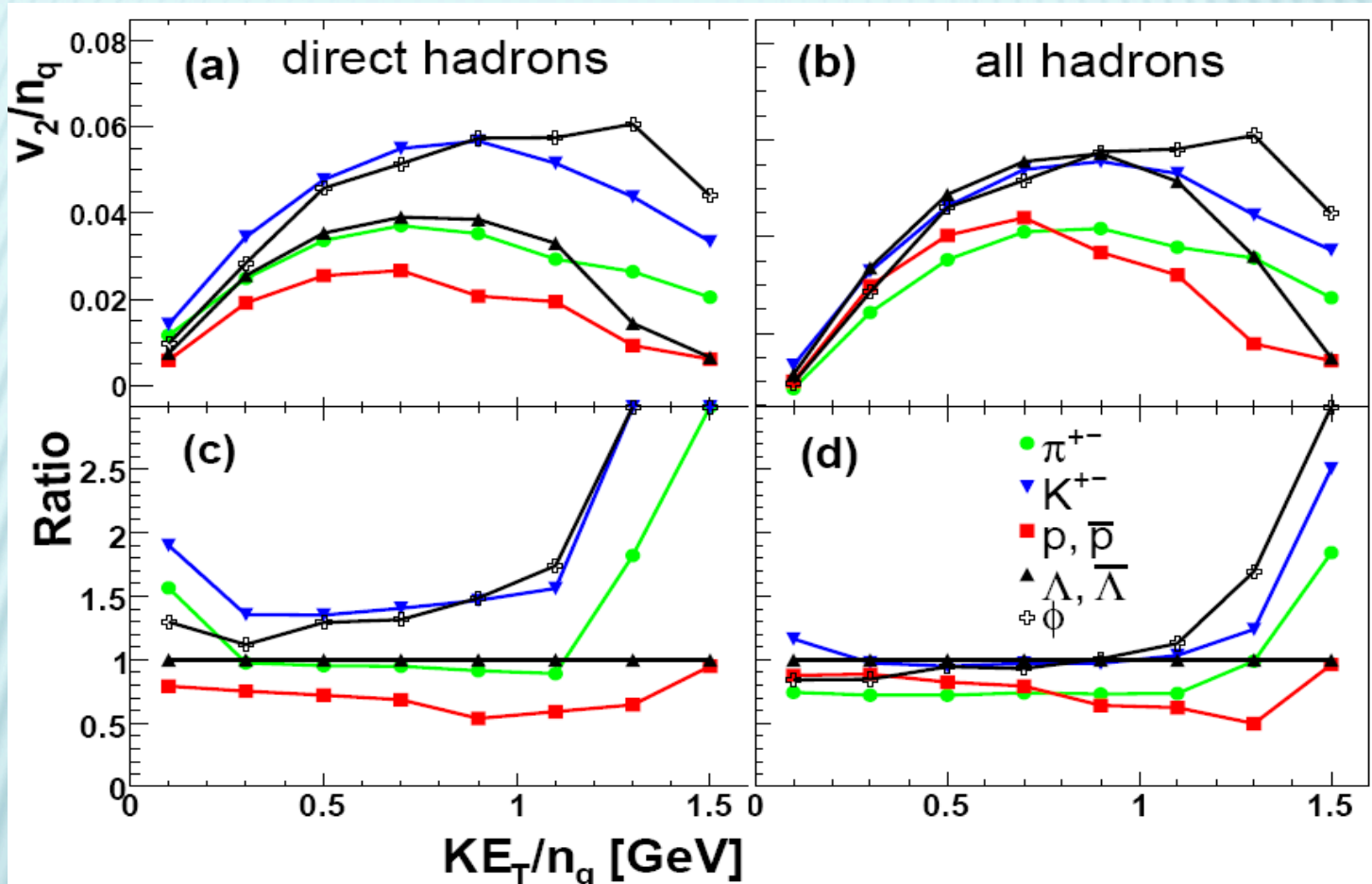


One of the explanations of  $KE_T/n_q$  scaling is partonic origin of the elliptic flow. *However, final state effects (such as resonance decays and jets) may also lead to appearance of the scaling*

# NCQ scaling at LHC

No scaling for direct particles

Appearance of the approximate scaling for all particles



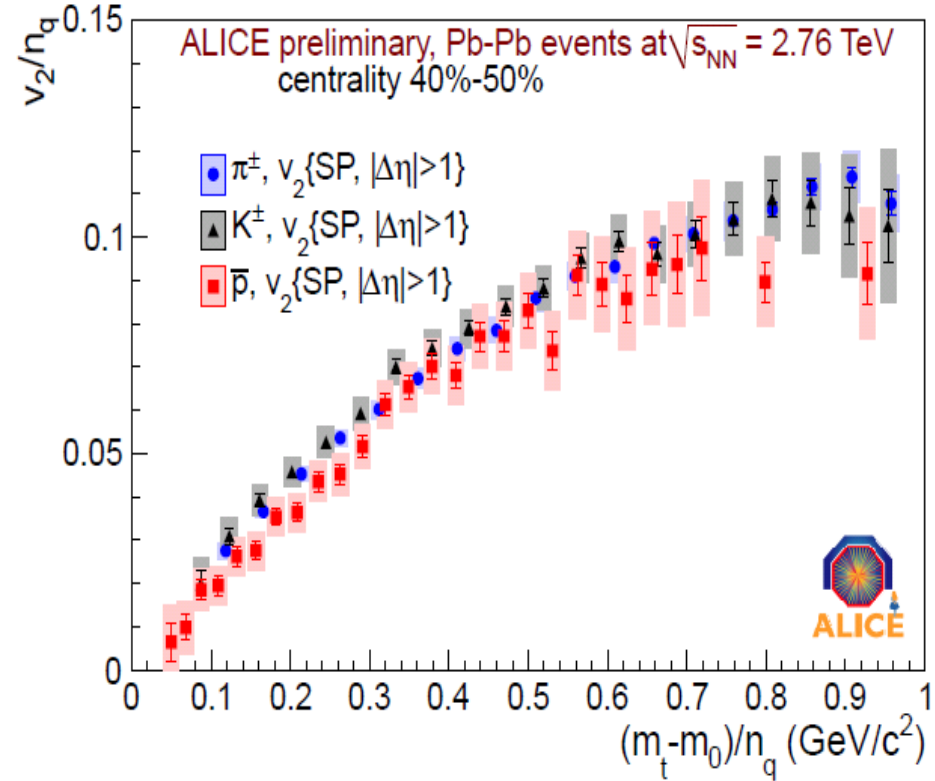
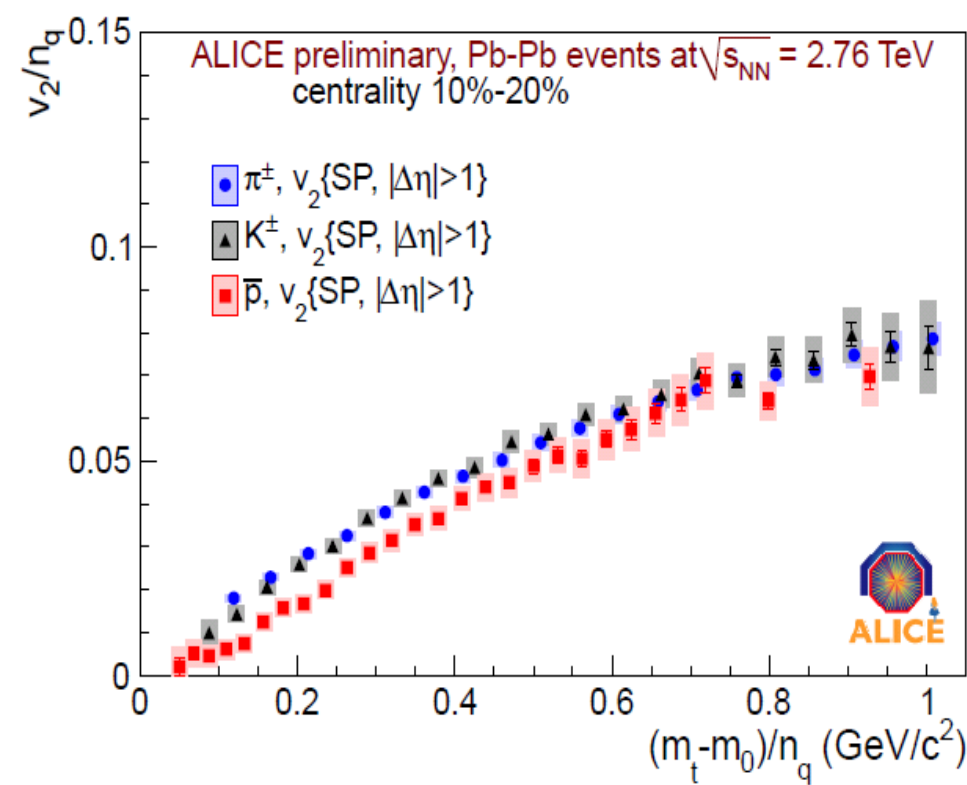
LHC: NCQ scaling will be only approximate (prediction, 2009)

# Experimental results (LHC)

ALICE Collaboration, M. Krzewicki et al., JPG 38 (2011) 124047

## Semi-central collisions

## Semi-peripheral collisions



The NCQ scaling is indeed only approximate (2011)

**V.  $V^4/(V^2*V^2)$  RATIO**

# Predictions

*N. Borghini, J.-Y. Ollitrault, PLB 642 (2006) 227*

- Within the *approximation* that the particle momentum  $\mathbf{p}$  and the fluid velocity  $\mathbf{v}$  are parallel (valid for *large momentum*  $p_{\perp}$  and *low freeze-out temperature*  $T$ )

$$dN/d\varphi = \exp(2\varepsilon p_{\perp} \cos(2\varphi)/T)$$

- Expanding to order  $\varepsilon$ , the  $\cos(2\varphi)$  term is

$$v_2 = \varepsilon p_{\perp} / T$$

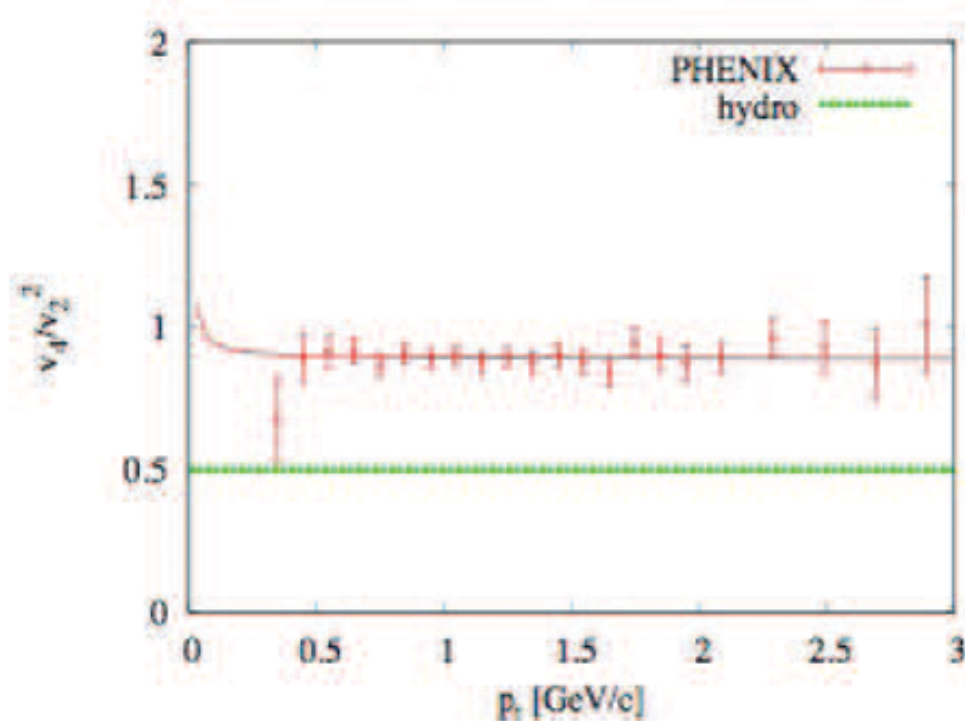
- Expanding to order  $\varepsilon^2$ , the  $\cos(4\varphi)$  term is

$$v_4 = \frac{1}{2} (v_2)^2$$

Hydrodynamics has a universal prediction for  $v_4/(v_2)^2$  !

Should be independent of equation of state, initial conditions, centrality, rapidity, particle type

## Comparison with data



PHENIX data for charged pions

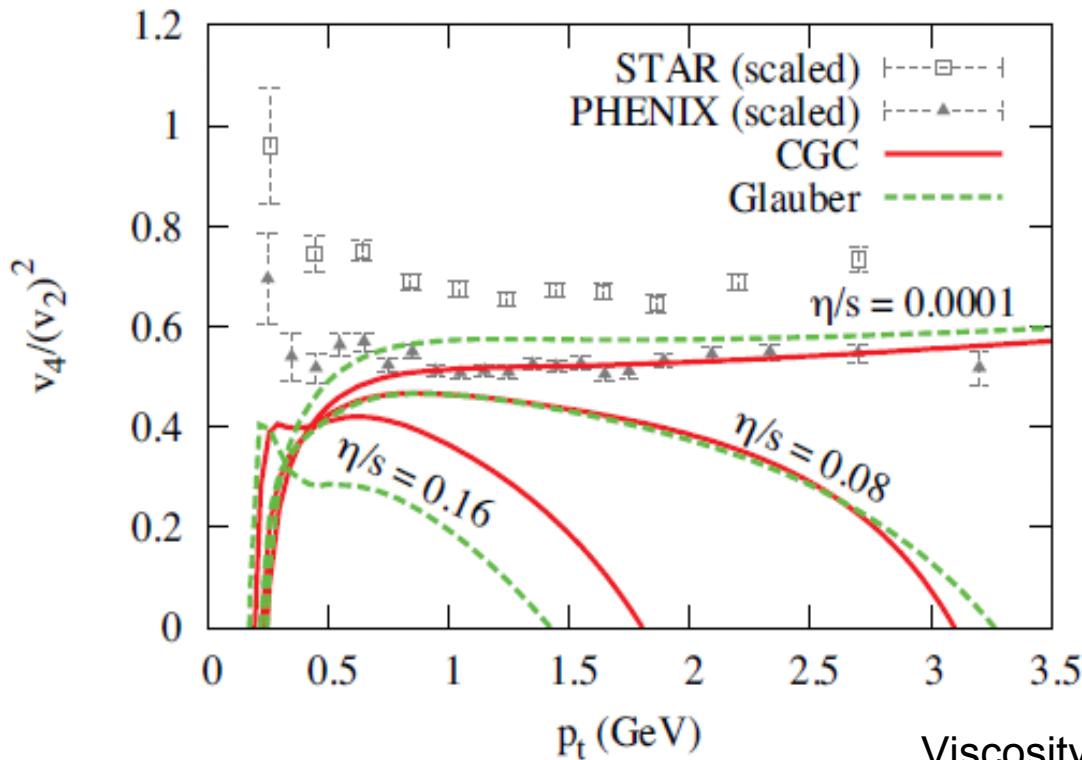
Au-Au collisions at 100+100 GeV

20-60% most central

The ratio is significantly larger than 0.5.  
Can this be explained by viscous corrections?



# Effects of initial profile and viscosity

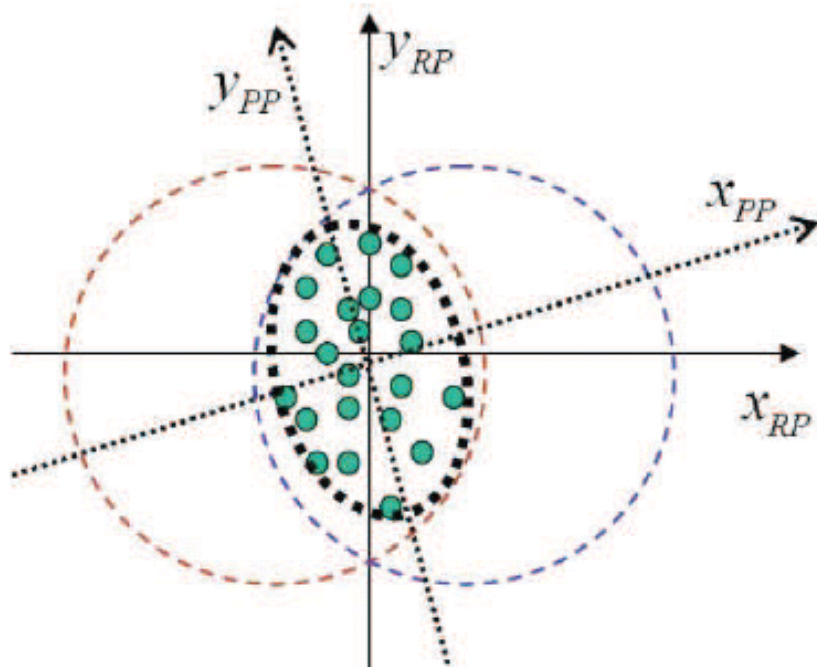


Initial profile has little effect although eccentricities differ.

results strongly depend on viscosity

Viscosity lowers  $v_4/(v_2)^2$  for a realistic  $T_f$

## Eccentricity fluctuations



Depending on where the participant nucleons are located within the nucleus at the time of the collision, the actual shape of the overlap area may vary: the **orientation and eccentricity** of the ellipse defined by participants fluctuates.

Assuming that  $v_2$  scales like the eccentricity, **eccentricity fluctuations** translate into  **$v_2$  fluctuations**

Eccentricity fluctuation can be computed in MC Glauber model or derived from experiment by comparing different methods for flow calculation.

# Why $\varepsilon$ fluctuations change $v_4/v_2^2$

Experimentally, no direct measure of  $v_2$  and  $v_4$

$v_2$  and  $v_4$  are measured via azimuthal correlations

$$v_2 \text{ from } \langle \cos(2\phi_1 - 2\phi_2) \rangle = \langle (v_2)^2 \rangle$$

$$v_4 \text{ from } \langle \cos(4\phi_1 - 2\phi_2 - 2\phi_3) \rangle = \langle v_4 (v_2)^2 \rangle$$

Experimentally measured

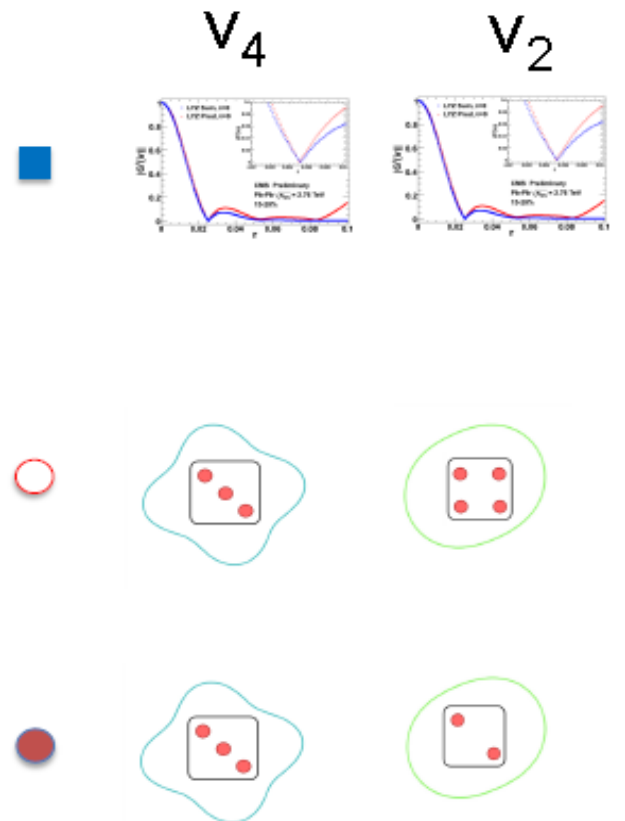
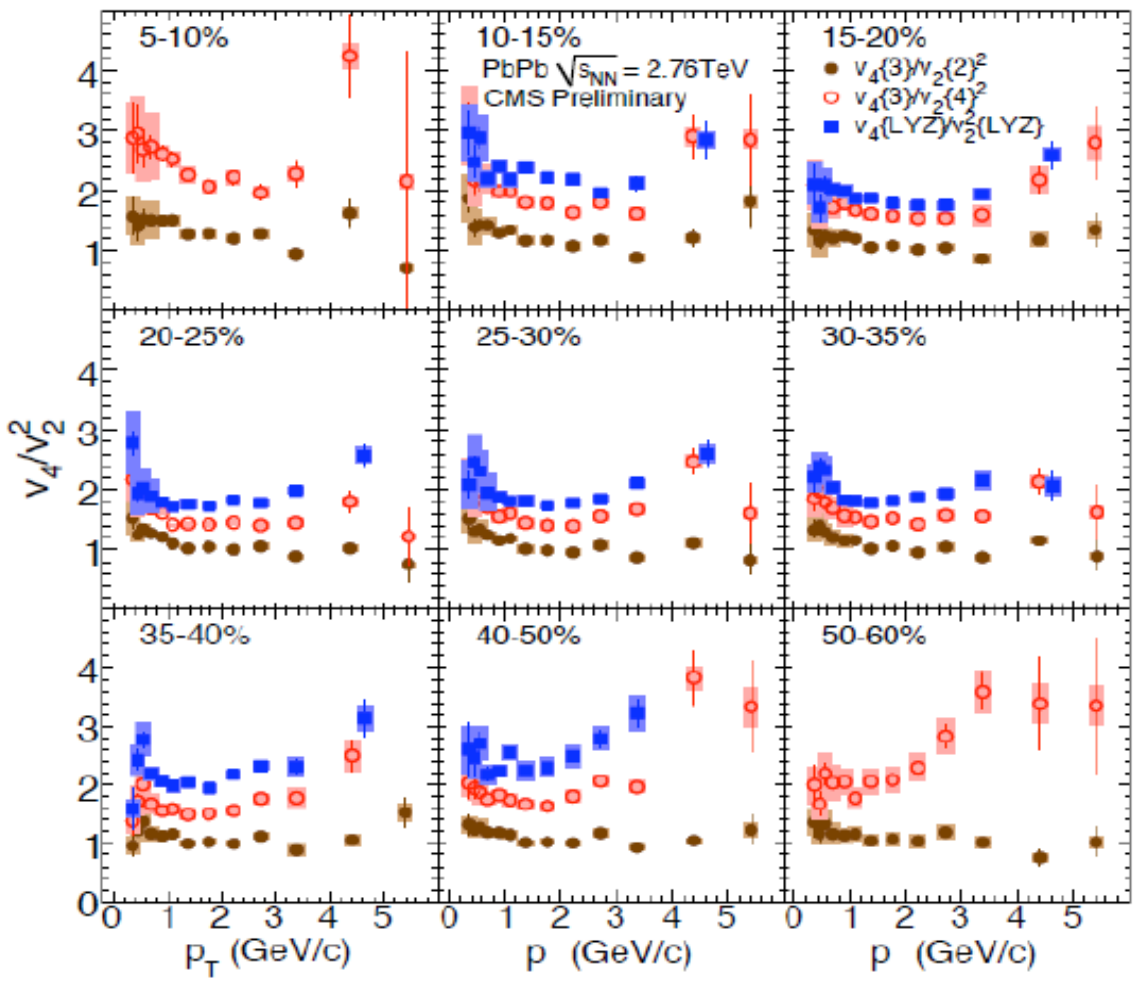
$$\frac{v_4}{v_2^2} = \frac{\langle v_4 (v_2)^2 \rangle}{\langle (v_2)^2 \rangle^2} = \frac{1}{2} \frac{\langle (v_2)^4 \rangle}{\langle (v_2)^2 \rangle^2} > \frac{1}{2}$$

fluctuations

hydro

Similar results obtained using Event Plane method

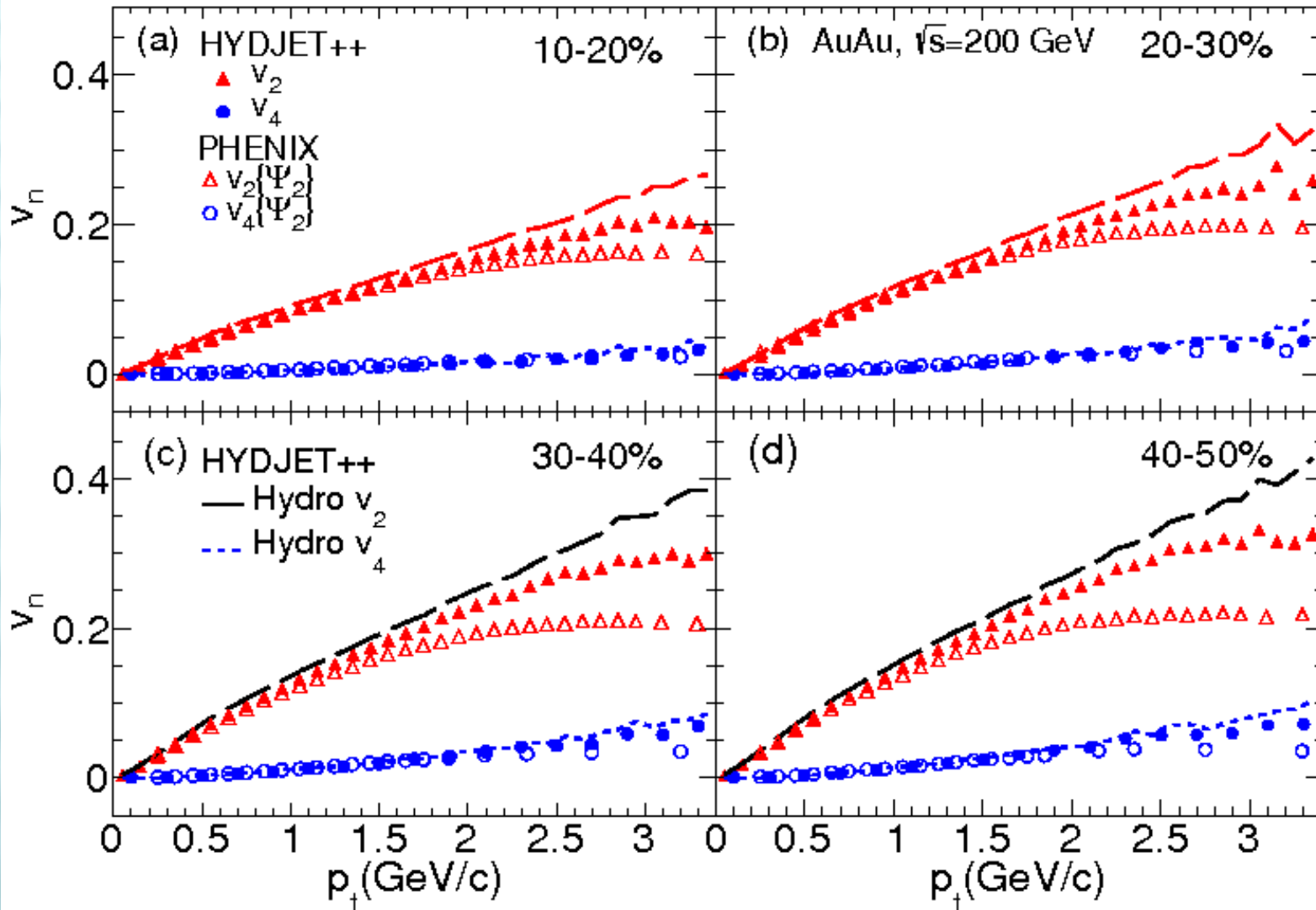
# $v_4 / v_2^2(p_T)$ at mid-rapidity $|\eta| < 0.8$



Significantly higher than RHIC: experimental method dependent

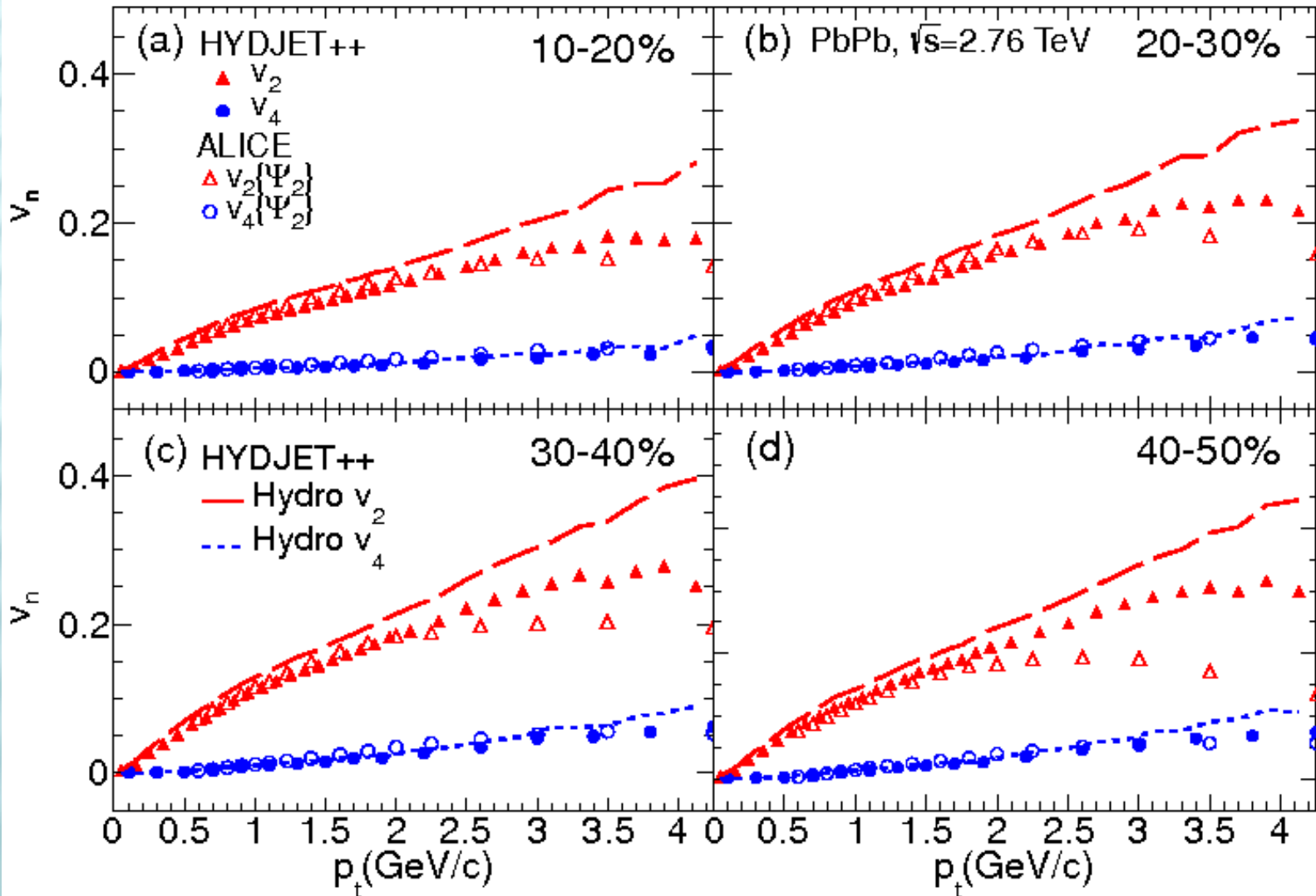
# HYDJET++ (RHIC)

Effects to be studied: resonance decay and hard part influence

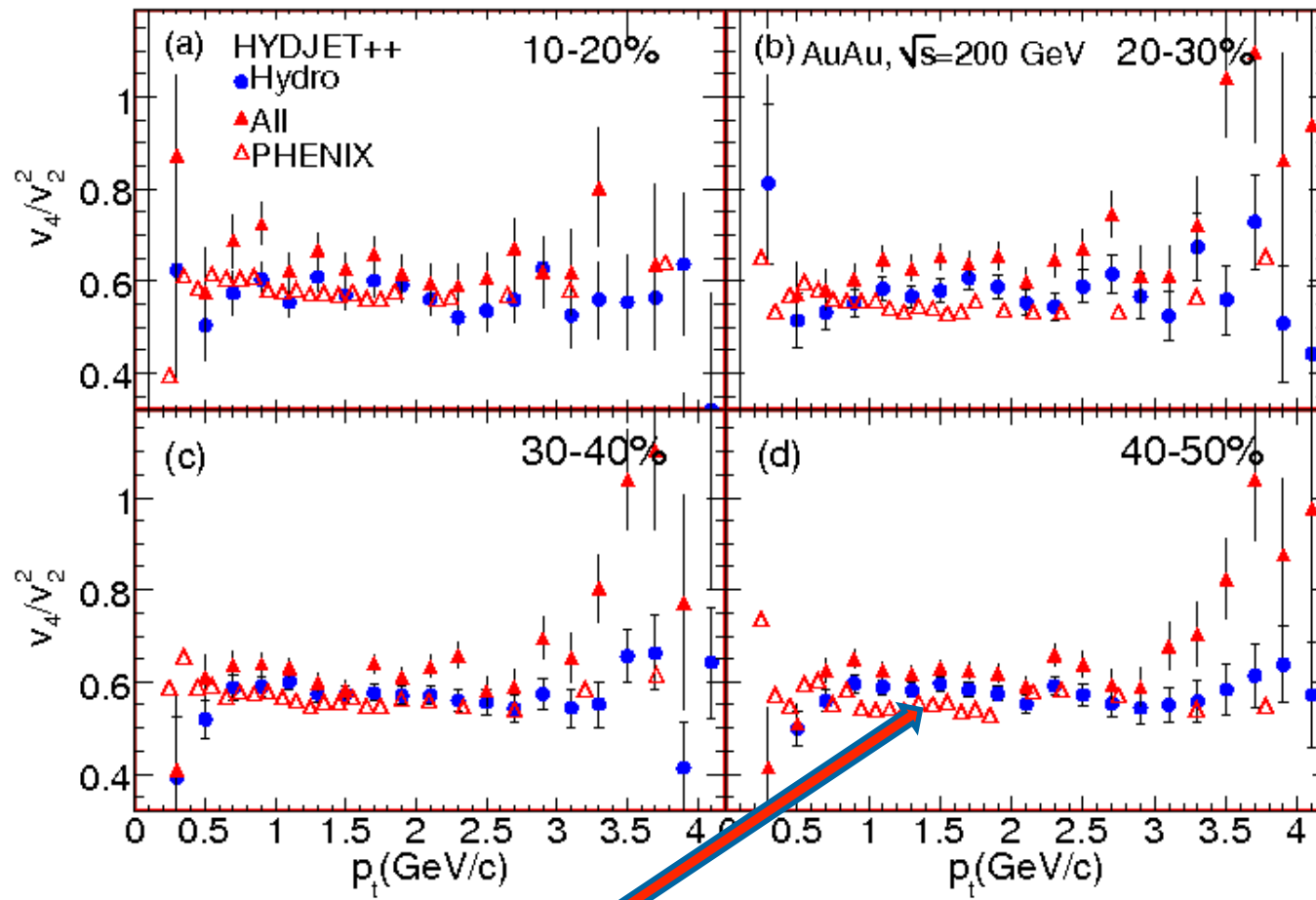


# HYDJET++ (LHC)

Pure hydrodynamics vs hydro+jets

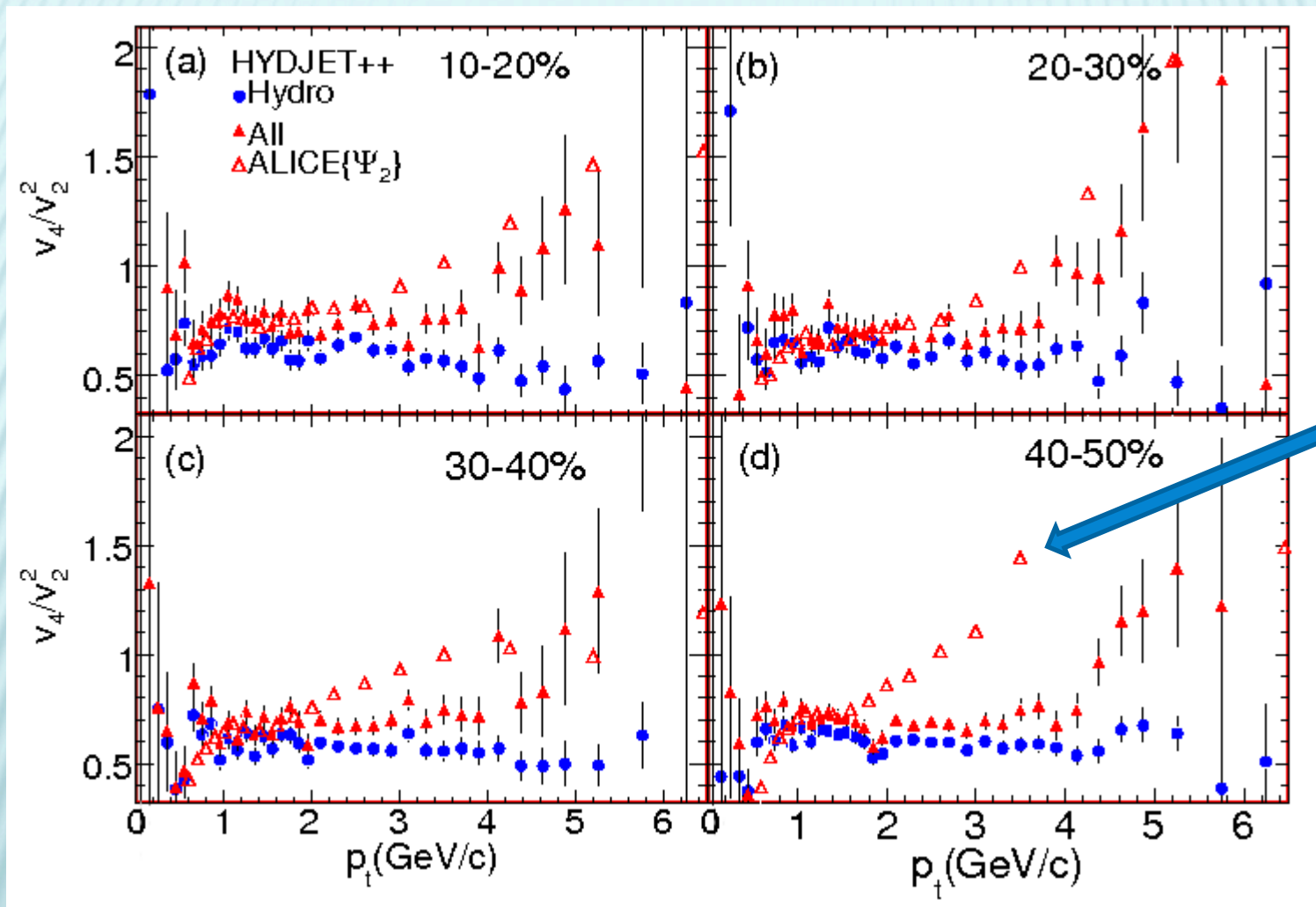


# HYDJET++ RESULTS FOR RHIC



**Jets increase the ratio**

# HYDJET++ RESULTS FOR LHC



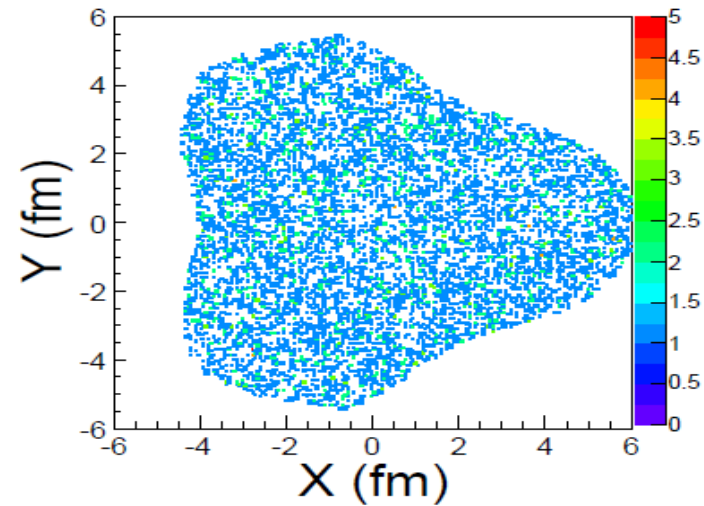
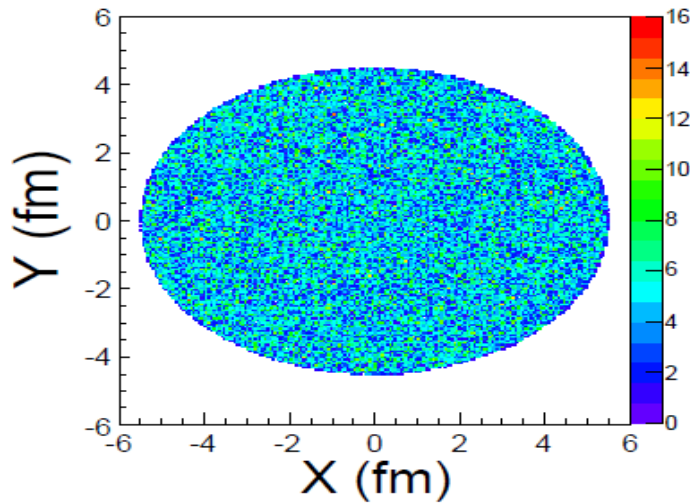
The same tendency is observed in Pb+Pb at LHC



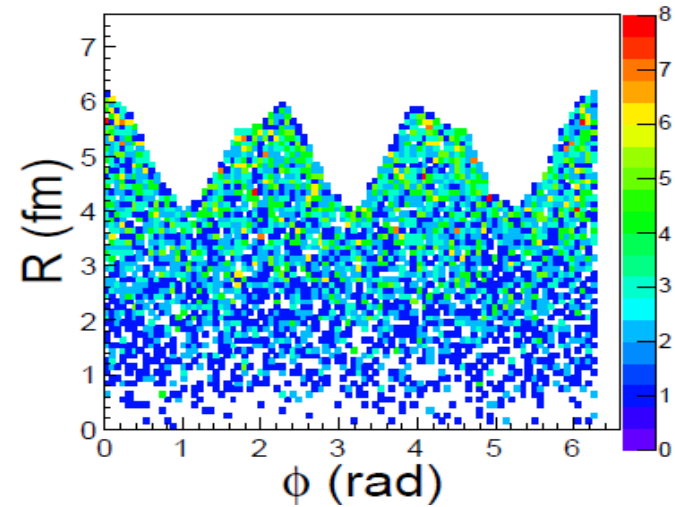
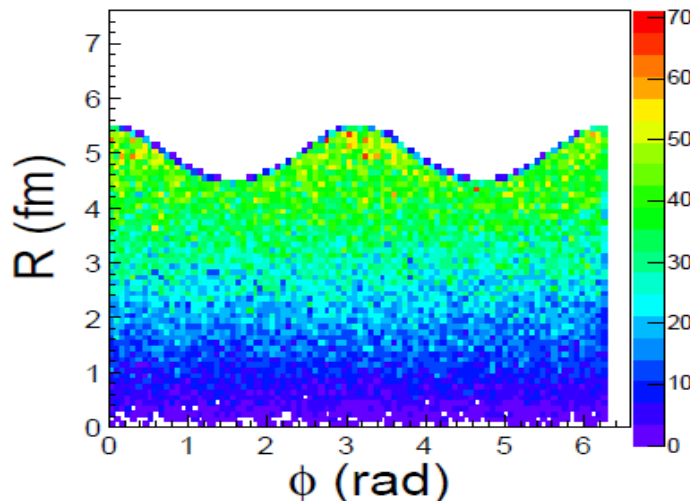
# **VI. Higher harmonics**

# GENERATION OF TRIANGULAR FLOW

V2



V3



$$R(b, \phi) = R_f(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos 2\phi}} [1 + \epsilon_3(b) \cos 3(\phi + \Psi_3^{\text{RP}})]$$

# GENERATION OF TRIANGULAR FLOW

## Space and Momentum Anisotropy

$$\epsilon(b) = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}, \quad v_2 \propto \frac{2(\delta - \epsilon)}{(1 - \delta^2)(1 - \epsilon^2)}, \quad \tan \varphi_* = \sqrt{\frac{1 - \delta(b)}{1 + \delta(b)}} \tan \varphi.$$

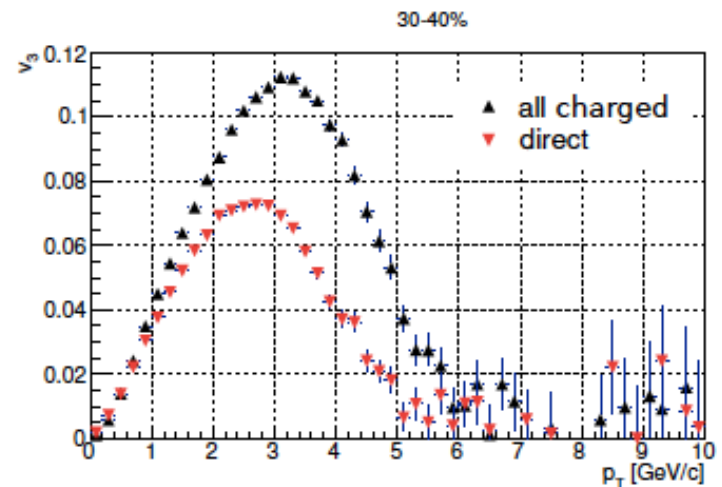
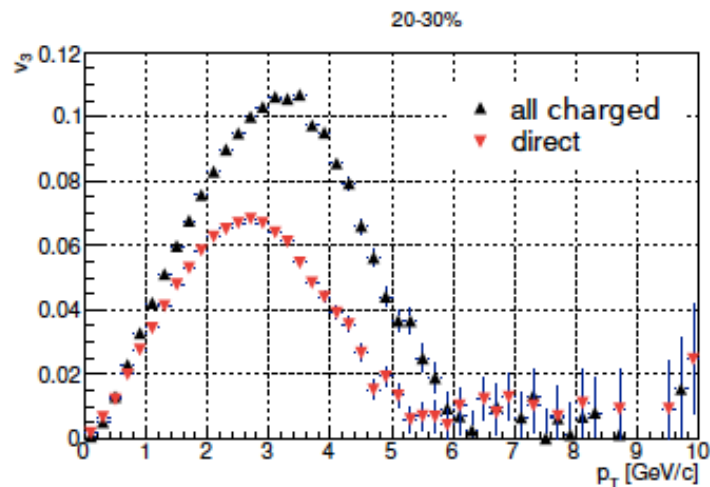
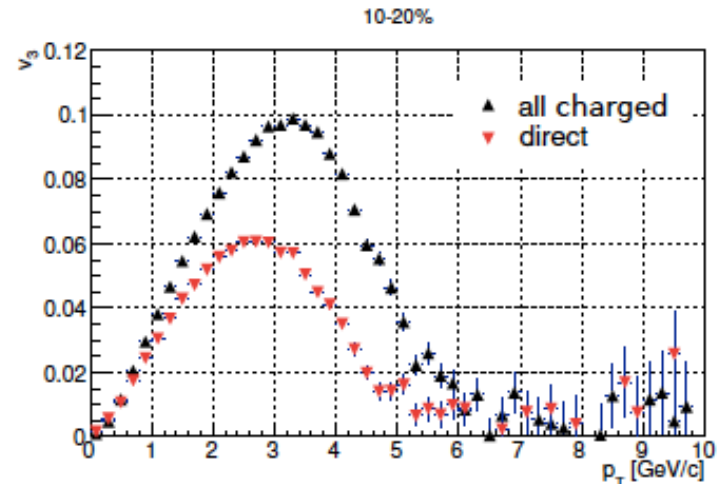
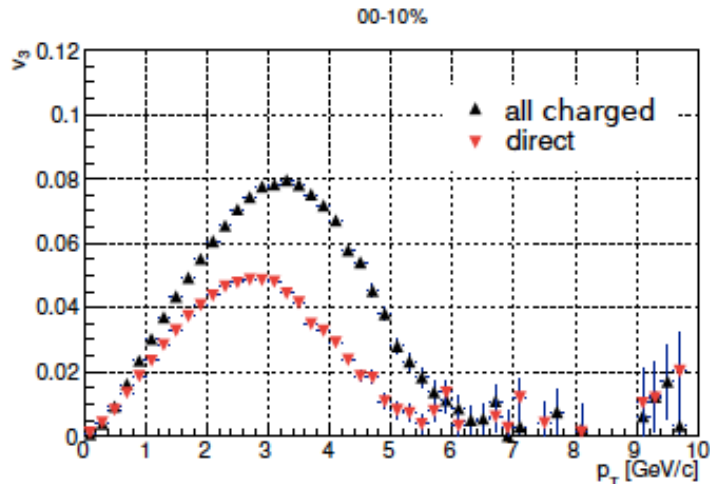
**Third component is generated as**

$$R(b, \phi) = R_f(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos 2\phi}} [1 + \epsilon_3(b) \cos 3(\phi + \Psi_3^{\text{RP}})]$$

Parameters  $\epsilon(b_0), \epsilon_3(b_0), \delta(b_0)$  are fitted to the experimental data

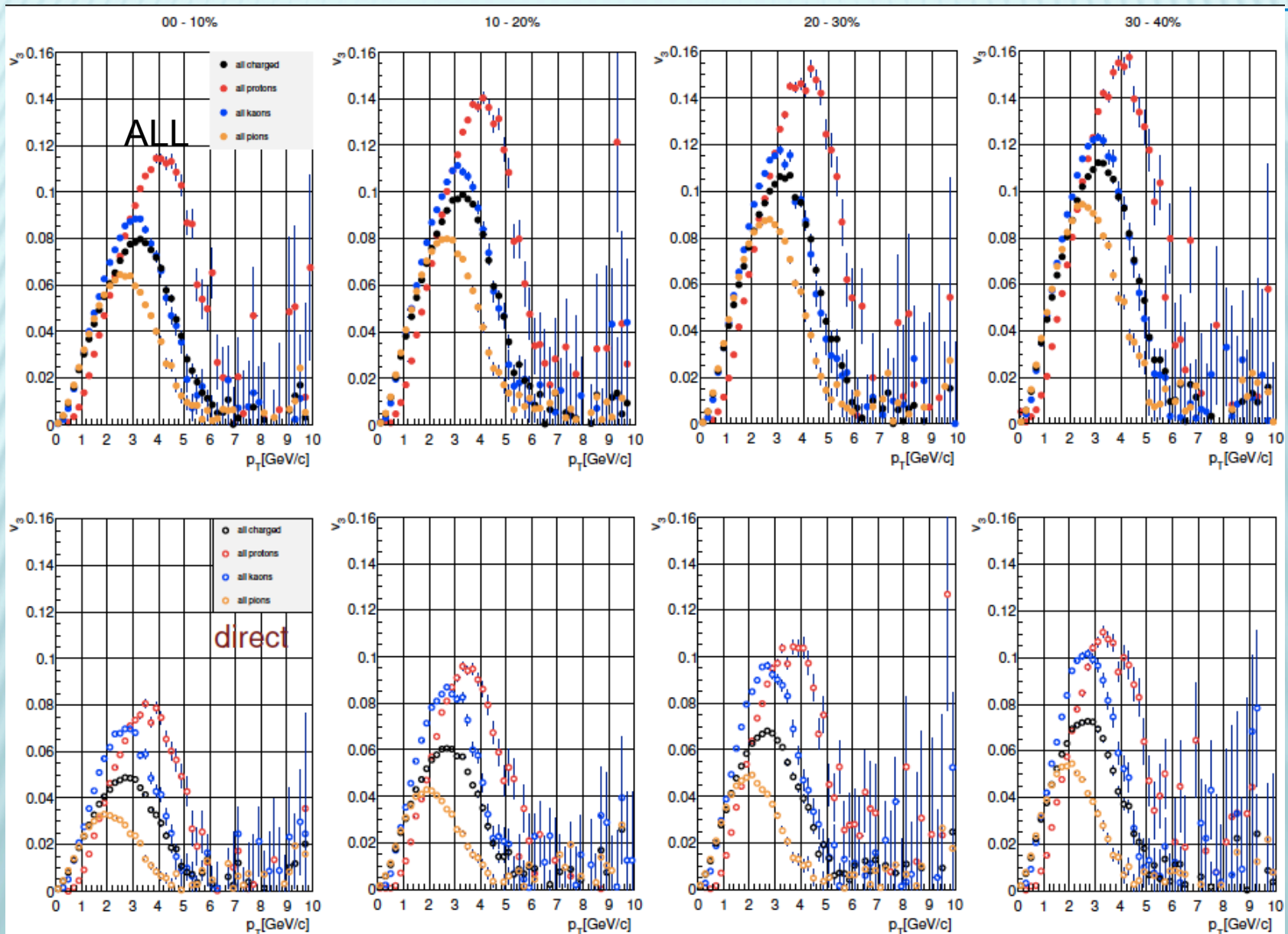
# GENERATION OF TRIANGULAR FLOW

V3



$$R(b, \phi) = R_f(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos 2\phi}} [1 + \epsilon_3(b) \cos 3(\phi + \Psi_3^{\text{RP}})]$$

# V3 for all and directly produced particles at different centralities at LHC



# NUMBER-OF-CONSTITUENT QUARK SCALING

## FROM XU SUN (QM2014)

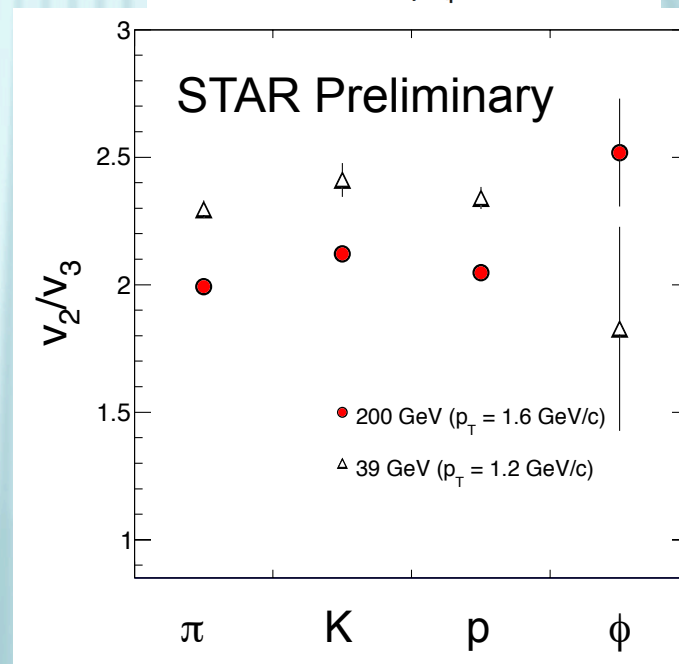
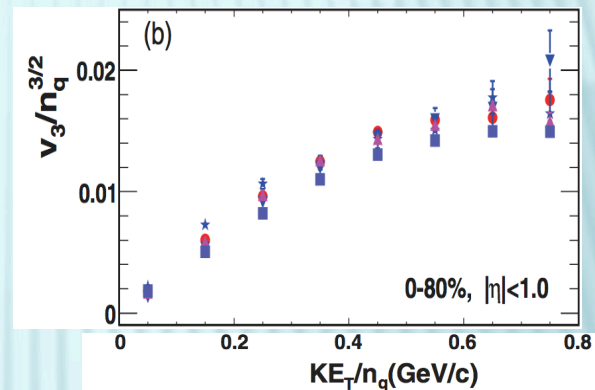
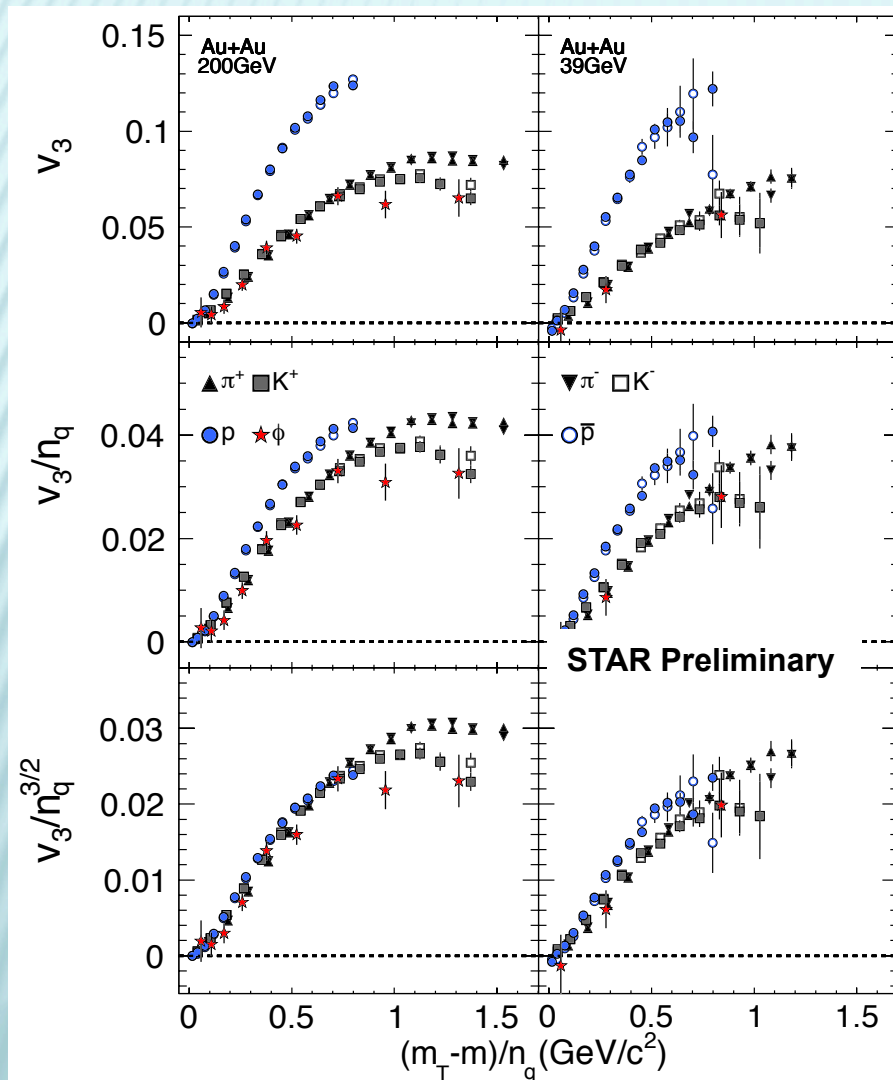
200 GeV

39 GeV

R. Lacey, J. Phys. G: Nucl. Part. Phys. 38 (2011) 124048

AMPT: PRC 84, 064907 (2011)

Harri Niemi, QM14, Friday

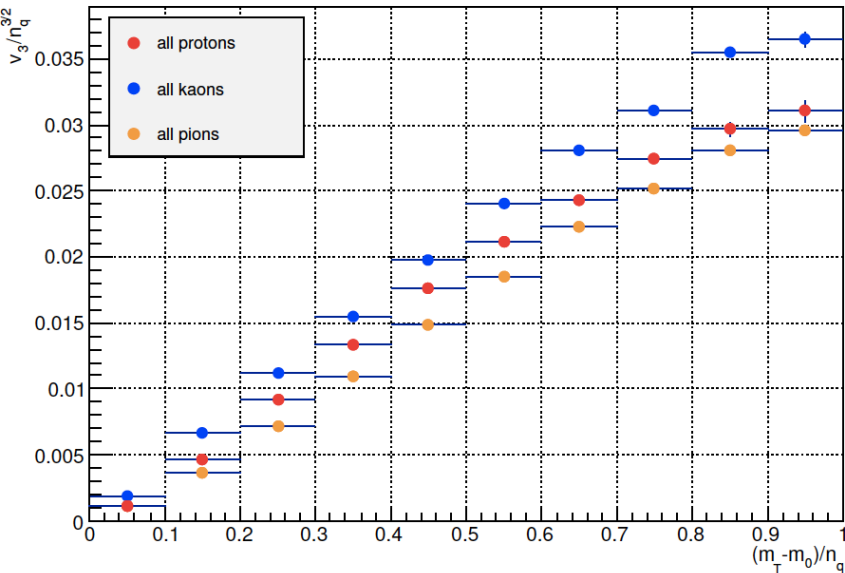


# NUMBER-OF-CONSTITUENT QUARK SCALING FROM XU SUN QM TALK FROM STAR COLLABORATION

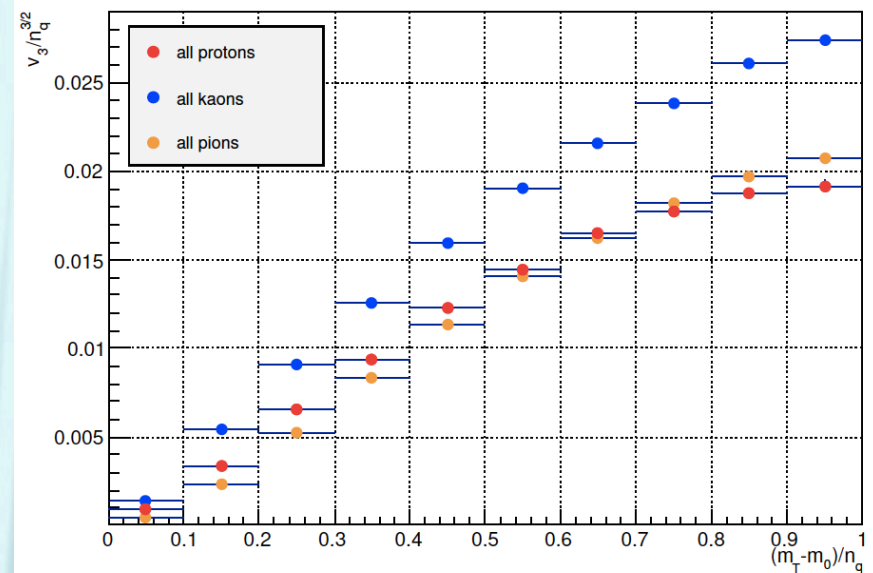
- Collective behavior for  $v_3$  at 39 and 200 GeV
  - Mass ordering at low  $p_T$ , similar to  $v_2$
  - $v_2/v_3 > 1$ :  $v_2$  geometry + fluctuation  
 $v_3$  fluctuation
- More energies (19.6, 27, ...) and particle species ( $K^0_s, \Lambda, \dots$ ) coming soon

# $V_3/NQ^{3/2}$ SCALING AT 200 AND 5500 AGEV IN HYDJET++

AuAu 200 GeV 10 - 40%



PbPb 5.5 TeV - 40%



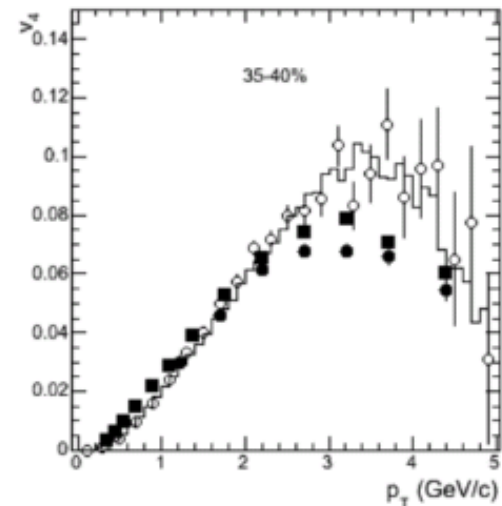
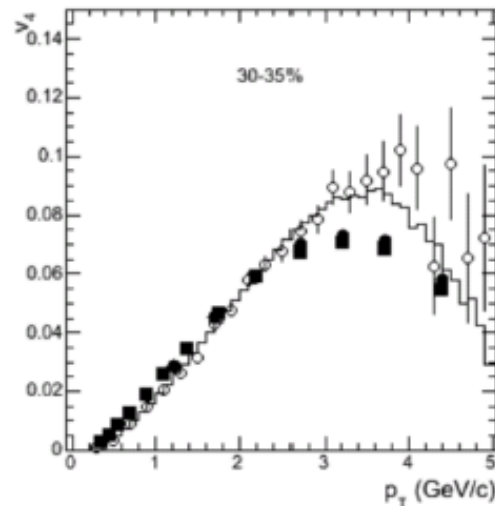
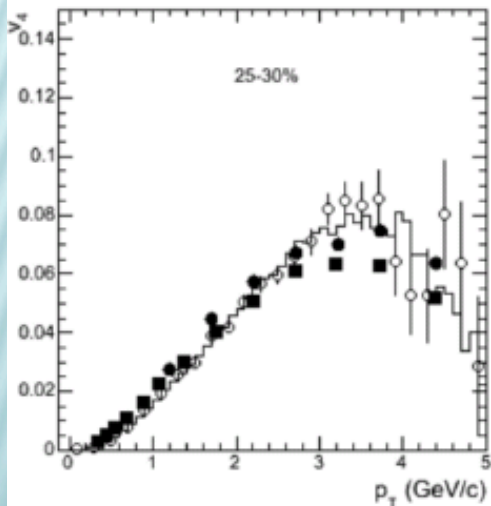
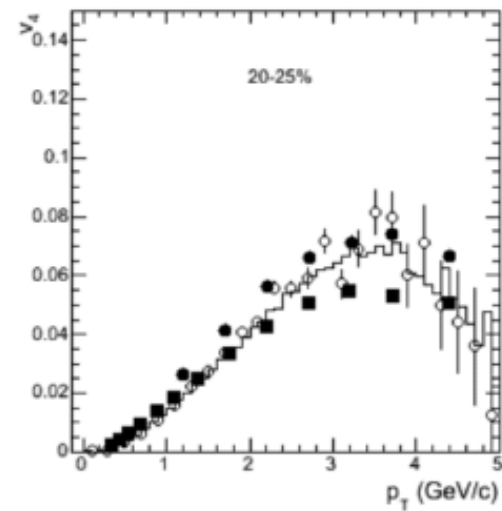
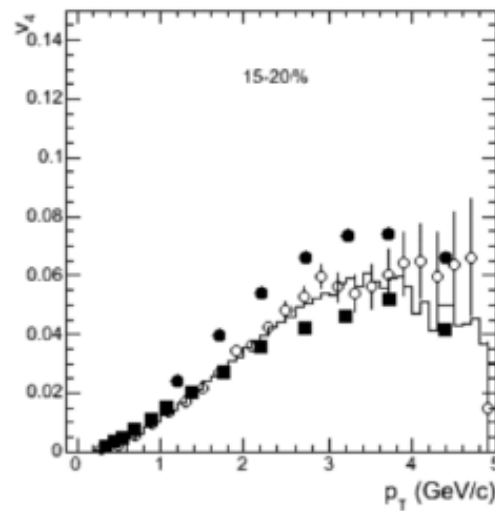
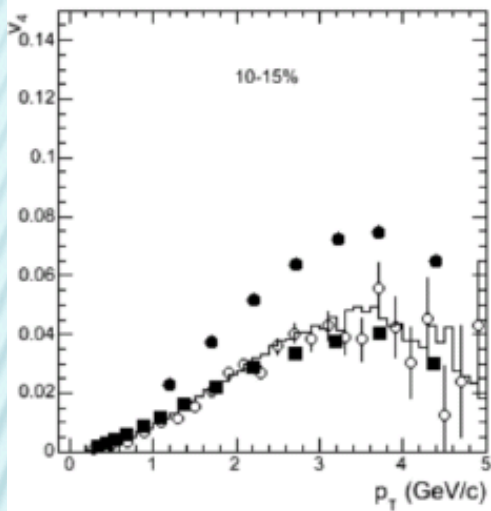
HYDJET++ shows deviation from scaling at higher energies  
Is it an accident scaling, found in START at 200 AGeV ?



# LHC DATA VS. HYDJET++ MODEL

## Quadrangular flow

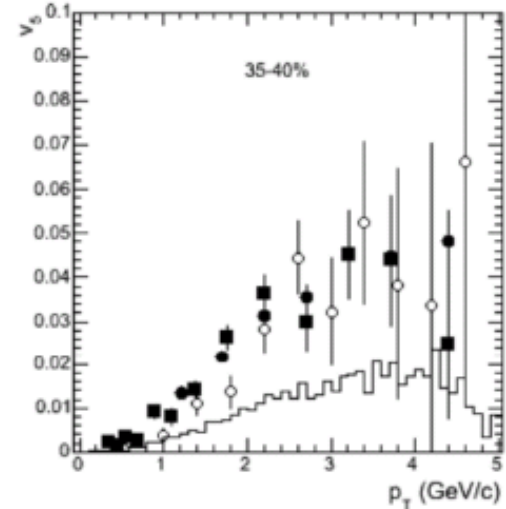
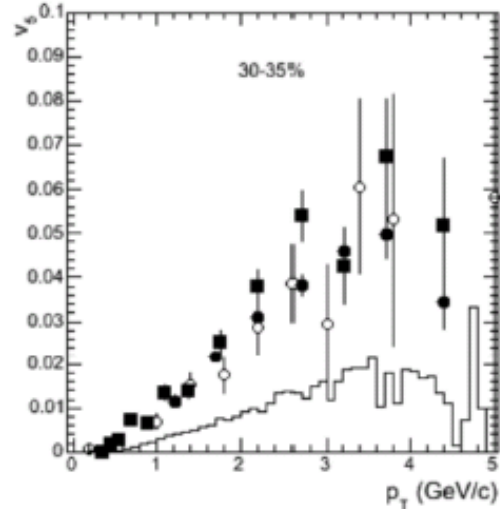
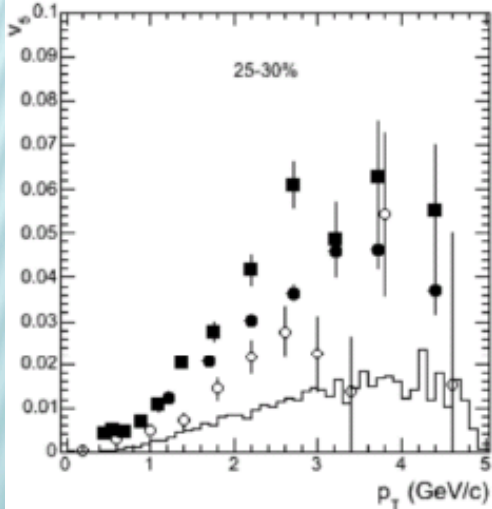
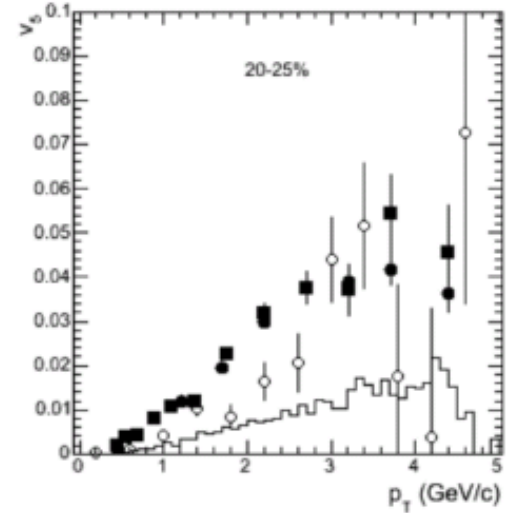
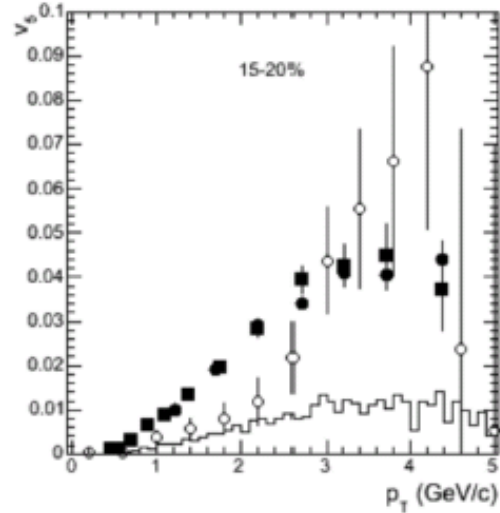
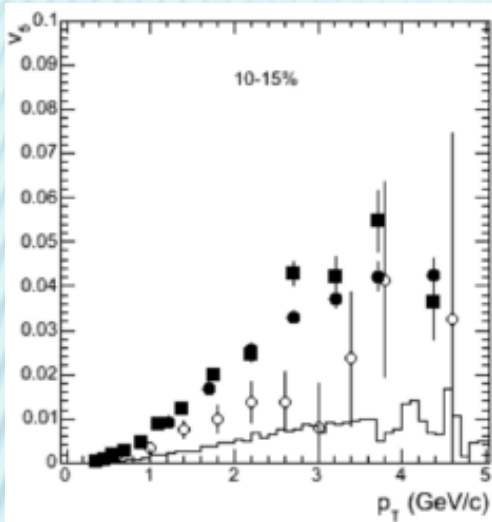
Pb+Pb @ 2.76 ATeV



# LHC DATA VS. HYDJET++ MODEL

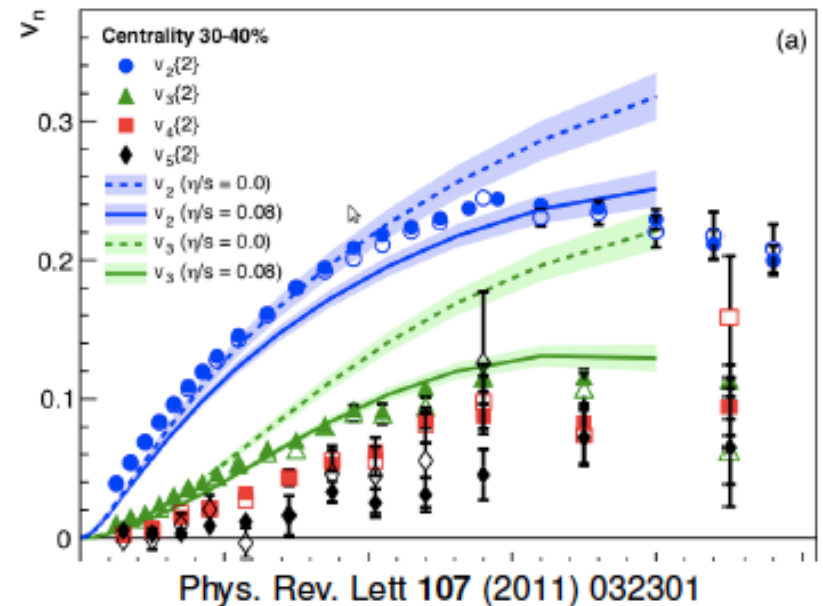
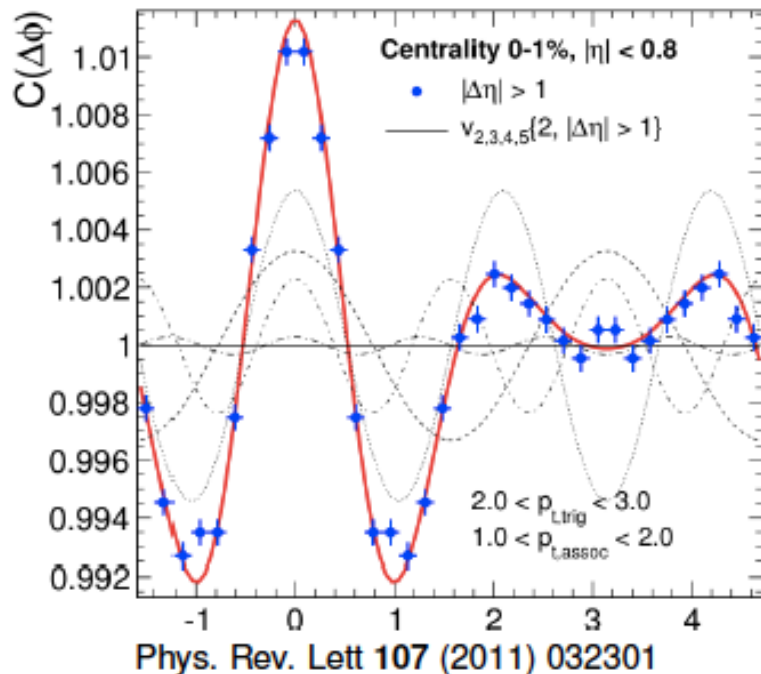
## Pentagonal flow

Pb+Pb @ 2.76 ATeV

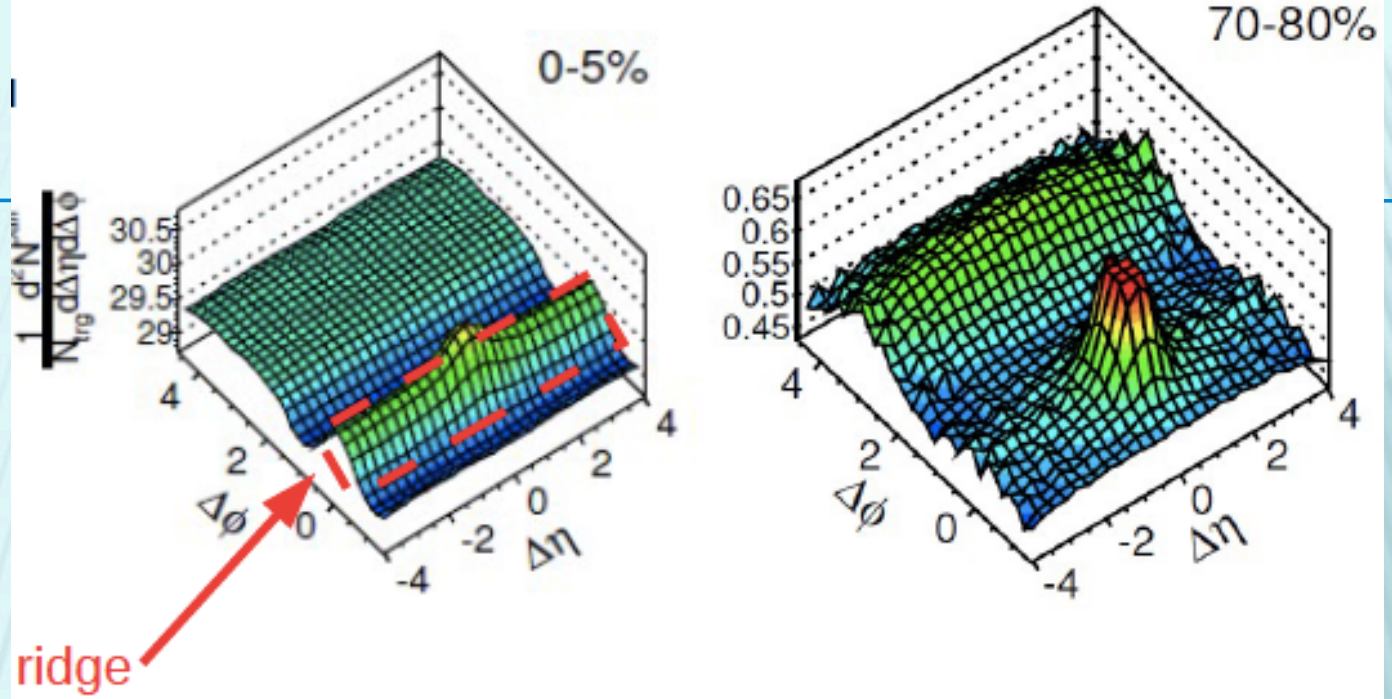
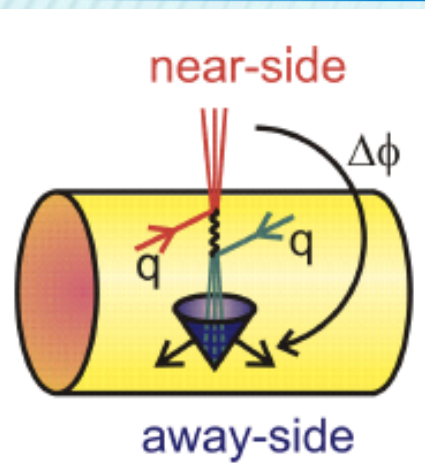


# Higher harmonics

- information on initial geometry
- more sensitive to viscosity
- explain shape of 2-particle correlation



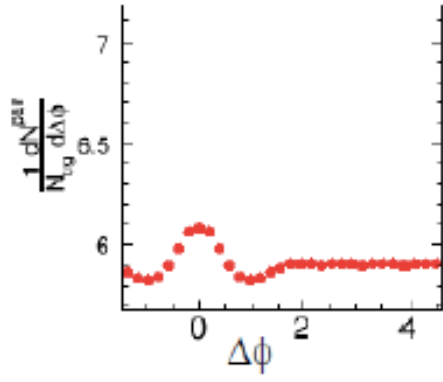
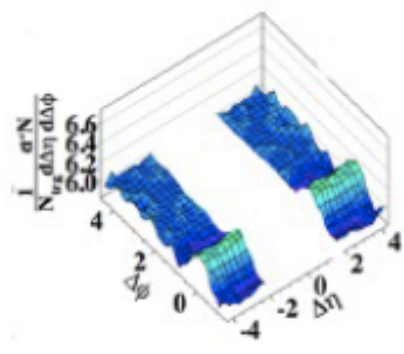
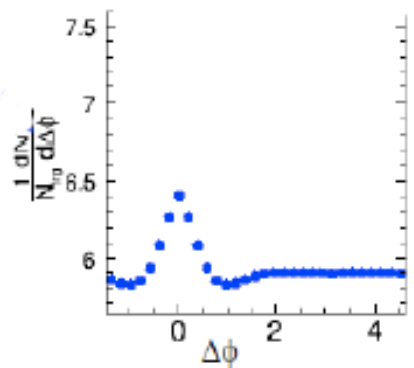
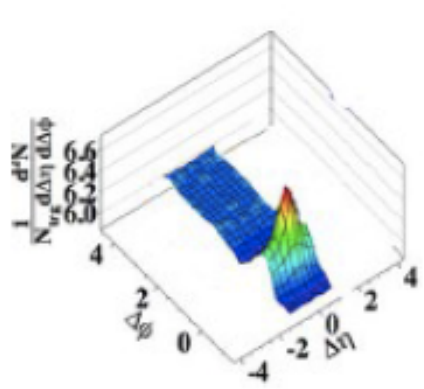
# RIDGE



CMS, PbPb 2.76 TeV, Eur. Phys. C 72 (2012) 10052,  $3 < p_{t}^{trig} < 3.5$  GeV/c,  $1 < p_{t}^{assoc} < 1.5$  GeV/c

## THE JET REGION

## RIDGE REGION



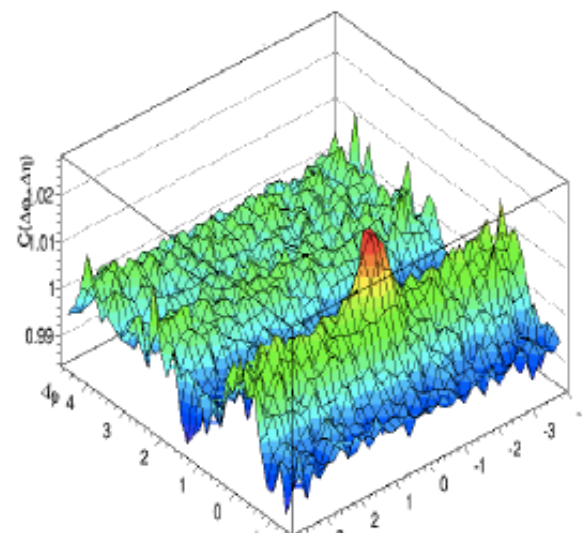
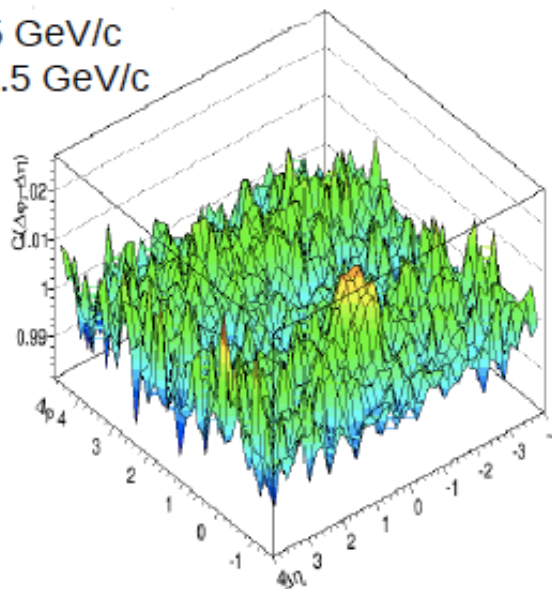
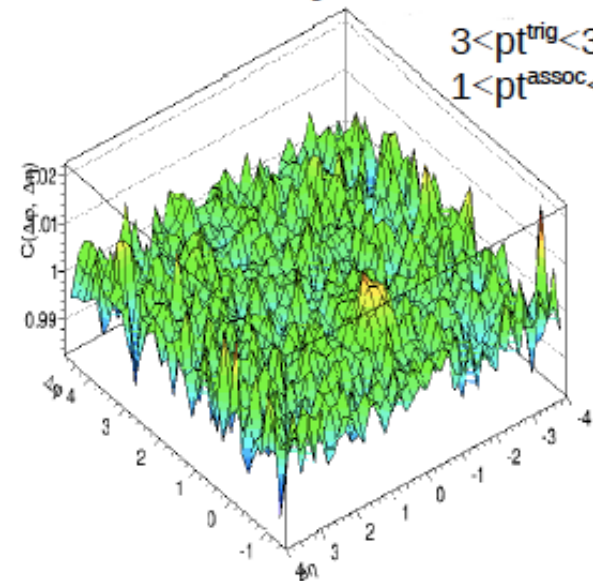
# RIDGE AND HIGHER HARMONICS

0% centrality

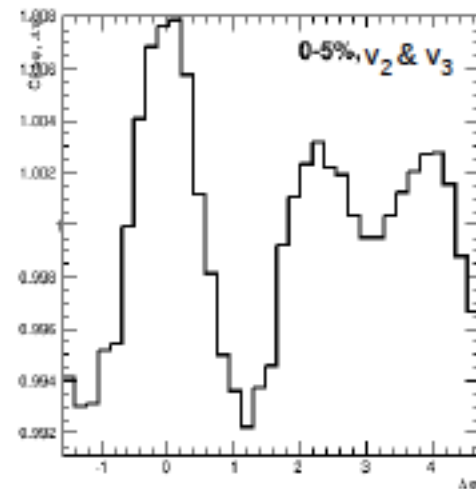
0-5% centrality,  $v_2$  only

0-5% centrality,  $v_2$  &  $v_3$

$3 < p_t^{\text{trig}} < 3.5 \text{ GeV}/c$   
 $1 < p_t^{\text{assoc}} < 1.5 \text{ GeV}/c$



RIDGE IN HYDJET++ APPEARS  
DUE TO  $v_2$  AND  $v_3$



# CONCLUSIONS

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The HYDJET++ model allows to investigate flow of hydro and jet parts separately, to look at reconstruction of pure hydro flow and its modification due to jet part.

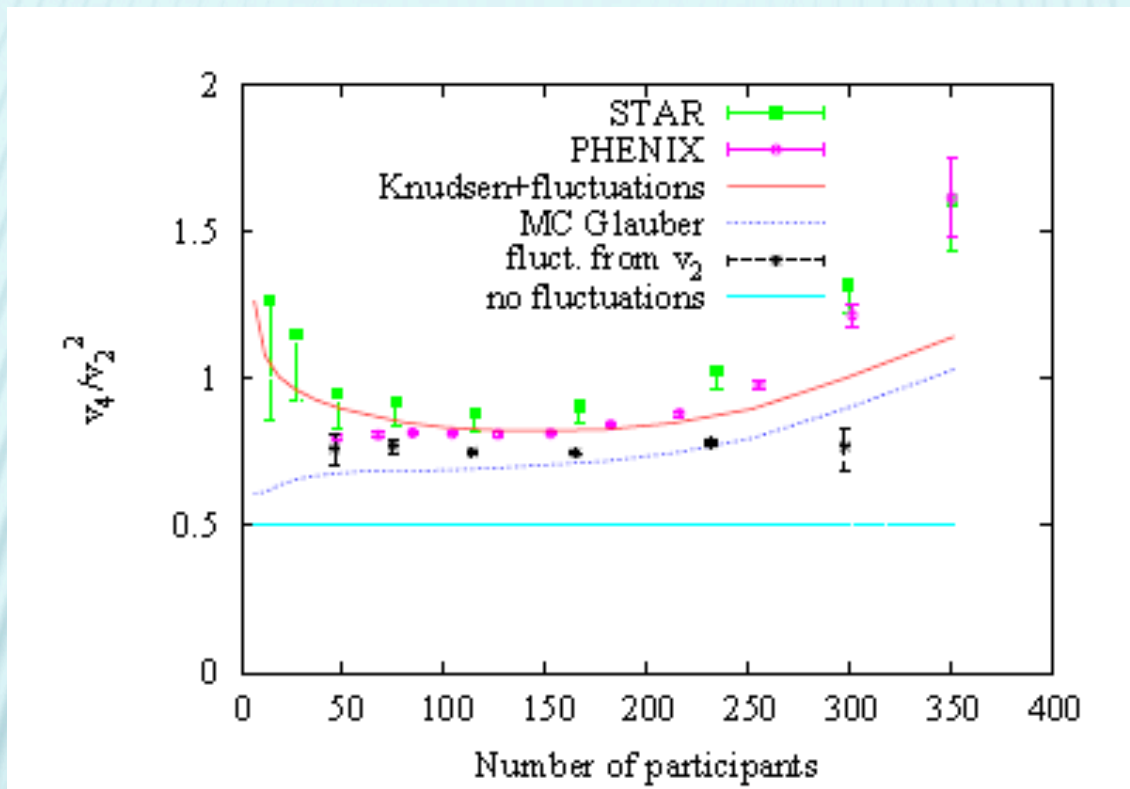
- *Jets and decays of resonances are important components of the anisotropic flow description*
- *Jets + eccentricity fluctuations are enough to explain both RHIC and LHC data on  $v_4/(v_2*v_2)$  ratio, including the high- $p_T$  tail*
- *The predicted violation of the NCQ scaling at LHC is observed*
- *Higher order harmonics and ridge – just an interplay of elliptic and triangular flows ?*

---

# Back-up Slides

# Effects of flow fluctuations and partial thermalization

M. Luzum, C. Gombeaud, J.-Y. Ollitrault, Phys.Rev.C81:054910,2010.



Stars: with fluctuations inferred from the difference between  $v_2\{2\}$  and  $v_2\{\text{LYZ}\}$ .

Dotted line: eccentricity fluctuations from a Monte-Carlo Glauber



# **V . PARAMETERS OF THE MODEL**

# Methods for $v_2$ calculation

## (1) Event plane method

$$v_2^{obs} \{EP\} = \langle \cos 2(\varphi_i - \Psi_2) \rangle$$

$\Psi_2$  is the calculated reaction plane angle:  $\tan n\psi_n = \frac{\sum_i \omega_i \sin n\varphi_i}{\sum_i \omega_i \cos n\varphi_i}$ ,  $n \geq 1$ ,  $0 \leq \psi_n < 2\pi/n$

$$v_2 \{EP\} = \frac{v_2^{obs} \{EP\}}{R} = \frac{v_2^{obs} \{EP\}}{\langle \cos 2(\Psi_2 - \Psi_R) \rangle}$$

## (2) Two particle correlation method

$$v_2 \{2\} = \sqrt{\langle \cos 2(\varphi_i - \varphi_j) \rangle}$$

## (3) Lee-Yang zero method

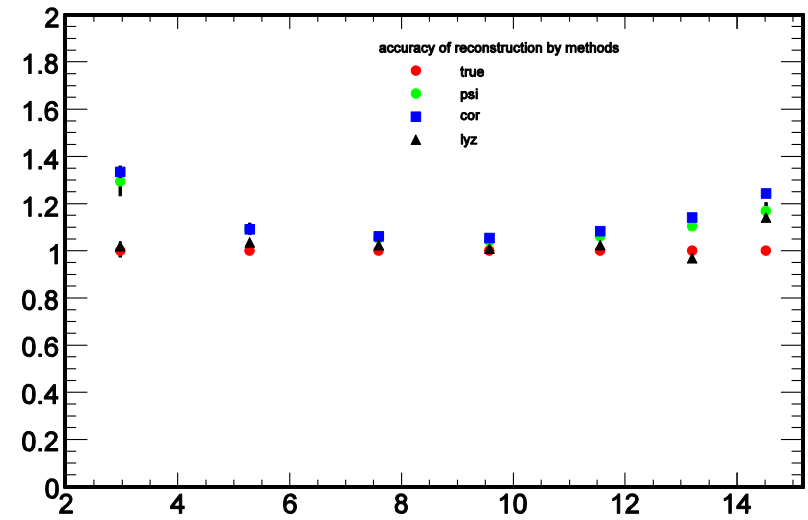
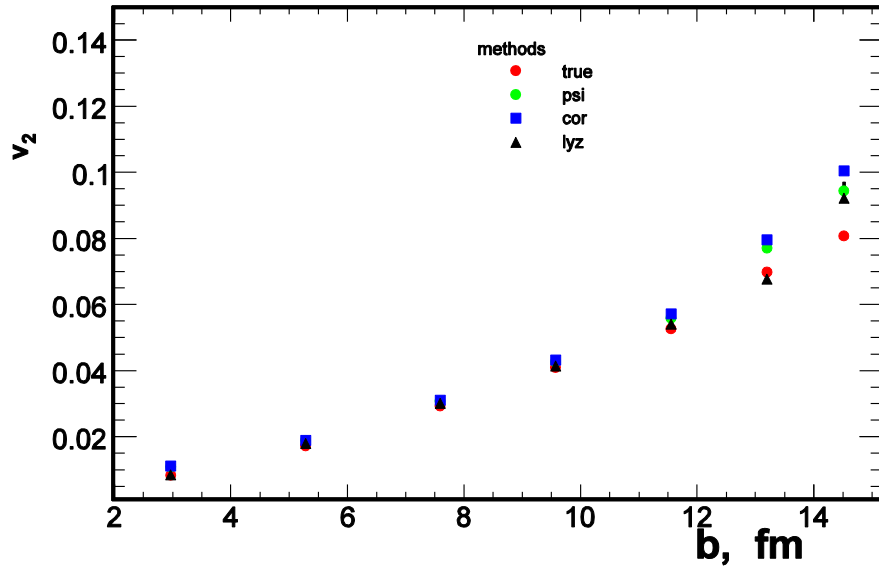
$$G(ir) = \langle e^{irQ} \rangle, Q = \sum \cos(2\varphi)$$

Integral  $v_2$  is connected with the first minimum  $r_0$  of the module of the  $G(ir)$ :

$$v_2 = \frac{j_0}{Nr_0}$$

Differential flow is calculated by the formula: 
$$\frac{v_2(p_T)}{Nv_2} = \text{Re} \left( \frac{\langle \cos(2\varphi) e^{ir_0 Q} \rangle}{\langle Q e^{ir_0 Q} \rangle} \right)$$

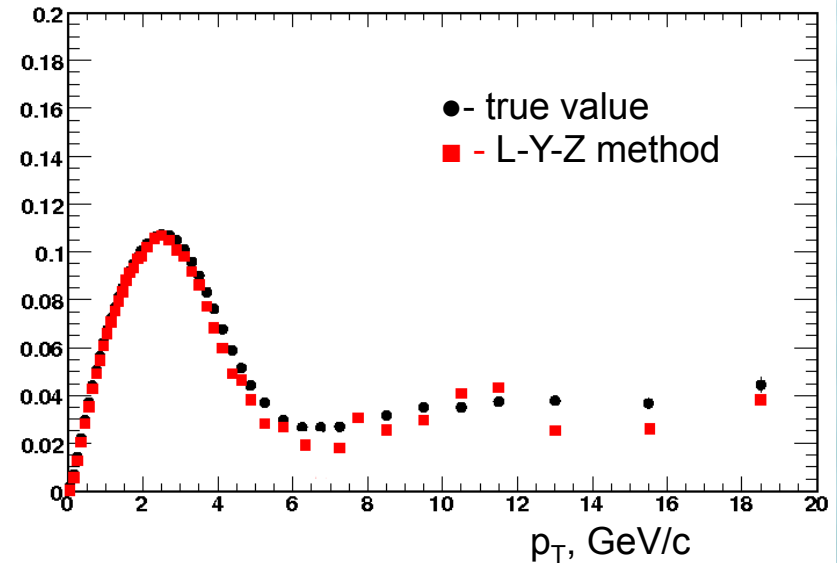
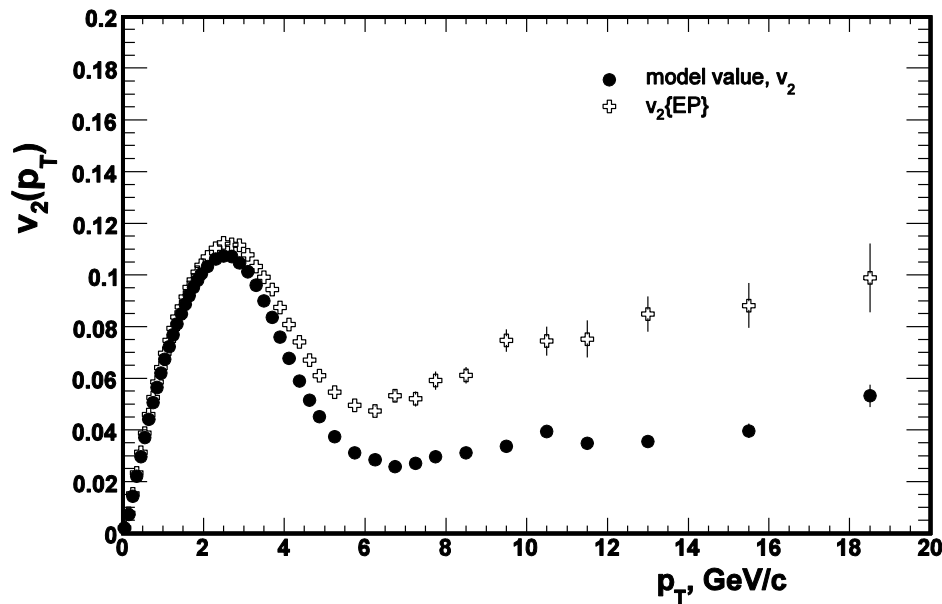
# RECONSTRUCTION OF INTEGRAL VALUE OF $V_2$ BY THE METHODS



The better reconstruction is achieved in midcentral collision for the methods, while Lee-Yang zero method tends to reconstruct true value at more central and more peripheral collision.

# Comparison of Event Plane and Lee-Yang zeroes methods ( $c=30\%$ )

## EventPlane method



## Lee-Yang zeroes Method

Event Plane method overestimates  $v_2$  at high  $p_t$  due to non-flow correlation (mostly because of jets).