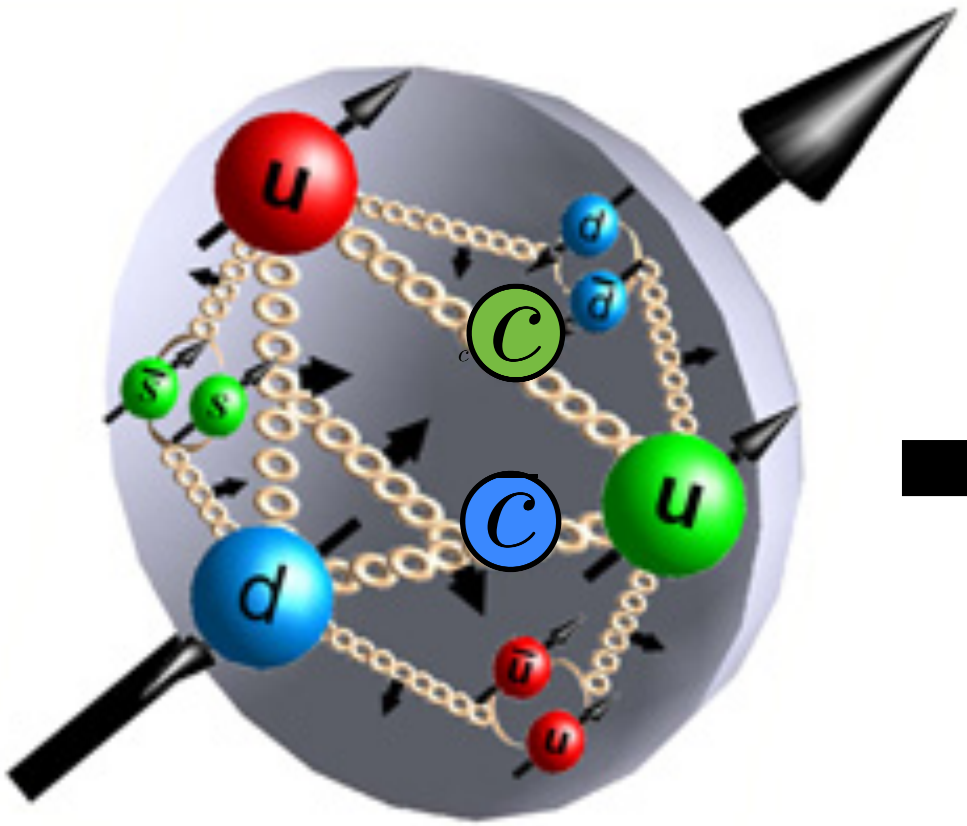
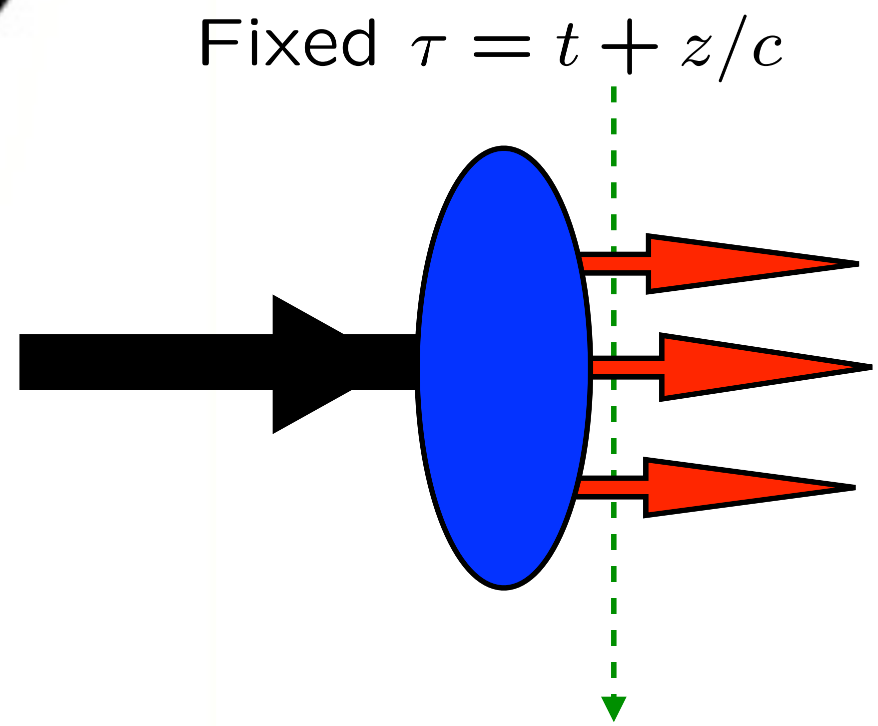
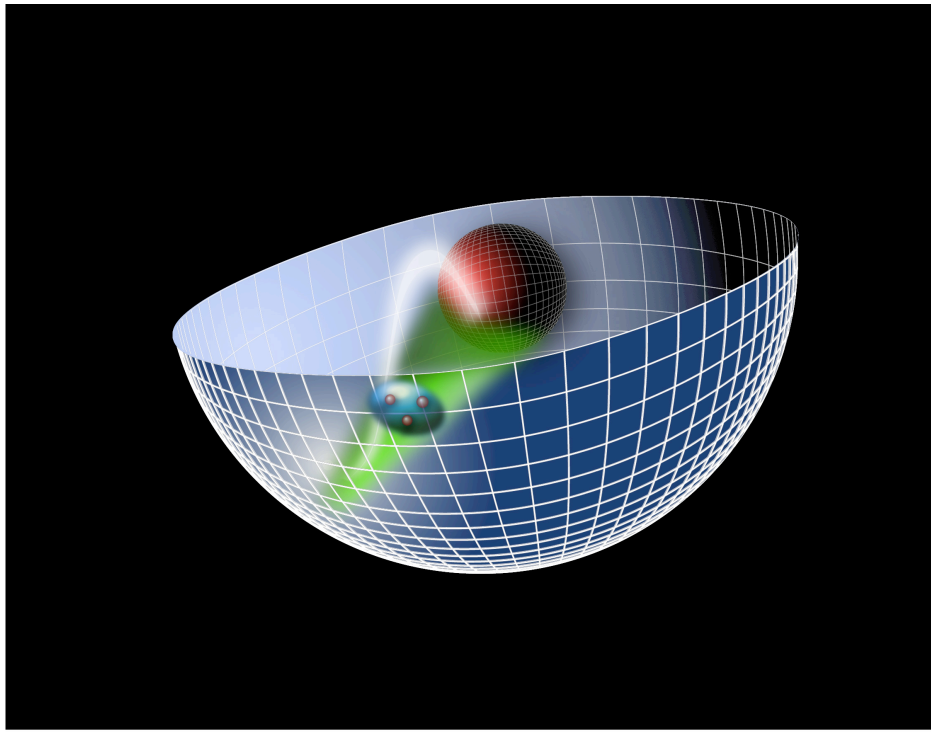
Introduction to Light-Front Quantization



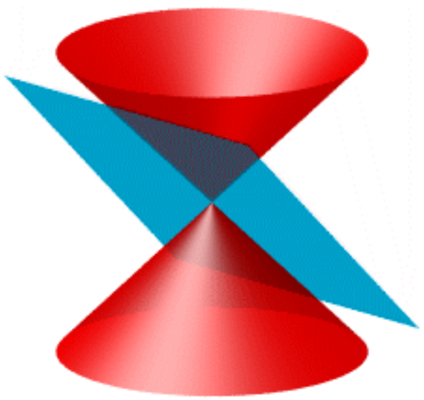




Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY
DIE LUFT DER FREIHEIT WEHT
1891

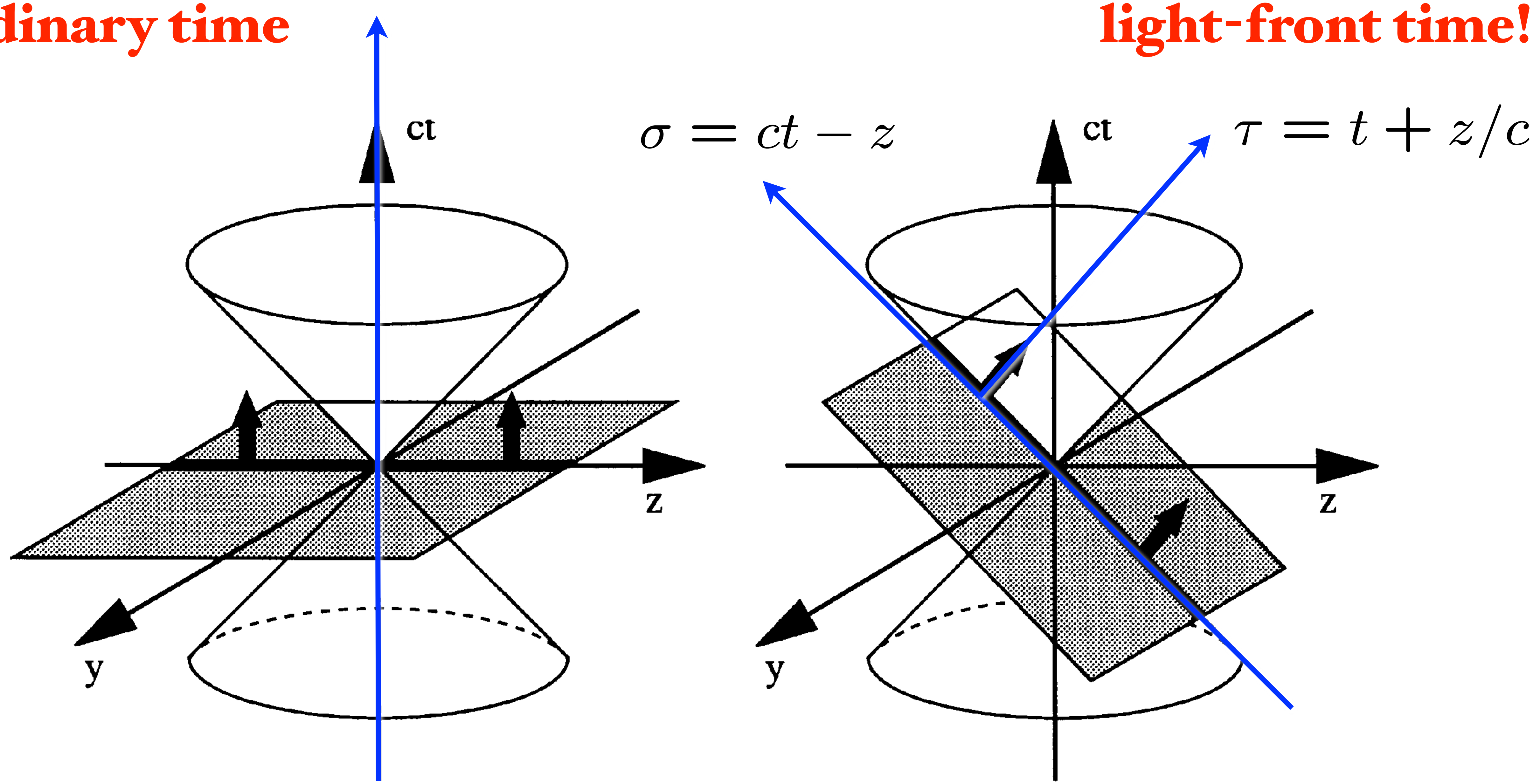


P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

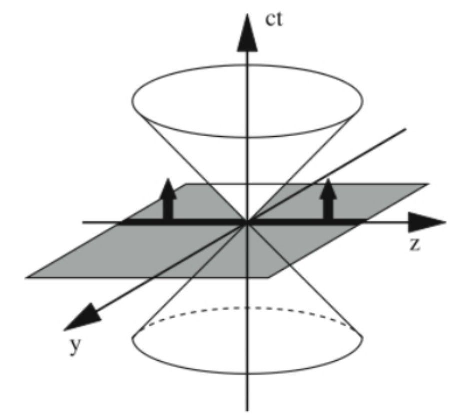
Evolve in light-front time!



Instant Form

Front Form

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



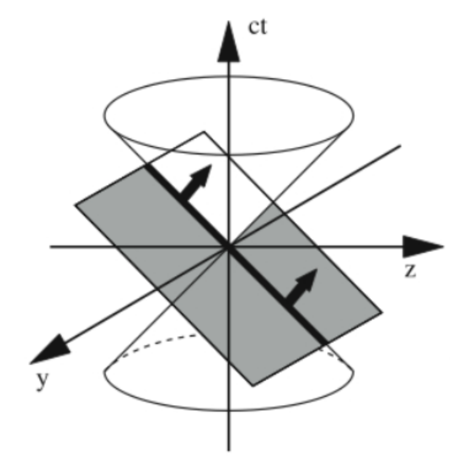
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

$$\tau = \quad x^+ = x^0 + x^3 \quad \text{light-front time}$$

$$\sigma = \quad x^- = x^0 - x^3 \quad \text{longitudinal space variable}$$

$$k^+ = k^0 + k^3 \quad \text{longitudinal momentum} \quad (k^+ \geq 0)$$

$$k^- = k^0 - k^3 \quad \text{light-front energy}$$



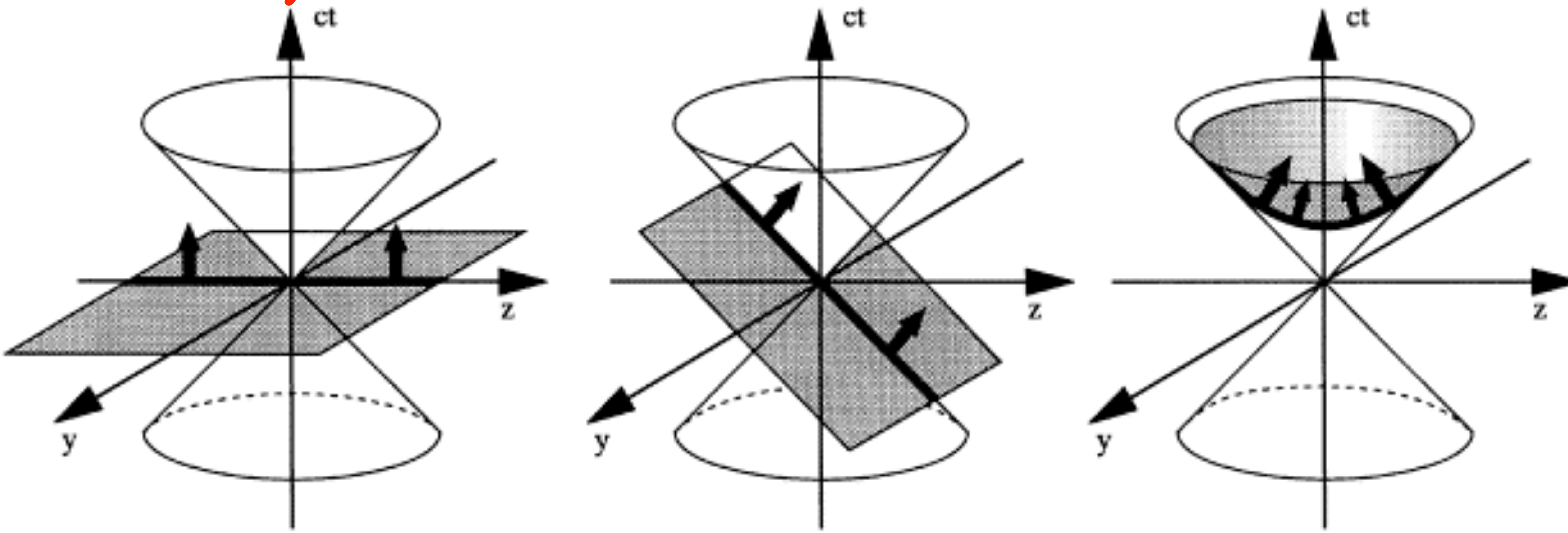
$$k \cdot x = \tfrac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Quantum chromodynamics and other field theories on the light cone.
Stanley J. Brodsky (SLAC), Hans-Christian Pauli (Heidelberg, Max Planck Inst.),
Stephen S. Pinsky (Ohio State U.). SLAC-PUB-7484, MPIH-V1-1997. Apr 1997. 203 pp.
Published in **Phys.Rept. 301 (1998) 299-486**
e-Print: **hep-ph/9705477**

P.A.M Dirac,
Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time	Evolve in light-front time	Evolve in point-form time



The instant form

$$\tilde{x}^0 = ct,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The front form

$$\tilde{x}^0 = ct+z,\quad \tilde{x}^1 = x,\quad \tilde{x}^2 = y,\quad \tilde{x}^3 = ct-z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

The point form

$$\tilde{x}^0 = \tau \quad , \quad ct = \tau\cosh\omega$$
$$\tilde{x}^1 = \omega \quad , \quad x = \tau\sinh\omega\,\sin\theta\,\cos\phi$$
$$\tilde{x}^2 = \theta \quad , \quad y = \tau\sinh\omega\,\sin\theta\,\sin\phi$$
$$\tilde{x}^3 = \phi \quad , \quad x = \tau\sinh\omega\,\cos\theta$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\tau^2 & 0 & 0 \\ 0 & 0 & -\tau^2\sinh^2\omega & 0 \\ 0 & 0 & 0 & -\tau^2\sinh^2\omega\,\sin^2\theta \end{pmatrix}$$

Each element of
flash photograph
illuminated
at same LF time

$$\tau = t + z/c$$

Images in a photograph show object captured at a single light-front time

HELEN BRADLEY - PHOTOGRAPHY

We view the universe as light reaches us along the light-front at fixed

$$\tau = t + z/c$$

Front Form Vacuum Describes the Empty, Causal Universe

‘Tis a mistake / Time flies not
It only hovers on the wing
Once born the moment dies not
‘tis an immortal thing

Montgomery

Each element of flash photograph illuminated along the light front **at a fixed**

$$\tau = t + z/c$$

Movie: Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenvalue

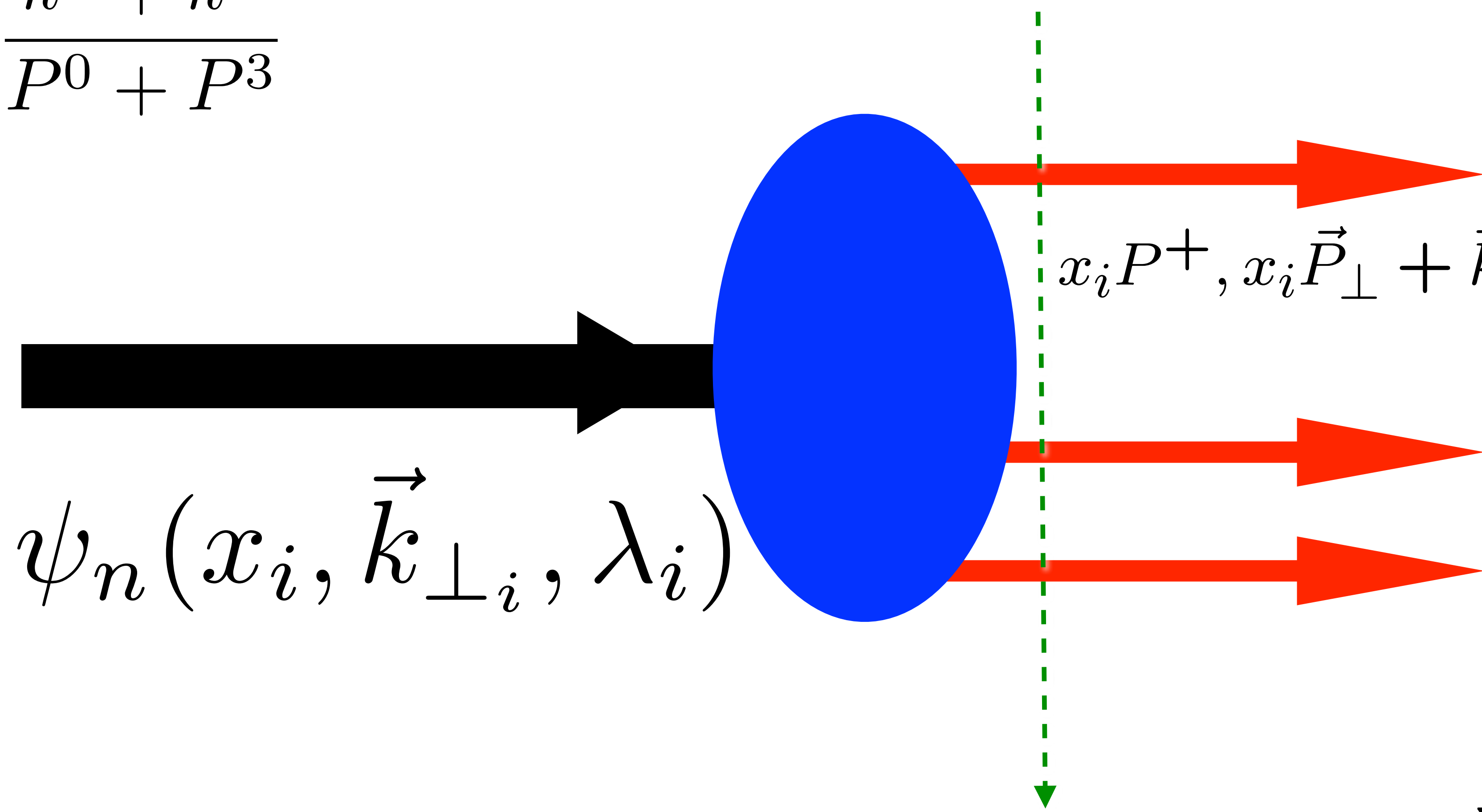
$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

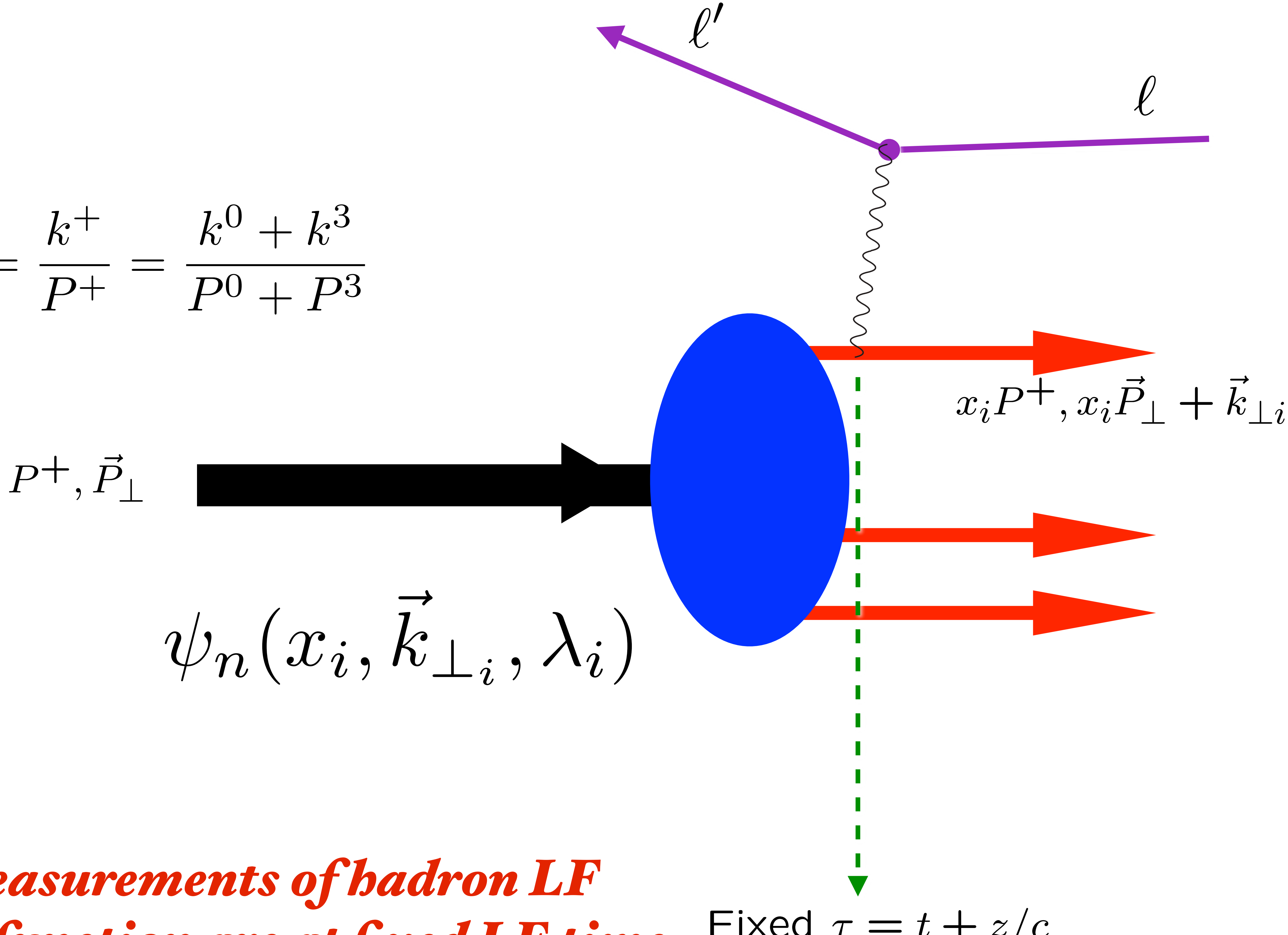
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Silas R. Beane

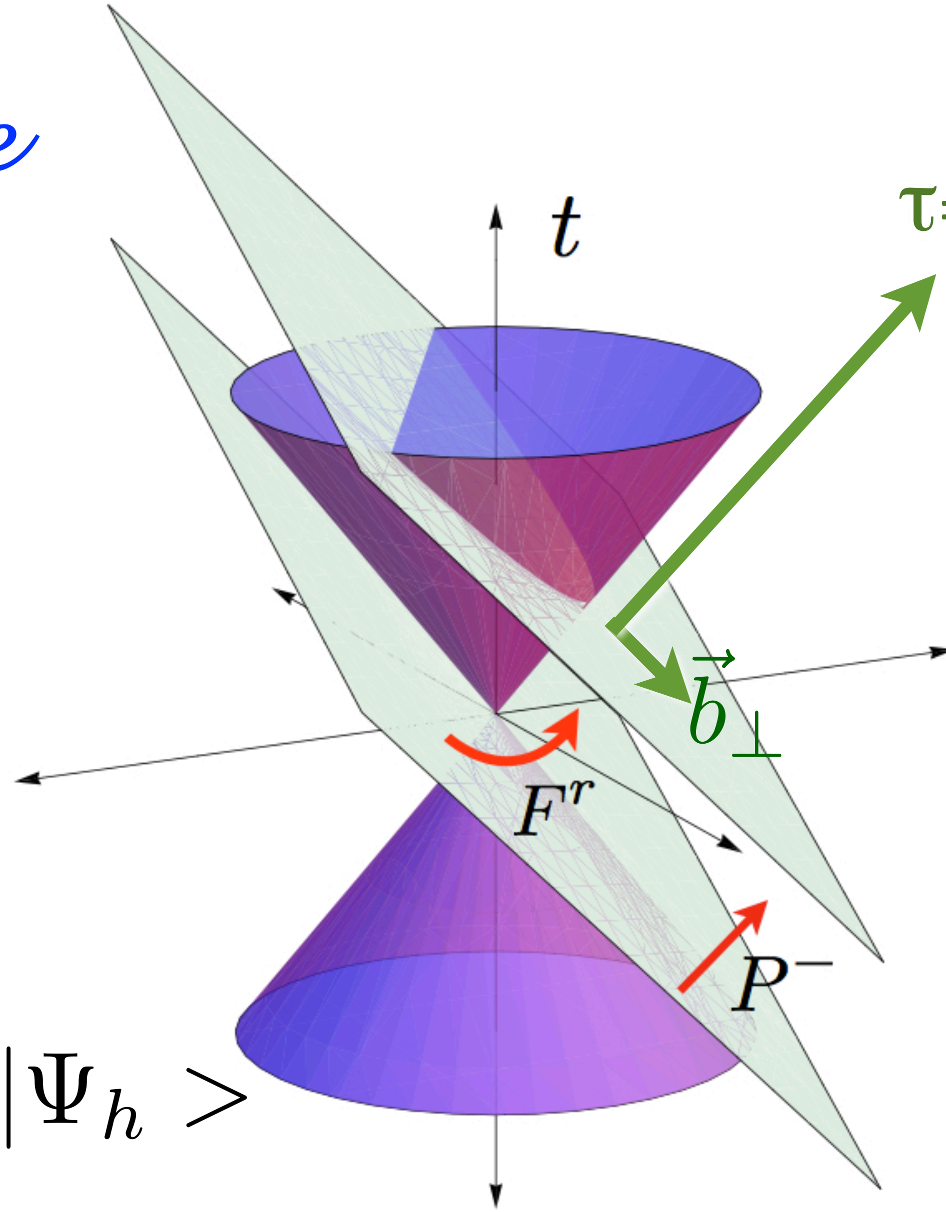
Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$



$$\zeta_\perp^2 = b_\perp^2 x(1-x)$$

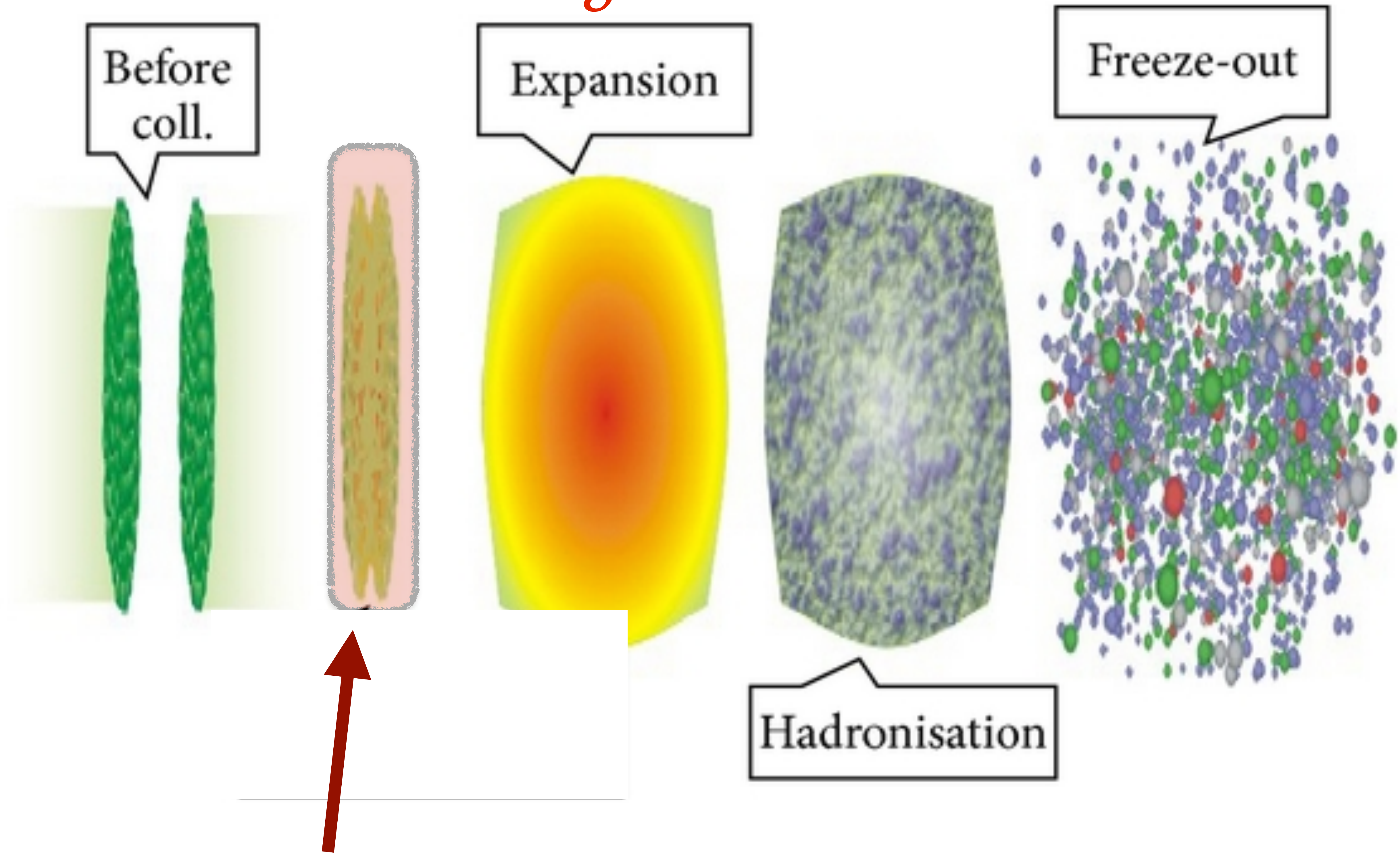
$$-\frac{d^2}{d\zeta_\perp^2} = \frac{k_\perp^2}{x(1-x)}$$

Null plane: a surface tangent to the light cone.

The null-plane Hamiltonian maps the initial light-like surface onto some other surface, and therefore describe the dynamical evolution of the system.

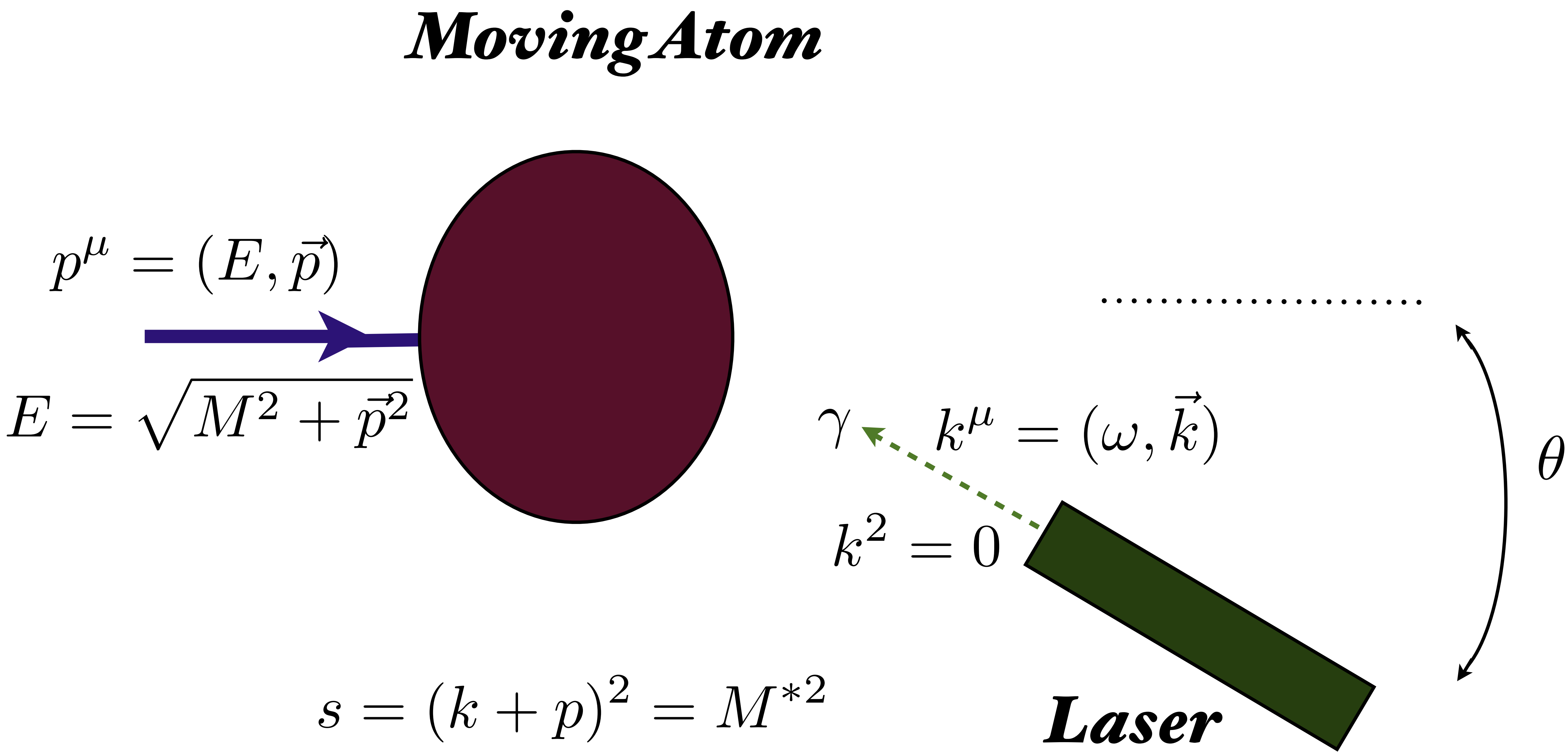
The energy P^- translates the system in the null-plane time coordinate x^+, whereas the spin Hamiltonians F_r rotate the initial surface about the surface of the light cone.

Heavy-Ion Collisions Visualized as Colliding "Pancakes"



Small longitudinal size Δz of source implies longitudinal momentum p_L distribution is constant

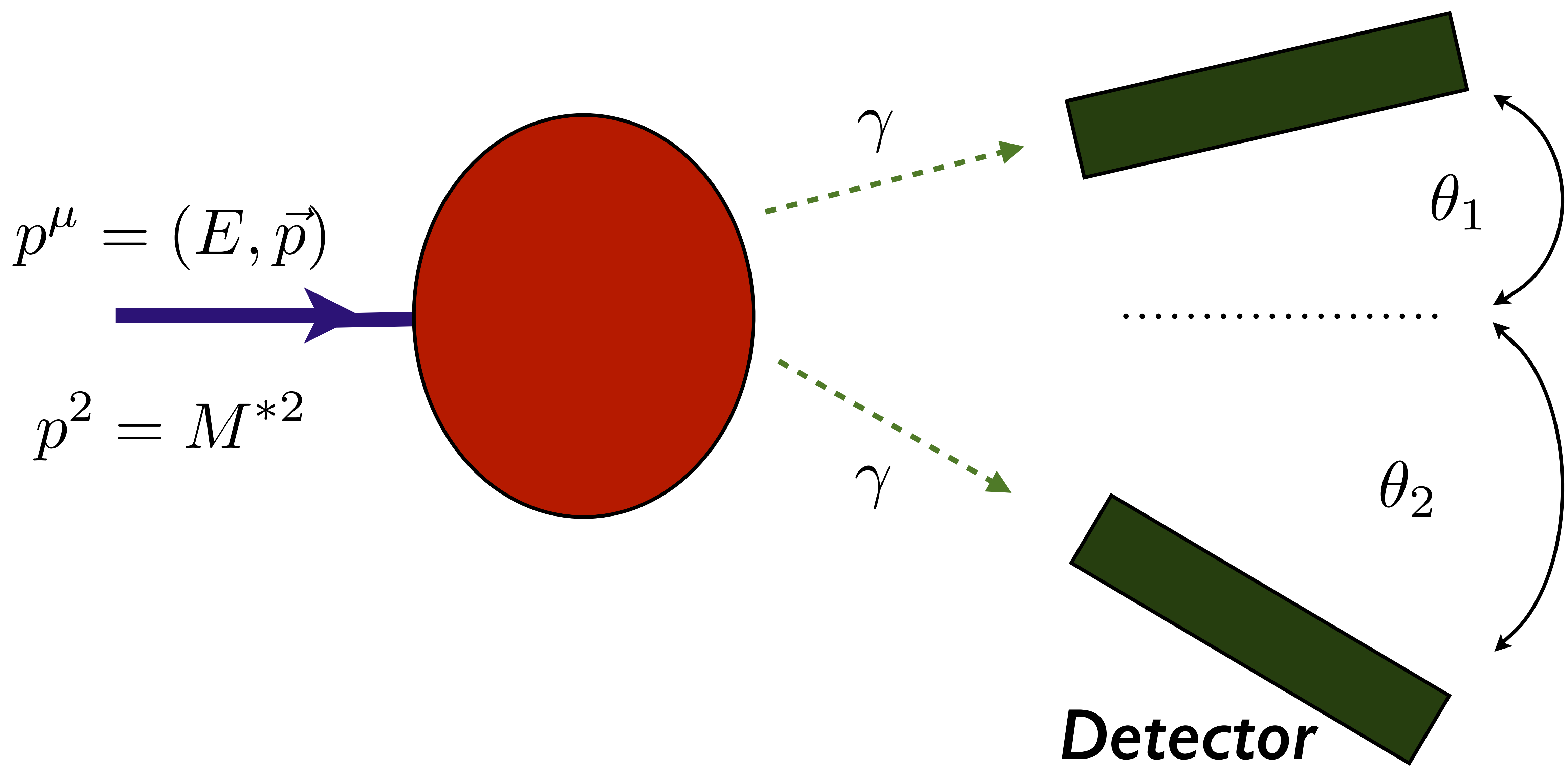
Einstein: Transverse Doppler Shift



The laser energy ω needed to excite the atom is reduced by the factor:

$$\frac{M}{(E + |\vec{p}| \cos\theta)}.$$

Detecting Decays of a Moving Excited Atom

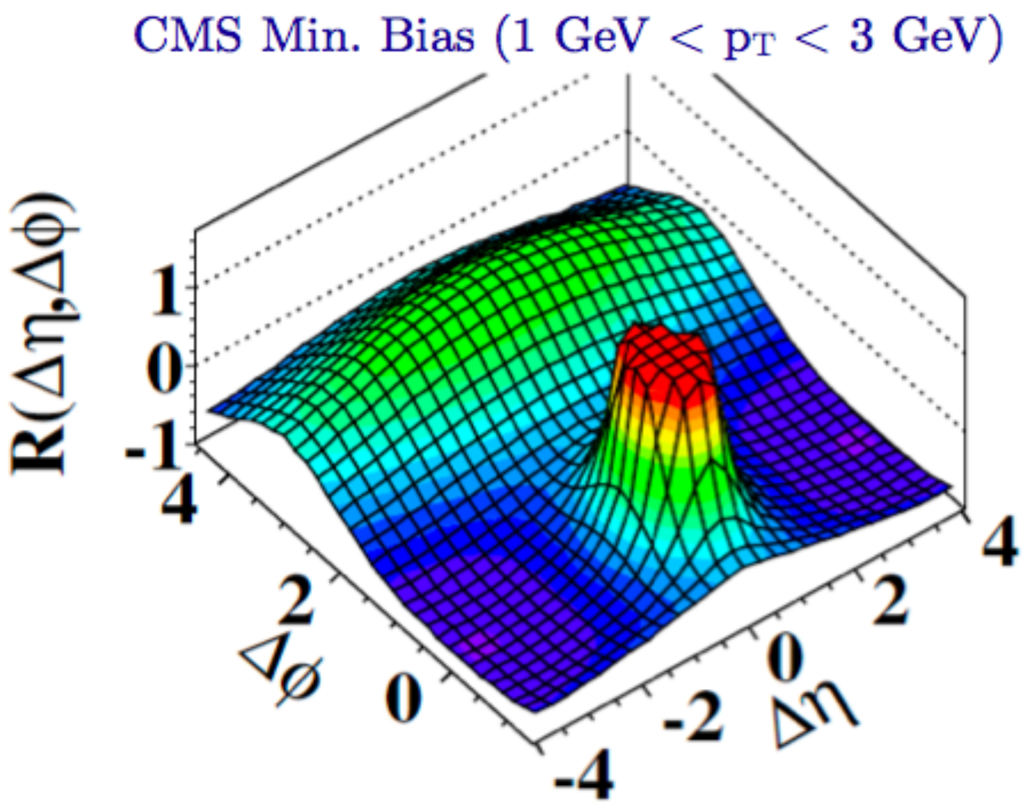


Each detector sees a different boost factor

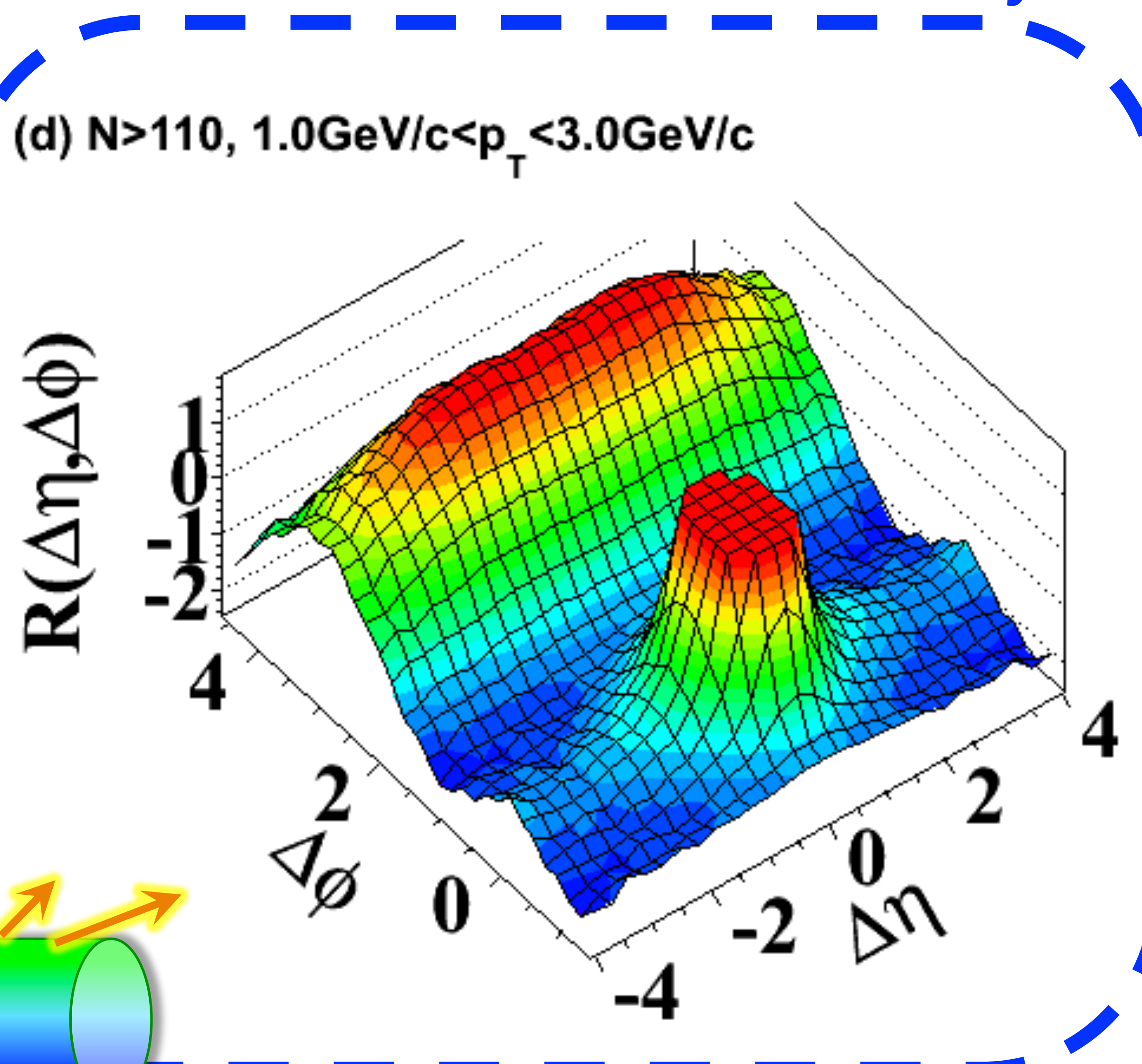
$$\frac{M}{(E + |\vec{p}| \cos\theta_i)}.$$

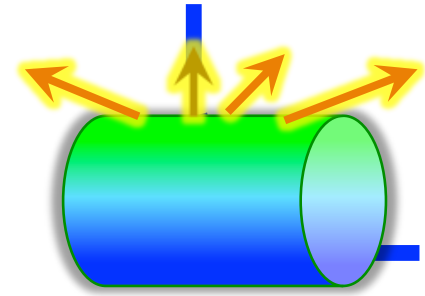
Ridge in high-multiplicity p p collisions

Two-particle correlations: CMS results



"Discovery"



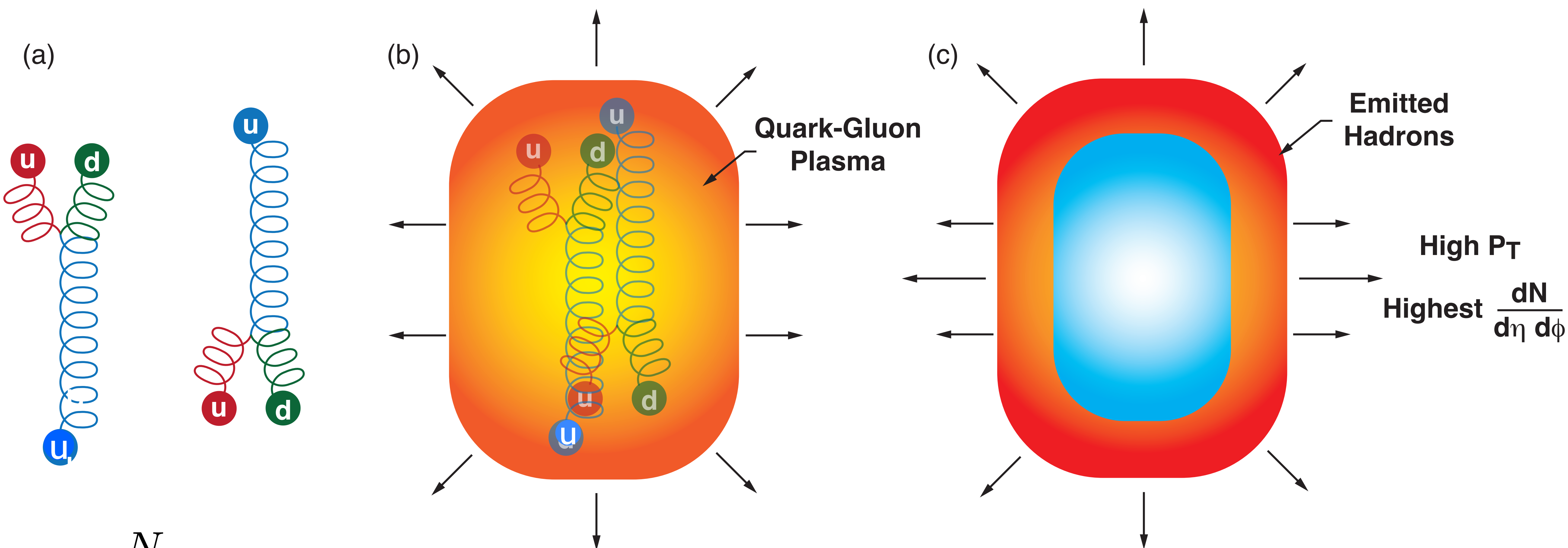


- **Ridge: Distinct long range correlation in η collimated around ΔΦ≈ 0 for two hadrons in the intermediate $1 < p_T, q_T < 3$ GeV**

Raju Venugopalan

Possible origin of same-side CMS ridge in p p Collisions

Bjorken, Goldhaber, sjb



$$\vec{V} = \sum_{i=1}^{N} [\cos 2\phi_i \hat{x} + \sin 2\phi_i \hat{y}]$$

Events spread over all longitudinal momenta, all rapidities

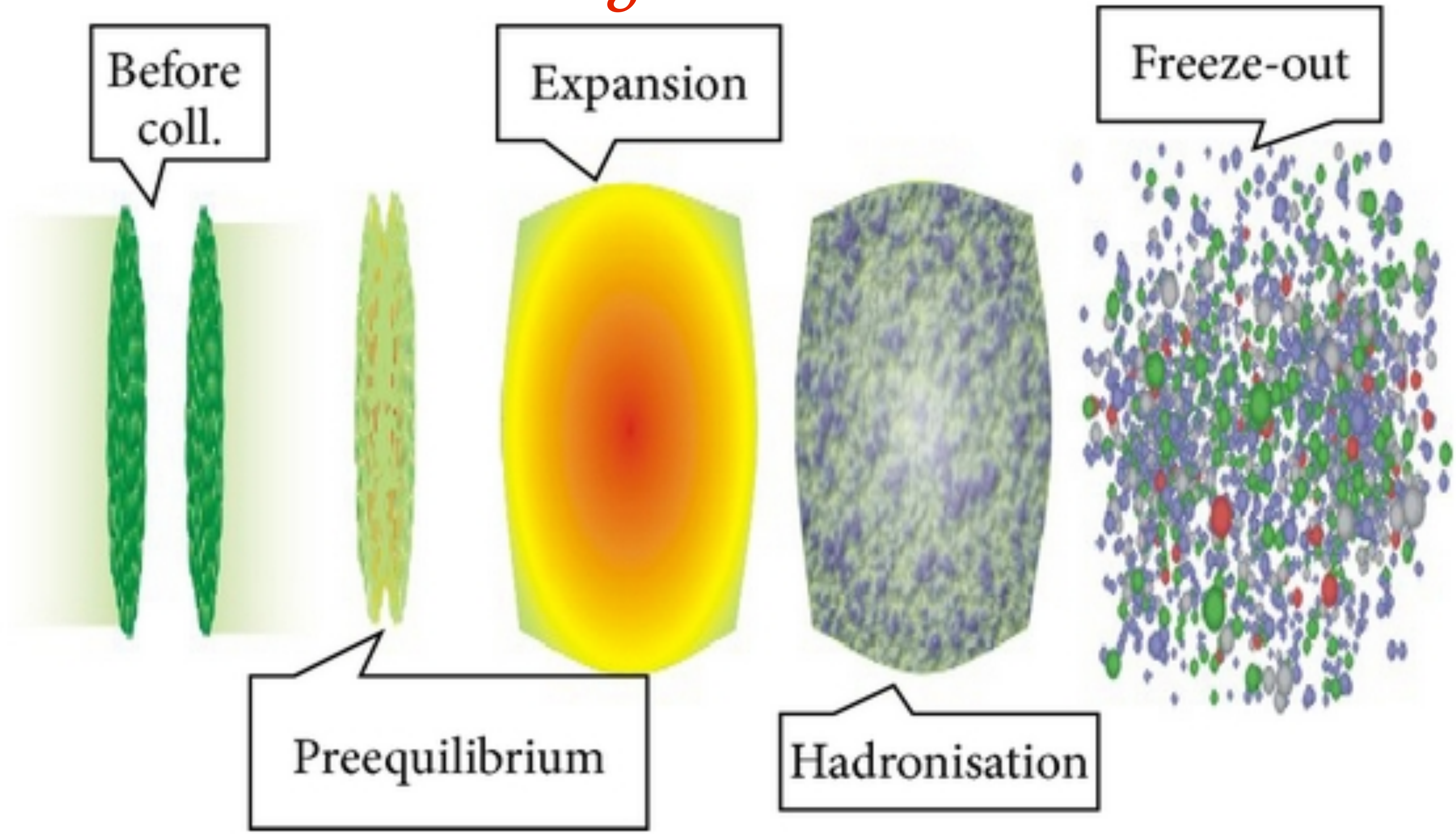
Possible multiparticle ridge-like correlations in very high multiplicity proton-proton collisions

Bjorken, Goldhaber, sjb

We suggest that this "ridge"-like correlation may be a reflection of the rare events generated by the collision of aligned flux tubes connecting the valence quarks in the wave functions of the colliding protons.

The "spray" of particles resulting from the approximate line source produced in such inelastic collisions then gives rise to events with a strong correlation between particles produced over a large range of both positive and negative rapidity.

Heavy-Ion Collisions Visualized as Colliding "Pancakes"



Misleading! No "squashing".
Lorentz contraction is mutual property of detector and emitter.

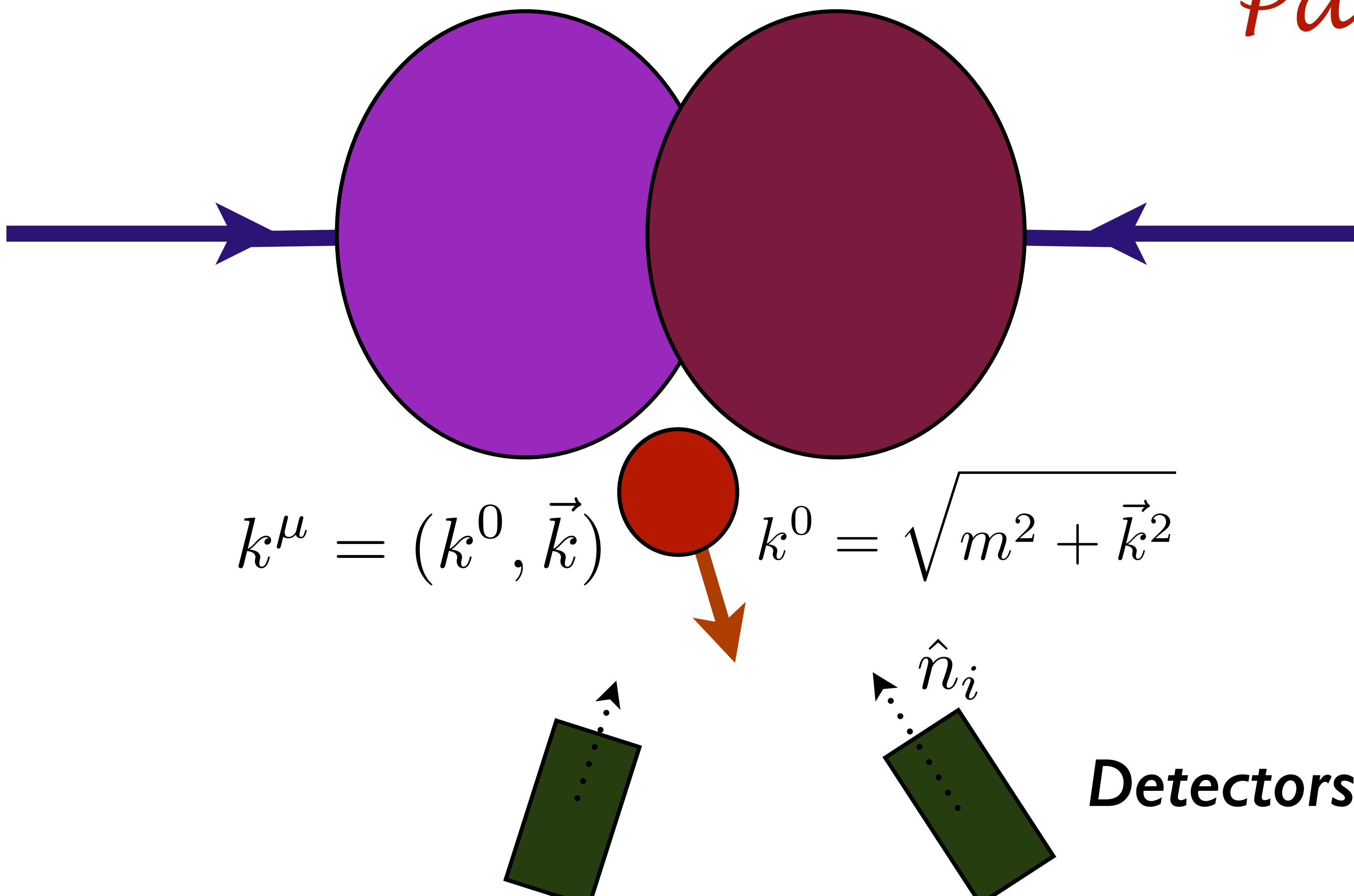
Hadron wavefunctions: dx/x parton distribution

Problems with Colliding "Pancakes"

- Assumes Lorentz boosts in the instant form
- Measurement requires infinite synchronization
- Occupants of a fast-moving spaceship not squashed
- Distribution of partons dx/x — no single boost
- Can one get the same result in rest frame of one ion?
- Lorentz boost of instant form wavefunctions dynamical — even particle number changes!
- Vacuum induced contributions needed

Colliding Protons or Ions

Not “Pancakes”



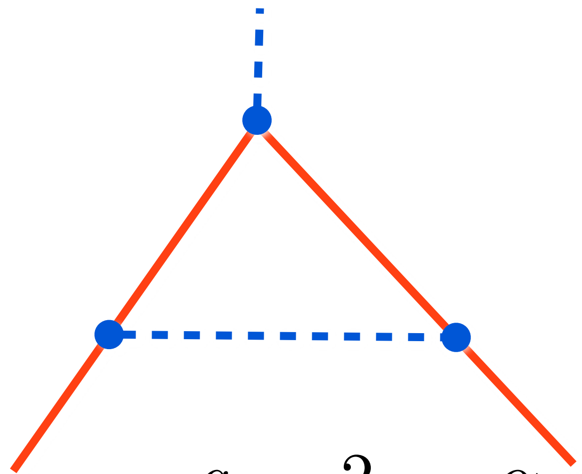
$$k^\mu = (k^0, \vec{k}) \qquad k^0 = \sqrt{m^2 + \vec{k}^2}$$

Each detector sees a different boost for each emitted particle

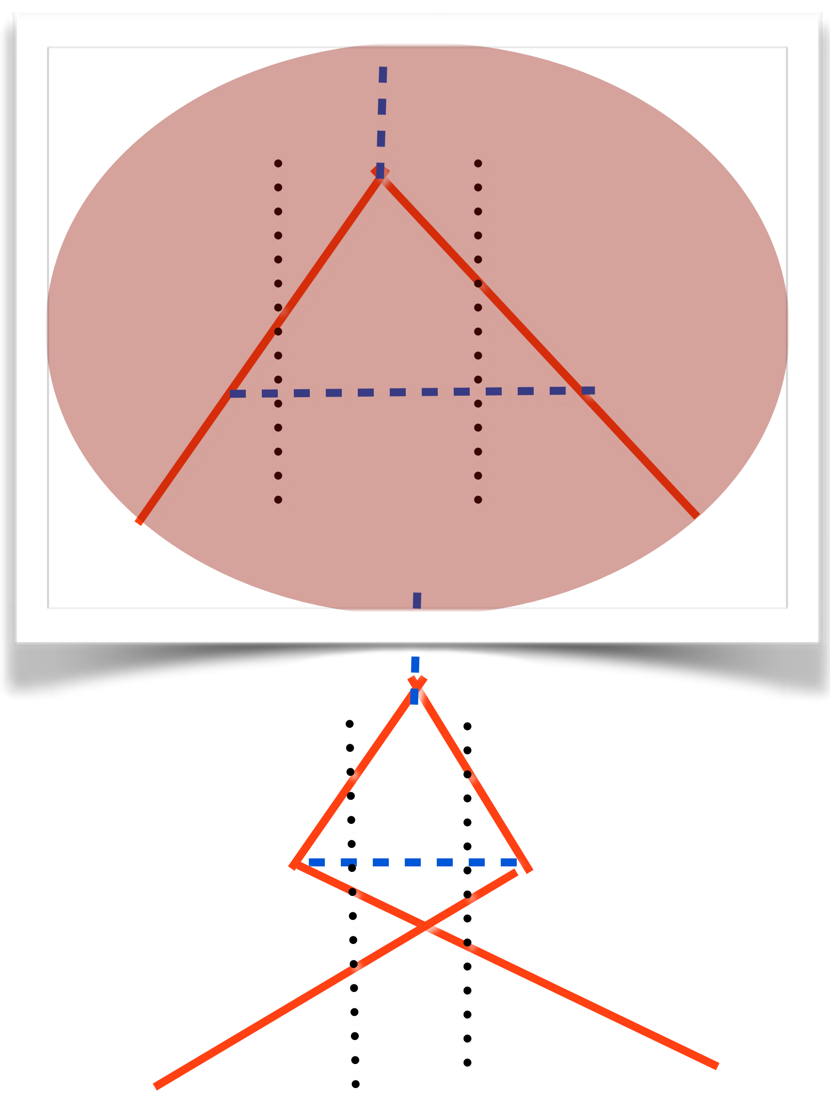
$$\frac{m}{k^0 + \vec{k} \cdot \hat{n}_i}.$$

Wick Theorem

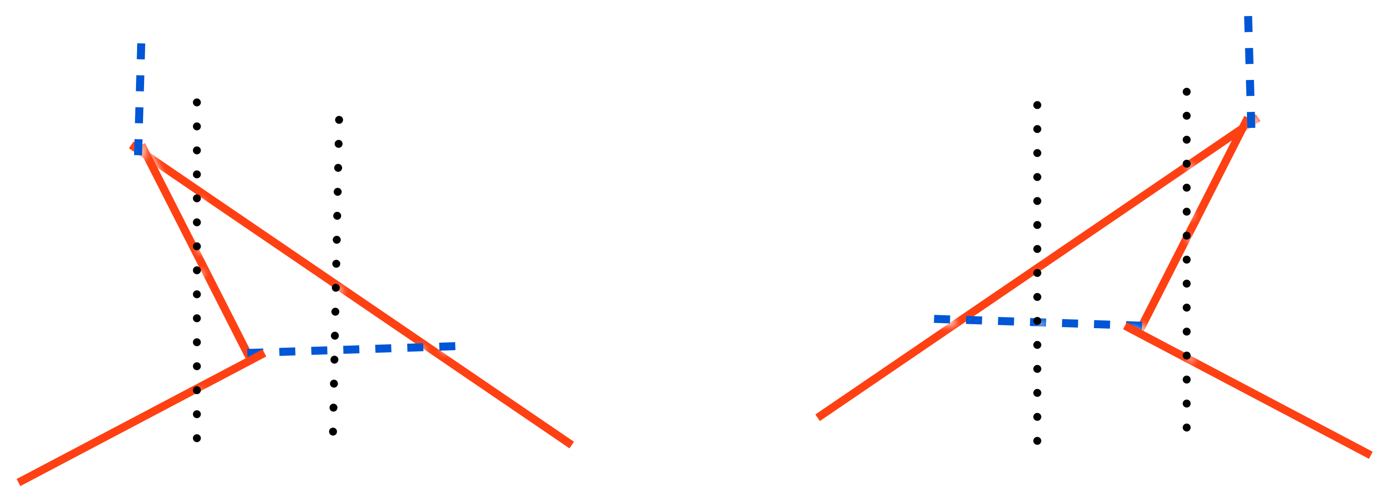
Feynman diagram =
single front-form time-ordered diagram!



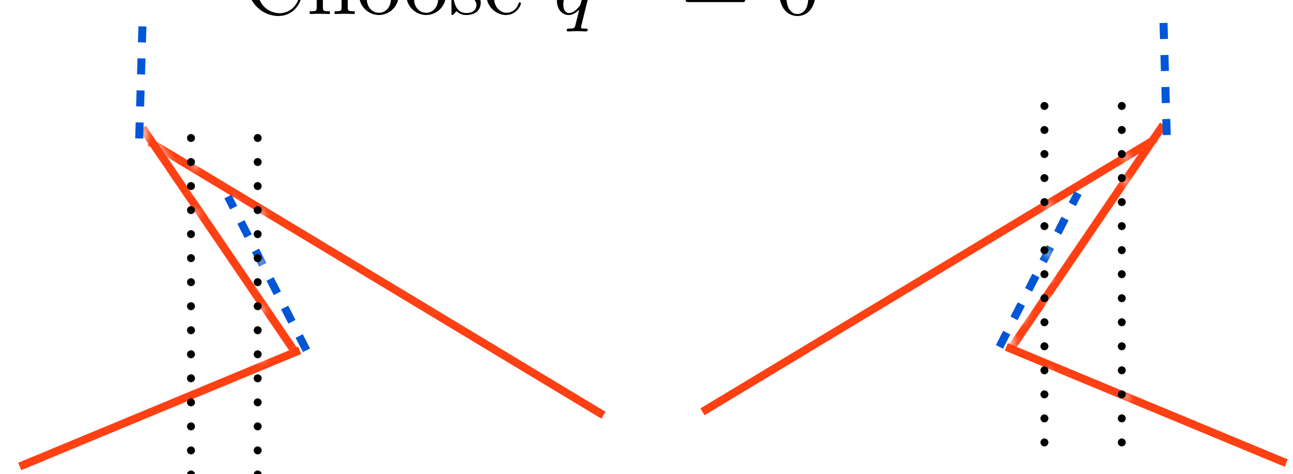
$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$



Also $P \to \infty$ observer frame (Weinberg)

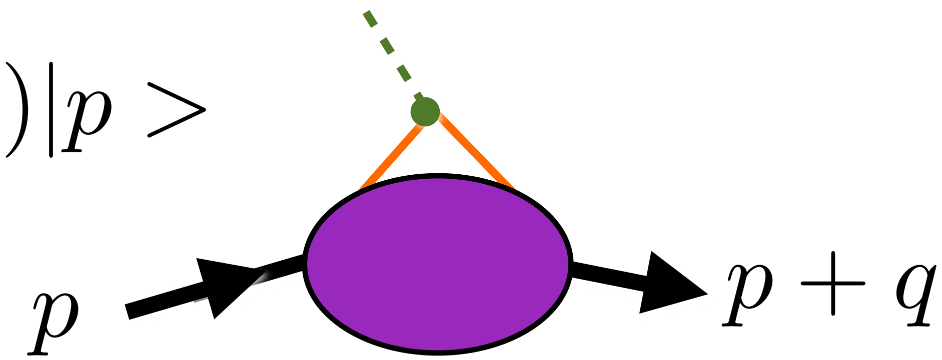


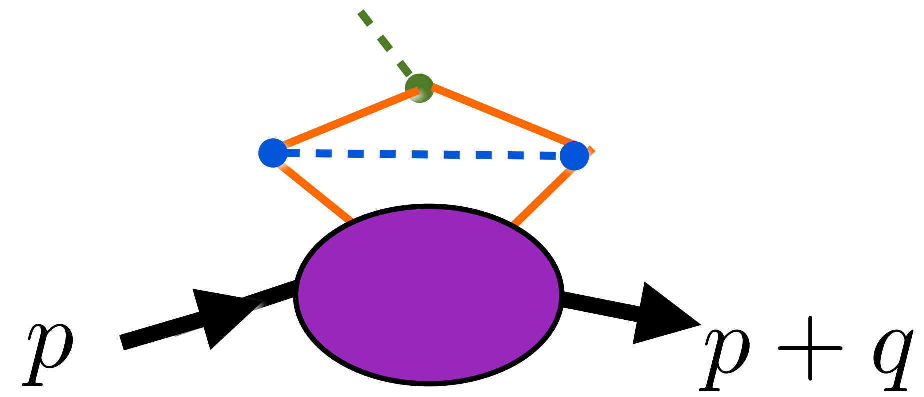
Choose $q^+ = 0$



Calculation of proton form factor in Instant Form

$$< p+q|J^\mu(0)|p >$$





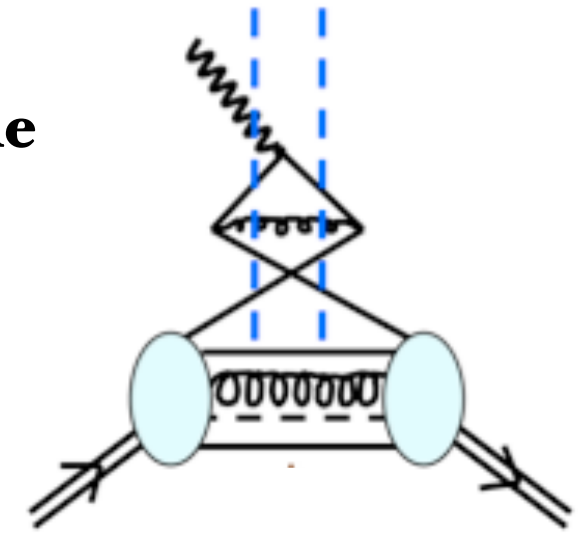
- **Need to boost proton wavefunction: p to p+q. Extremely complicated dynamical problem; particle number changes!**
- **Need to couple to all currents arising from vacuum!! Remain even after normal-ordering**
- **Instant-form WFs insufficient to calculate form factors**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**

Boost of a Composite System

- Boost is not product of independent boosts of constituents since constituents are already moving
- Only known at weak binding
- Dirac: Boosts are dynamical
- Correct form needed to prove Low Energy Theorem for Compton scattering and Drell-Hearn Gerasimov Sum Rule

Disadvantages of the Instant Form

- Boosts are dynamical, change particle number: not Melosh!
- Famous wrong proof showing violation of LET and DHG sum rule
- Each Amplitude is Frame-Dependent
- States defined at one instant of time over all space - acausal!
- Current matrix elements involve connected vacuum currents -- eigensolutions insufficient!
- N! time-ordered graphs, each frame-dependent
- Vacuum is complex: apparently gives huge vacuum energy density
- Cluster decomposition theorem fails in relativistic systems
- Virtually no valid calculations of dynamics of relativistic composite systems use the instant form
- Why Feynman invented Feynman diagrams!

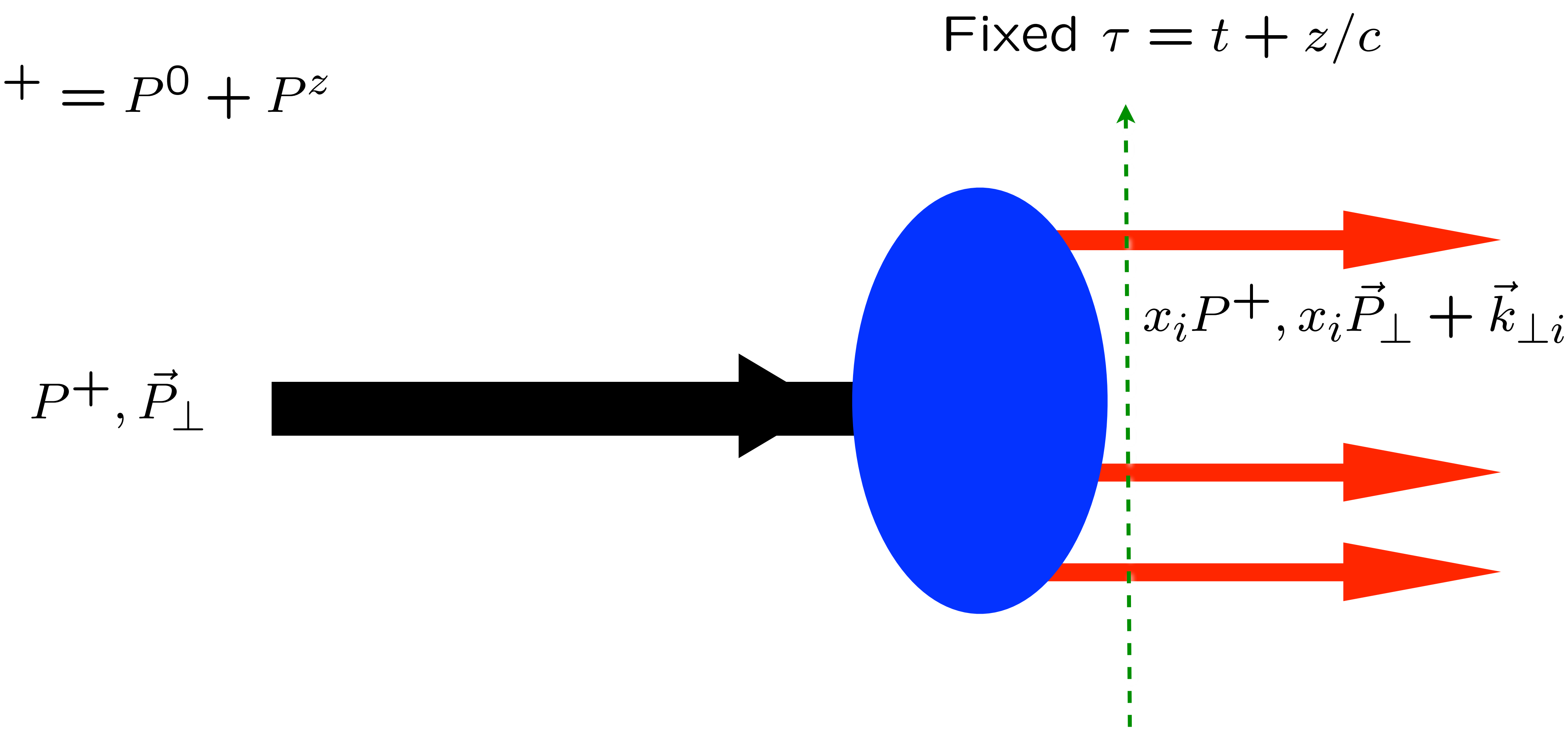


Light-Front vs. Instant Form

- Light-Front Wavefunctions are frame-independent
- Boosting an instant-form wavefunctions dynamical problem -- extremely complicated even in QED
- Vacuum state is lowest energy eigenstate of Hamiltonian
- Light-Front Vacuum same as vacuum of free Hamiltonian
- Zero anomalous gravitomagnetic moment
- Instant-Form Vacuum infinitely complex even in QED
- n! time-ordered diagrams in Instant Form
- Causal commutators using LF time; cluster decomposition

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$P^+ = P^0 + P^z$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

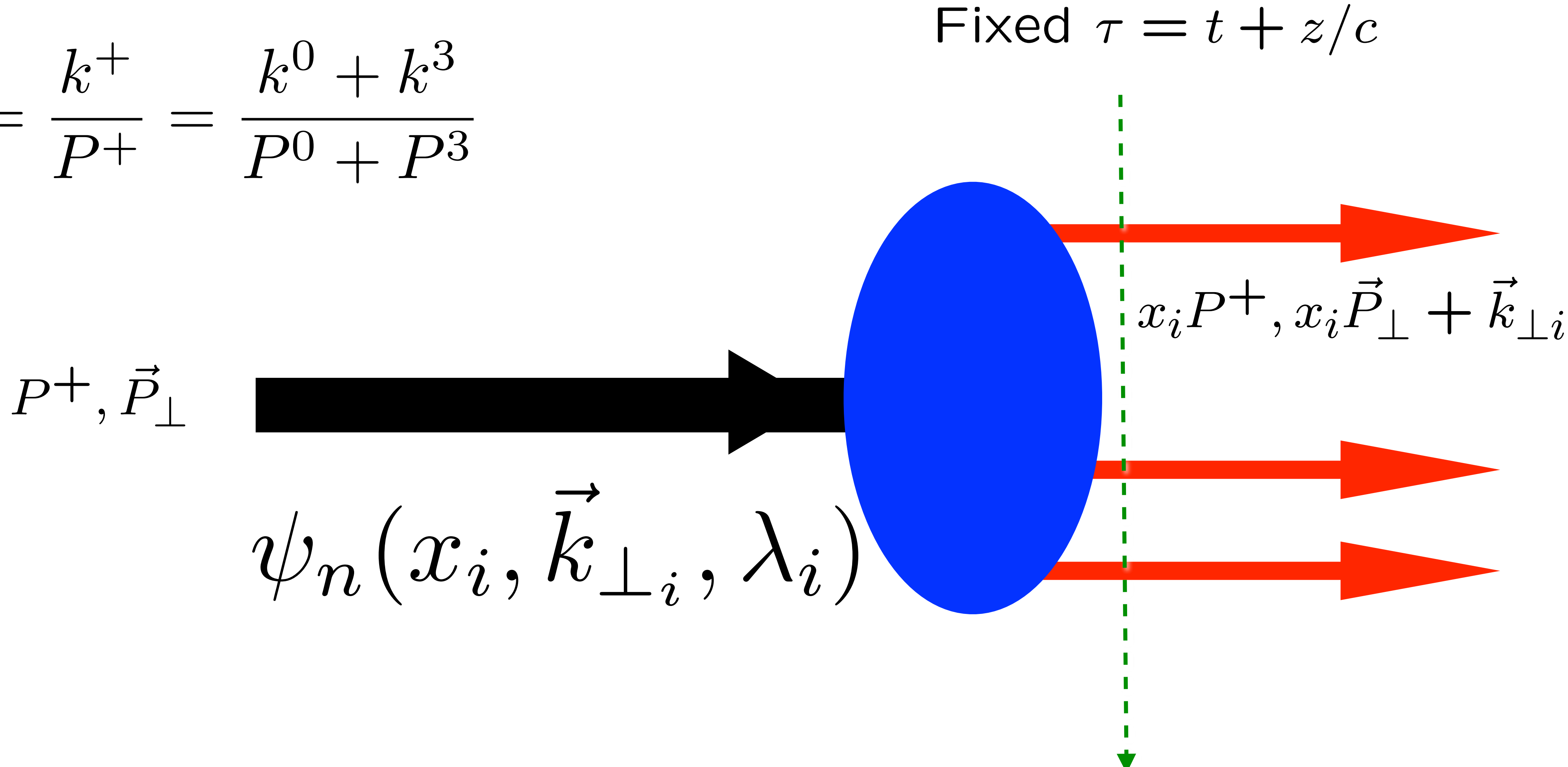
$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

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Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Angular Momentum on the Light-Front

LC gauge $A^+=0$

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

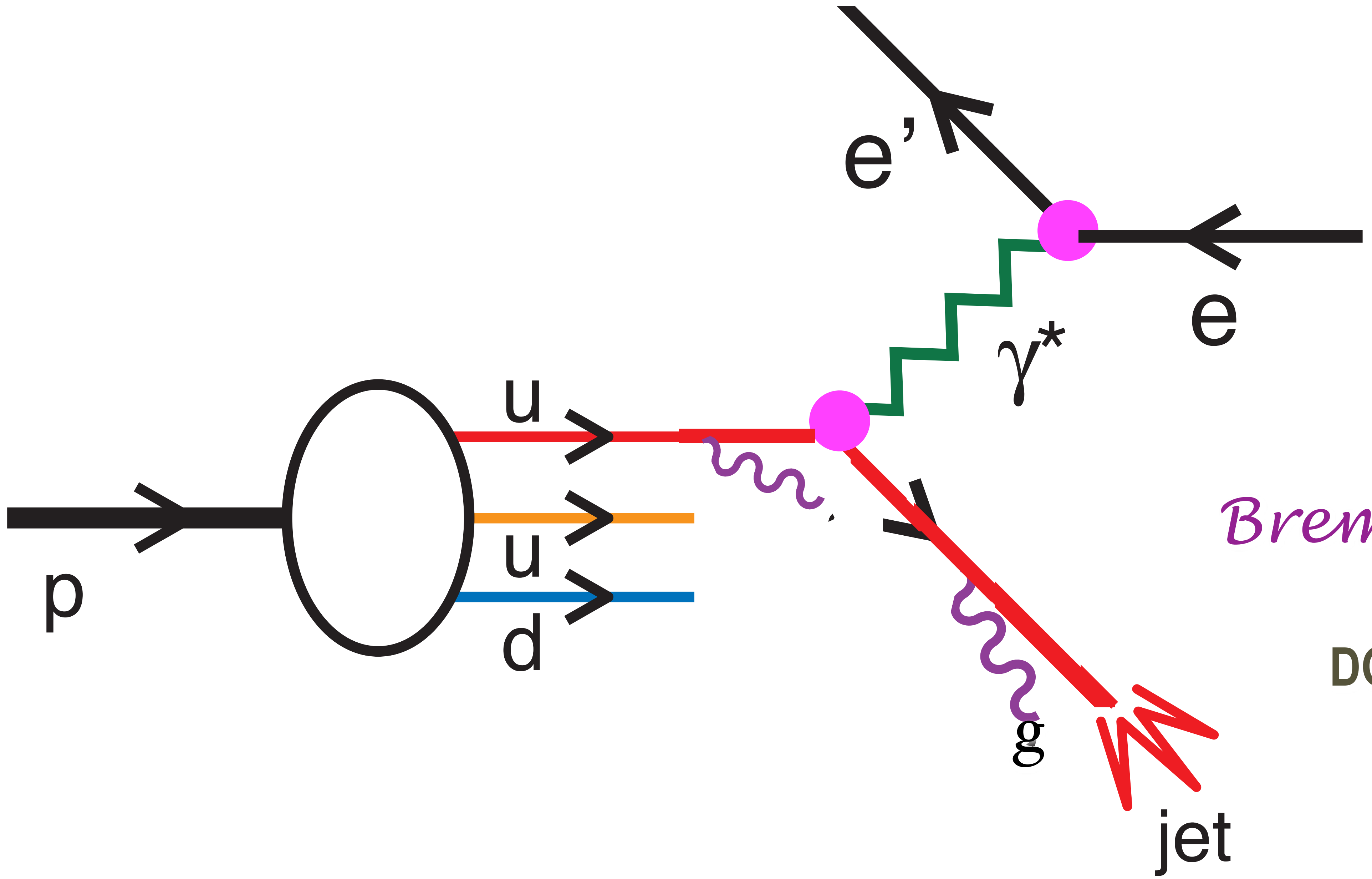
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

Deep Inelastic Electron-Proton Scattering



Gluonic Bremmstrahlung

DGLAP Evolution

$$\left|\psi_p(P^+, \vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\, 16\pi^3} 16\pi^3 \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{\perp i}\right)$$

$$\times \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \left| n;\, x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i \right\rangle.$$

$$q_{\lambda_q/\Lambda_p}(x, \Lambda) = \sum_{n, q_a} \int \prod_{j=1}^{n} \mathrm{d}x_j\, \mathrm{d}^2\vec{k}_{\perp j} \sum_{\lambda_i} \left|\psi_{n/H}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$

$$\times \delta\left(1 - \sum_{i}^{n} x_i\right) \delta^{(2)}\left(\sum_{i}^{n} \vec{k}_{\perp i}\right) \delta(x - x_q) \delta_{\lambda_a \lambda_q} \Theta\left(\Lambda^2 - \mathcal{M}_n^2\right),$$

Obeys DGLAP Evolution *Defines quark distributions*

Connection to Bethe-Salpeter:

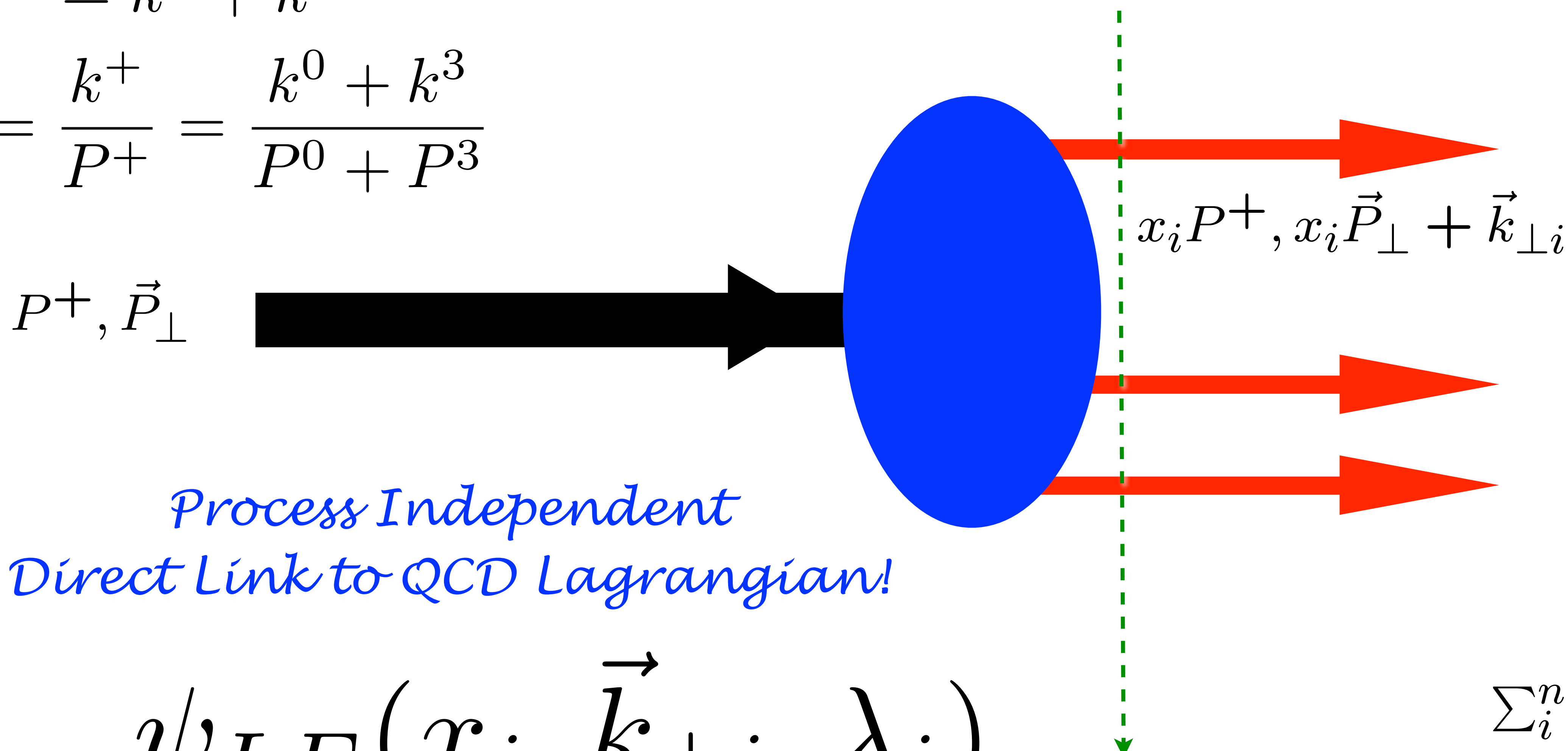
$$\int dk^- \Psi_{BS}(k, P) \rightarrow \psi_{LF}(x, \vec{k}_\perp) \qquad \Psi_{BS}(x, P)\big|_{x^+=0}$$

$$\int \Psi_{BS}(P,k)dk^0 = \psi_{IF}(\vec{k}) \qquad \int \Psi_{BS}(P,k)dk^- = \psi_{LF}(\vec{k})$$

Fixed $\tau = t + z/c$

$$k^{\pm} = k^0 + k^3$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Process Independent
Direct Link to QCD Lagrangian!

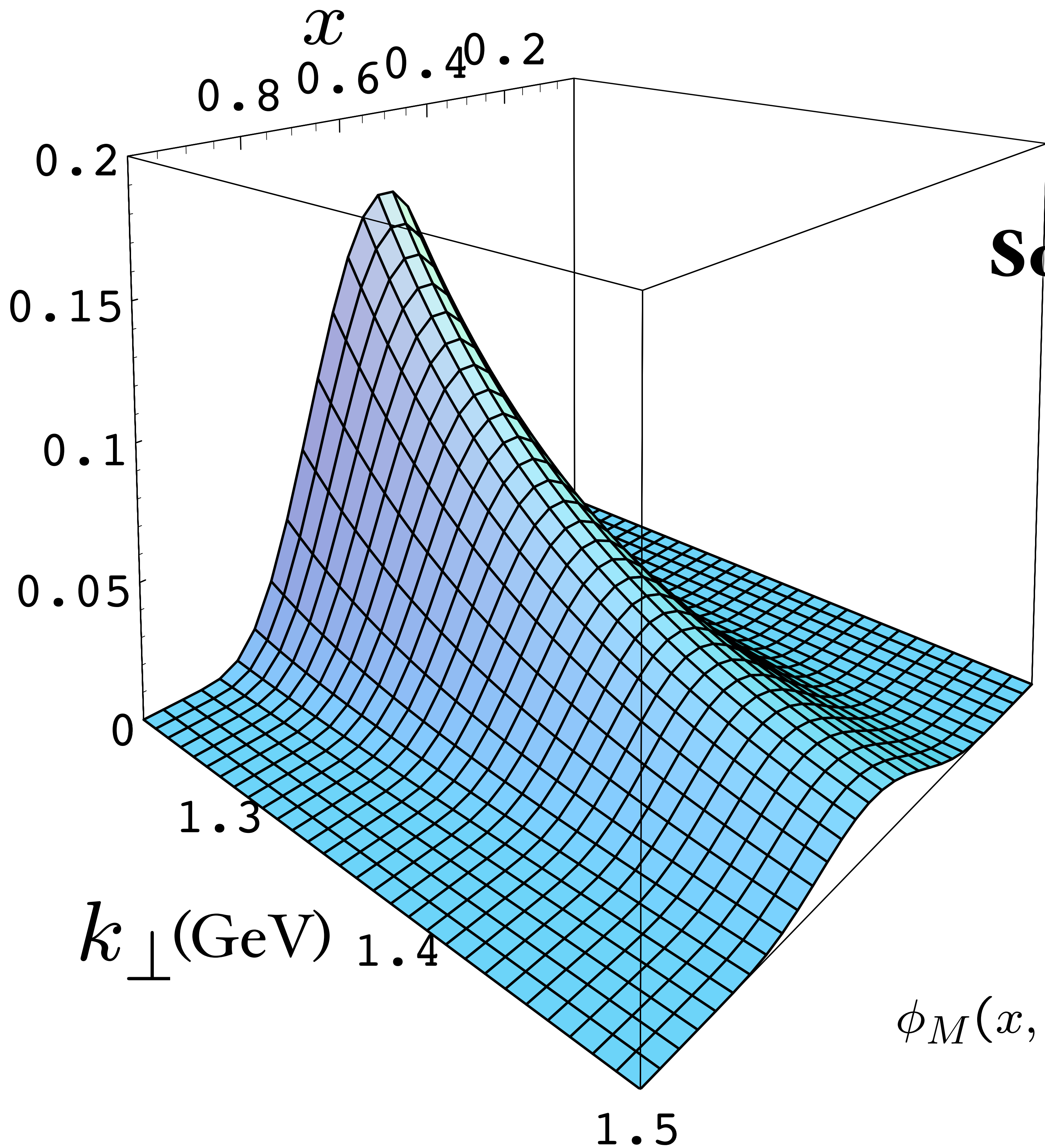
$$\psi_{LF}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Prediction from AdS/CFT: Meson LFWF



Soft Wall Model

de Teramond, sjb

$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$

Increases PQCD prediction for $F_\pi(Q^2)$ by 16/9

J/ψ

LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

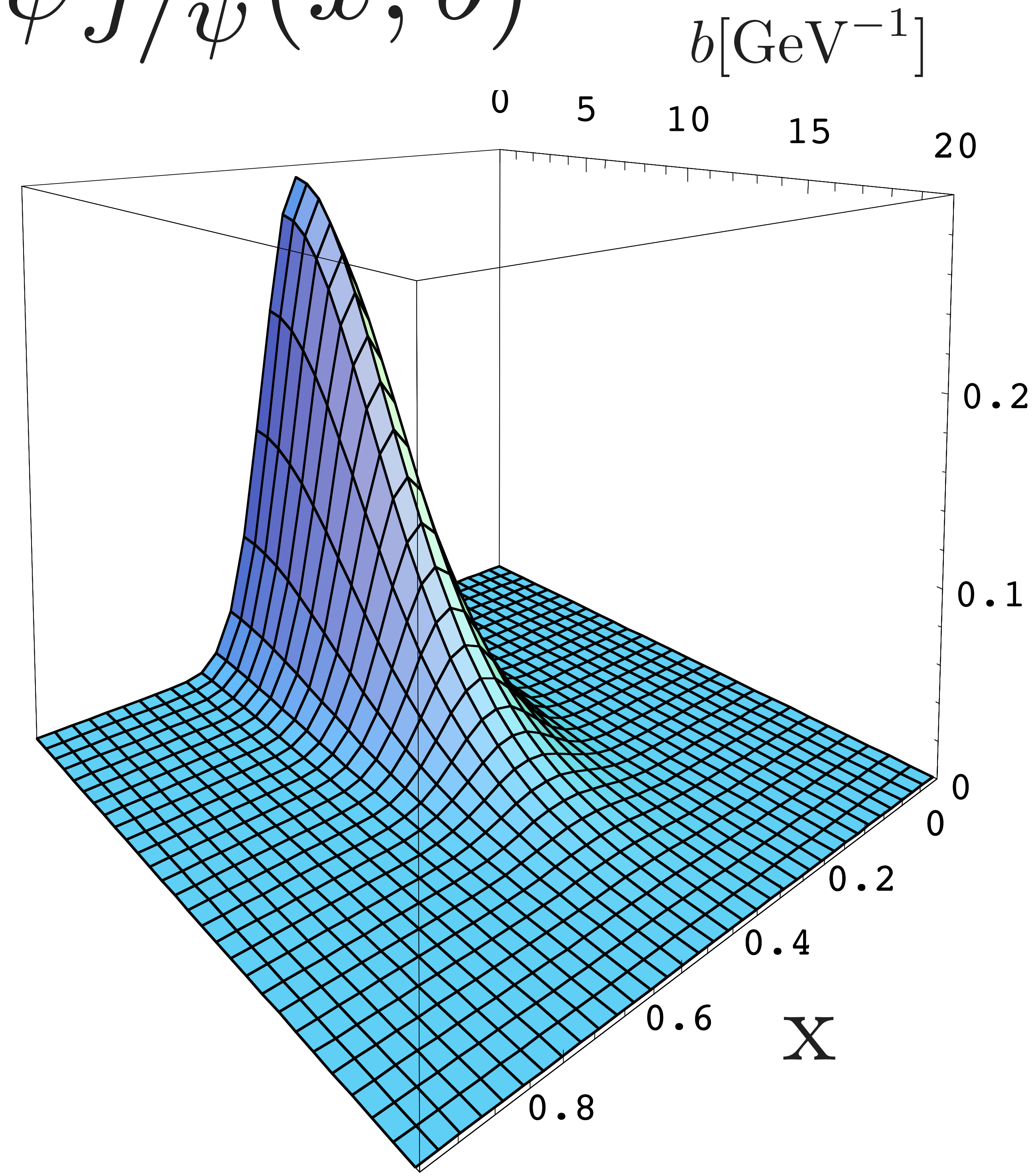
where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

minimum of LF energy denominator

$$\kappa = 0.375 \text{ GeV}$$

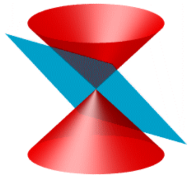
$$\psi_{J/\psi}(x, b)$$



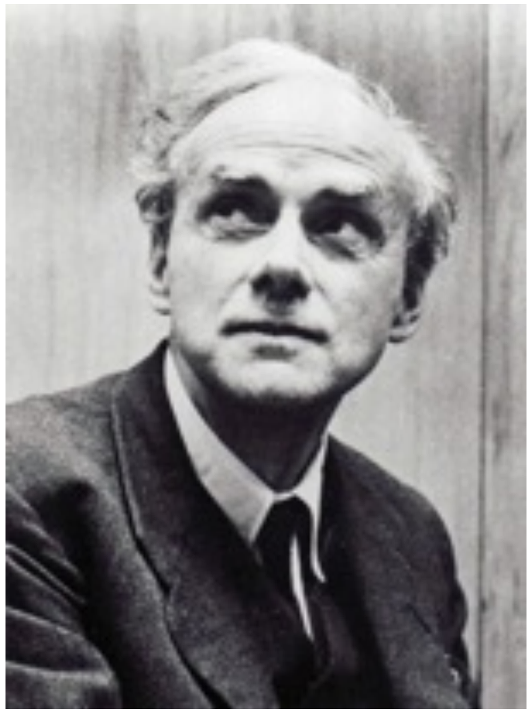
$$m_a = m_b = 1.25 \text{ GeV}$$

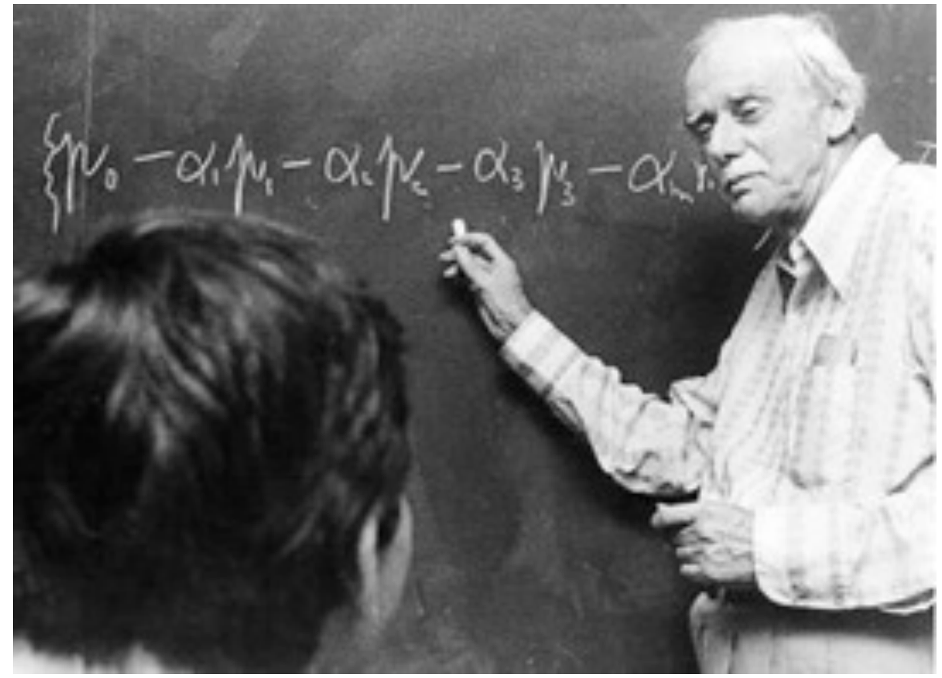
Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts!
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant



P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)





"Working with a front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out " -

P.A.M. Dirac (1977)

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics — quark kinetic energy + quark-gluon dynamics — quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

QCD Mass Scale from Confinement not Explicit

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

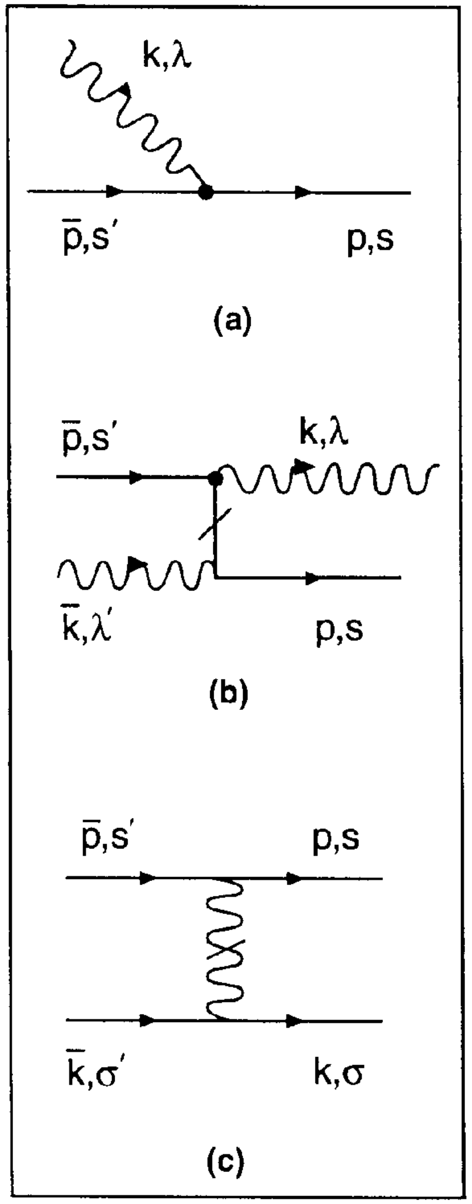
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

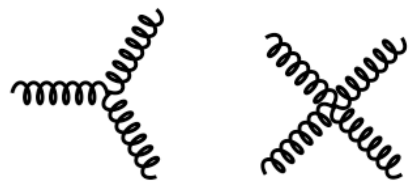
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

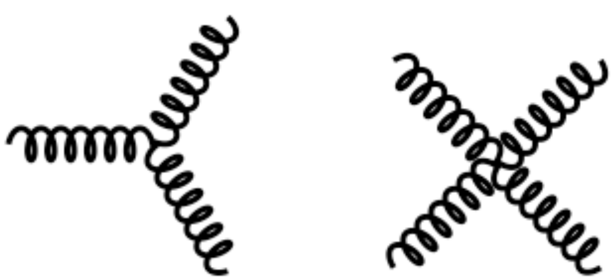


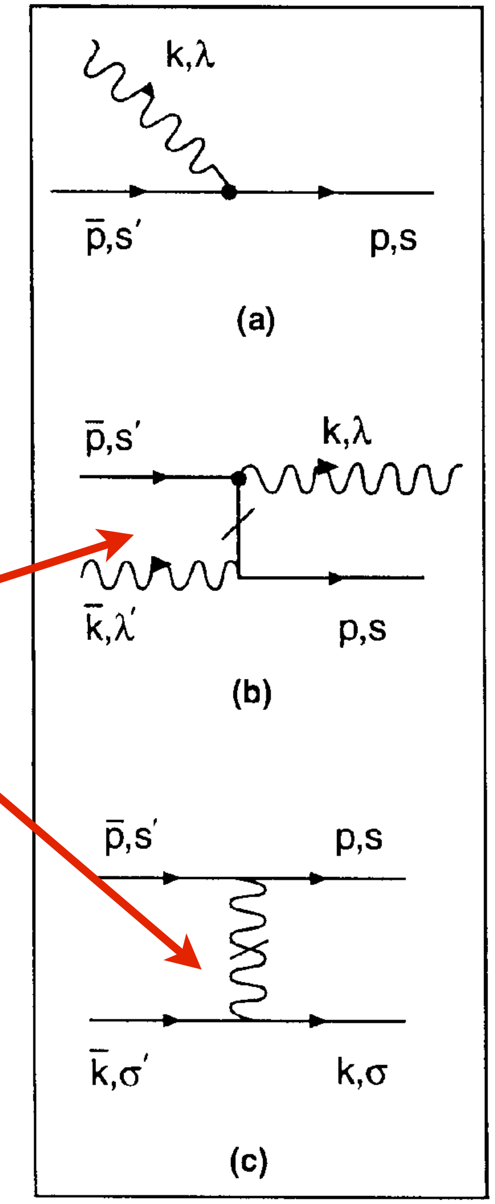
H^{int}_{LF}

LFWFs: Off-shell in P- and invariant mass

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$\begin{aligned}
H^{LF}_{QCD} &= \frac{1}{2}\int d^3x \overline{\tilde{\psi}}\gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+}\tilde{\psi} - A^i_a (i\partial^\perp)^2 A_{ia} \\
&- \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(i\partial^+)^2} \overline{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\
&- g^2 \int d^3x \overline{\tilde{\psi}}\gamma^+ \left(\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\
&+ g^2 \int d^3x \mathbf{Tr}\left(\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(i\partial^+)^2}\left[i\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\
&+ \frac{1}{2}g^2 \int d^3x \overline{\tilde{\psi}} \tilde{\not{A}} \frac{\gamma^+}{i\partial^+} \tilde{\not{A}} \tilde{\psi} \\
&+ g \int d^3x \overline{\tilde{\psi}} \tilde{\not{A}} \tilde{\psi} \\
&+ 2g \int d^3x \mathbf{Tr}\left(i\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right)
\end{aligned}$$





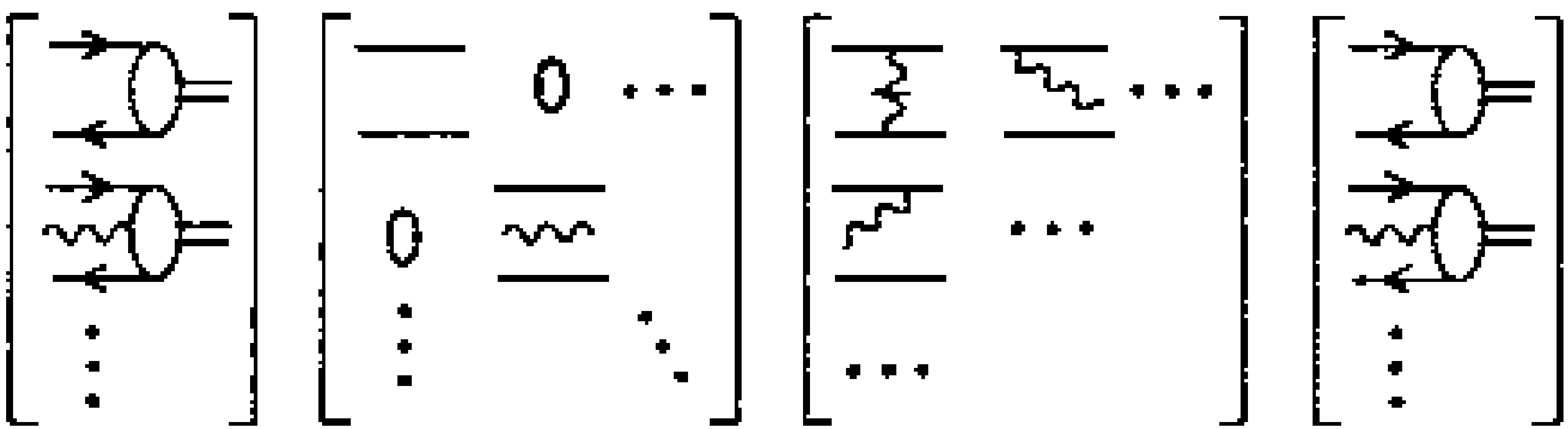
Physical gauge: $A^+ = 0$

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V \,|q\bar{q}\rangle & \langle q\bar{q}|\, V \,|q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V \,|q\bar{q}\rangle & \langle q\bar{q}g|\, V \,|q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

$$A^+ = 0$$



Minkowski space; frame-independent; no fermion doubling; no ghosts

- Light-Front Vacuum = vacuum of free Hamiltonian!

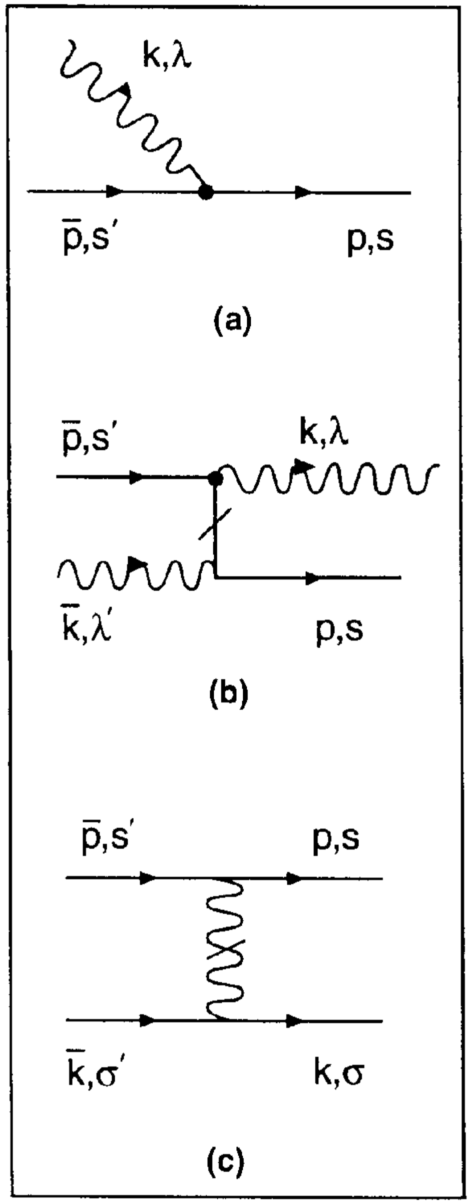
Light-Front QCD

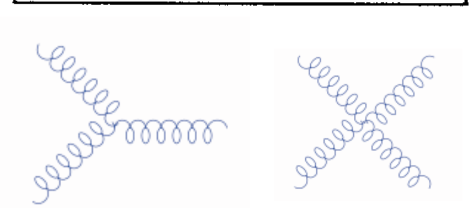
Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb

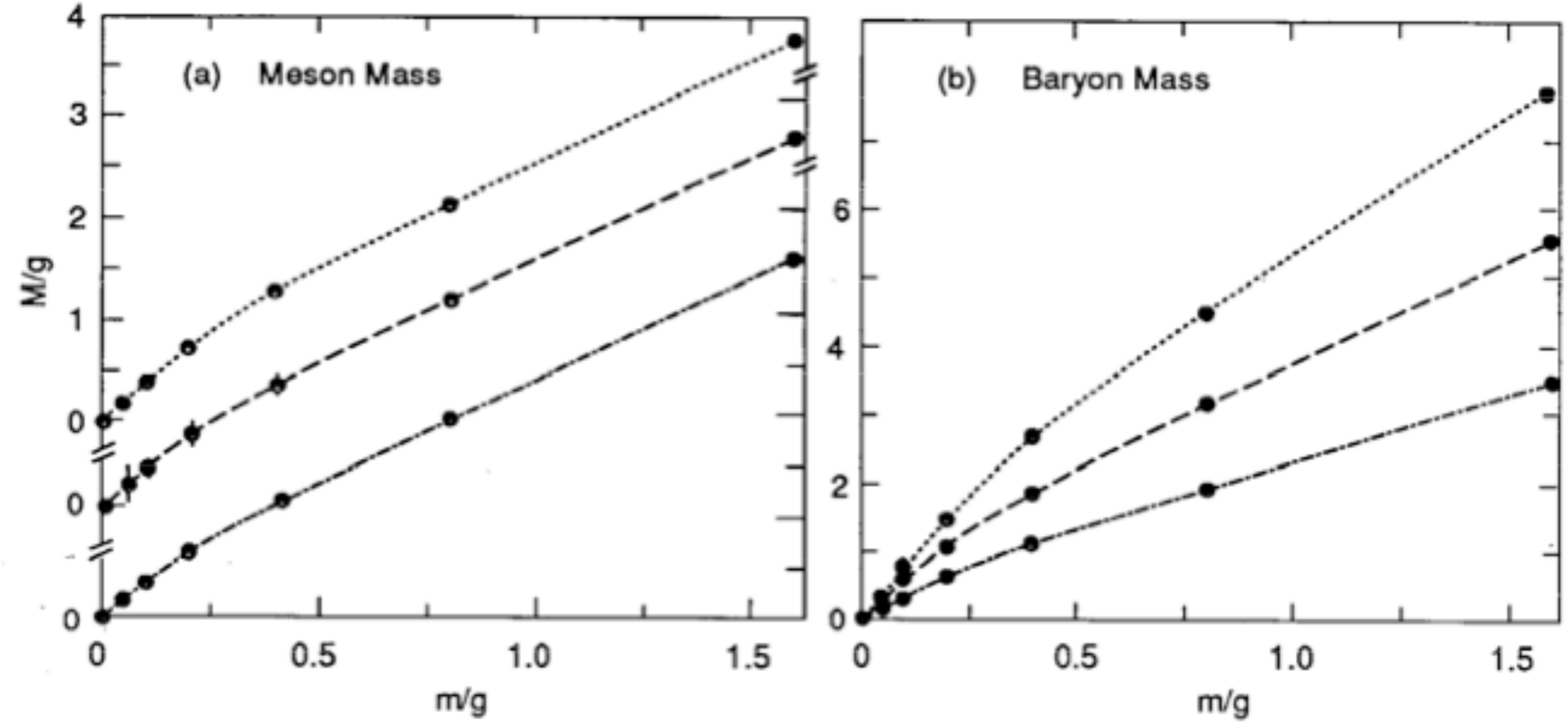




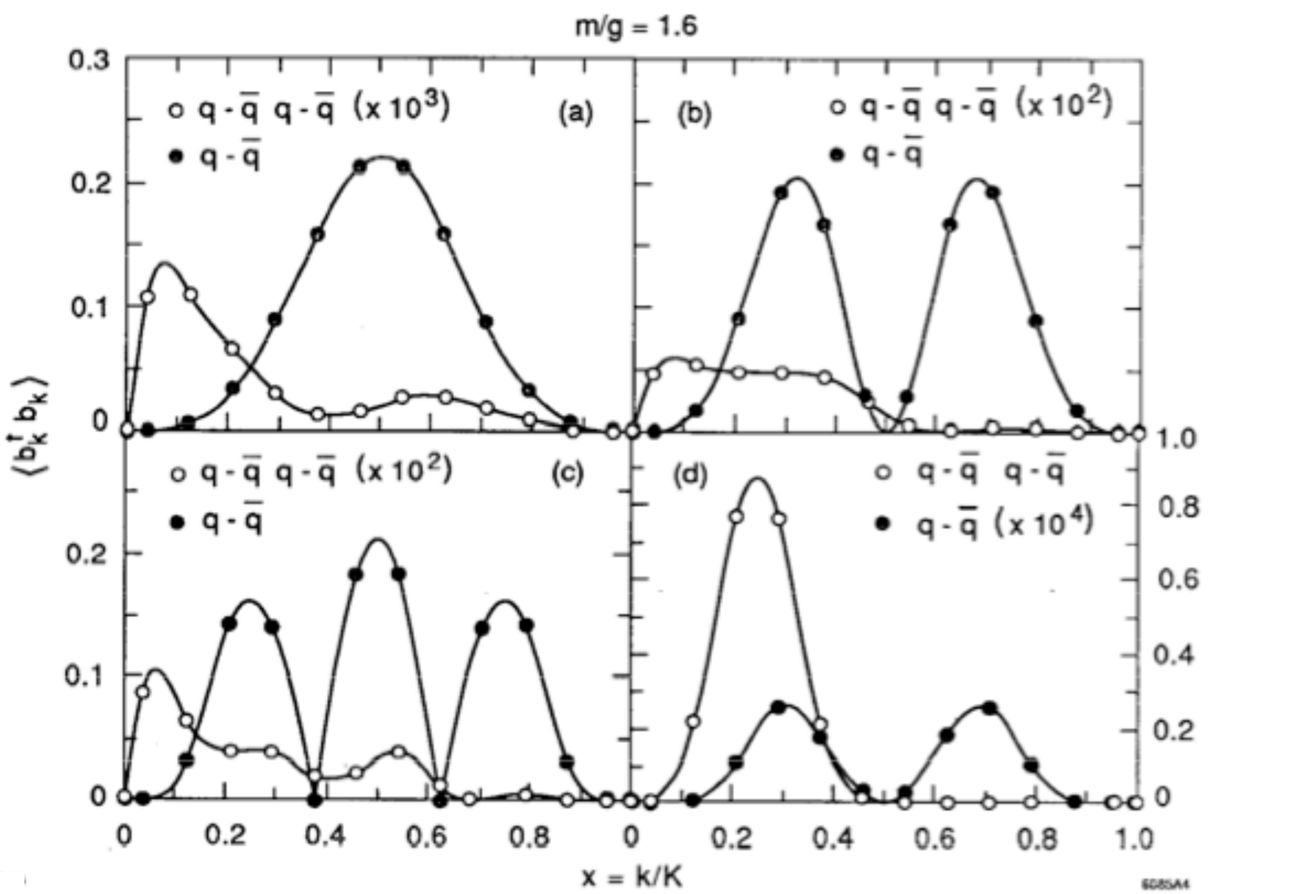
n	Sector	1 $q\bar{q}$	2 gg	3 $q\bar{q}\,g$	4 $q\bar{q}\,q\bar{q}$	5 $gg\,g$	6 $q\bar{q}\,gg$	7 $q\bar{q}\,q\bar{q}\,g$	8 $q\bar{q}\,q\bar{q}\,q\bar{q}$	9 $gg\,gg$	10 $q\bar{q}\,gg\,g$	11 $q\bar{q}\,q\bar{q}\,gg$	12 $q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	13 $q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$
1	$q\bar{q}$					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	$q\bar{q}\,g$								•	•		•	•	•
4	$q\bar{q}\,q\bar{q}$		•			•				•	•		•	•
5	$gg\,g$	•			•			•	•			•	•	•
6	$q\bar{q}\,gg$								•				•	•
7	$q\bar{q}\,q\bar{q}\,g$	•	•			•				•				•
8	$q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•		•	•			•	•			
9	$gg\,gg$	•		•	•			•	•			•	•	•
10	$q\bar{q}\,gg\,g$	•	•		•				•				•	•
11	$q\bar{q}\,q\bar{q}\,gg$	•	•	•		•				•				•
12	$q\bar{q}\,q\bar{q}\,q\bar{q}\,g$	•	•	•	•	•	•			•	•			
13	$q\bar{q}\,q\bar{q}\,q\bar{q}\,q\bar{q}$	•	•	•	•	•	•	•		•	•	•		

Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

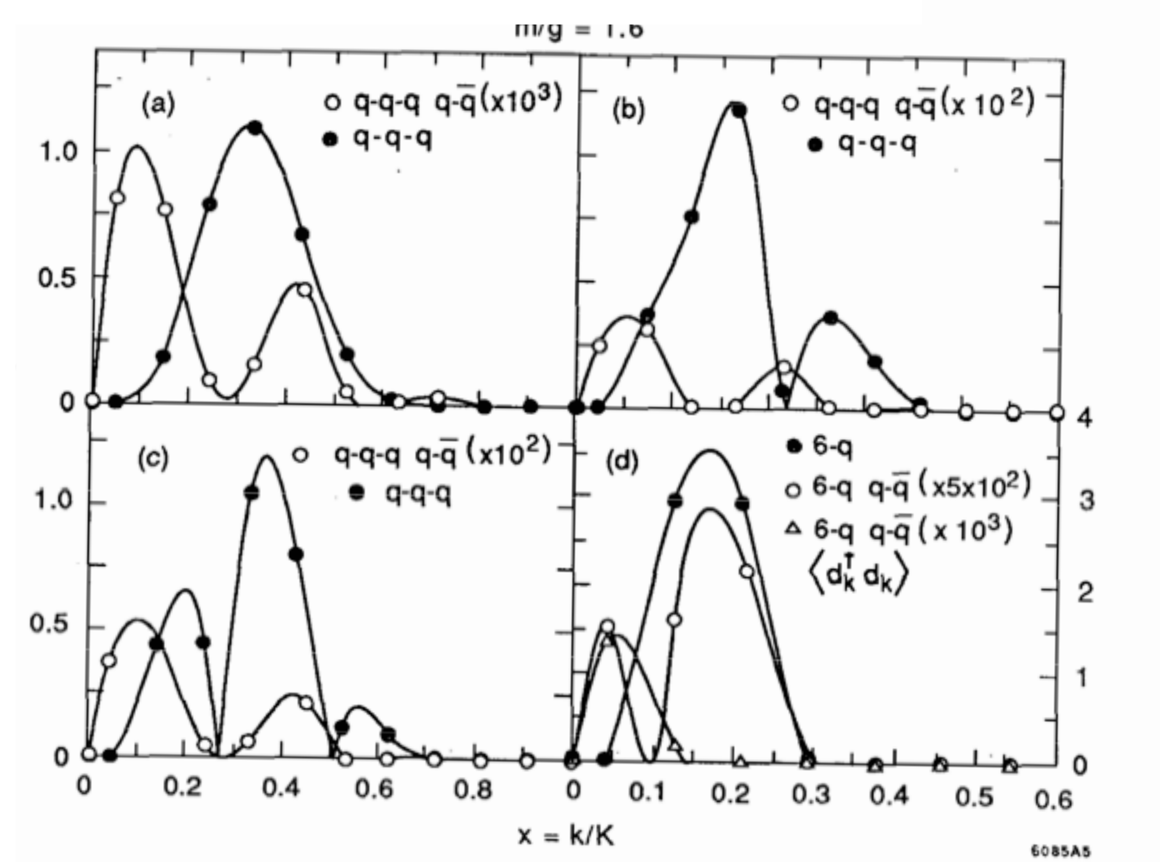
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

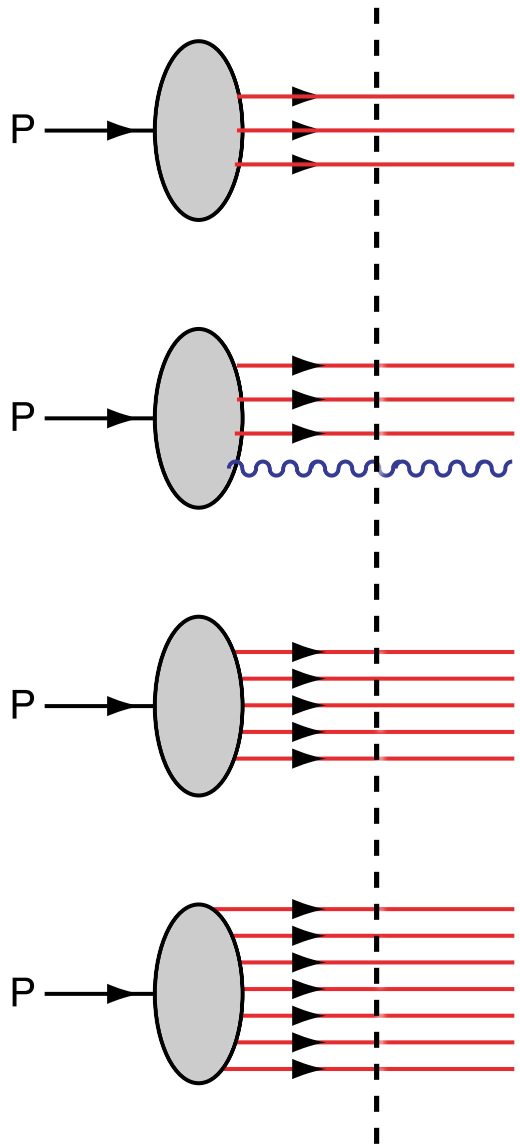
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

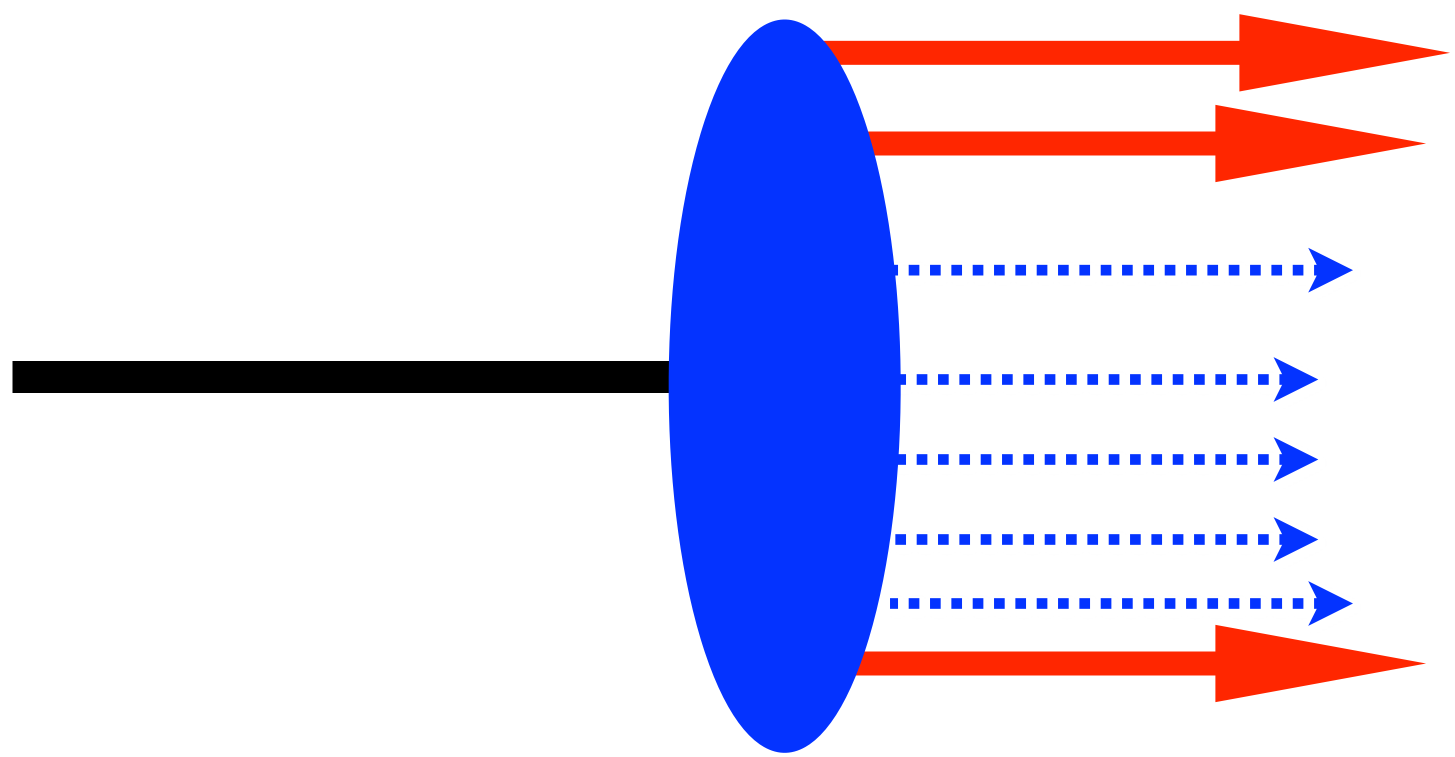
Mueller: gluon Fock states **BFKL Pomeron**

Hidden Color

Soft gluons in the infinite momentum wave function and the BFKL **pomeron.**
Alfred H. Mueller (SLAC & Columbia U.) . SLAC-PUB-10047, CU-TP-609, Aug 1993. 12pp.
Published in **Nucl.Phys.B415:373-385,1994**.

Light cone wave functions at small x.
F. Antonuccio (Heidelberg, Max Planck Inst. & Heidelberg U.) , S.J. Brodsky (SLAC) , S. Dalley (CERN) .
Phys.Lett.B412:104-110,1997.
e-Print: **hep-ph/9705413**

Mueller: BFKL derived from multi-gluon Fock State



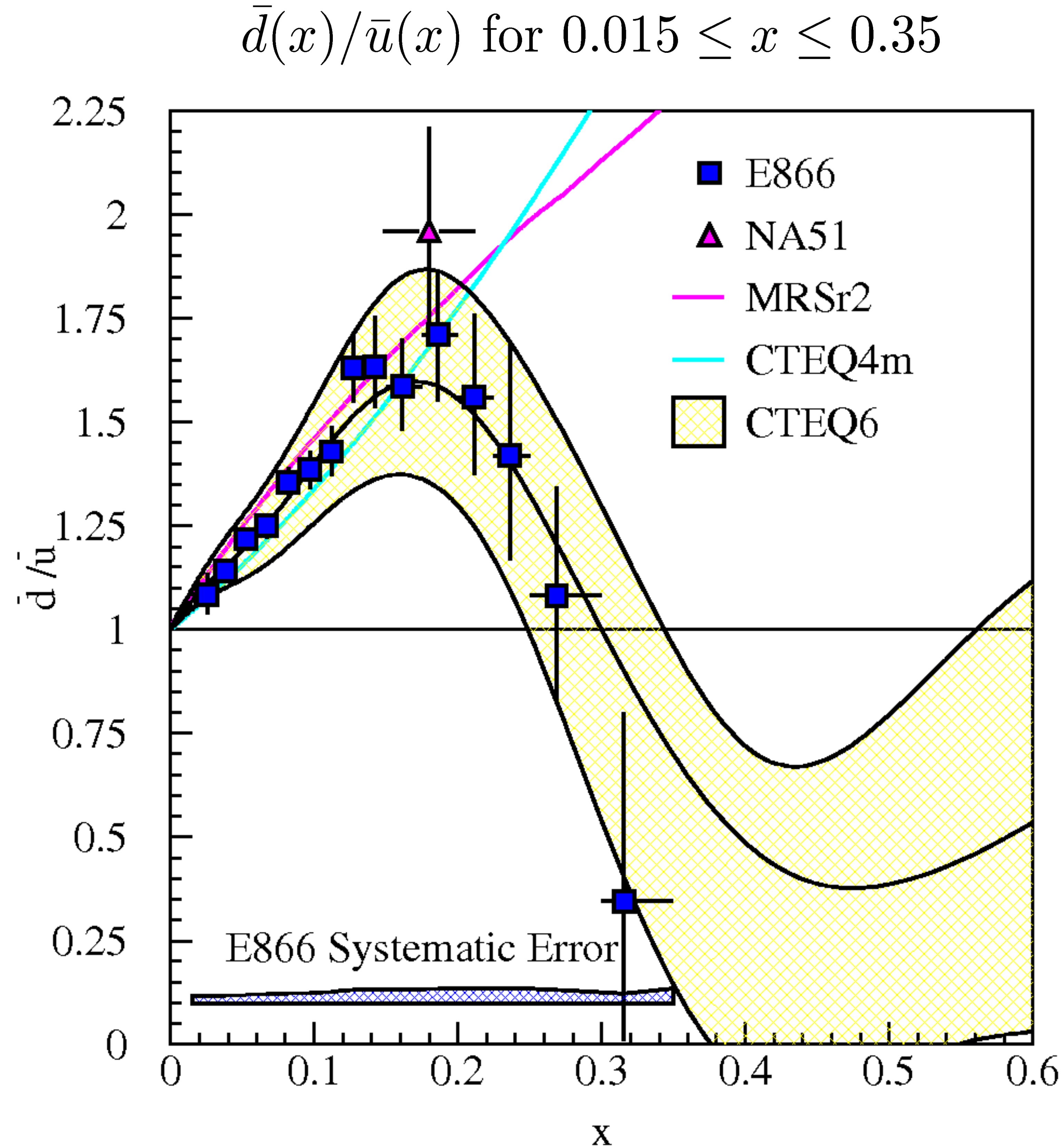
Antonuccio, Dalley, sjb: Ladder Relations

E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

$$s(x) \neq \bar{s}(x)$$

Intrinsic glue, sea, heavy quarks



Do heavy quarks exist in the proton at high x?

Conventional wisdom: impossible!

Heavy quarks generated only at low x
via DGLAP evolution
from gluon splitting

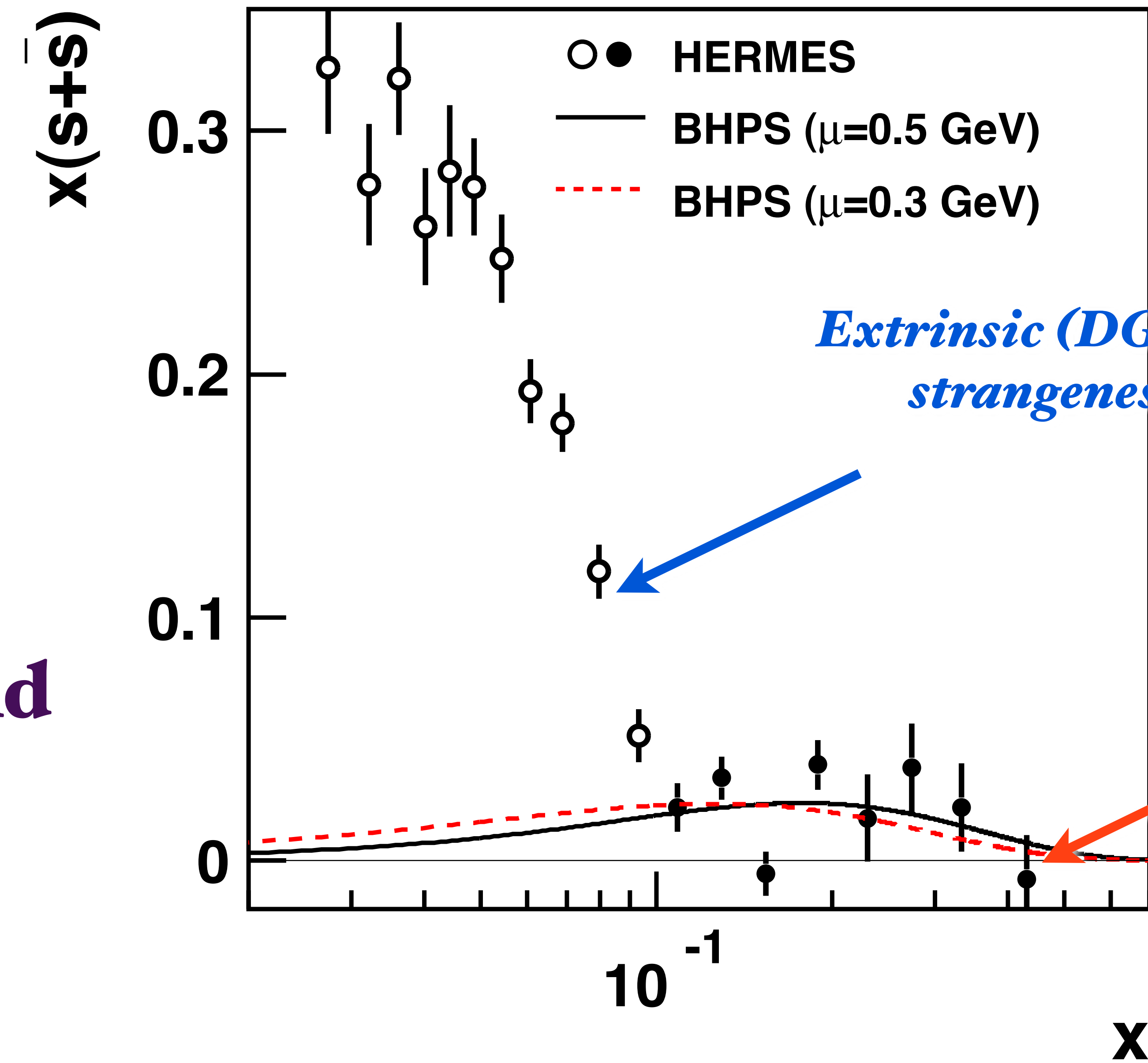
$$s(x, \mu_F^2) = c(x, \mu_F^2) = b(x, \mu_F^2) \equiv 0$$

$$\text{at starting scale } \mu_F^2$$

Conventional wisdom is wrong even in QED!

HERMES: Two components to $s(x,Q^2)$!

BHPS: Hoyer, Sakai, Peterson, sjb



Extrinsic (DGLAP) strangeness!

Intrinsic strangeness!

Consistent with intrinsic charm data

QCD: $\frac{1}{M_Q^2}$ scaling

W. C. Chang and J.-C. Peng

arXiv:1105.2381

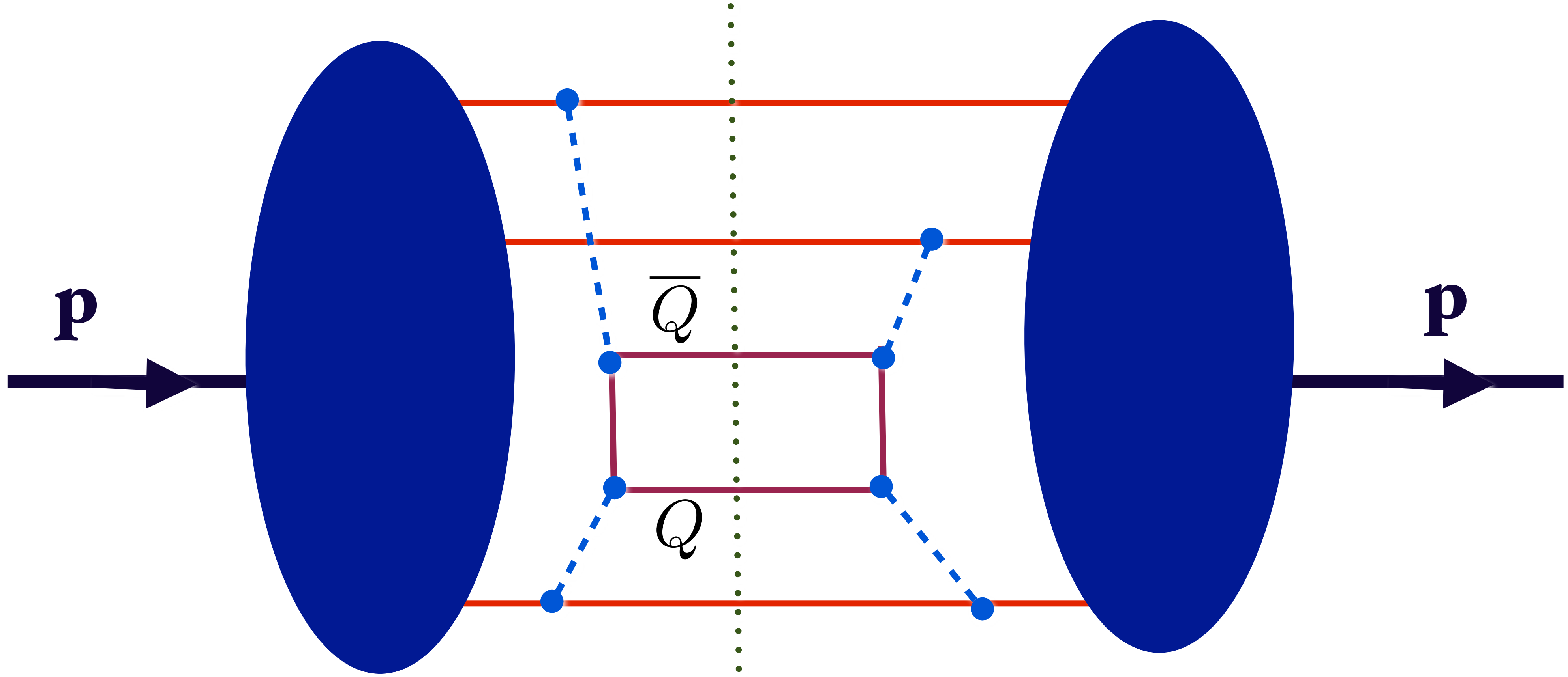
Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x,Q^2) = s(x,Q^2)_{\text{extrinsic}} + s(x,Q^2)_{\text{intrinsic}}$$

Fixed LF time

Proton Self Energy
Intrinsic Heavy Quarks

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

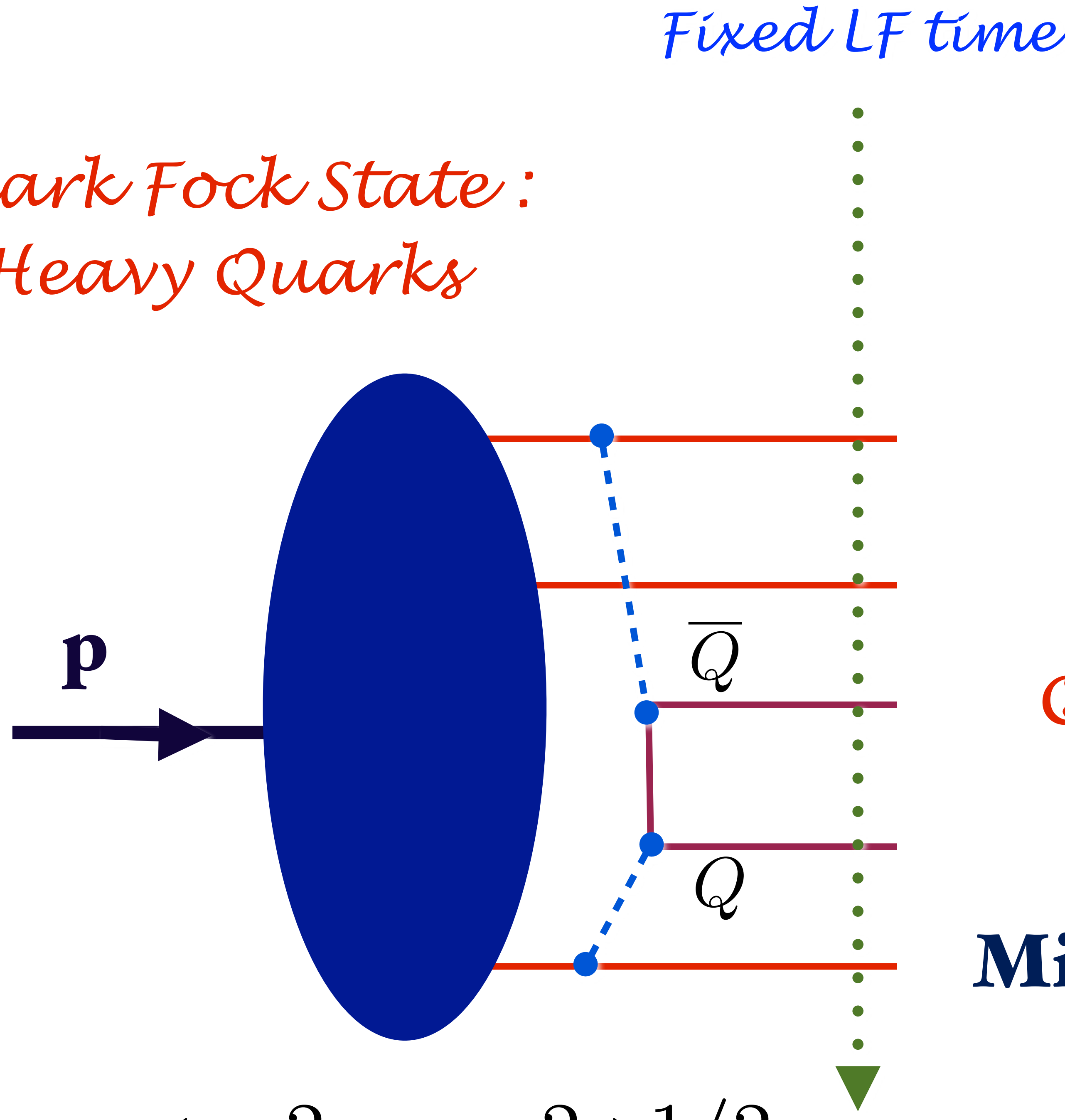


Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov, et al.

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



QCD predicts Intrinsic Heavy Quarks at high x!

Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

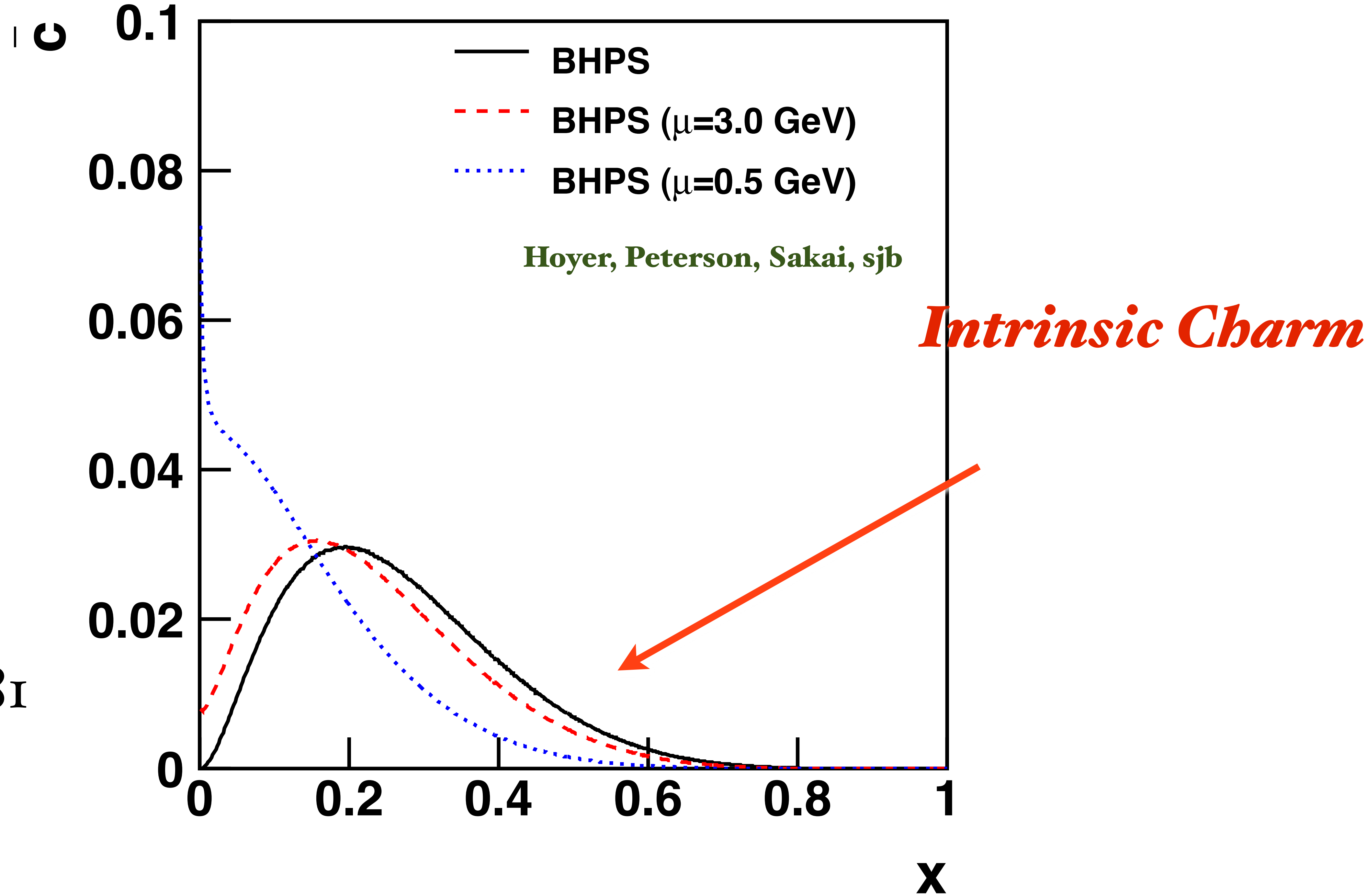
$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

QCD ($1/m_Q^2$) scaling: predict IC !

W. C. Chang and J.-C. Peng

arXiv:1105.2381



Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV^2 using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

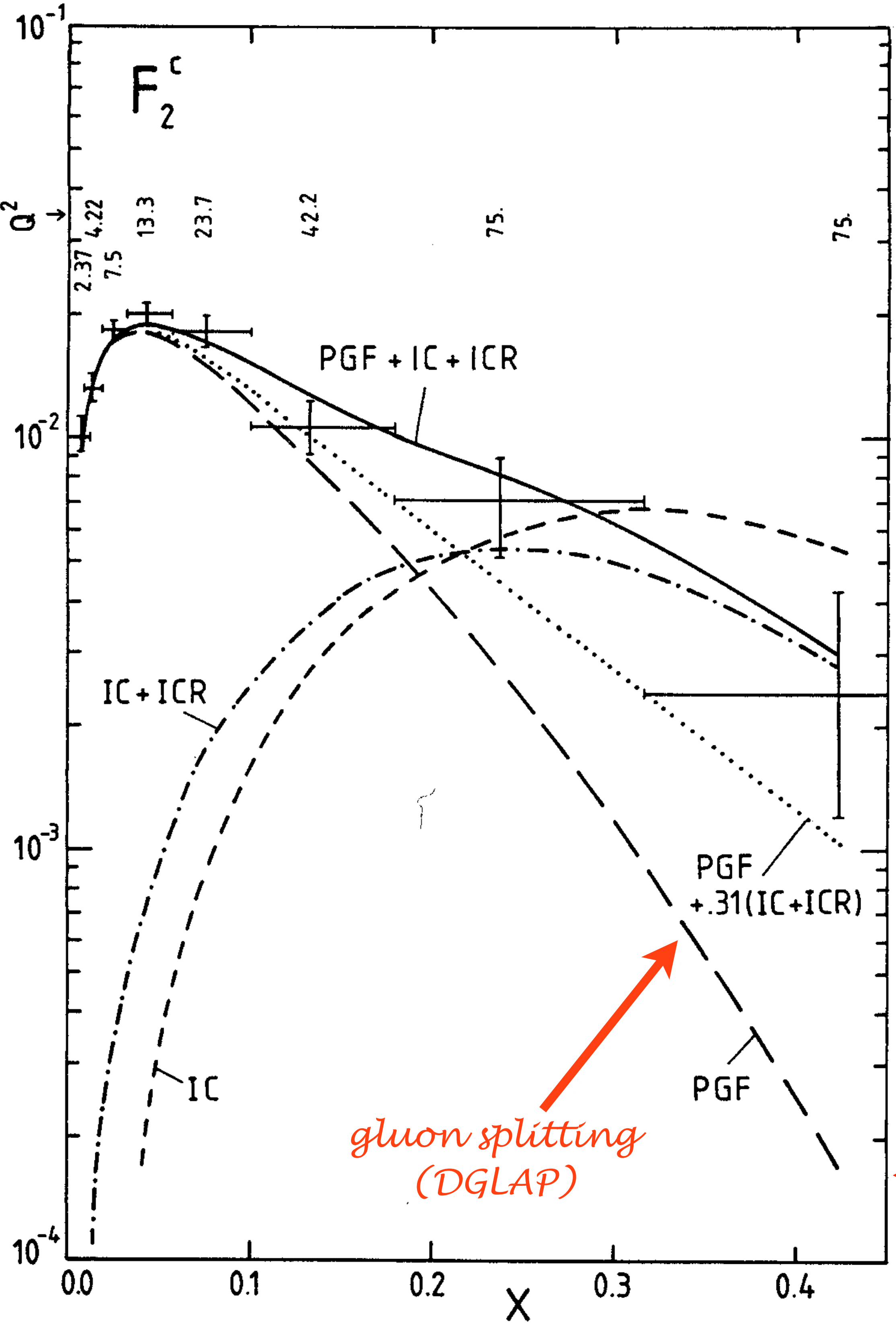
Consistent with EMC

Measurement of Charm Structure Function!

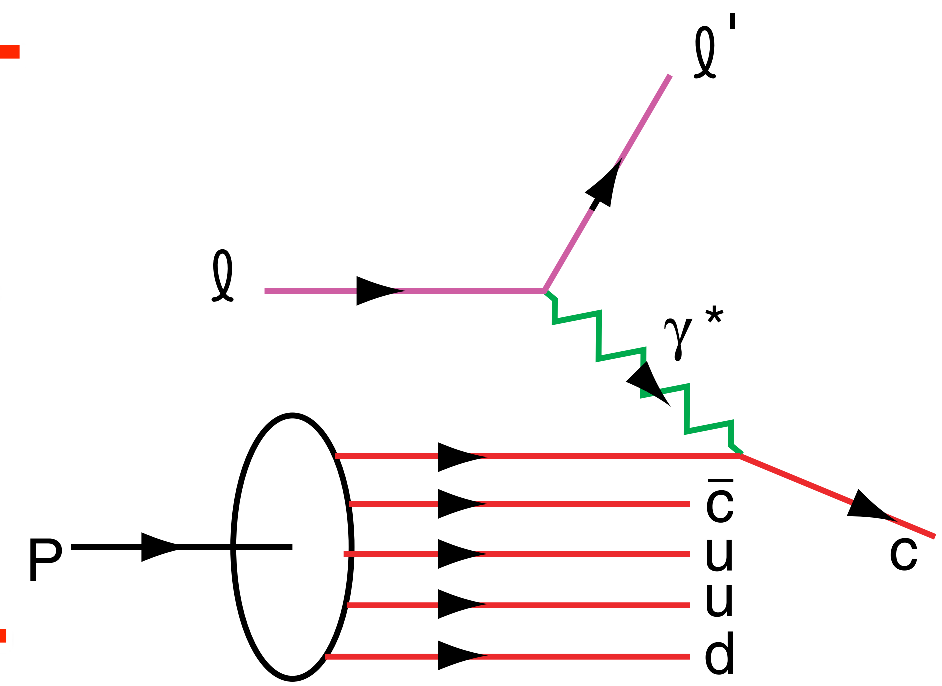
J. J. Aubert et al. [European Muon Collaboration], “Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions,” Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb



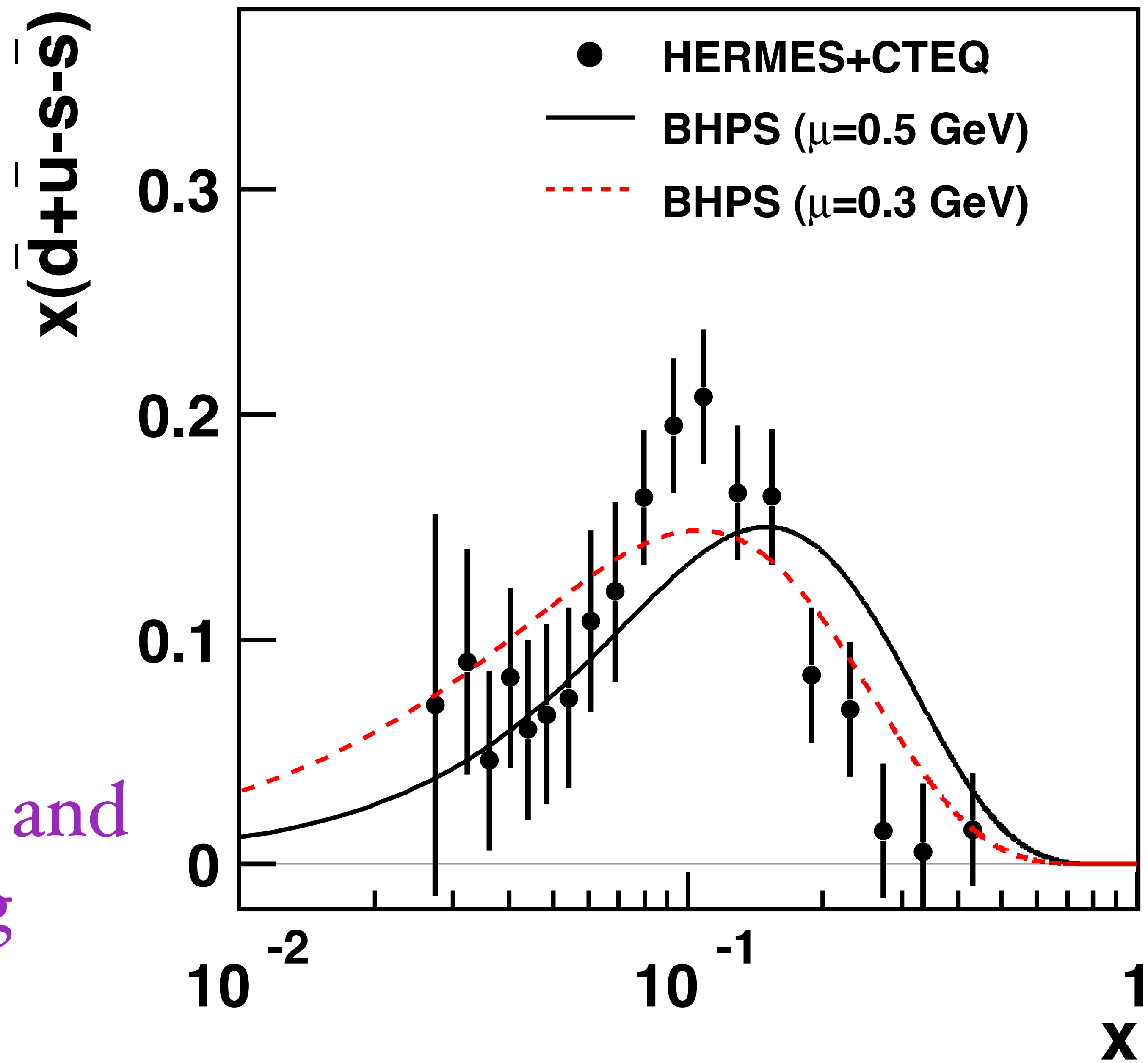
factor of 30 !



DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

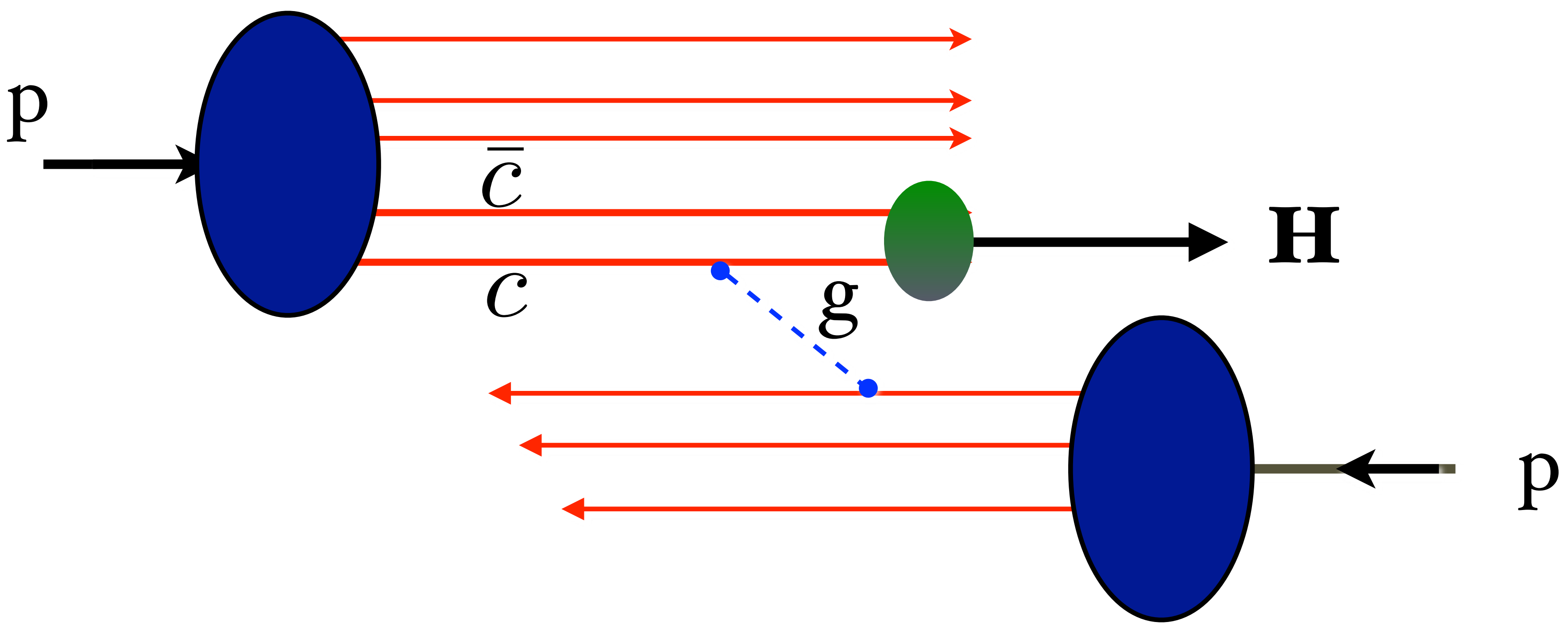


W. C. Chang and J.-C. Peng

Comparison of the $x(\bar{d}(x)+\bar{u}(x)-s(x)-\bar{s}(x))$ data with the calculations based on the BHPS model. The values of $x(s(x)+\bar{s}(x))$ are from the HERMES experiment [6], and those of $x(\bar{d}(x)+\bar{u}(x))$ are obtained from the PDF set CTEQ6.6 [11]. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalization of the calculations are adjusted to fit the data.

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \to H X$$

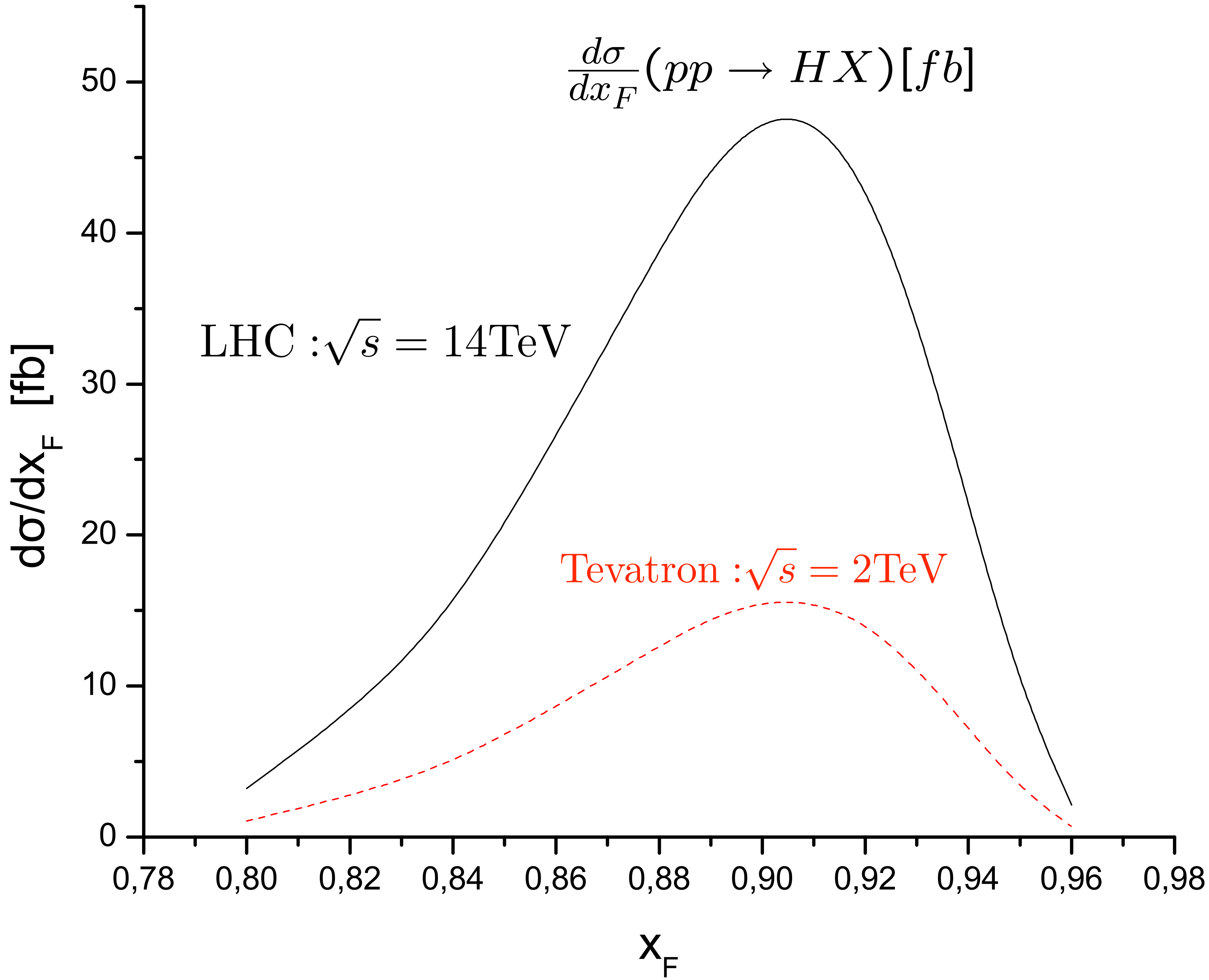
Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

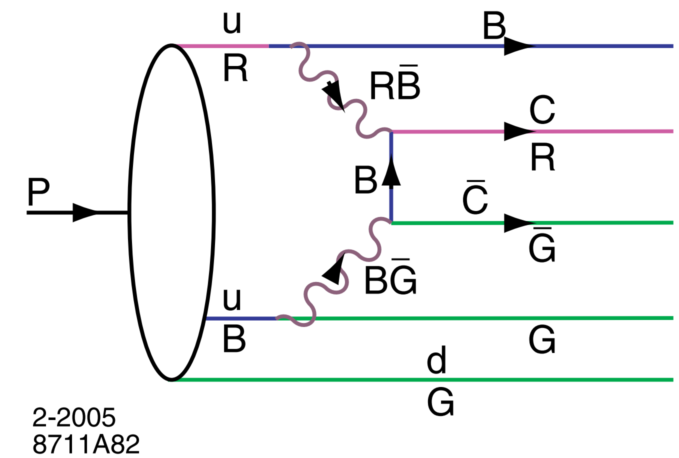
AFTER: Higgs production at threshold!

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production



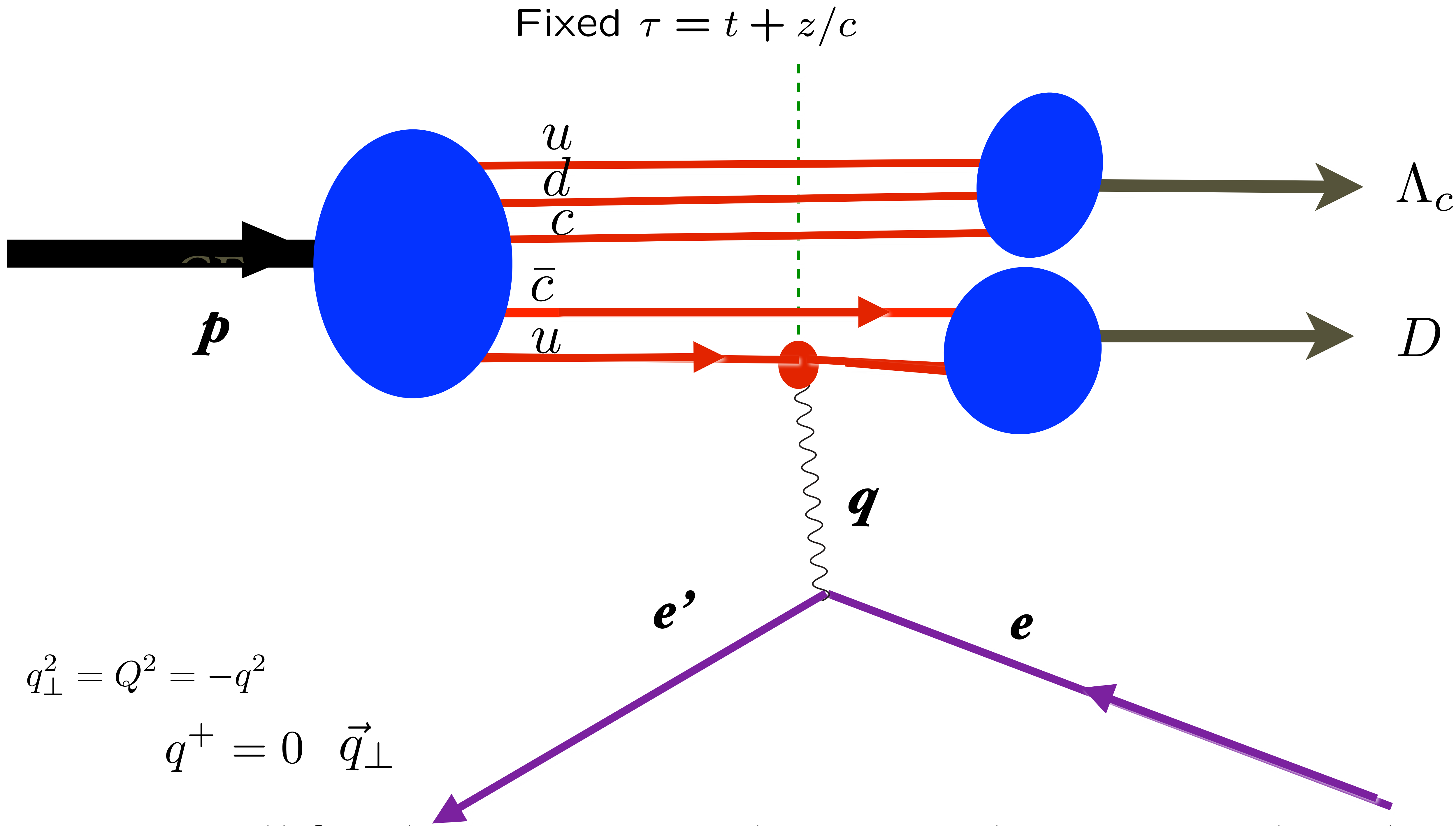
Goldhaber, Kopeliovich, Schmidt, sjb

Intrinsic Heavy-Quark Fock States



- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Cannot use $c(x,Q^2)$ to determine $g(x,Q^2)$

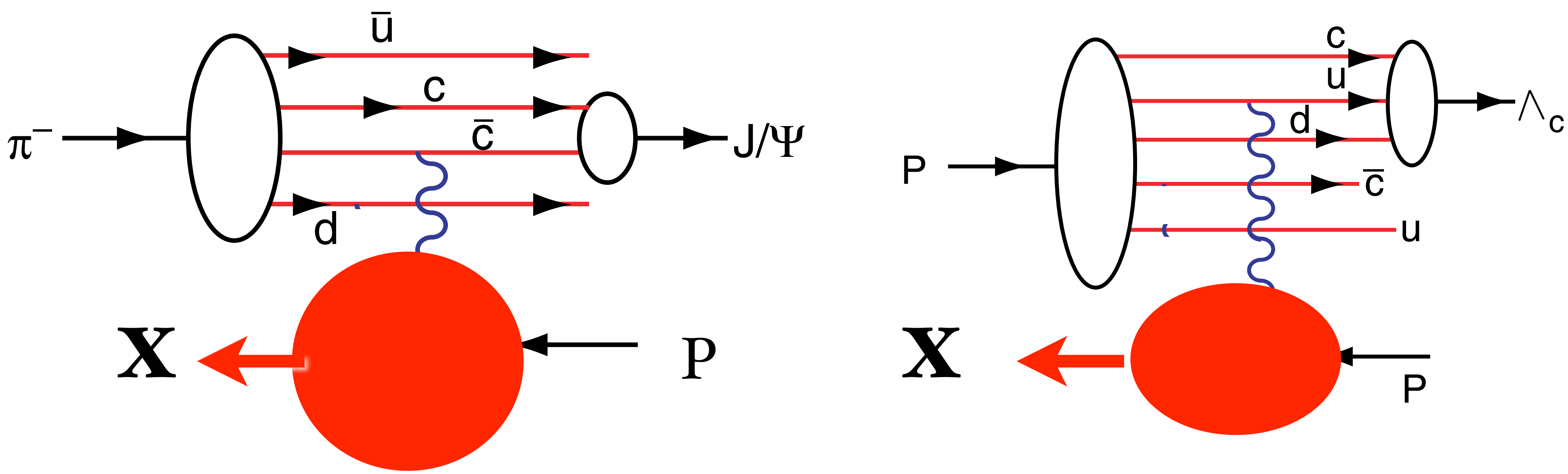
Light-Front Wavefunctions and Electron-Proton Collisions



All final states |F> in electroproduction produced from n to n' overlap of LFWFs

Coalescence of comovers produces $|F >= |\Lambda_c D >$ final state

Leading Hadron Production from Intrinsic Charm



Spectator counting rules

$$\frac{dN}{dx_F} \propto (1-x_F)^{2n_{spect}-1}$$

Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ, Λ_c and other Charm Hadrons at High x_F

- EMC data: $c(x,Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

C.H. Chang, J.P. Ma, C.F. Qiao and X.G.Wu

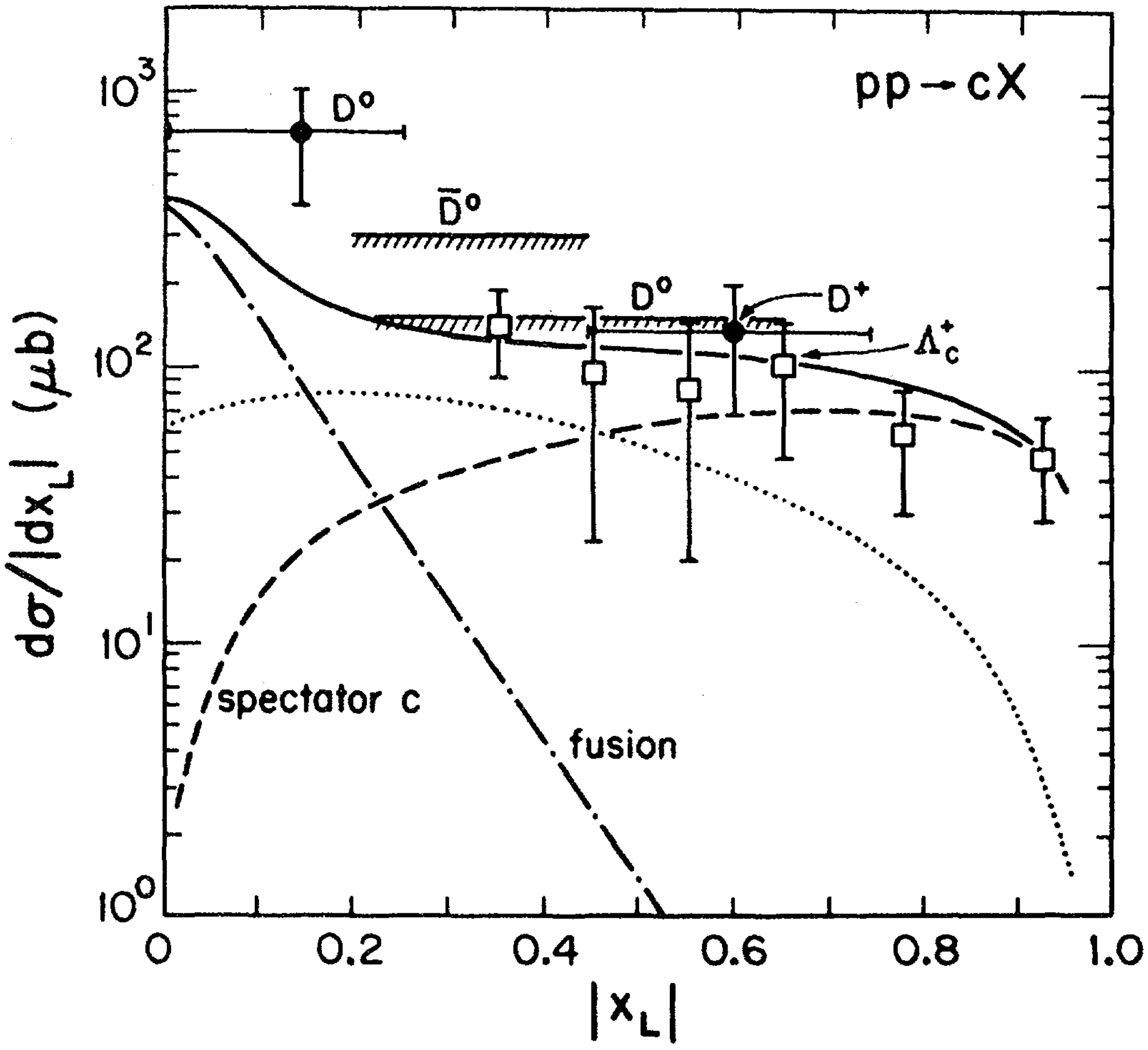
- High x_F $pp \to \Xi(ccd) X$ (SELEX)

Critical Measurements at threshold for JLab, PANDA

Interesting spin, charge asymmetry, threshold, spectator effects

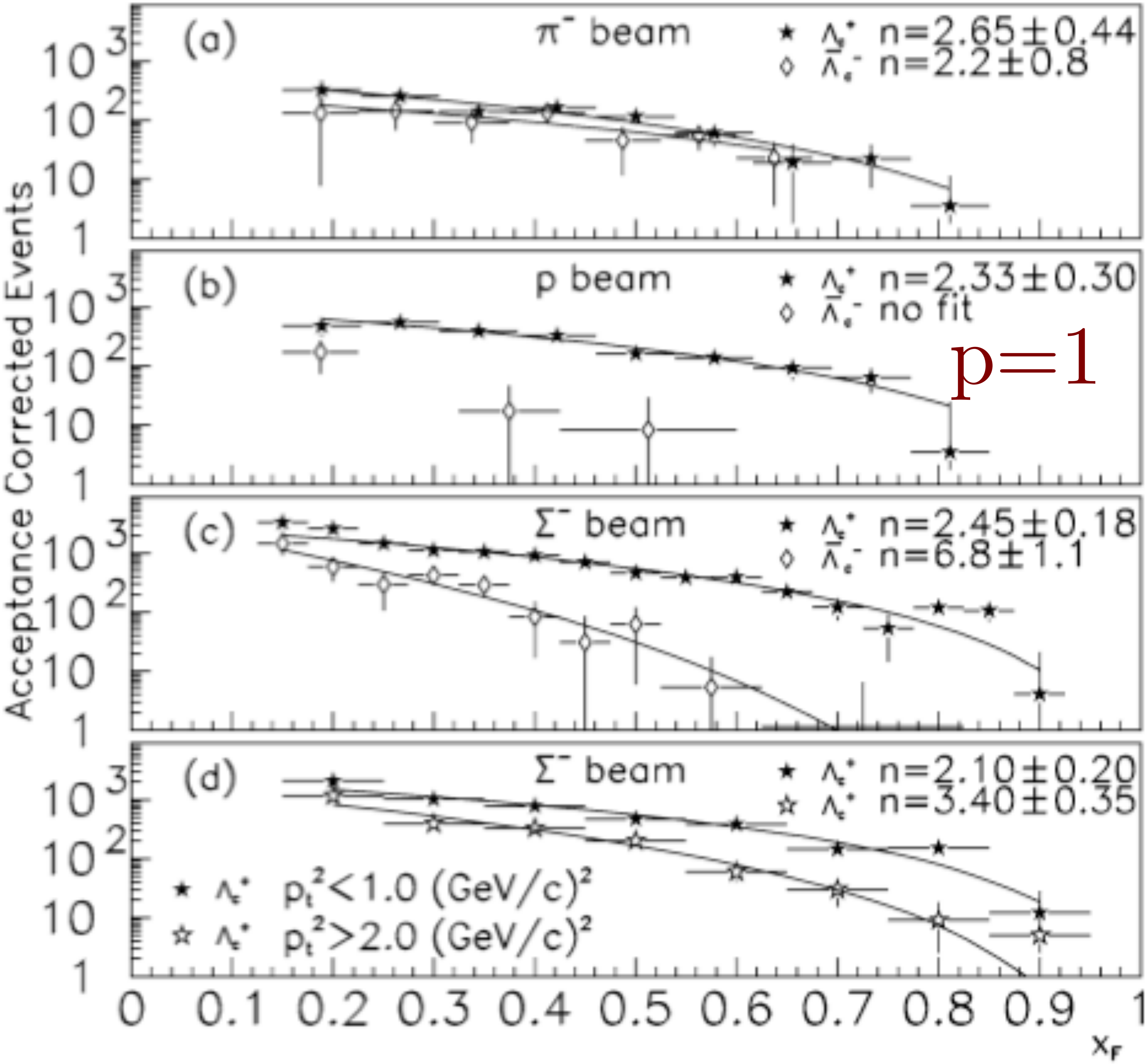
Important corrections to B decays; Quarkonium decays

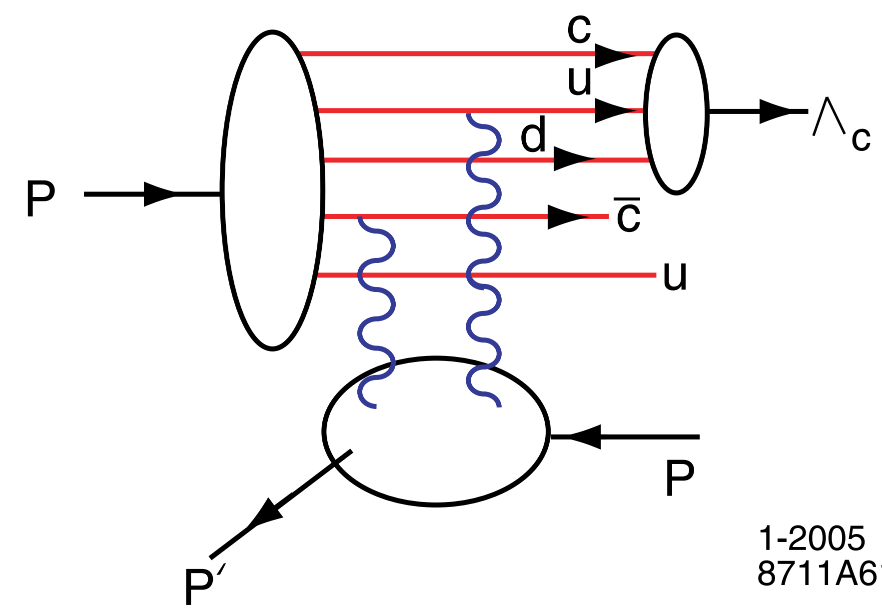
Gardner, Karliner, sjb



Barger, Halzen, Keung

Evidence for charm at large x





$$p(uudc\bar{c})$$

$$\rightarrow \Lambda_c(cud)$$

$$n_s = 2$$

Phase space gives minimum power p

$$(1 - x_F)^p, p = n_s - 1$$

Excludes PYTHIA 'color drag' model

All events have $x^F_{\psi\psi} > 0.4$!

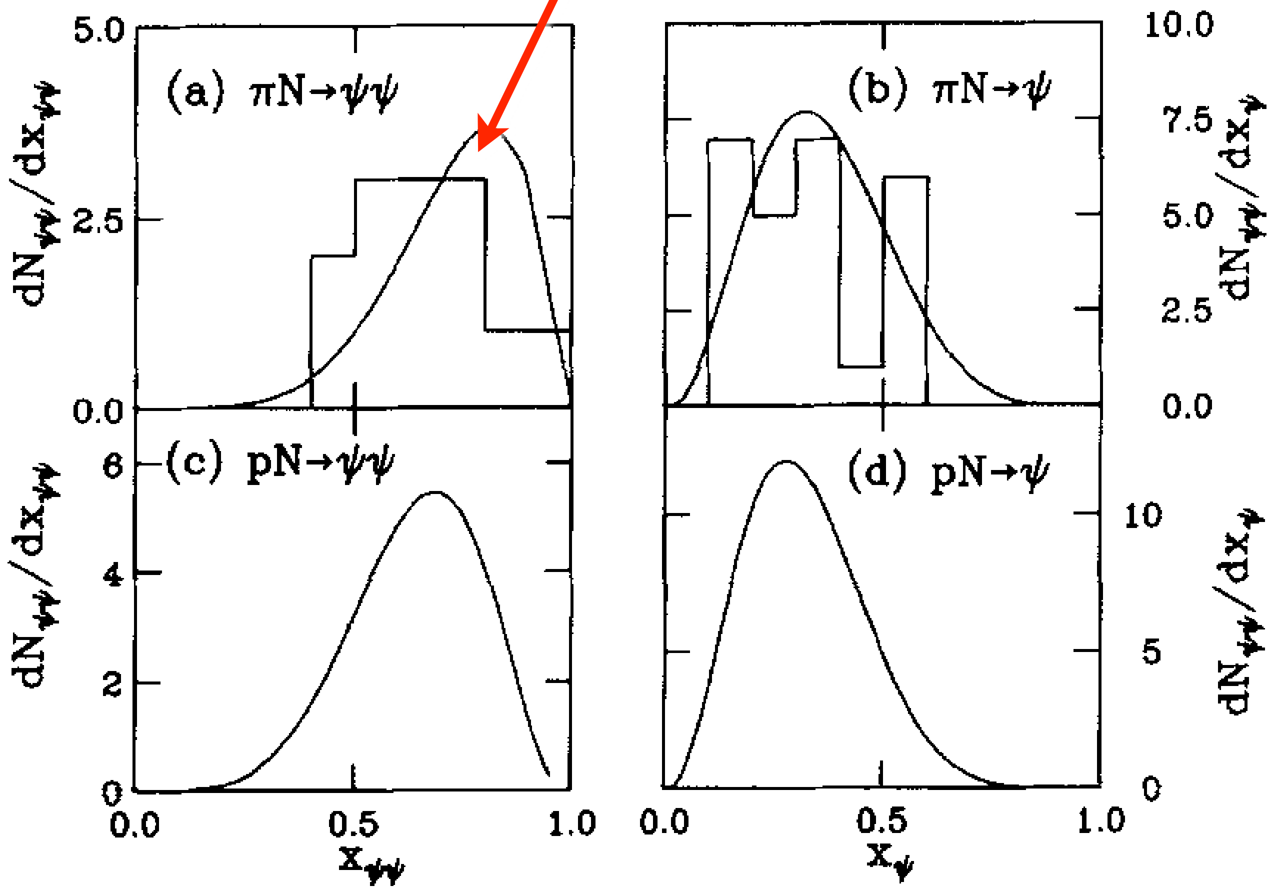


Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of J/ψ's from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/c [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single J/ψ's is twice the number of pairs.

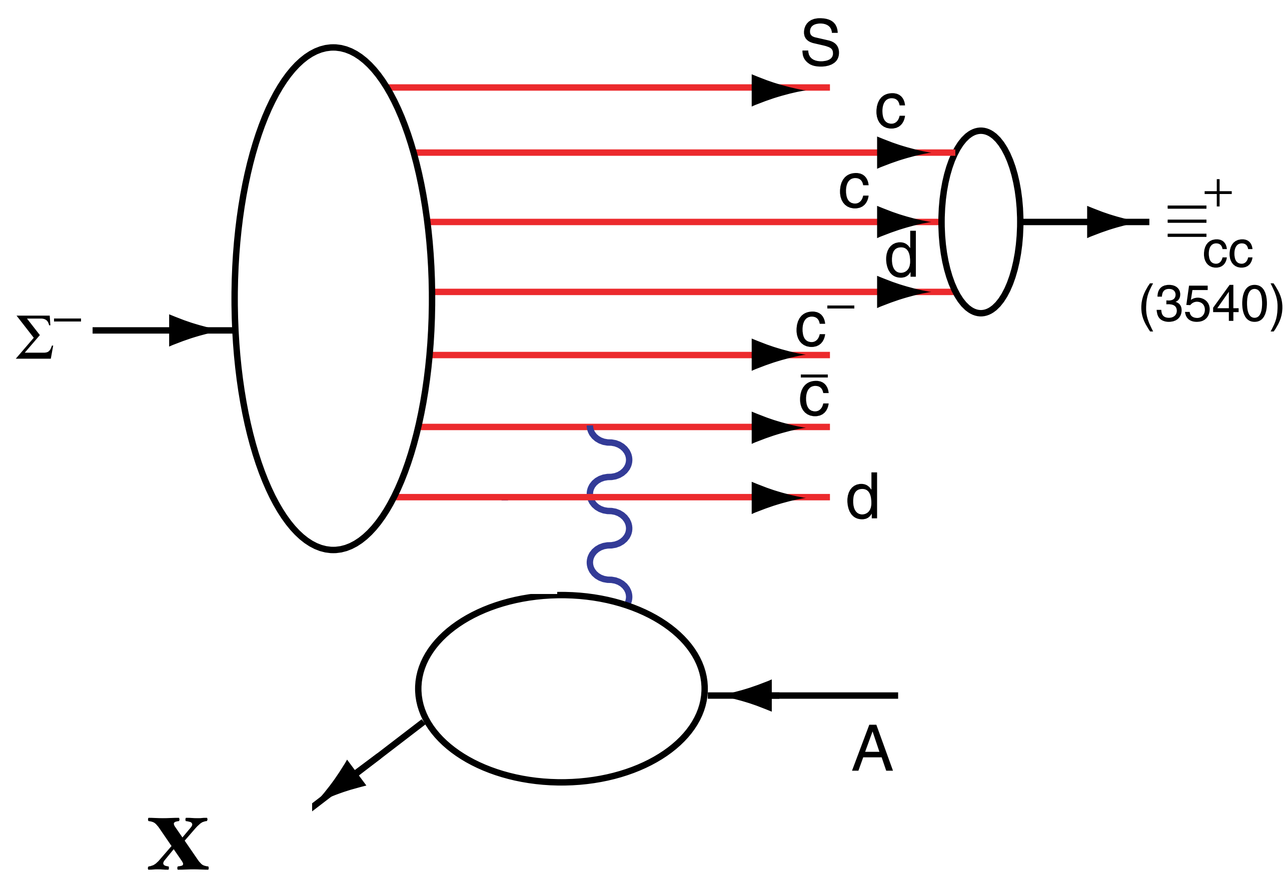
NA3 Data

$$\pi A \rightarrow J/\psi J/\psi X$$

R. Vogt, sjb

The probability distribution for a general n-particle intrinsic $c\bar{c}$ Fock state as a function of x and $\boldsymbol{k}_T$ is written as

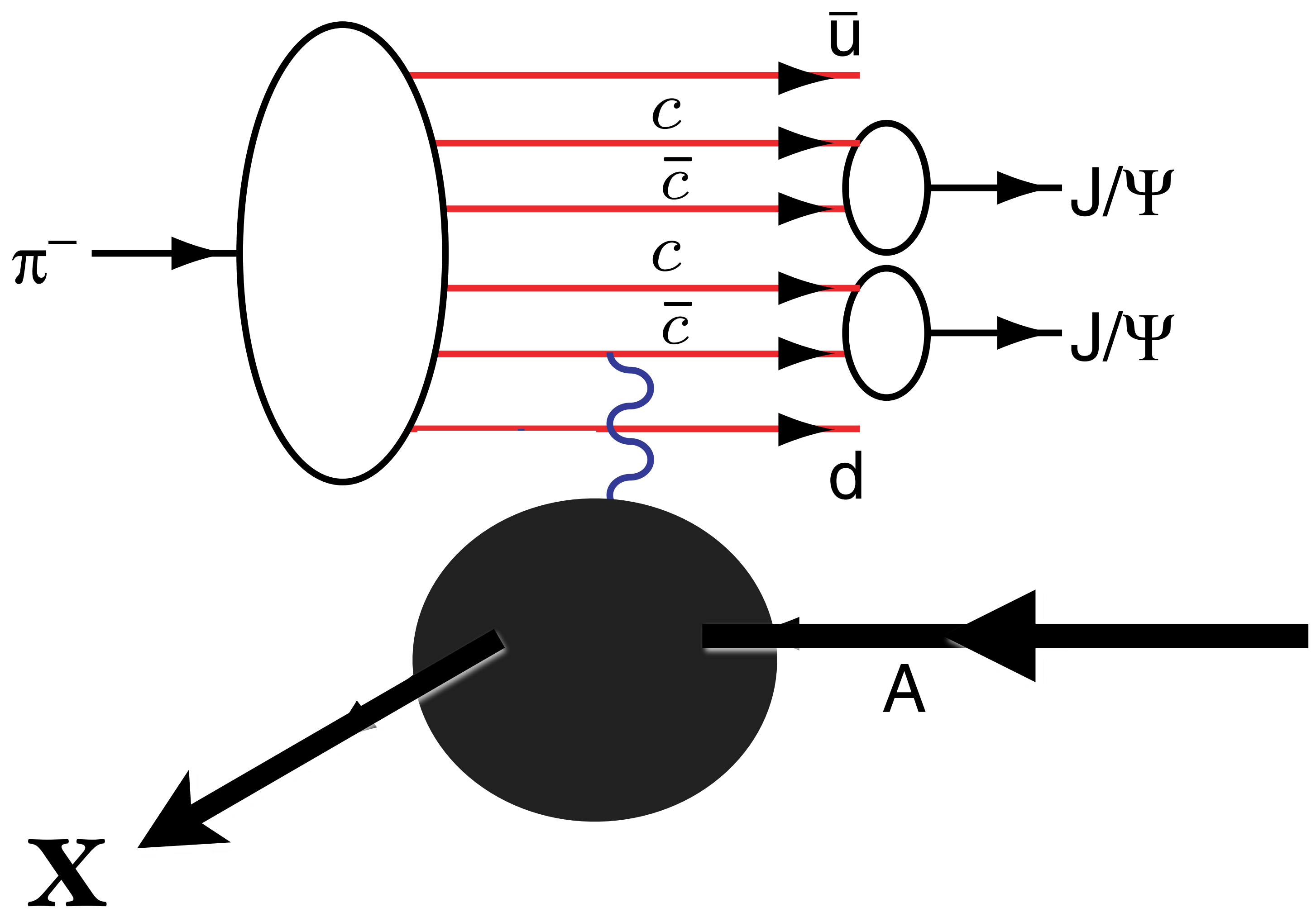
$$\frac{dP_{\rm ic}}{\prod_{i=1}^n dx_i d^2k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n \boldsymbol{k}_{T,i})\delta(1-\sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$



Production of a Double-Charm Baryon

SELEX high x_F $< x_F >= 0.33$

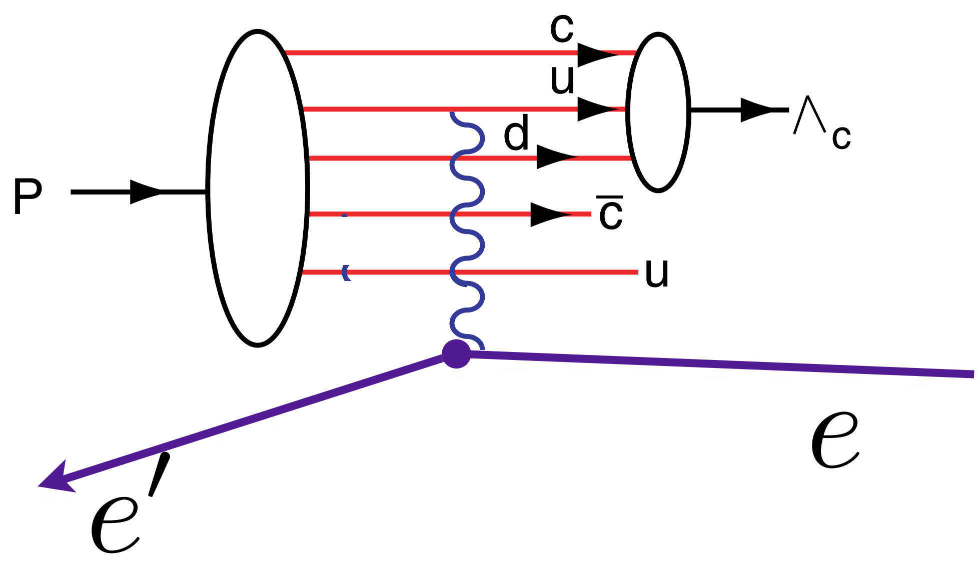
Production of Two Charmonia at High x_F



Leading charm production in proton fragmentation region at the EIC

Intrinsic charm and bottom quarks
have same rapidity as valence quarks

Produce $\Xi(ccd), B(\bar{b}u), \Lambda(cbu), \Xi(bbu)$



Coalescence of Comoving Charm and Valence Quarks
Produce J/ψ, Λ_c and other Charm Hadrons at High x_F

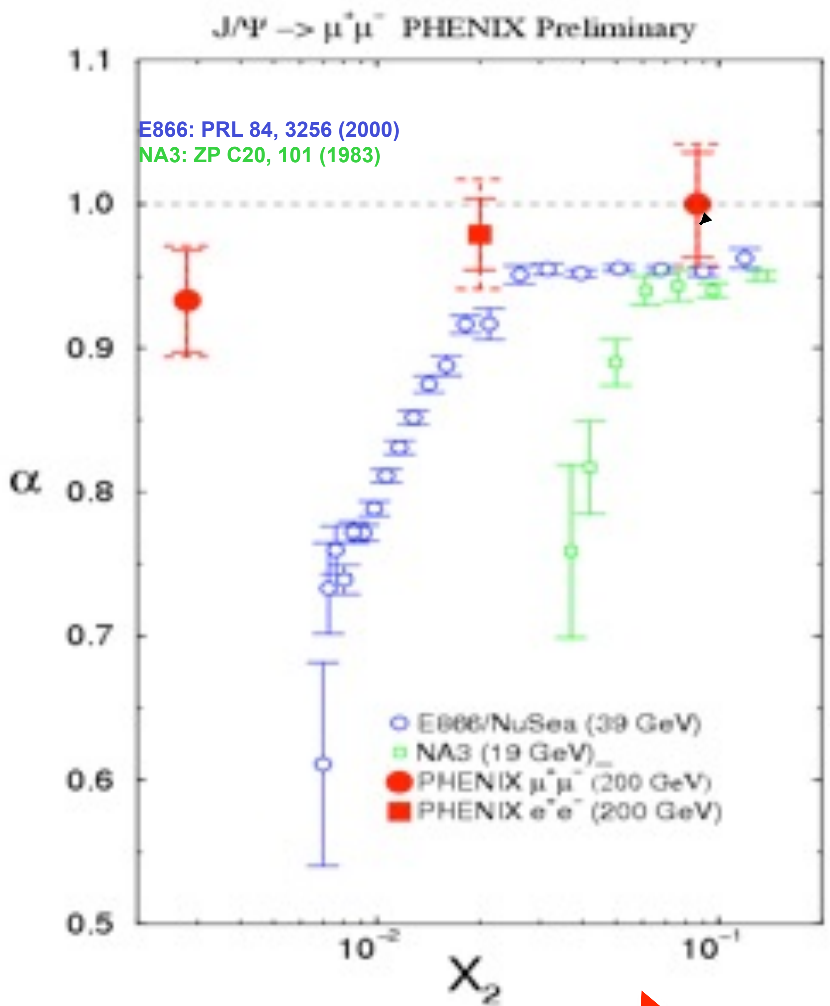
Key QCD Issues in Electroproduction

- **Intrinsic Heavy Quarks**
- **Role of Color Confinement in DIS**
- **Hadronization at the Amplitude Level**
- **Leading-Twist Lensing: Sivers Effect**
- **Diffractive DIS**
- **Static versus Dynamic Structure Functions**
- **Origin of Shadowing and Anti-Shadowing**
- **Is Anti-Shadowing Non-Universal: Flavor Specific?**
- **Nature of Nuclear Correlations**
- $1 < x < A$

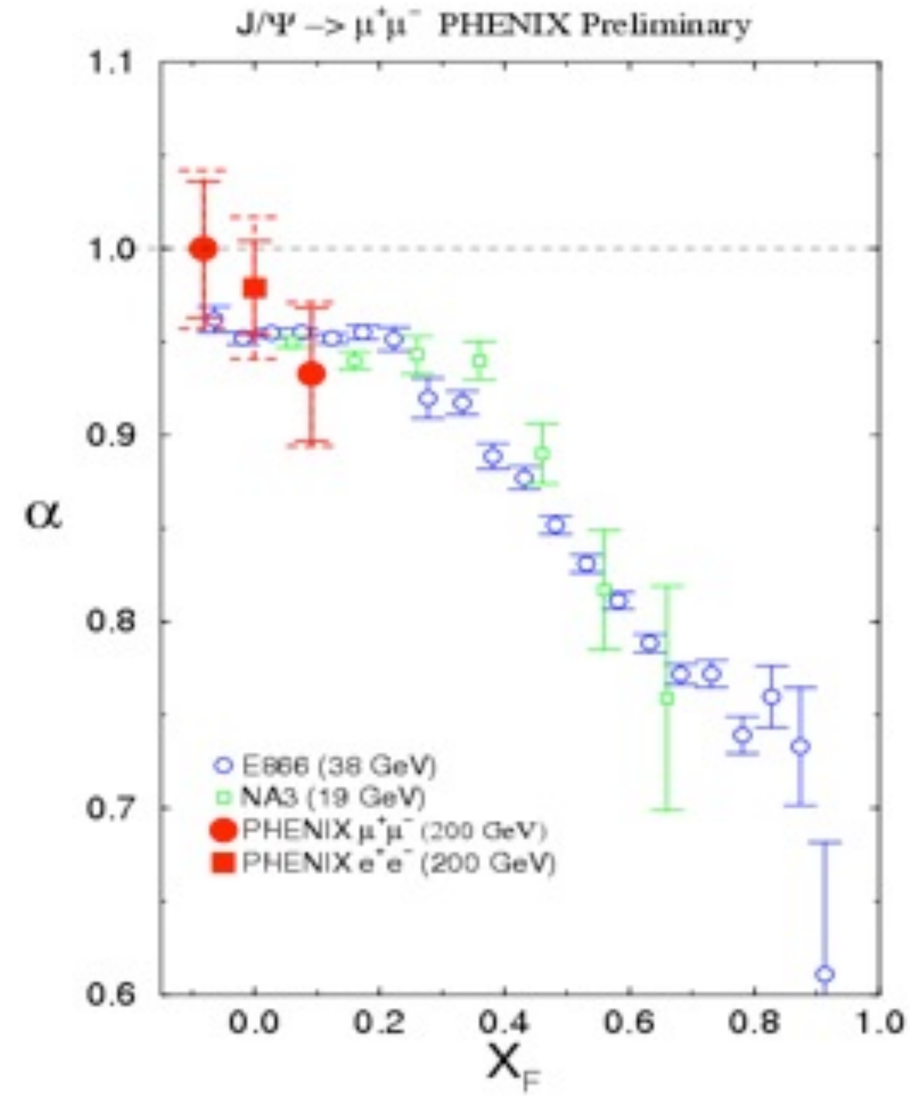
J/ψ nuclear dependence vrs rapidity, x_{Au}, x_F

M.Leitch

PHENIX compared to lower energy measurements



Klein,Vogt, PRL 91:142301,2003
Kopeliovich, NP A696:669,2001



Huge "absorption" effect

Violates PQCD factorization!

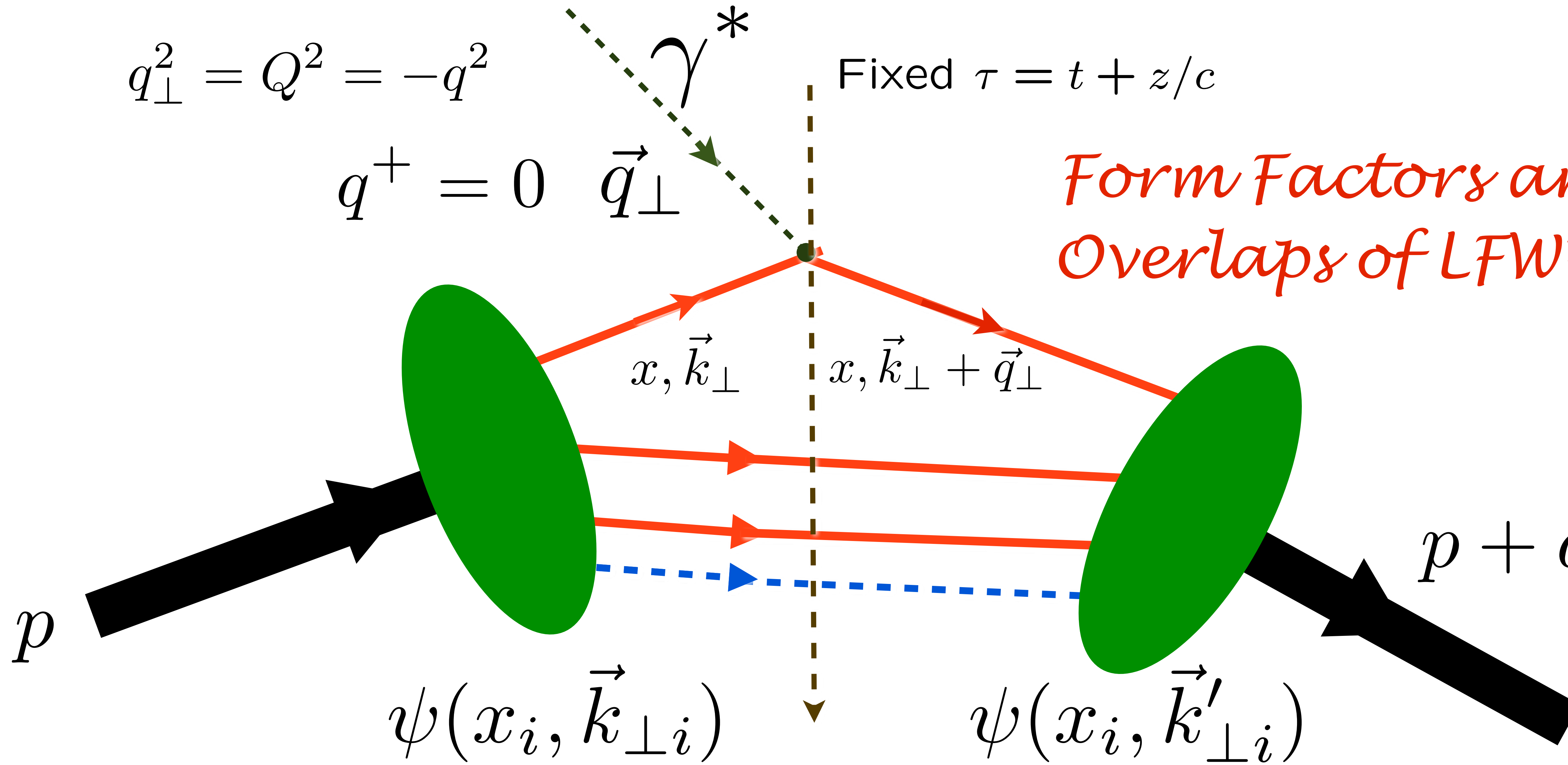
$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

Hoyer, Sukhatme, Vanttinen

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!
Drell, sjb

Exact LF Formula for Pauli Form Factor

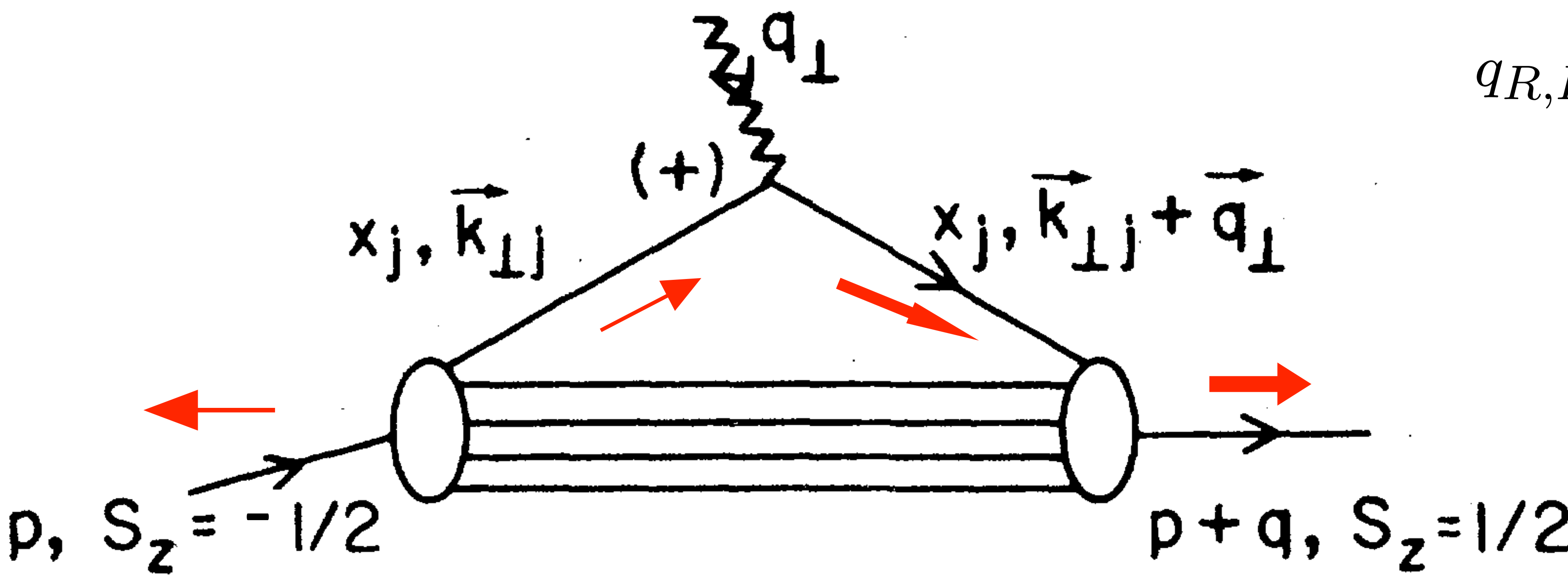
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Gravitational Form Factors

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \overline{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P)\ ,$$

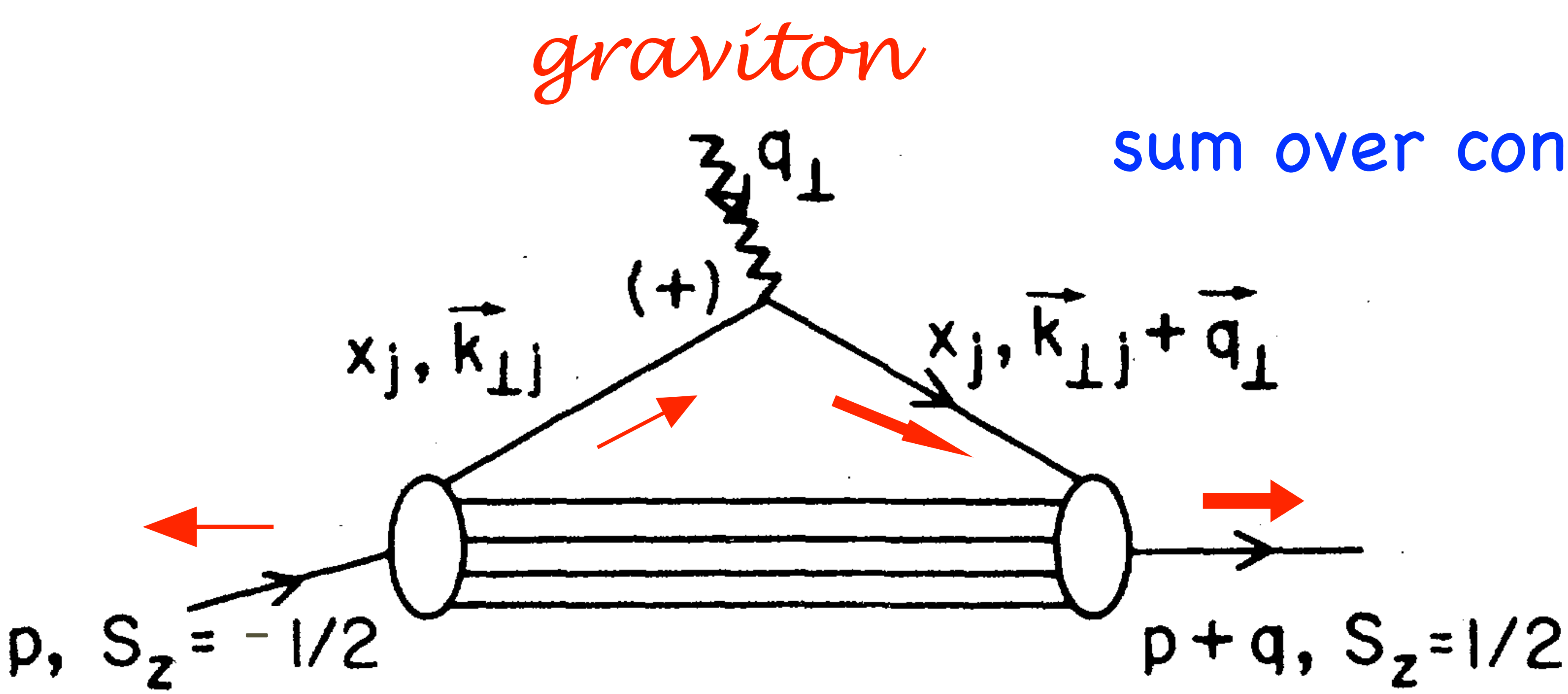
where $q^\mu = (P' - P)^\mu$, $\overline{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\right\rangle = A(q^2)\ ,$$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M}\ .$$

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

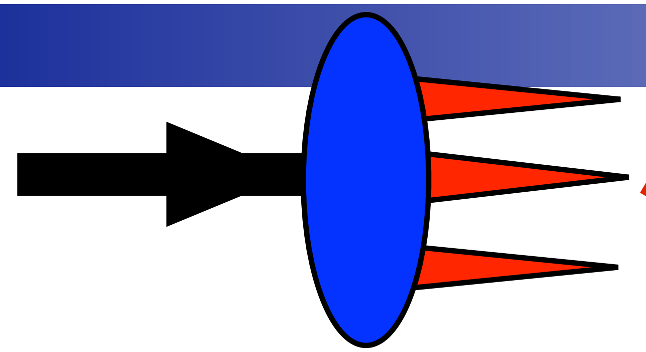


sum over constituents

Hwang, Schmidt, sjb; Holstein et al

$$B(0) = 0$$

Each Fock State



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

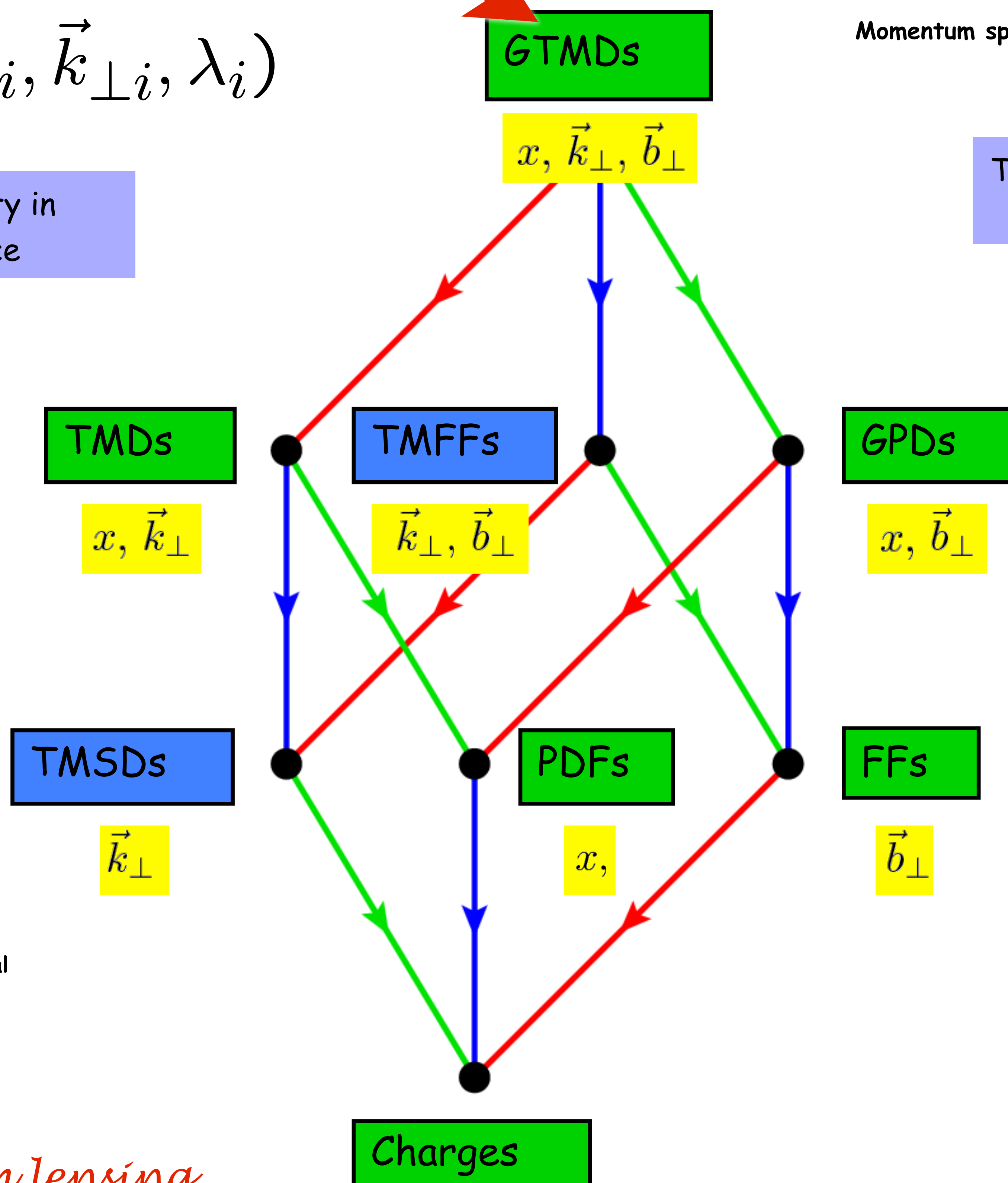
- *Light Front Wavefunctions:*

Transverse density in momentum space

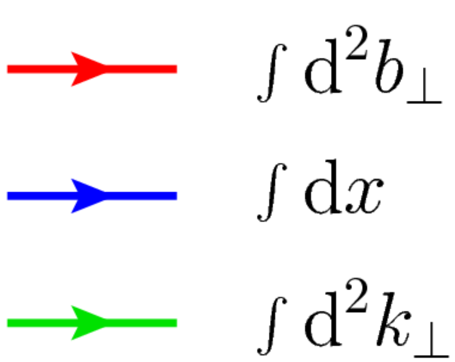
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

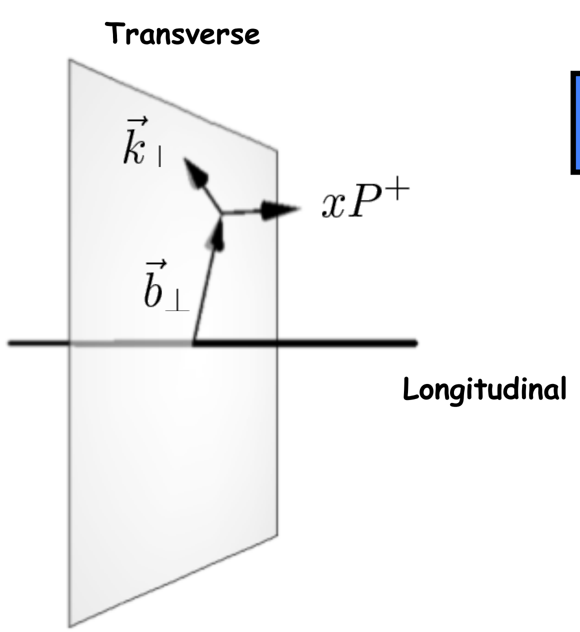
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



Lorce, Pasquini

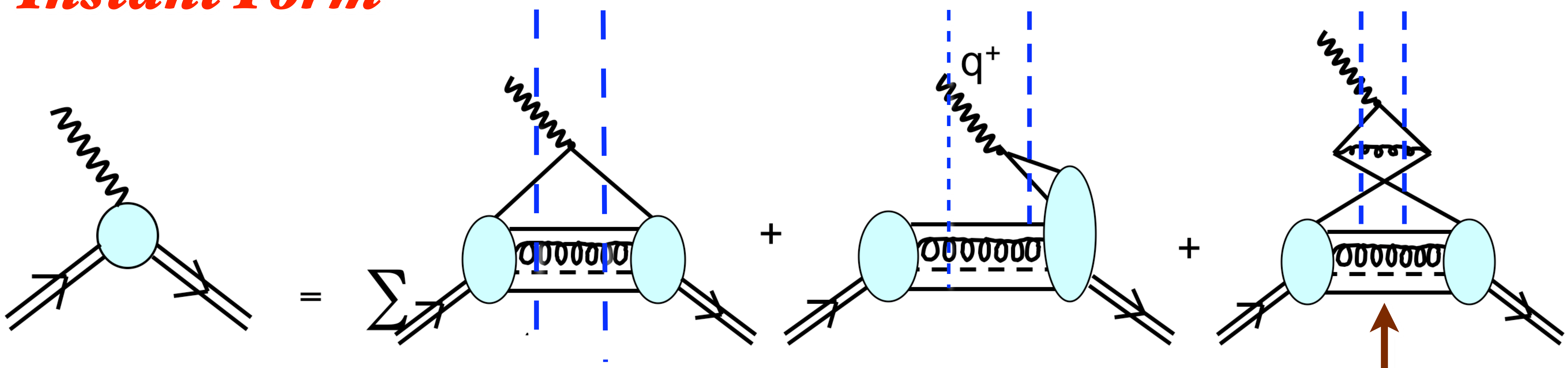




Sivers, T-odd from lensing

Calculation of Form Factors in Equal-Time Theory

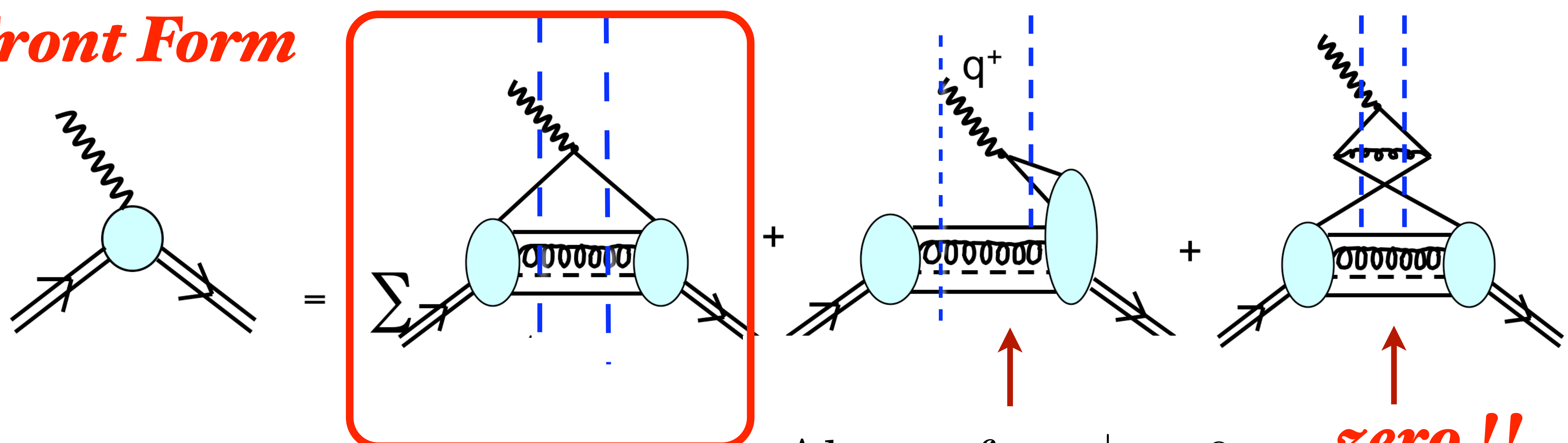
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Complete Answer

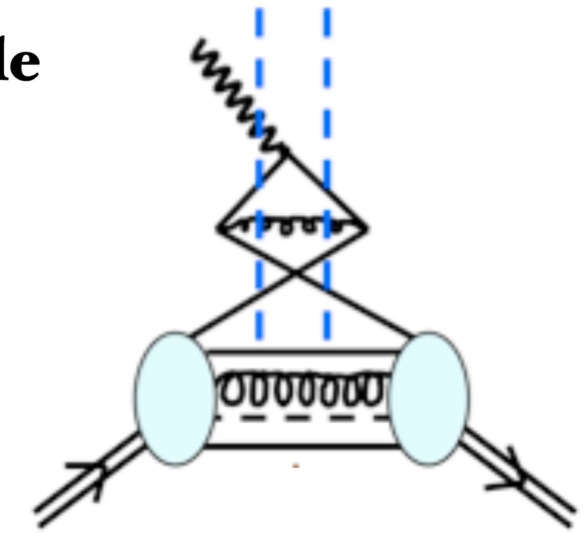
Absent for $q^+ = 0$

zero !!

No vacuum graphs

Disadvantages of the Instant Form

- Boosts are dynamical, change particle number: not Melosh!
- Famous wrong proof showing violation of LET and DHG sum rule
- Each Amplitude is Frame-Dependent
- States defined at one instant of time over all space - acausal!
- Current matrix elements involve connected vacuum currents -- eigensolutions insufficient!
- N! time-ordered graphs, each frame-dependent
- Vacuum is complex: apparently gives huge vacuum energy density
- Normal-ordering required to compute observables
- Cluster decomposition theorem fails in relativistic systems
- Virtually no valid calculations of dynamics of relativistic composite systems use the instant form
- Why Feynman invented Feynman diagrams!



PHYSICAL REVIEW VOLUME 174, NUMBER 5 25 OCTOBER 1968

Electromagnetic Interactions of Loosely-Bound Composite Systems*

STANLEY J. BRODSKY AND JOEL R. PRIMACK
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 13 June 1968)

Contrary to popular assumption, the interaction of a composite system with an external electromagnetic field is not equal to the sum of the individual Foldy-Wouthyusen interactions of the constituents if the constituents have spin. We give the correct interaction, and note that it is consistent with the Drell-Hearn-Gerasimov sum rule and the low-energy theorem for Compton scattering. We also discuss the validity of additivity of the individual Dirac interactions, and the corrections to this approximation, with particular reference to the atomic Zeeman effect, which is of importance in the fine-structure and Lamb-shift measurements.

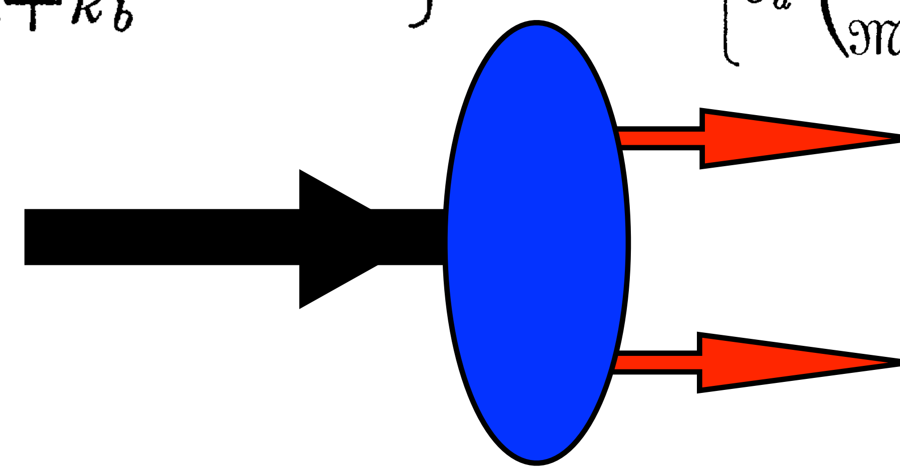
Dynamical boost contribution

$$\left\{\begin{matrix} 1 \\ \dfrac{1}{2m_a+k_a}\boldsymbol{\sigma}_a\cdot\mathbf{p} \end{matrix}\right\} \otimes \left\{\begin{matrix} 1 \\ \dfrac{1}{2m_b+k_b}\boldsymbol{\sigma}_b\cdot(-\mathbf{p}) \end{matrix}\right\}$$

$$\vec{P} \neq 0$$

$$\left\{\begin{matrix} 1+\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathfrak{M}+E}\,\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{matrix}\right\} \otimes \left\{\begin{matrix} 1-\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathfrak{M}+E}\,\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{matrix}\right\}$$

Instant Form WF



Also: Hugh Osborne

Single particle wave-packet

Primack, sjb

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}}\, u(p)\, \phi(p)\, e^{-ip.x}$$

$$u(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p^0+m} \end{pmatrix} \chi.$$

Wavefunction of moving bound state:

Not product of independent boosts!

$$\varphi_{E\mathbf{P}}(\mathbf{x}_a \quad \mathbf{x}_b\,,\, X^0)_{SM}$$

$$= \frac{E+\mathcal{M}}{2\mathcal{M}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a{}^0+m_a}{2p_a{}^0}\,\frac{p_b{}^0+m_b}{2p_b{}^0}\right)^{1/2}$$

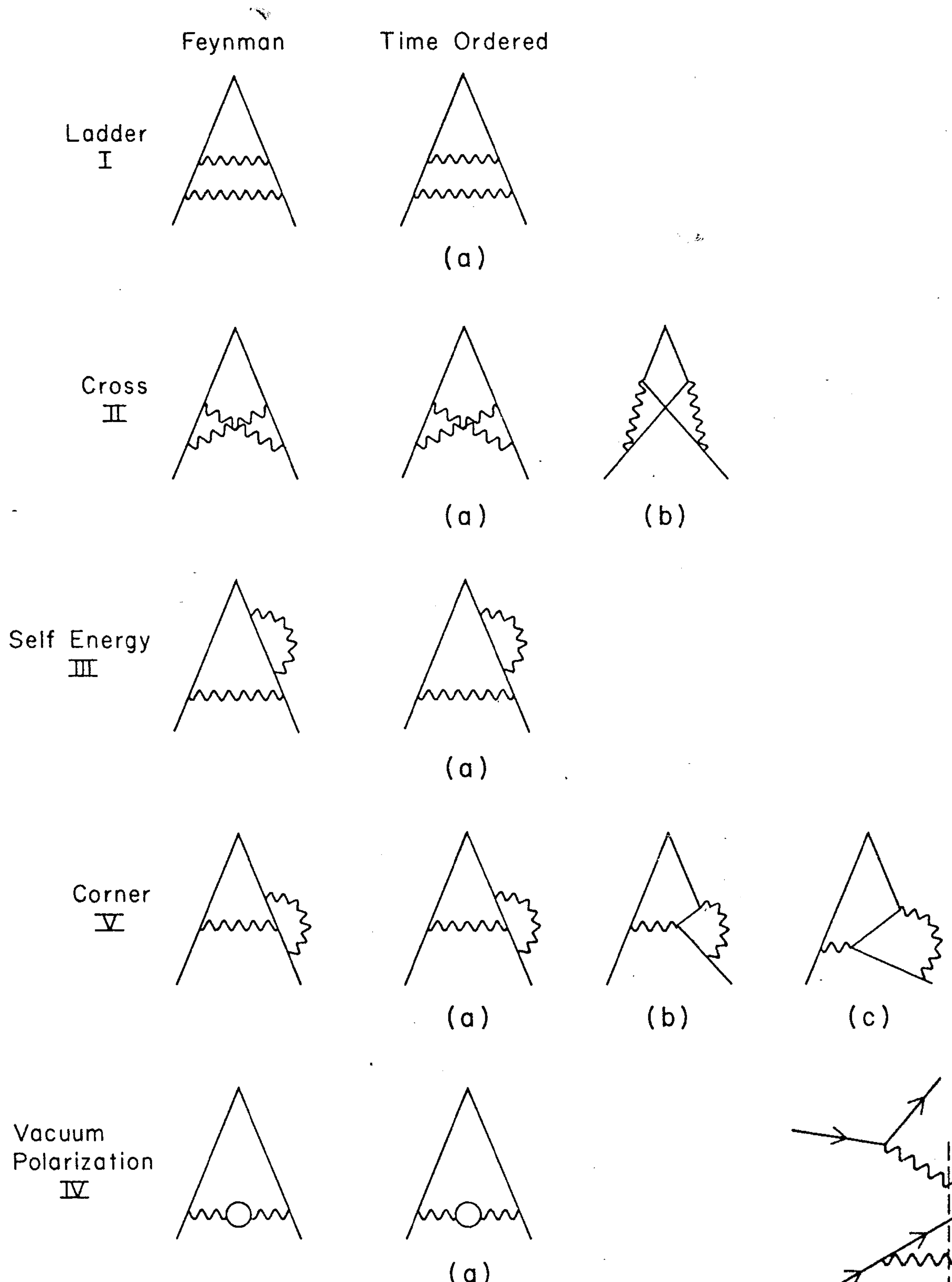
$$\times \begin{pmatrix} 1 + \dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathcal{M}+E}\,\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathcal{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{pmatrix} \otimes \begin{pmatrix} 1 - \dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathcal{M}+E}\,\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathcal{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{pmatrix}$$

$$\times\ \phi_{\mathcal{M}}(\mathbf{p})\,\chi_{SM}\exp[i\mathbf{p}\cdot\tilde{\mathbf{x}}+i\mathbf{P}\cdot\mathbf{X}]\exp[-iEX^0].$$

$$\tilde{\mathbf{x}} = \mathbf{x} + (\gamma-1)\,\hat{\mathbf{V}}\hat{\mathbf{V}}\cdot\mathbf{x} \quad ; \quad p^0_{a,b} = \sqrt{\mathbf{p}^2+m^2_{a,b}}\,, \quad k_{a,b} \equiv -\tau_{b,a}(U+W).$$

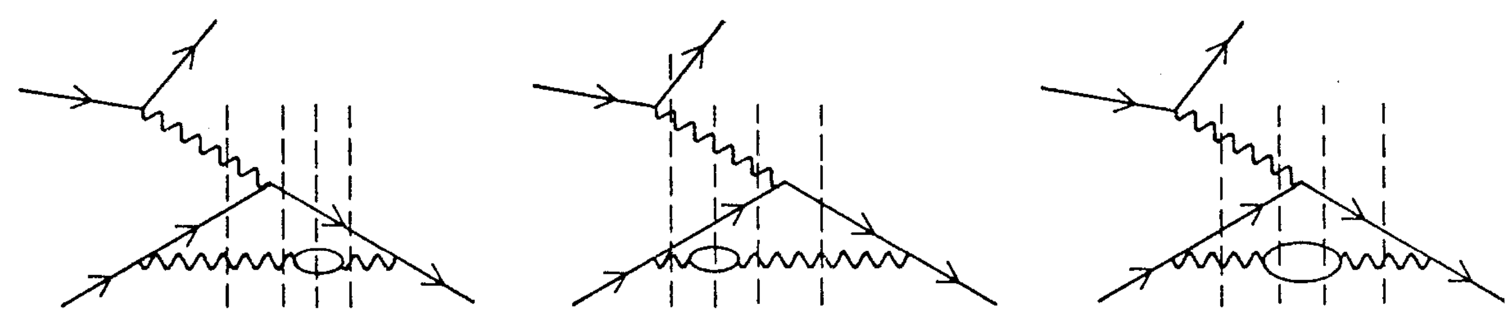
Correct Boosted Wavefunction needed for LET, DGH!

QED g-2 in LFPth



Roskies, Suaya, and sjb

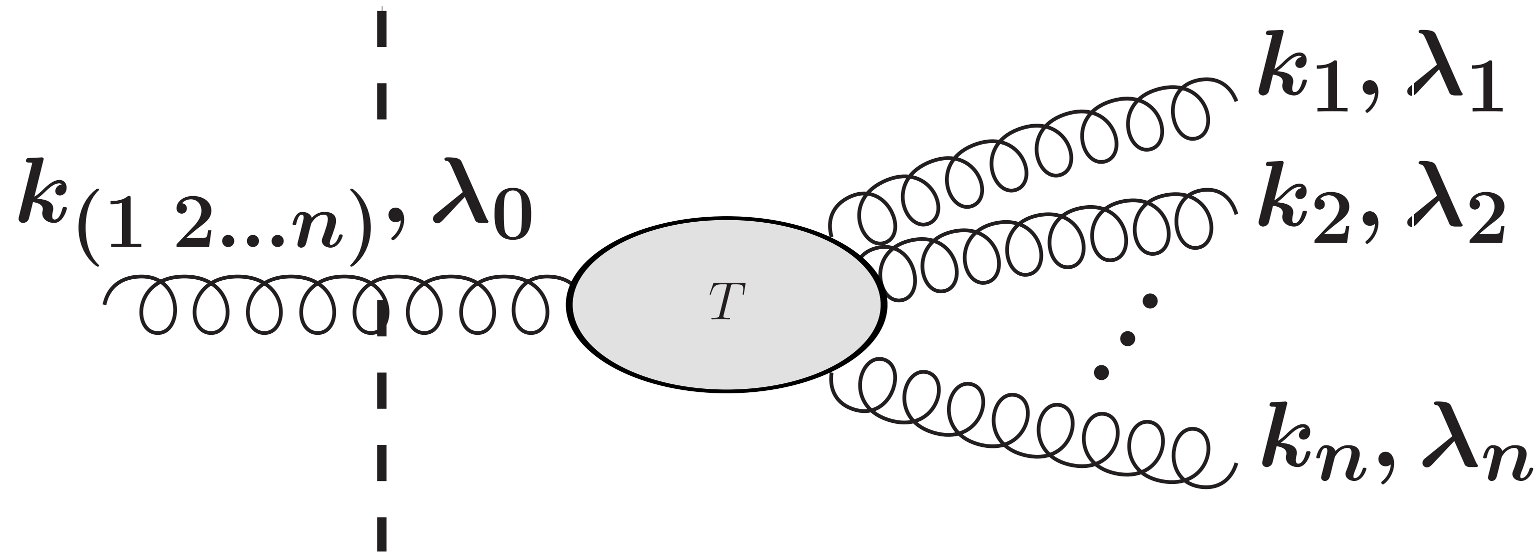
Alternate denominator renormalization

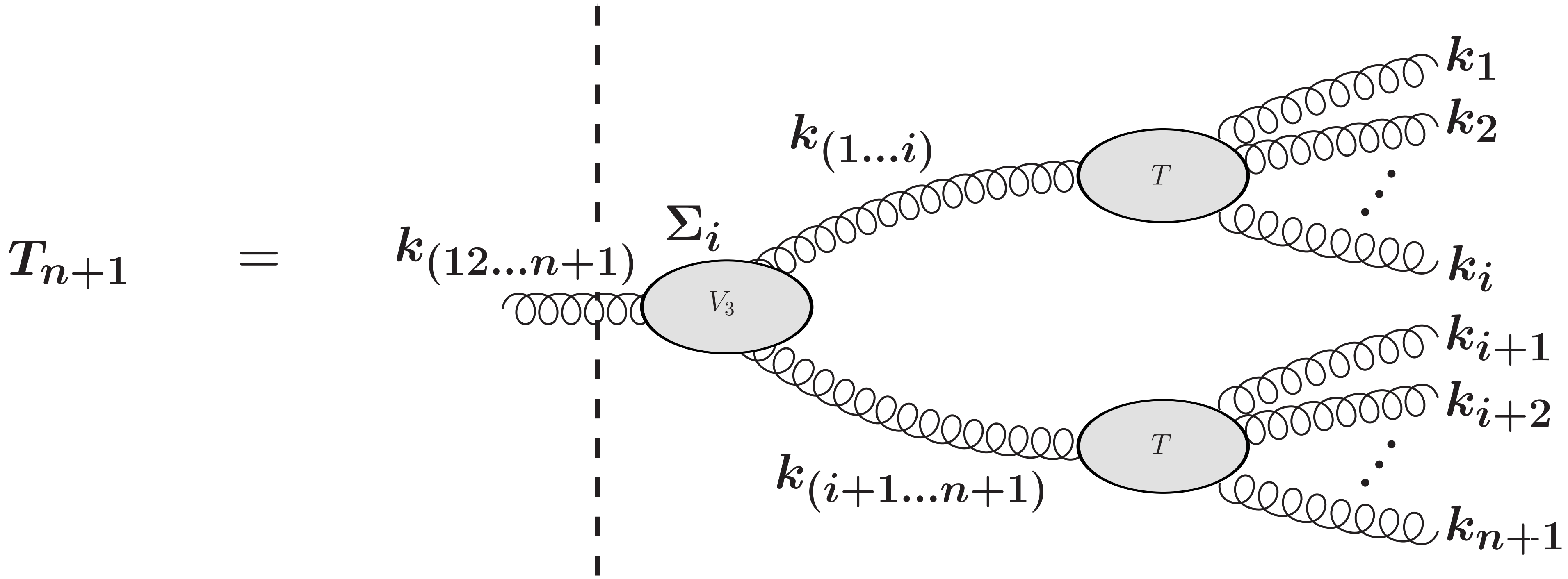


Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: C. Ji & sjb





Parke-Taylor amplitudes reflect LF angular momentum conservation

$$\langle i j \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) :$$

Single-spin asymmetries

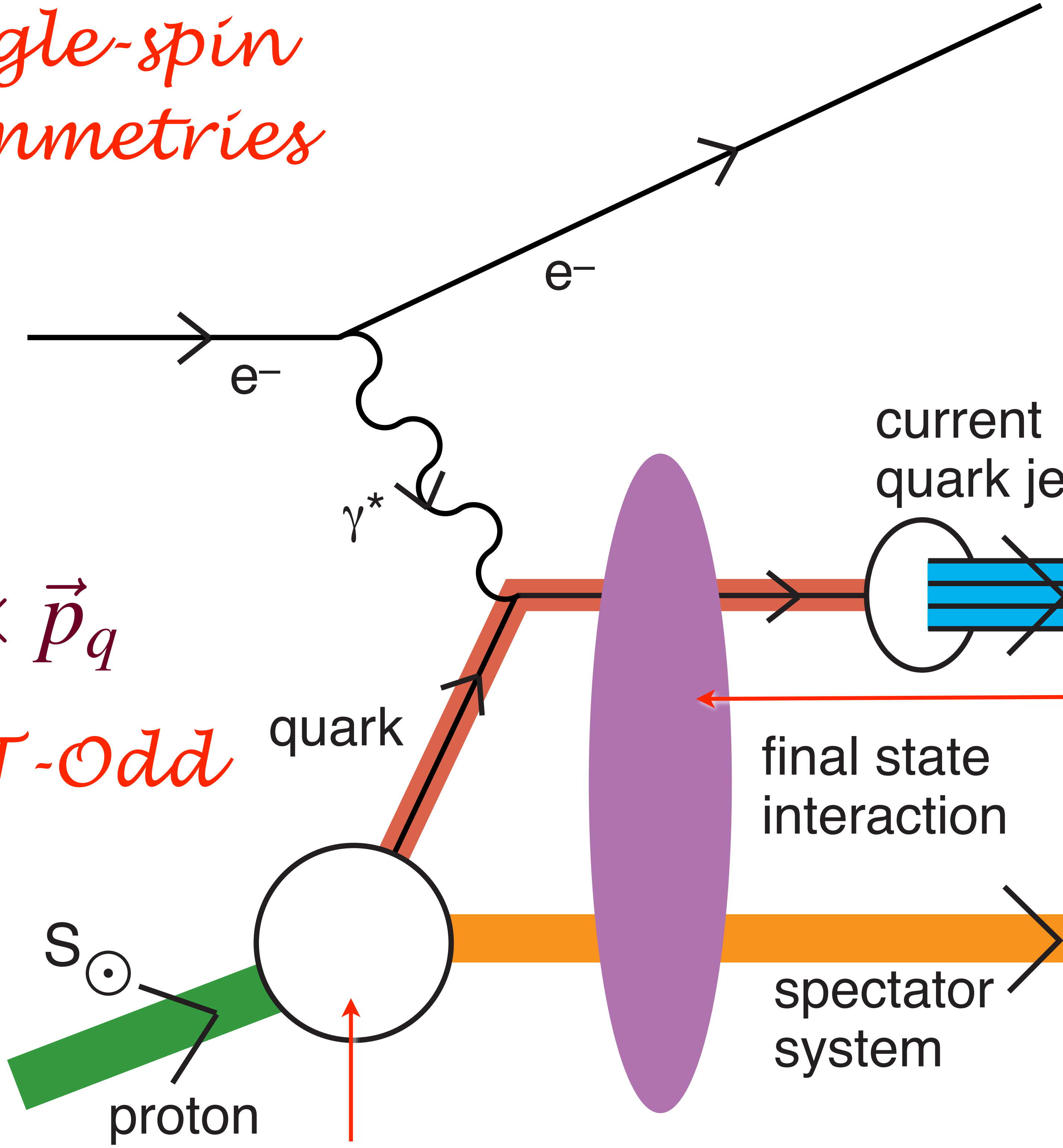
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



Light-Front Wavefunction
S and P- Waves!

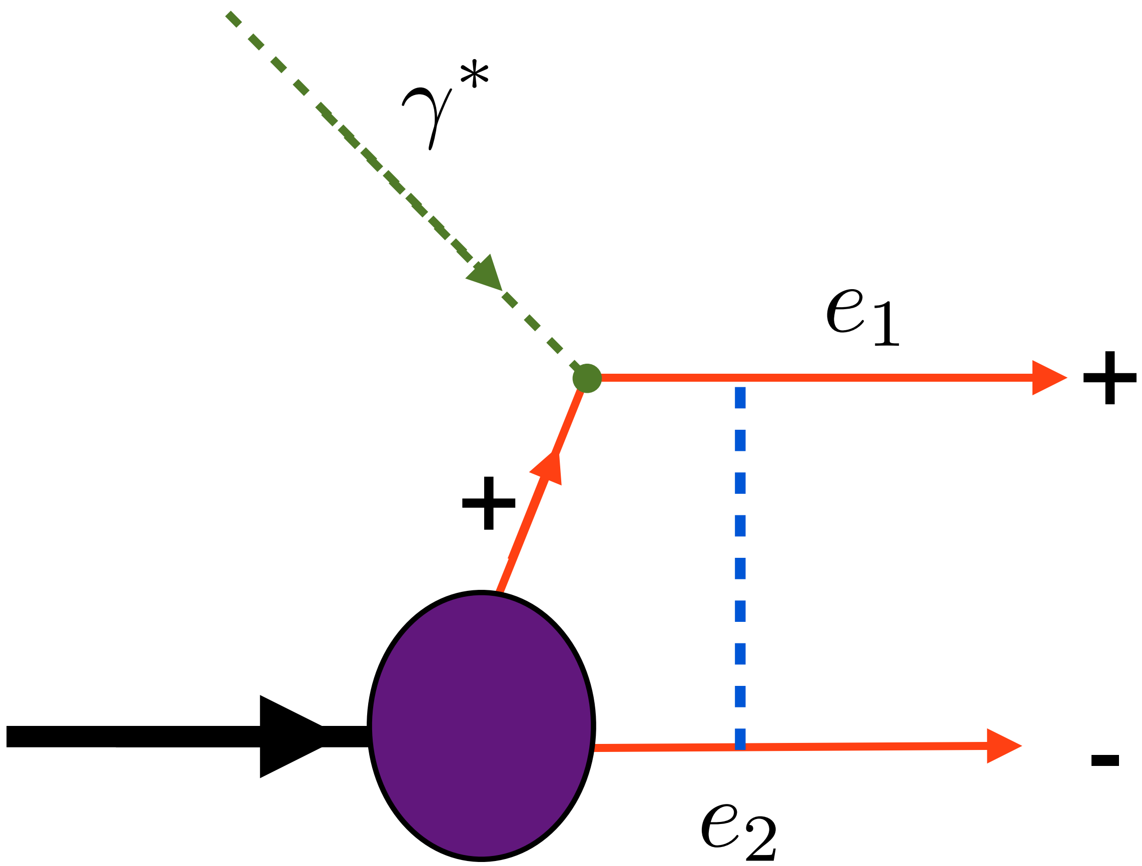
QCD S- and P-
Coulomb Phases
--Wilson Line

“Lensing Effect”

QED: Lensing involves soft scales

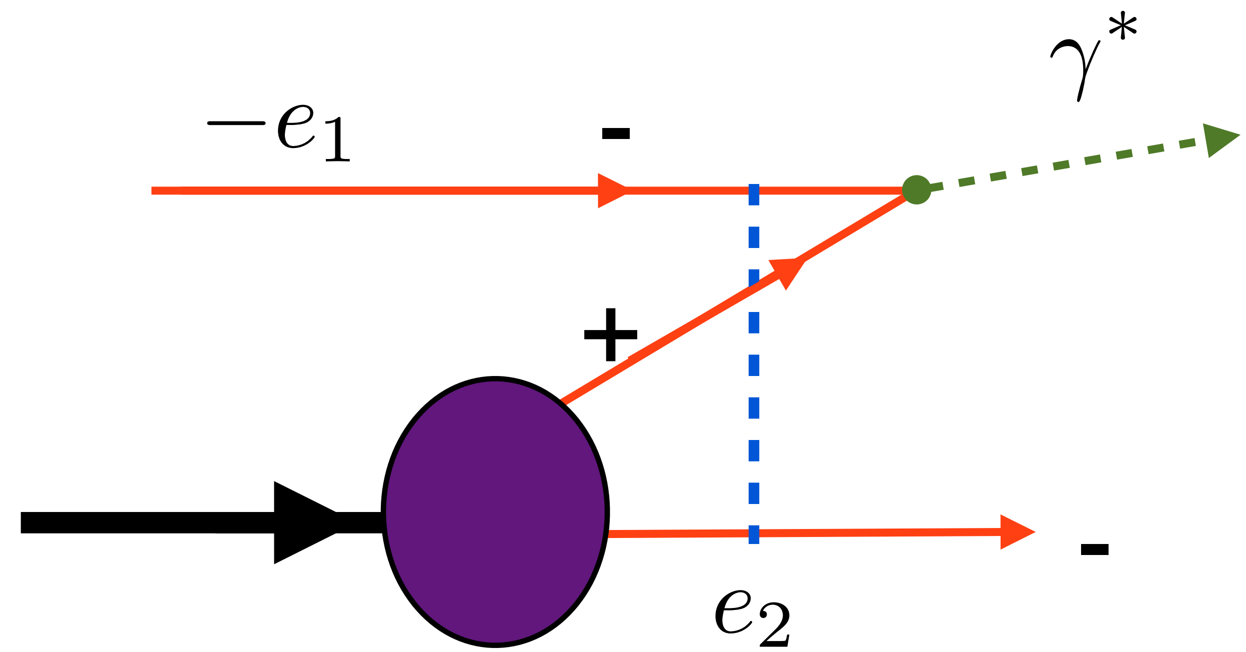
Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!



DIS

Attractive, opposite-sign rescattering potential



DY

Repulsive, same-sign scattering potential

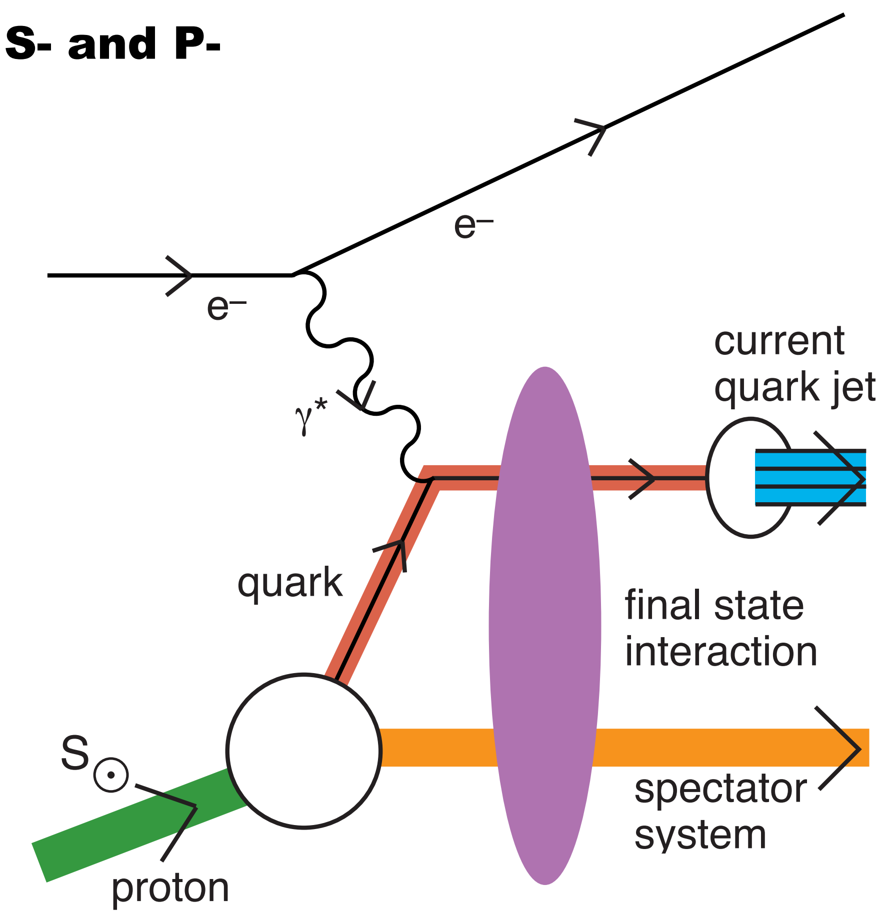
Dae Sung Hwang, Yuri V. Kovchegov,
Ivan Schmidt, Matthew D. Sievert, sjb

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;**
- **Wilson line effect -- lc gauge prescription**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**

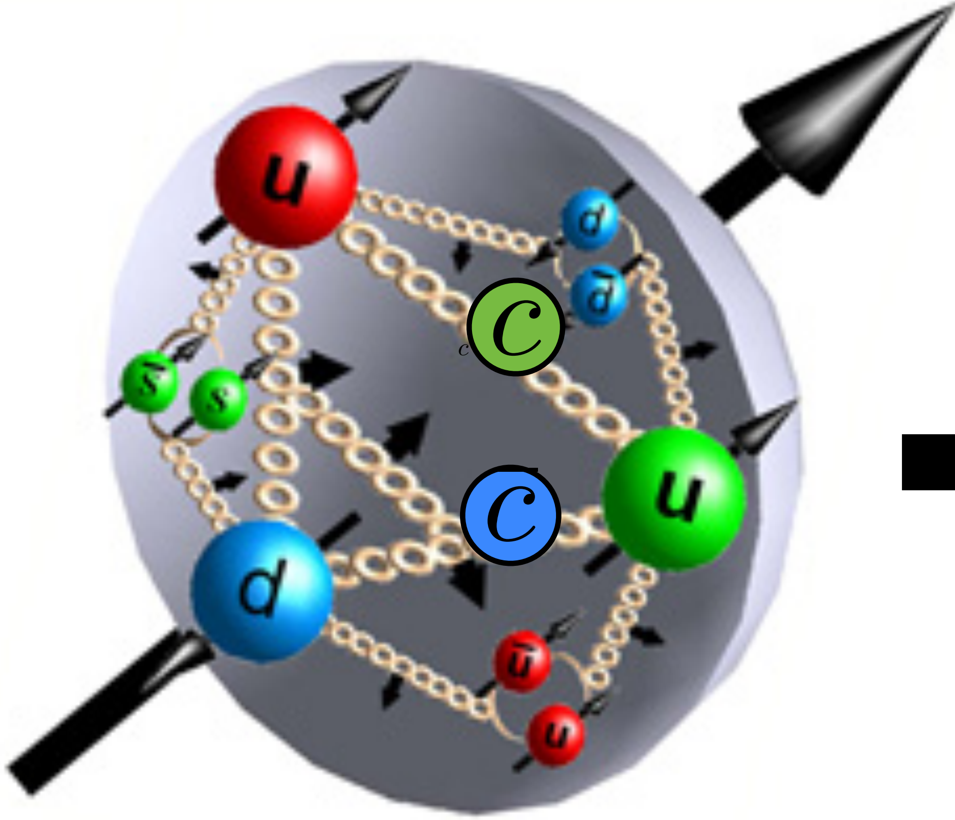


Mulders, Boer **Qiu, Sterman**

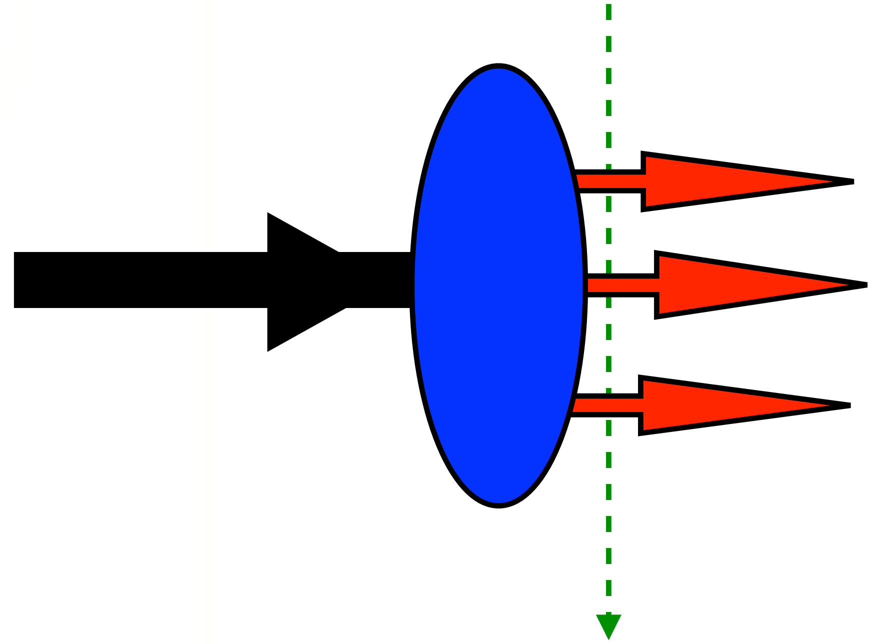
Pasquini, Xiao, Yuan, sjb

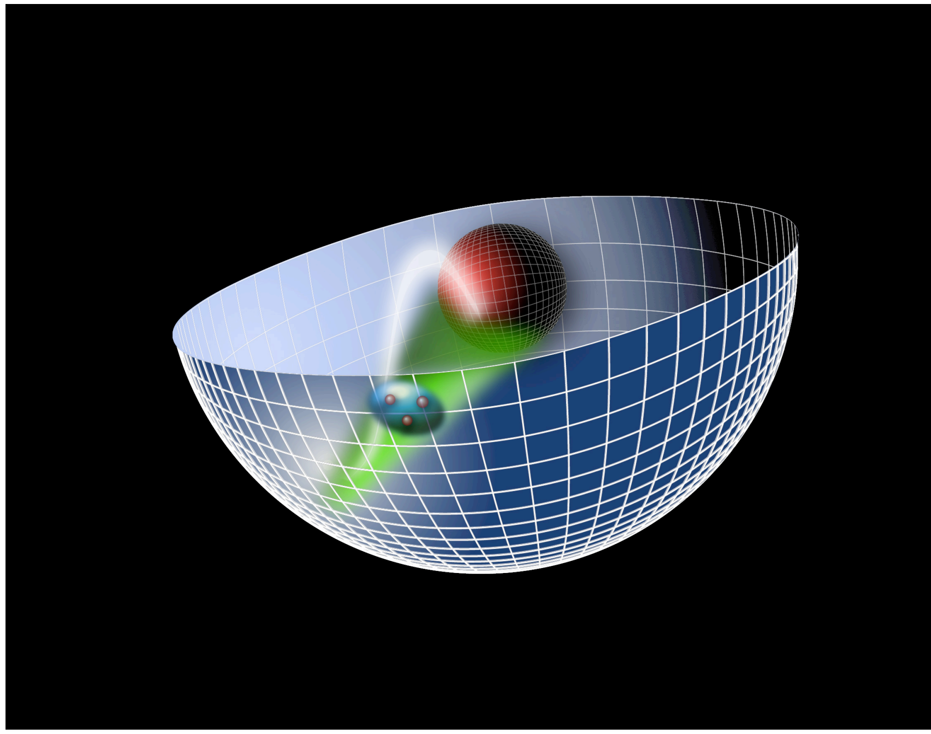
Dae Sung Hwang, Yuri V. Kovchegov, Ivan Schmidt, Matthew D. Sievert, sjb

Introduction to Light-Front Quantization



Fixed $\tau = t + z/c$

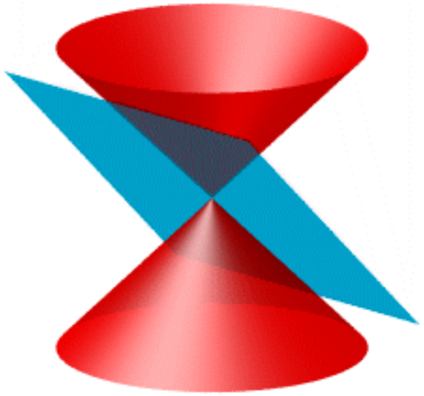




Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY 1891



NeD TURIC

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