Numerical solution of the Boltzmann equation for ultracold fermions

Pierre-Alexandre PANTEL, Dany Davesne, Michael Urban

Institut de Physique Nucléaire de Lyon – France

NeD/TURIC 2014

14 June 2014

Why cold atom gases?

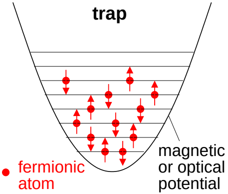
- Explore the limits of many-body theories used in nuclear physics by applying them to systems of cold atoms
- Atomic clouds can simulate strongly interacting systems
- Only one parameter to describe the interaction strength: a (the scattering length in s-wave channel):

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

B is the external magnetic field applied to the trapped atomic cloud

- Experiments on cold atoms don't require big and expensive experimental set-up

From an atomic gas to a nucleus



$$N \sim 10^3 - 10^6 \ \text{(3D)}$$

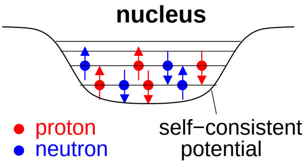
$$T_\text{F} = \varepsilon_F / k_B \sim 10^{-6}\ \mathrm{K}$$

$$k_\text{F}^{-1} \sim 10^{-6}\ \mathrm{m}$$

$$\text{int. range } r \sim 10^{-4} k_\text{F}^{-1}$$

$$T_{exp} \to 0.1\, T_\text{F}$$

$$\Leftrightarrow$$



$$A \sim 10 - 10^2$$

$$T_\text{F} \sim 10^{11}\ \mathrm{K}$$

$$k_\text{F}^{-1} \sim 10^{-15}\ \mathrm{m}$$

$$r \sim k_\text{F}^{-1}$$

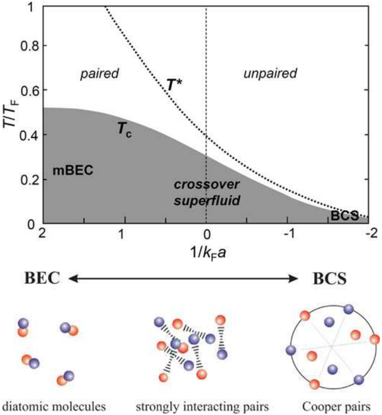
Superfluid
phase!

Phase diagram of the BEC–BCS crossover

BOSONS

Phase diagram

FERMIONS



[R. Grimm, 2007]

Repulsive
($a > 0$)

Attractive
($a < 0$)

$^6\mathrm{Li}_2$

$^6\mathrm{Li}$

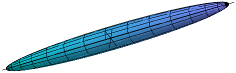
From a Bosonic to a Fermionic behaviour with the same system!

What kind of systems can be produced?

- Preparing an atomic ($^6\mathrm{Li}$, $^{40}\mathrm{K}$,…) gas into 2 hyperfine levels (pseudospin): $|\uparrow\rangle$ and $|\downarrow\rangle$
- Optical trap (laser) or magneto-optical trap
- Simulating polarized matter: $N_\uparrow \neq N_\downarrow$
- Systems of two different atom species (different masses): useful for quark matter
- And more: neutron matter, color superconductivity (gases with atoms in three different hyperfine levels),…

Our systems

- Harmonic trap potential with frequencies inducing a cigar-shaped gas:



$$\omega_x = \omega_y \gg \omega_z$$

- Near Feshbach resonance: $(k_F a)^{-1} \to 0^-$

$$\Rightarrow \text{strongly correlated Fermions}$$

- Collective modes in the normal phase

Boltzmann equation: resolution

$$\frac{\partial f}{\partial t} + \dot{\boldsymbol{r}} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \dot{\boldsymbol{p}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = -I[f]$$

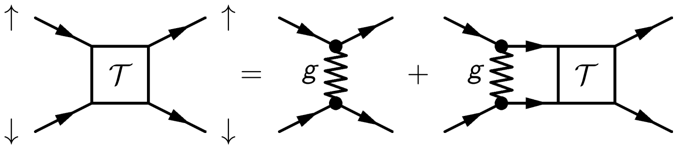
to be solved for $N = 6.10^5$ atoms!

- Distribution function $f = f(\boldsymbol{r}, \boldsymbol{p}, t)$
- $\dot{\boldsymbol{r}}$ and $\dot{\boldsymbol{p}}$ considered classical
- Trapping potential $V_{\text{trap}}(\boldsymbol{r})$: harmonic

- Numerical: test particles method
- Gaussian extension
- Check through collision rate

Meanfield potential $U(\boldsymbol{r})$ in $f(\boldsymbol{r}, \boldsymbol{p}, t)$
AND
in-medium cross section in $I[f]$

$\mathcal{T}$-matrix approximation



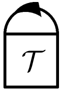
- $\mathcal{T}$-matrix:

$$\Gamma(\omega, \boldsymbol{k}) = \frac{g}{1 - gJ(\omega, \boldsymbol{k})}$$

- J is the 2-particles propagator
- $g = 4\pi a/m$

- Self-energy: NSR theory

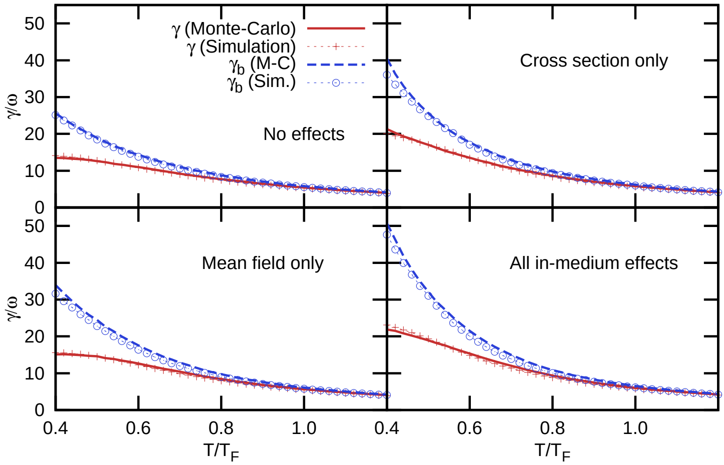
$\Sigma =$



- Mean field: $U = \mathrm{Re}\,\Sigma(0, k_\mathrm{F})$
- In-medium cross-section:

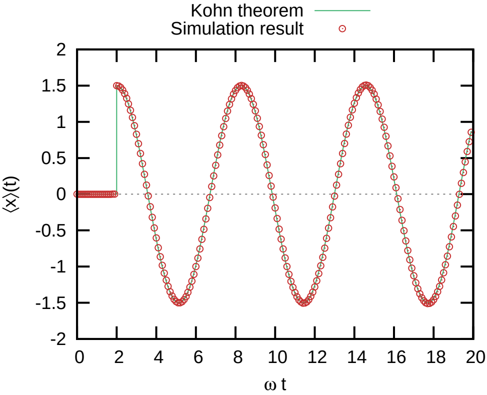
$$\sigma_{\text{in-med}}(\boldsymbol{k}, \boldsymbol{q}) \propto |\Gamma|^2$$

Numerical code and collision rate



Sloshing mode

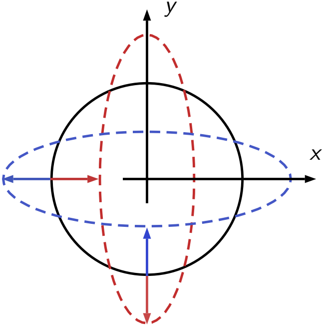
- Global oscillation of the cloud along one direction: $\langle x \rangle (t)$
- Harmonic trap: Kohn's theorem
  frequency $= \omega_x$ whatever the interaction



Quadrupole mode

Radius compression related to hydrodynamic behavior (superfluid?)

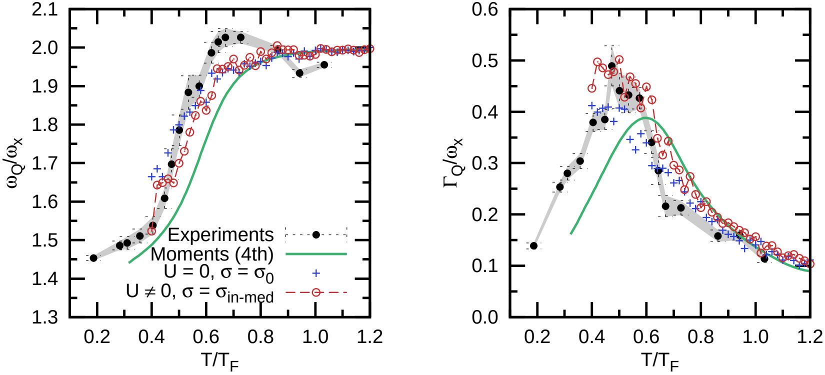
$$\langle x^2 - y^2 \rangle (t) \propto e^{-\Gamma_Q t} \sin(\omega_Q t)$$



Quadrupole mode

Radius compression related to hydrodynamic behavior (superfluid?)

$$\langle x^2 - y^2 \rangle(t) \propto e^{-\Gamma_Q t} \sin(\omega_Q t)$$

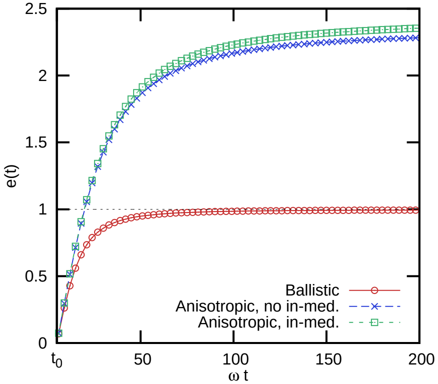


[Riedl et al., PRA 78, 053639; Chiacchiera et al., PRA 84, 043634]

Expanding the gas

- When $t \geqslant t_0$, $V_{\text{trap}} = 0$
- Experimental method to determine the temperature and density
- Significant of the collision regime: hydrodynamic? collision-less?
- Anisotropic traps ⇔ shear viscosity η/s

Looking at $e(t) = \sqrt{\langle r^2 \rangle / \langle z^2 \rangle}$



[C. Cao *et al.*, Science 331, 58 (2011)]

Summary

- Boltzmann code with in-medium effects
- Study physics of some particular state and behaviour of the matter with the collective modes

Outlook (WIP)

- Colliding polarized clouds: new collective modes

[A. Sommer et al., Nature 472, 7342 (2011)]

- Polarized gases: new superfluid phase (FFLO)
- Neutron stars, color superconductivity,. . .

Advertising

Trapped atomic gases:
a laboratory for thermodynamic and non-equilibrium processes
for strongly correlated particles and with a lot of available data!