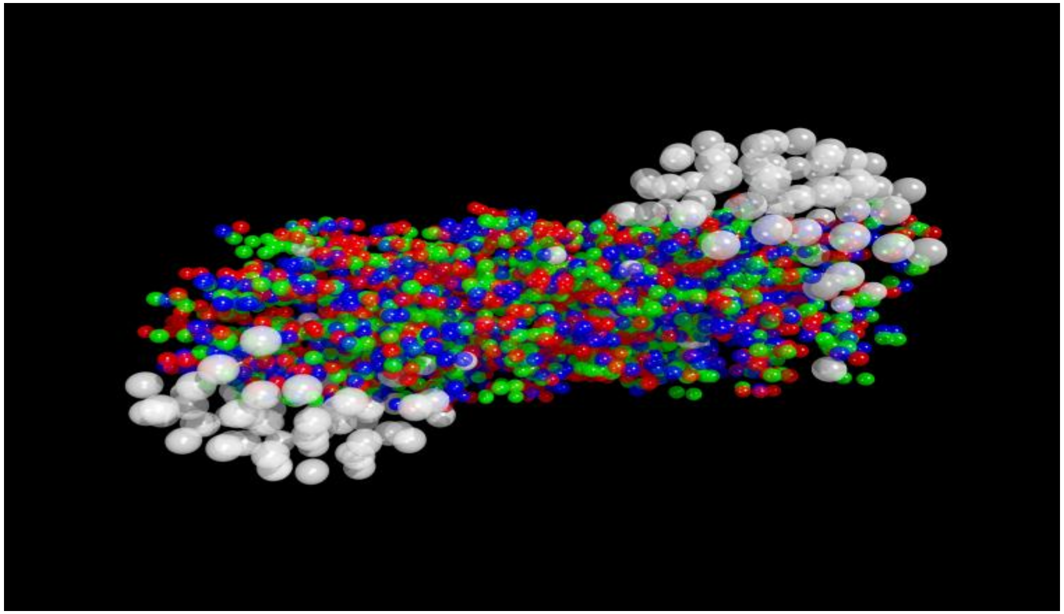
University of Catania
INFN-LNS

Heavy flavor in medium momentum evolution : Langevin vs Boltzmann



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In collaboration with: Vincenzo Greco
Francesco Scadina
Salvatore Plumari

NeD & TURIC-2014, Hersonissos,Crete, Greece

OUTLINE OF MY TALK..............

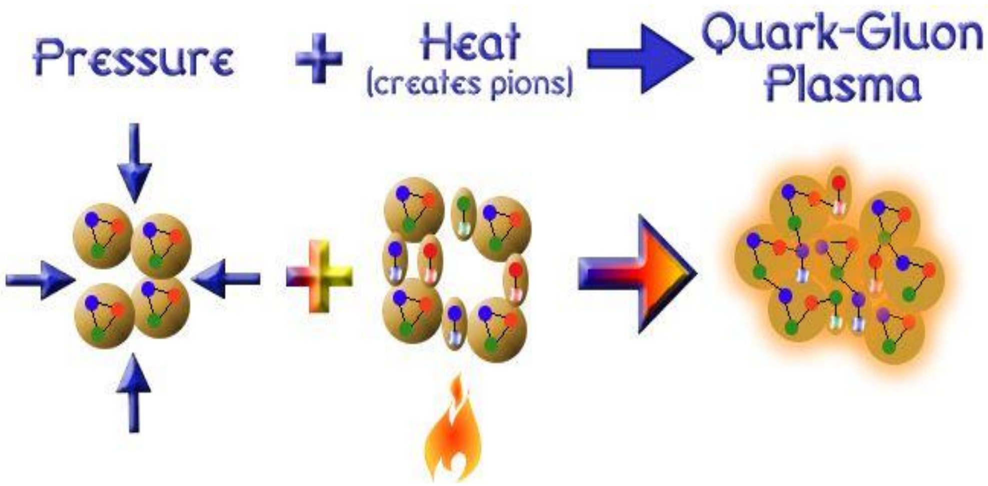
- Introduction
- Similarities and differences between the two approaches in a static medium (Langevin and Boltzmann)

  1) Spectra
  2) Momentum spreading
  2) Back to back azimuthal correlation

- Comparison with the experimental observables (RAA and v2)
- Summary and outlook

Introduction

At very high temperature and density hadrons melt to a new phase of matter called Quark Gluon Plasma (QGP).

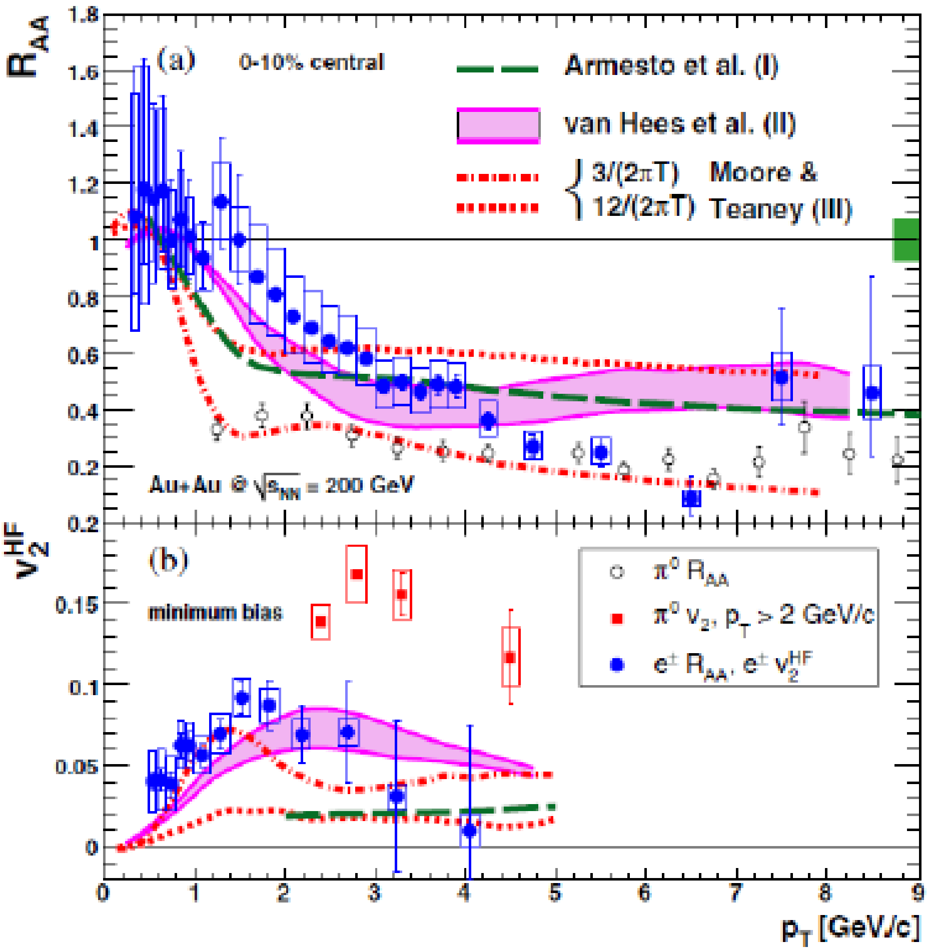


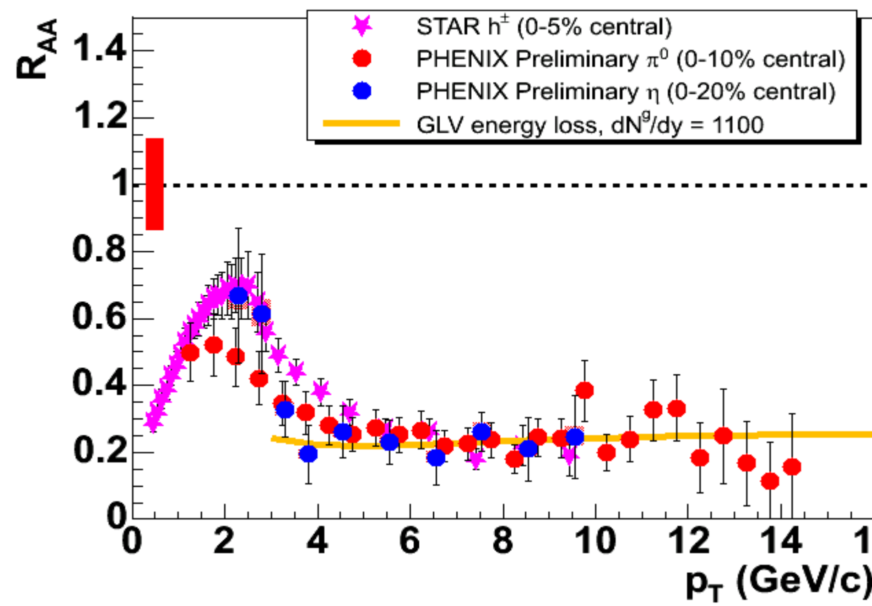
$$M_{c,b} >> \Lambda_{QCD}$$ **Produced by pQCD process (out of Equil.)**

$$\tau_{c,b} << \tau_{QGP}$$ **They go through all the QGP life time**

$$M_{c,b} >> T_0$$ **No thermal production**

Heavy flavor at RHIC

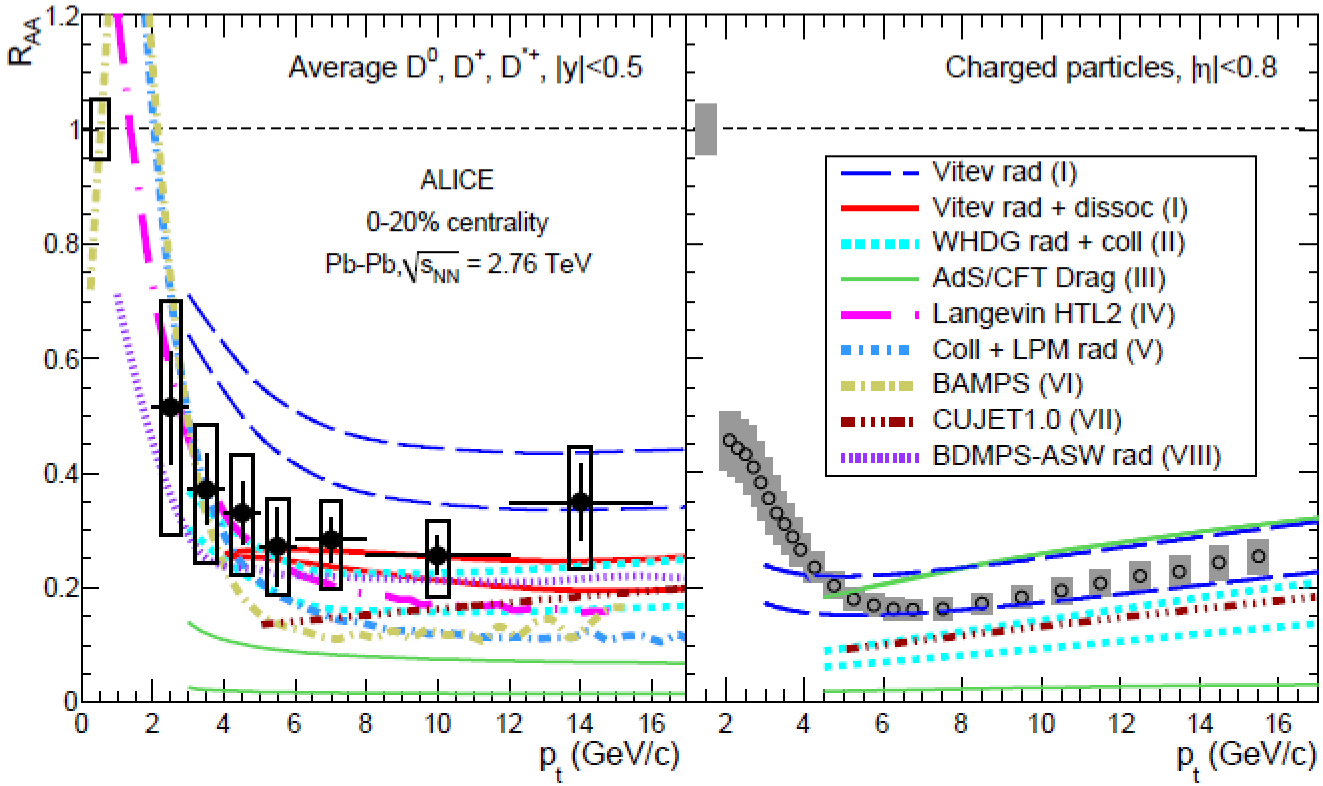




At RHIC energy heavy flavor suppression is similar to light flavor

Simultaneous description of RAA and v2 is a tough challenge for all the models.

Heavy Flavors at LHC



Again at LHC energy heavy flavor suppression is similar to light flavor

Is the momentum transfer really small !

Can one describe both RAA and v2 simultaneously?

Boltzmann Kinetic equation

$$\left(\frac{\partial}{\partial t}+\frac{P}{E}\frac{\partial}{\partial x}+\mathrm{F}.\frac{\partial}{\partial p}\right)f\left(x,p,t\right)=\left(\frac{\partial f}{\partial t}\right)_{col}$$

$$R(p,t)=\left(\frac{\partial f}{\partial t}\right)_{col}=\int d^3k\left[\omega(p+k,k)f(p+k)-\omega(p,k)f(p)\right]$$

$$\omega(p,k)=g\int\frac{d^3q}{(2\pi)^3}f'(q)v_{q,p}\sigma_{p,q\to p-k,q+k}$$ → is rate of collisions which change the momentum of the charmed quark from p to p-k

$$\omega(p+k,k)f(p+k)\approx\omega(p,k)f(p)+k.\frac{\partial}{\partial p}(\omega f)+\frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i\partial p_j}(\omega f)$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}}=\frac{\partial}{\partial \mathbf{p_i}}\left[\mathbf{A_i}(\mathbf{p})\mathbf{f}+\frac{\partial}{\partial \mathbf{p_j}}\left[\mathbf{B_{ij}}(\mathbf{p})\mathbf{f}\right]\right]$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

,

$$\mathbf{A_i}=\int \mathbf{d^3k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k_i}\quad\to \textbf{Drag Coefficient}$$

$$\mathbf{B_{ij}}=\int \mathbf{d^3k}\,\omega(\mathbf{p},\mathbf{k})\mathbf{k_i}\mathbf{k_j}\quad\to \textbf{Diffusion Coefficient}$$

$$\omega(p+k,k)f(p+k) \approx \omega(p,k)f(p) + k.\frac{\partial}{\partial p}(\omega f) + \frac{1}{2}k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}(\omega f)$$

Boltzmann Equation

Fokker Planck

It will be interesting to study both the equation in a identical environment to ensure the validity of this assumption at different momentum transfer and their subsequent effects on RAA and v2.

Langevin Equation

$$dx_j = \frac{p_j}{E} dt$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k$$

where Γ **is the deterministic friction (drag) force**

C_{ij} **is stochastic force in terms of independent**

Gaussian-normal distributed random variable

$$\rho = (\rho_x, \rho_y, \rho_z) \quad , \quad P(\rho) = \left(\frac{1}{2\pi}\right)^3 \exp(-\frac{\rho^2}{2})$$

With $< \rho_i(t)\rho_k(t') >= \delta(t-t')\delta_{jk}$

H. v. Hees and R. Rapp
arXiv:0903.1096

$\xi = 0$ **the pre-point Ito**

interpretation of the momentum argument of the covariance matrix.

Langevin process defined like this is equivalent to the Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + \frac{p_j}{E}\frac{\partial f}{\partial x_j} = \frac{\partial}{\partial p_j}\left[\left(p_j\Gamma - \xi C_{lk}\frac{\partial C_{jk}}{\partial p_l}\right) f\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}(C_{jl}C_{kl}f)$$

the covariance matrix is related to the diffusion matrix by

$$C_{jk} = \sqrt{2B_0}P^{\perp}_{jk} + \sqrt{2B_1}P^{\parallel}_{jk}$$

and $$A_i = p_j\Gamma - \xi C_{lk}\frac{\partial C_{ij}}{\partial p_l}$$

With $B_0 = B_1 = D$ $$C_{jk} = \sqrt{2D(E)}\delta_{jk}$$

Relativistic dissipation-fluctuation relation

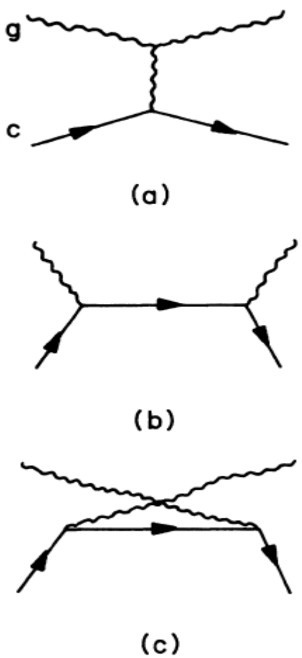
$$A(E)ET - D(E) + T(1-\xi)D'(E) = 0$$

For Collision Process the A_i and B_{ij} can be calculated as following :

$$A_i = \frac{1}{2E_p}\int\frac{d^3q}{(2\pi)^3 2E_q}\int\frac{d^3q'}{(2\pi)^3 2E_{q'}}\int\frac{d^3p'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_c}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')f(q)[(p-p')_i] = \langle\langle(p-p')_i\rangle\rangle$$

$$B_{ij} = \frac{1}{2}\langle\langle(p-p')_i(p'-p)_j\rangle\rangle$$

Elastic processes



$$gc \rightarrow gc$$

✓ We have introduce a mass into the internal gluon propagator in the t and u-channel-exchange diagrams, to shield the infrared divergence.

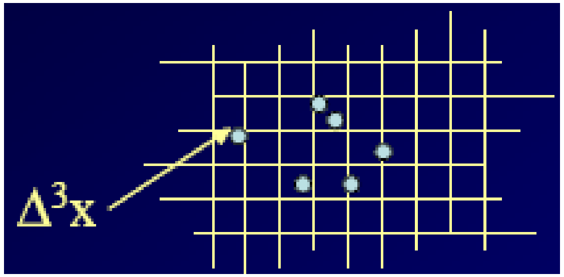
B. Svetitsky PRD 37(1987)2484

Transport theory

$$p^\mu \partial_\mu f(x,p) = C_{22}$$

We consider two body collisions

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2)$$
$$-\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2)$$



$$\Delta t \to 0$$
$$\Delta^3 x \to 0$$

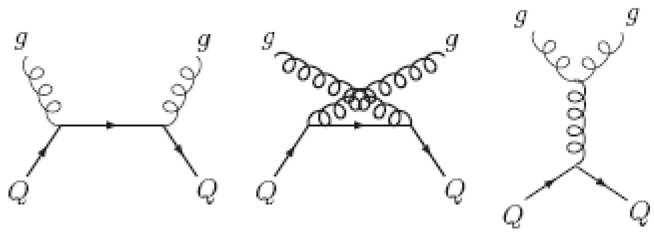
Exact solution

Collision integral is solved with a local stochastic sampling

[Z. Xhu, et al. PRC71(04)]
Greco et al PLB670, 325 (08)]

$$P_{22} = \frac{\Delta N_{\text{coll}}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{\text{rel}}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Cross Section gc -> gc

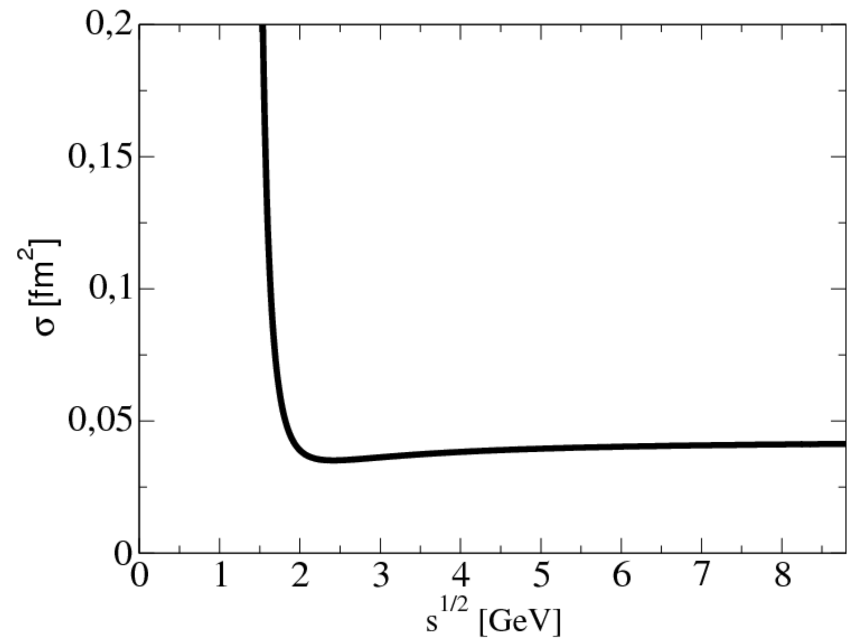


The infrared singularity is regularized introducing a Debye-screaning-mass μ_D

$$\frac{1}{t} \rightarrow \frac{1}{t - m_D^2} \qquad m_D = \sqrt{4\pi\alpha_s}\,T$$

$$\sum|\mathcal{M}|^2 = \pi^2\alpha^2(Q^2)\left[\frac{32(s-M^2)(M^2-u)}{t^2} + \frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2}\right.$$
$$+\frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(M^2+u)}{(M^2-u)^2} + \frac{16}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)}$$
$$\left.+16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)} - 16\frac{(s-M^2)(M^2-u)-M^2(s-u)}{t(M^2-u)}\right]$$

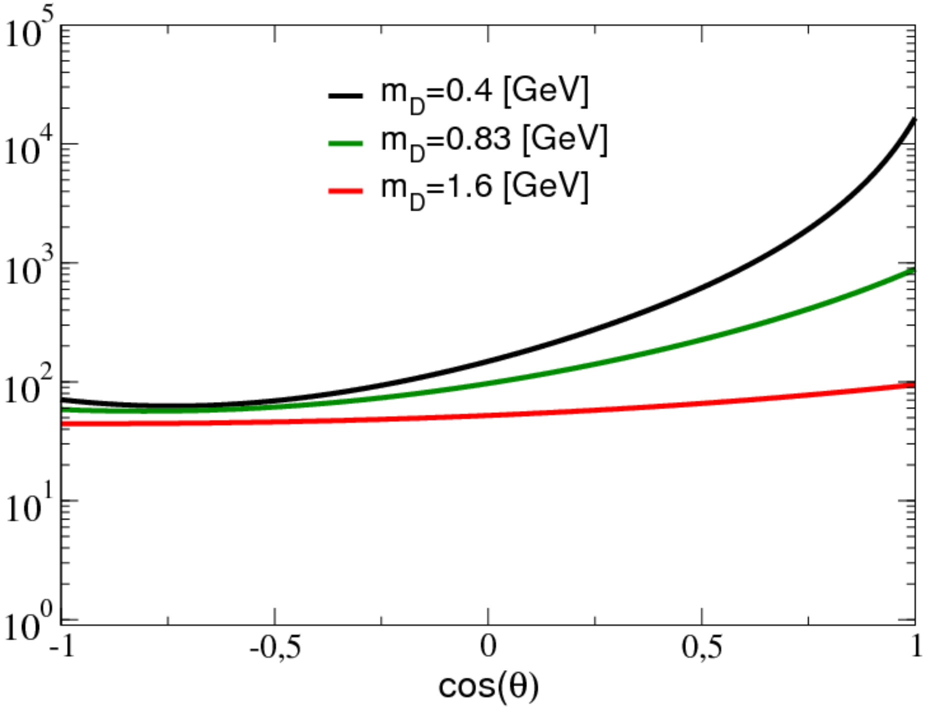
$$\hat{\sigma} = \frac{1}{16\pi(s-M^2)^2}\int_{-(s-M^2)^2/s}^{0} dt \sum|\mathcal{M}|^2 \longrightarrow$$



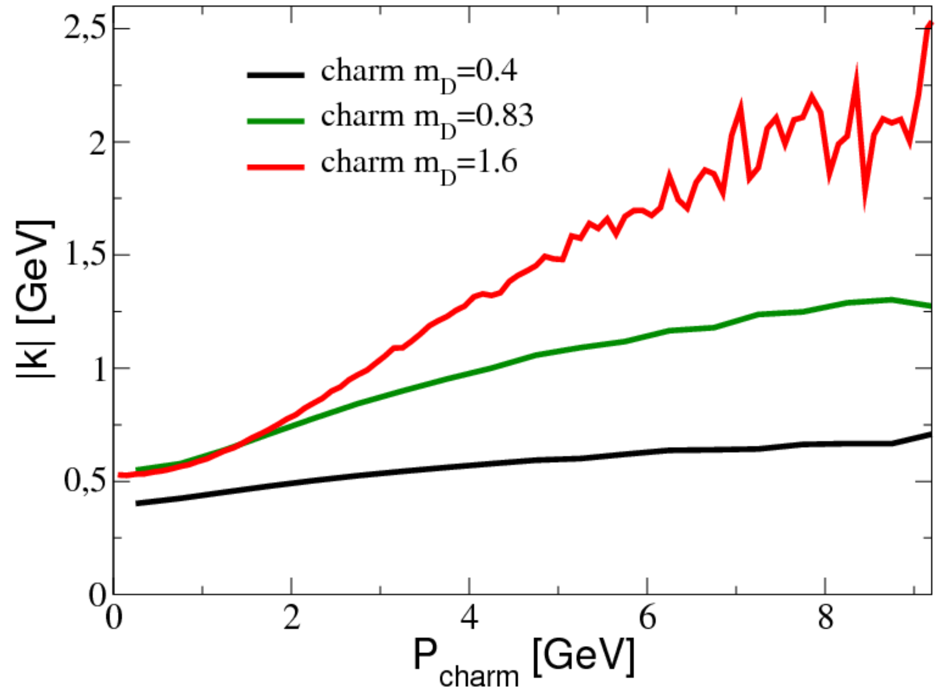
L. Combridge, Nucl. Phys. B151, 429 (1979)]
[B. Svetitsky, Phys. Rev. D 37, 2484 (1988)]

Boltzmann vs Langevin (Charm)

Angular dependence of σ



Mometum transfer vs P

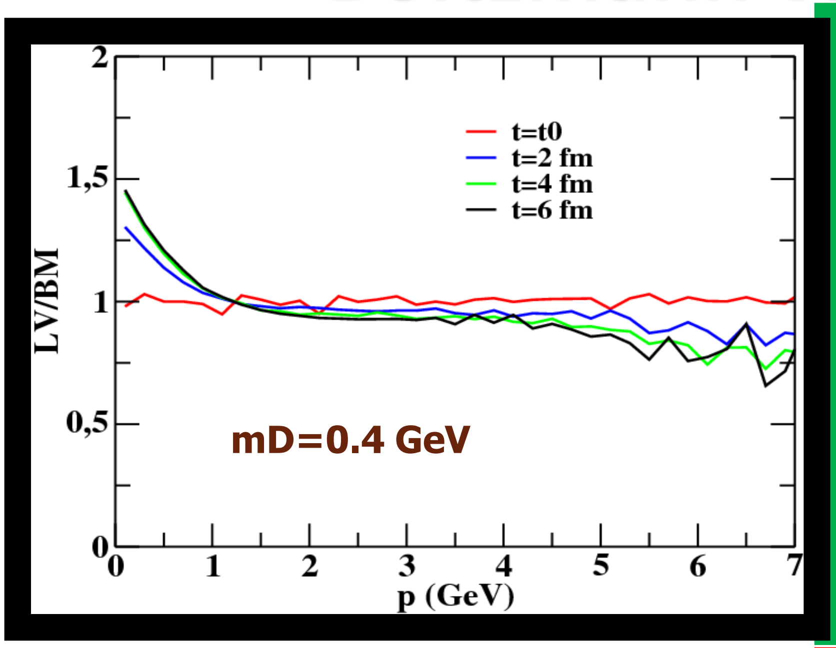


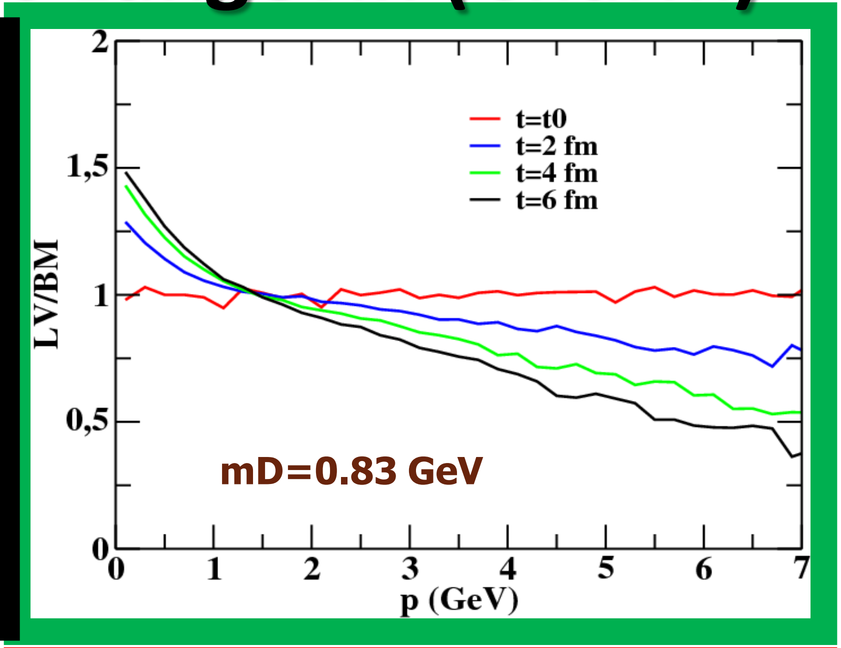
Decreasing m_D makes the σ more anisotropic ➡ **Smaller average momentum transfer**

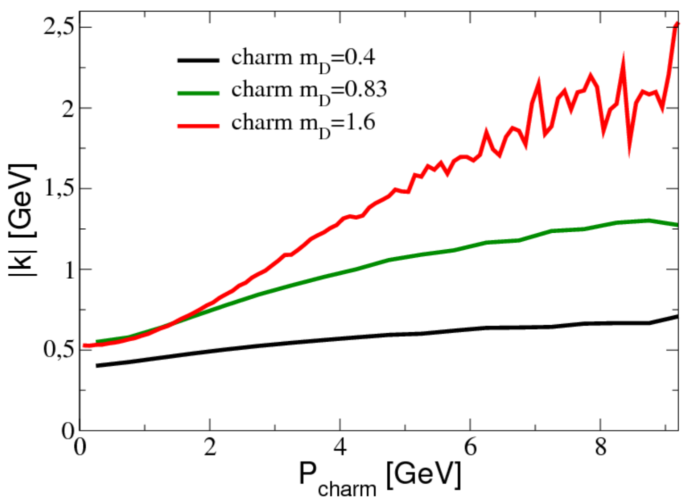
Hees, Mannarelli,Greco,Rapp, PRL100(2008)
Hees,Greco,Rapp. PRC73 (2006) 034913

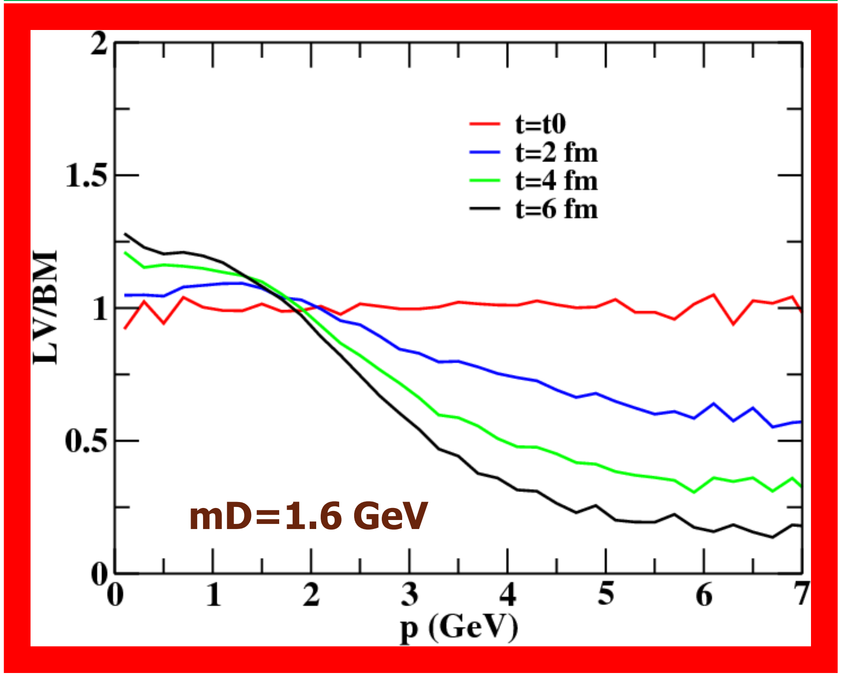
Talk by Hamza Berrehrah
NeD & TURIC-2014

Boltzmann vs Langevin (Charm)

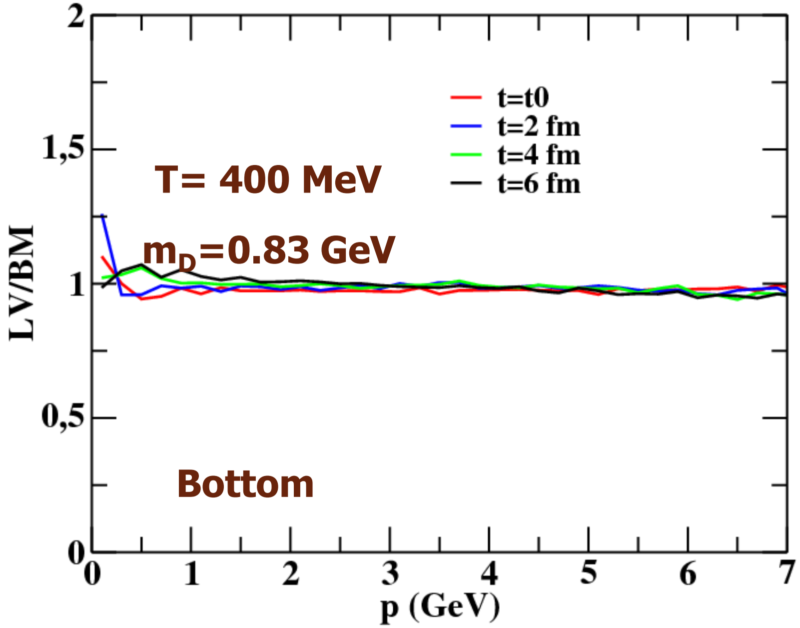


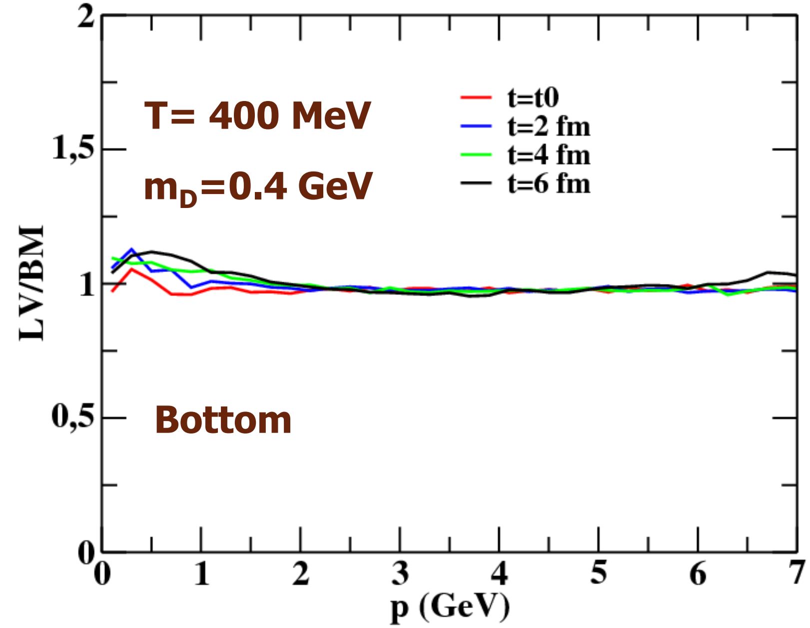


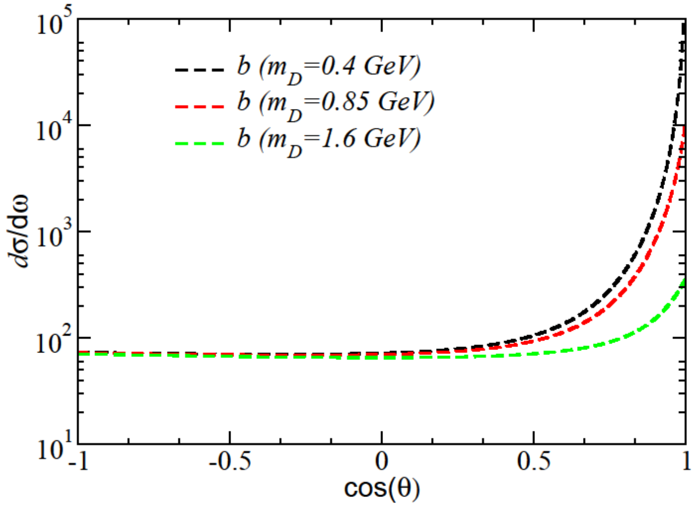




Bottom: Boltzmann = Langevin

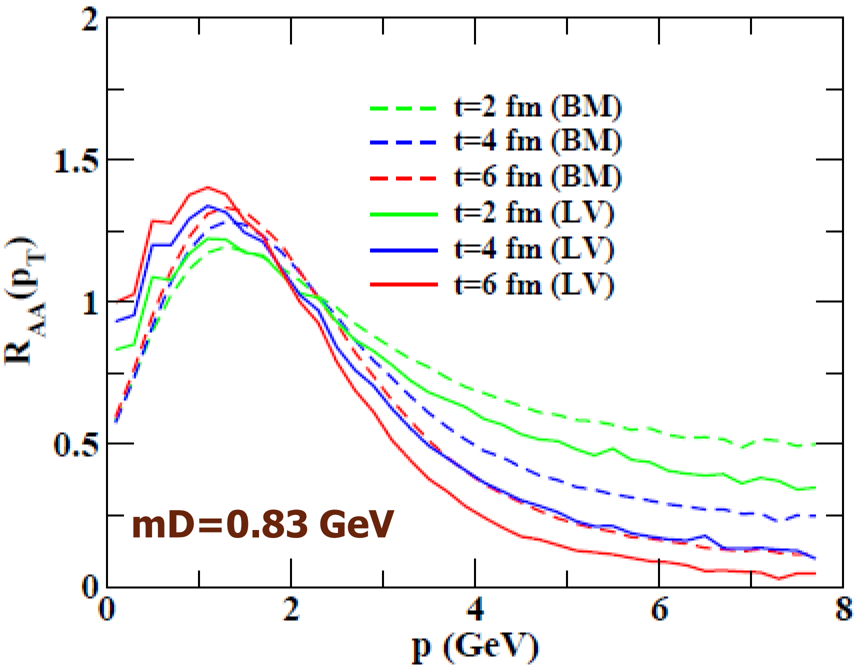


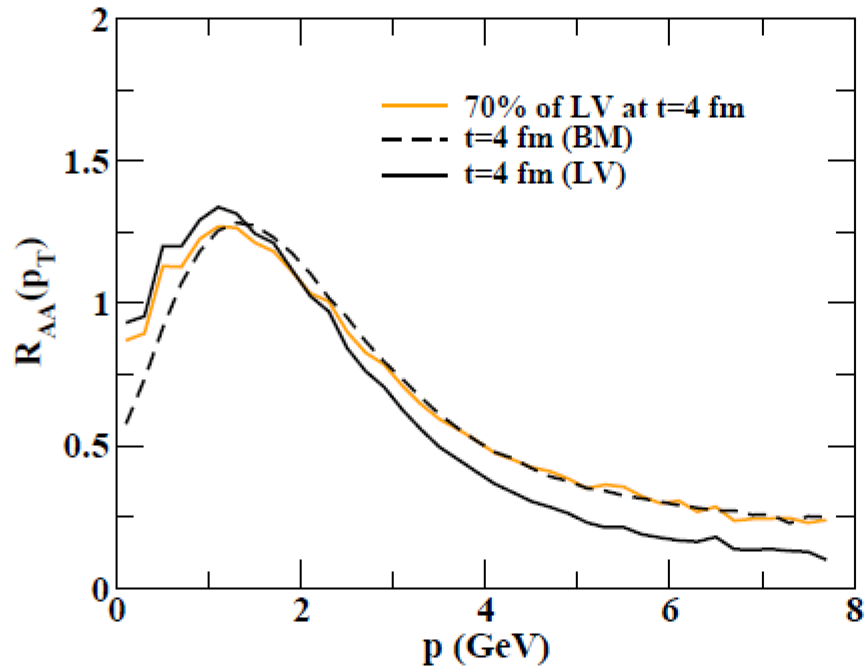




But Larger M_b/T (≈ 10) the better Langevin approximation works

Implication for observable, R_{AA}?





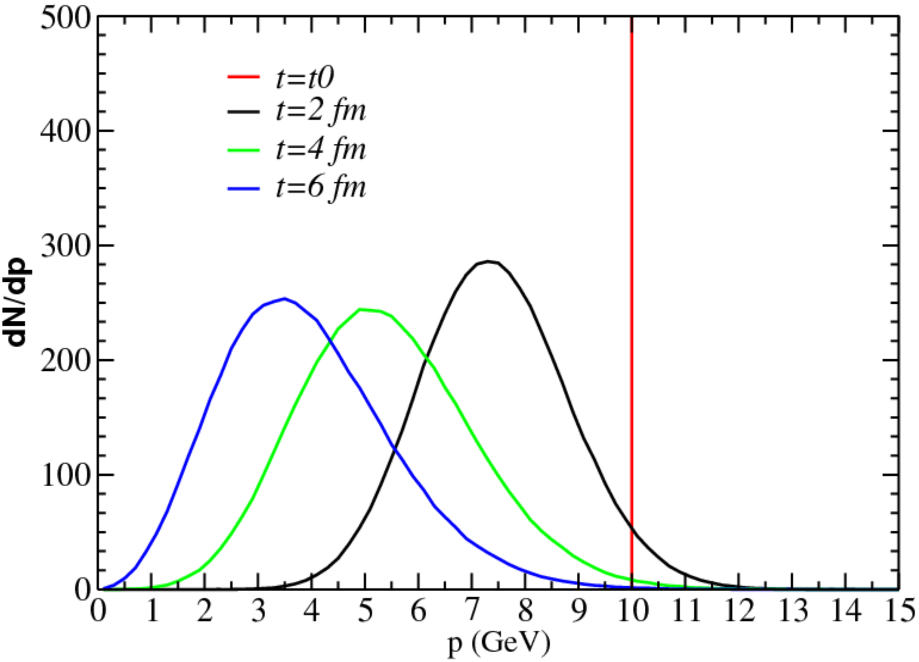
The Langevin approach indicates a smaller R_{AA} thus a larger suppression.

However one can mock the differences of the microscopic evolution and reproduce the same R_{AA} of Boltzmann equation just changing the diffusion coefficient by about a 30 %

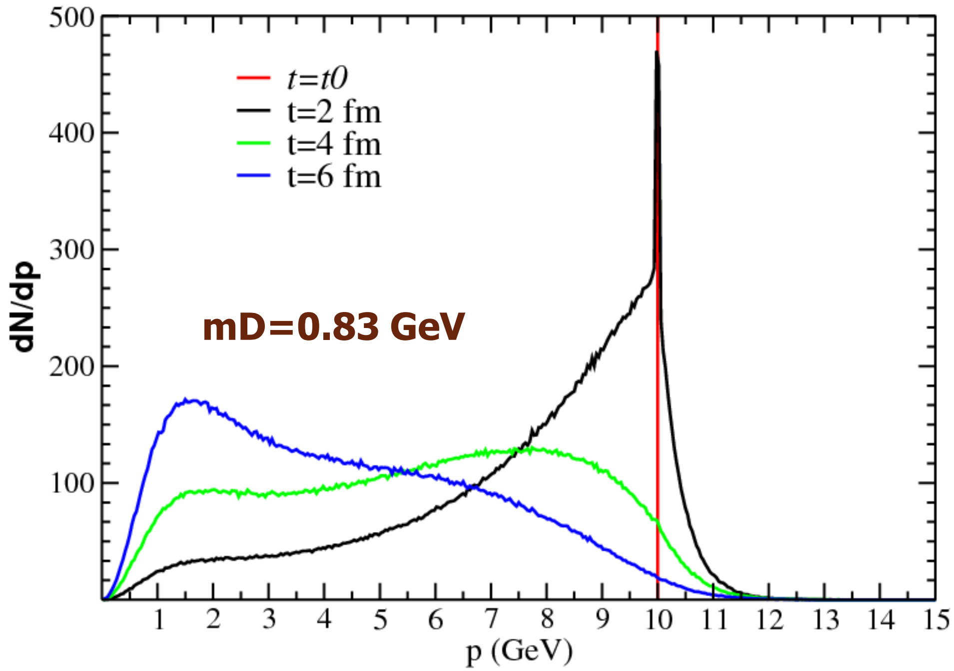
Momentum evolution starting from a δ (Charm) in a Box

$$\left.\frac{dN}{d^3p}\right|_{initial} = \delta(p - 10GeV)$$

Langevin



Boltzmann



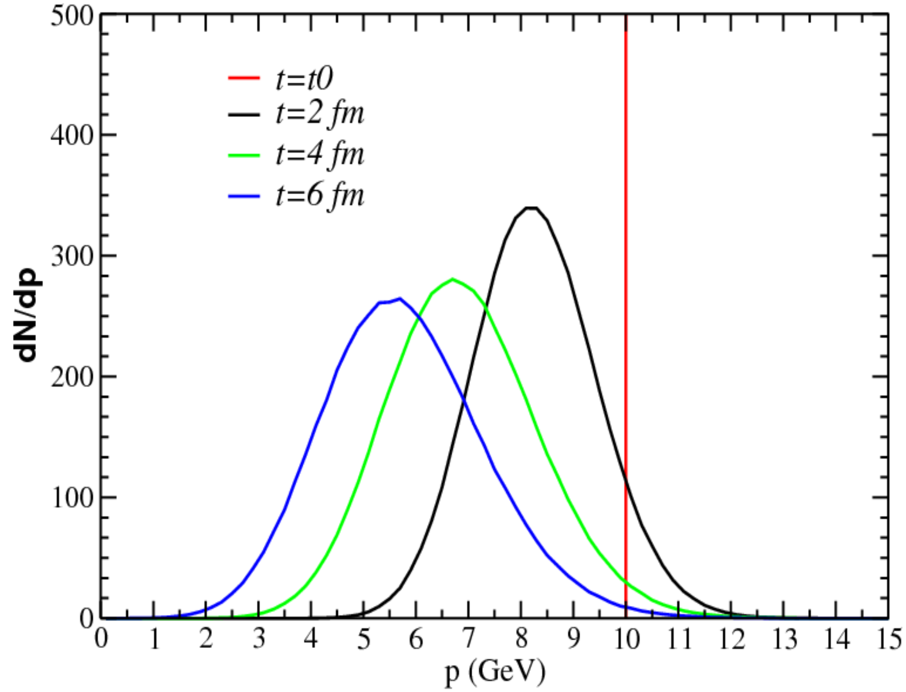
- Clearly appears the shift of the average momentum with t due to the drag force
- The gaussian nature of diffusion force reflect itself in the gaussian form of p-distribution
- Boltzmann approach can throw partic at low p instead Langevin can not
- A part of dynamic evolution involving large momentum transfer is discarded with Langevin approach

Momentum evolution starting from a δ (Bottom)

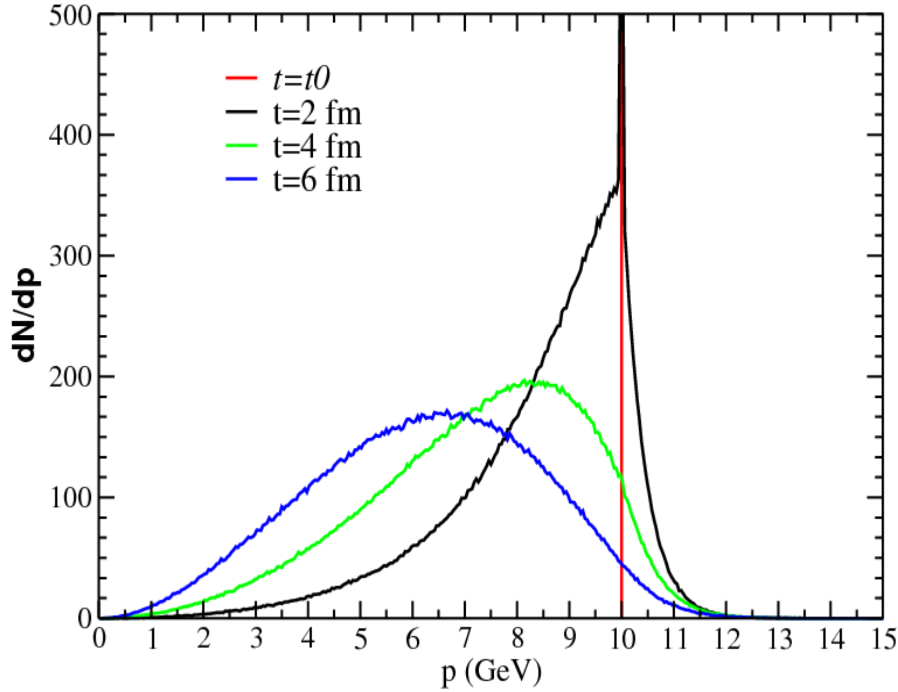
In a Box

$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$

Langevin

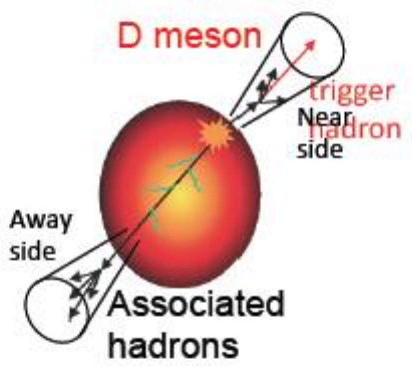


Boltzmann

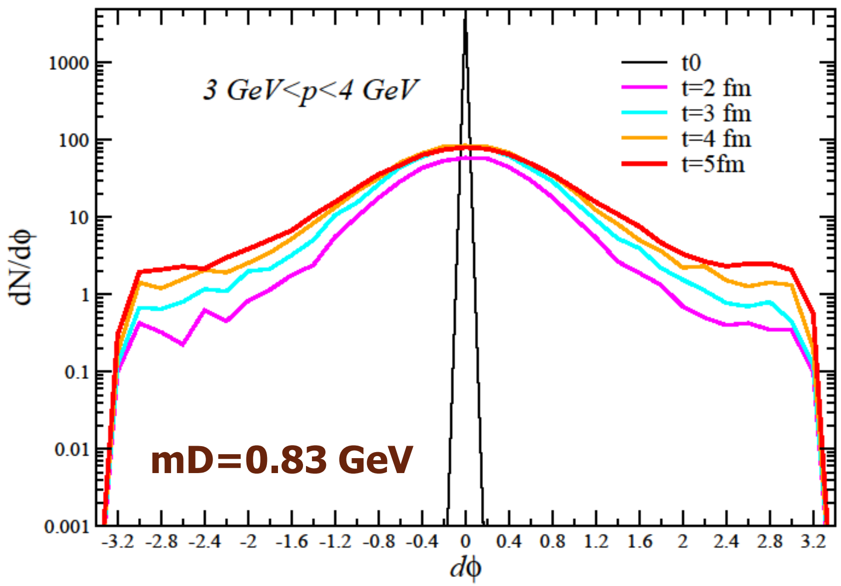


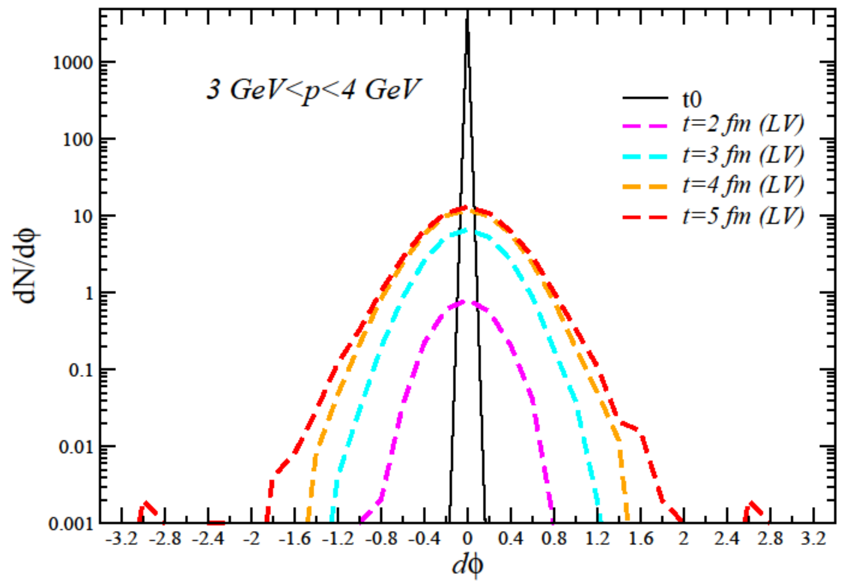
T=400 MeV Mc/T≈3 Mb/T≈10

Back to Back correlation in a Box



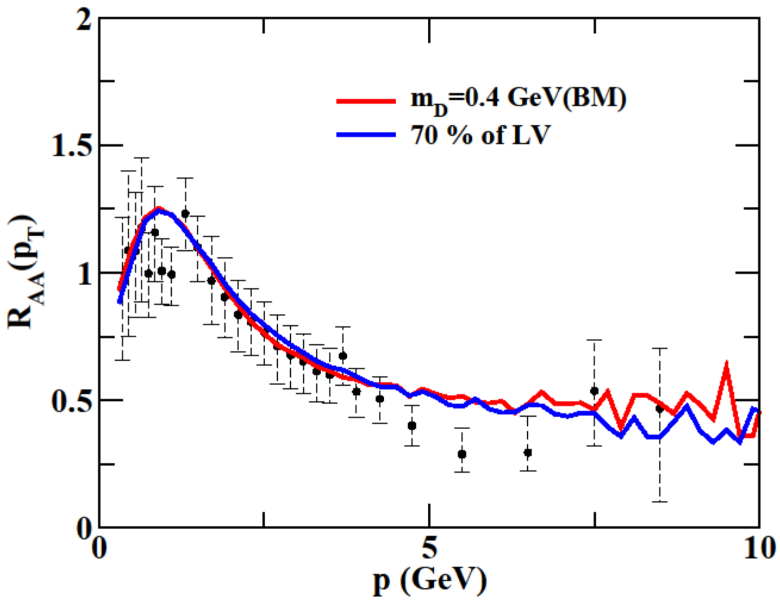
Initialization: px=pz=0, py=10 GeV
x=z=0, y=-2.5 fm

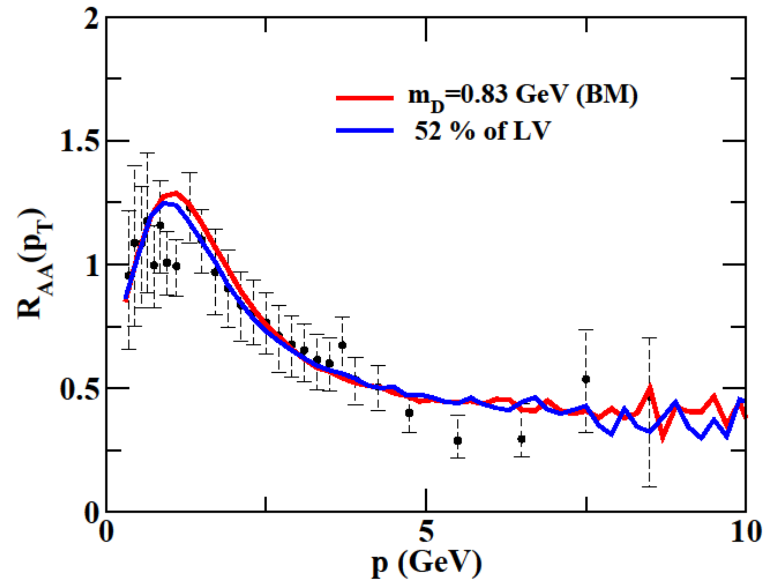


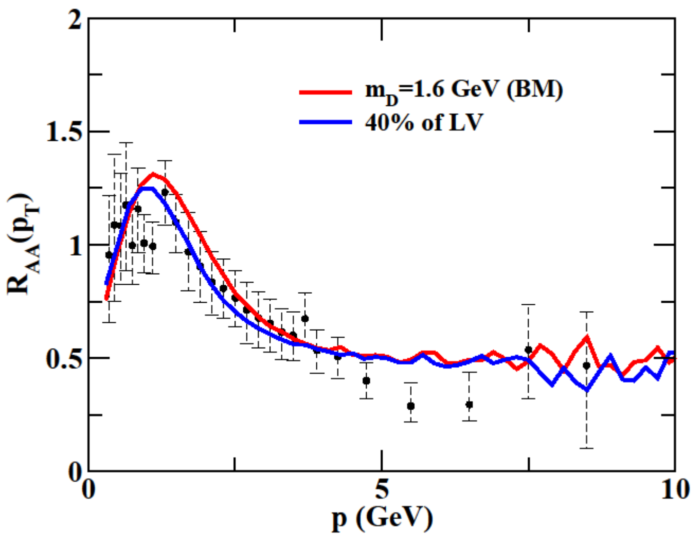


The larger spread of momentum with the Boltzmann implicates a large spread in the angular distributions of the Away side charm

R_{AA} at RHIC for different <k>





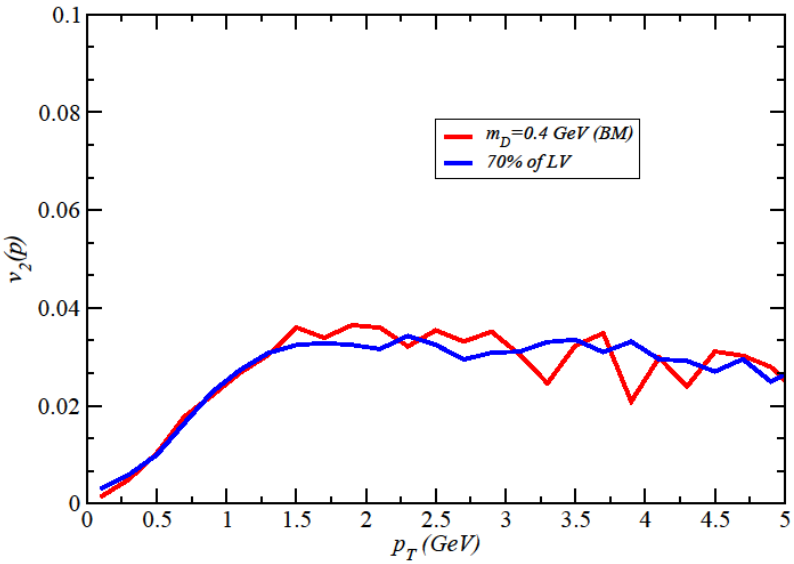


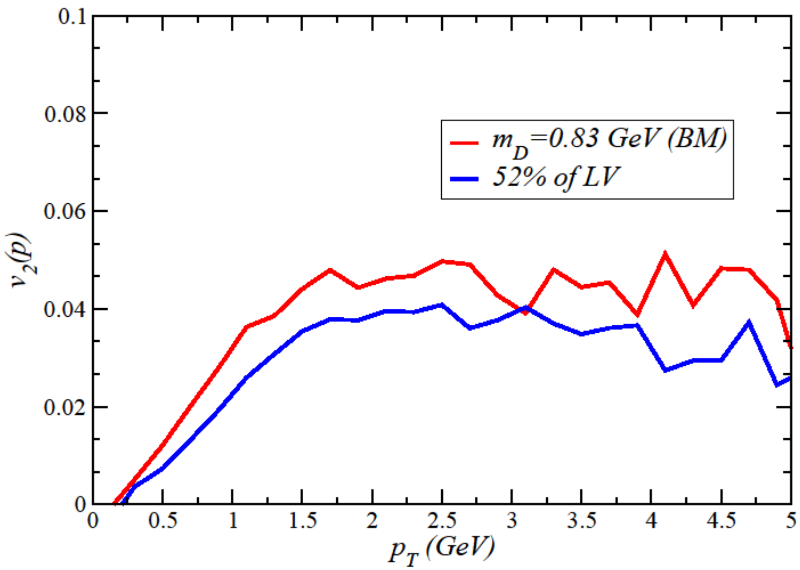
The Langevin approach indicates a smaller R_{AA} thus a larger suppression.

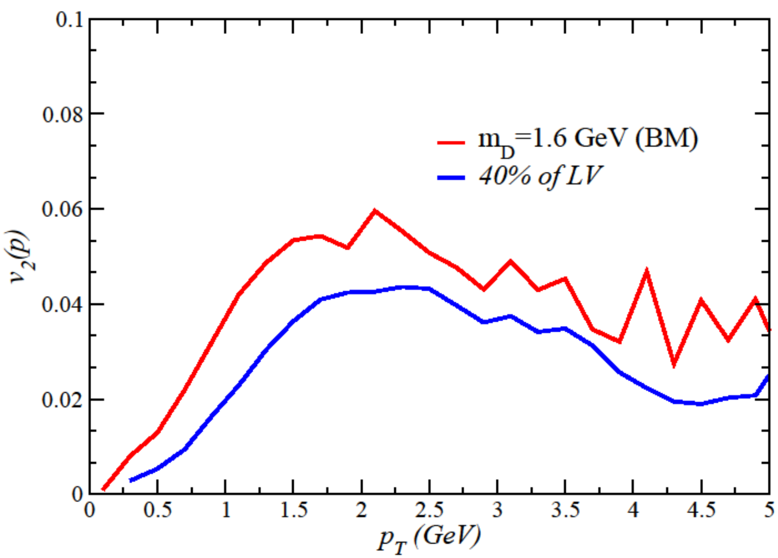
One can get very similar R_{AA} for both the approaches just reducing the diffusion coefficent

The smaller averege transfered momentum the better Langevin works

v_2 at RHIC centrality 20-30 %



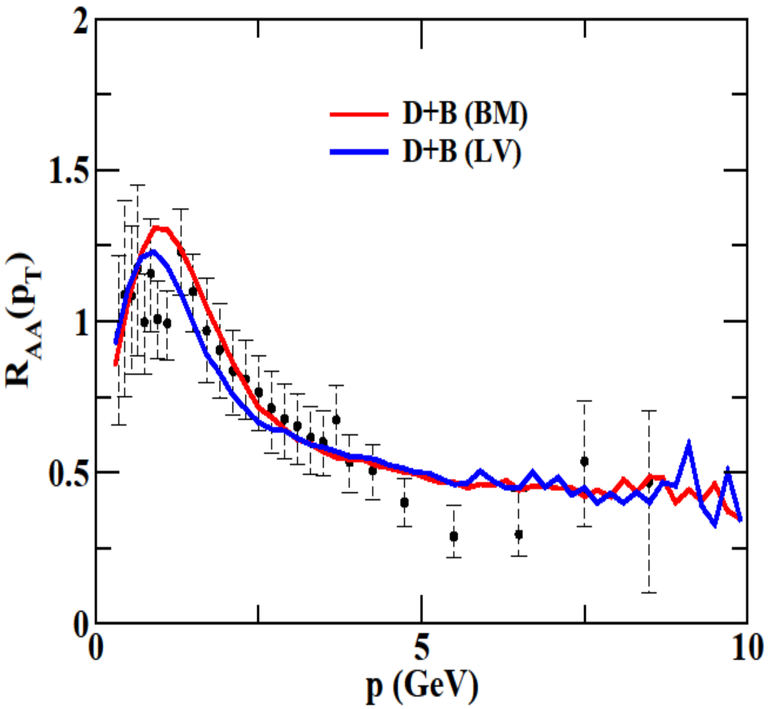


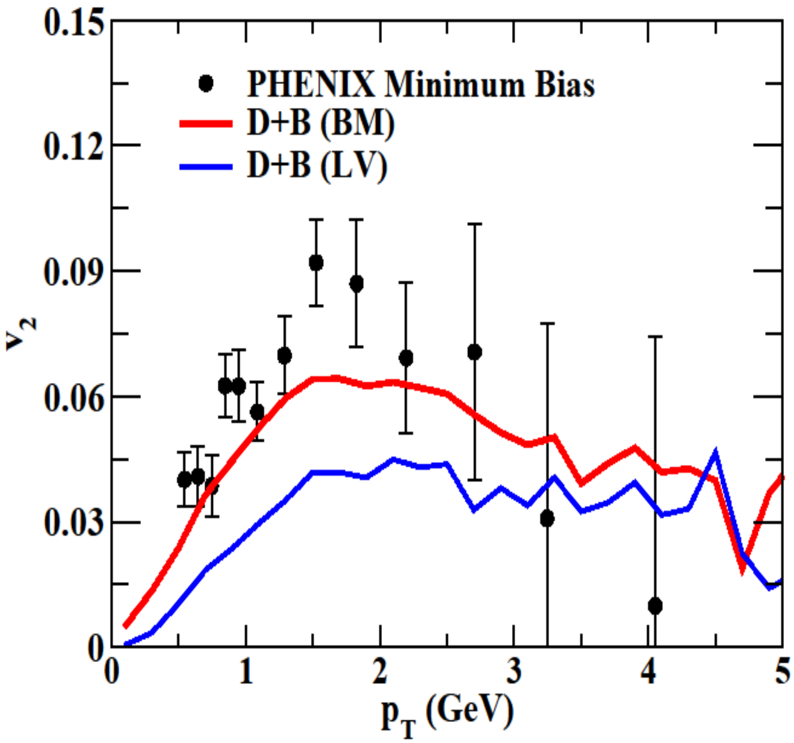


Also for v_2 the smaller averege transfered momentum the better Langevin works

Boltzmann is more efficient in producing v_2 for fixed R_{AA}

R_{AA} and v2 at RHIC at mD=1.6 GeV

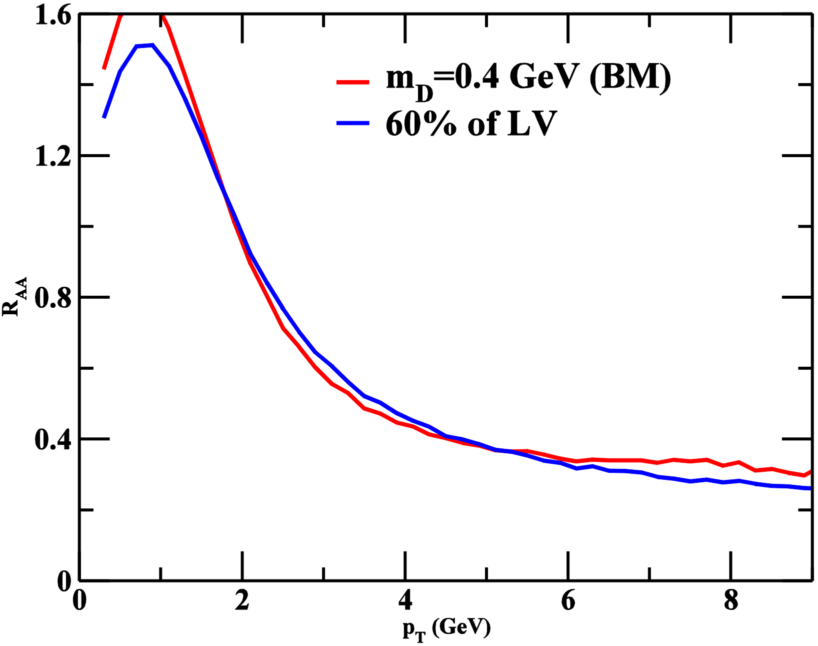


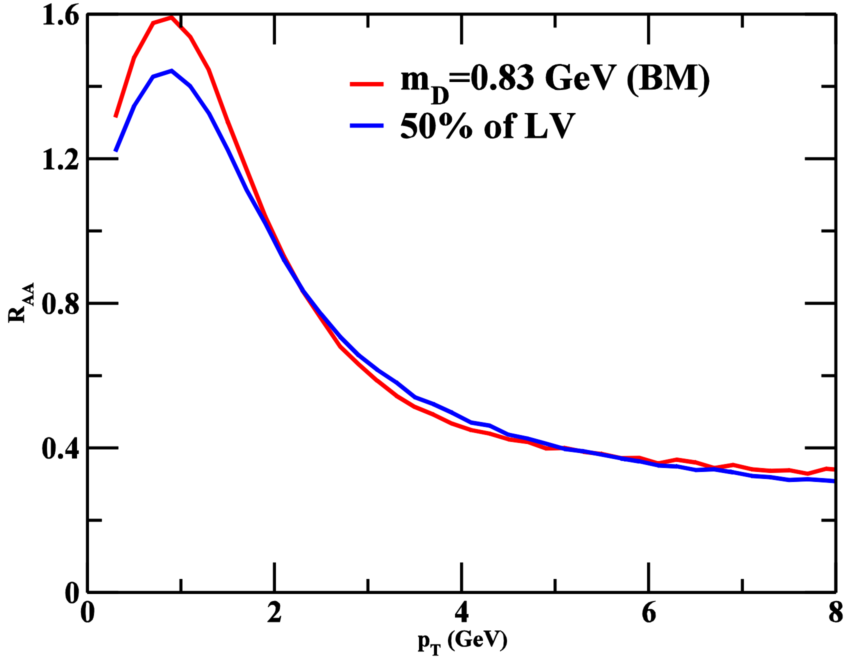


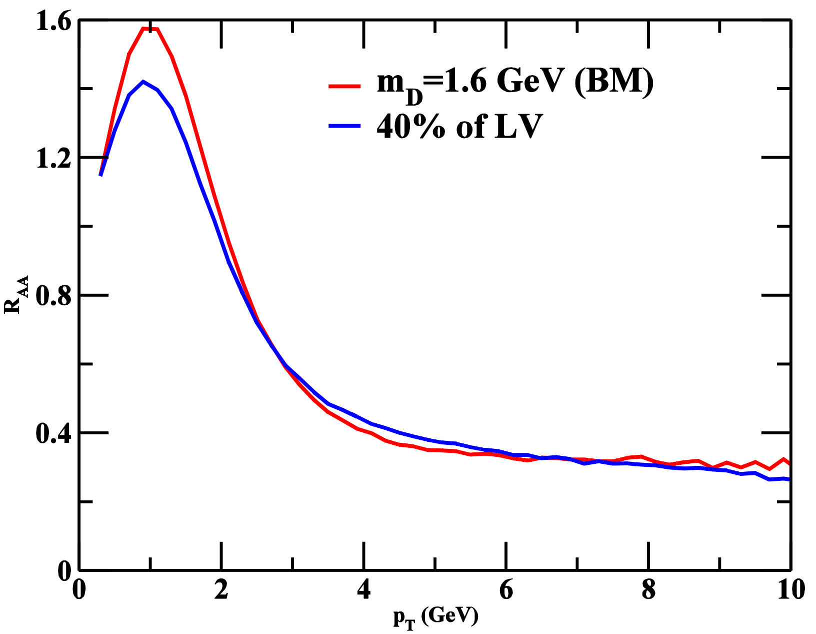
Our results can be further improved by implementing Coalescence + Fragmentation for hadronisation.

With isotropic cross section one may describe both RAA and V2 simultaneously within the Boltzmann approach at RHIC!

R_{AA} @LHC centrality 30-50%

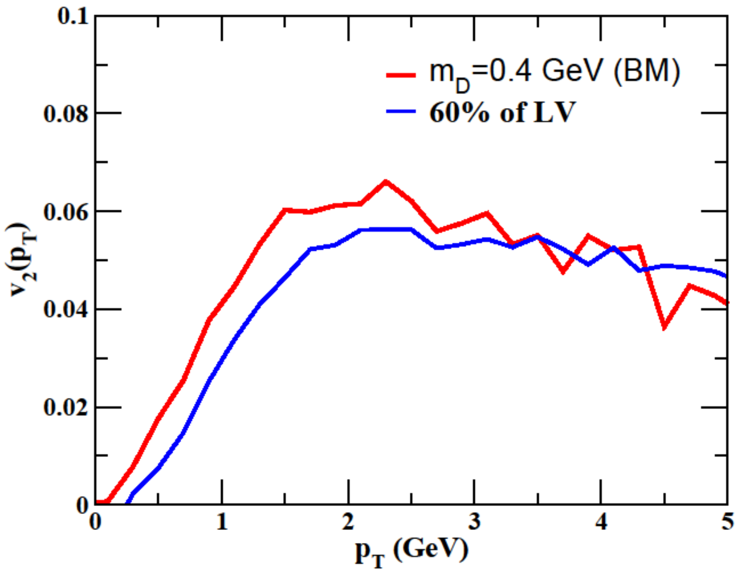


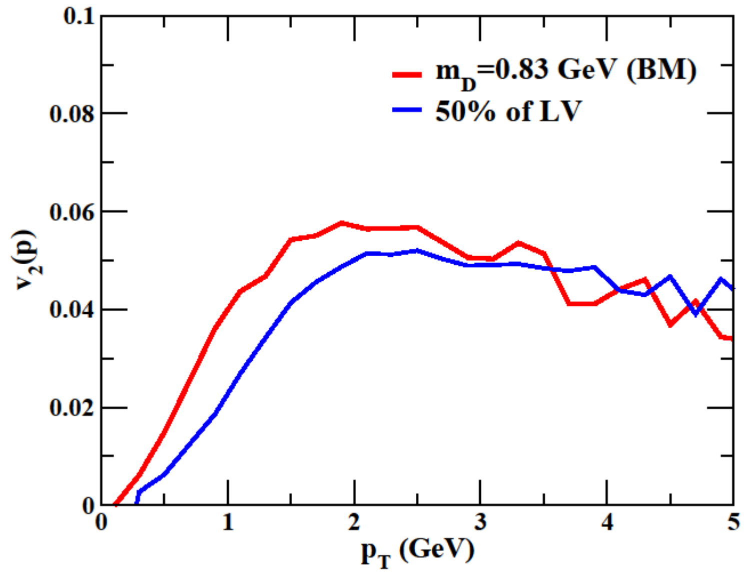


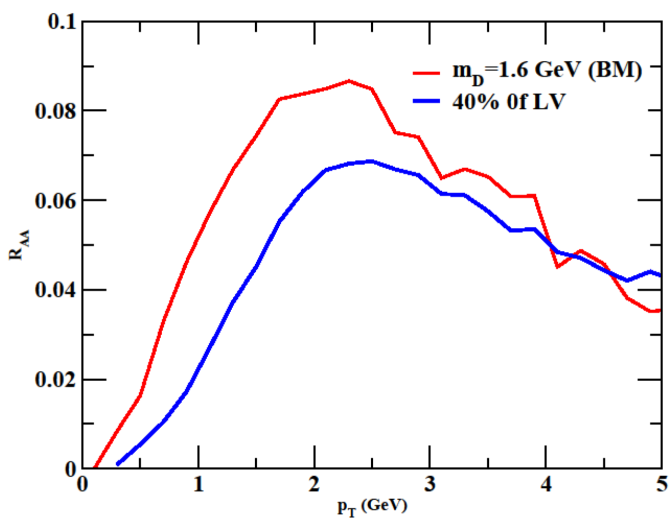


One can get very similar R_{AA} for both the approaches just reducing the diffusion coefficent

V_2 @ LHC centrality 30-50%







Also for v_2 the smaller averege transfered momentum the better Langevin works

Boltzmann is more efficient in producing v_2 for fixed R_{AA}

Summary & Outlook ……

- Both Langevin and Boltzmann equation has been solved in a box for heavy quark propagating in a thermal bath composed of gluon at T= 400 MeV as well as for a expanding medium at RHIC and LHC energies.

- We found charm quarks does not follow the Brownian motion at RHIC and LHC energies.

- Langevin dynamics overestimate the suppression than the Boltzmann approach

- For a given RAA Boltzmann approach develop larger v2.

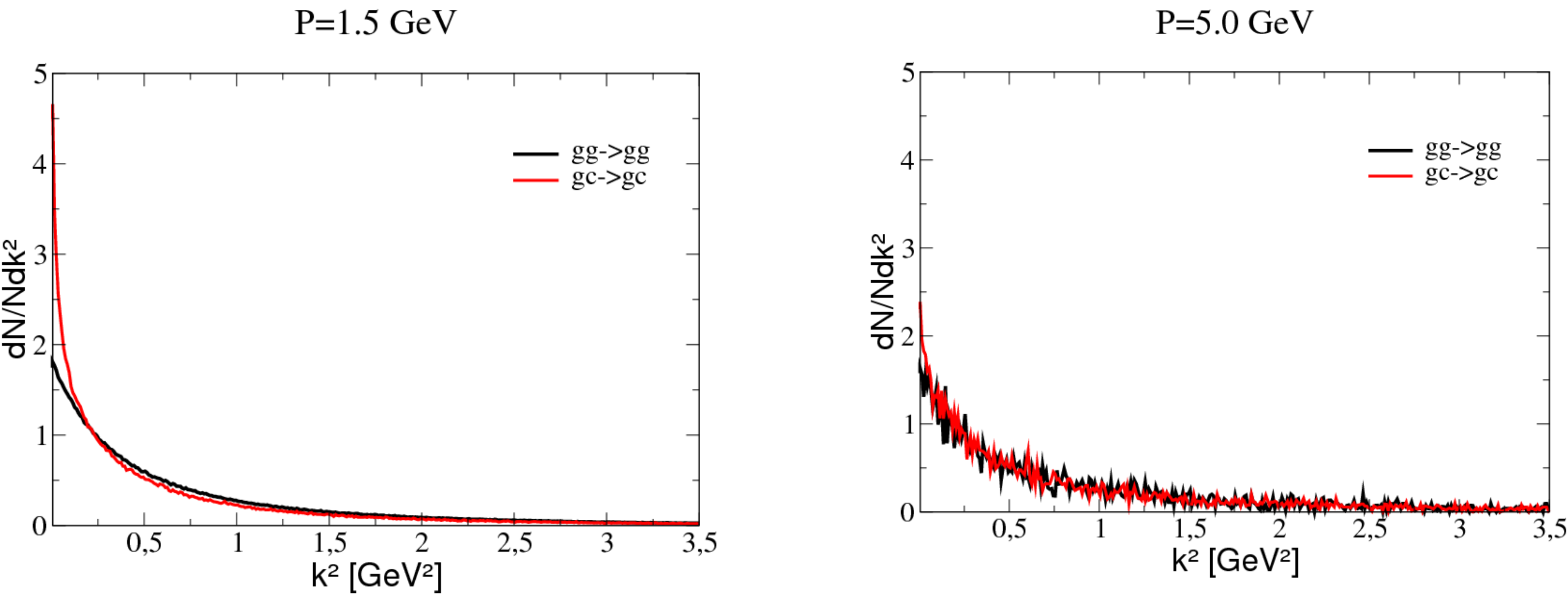
- With isotropic cross-section one can reproduce RAA and v2 simultaneously within the Boltzmann approached at RHIC energy.

- Implementation of T-matrix and radiative process is under progress.

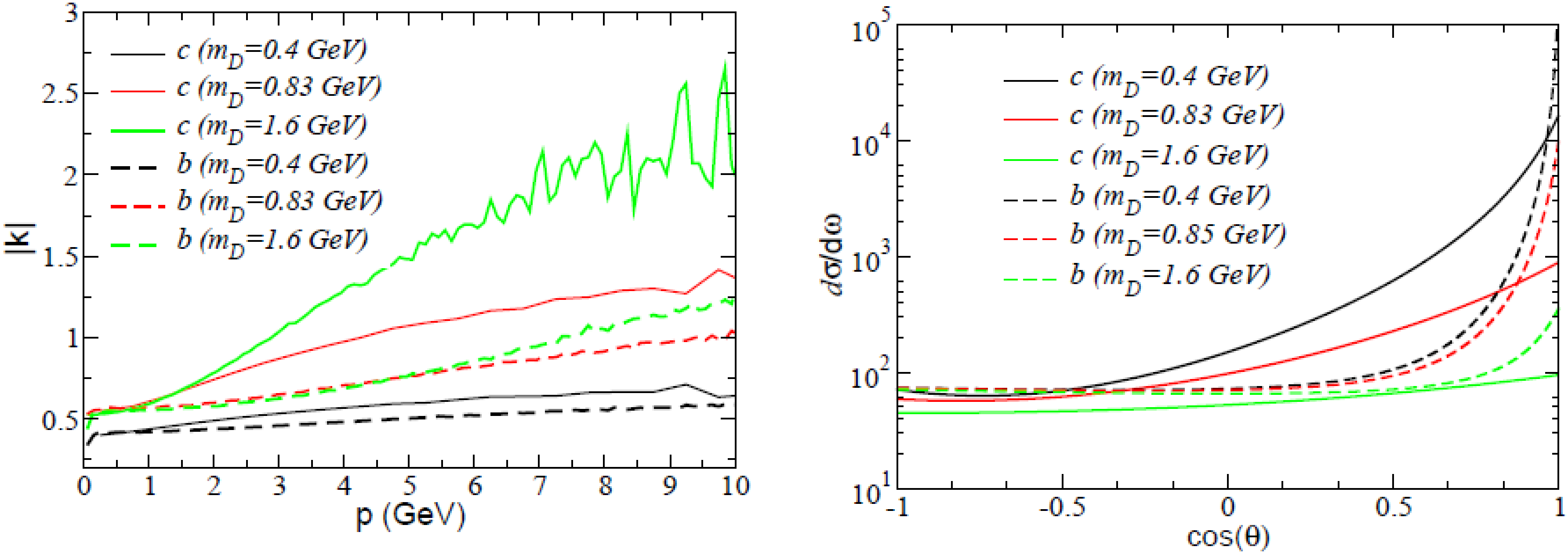

Thank You . . . .

Momentum transfer

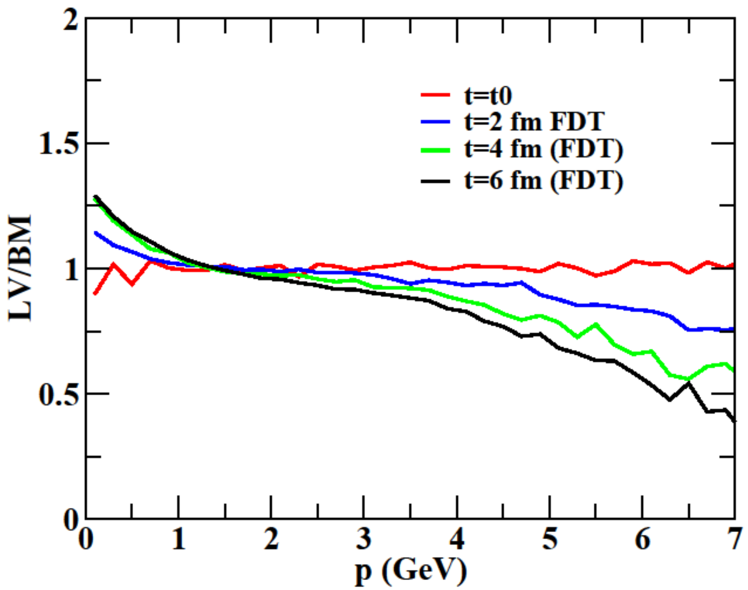
Distribution of the squared momenta transfer k^2 for fixed momentum P of the charm



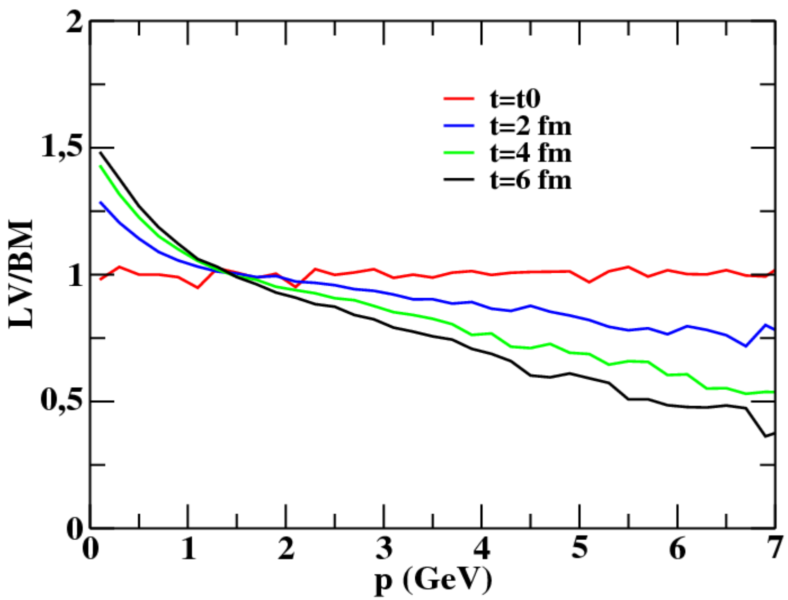
The momenta transfer of gg->gg and gc-> gc are not so different



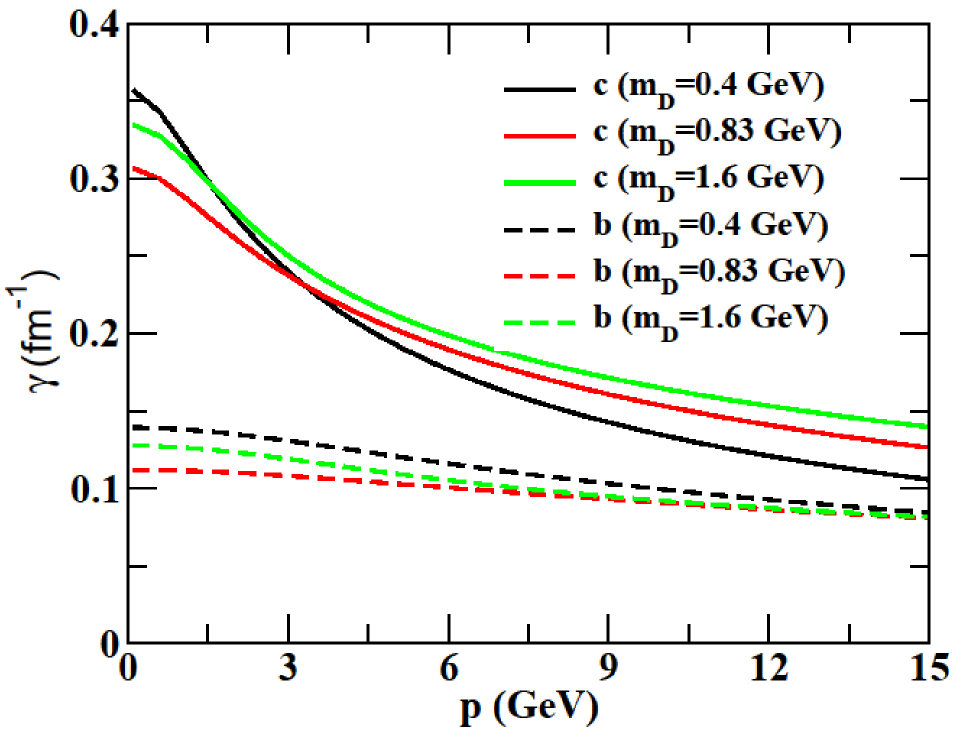
c (m_D=0.4 GeV)
c (m_D=0.83 GeV)
c (m_D=1.6 GeV)
b (m_D=0.4 GeV)
b (m_D=0.83 GeV)
b (m_D=1.6 GeV)
|k|
p (GeV)
c (m_D=0.4 GeV)
c (m_D=0.83 GeV)
c (m_D=1.6 GeV)
b (m_D=0.4 GeV)
b (m_D=0.85 GeV)
b (m_D=1.6 GeV)
dσ/dω
cos(θ)



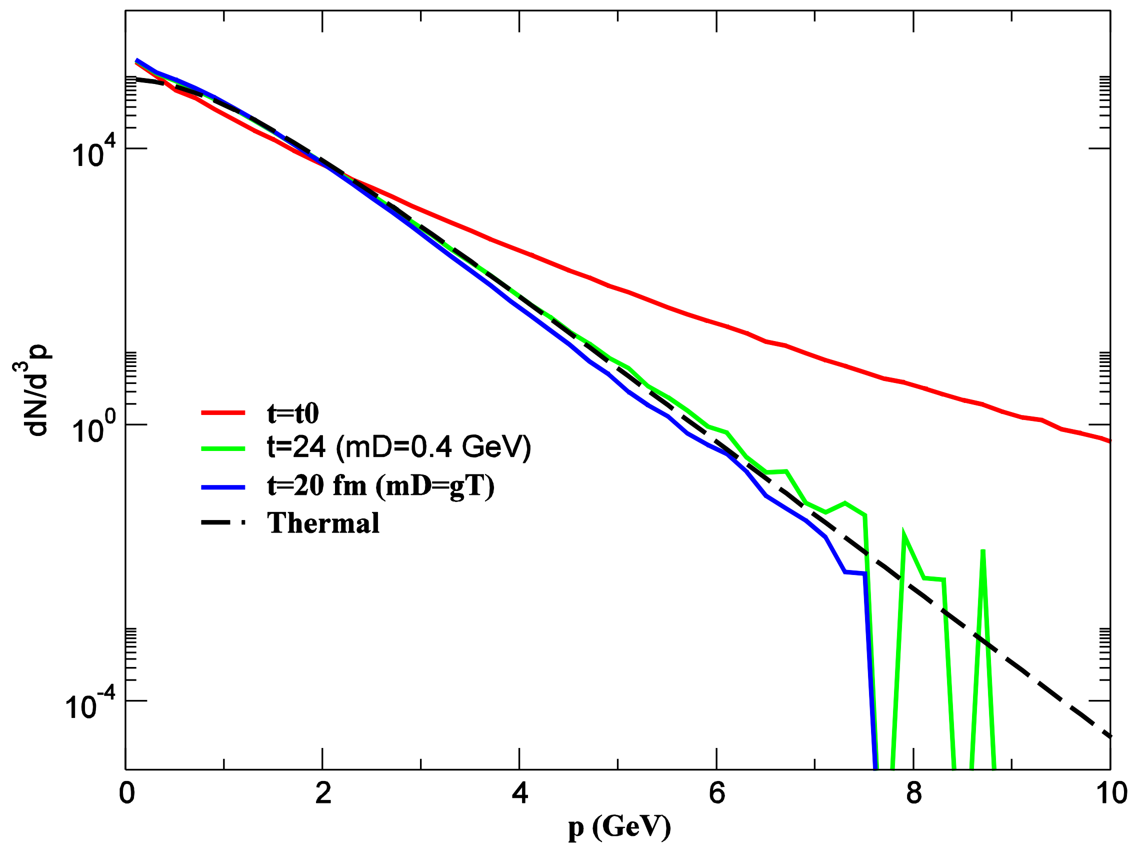
With FDT



With pQCD

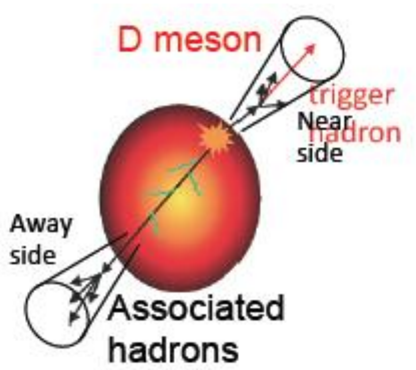


0.4
0.3
0.2
0.1
0
γ (fm⁻¹)
0
3
6
9
12
15
p (GeV)
c (m_D=0.4 GeV)
c (m_D=0.83 GeV)
c (m_D=1.6 GeV)
b (m_D=0.4 GeV)
b (m_D=0.83 GeV)
b (m_D=1.6 GeV)

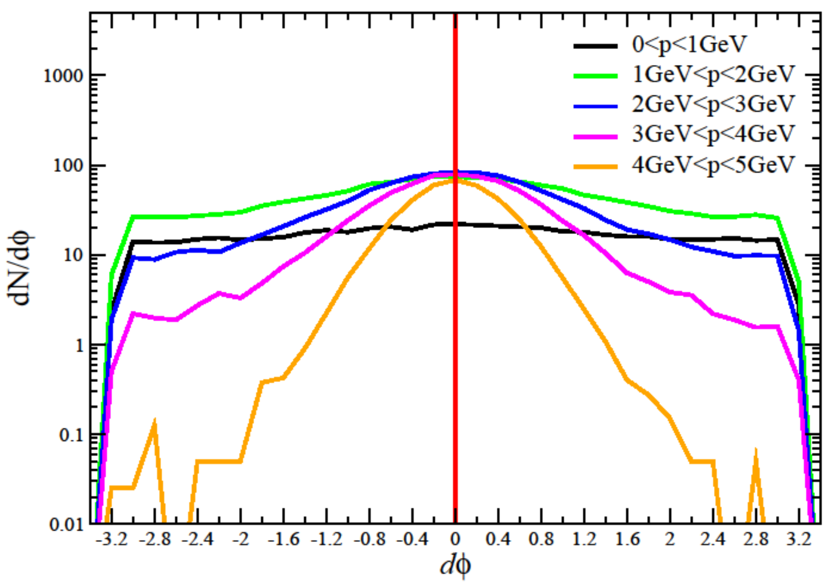


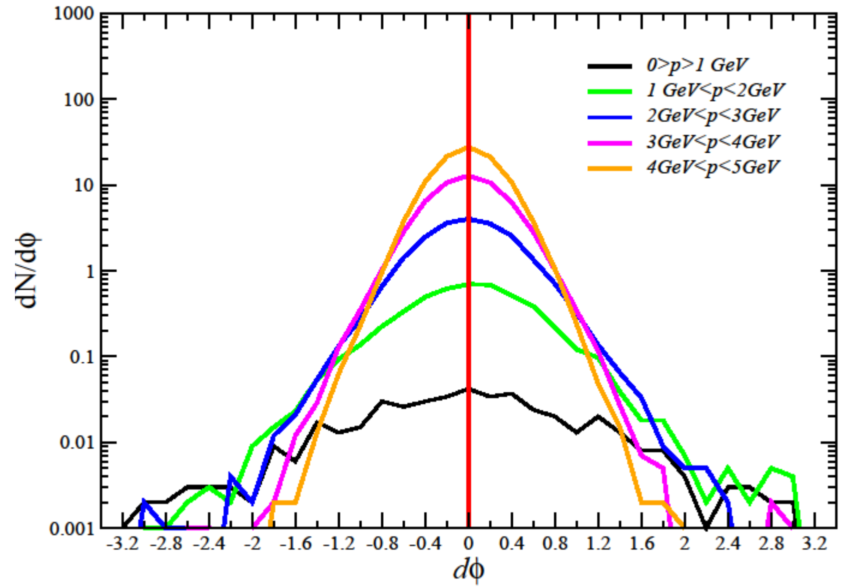
dN/d^3p
10^4
10^0
10^-4
0
2
4
6
8
10
p (GeV)
t=t0
t=24 (mD=0.4 GeV)
t=20 fm (mD=gT)
Thermal

Back to Back correlation



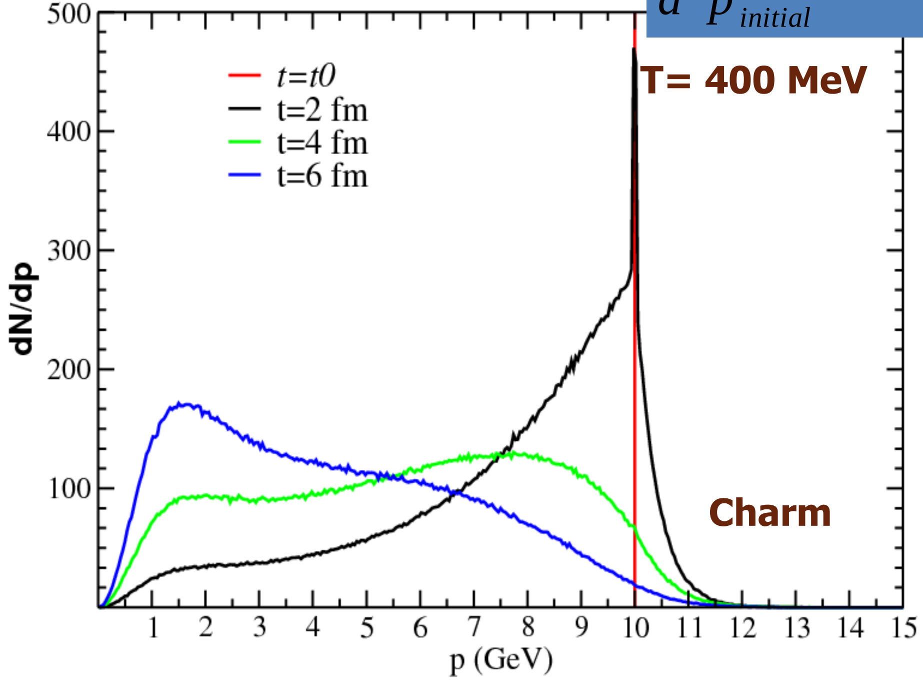
The larger spread of momentum with the Boltzmann implicates a large spread in the angular distributions of the Away side charm

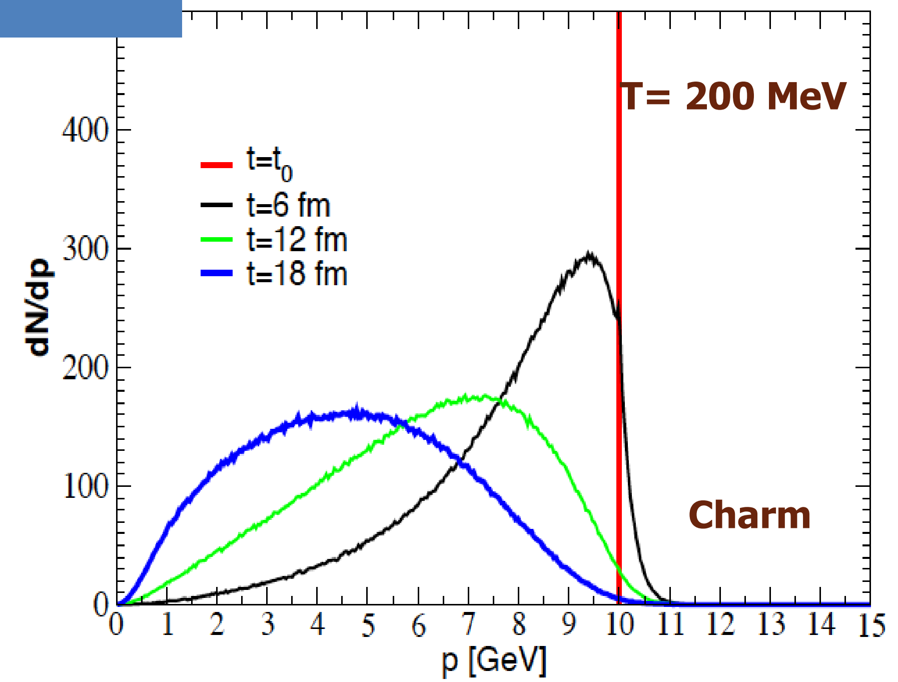


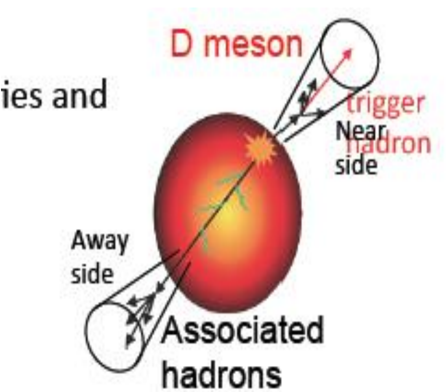


Momentum evolution for charm vs temperature

$$\frac{dN}{d^3p}_{initial} = \delta(p - 10GeV)$$



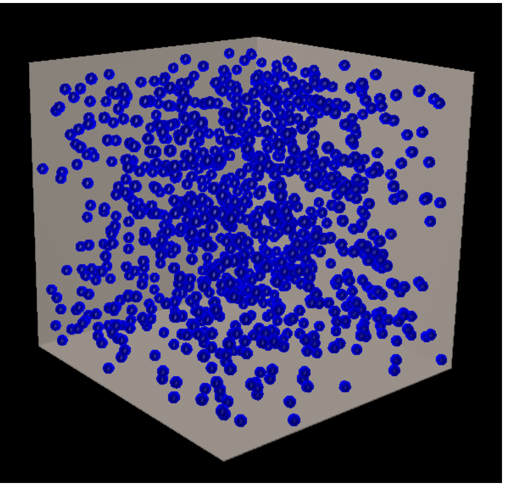




- At 200 MeV Mc/T= 6 -> start to see a peak with a width

Such large spread of momentum implicates a large spread in the angular distributions that could be experimentally observed studying the back to back **Charm-antiCharm** angular correlation

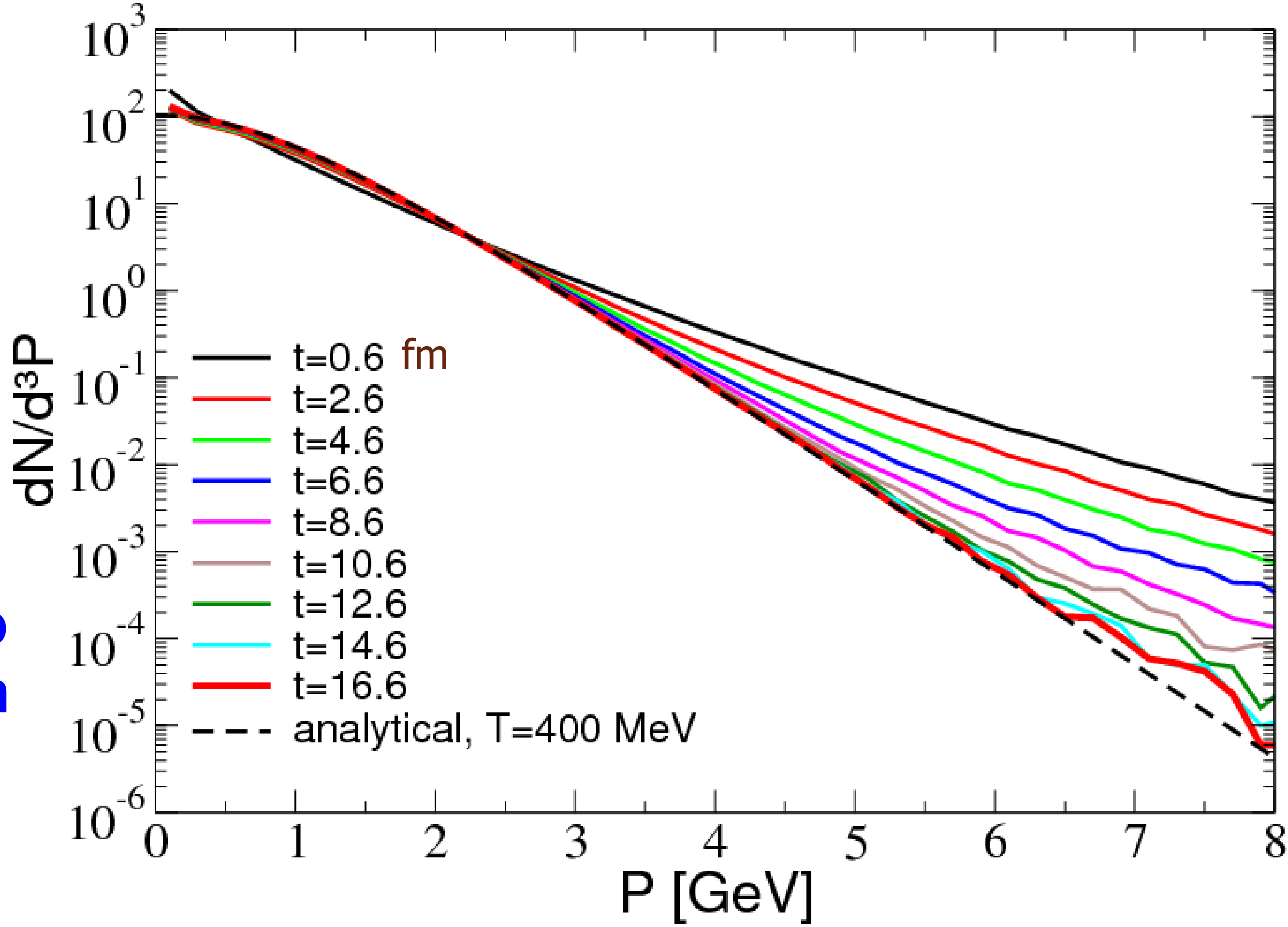
Charm evolution in a static medium



Simulations in which a particle ensemble in a **box** evolves dynamically

Bulk composed only by **gluons in thermal equilibrium** at **T=400 MeV**

C and Cbar are initially distributed: uniformily in r-space, while in p-space



Due to collisions charm approaches to thermal equilibrium with the bulk