Free energy vs. internal energy potential for heavy quark systems at finite temperature

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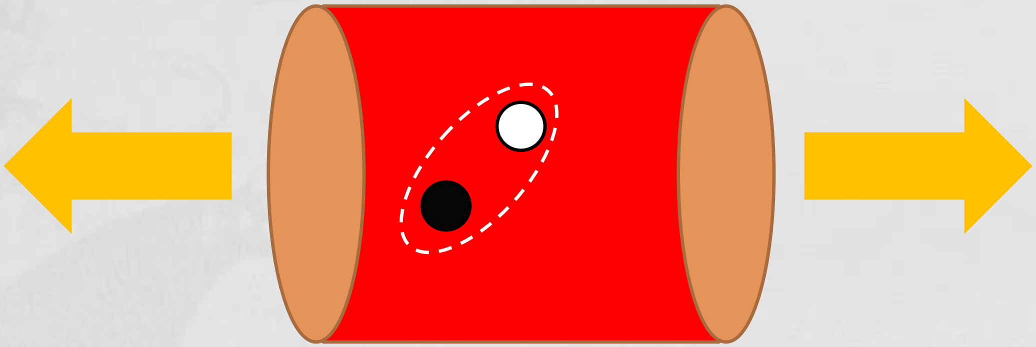
1. motivation

- Quarkonium has been an important probe for QGP since Matsui and Satz (1986)

  ; a thermometer of hot QCD matter
- Most phenomenological studies use potential model

  to explain experimental data.
- Should we use Free energy (weak binding) or internal energy (strong binding) for heavy quark potential in QGP?



2. Method

- QCD sum rule is a successful method to study hadron physics.
  M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, NPB 147, 385 (1979)

- It was extended to finite density & temperature.
  T. Hatsuda, S. H. Lee, PRC 46, 34 (1992)
  T. Hatsuda, Y. Koike, S. H. Lee, NPB 394, 221 (1993)

- Recently it was combined with lQCD data to study quarkonium properties in hot QCD matter.
  K. Morita, S. H. Lee, Phys. Rev. Lett. 100, 022301 (2008)
  K. Morita, S. H. Lee, Phys. Rev. C77, 064904 (2008)
  K. Morita, S. H. Lee, Phys. Rev. D79, 011501 (2009)
  Y. H. Song, S. H. Lee, K. Morita, Phys. Rev. C79, 014907 (2009)
  …

- In this study we calculate J/Ψ wavefunction, |Ψ(0,T)|, from the QCD sum rule and compare with the results from free energy & internal energy potentials.

3. QCD sum rule

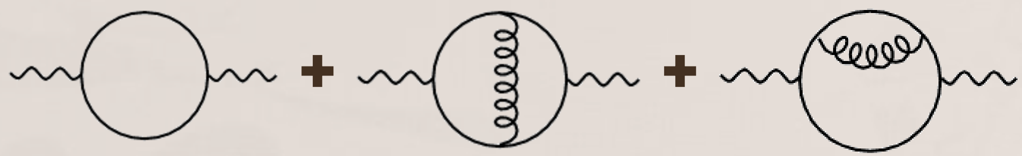
$$\Pi_{\mu\nu}(q^2) = i\int d^4x\, e^{iq\cdot x}\langle T[j_\mu(x), j_\nu(0)]\rangle$$

$$= (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2)$$

$$j_\mu(x) = \bar{c}(x)\gamma_\mu c(x)$$

Operator product expansion (OPE) – real part -

Perturbative part

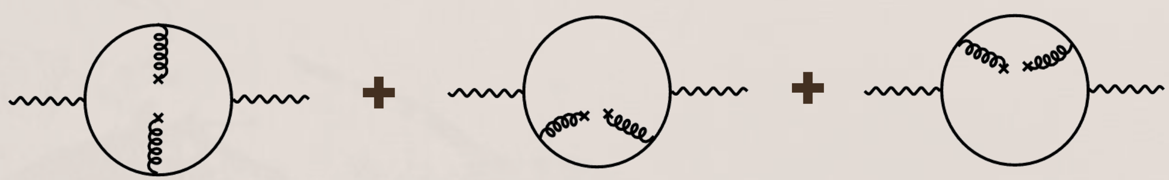


Scalar condensate

Twist-2 condensate

$$\mathrm{Re}\ \Pi(q^2) = C_I + C_{G0}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + C_{G2}\frac{q_\mu q_\nu}{q^2}\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\alpha}_{\nu} \right\rangle$$

Non-perturbative part



Condensate : soft interaction with vacuum or nuclear medium

Gluon condensate from lattice

From energy-momentum tensor

$$T_{\mu\nu} = (e+p)\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) + \frac{1}{4}(e-3p)g_{\mu\nu}$$

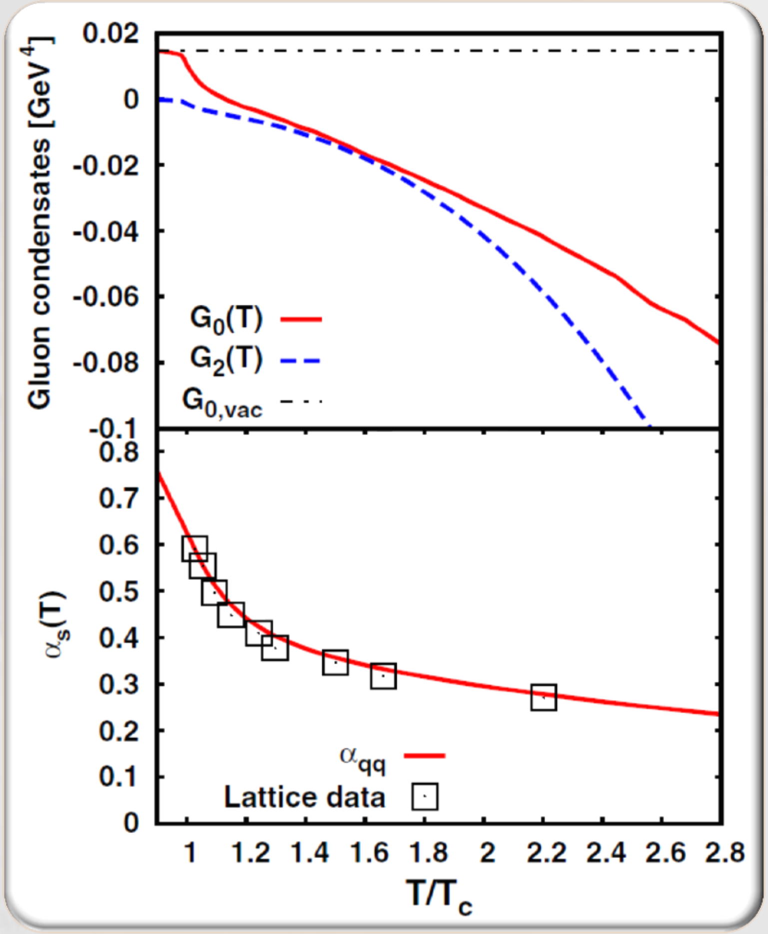
Trace part of $T_{\mu\nu}$

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \equiv G_0(T) = G_0^{vac.} - \frac{8}{11}(e-3p)$$

Symmetric and traceless part of $T_{\mu\nu}$

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\alpha}_{\nu} \right\rangle \equiv \left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) G_2(T)$$

$$G_2(T) = -\frac{\alpha_s(T)}{\pi}(e+p)$$

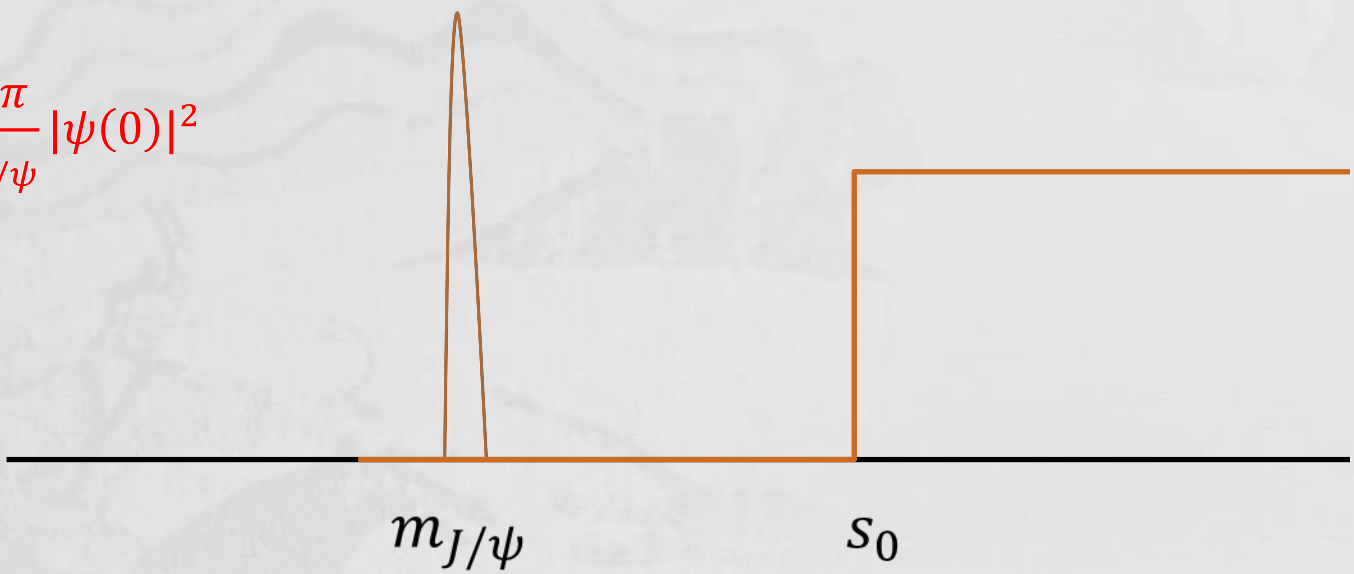


Spectral function – imaginary part -

- $\mathrm{Im}\ \Pi(q^2) = f_0 \delta\left(q^2 - m_{J/\psi}^2\right) + \theta(q^2 - s_0)\mathrm{Im}\ \Pi^{pert.}(q^2)$

$$\frac{f_0 \Gamma \sqrt{q^2}}{(q^2 - m_{J/\psi}^2)^2 + q^2\Gamma^2}$$

$$f_0 = \frac{12\pi}{m_{J/\psi}} |\psi(0)|^2$$



Dispersion relation

$$\text{Re}\ \Pi(q^2) = \frac{1}{\pi}\int \frac{\text{Im}\ \Pi(s)}{s-q^2}\, ds$$

- QCD parameters

$$m_c, \alpha_s,$$
$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\alpha}_{\nu} \right\rangle$$

- Physical parameters

$$m_{J/\psi}, \Gamma,$$
$$f_0 = \frac{12\pi}{m_{J/\psi}} |\psi(0)|^2$$

Borel transformation
to improve the dispersion relation

$$\hat{B} \equiv \lim_{\substack{Q^2=-q^2\to\infty \\ n\to\infty \\ Q^2/n=M^2}} \frac{1}{(n-1)!}(Q^2)^n\left(-\frac{d}{dQ^2}\right)^n$$

$$\mathrm{Re}\,\Pi(q^2) = \frac{1}{\pi}\int \frac{\mathrm{Im}\,\Pi(s)}{s-q^2}\,ds$$

$\hat{B}$

$$\hat{B}\left(\frac{1}{Q^2}\right)^N = \frac{1}{(N-1)!}\left(\frac{1}{M^2}\right)^N$$

suppresses high-dimensional terms in OPE

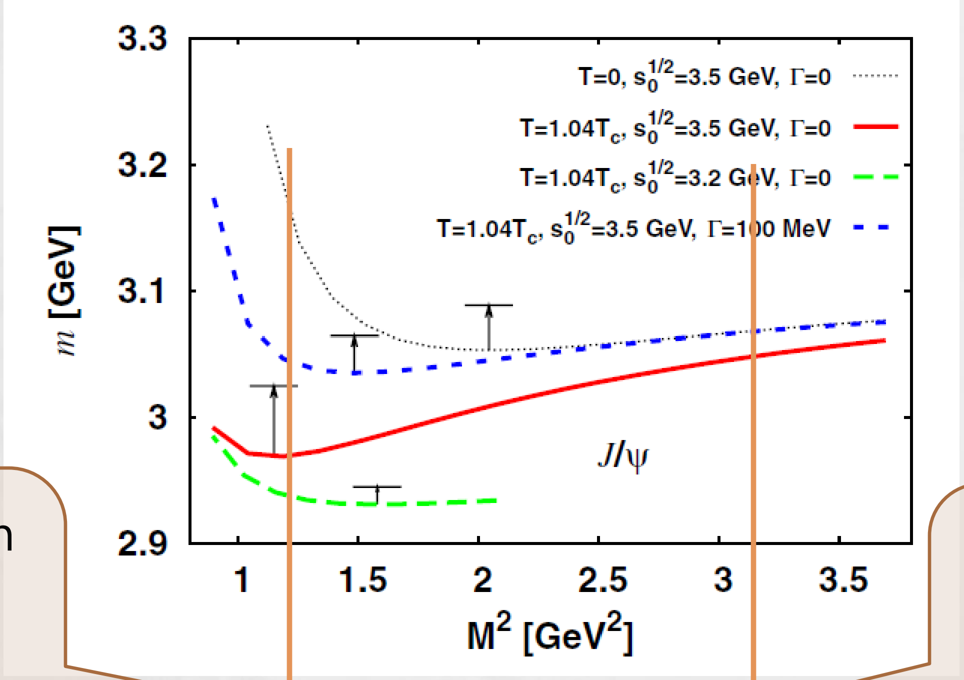
$\hat{B}$

$$\frac{1}{\pi}\oint_C \mathrm{Im}\,\Pi(s)\,e^{-s/M^2}\,ds$$

suppresses continuum part

Borel curve for J/Ψ mass

$$m_{J/\psi}^2 = -\frac{\partial(1/M^2)[\Pi^{OPE}(M^2) - \Pi^{cont}(M^2; s_0)]}{\Pi^{OPE}(M^2) - \Pi^{cont}(M^2; s_0)}$$



Adjust s_0 to find the most flat Borel curve (independent of M^2)

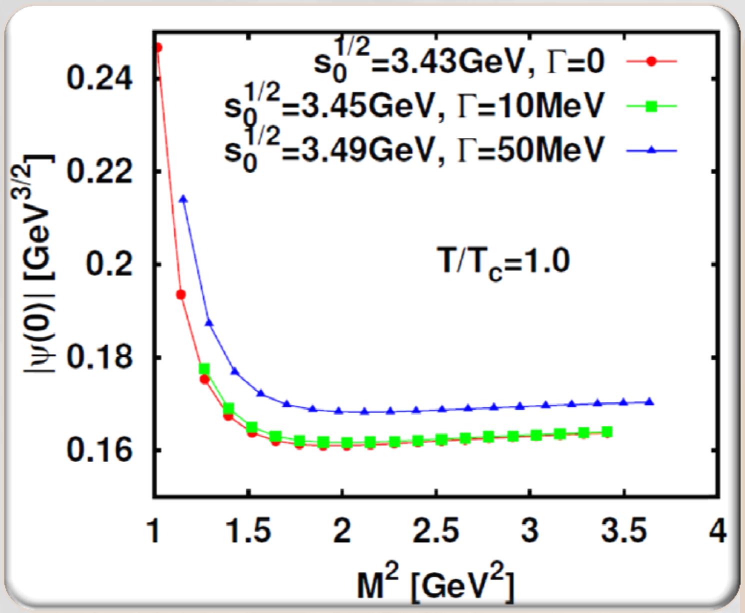
The contribution of dimension 4 condensates to OPE should be less than 30 %

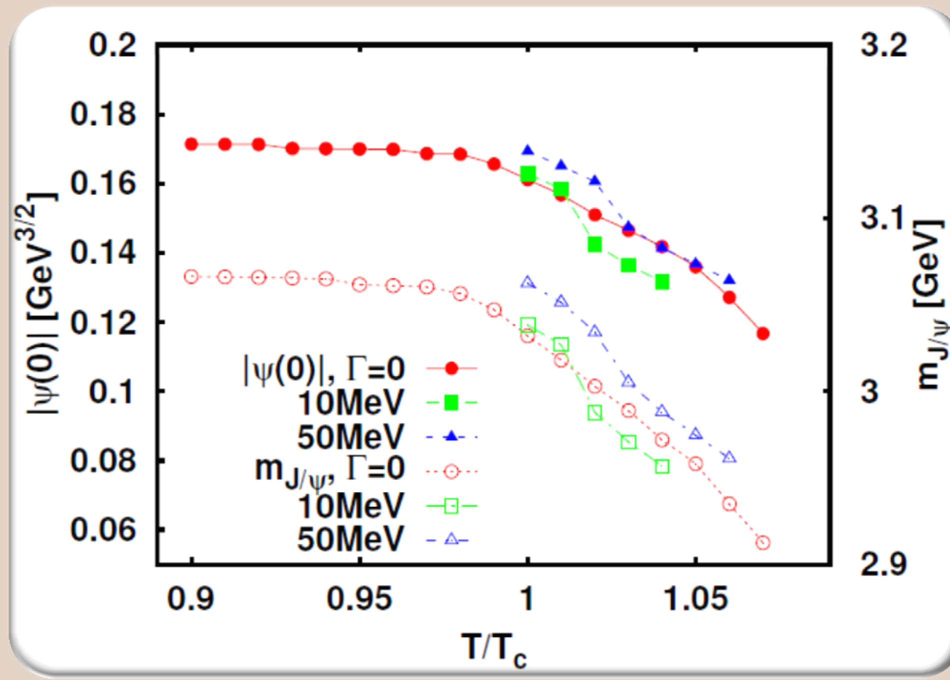
Borel window

The contribution of continuum should be less than 30 %

J/Ψ wavefunction |Ψ(0)|

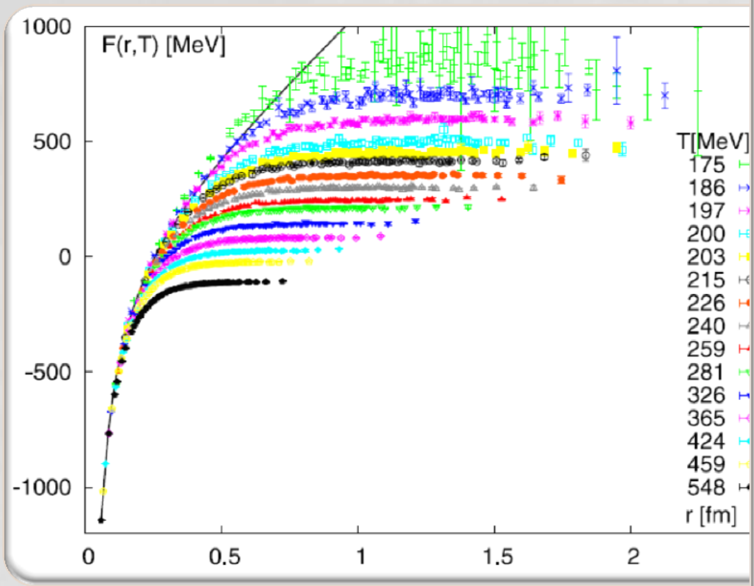
$$f_0 = \frac{12\pi}{m_{J/\psi}}|\psi(0)|^2 = \exp\left[\frac{m^2_{J/\psi}(M^2)}{M^2}\right] \times [\Pi^{OPE}(M^2) - \Pi^{cont}(M^2; s_0)]$$

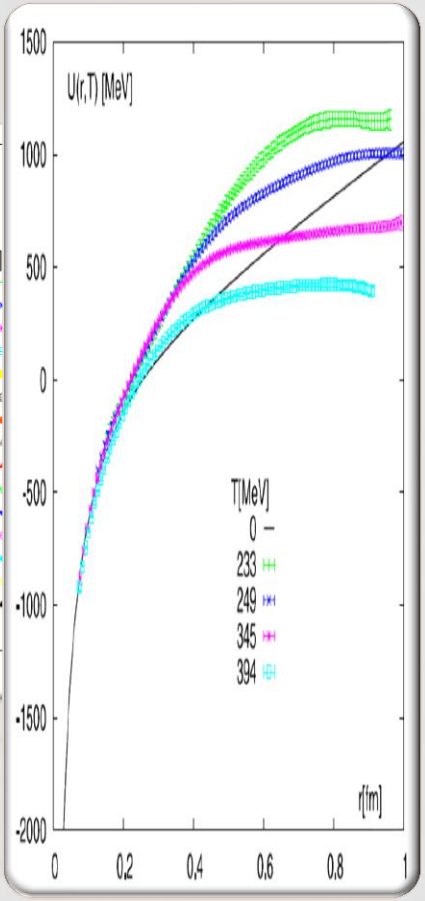




1. Both |Ψ(0)| and J/Ψ mass decrease with T
2. Width effect is small

4. $|\Psi(0)|$ from free & internal energy lattice potentials





U (internal energy) =F+TS

→ strongly bound

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F (free energy)

→ weakly bound

Schrödinger equation

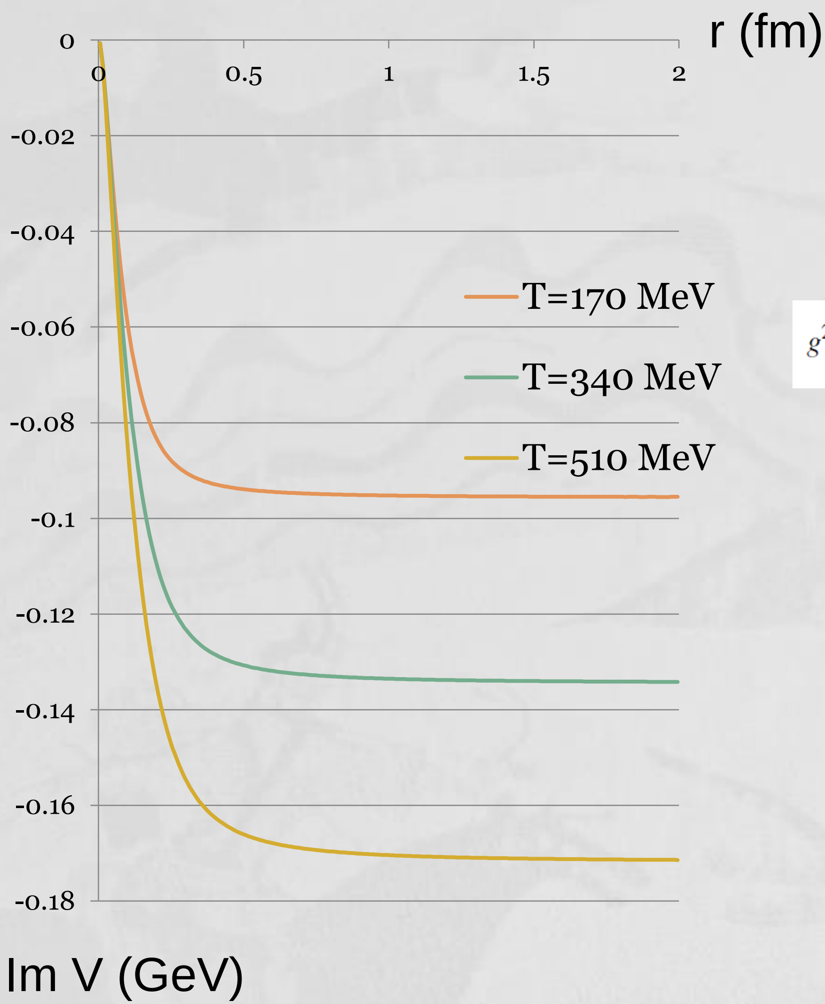
- $\left[2m_c - \frac{1}{m_c}\nabla^2 + V(r,T)\right]\psi(r,T) = M_{J/\psi}\psi(r,T)$
- $\left[-\frac{1}{m_c}\nabla^2 + \tilde{V}(r,T)\right]\psi(r,T)$

  $= -\{2\tilde{m}_c - M_{J/\psi}\}\psi(r,T) = \varepsilon\,\psi(r,T),$

where $\tilde{V}(r,T) = V(r,T) - V(r=\infty,T)$

$\tilde{m}_c = m_c + V(r=\infty,T)/2.$

Imaginary potential energy from hard thermal loop (HTL)



$$g^2(T) = \frac{8\pi^2}{9\ln(9.082T/\Lambda_{\overline{MS}})}, \quad m_D^2(T) = \frac{8\pi^2}{3\ln(7.547T/\Lambda_{\overline{MS}})}, \quad \Lambda_{\overline{MS}} = 300\mathrm{MeV}.$$

$$\mathrm{Im}V^{HTL}(r,T) = -\frac{ig^2TC_F}{4\pi}\phi(m_D r), \quad \phi(x) = 2\int_0^\infty \frac{dzz}{(z^2+1)^2}\left|1 - \frac{sin(zx)}{zx}\right|,$$

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Complex Schrödinger equation

$$\frac{d^2}{dr^2}u_R + m_c(\varepsilon_R - V_R)u_R - m_c(\varepsilon_I - V_I)u_I = 0$$
$$\frac{d^2}{dr^2}u_I + m_c(\varepsilon_R - V_R)u_I + m_c(\varepsilon_I - V_I)u_R = 0$$

where

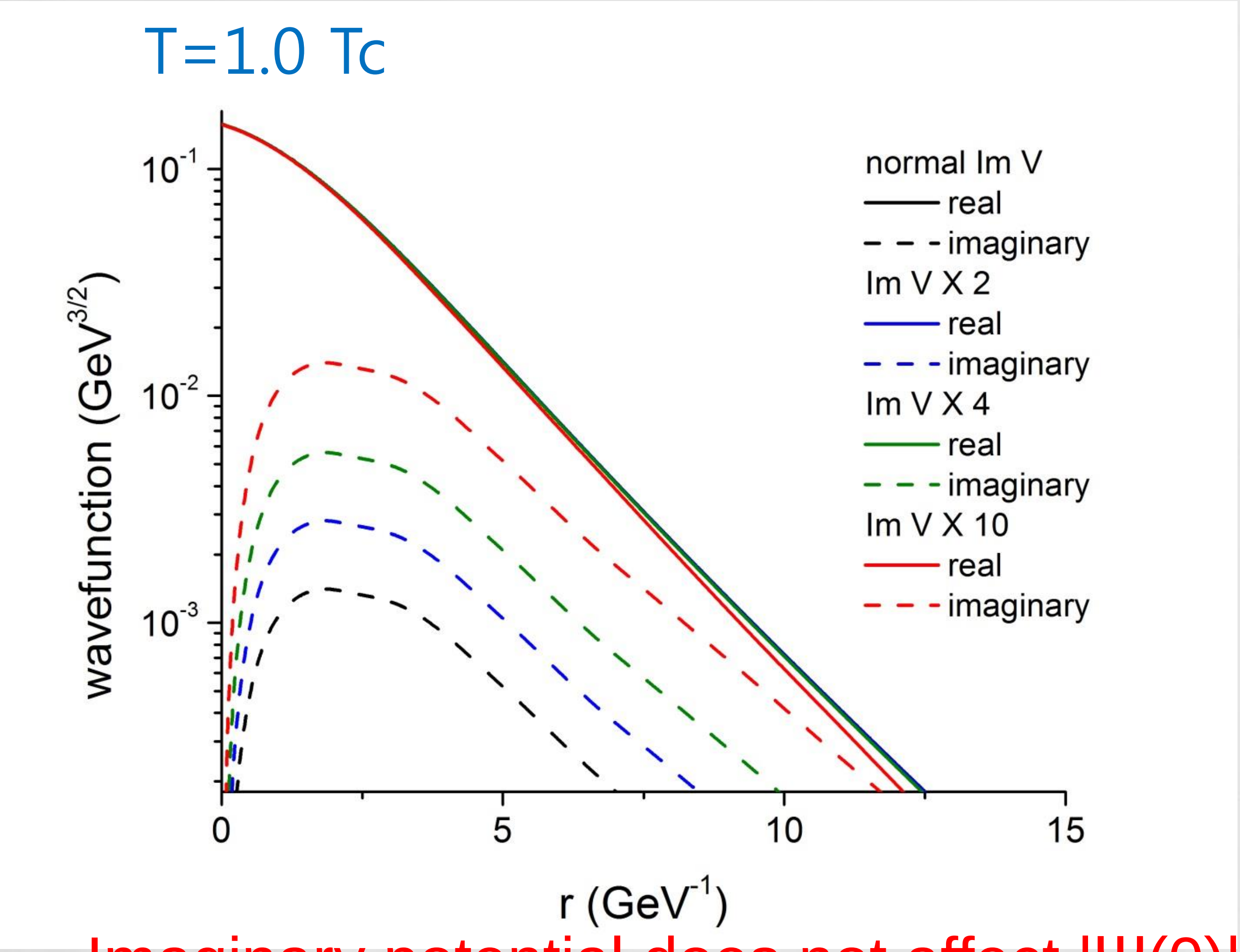
$$\psi = \frac{1}{r}(u_R + iu_I), \quad \varepsilon = \varepsilon_R + i\varepsilon_I$$

$$u_R(h) = h + \cdots,$$
$$u_I(h) = -\frac{m_c\varepsilon_I}{6}h^3 + \cdots \quad (h \ll 1)$$

Solved numerically by the Runge-Kutta method in iterative way

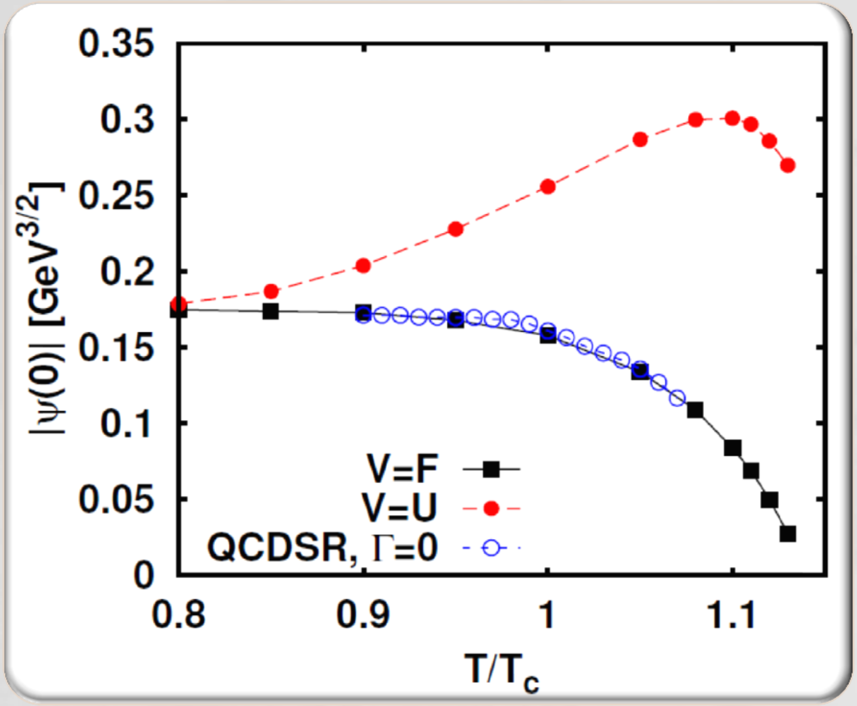
J/Ψ wavefunction

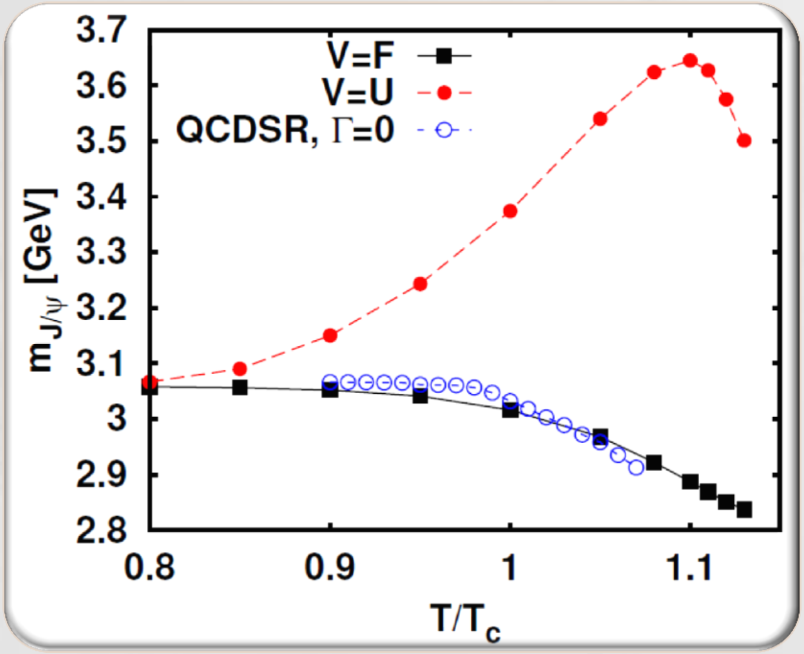
from the complex Schrödinger equation



Imaginary potential does not affect |Ψ(0)| !!

5. Comparison of the results from QCD sum rule & Schrödinger equation





|Ψ(0)| as well as J/Ψ mass from QCD sum rule closely follow those from free energy potential.

6. Summary

- Whether free energy or internal energy as heavy quark potential is a very important question in quarkonium study in hot QCD matter
- The strength of J/Ψ wavefunction at origin, $|\Psi(0,T)|$, was calculated by using QCD sum rule.
- It was also calculated by solving Schrödinger equations with lattice free energy & internal energy potentials.
- Comparing them, we found that QCD sum rule results are very close to those from the lattice free energy potential.
- It implies that J/ψ seems to dissociate near Tc.