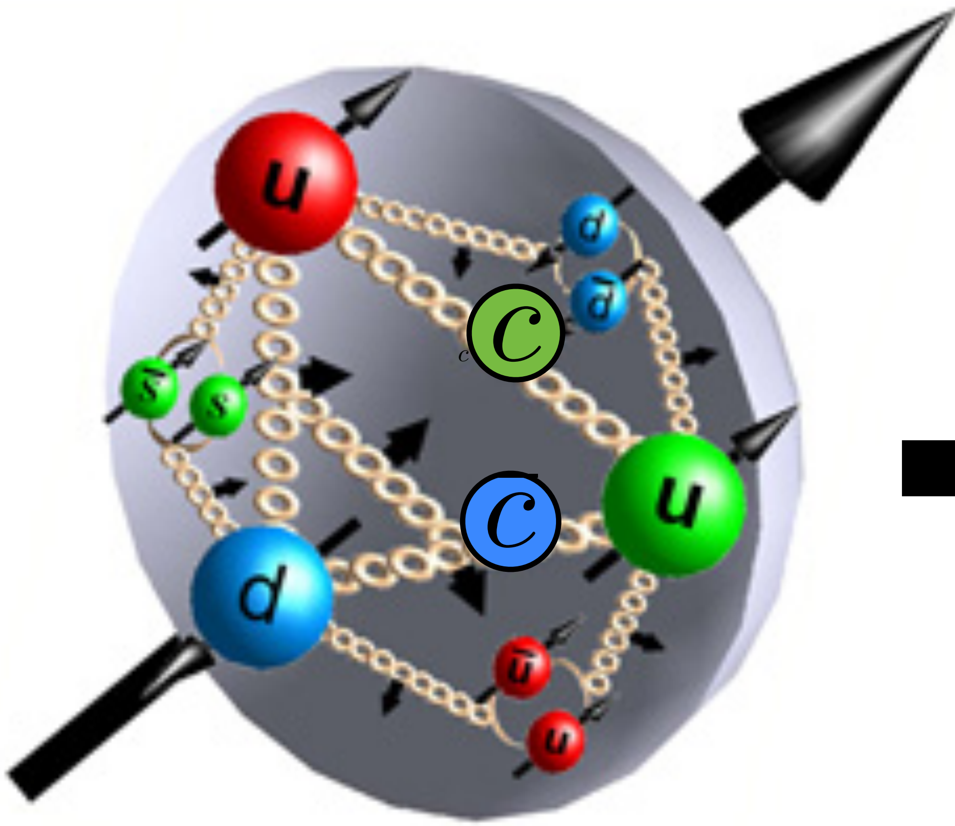
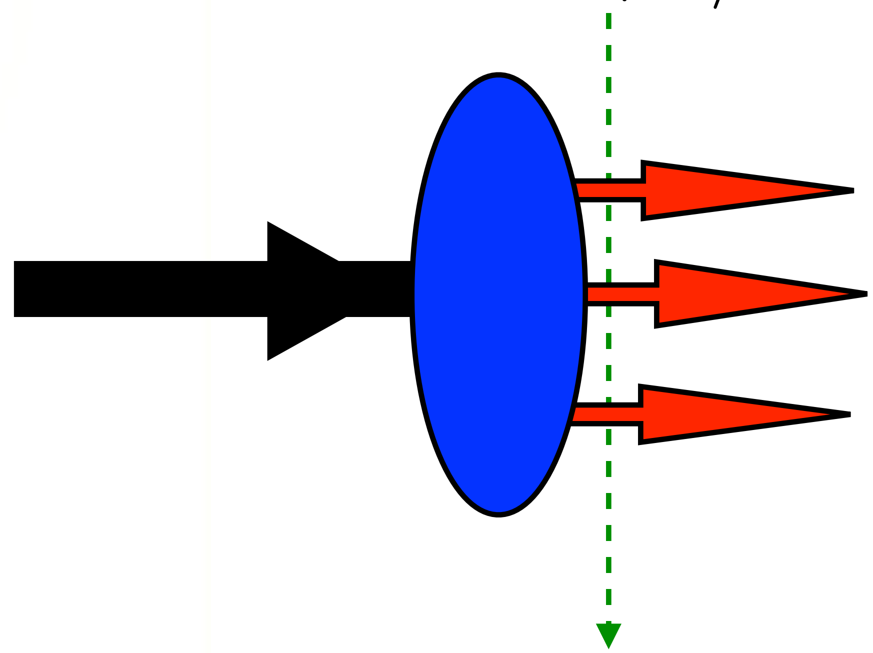
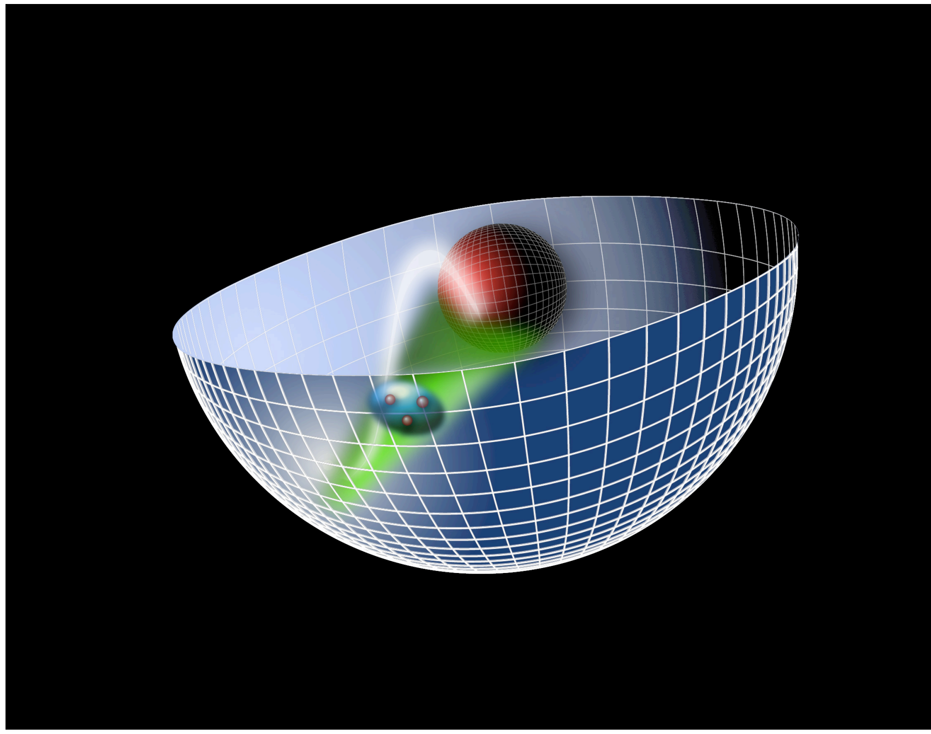
Introduction to Light-Front Quantization

Lecture II



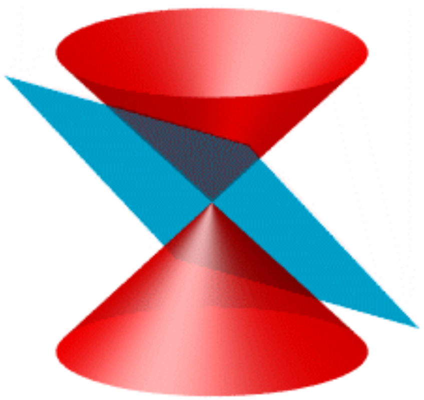
Fixed $\tau = t + z/c$





Stan Brodsky

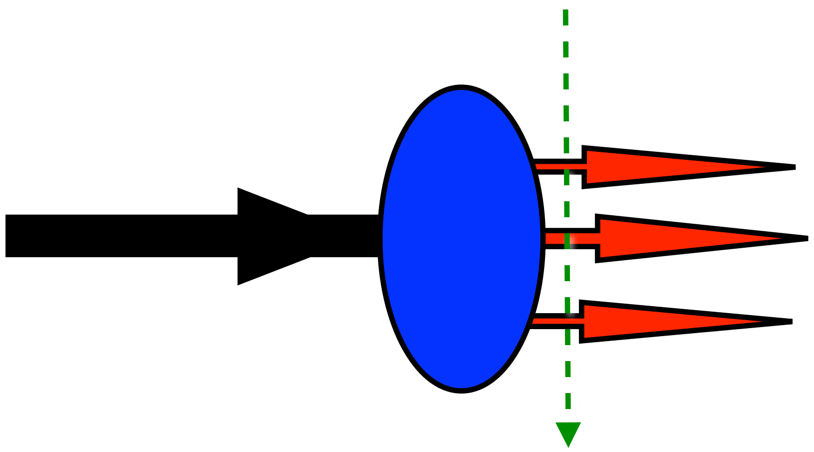
SLAC NATIONAL ACCELERATOR LABORATORY



Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

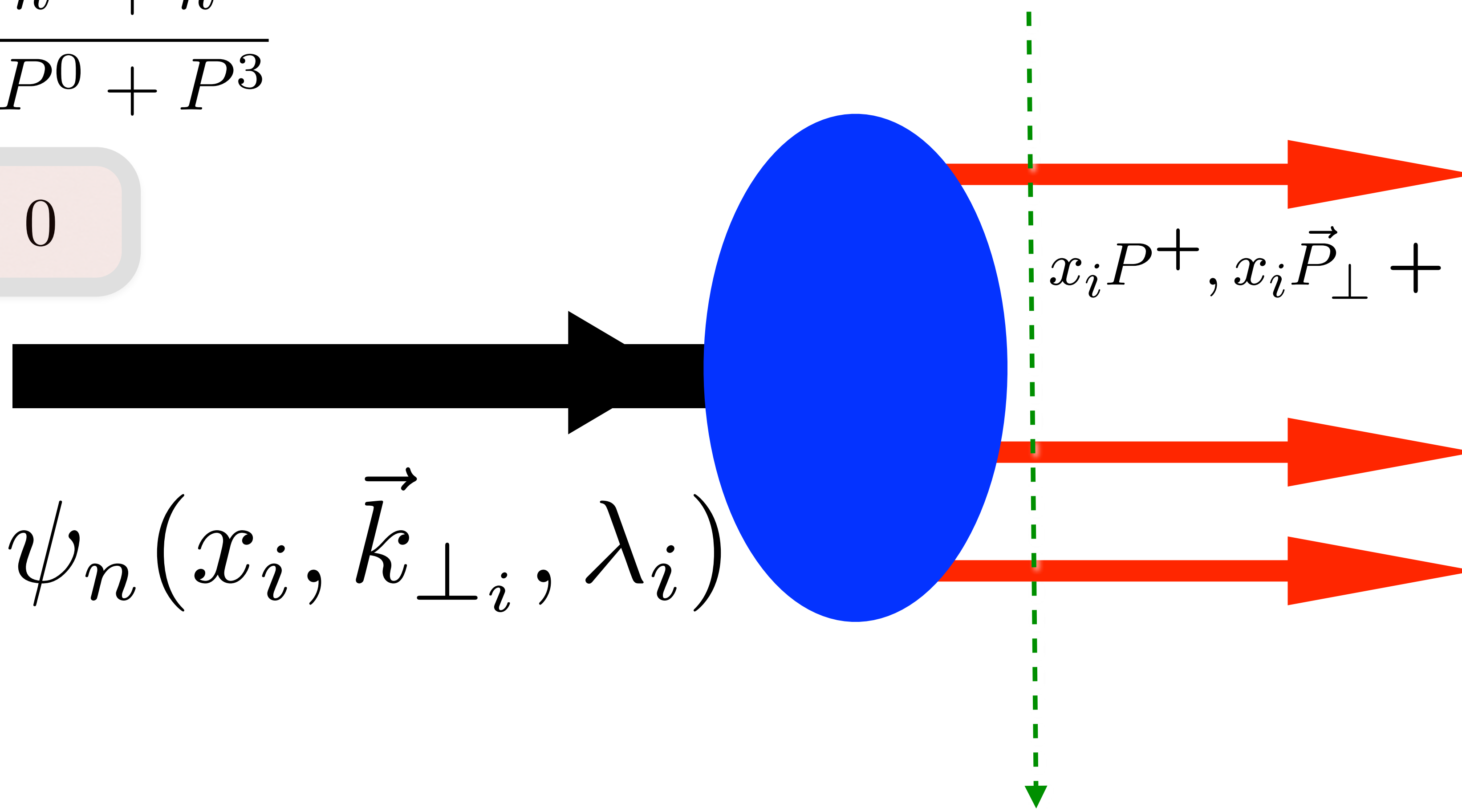
Eigenstate of LF Hamiltonian

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^3 > 0, P^3 < 0$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; x_i, \vec{k}_{\perp i}, \lambda_i >$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

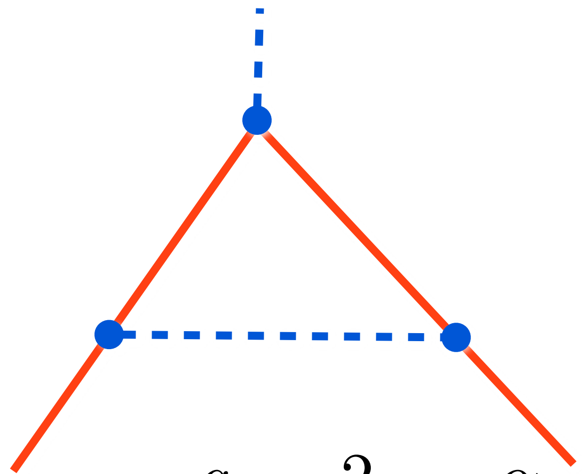
Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Advantages of the Dirac's Front Form for Hadron Physics

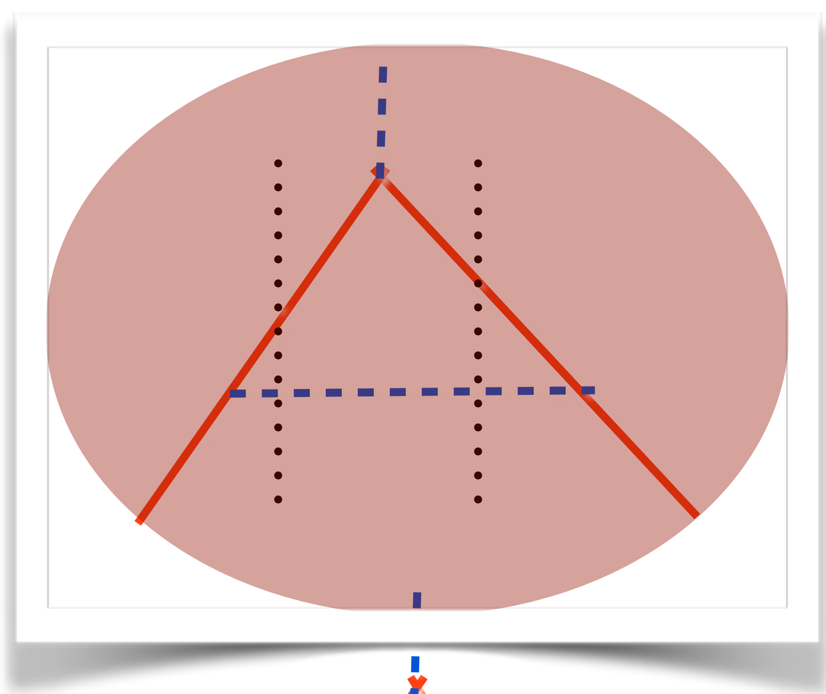
- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts!
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

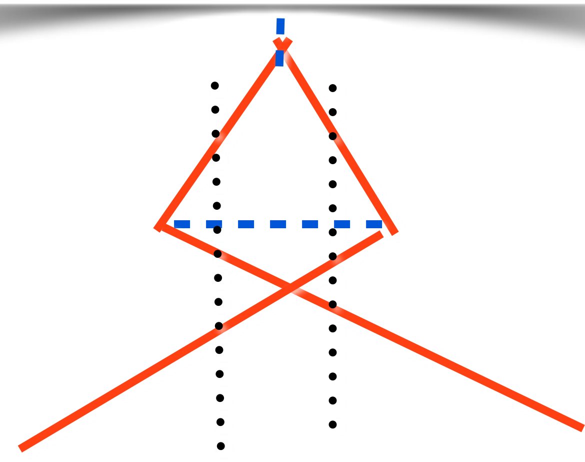
Wick Theorem

Feynman diagram =
single front-form time-ordered diagram!

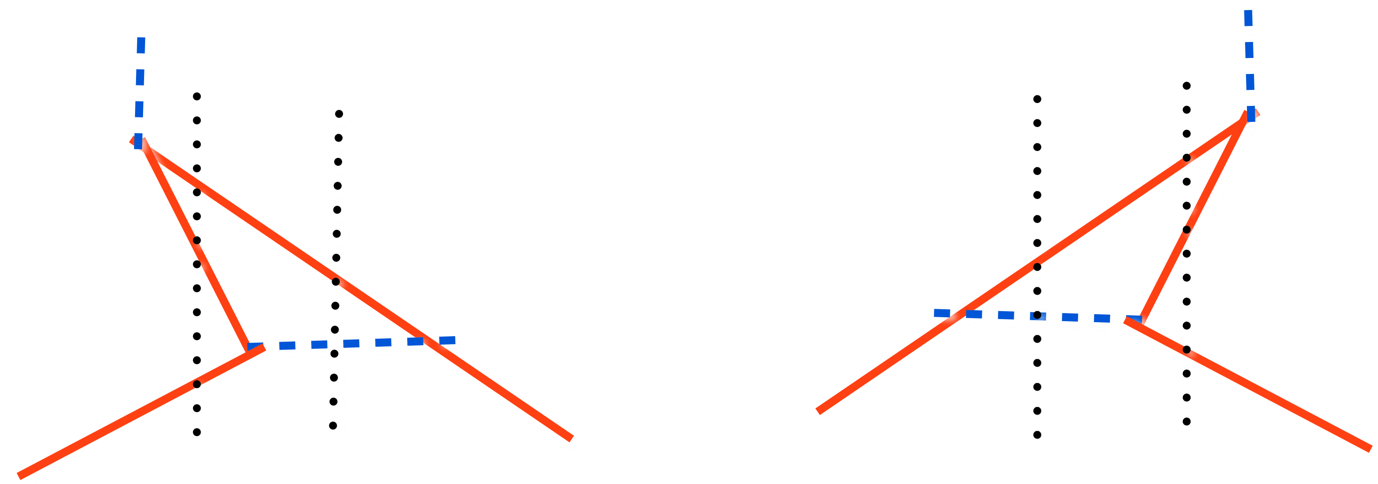


$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

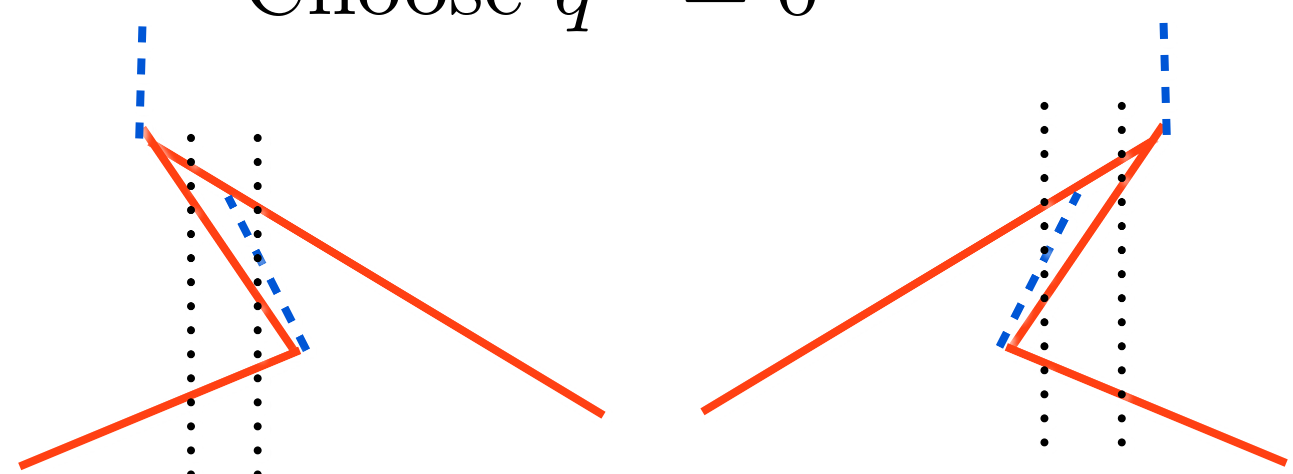




Also $P \to \infty$ observer frame (Weinberg)



Choose $q^+ = 0$



Remarkable Advantages of the Front Form

- **Light-Front Time-Ordered Perturbation Theory: Elegant, Physical**
- **Frame-Independent**
- **Few LF Time-Ordered Diagrams (not n!) -- all k^+ must be positive**
- **$J^z = L^z + S^z$ conserved at each vertex**
- **Automatically normal-ordered; LF Vacuum trivial up to zero modes**
- **Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED**
- **Reproduces Parke-Taylor Rules and Amplitudes (Stasto)**
- **Hadronization at the Amplitude Level with Confinement**

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

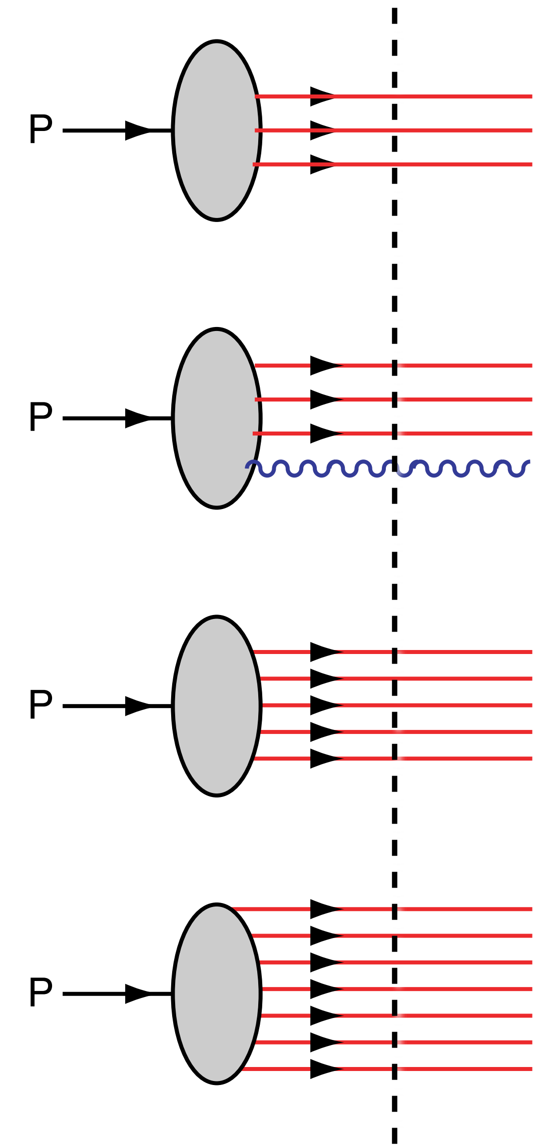
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

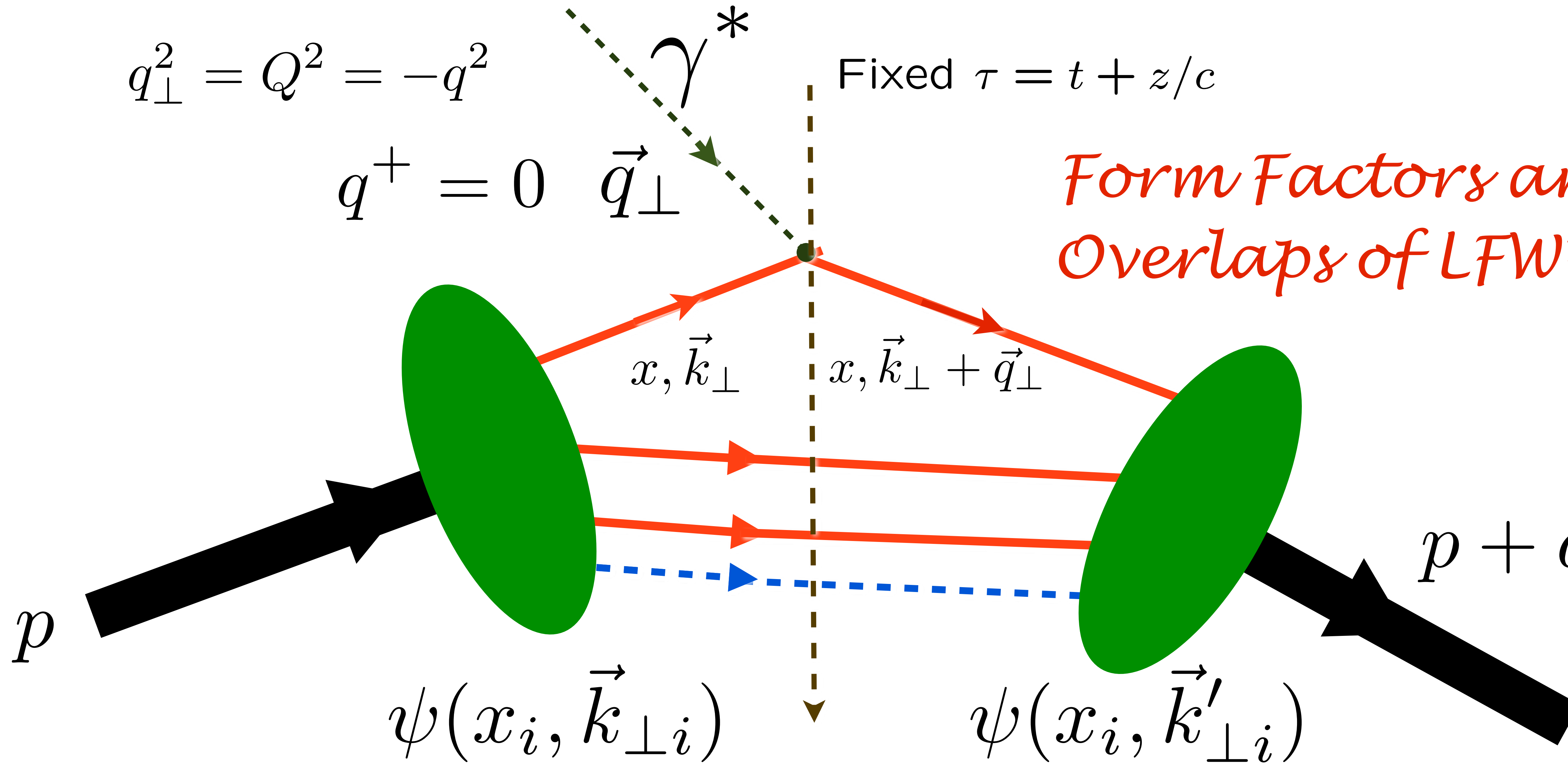
Mueller: gluon Fock states **BFKL Pomeron**

Hidden Color

$$< p+q|j^+(0)|p> = 2p^+F(q^2)$$

Interaction picture

$$q_\perp^2 = Q^2 = -q^2$$



Form Factors are Overlaps of LFWFs

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!
Drell, sjb

Exact LF Formula for Pauli Form Factor

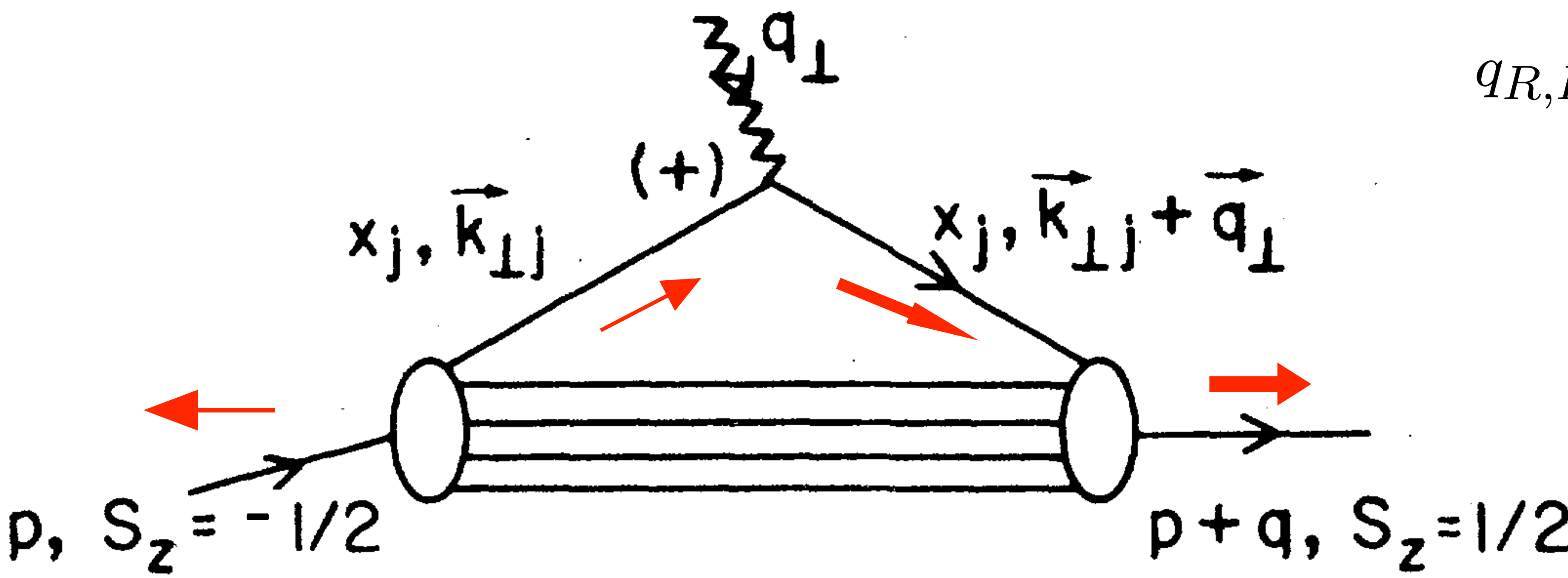
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

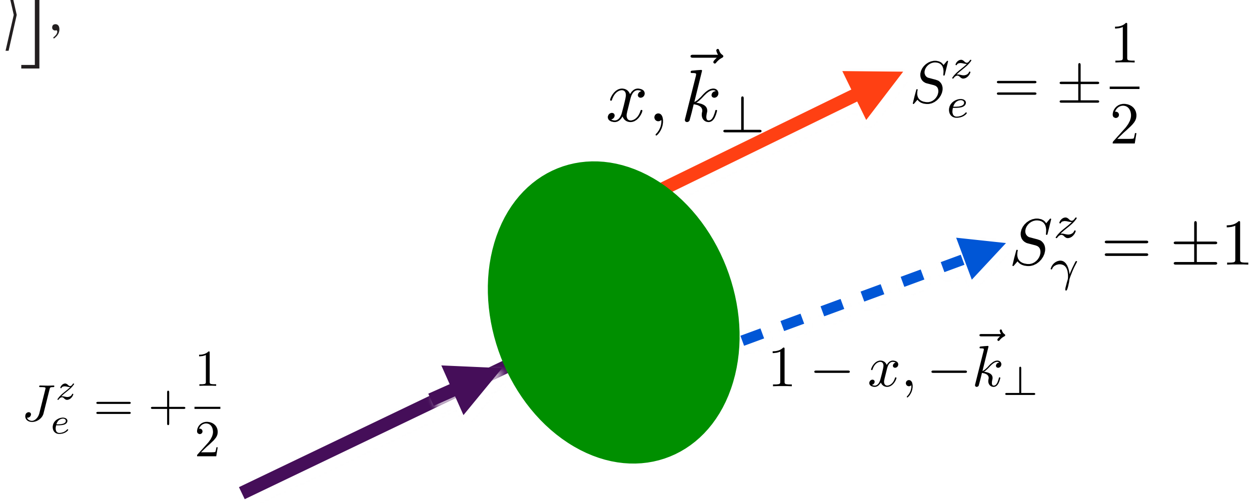


Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

The two-particle Fock state for an electron with $J^z = +\frac{1}{2}$ has four possible spin combinations:

$$
\begin{aligned}
&\left|\Psi^{\uparrow}_{\text{two particle}}(P^+, \vec{P}_\perp = \vec{0}_\perp)\right\rangle \\
&= \int \frac{d^2\vec{k}_\perp\, dx}{\sqrt{x(1-x)}\,16\pi^3}\Big[\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp)\left|+\tfrac{1}{2}+1;\, xP^+,\, \vec{k}_\perp\right\rangle \\
&\quad + \psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp)\left|+\tfrac{1}{2}-1;\, xP^+,\, \vec{k}_\perp\right\rangle + \psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp)\left|-\tfrac{1}{2}+1;\, xP^+,\, \vec{k}_\perp\right\rangle \\
&\quad + \psi^{\uparrow}_{-\frac{1}{2}-1}(x,\vec{k}_\perp)\left|-\tfrac{1}{2}-1;\, xP^+,\, \vec{k}_\perp\right\rangle\Big],
\end{aligned}
$$



$$
\begin{cases}
\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\dfrac{(-k^1 + ik^2)}{x(1-x)}\,\varphi, \\
\psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\dfrac{(+k^1 + ik^2)}{1-x}\,\varphi, \\
\psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp) = -\sqrt{2}\left(M - \dfrac{m}{x}\right)\varphi, \\
\psi^{\uparrow}_{-\frac{1}{2}-1}(x,\vec{k}_\perp) = 0,
\end{cases}
$$

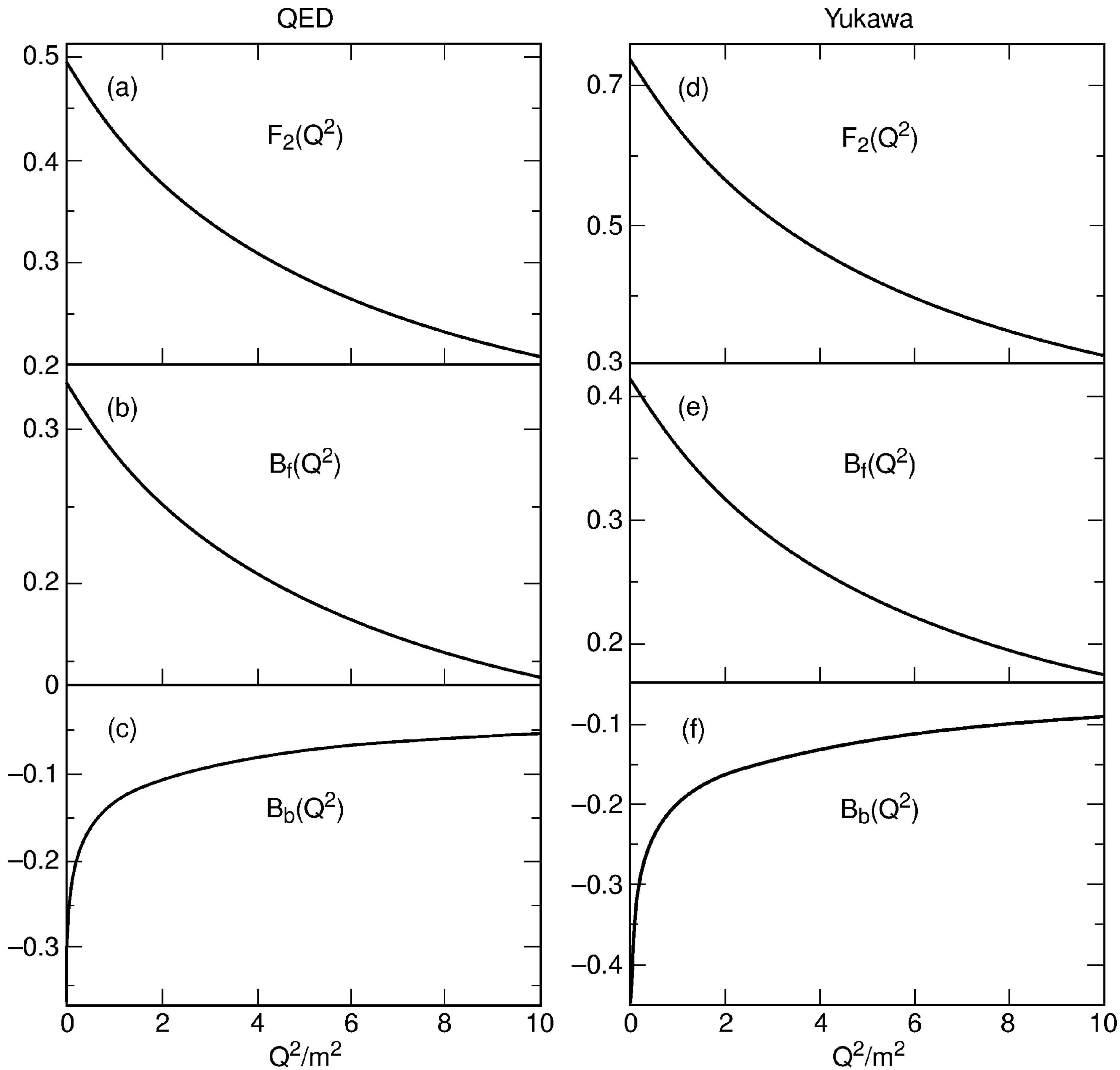
$$
\varphi = \varphi(x,\vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}.
$$

Electron LFWF
Structure Function

Hwang, Schmidt, Ma, sjb

$$
\begin{aligned}
F_2(q^2) &= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \langle \Psi^{\uparrow}(P^+, \vec{P}_\perp = \vec{q}_\perp) | \Psi^{\downarrow}(P^+, \vec{P}_\perp = \vec{0}_\perp) \rangle \\
&= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \int \frac{\mathrm{d}^2 \vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \Big[\psi^{\uparrow\,*}_{+\frac{1}{2}-1}(x, \vec{k}'_\perp) \psi^{\downarrow}_{+\frac{1}{2}-1}(x, \vec{k}_\perp) \\
&\qquad\qquad + \psi^{\uparrow\,*}_{-\frac{1}{2}+1}(x, \vec{k}'_\perp) \psi^{\downarrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) \Big] \\
&= 4M \int \frac{\mathrm{d}^2 \vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \frac{(m - Mx)}{x} \varphi(x, \vec{k}'_\perp)^* \varphi(x, \vec{k}_\perp) \\
&= 4Me^2 \int \frac{\mathrm{d}^2 \vec{k}_\perp \, \mathrm{d}x}{16\pi^3} \frac{(m - xM)}{x(1-x)} \\
&\quad \times \frac{1}{M^2 - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + \lambda^2)/(1-x)} \\
&\quad \times \frac{1}{M^2 - (\vec{k}_\perp^2 + m^2)/x - (\vec{k}_\perp^2 + \lambda^2)/(1-x)}. \qquad (30)
\end{aligned}
$$

$$
F_2(q^2) = \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}\alpha \int_0^1 \mathrm{d}x \, \frac{m - xM}{\alpha(1-\alpha)\frac{1-x}{x}\vec{q}_\perp^{\,2} - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}}.
$$



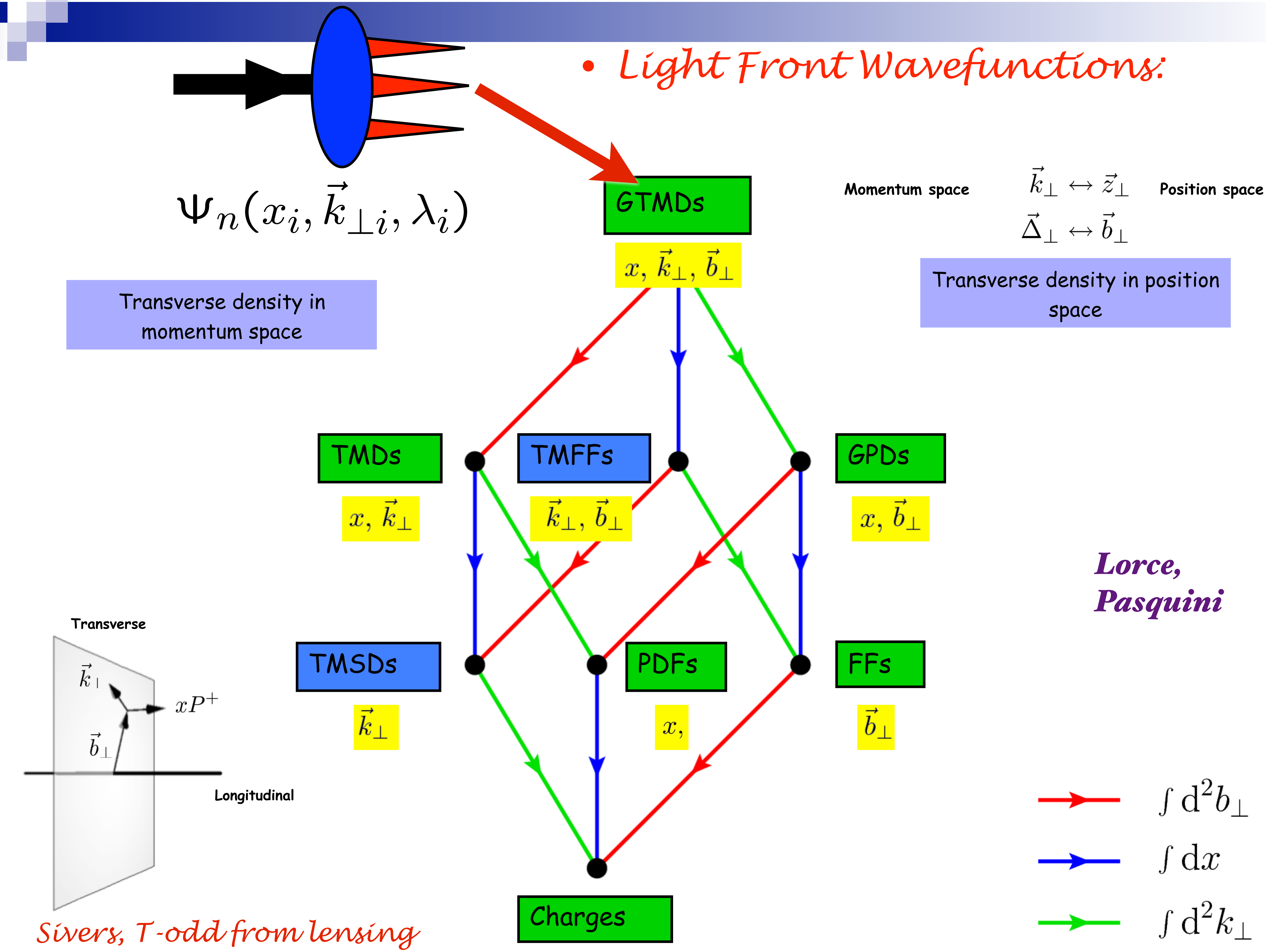
QED
Yukawa
(a)
F2(Q2)
(b)
Bf(Q2)
(c)
Bb(Q2)
(d)
F2(Q2)
(e)
Bf(Q2)
(f)
Bb(Q2)
0.5
0.4
0.3
0.2
0
−0.1
−0.2
−0.3
0.7
0.5
0.3
0.4
0.3
0.2
−0.1
−0.2
−0.3
−0.4
0
2
4
6
8
10
Q2/m2

The anomalous moment is obtained in the limit of zero momentum transfer:

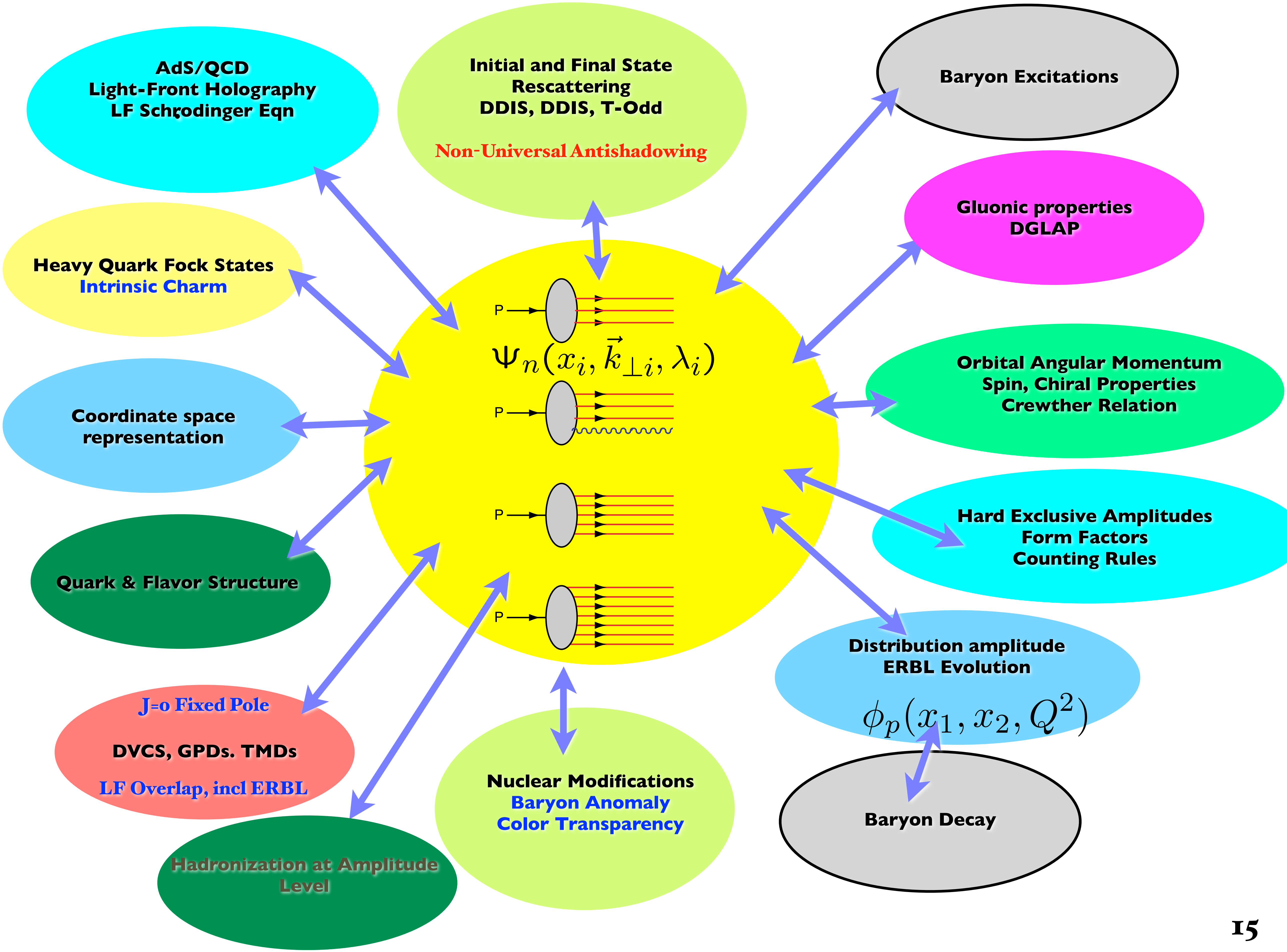
$$
\begin{aligned}
F_2(0) &= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3}\,\frac{(m-xM)}{x(1-x)}\,\frac{1}{[M^2-(\vec{k}_\perp^{\,2}+m^2)/x-(\vec{k}_\perp^{\,2}+\lambda^2)/(1-x)]^2} \\
&= \frac{Me^2}{4\pi^2}\int_0^1 \mathrm{d}x\,\frac{m-xM}{-M^2+\frac{m^2}{x}+\frac{\lambda^2}{1-x}},
\end{aligned}
\tag{32}
$$

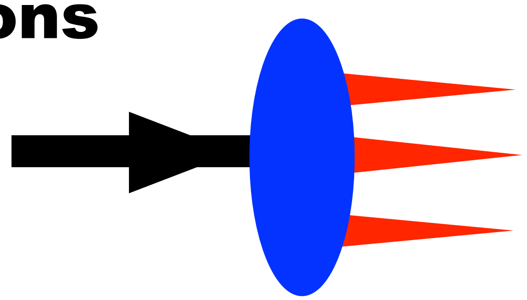
which is the result of Ref. [8]. For zero photon mass and $M = m$, it gives the correct order α Schwinger value $a_e = F_2(0) = \alpha/2\pi$ for the electron anomalous magnetic moment for QED.

- *Light Front Wavefunctions:*



QCD and the LF Hadron Wavefunctions

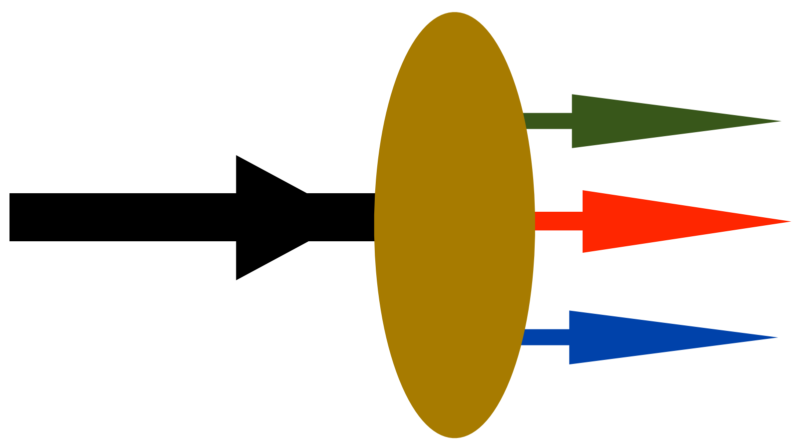




$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**
- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

- *Hadron Physics without LFWFs is like Biology without DNA!*



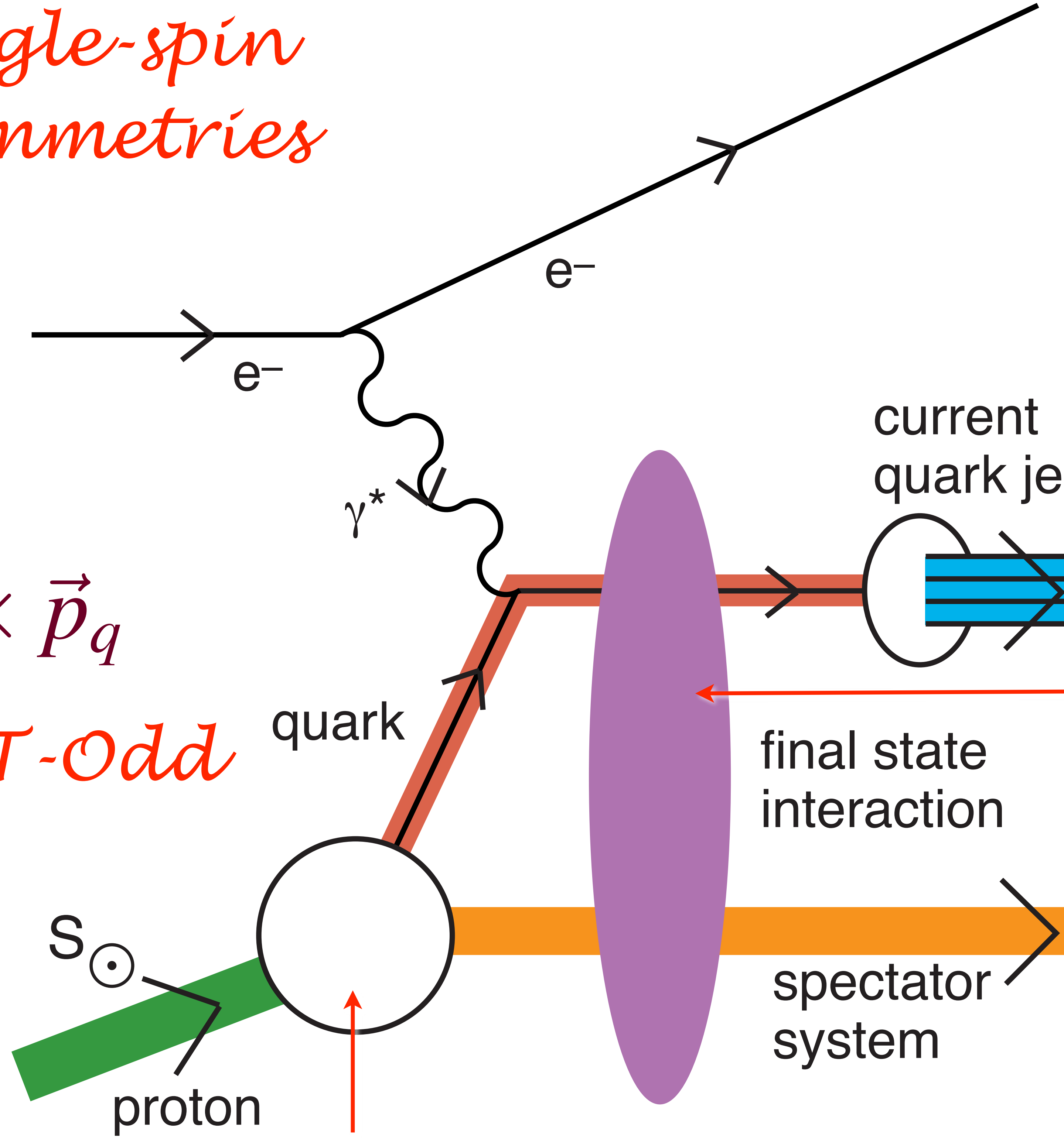
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$



Collins, Burkardt, Ji, Yuan. Pasquini, ...

QCD S- and P- Coulomb Phases --Wilson Line

Pseudo- T-Odd

"Lensing Effect"

QED: Lensing involves soft scales

Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

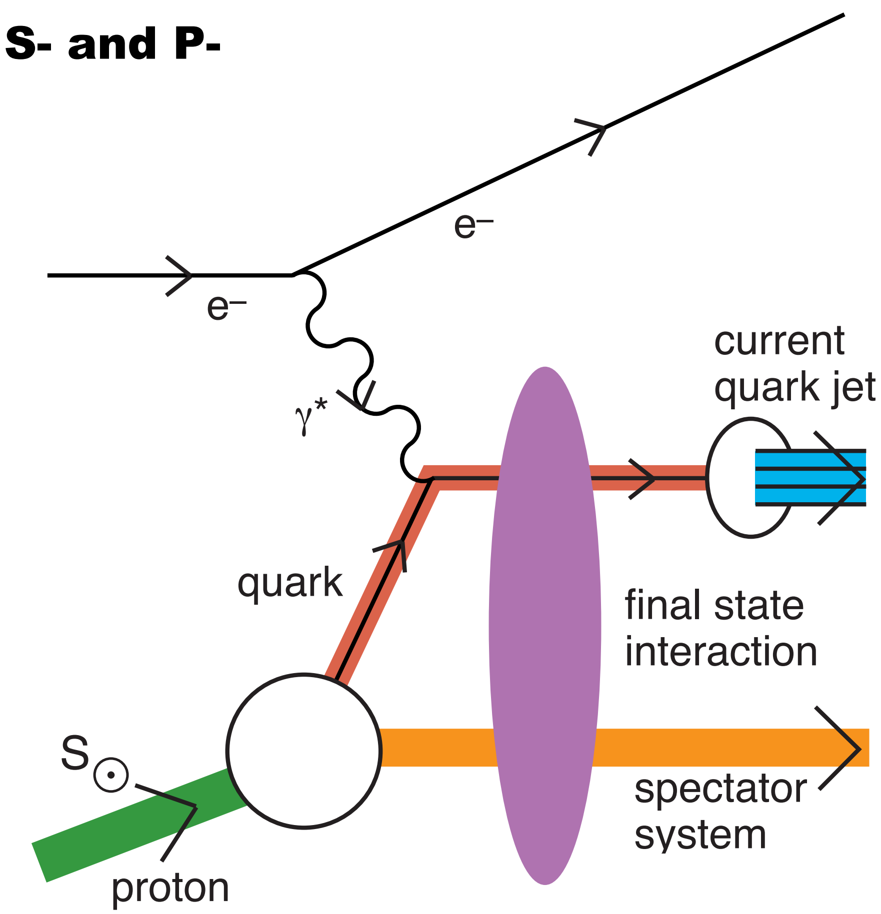
Sign reversal in DY!

Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

Hwang, Schmidt, sjb
Collins

$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;**
- **Wilson line effect -- lc gauge prescription**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**

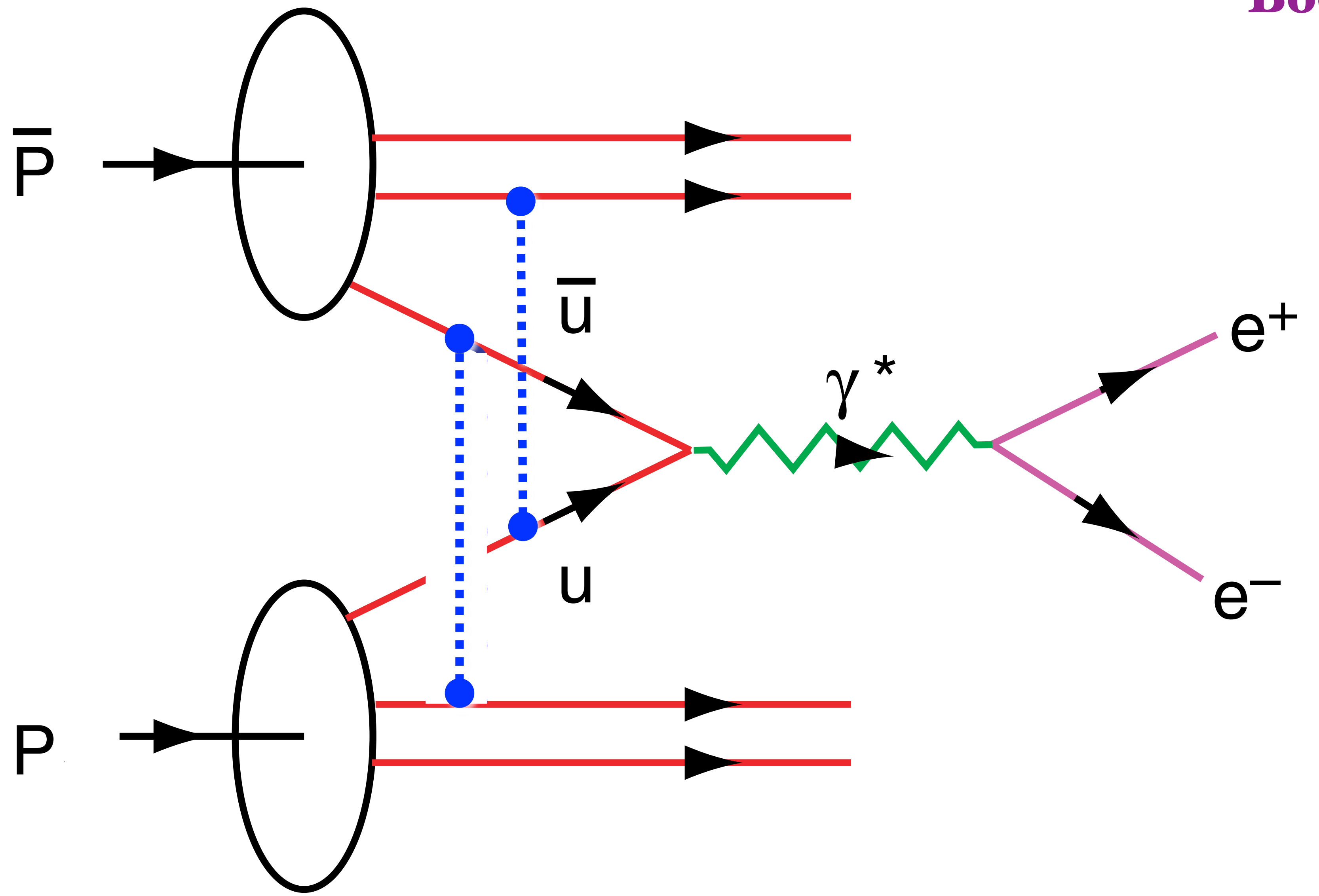


Mulders, Boer   Qiu, Sterman

Pasquini, Xiao, Yuan, sjb

Dae Sung Hwang, Yuri V. Kovchegov, Ivan Schmidt, Matthew D. Sievert, sjb

Boer, Hwang, sjb



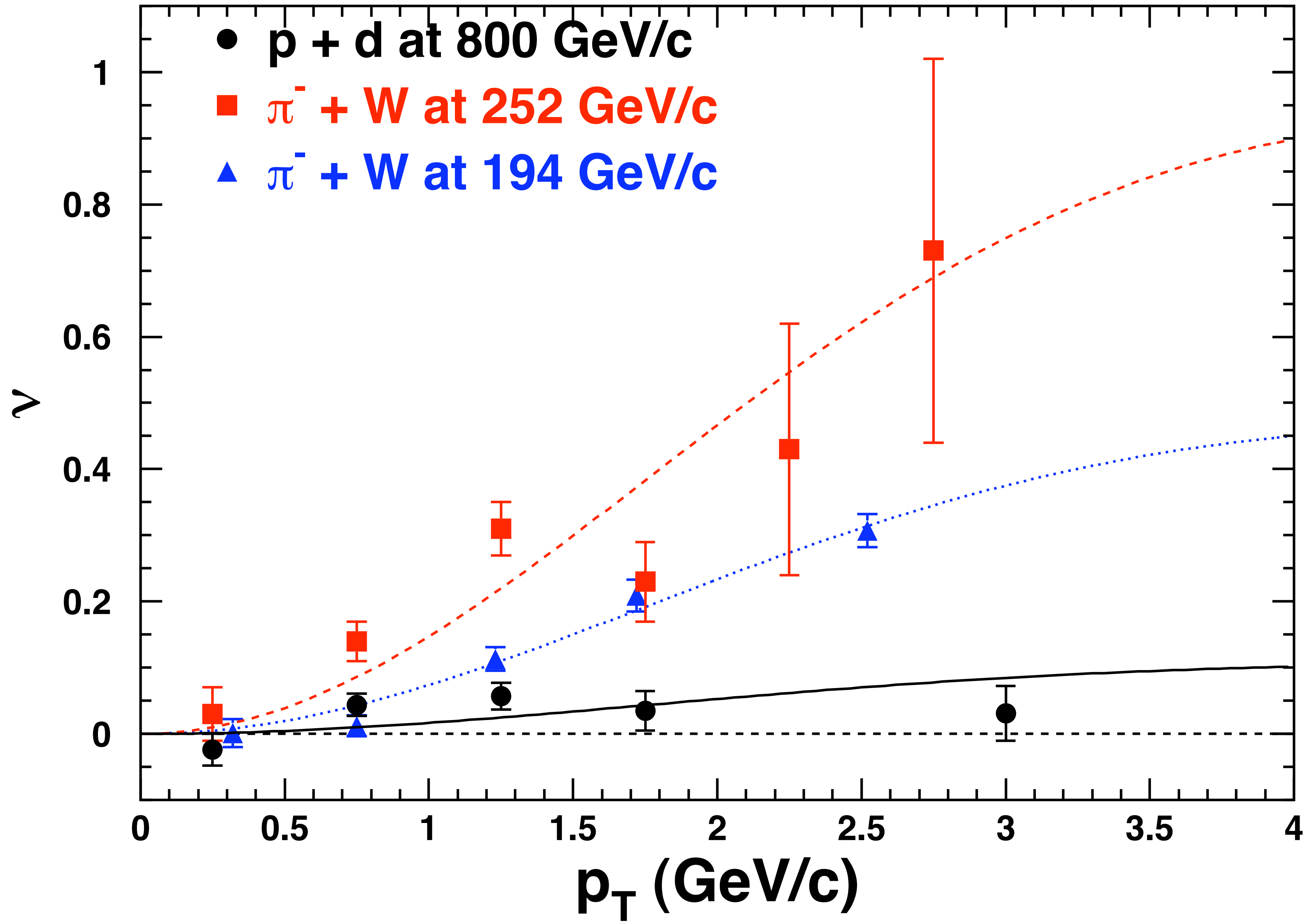
DY $\cos 2\phi$ **correlation at leading twist from double ISI**

Product of Boer - Mulders Functions

$$h_1^\perp(x_1, \boldsymbol{p}_\perp^2) \times \bar{h}_1^\perp(x_2, \boldsymbol{k}_\perp^2)$$

Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c

(FNAL E866/NuSea Collaboration)



Huge Effect in $\pi W \to \mu^+ \mu^- X$

Negligible Effect $pd \to \mu^+ \mu^- X$

Parameter ν vs. p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c^2 are also shown.

Double Initial-State Interactions generate anomalous $\cos 2\phi$

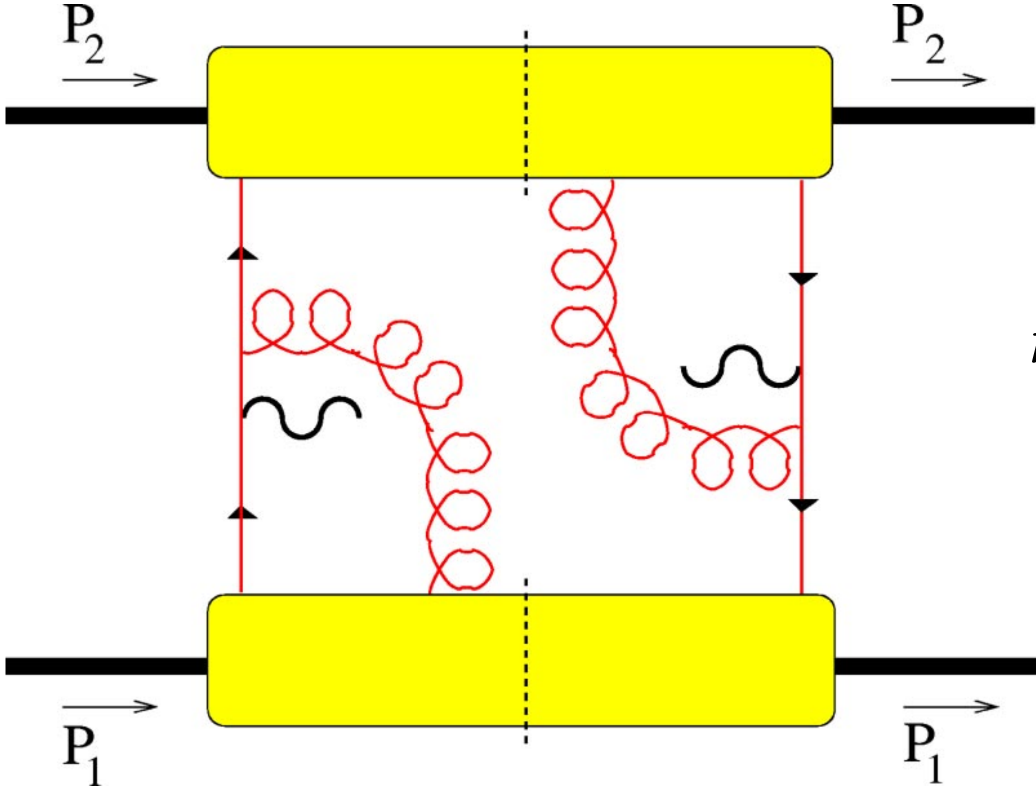
Drell-Yan planar correlations

Boer, Hwang, sjb

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \propto \left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

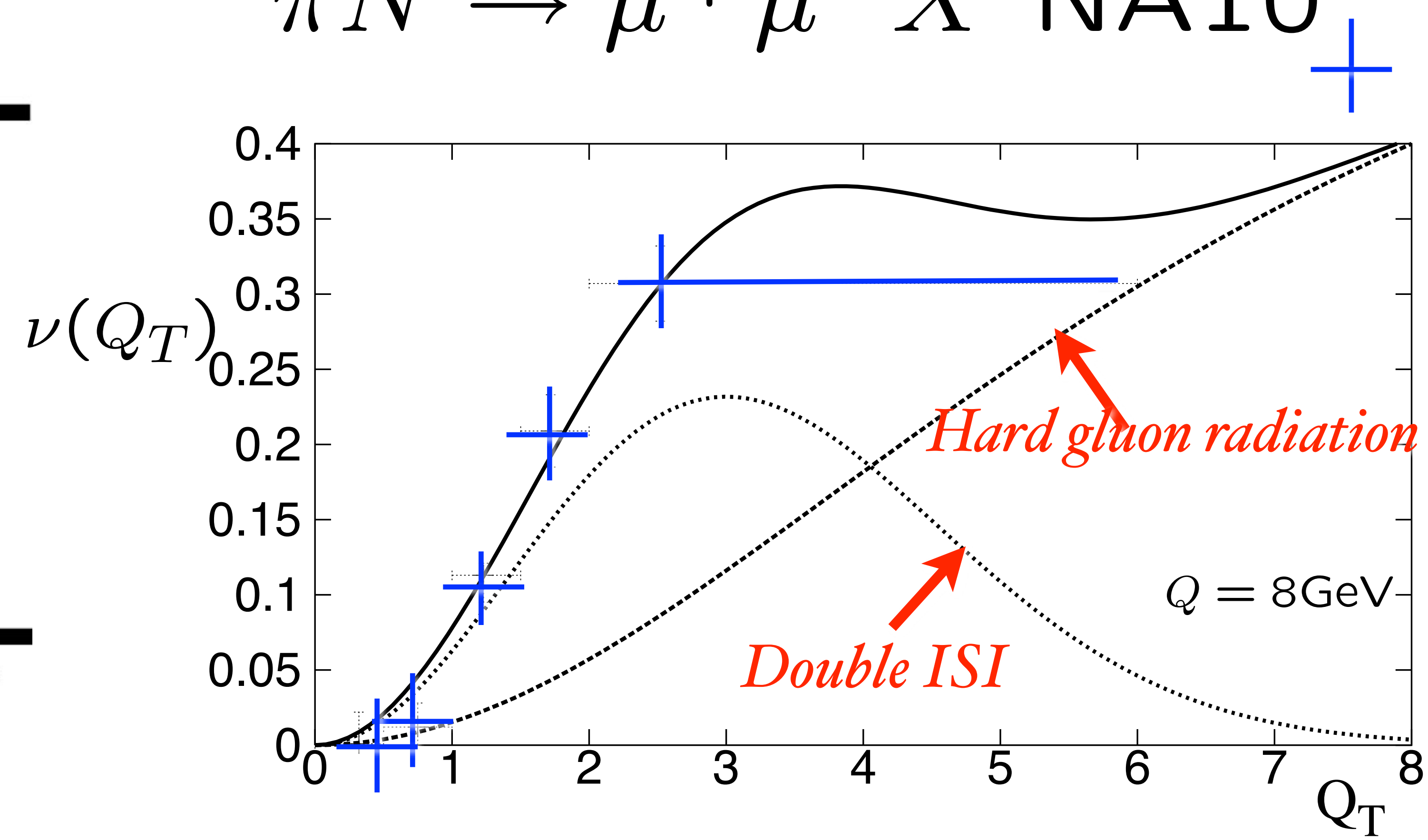
PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

$$\frac{\nu}{2} \propto h_1^{\perp}(\pi)\, h_1^{\perp}(N)$$



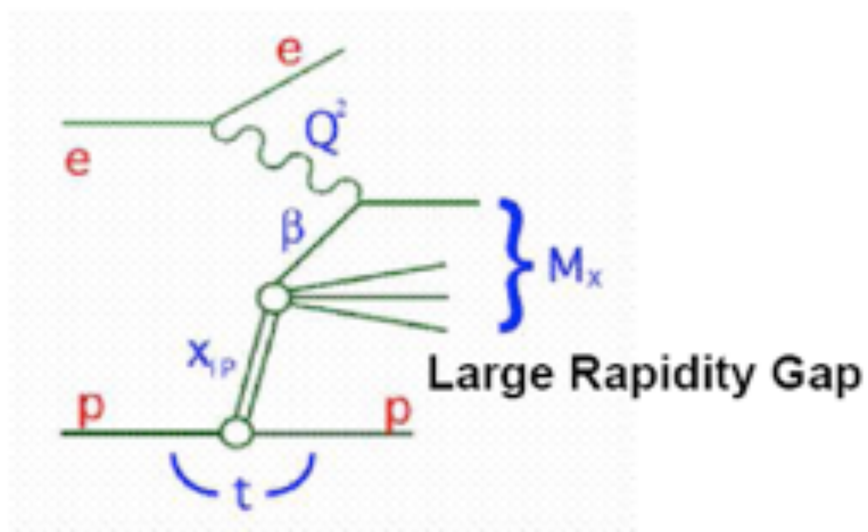
Violates Lam-Tung relation!

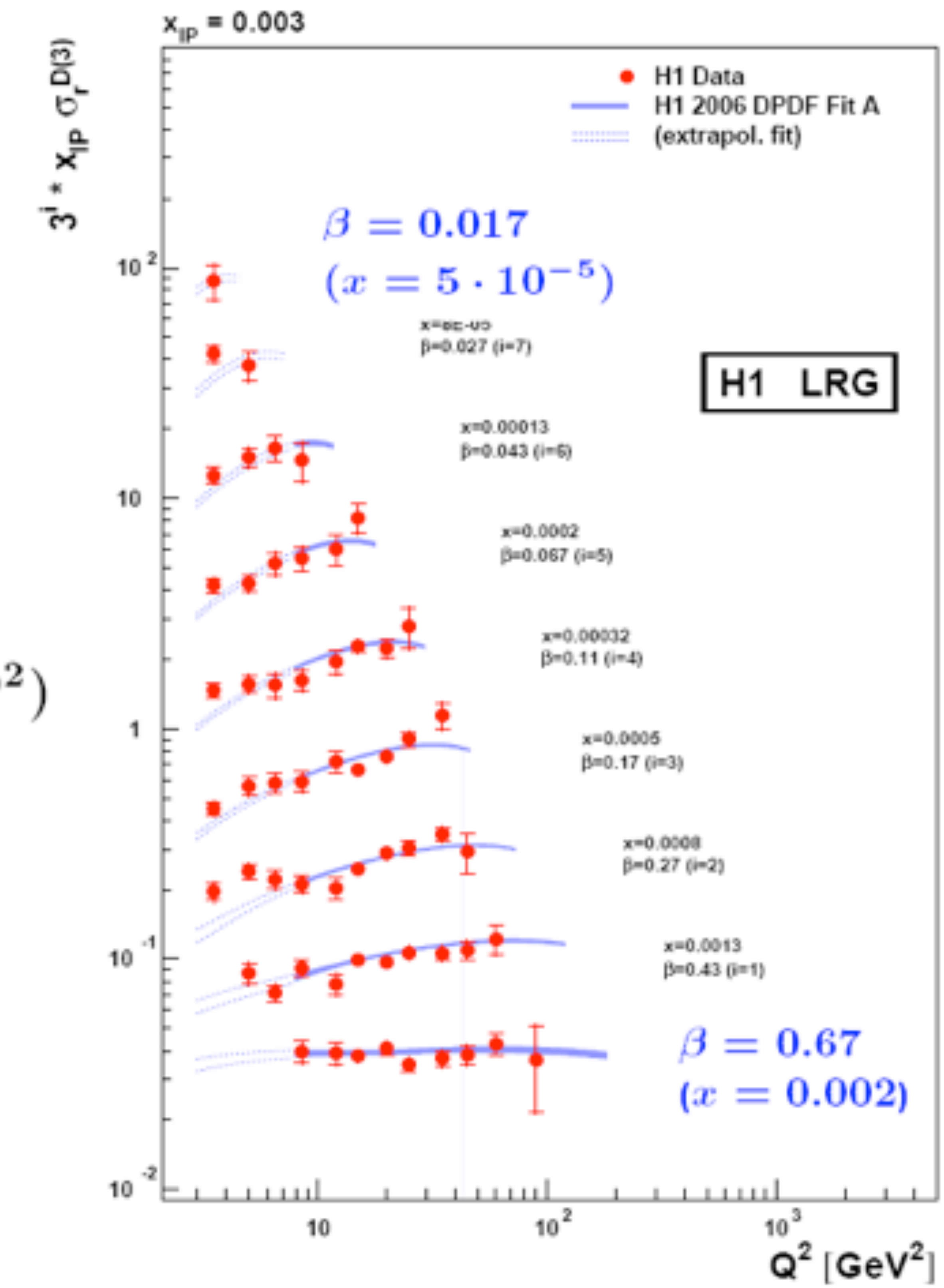
$\pi N \to \mu^+\mu^- X$ NA10



Model: Boer,

Diffractive Structure Function F_2^D





Diffractive inclusive cross section

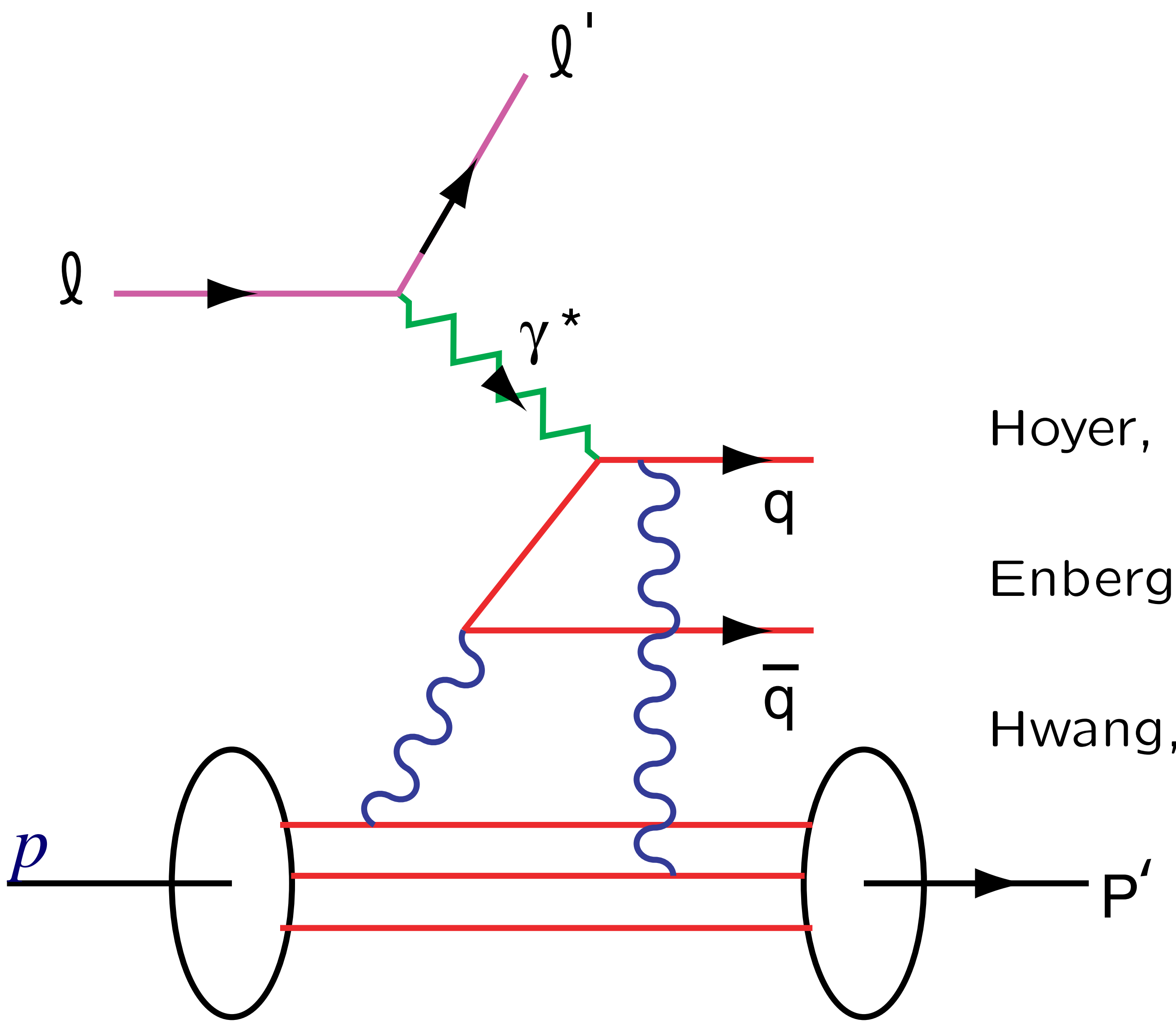
$$\frac{d^3\sigma_{NC}^{diff}}{dx_{I\!P}\, d\beta\, dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_{I\!P}, \beta, Q^2)$$

$$F_2^D(x_{I\!P}, \beta, Q^2) = f(x_{I\!P}) \cdot F_2^{I\!P}(\beta, Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

Large kinematic domain $3 < Q^2 < 1600\ \mathrm{GeV}^2$

Precise measurements *sys 5%, stat 5–20%*



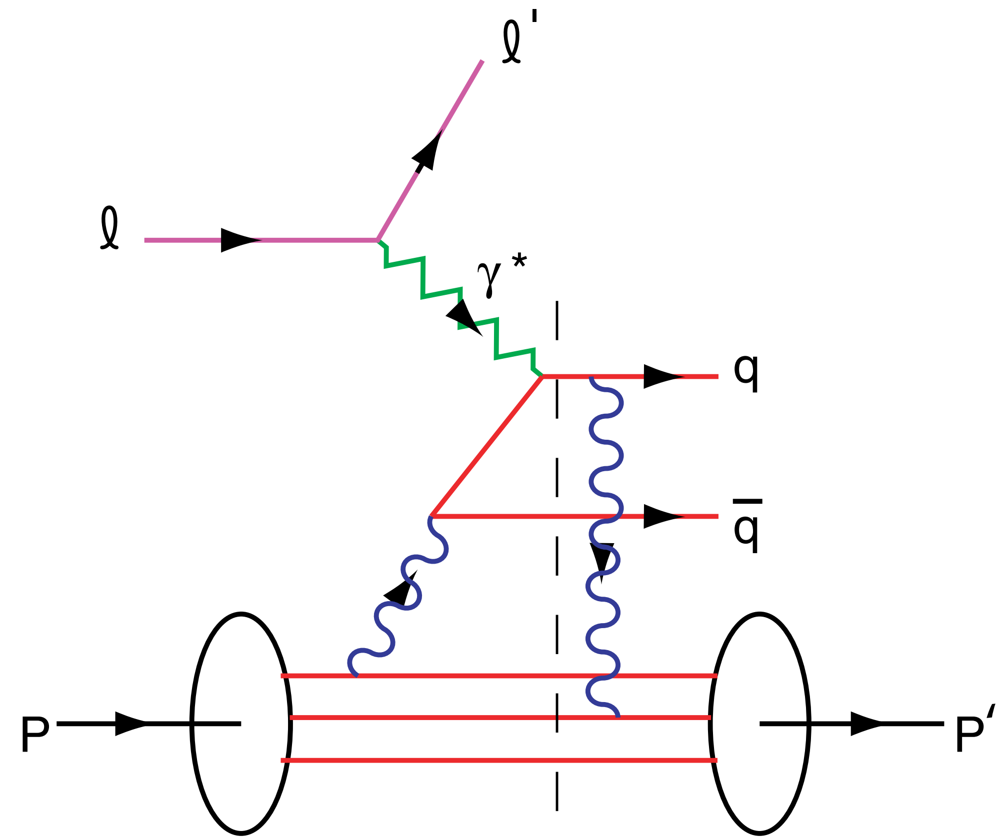
Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHMPS)

Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron



Integration over on-shell domain produces phase i

Need Imaginary Phase to Generate Pomeron and DDIS

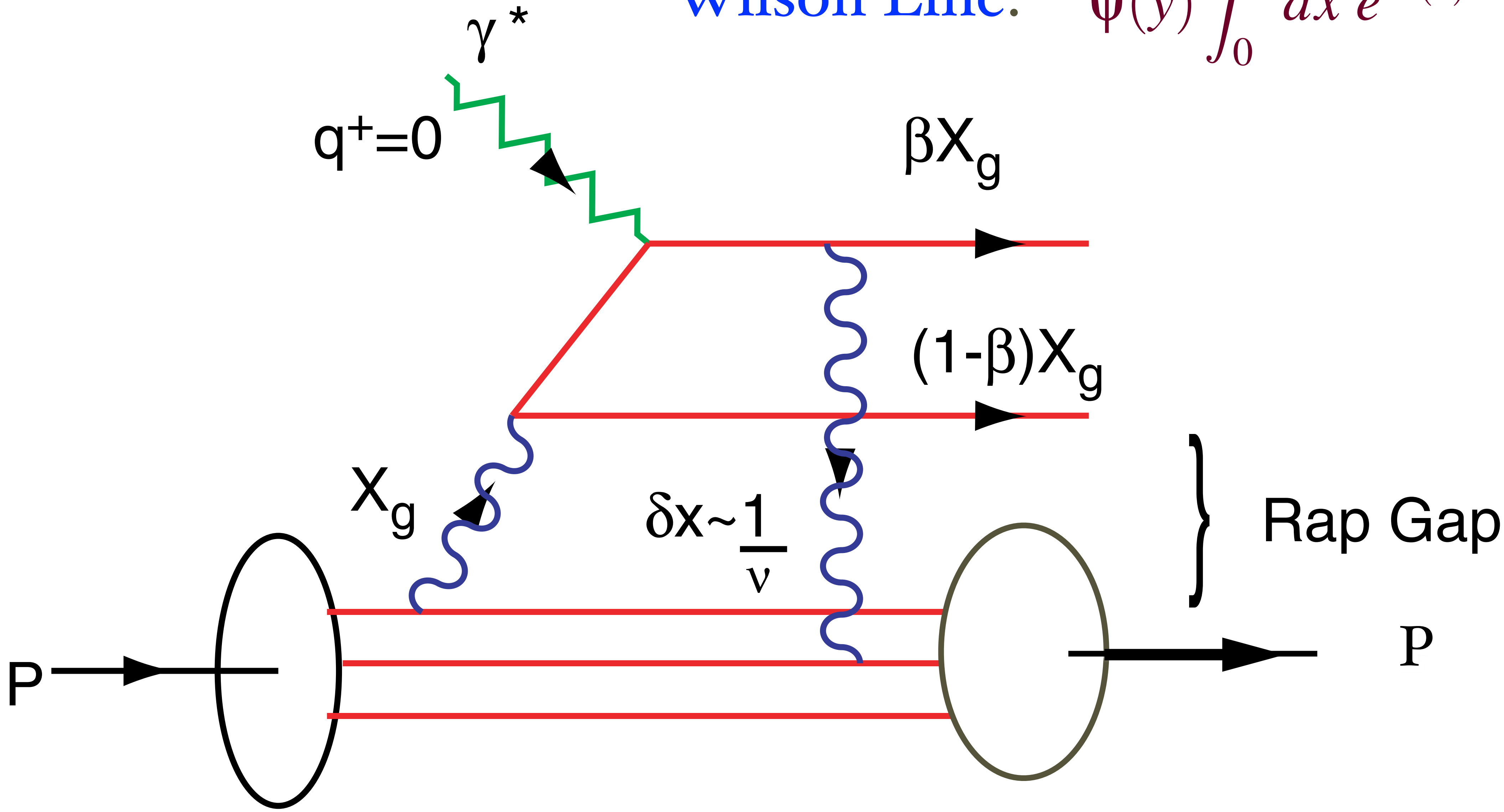
Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target!

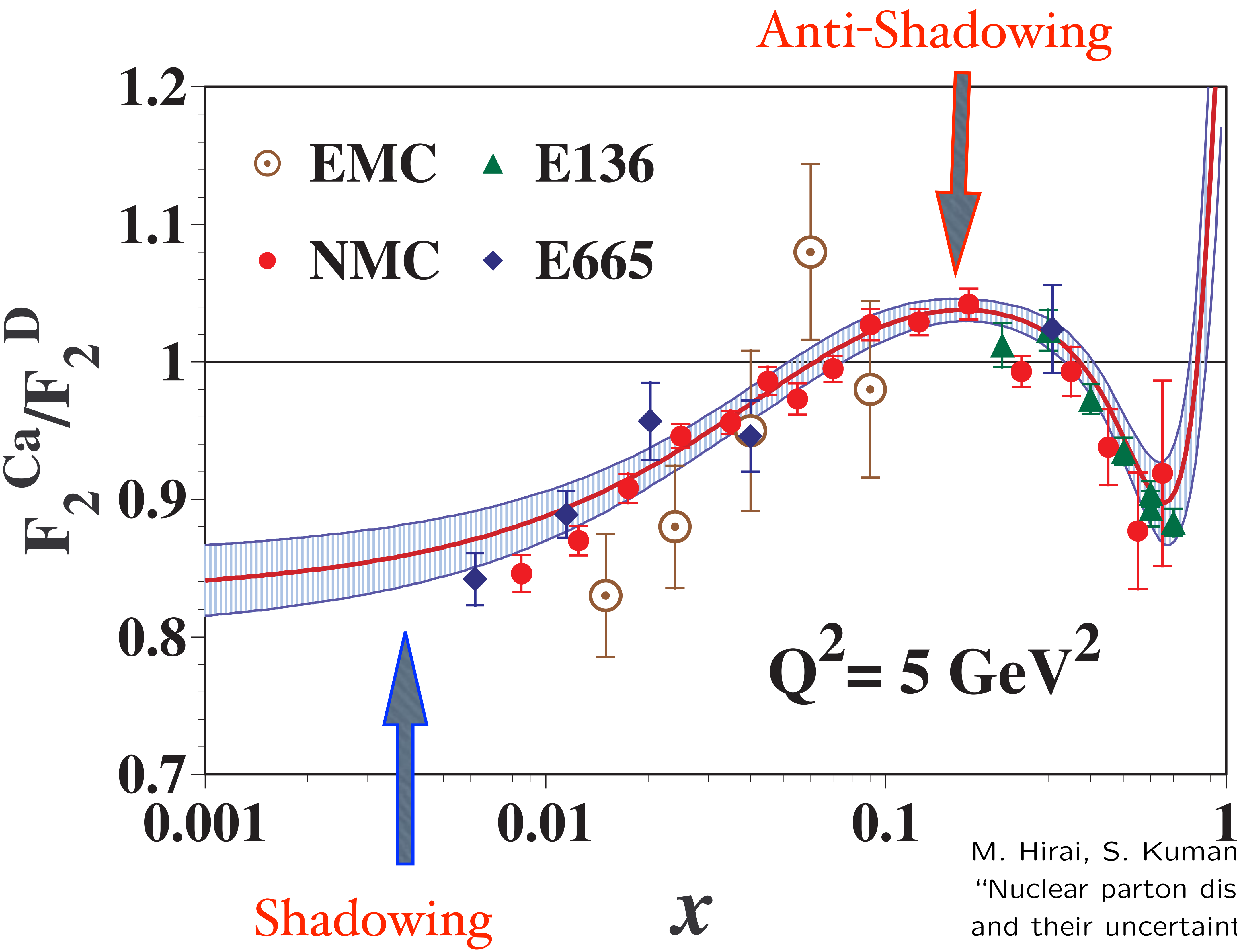
Hoyer, Marchal, Peigne, Sannino, sjb

QCD Mechanism for Rapidity Gaps

Wilson Line: $\overline{\psi}(y) \int_0^y dx\, e^{iA(x)\cdot dx}\, \psi(0)$



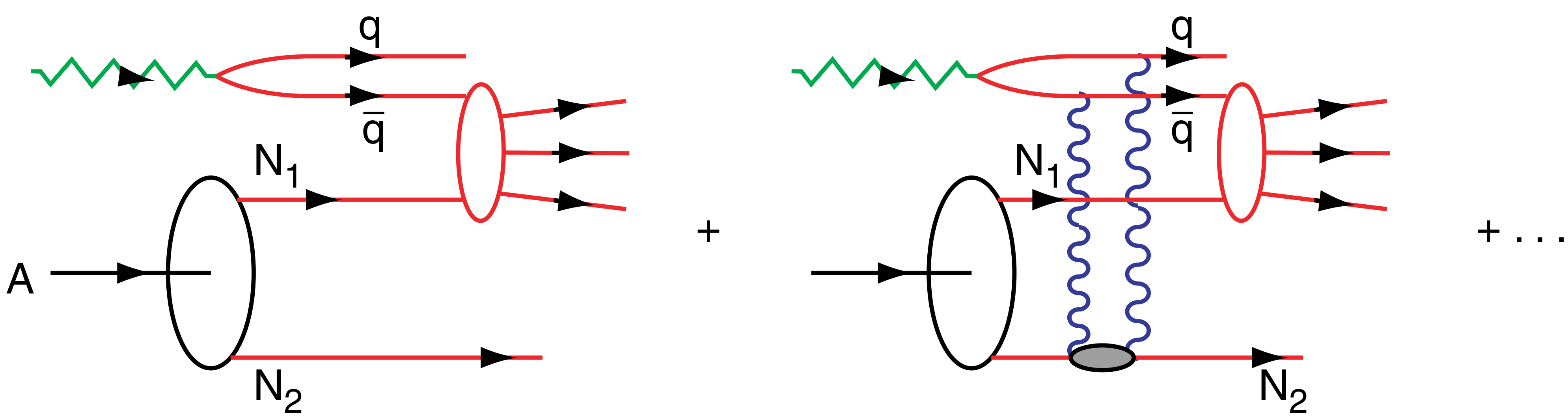
Reproduces lab-frame color dipole approach



M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

Stodolsky
Pumplin, sjb
Gribov

Nuclear Shadowing in QCD



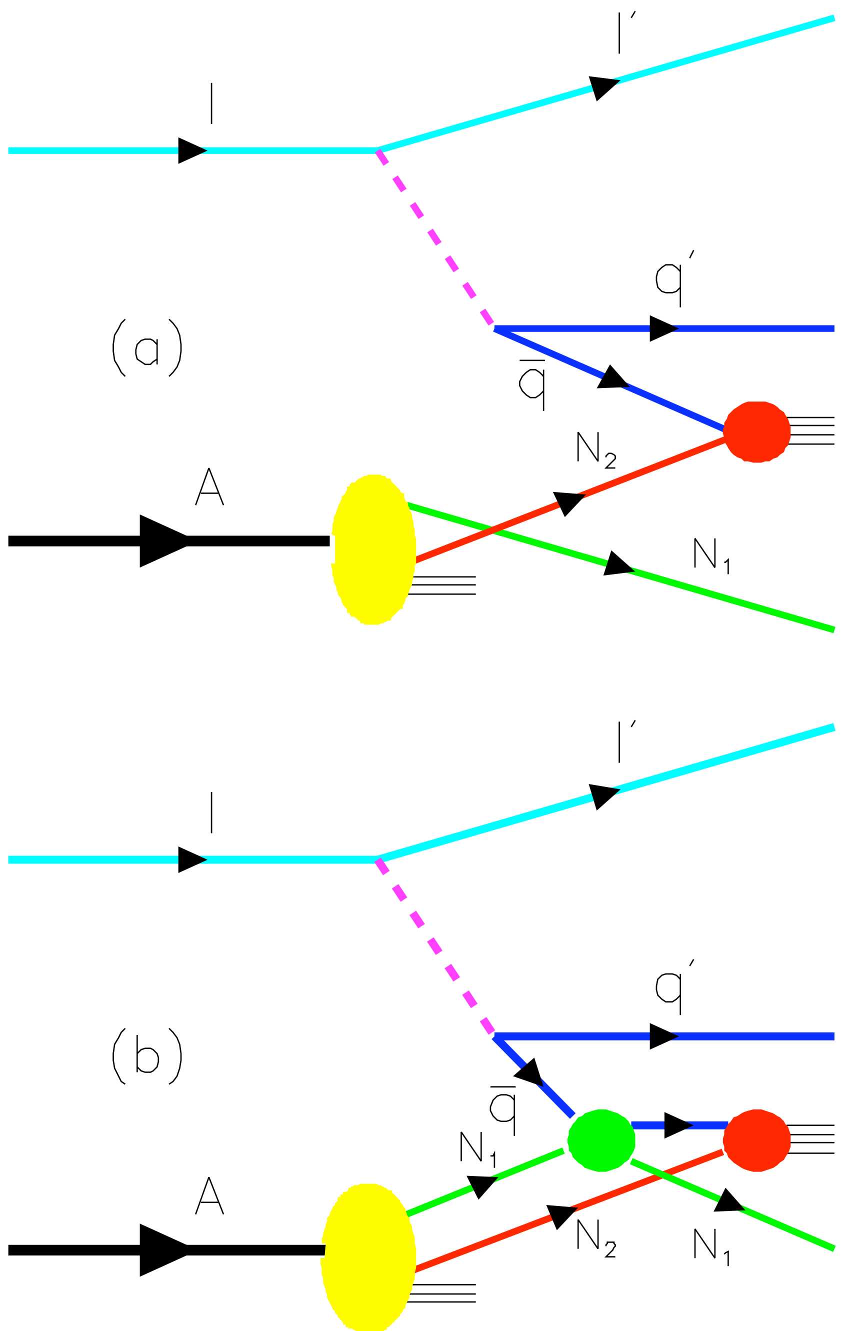
Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus

Diffraction via Reggeon gives constructive interference!

Anti-shadowing not universal



The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

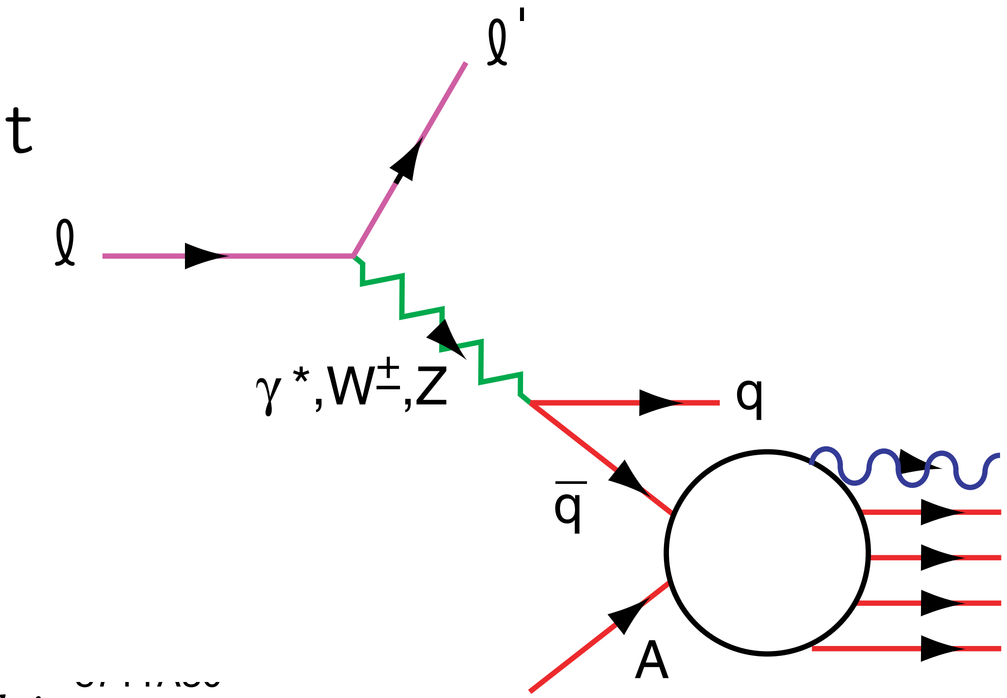
$\rightarrow$ Shadowing of the DIS nuclear structure functions.

Diffraction via Pomeron gives destructive interference!

Shadowing

Origin of Regge Behavior of Deep Inelastic Structure Functions

$$F_{2p}(x) - F_{2n}(x) \propto x^{1/2}$$



Antiquark interacts with target nucleus at energy $\widehat{s} \propto \frac{1}{x_{bj}}$

Regge contribution: $\sigma_{\bar{q}N} \sim \widehat{s}^{\alpha_R - 1}$

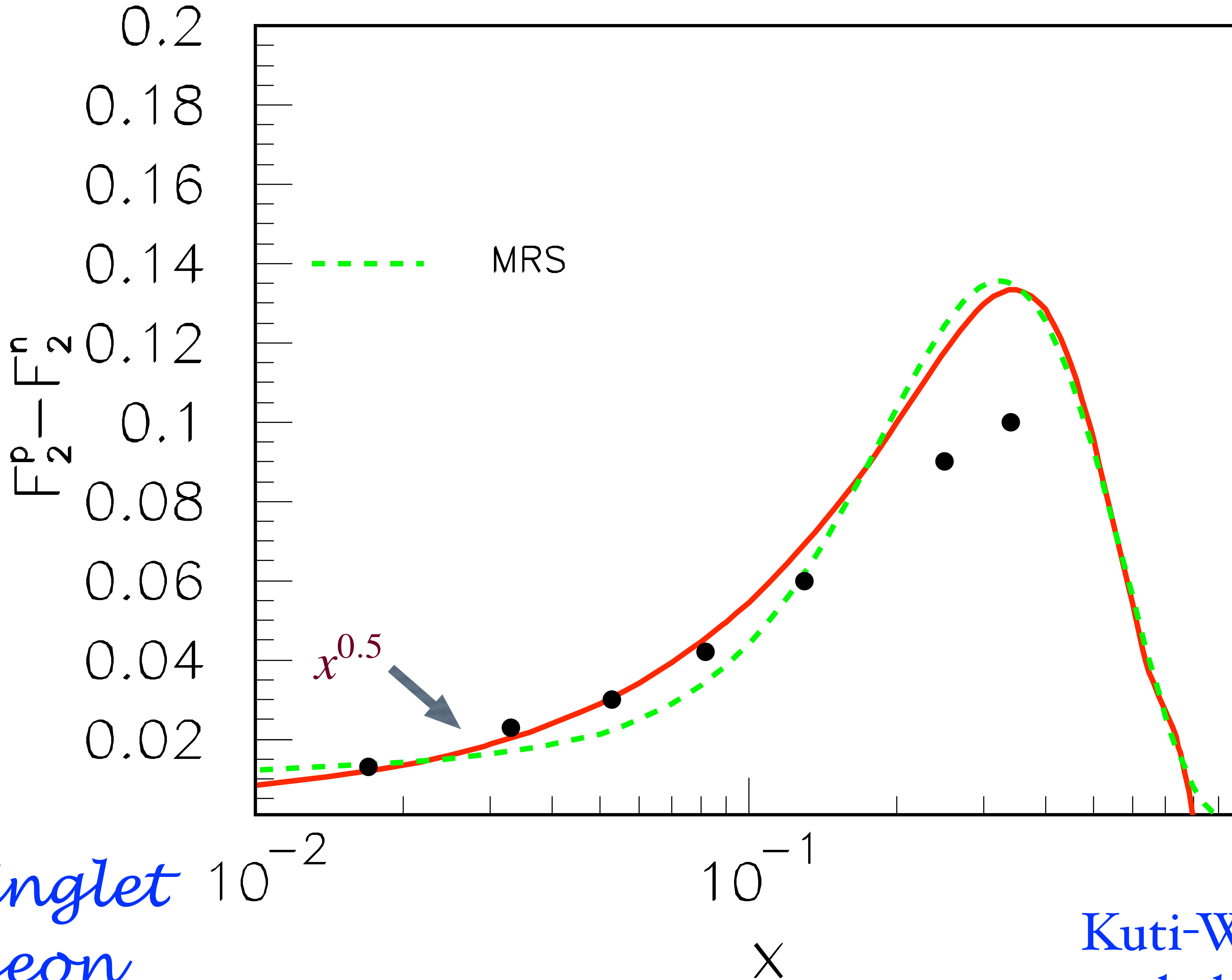
Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x_{bj}}$ at small x_{bj}.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Landshoff, Polkinghorne, Short

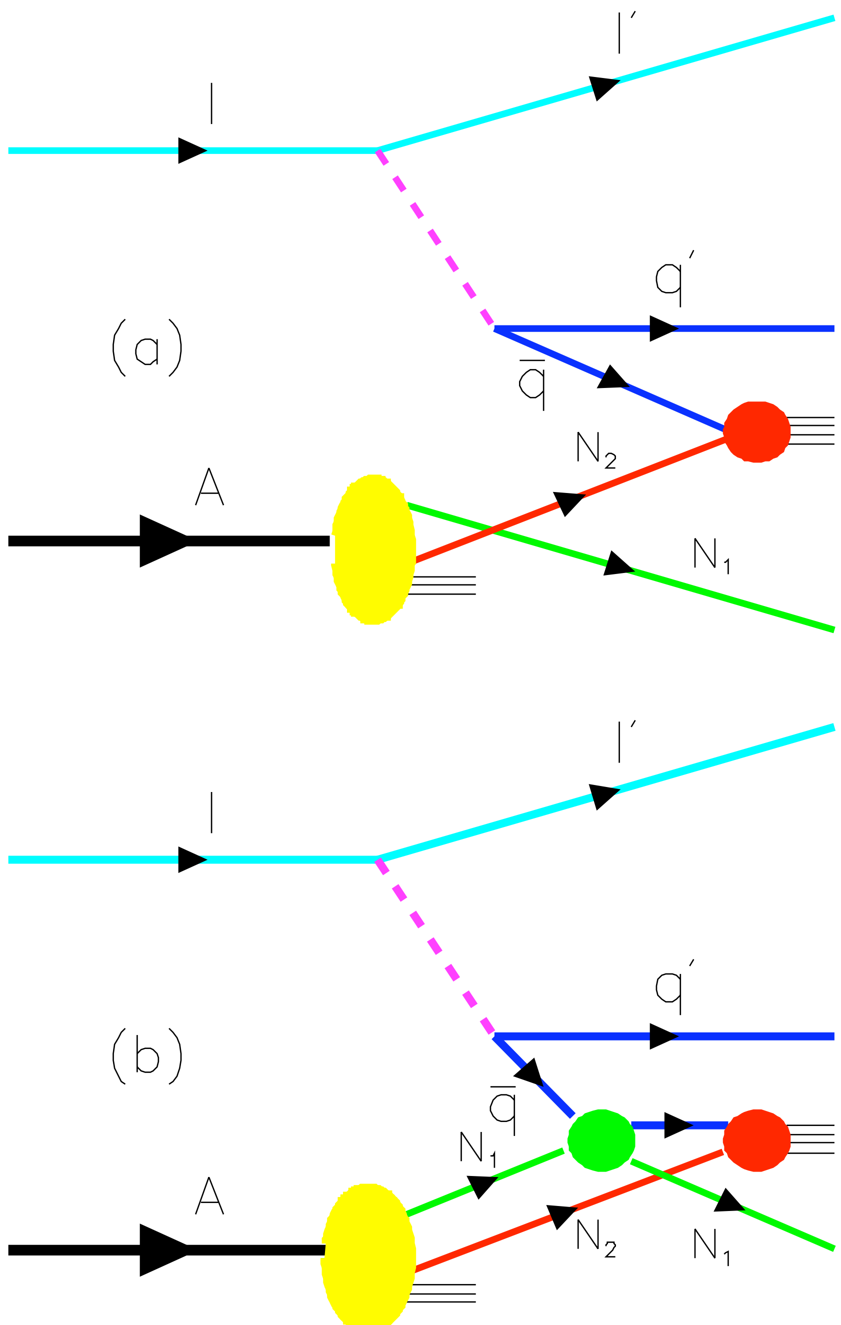
Close, Gunion, sjb

Schmidt, Yang, Lu, sjb



Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior



The one-step and two-step processes in DIS on a nucleus.

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Reggeon

If the scattering on nucleon N_1 is via ~~pomeron~~ exchange, the one-step and two-step amplitudes are ~~opposite~~ in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

Diffraction via Reggeon gives constructive interference!

Anti-shadowing

Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1-i) \times i = \frac{1}{\sqrt{2}}(i+1)$$

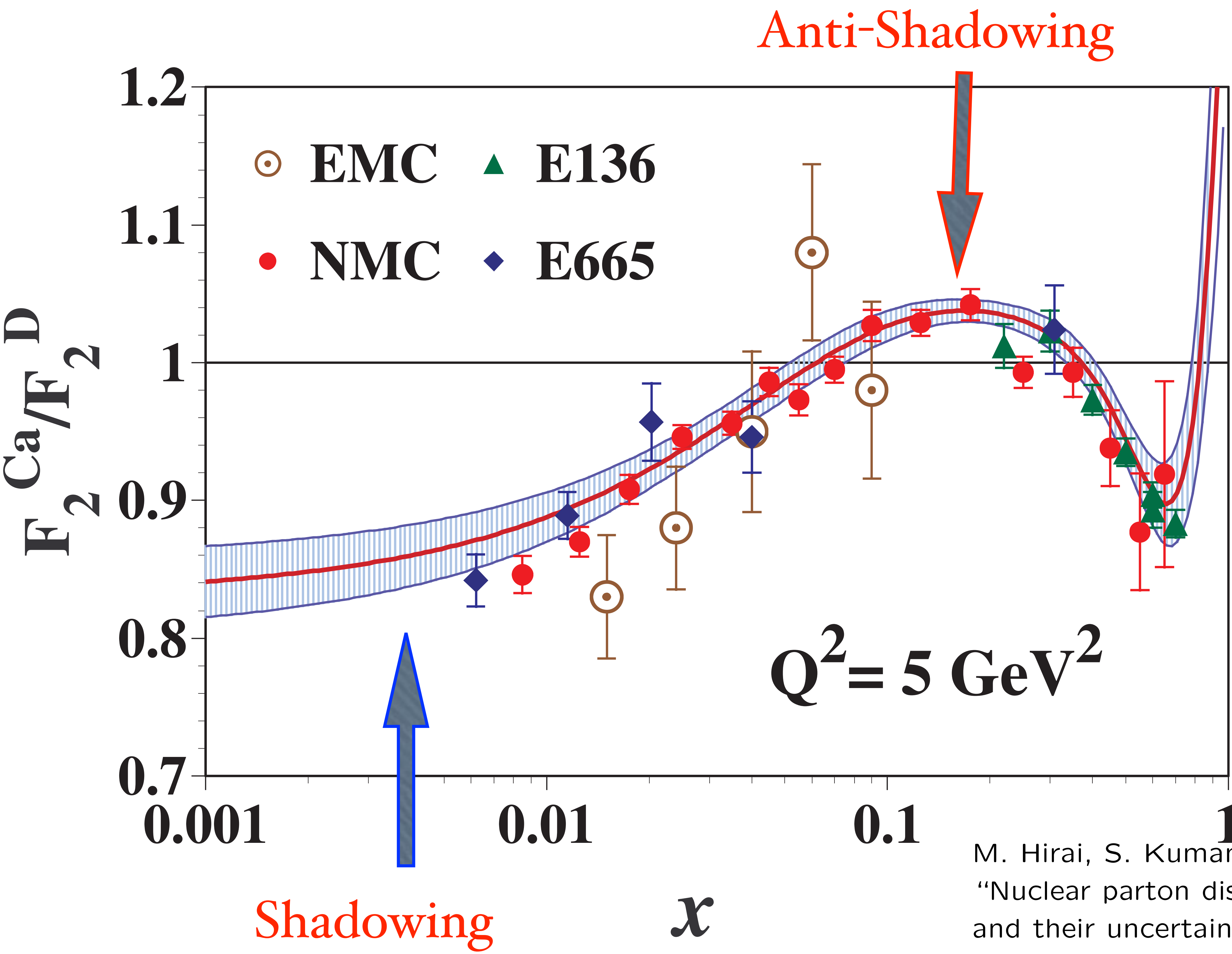
Constructive Interference

Depends on quark flavor!

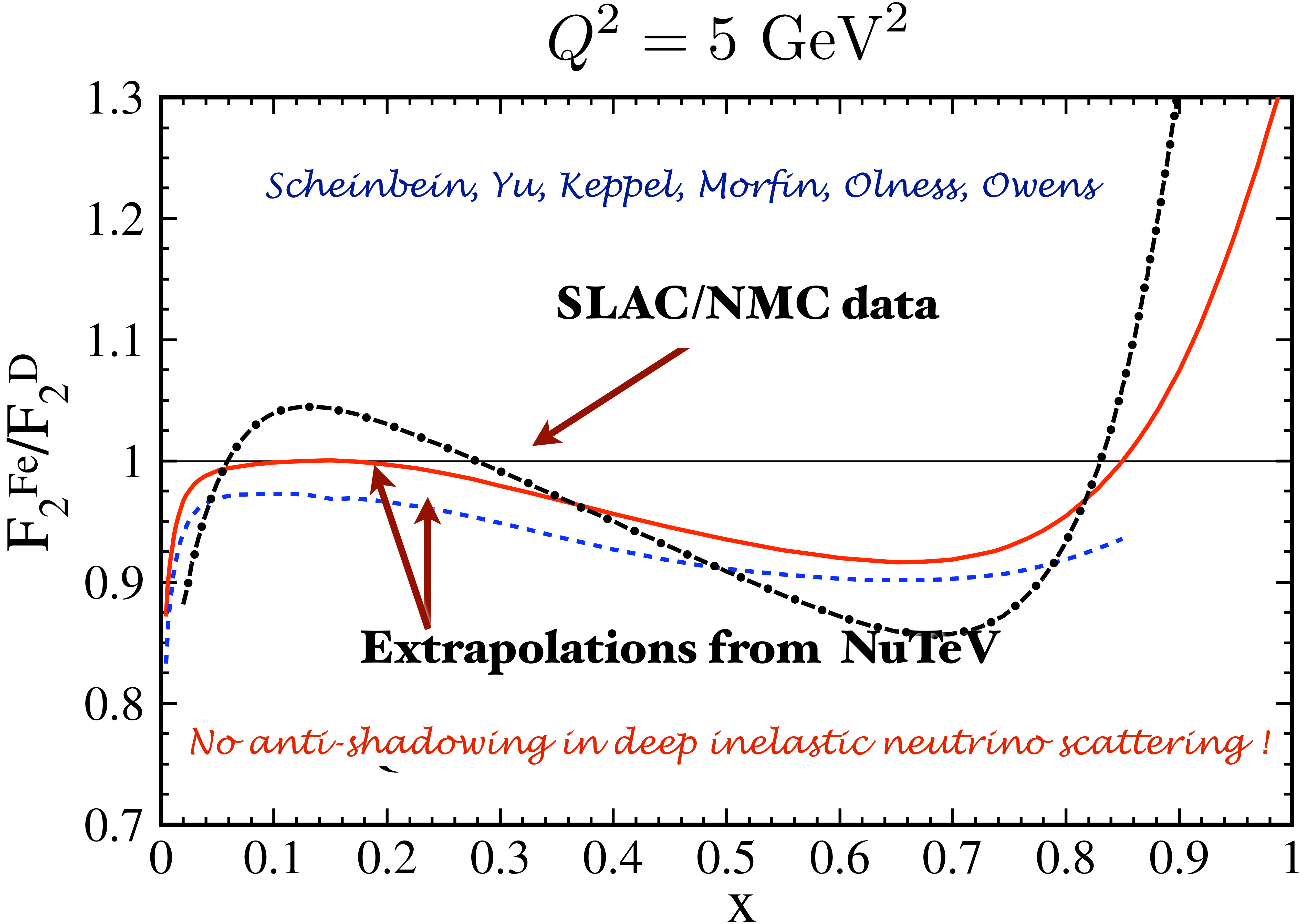
Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

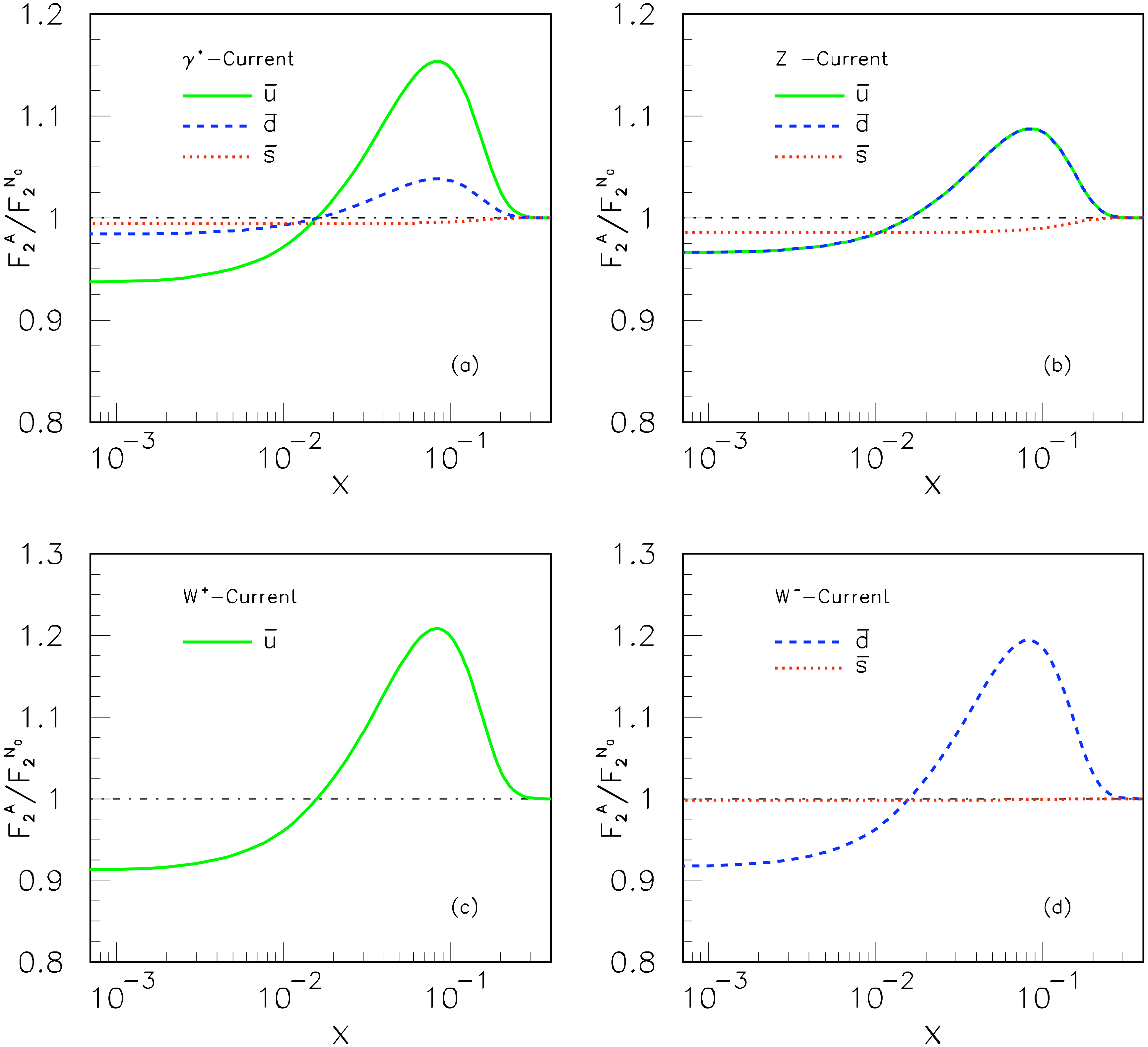
Critical test: Tagged Drell-Yan



M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].



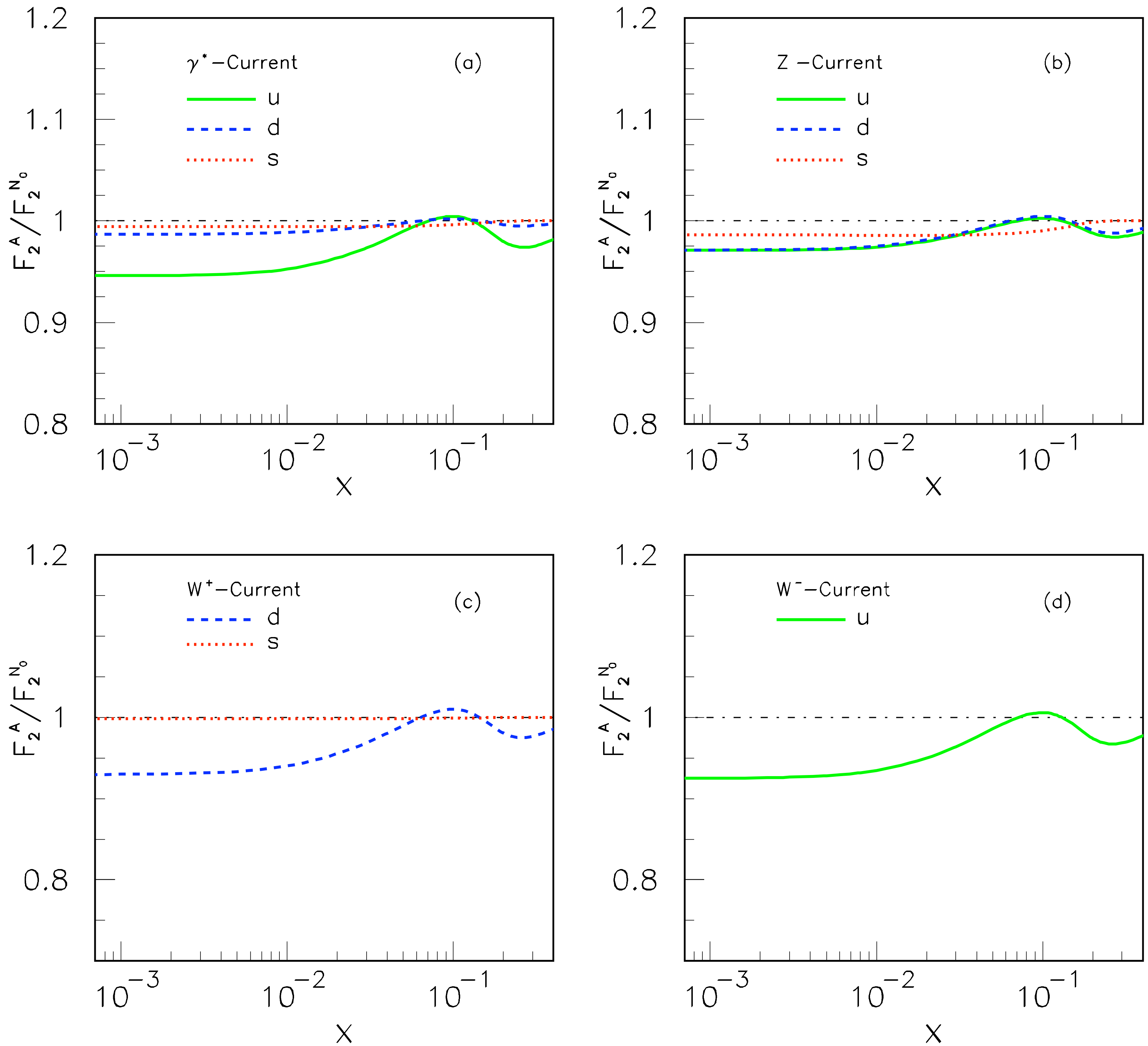
$Q^2 = 5\ \mathrm{GeV}^2$
Scheinbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
F_2^{Fe}/F_2^D
x
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Schmidt, Yang; sjb

Nuclear Antishadowing not universal !

Shadowing and Antishadowing of DIS Structure Functions



S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

Modifies NuTeV extraction of

$$\sin^2 \theta_W$$

Test in flavor-tagged lepton-nucleus collisions

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

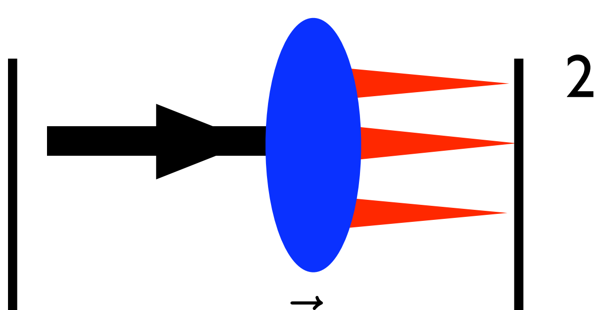
Hwang, Schmidt, sjb,

Mulders, Boer

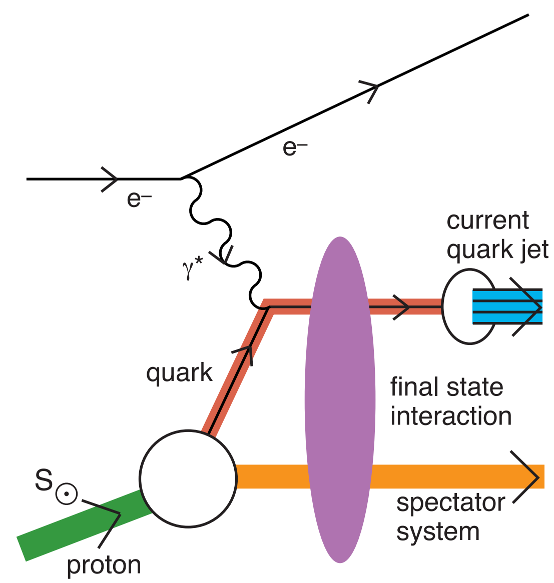
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb



$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



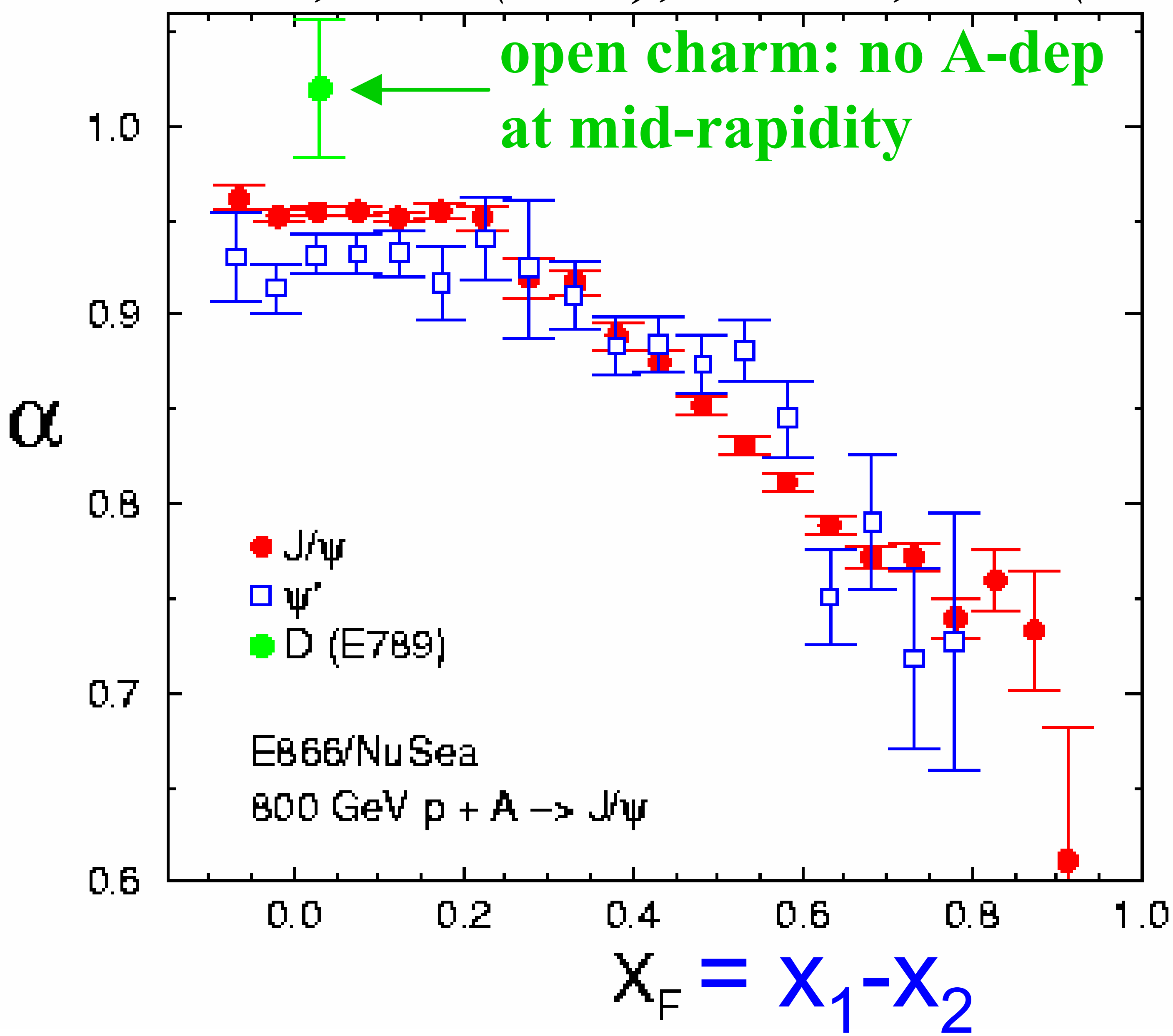
Key QCD Issues in Electroproduction

- Intrinsic Heavy Quarks
- Role of Color Confinement in DIS
- Hadronization at the Amplitude Level
- Leading-Twist Lensing: Sivers Effect
- Diffractive DIS
- Static versus Dynamic Structure Functions
- Origin of Shadowing and Anti-Shadowing
- Is Anti-Shadowing Non-Universal: Flavor Specific?
- Nature of Nuclear Correlations
- $1 < x < A$

M. Leitch

800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^\alpha$

PRL 84, 3256 (2000); PRL 72, 2542 (1994)



$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization

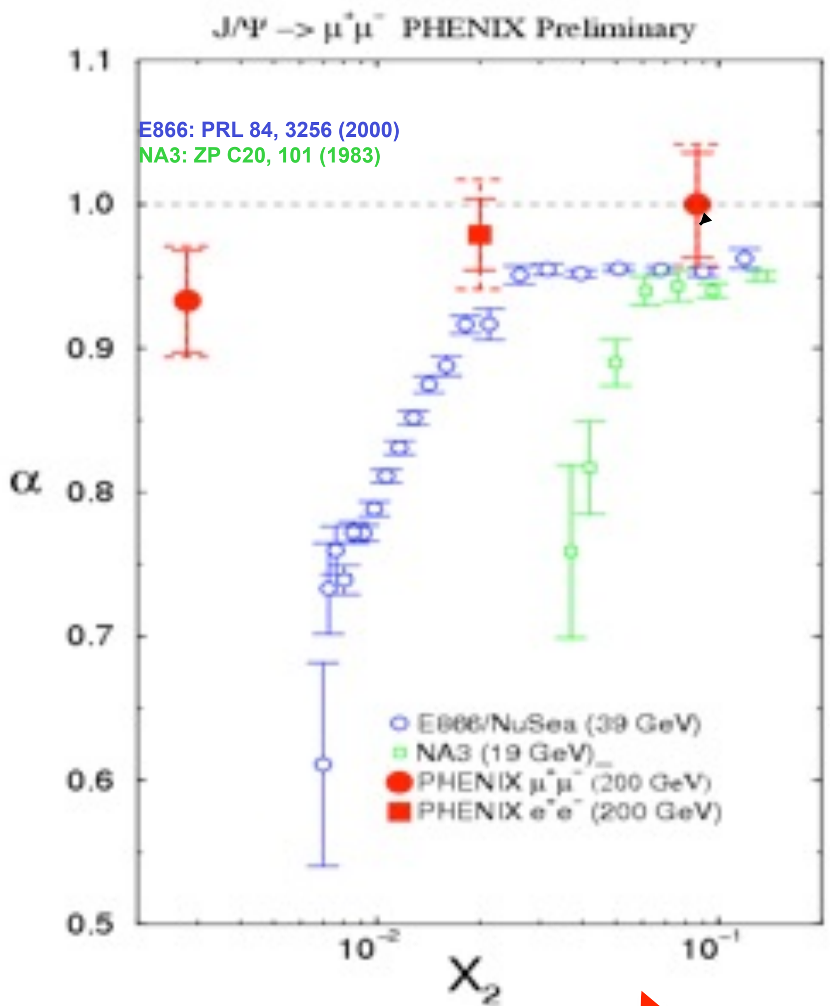
Violation of factorization in charm hadroproduction.
P. Hoyer, **M. Vanttinen** (**Helsinki U.**) , **U. Sukhatme** (**Illinois U., Chicago**) . **HU-TFT-90-14, May 1990. 7pp.**
Published in Phys.Lett.B246:217-220,1990

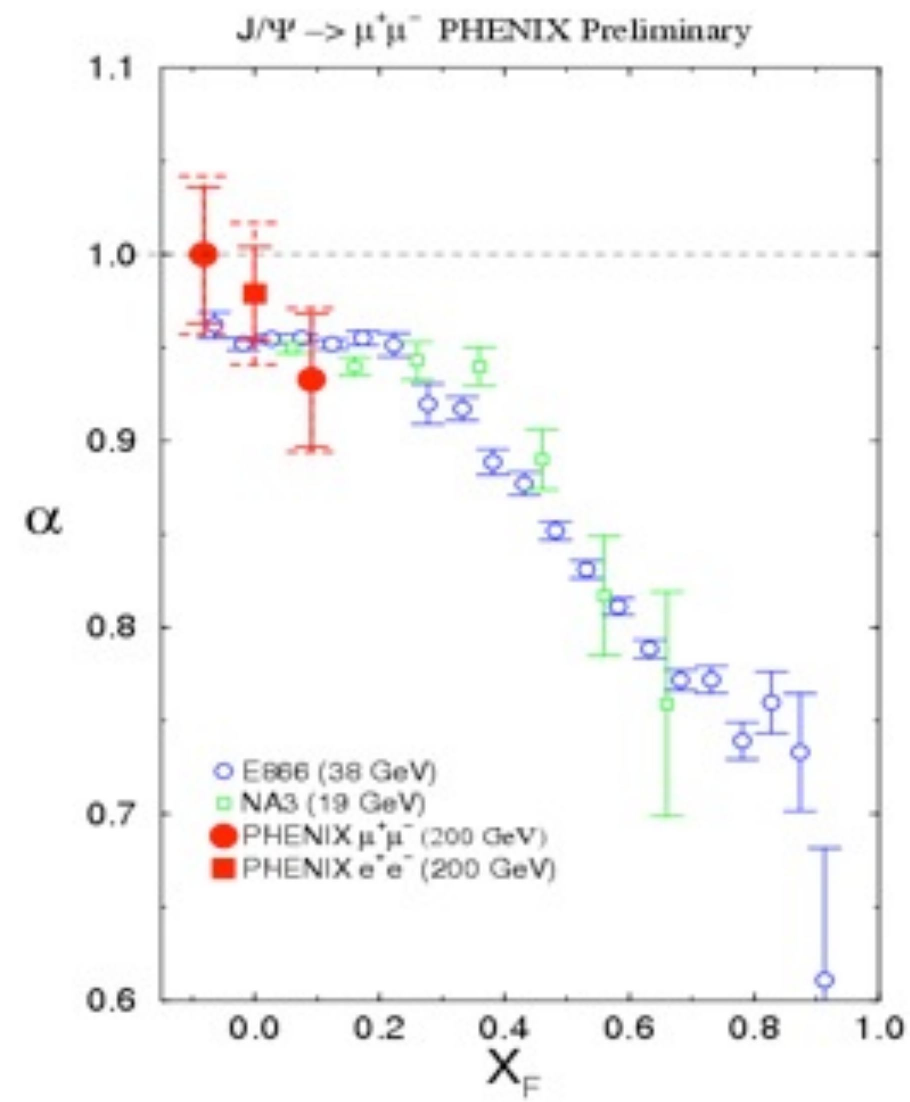
IC Explains large excess of quarkonia at large x_F, A-dependence

J/ψ nuclear dependence vrs rapidity, x_{Au}, x_F

M.Leitch

PHENIX compared to lower energy measurements





Huge "absorption" effect

Klein,Vogt, PRL 91:142301,2003
Kopeliovich, NP A696:669,2001

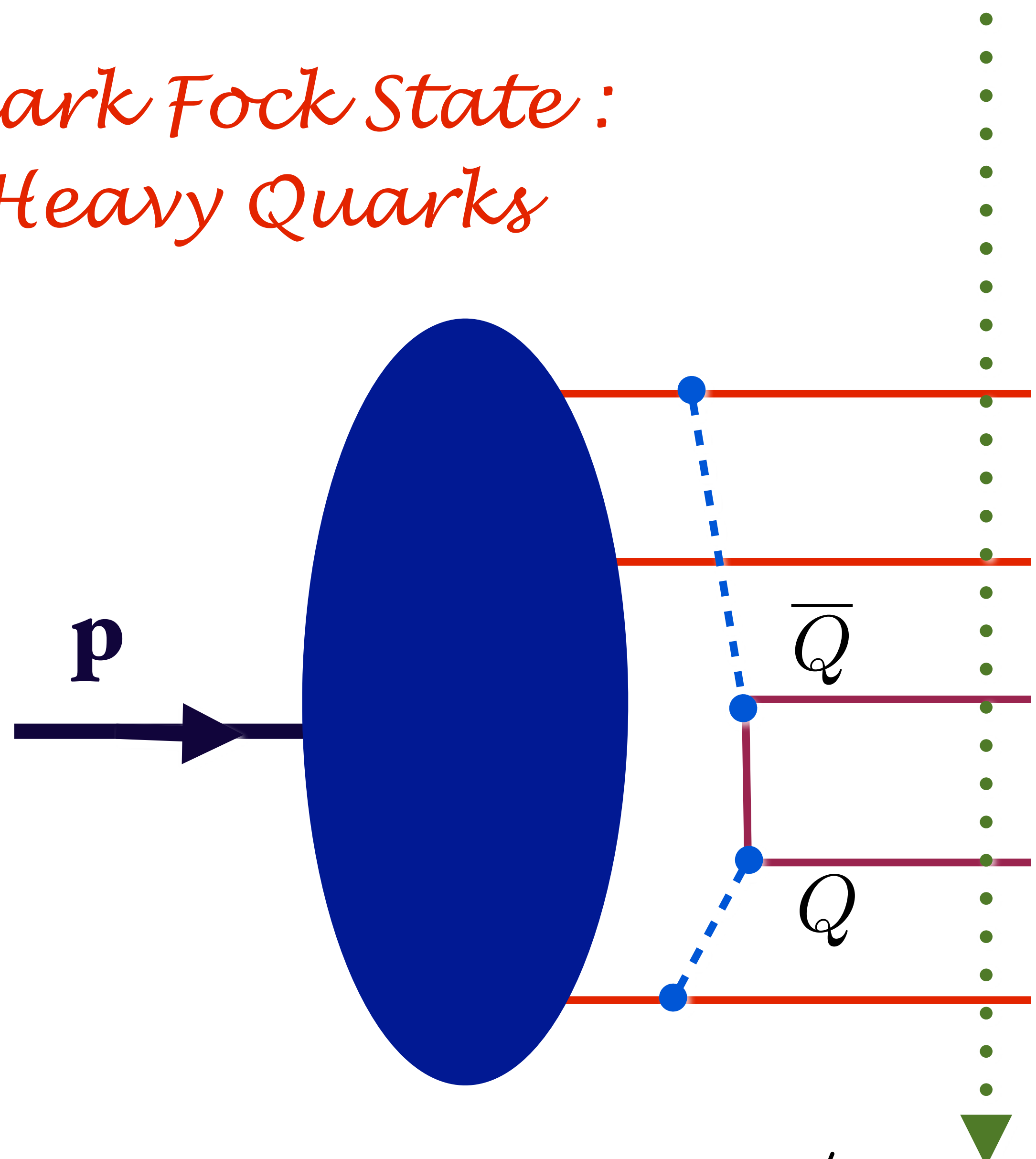
Violates PQCD factorization!

$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

Hoyer, Sukhatme, Vanttinen

Fixed LF time

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



QCD predicts Intrinsic Heavy Quarks at high x!

Minimal off-shellness

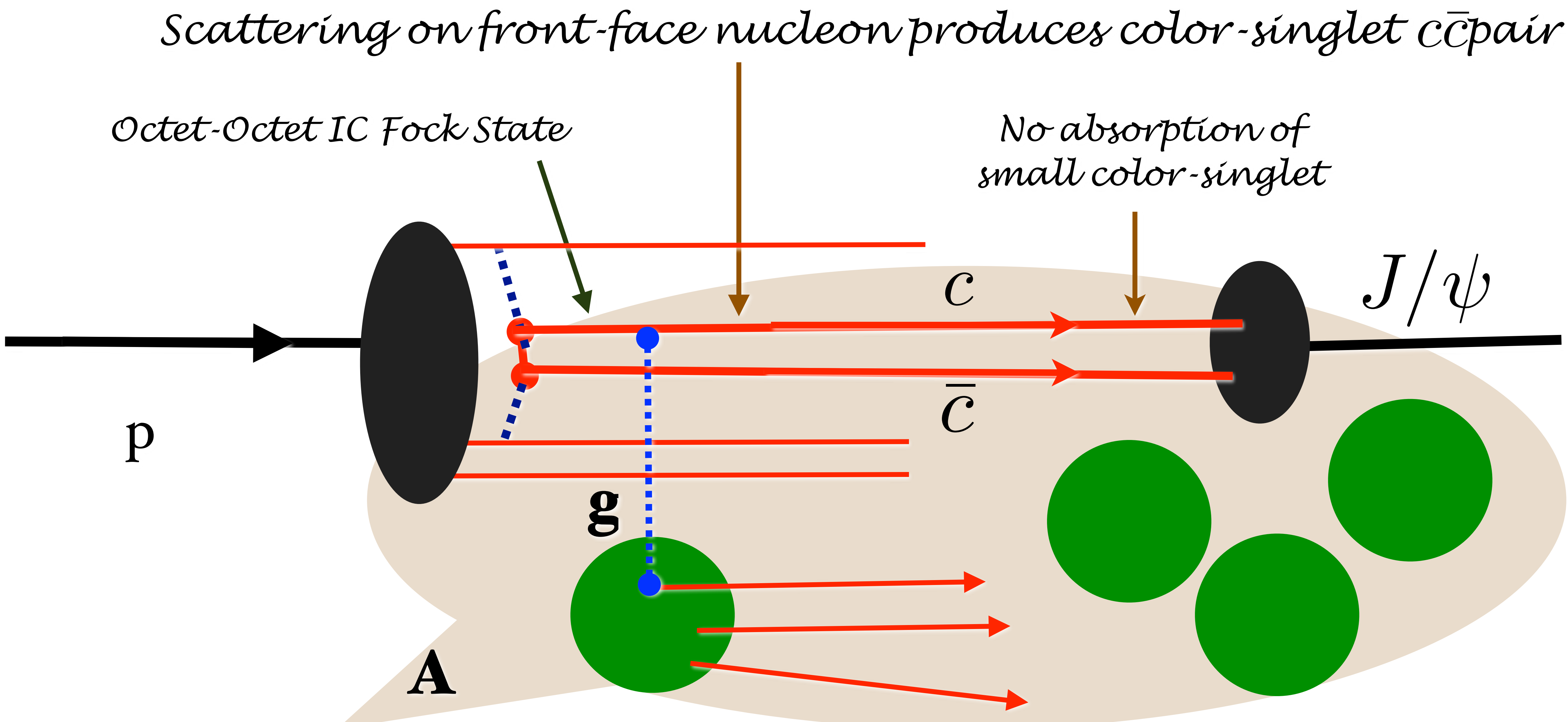
$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al.

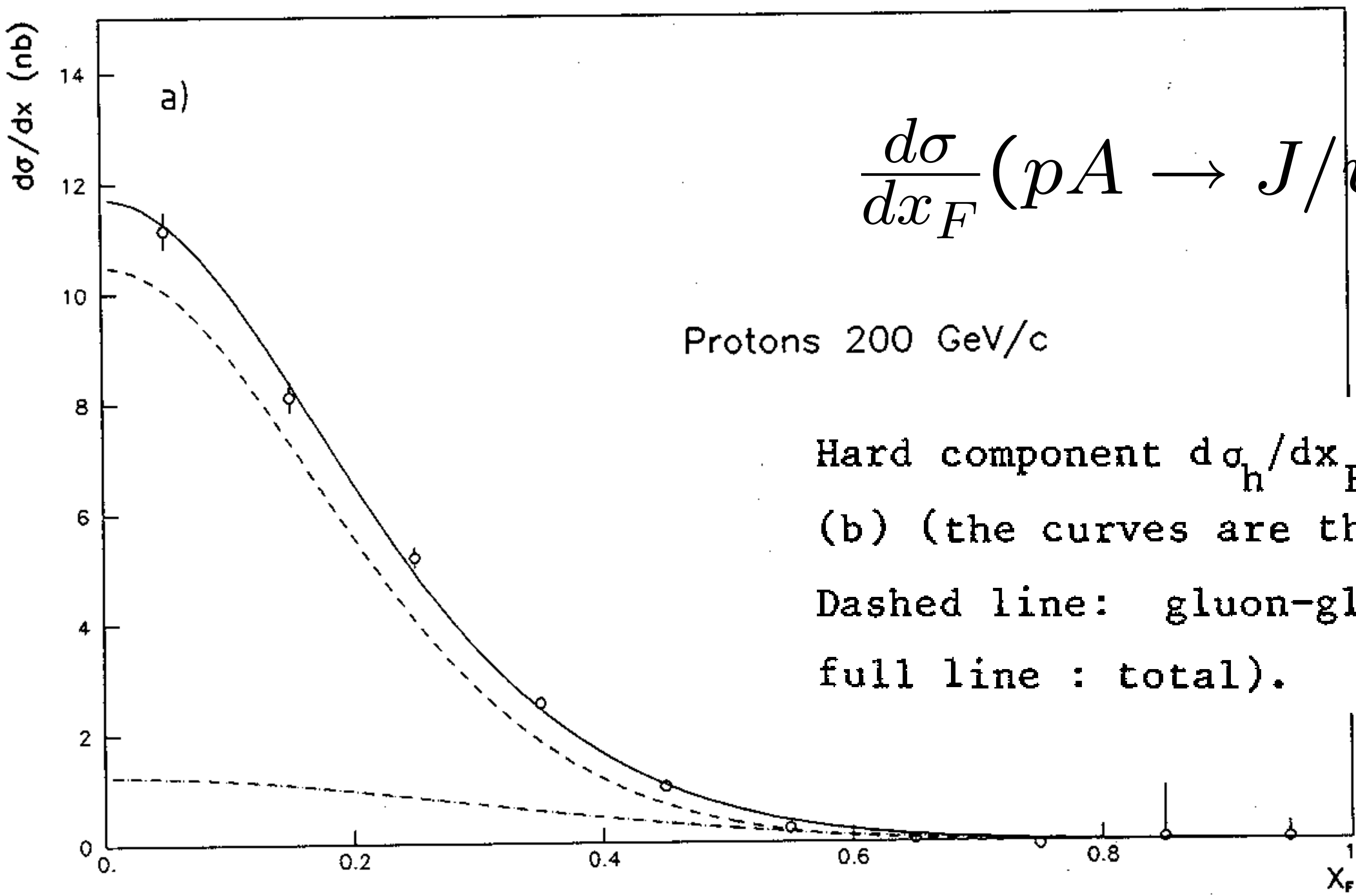
Color-Opaque IC Fock state interacts on nuclear front surface

Kopeliovich, Schmidt, Soffer, sjb

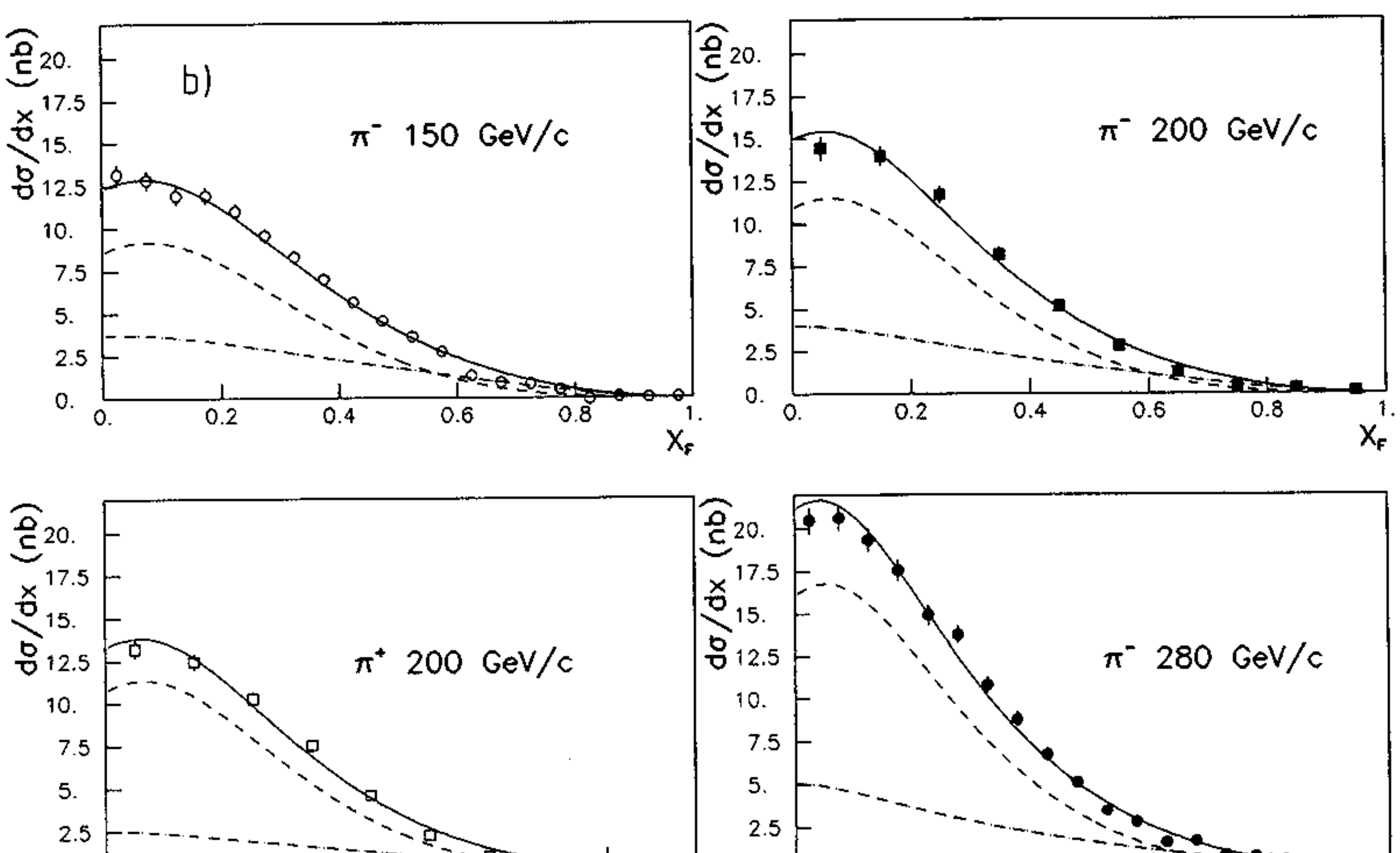


$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \to J/\psi X)$$

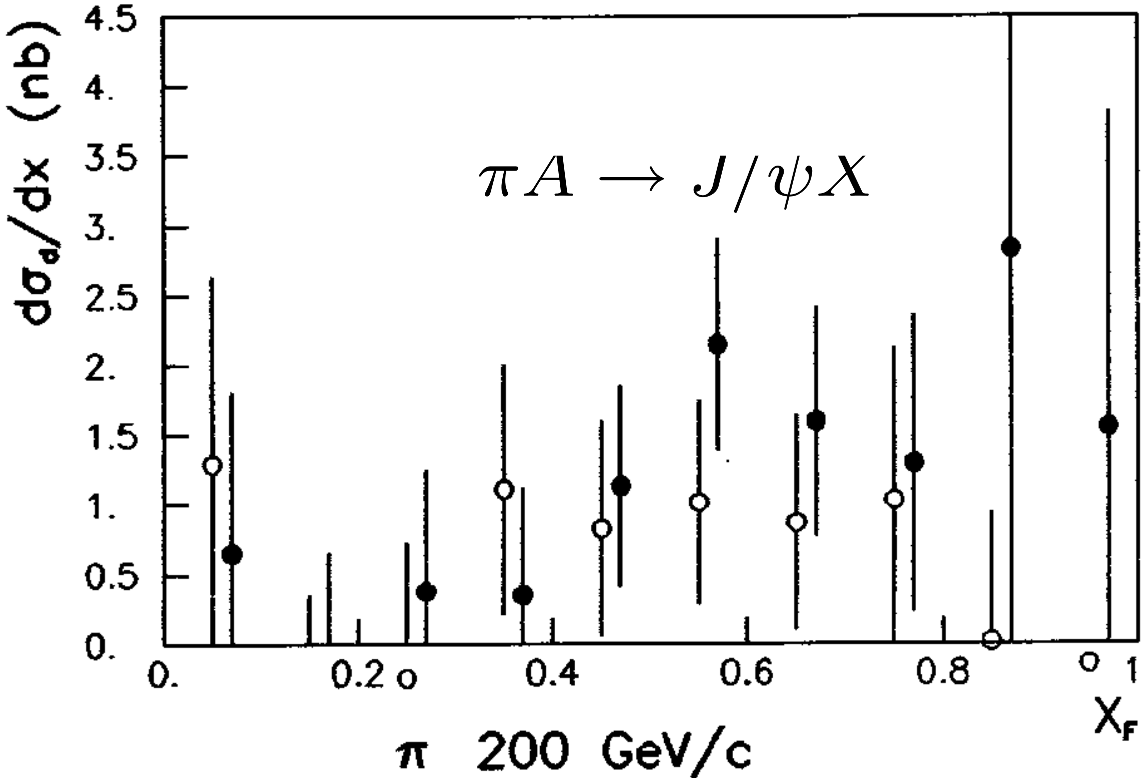
$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$



Hard component $d\sigma_h/dx_F$ for incident protons (a) and pions (b) (the curves are the result of the fit described in the text. Dashed line: gluon-gluon fusion; dash-dotted line : $q\bar{q}$ fusion; full line : total).

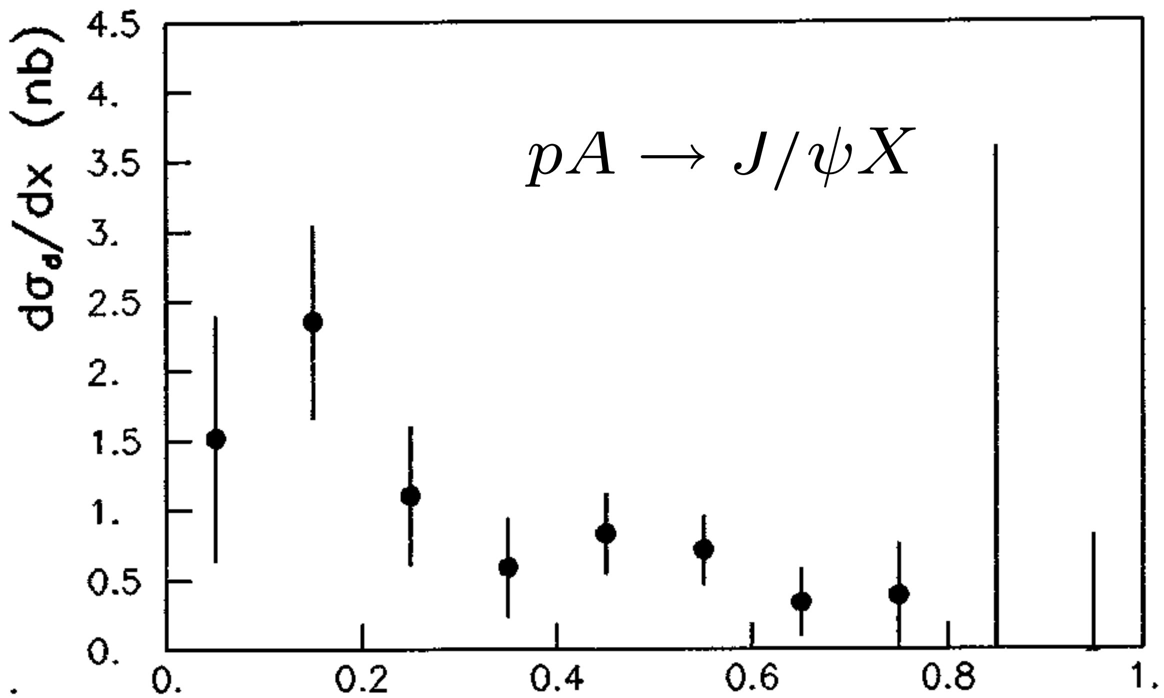


A^1 component consistent with sum of gg and $\bar{q}q$ fusion



$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3}\frac{d\sigma_{2/3}}{dx_F}$$

$A^{2/3}$ component



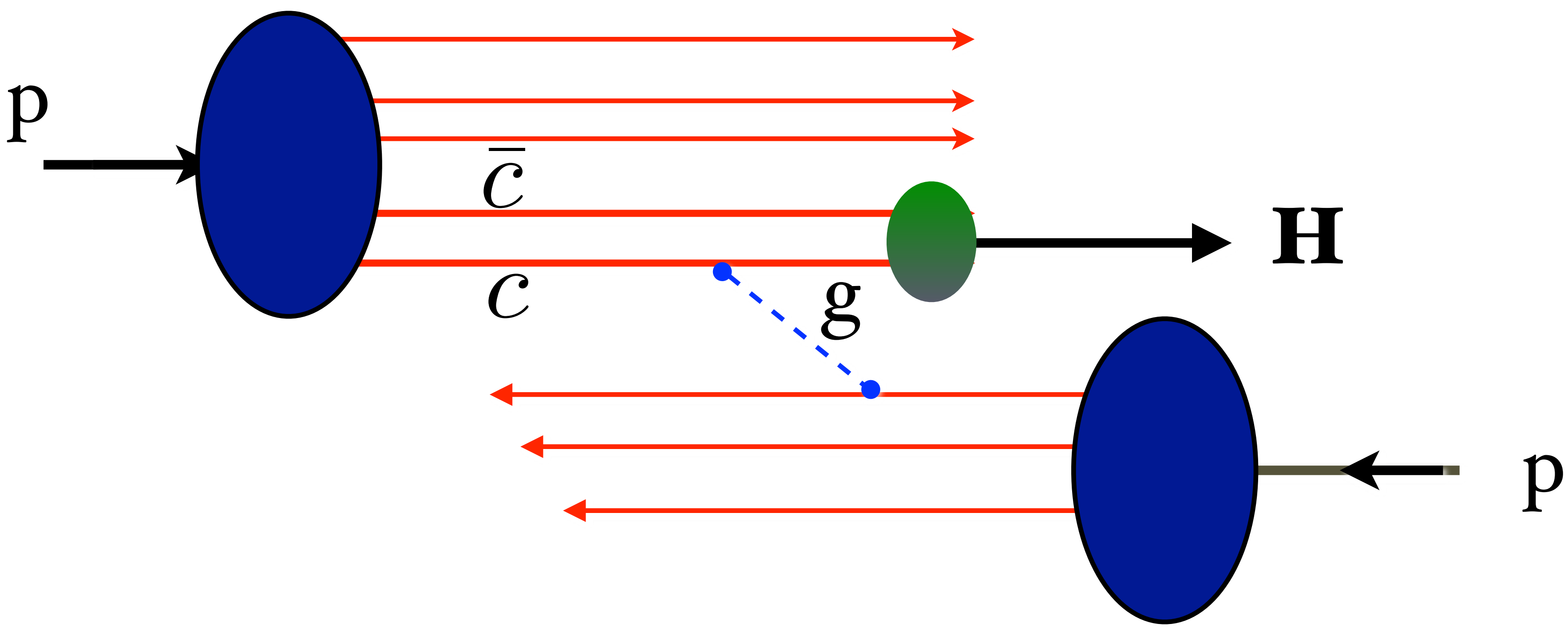
p 200 GeV/c

J. Badier et al, NA3

Excess beyond conventional PQCD subprocesses

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \to H X$$

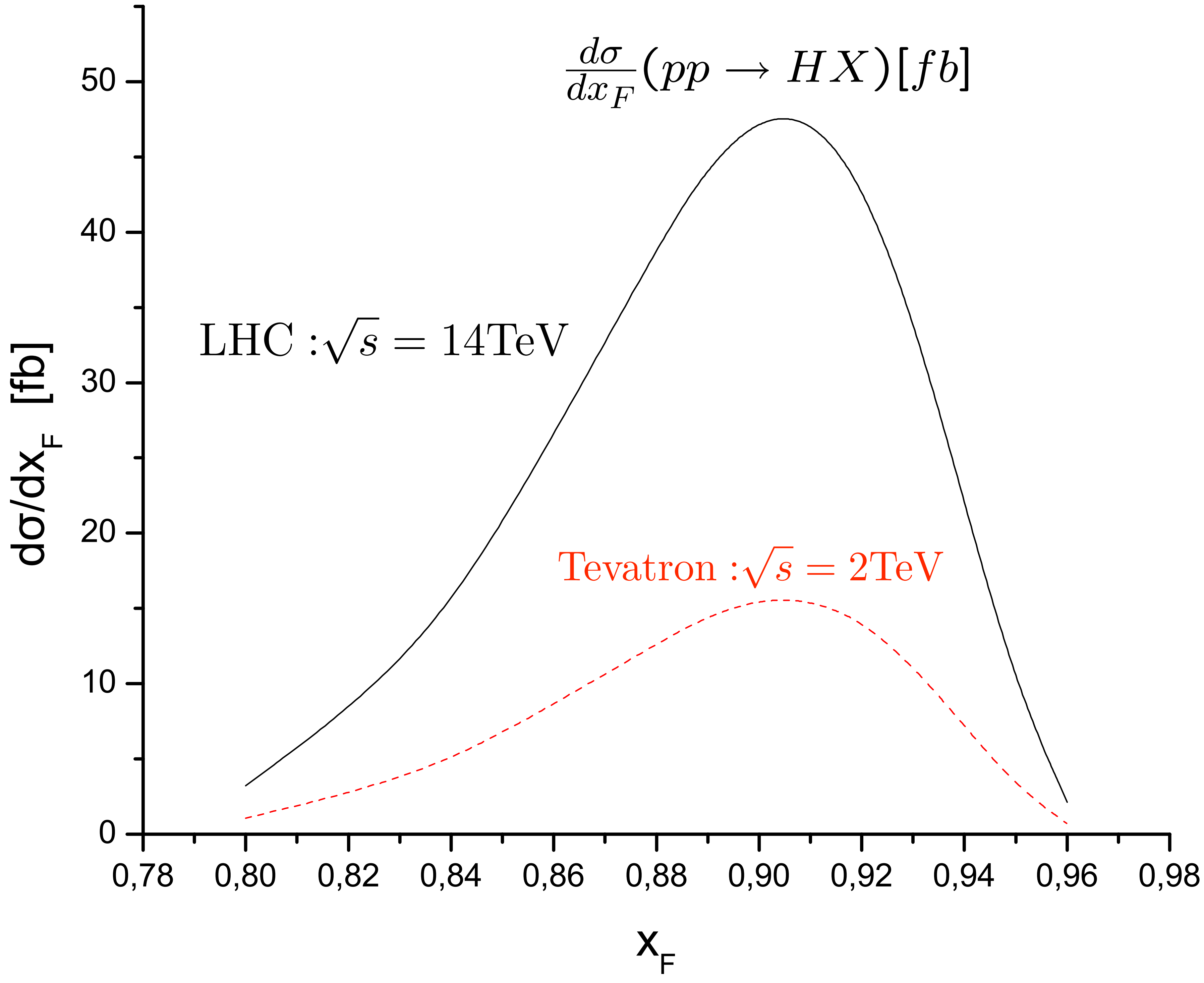
Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs

AFTER: Higgs production at threshold!

Intrinsic Heavy Quark Contribution to Inclusive Higgs Production

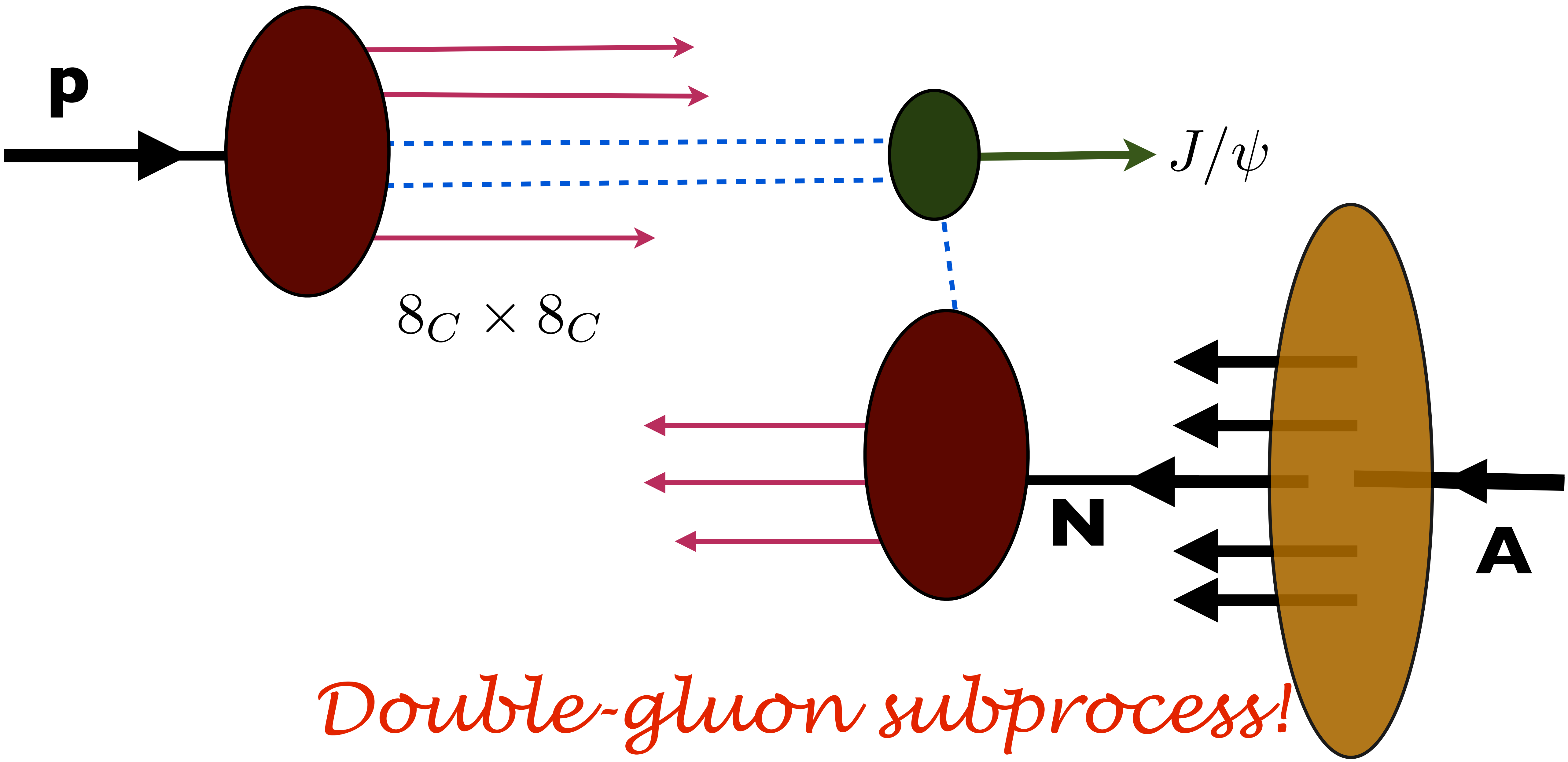


Goldhaber, Kopeliovich, Schmidt, sjb

$$pA \to J/\psi X$$

LHC

$$(gg)_{8_C} + g_{8_C} \to J/\psi$$



Double-gluon subprocess!

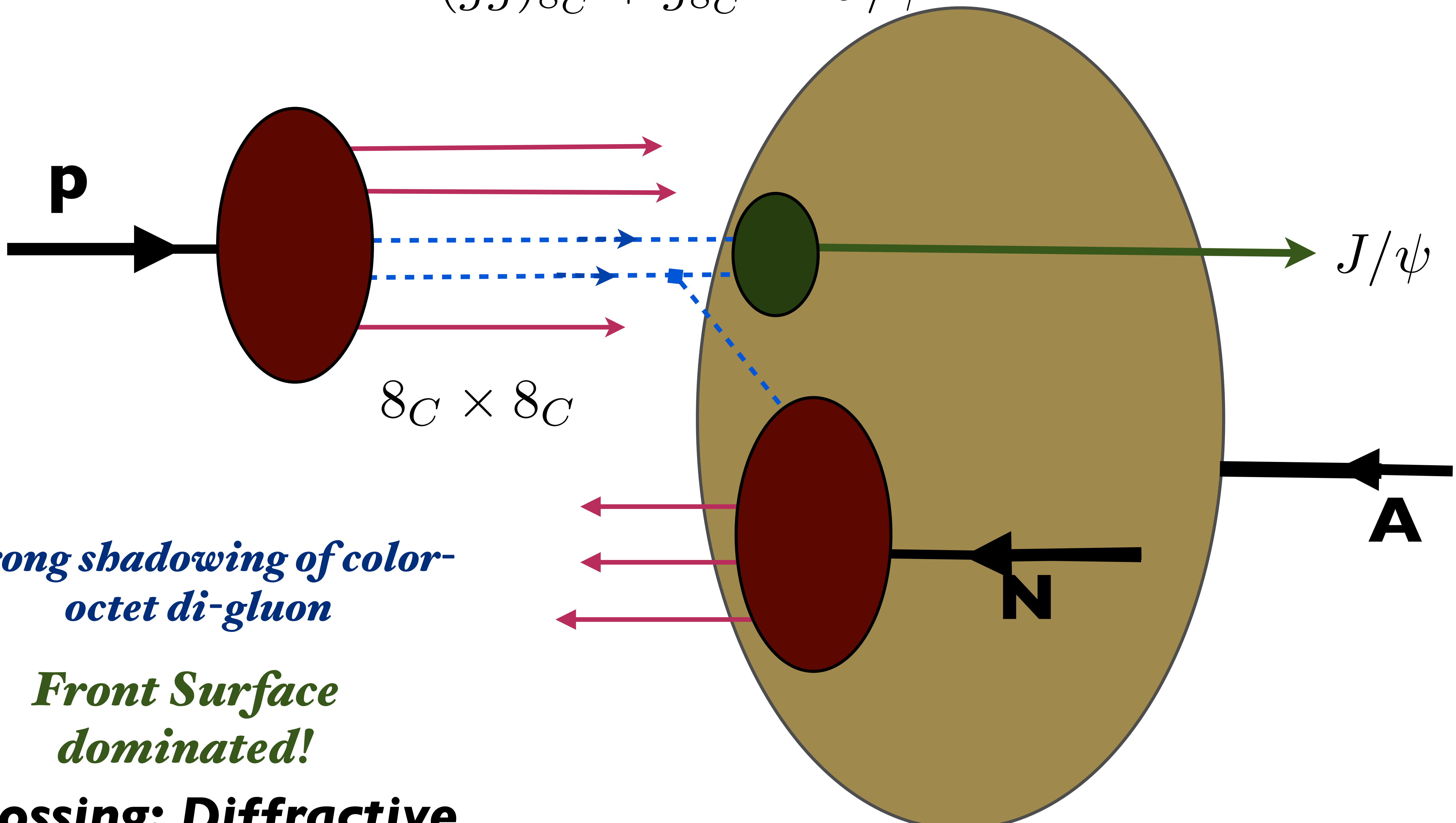
Higher-Twist but can dominate at forward rapidity, small p_T

Forward rapidity y ~4

$$pA \rightarrow J/\psi X$$

LHC

$$(gg)_{8_C} + g_{8_C} \rightarrow J/\psi$$



Strong shadowing of color-octet di-gluon

Front Surface dominated!

Crossing: Diffractive & pomeron exchange

Double-gluon subprocess

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

QCD Mass Scale from Confinement not Explicit

Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What sets the QCD mass scale?
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Structure Functions,Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable

Atomic Physics from First Principles

$\mathcal{L}_{QED} \rightarrow$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) \; |\Psi >= E \; |\Psi >$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)] \; \psi(r) = E \; \psi(r)$$

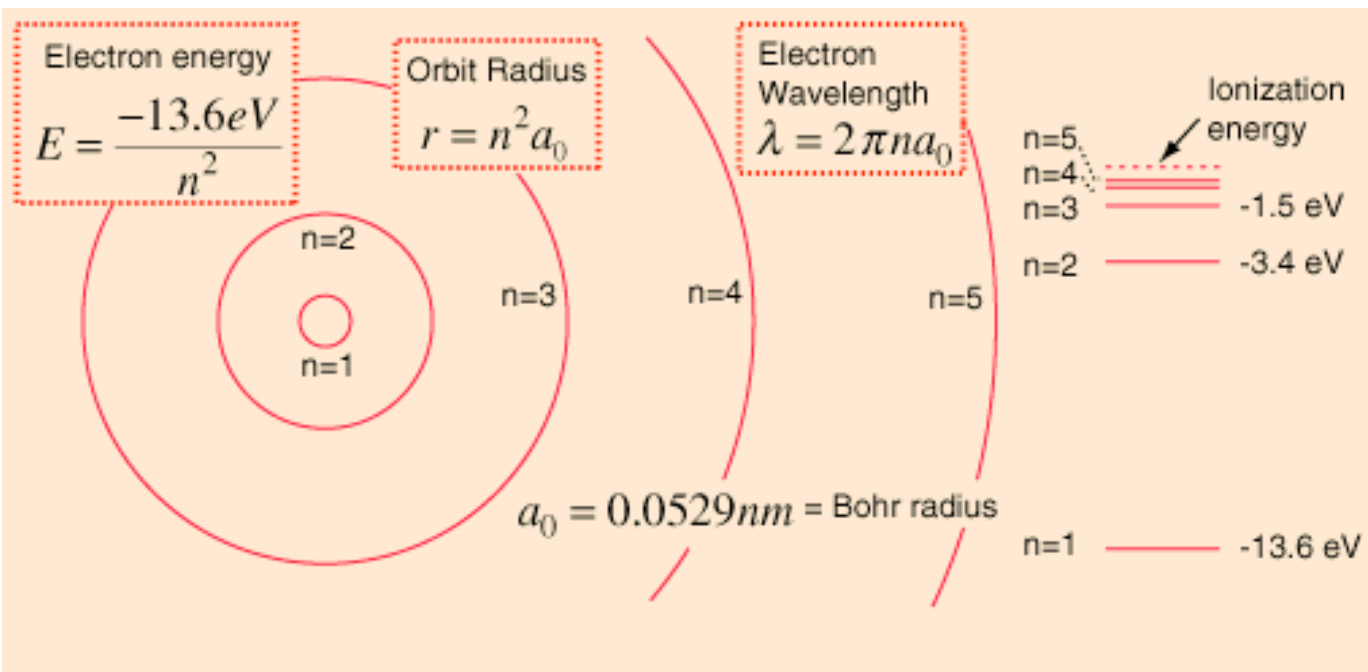
Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

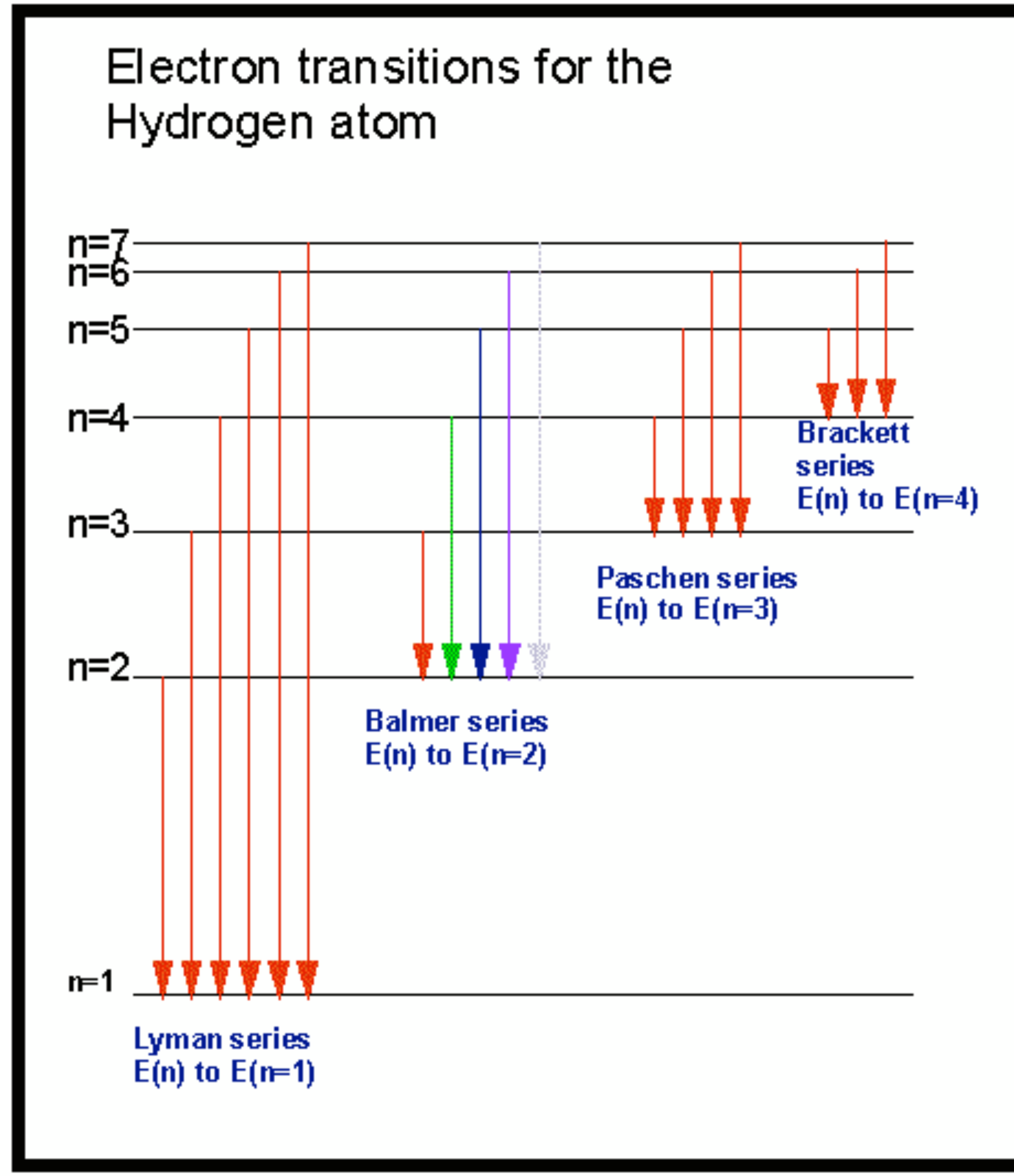
Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Bohr Atom



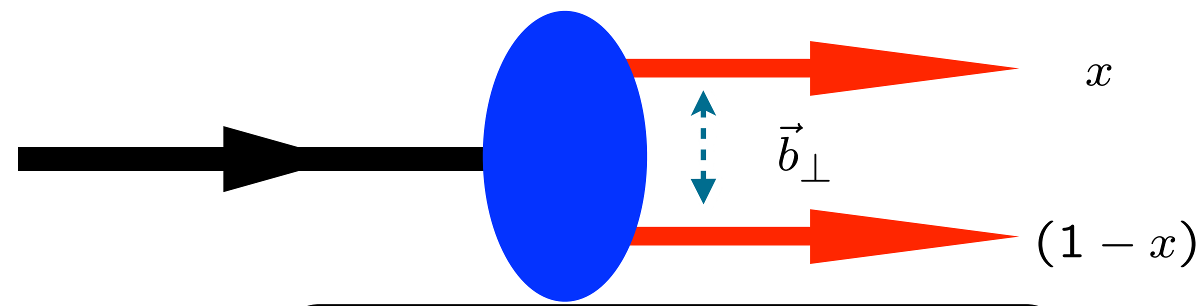
Electron energy
$E = \frac{-13.6eV}{n^2}$
Orbit Radius
$r = n^2 a_0$
Electron Wavelength
$\lambda = 2\pi n a_0$
Ionization energy
n=1
n=2
n=3
n=4
n=5
n=5
n=4
n=3
-1.5 eV
n=2
-3.4 eV
$a_0 = 0.0529nm$ = Bohr radius
n=1
-13.6 eV



Electron transitions for the Hydrogen atom
n=7
n=6
n=5
n=4
n=3
n=2
n=1
Brackett series
E(n) to E(n=4)
Paschen series
E(n) to E(n=3)
Balmer series
E(n) to E(n=2)
Lyman series
E(n) to E(n=1)

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}] \; \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)] \; \psi_{LF}(\zeta) = M^2 \; \psi_{LF}(\zeta)$$

Azimuthal Basis

ζ, ϕ

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

- Invariant mass $\mathcal{M}^2$ in terms of LF mode ϕ

$$\begin{aligned}\mathcal{M}^2 &= \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta) U(\zeta)\phi(\zeta)\\ &= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right)\phi(\zeta) + \int d\zeta\, \phi^*(\zeta) U(\zeta)\phi(\zeta)\end{aligned}$$

where the interaction terms are summed up in the effective potential $U(\zeta)$ and the orbital angular momentum in ∇^2 has the $SO(2)$ Casimir representation $SO(N) \sim S^{N-1}:\ L(L+N-2)$

$$-\frac{\partial^2}{\partial\varphi^2}|\phi\rangle = L^2|\phi\rangle$$

- LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)} \qquad m_q = 0$$

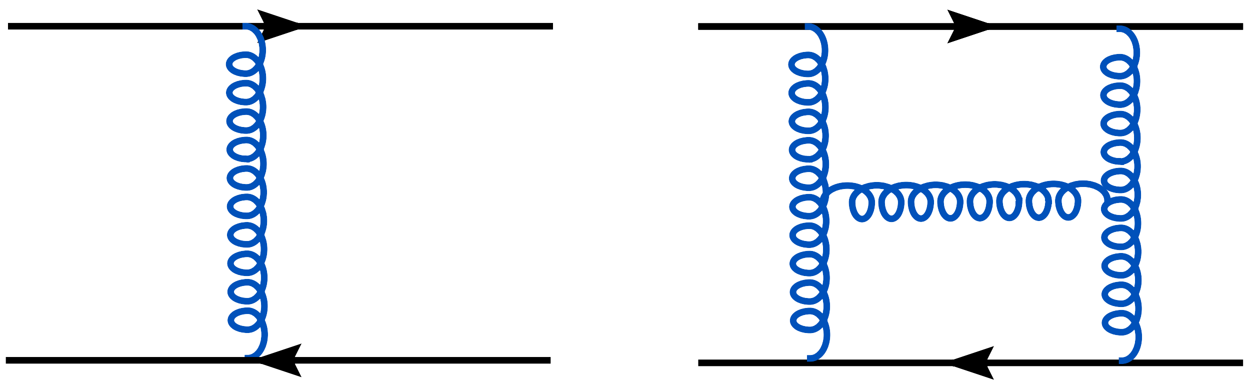
- Effective light-front Schrödinger equation: relativistic, covariant and analytically tractable.

Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$

$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



$\log \kappa^2 \zeta^2$

Summation of H graphs: confining potential

Confinement eliminates IR divergences

Self-consistent mass scale κ

Light-Front Schrödinger Equation

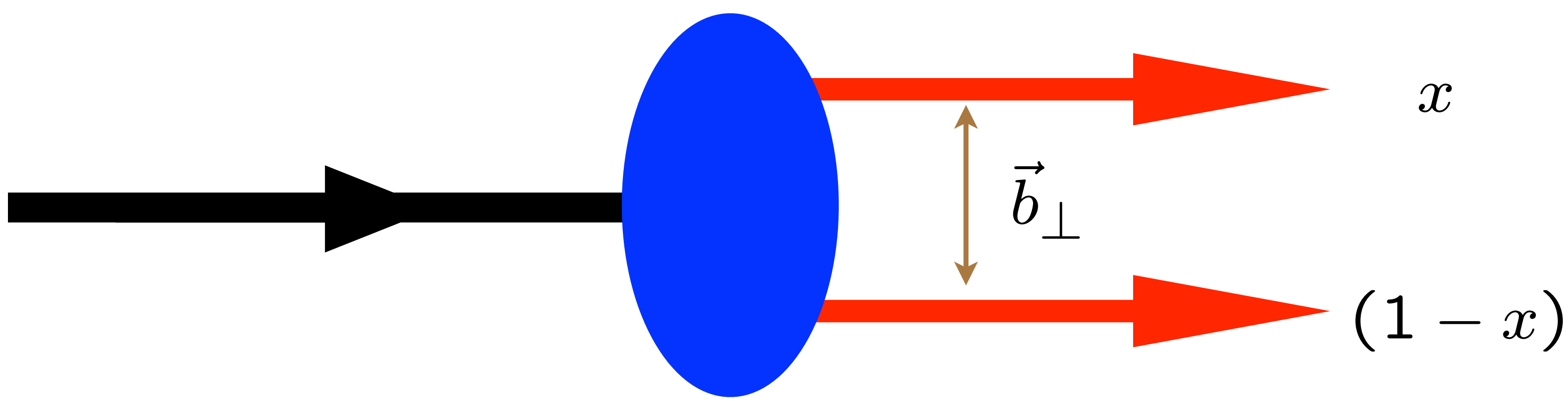
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

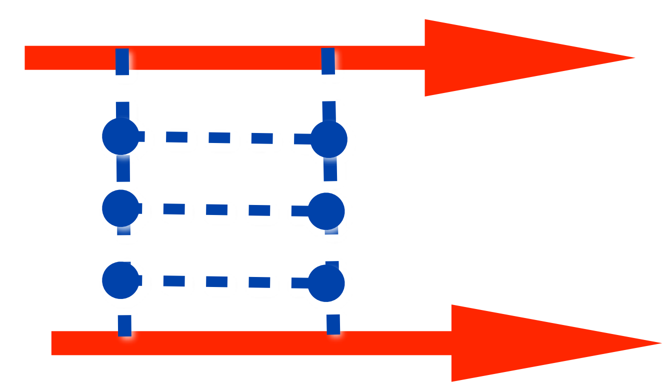
Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$



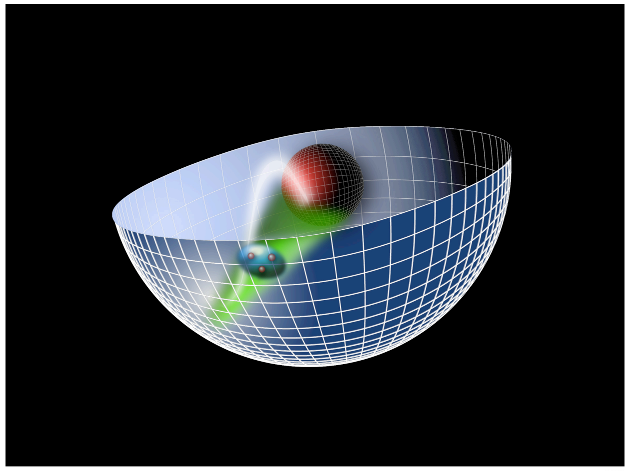
U is the confining QCD potential
Conjecture: 'H'-diagrams generate



$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model



Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6 \; GeV$$

$$1/\kappa \simeq 1/3 \; fm$$

- **de Alfaro, Fubini, Furlan:** *Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!*

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L

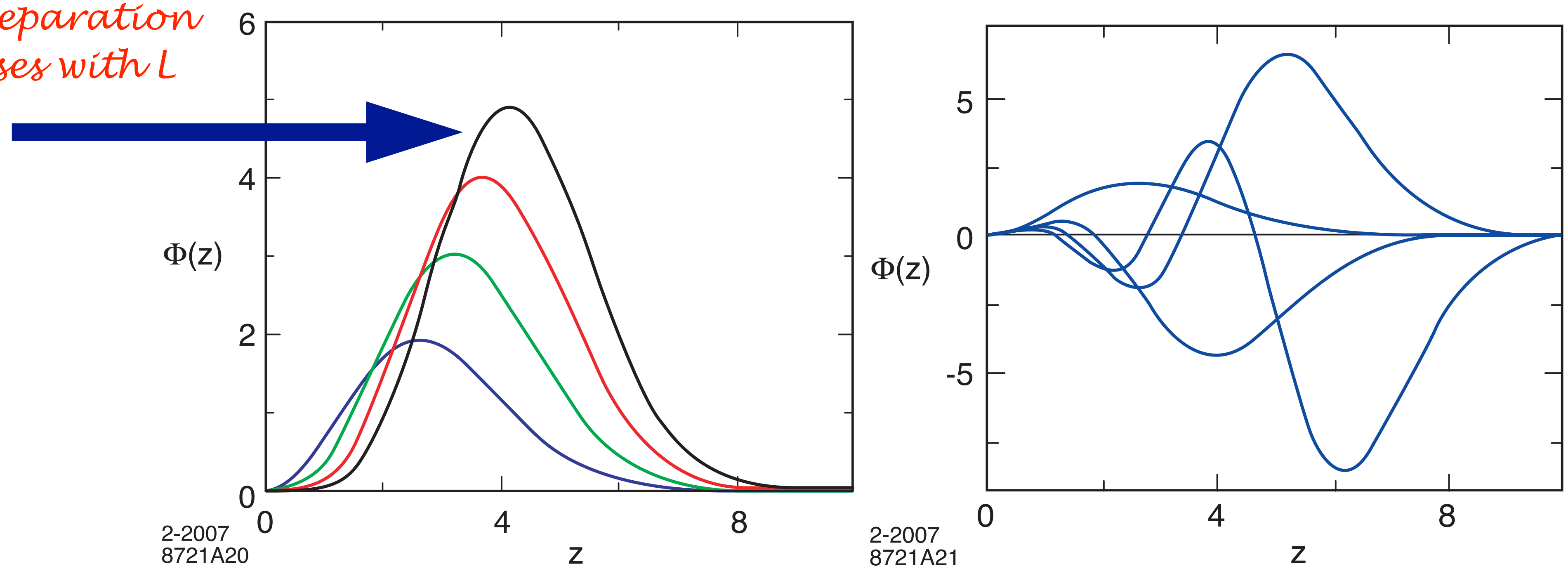
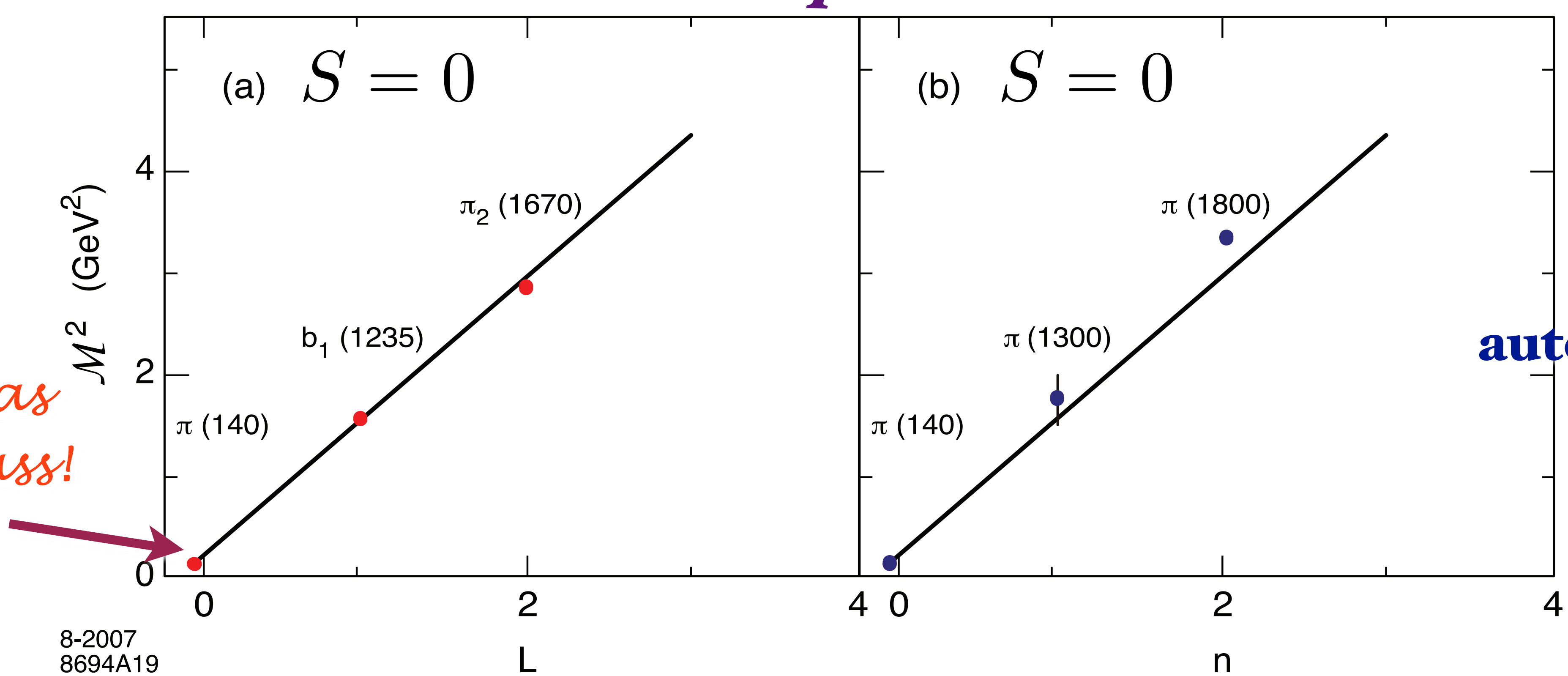


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

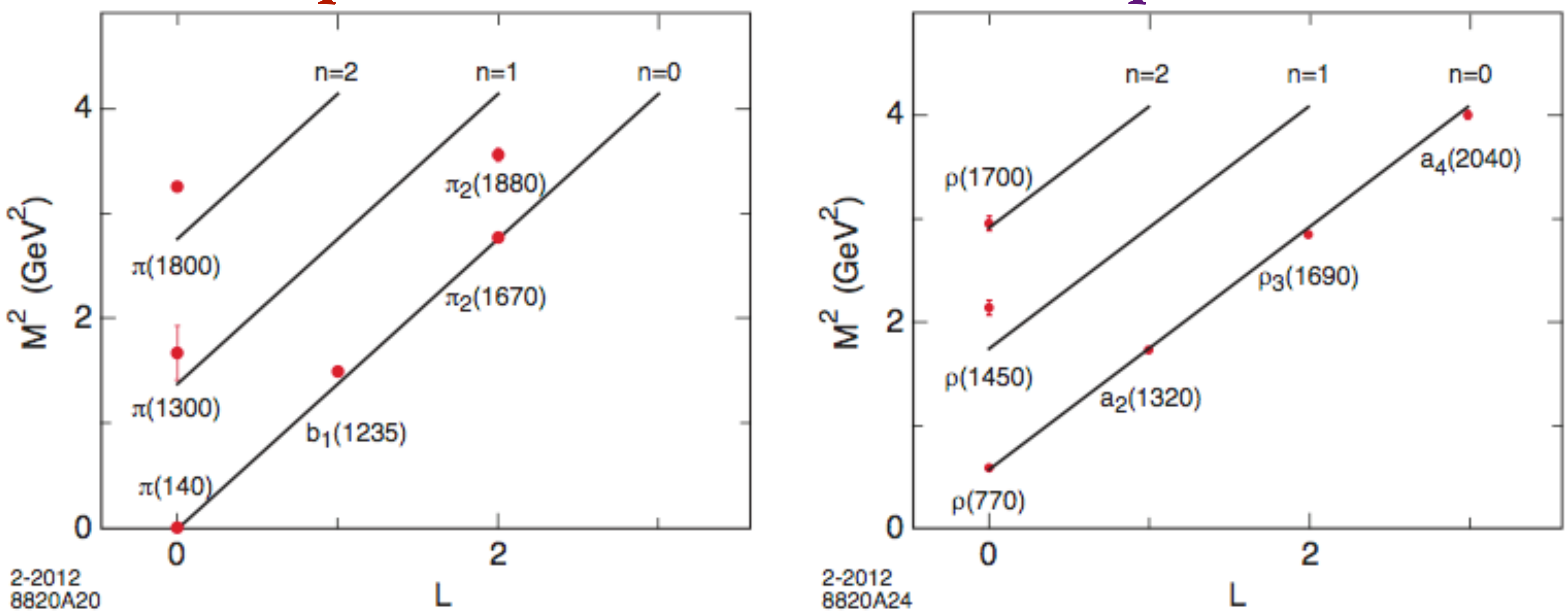
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***



I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

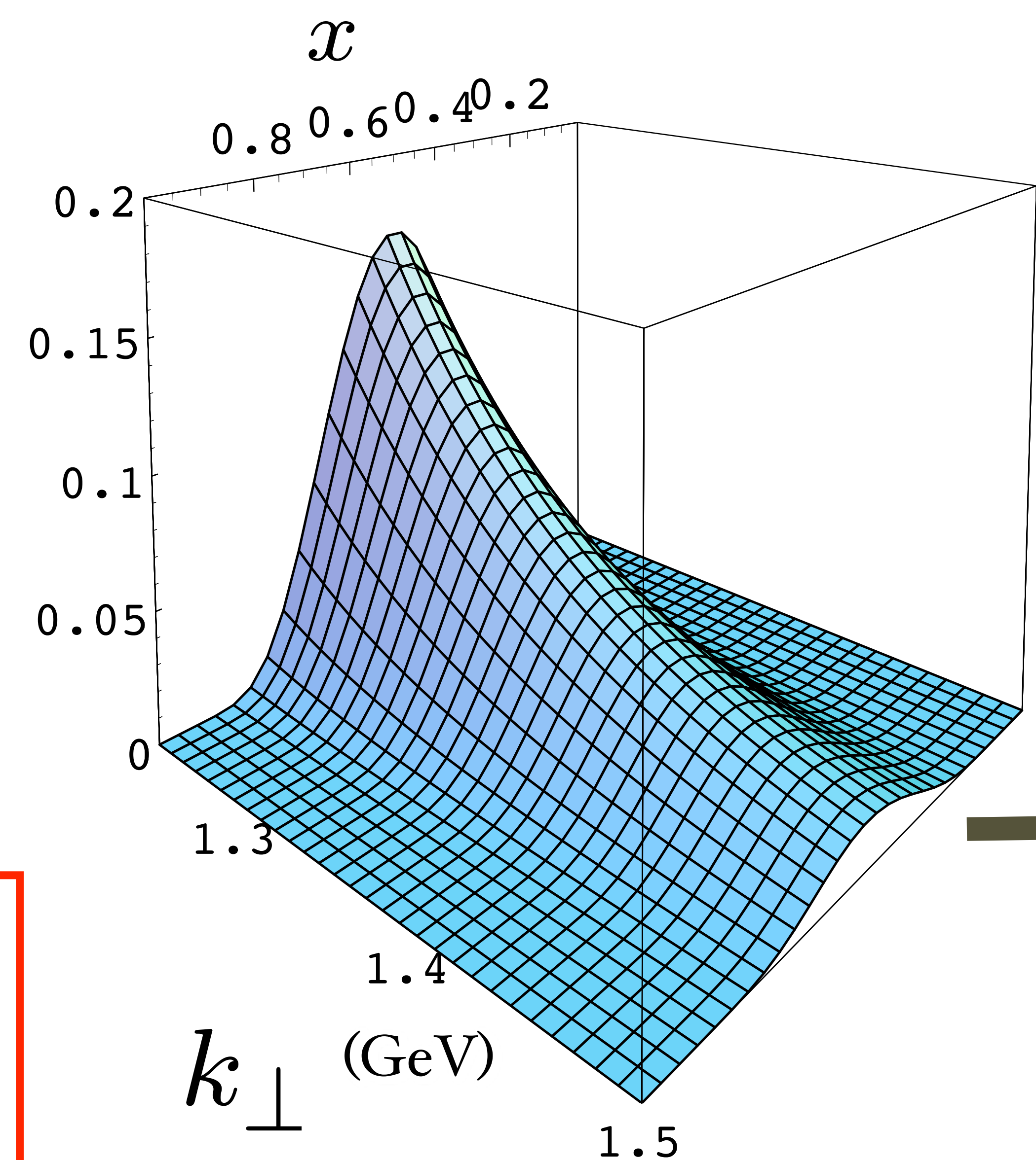
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

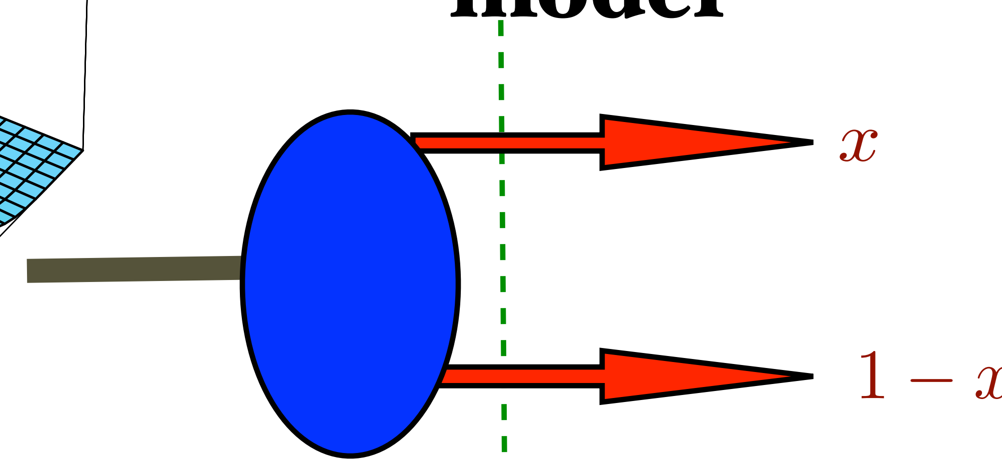
Prediction from AdS/QCD: Meson LFWF

$\psi_M(x, k_\perp^2)$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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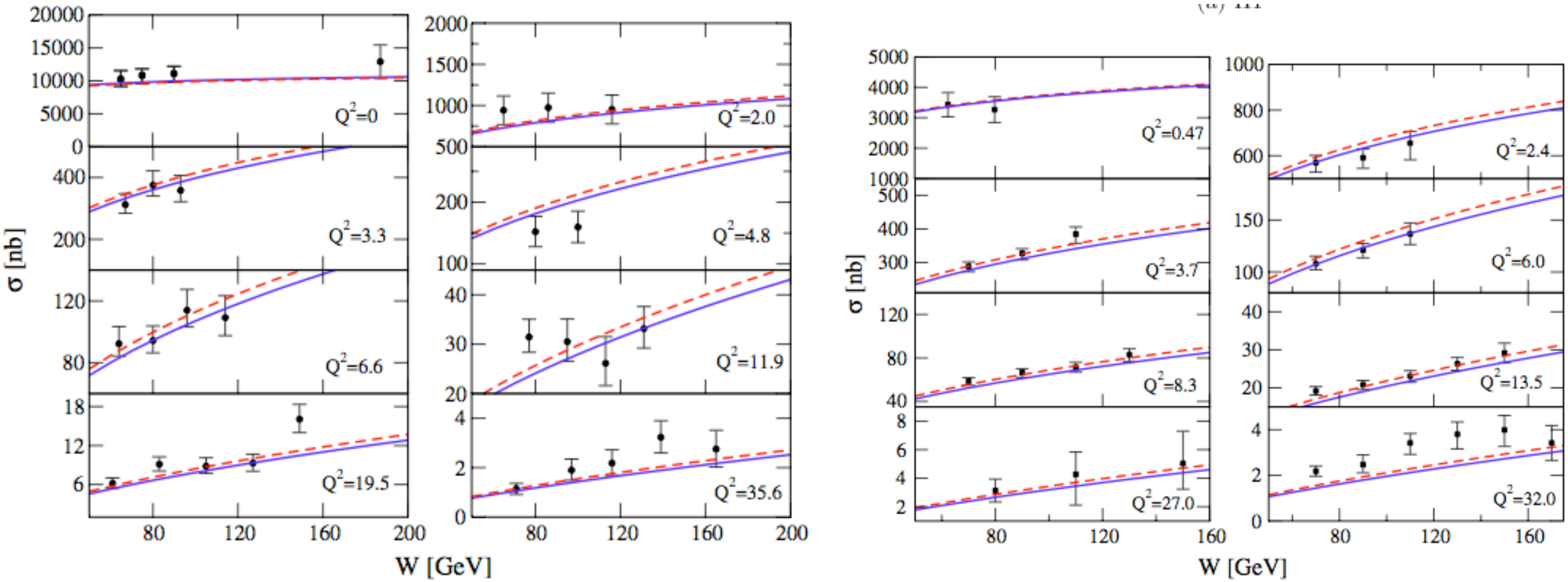
(Received 5 April 2012; published 20 August 2012)

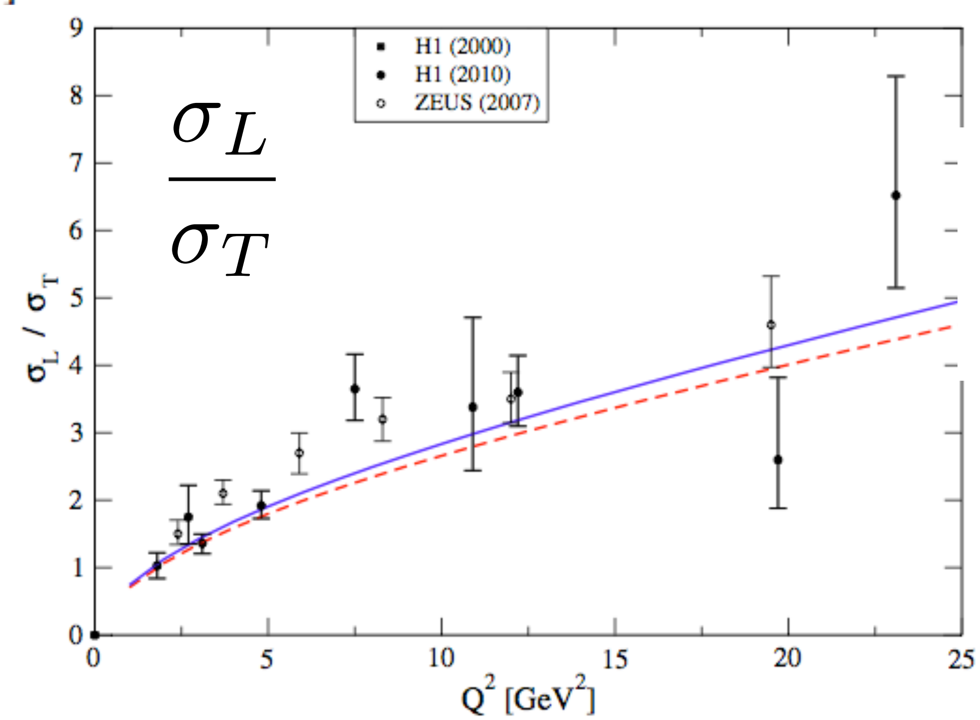
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x,k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H (b) ZEUS



J. R. Forshaw,
R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

PHYSICAL REVIEW D **88,** 014042 (2013)

Predicting the isospin asymmetry in $B \to K^* \gamma$ using holographic AdS/QCD distribution amplitudes for the K^*

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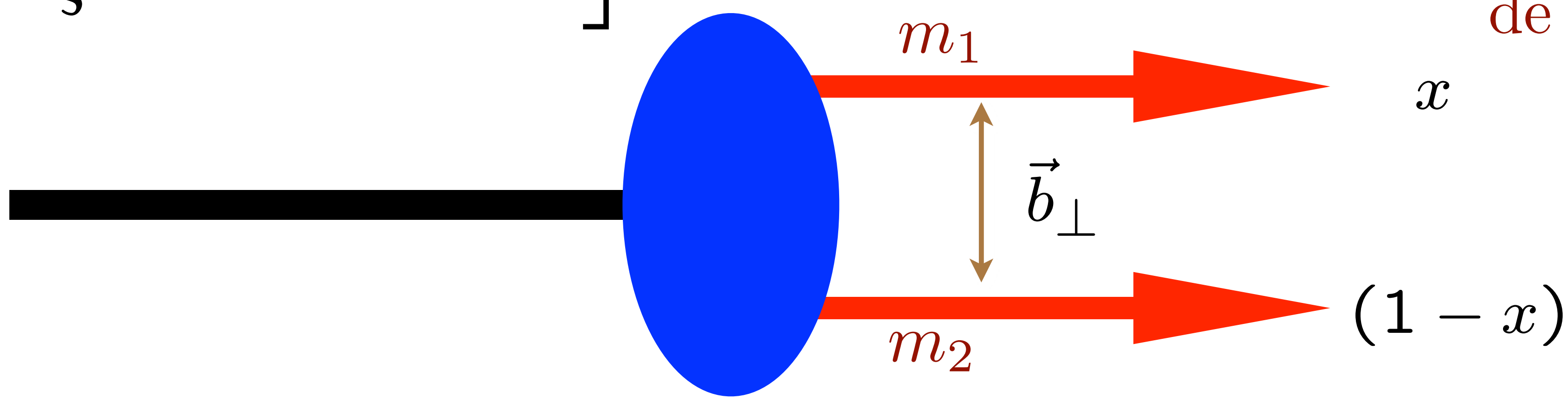
(Received 10 May 2013; published 26 July 2013)

$$\phi_\lambda(z,\zeta) = \mathcal{N}_\lambda \frac{\kappa}{\sqrt{\pi}} \sqrt{z(1-z)} \exp\left(-\frac{\kappa^2\zeta^2}{2}\right) \times \exp\left(-\frac{(1-z)m_q^2 + z m_{\bar{q}}^2}{2\kappa^2 z(1-z)}\right), \qquad \text{where } \zeta = \sqrt{z(1-z)}\,r$$

We predict the isospin asymmetry well as the branching ratio for the decay $B \to K^* \gamma$ within QCD factorization using new anti–de Sitter/quantum chromodynamics (AdS/QCD) holographic distribution amplitudes (DAs) for the K^* meson. Our prediction for the branching ratio agrees with that obtained using standard QCD sum-rules (SR) DAs and with experiment. More interestingly, our prediction for the isospin asymmetry using the AdS/QCD DA does not suffer from the end-point divergence encountered when using the corresponding SR DA. We predict an isospin asymmetry of 3.2% in agreement with the most recent average measured value of (5.2 ± 2.6)% quoted by the Particle Data Group.

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle \psi | \sum_a m_a^2 / x_a | \psi \rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x, \zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x} + \frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda \zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M^2_{n,L,S} = M^2_{K^\pm} + 4\lambda \left(n + \frac{J+L}{2}\right)$$

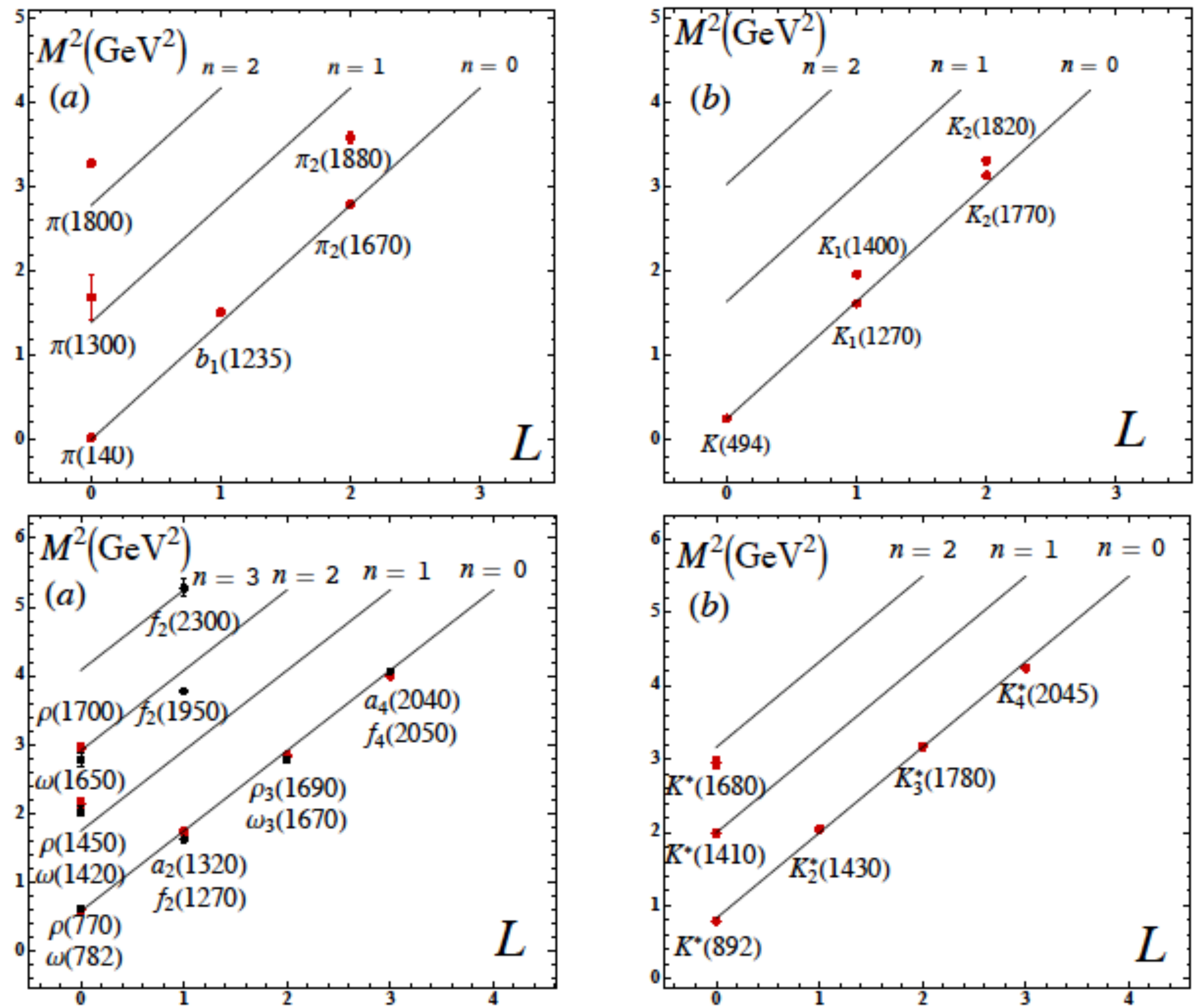
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

De Teramond, Dosch, sjb

$m_u = m_d = 46\text{ MeV}, \quad m_s = 357\text{ MeV}$

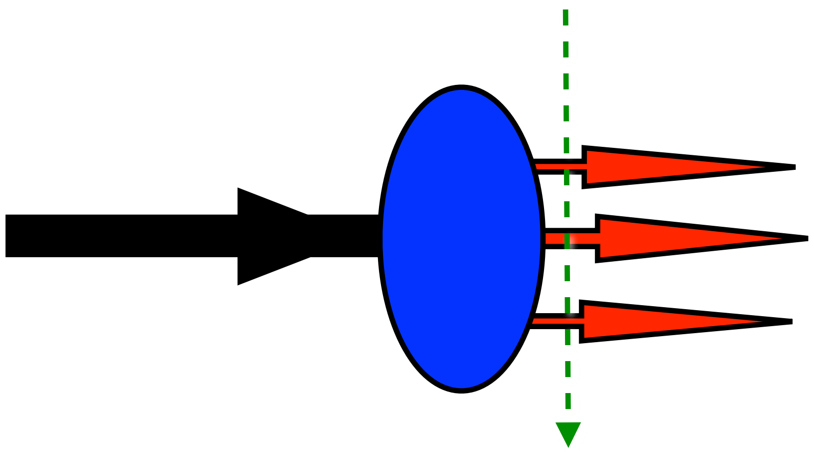
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

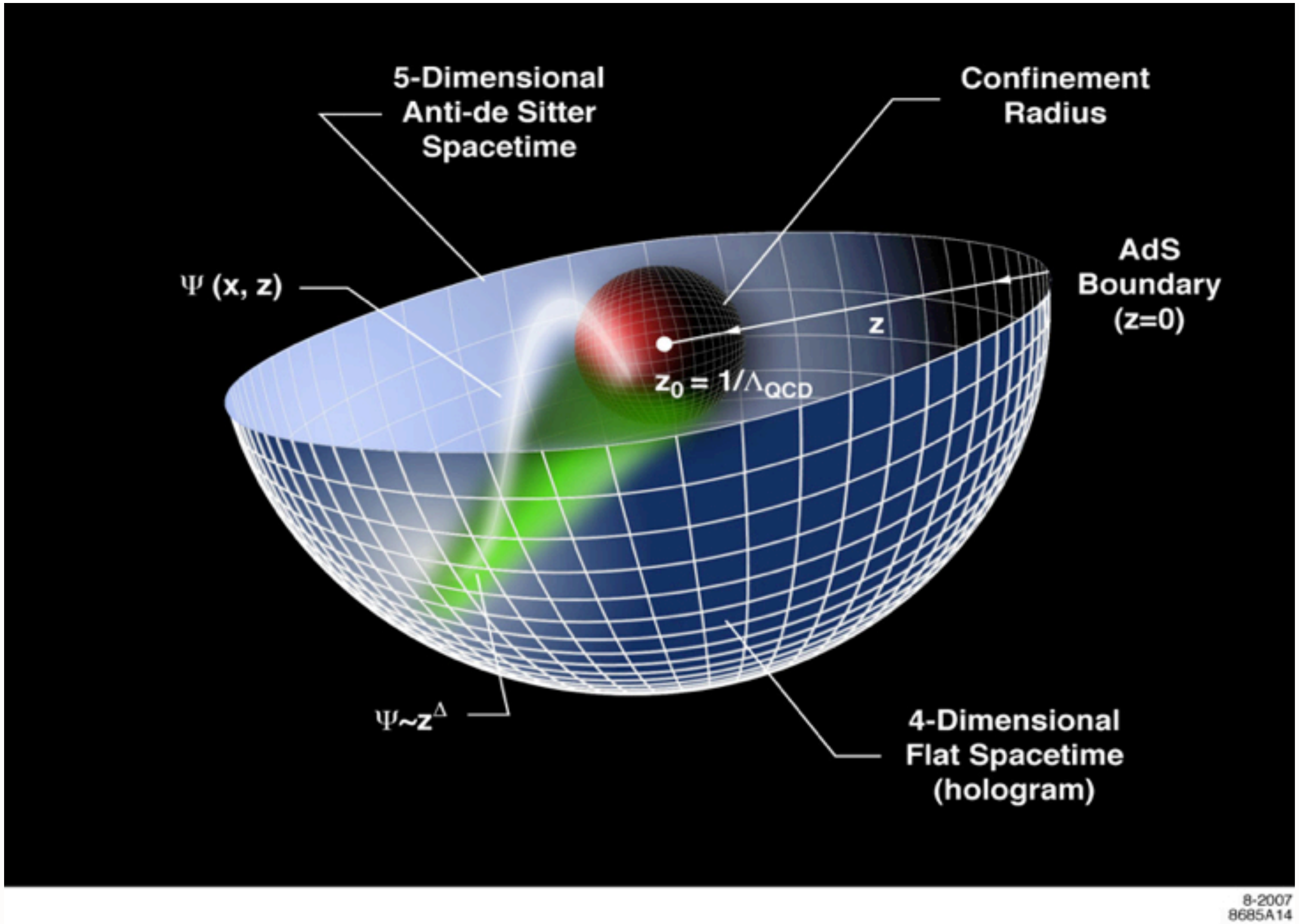
$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space



Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

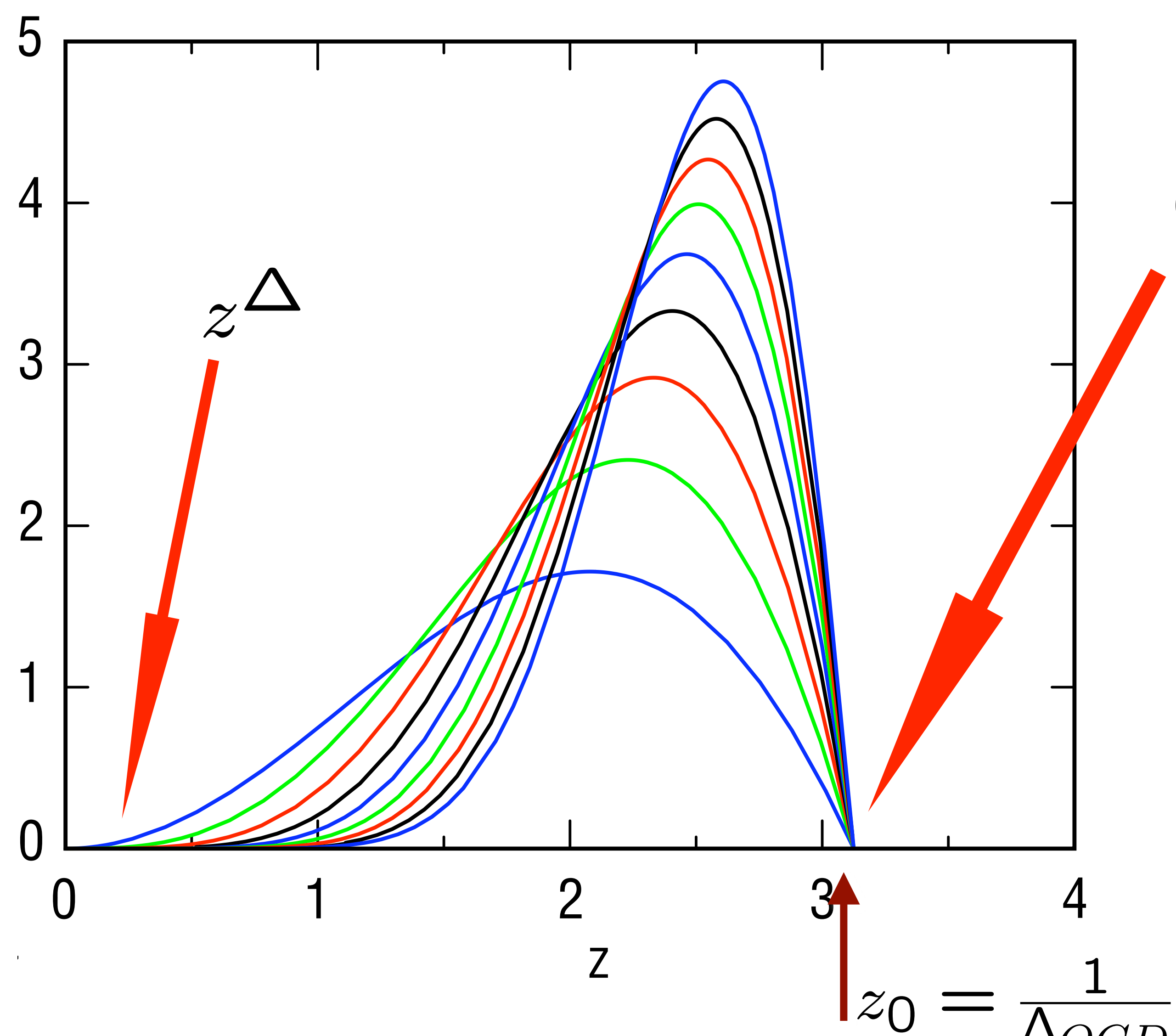
- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

- Physical AdS modes $\Phi_P(x,z) \sim e^{-iP\cdot x}\,\Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum P^μ and hadronic invariant mass states $P_\mu P^\mu = \mathcal{M}^2$.
- For small-z $\Phi(z) \sim z^\Delta$. The scaling dimension Δ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson

equivalent to dimensions of chiral superfields



Hard Wall

Confinement in the 5th dimension

de Teramond, sjb

Identify hadron by its interpolating operator at z --> 0

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement**
- **Introduces confinement scale** κ
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

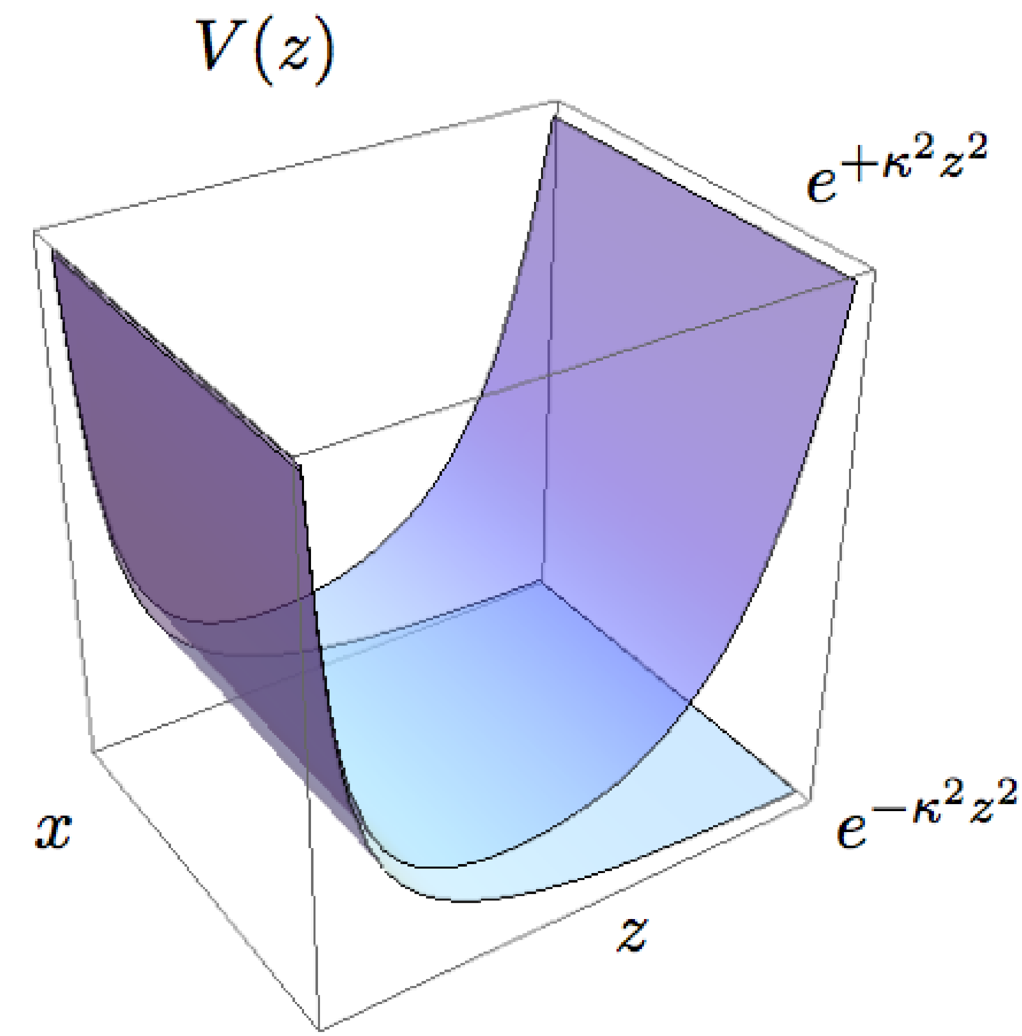
where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$

- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

Positive-sign dilaton

- de Teramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

z ⟷ $\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$

G. de Teramond and sjb, PRL 102 081601 (2009)

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

General dilaton profile

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\phi''(\zeta) + \frac{1}{4}\phi'(\zeta)^2 + \frac{2J-3}{2\zeta}\phi'(\zeta)$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

de Teramond, Dosch, sjb

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2\partial_z^2 - \left(3-2J - 2\kappa^2 z^2\right) z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2\zeta^2/2}\,\Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L+S-1)\right)\phi_{\mu_1\cdots\mu_J} = \mathcal{M}^2\phi_{\mu_1\cdots\mu_J}$$

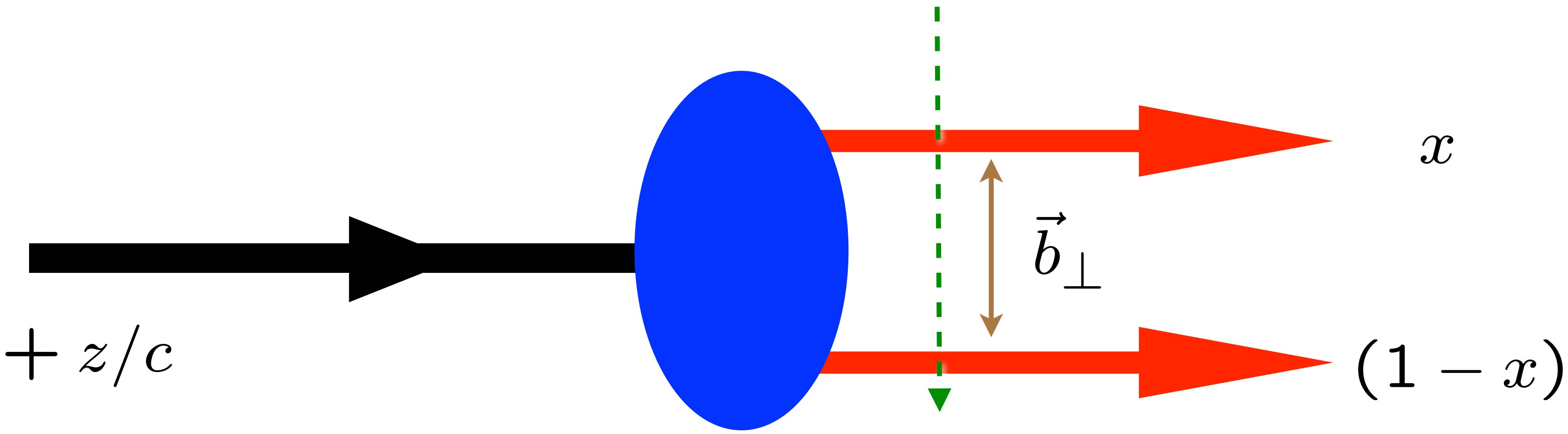
with $(\mu R)^2 = -(2-J)^2 + L^2$

de Teramond, sjb

LF(3+1) ⟷ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

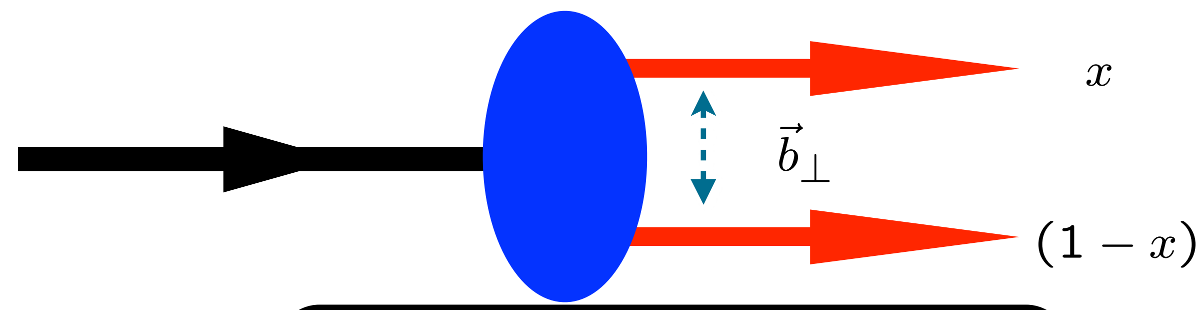
$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$H_{QCD}^{LF}$$

$$(H_{LF}^0 + H_{LF}^I)|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states (retarded interactions)

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\rm eff}^{LF}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

Azimuthal Basis

ζ, ϕ

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Quark separation increases with L

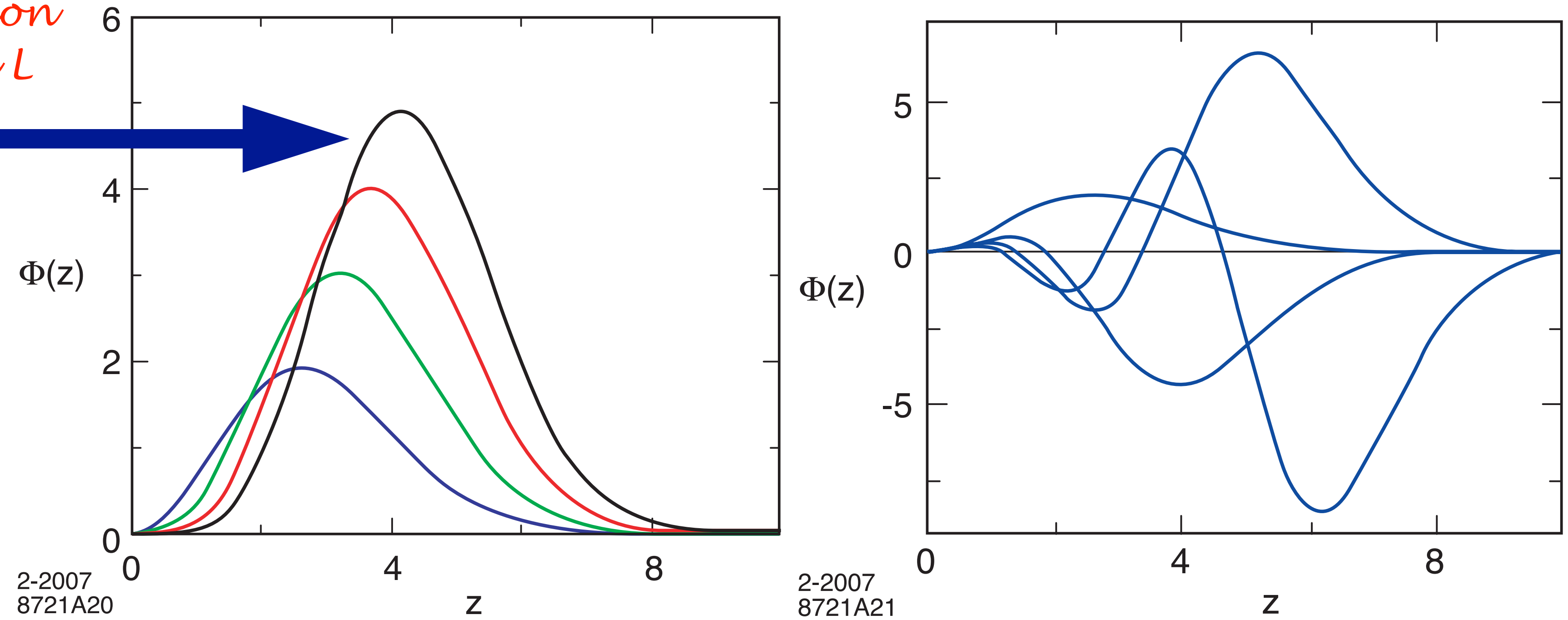
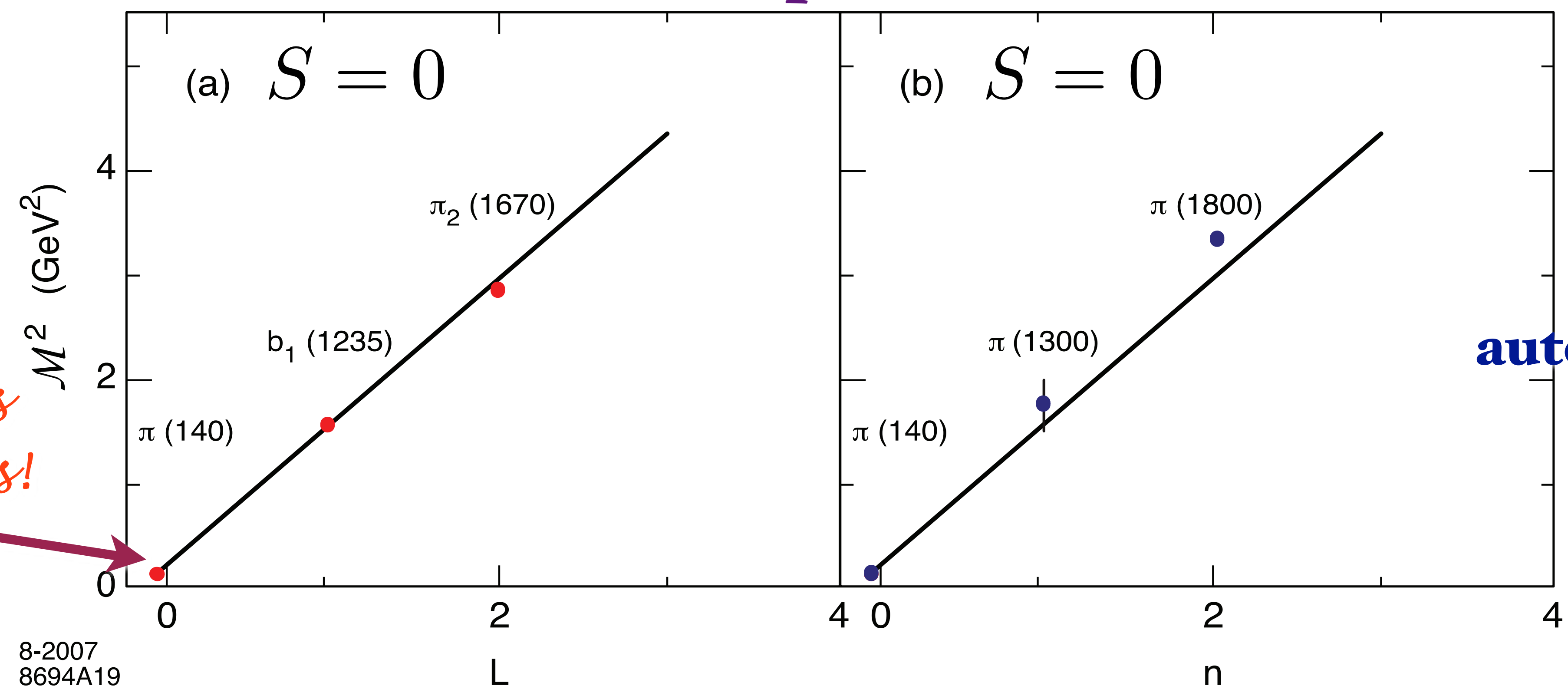


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

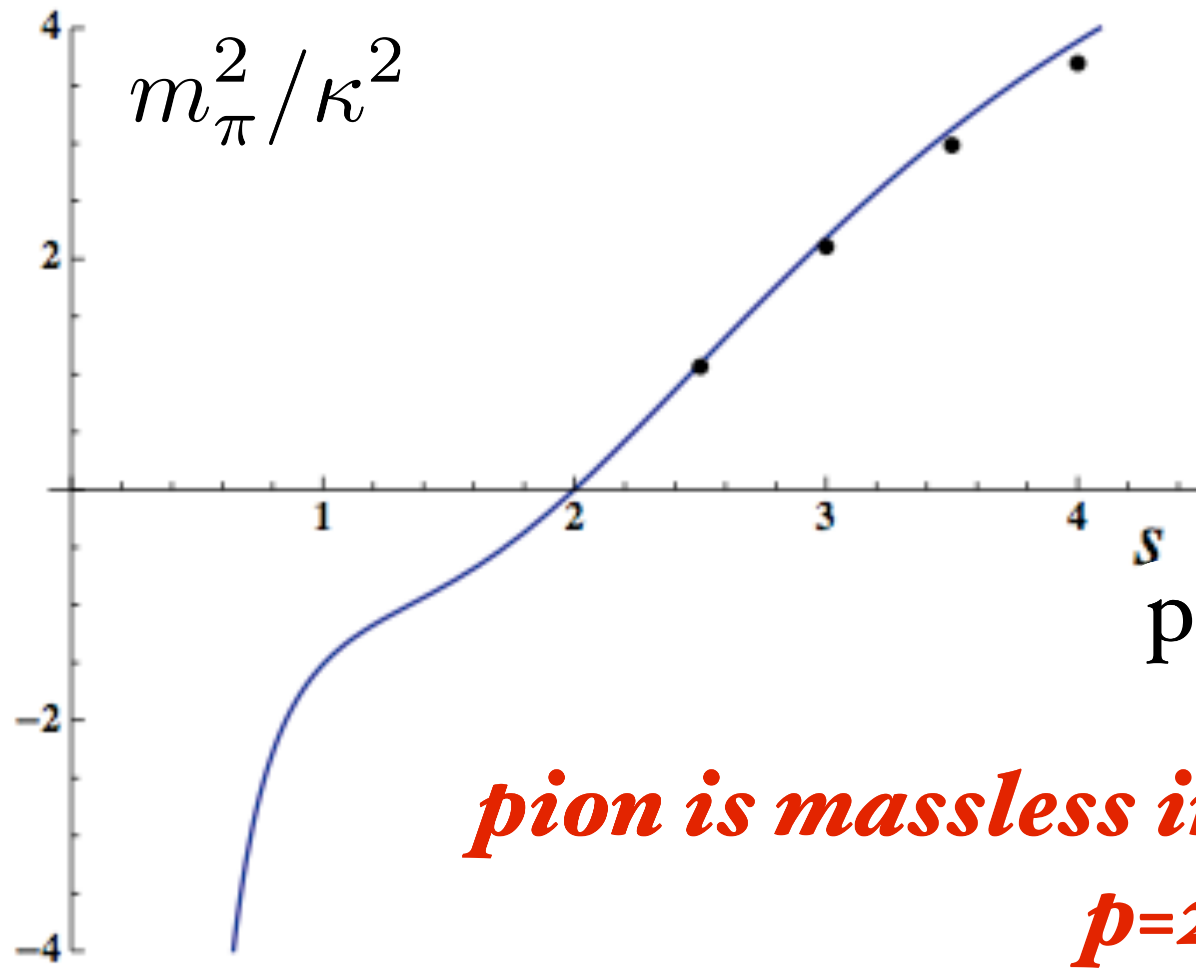
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Teramond, sjb**

Dilaton-Modified AdS₅

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- Soft-wall dilaton profile breaks conformal invariance $e^{\varphi(z)} = e^{+\kappa^2 z^2}$ **Positive-sign dilaton**
- Color Confinement, mass gap
- Introduces single confinement scale κ
- Uses AdS_5 as template for conformal theory

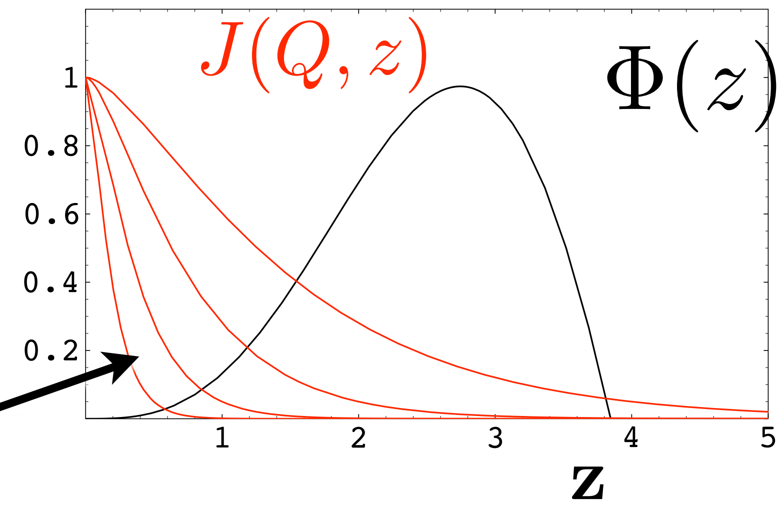
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q,z) \Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

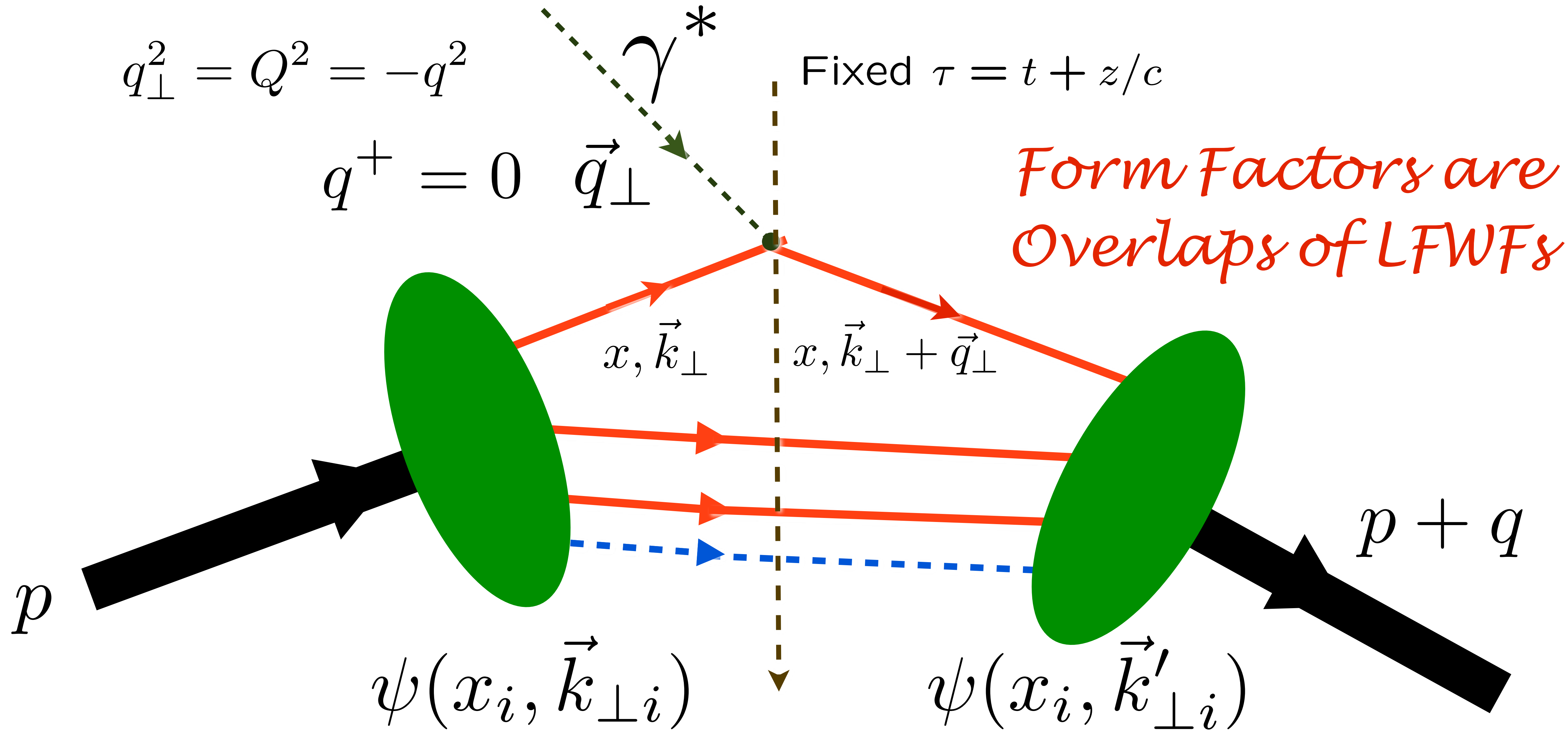
where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$.

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

Twist $\tau = n + L$

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Soper: DYW: Product of LFWFs in transverse space

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

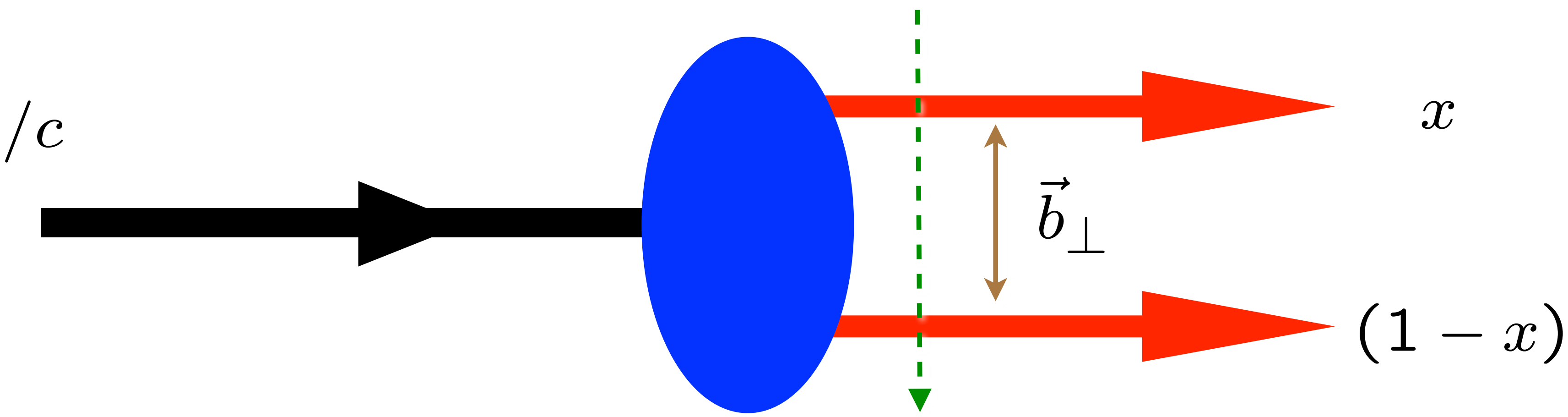
de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS$_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

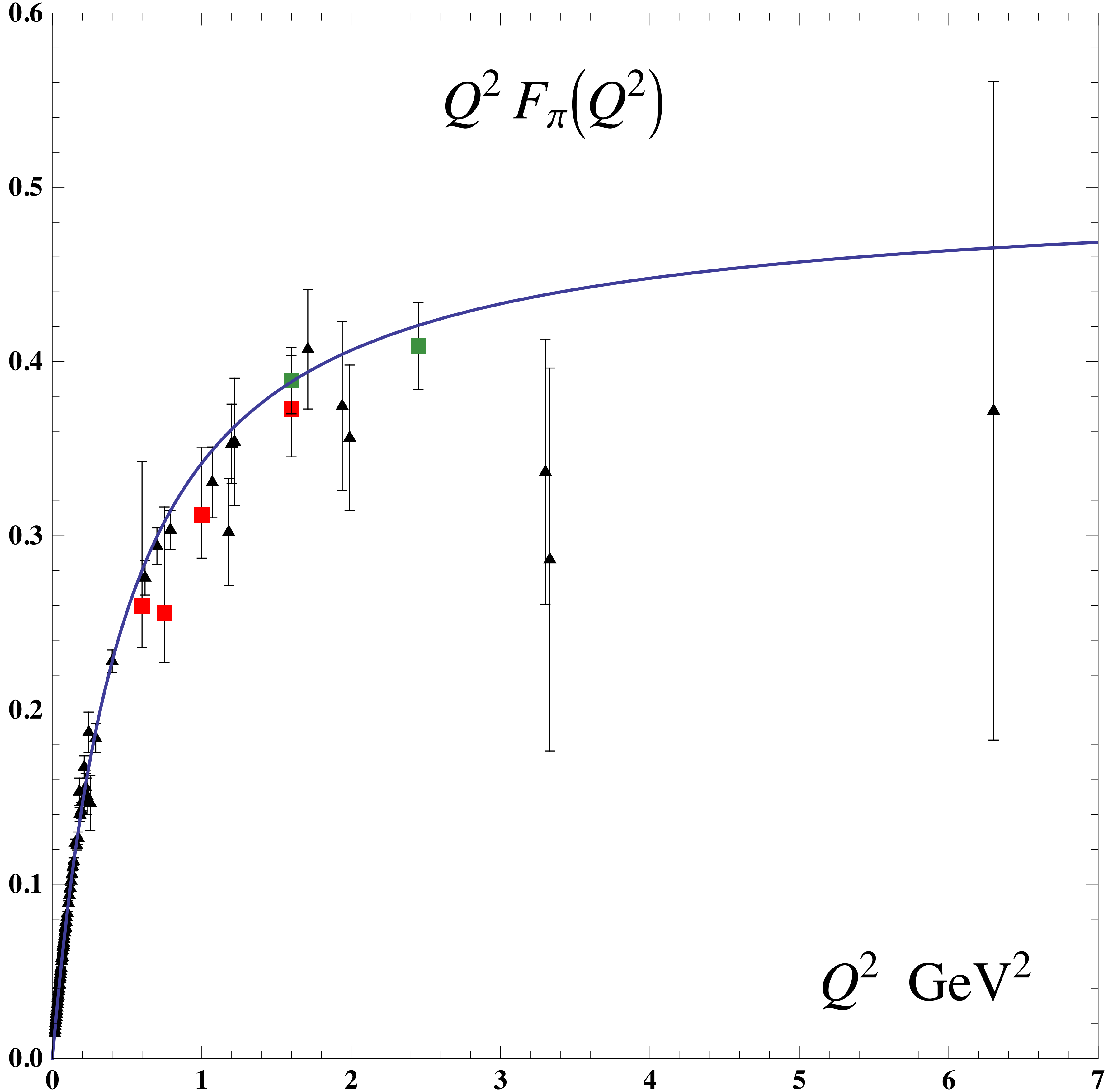
$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

Fixed $\tau = t + z/c$



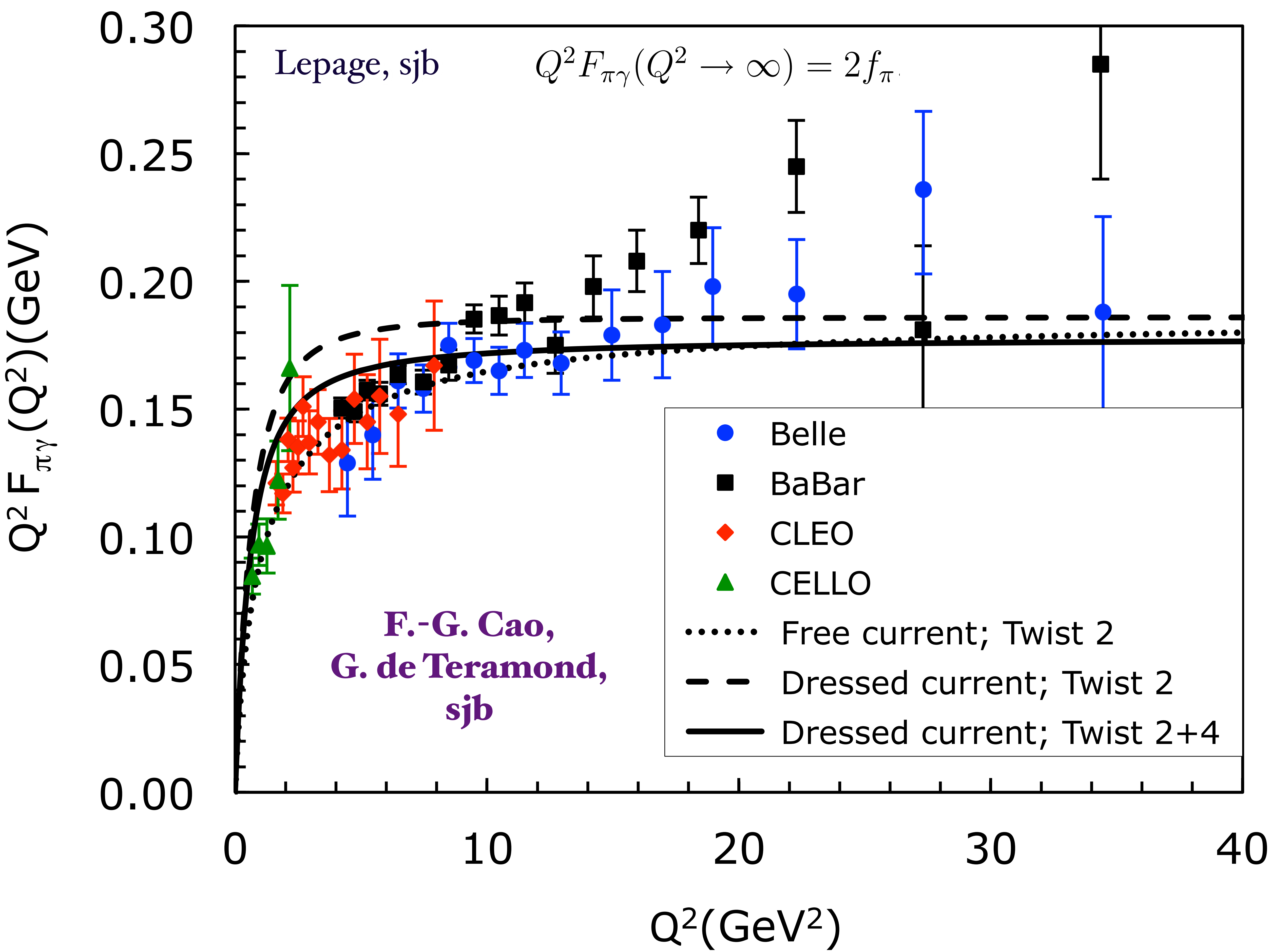
$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Photon-to-pion transition form factor



Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

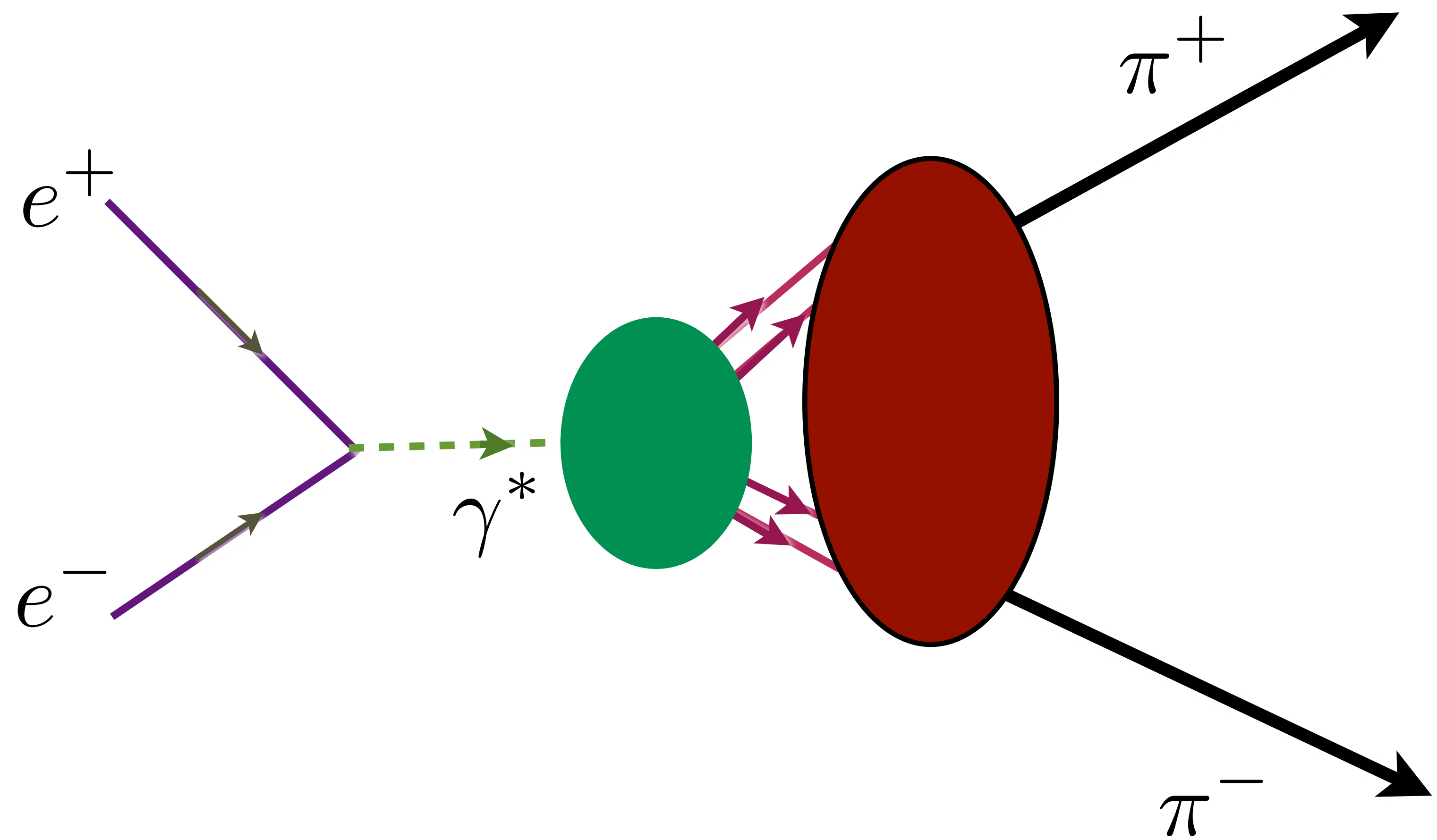
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\Phi(z) J_\kappa(Q, z)\Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

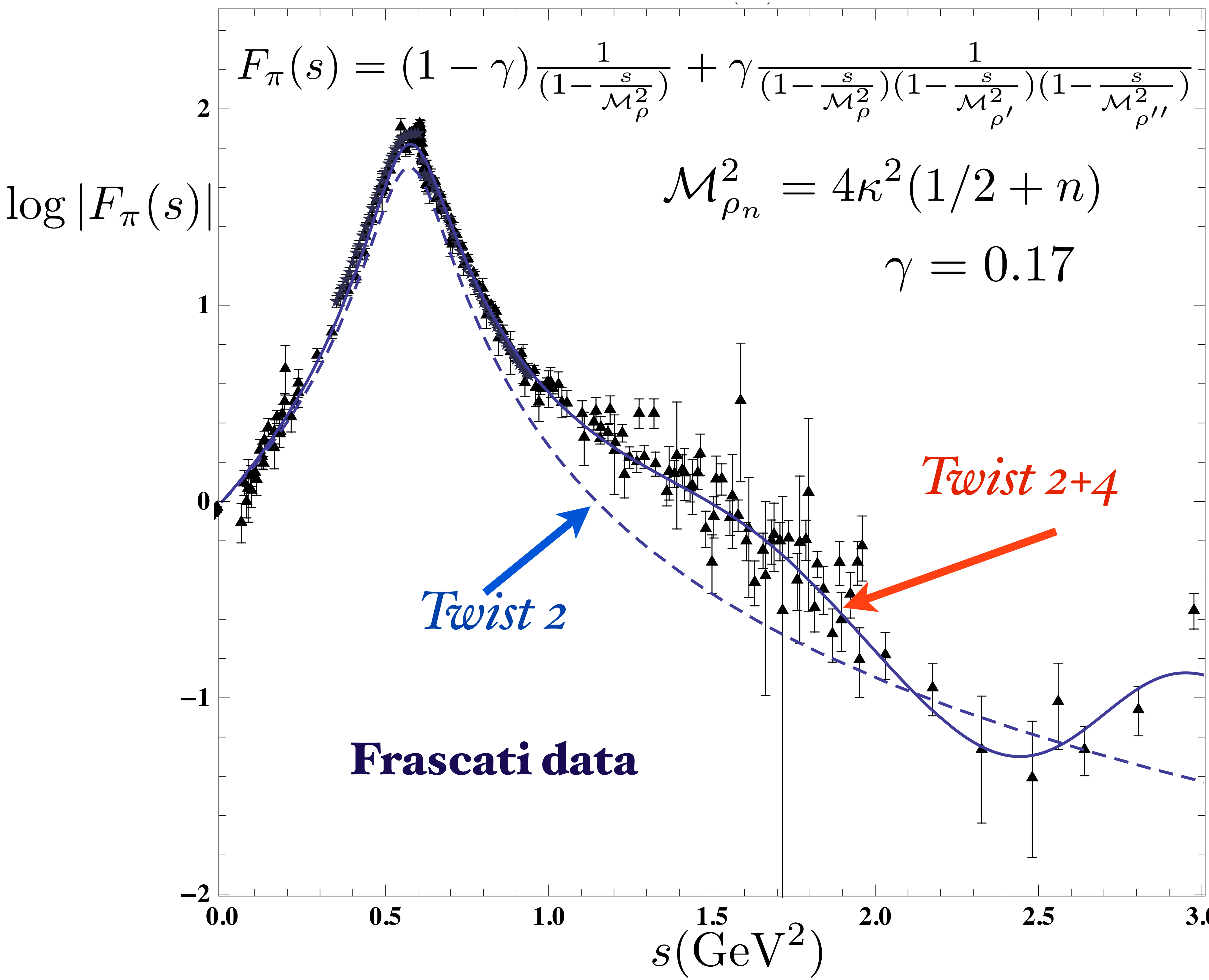
$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



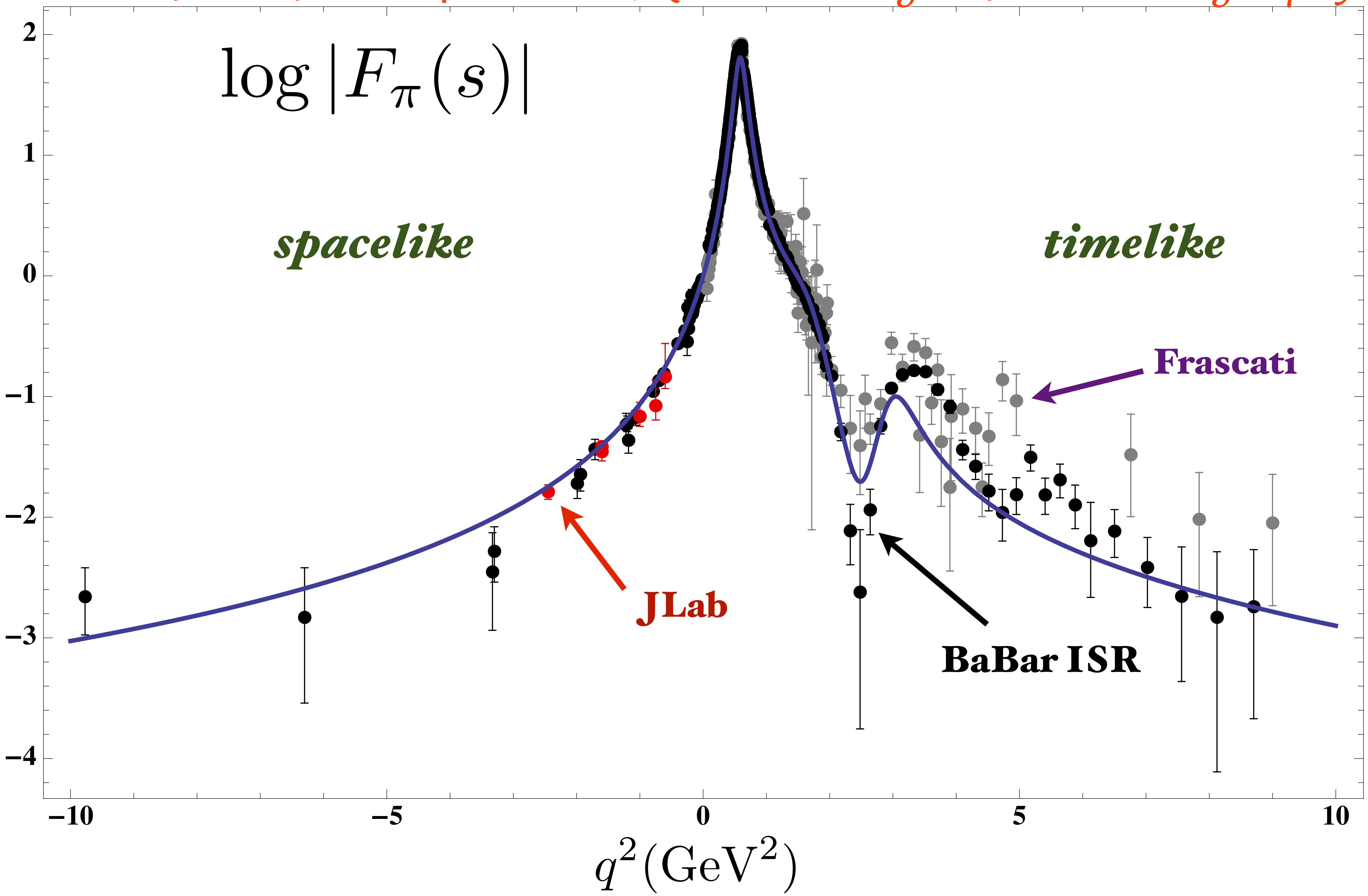
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

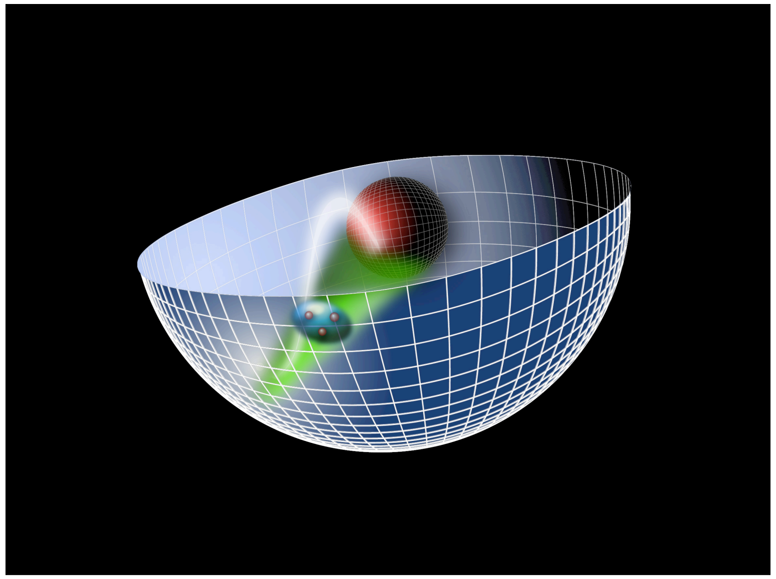
14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



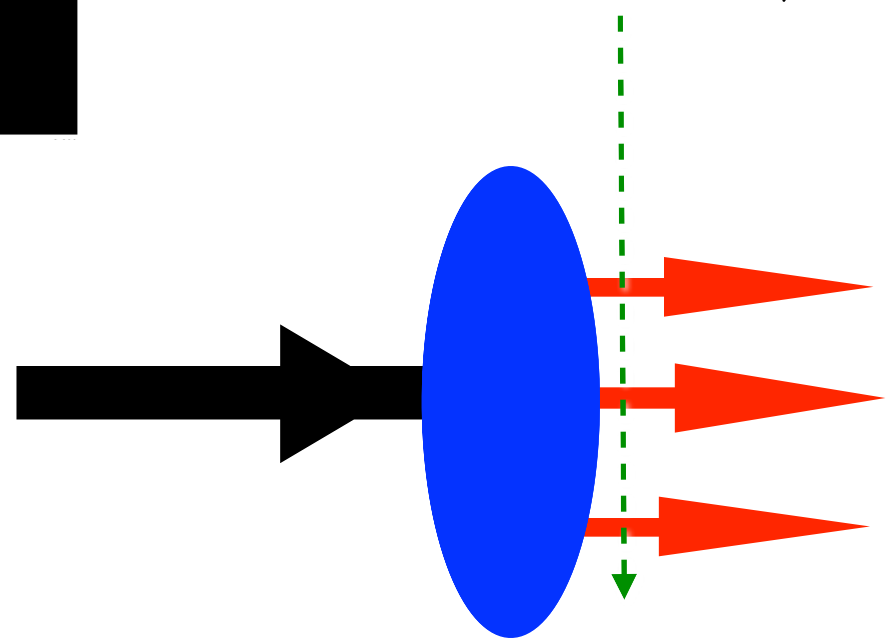
$\phi(z)$



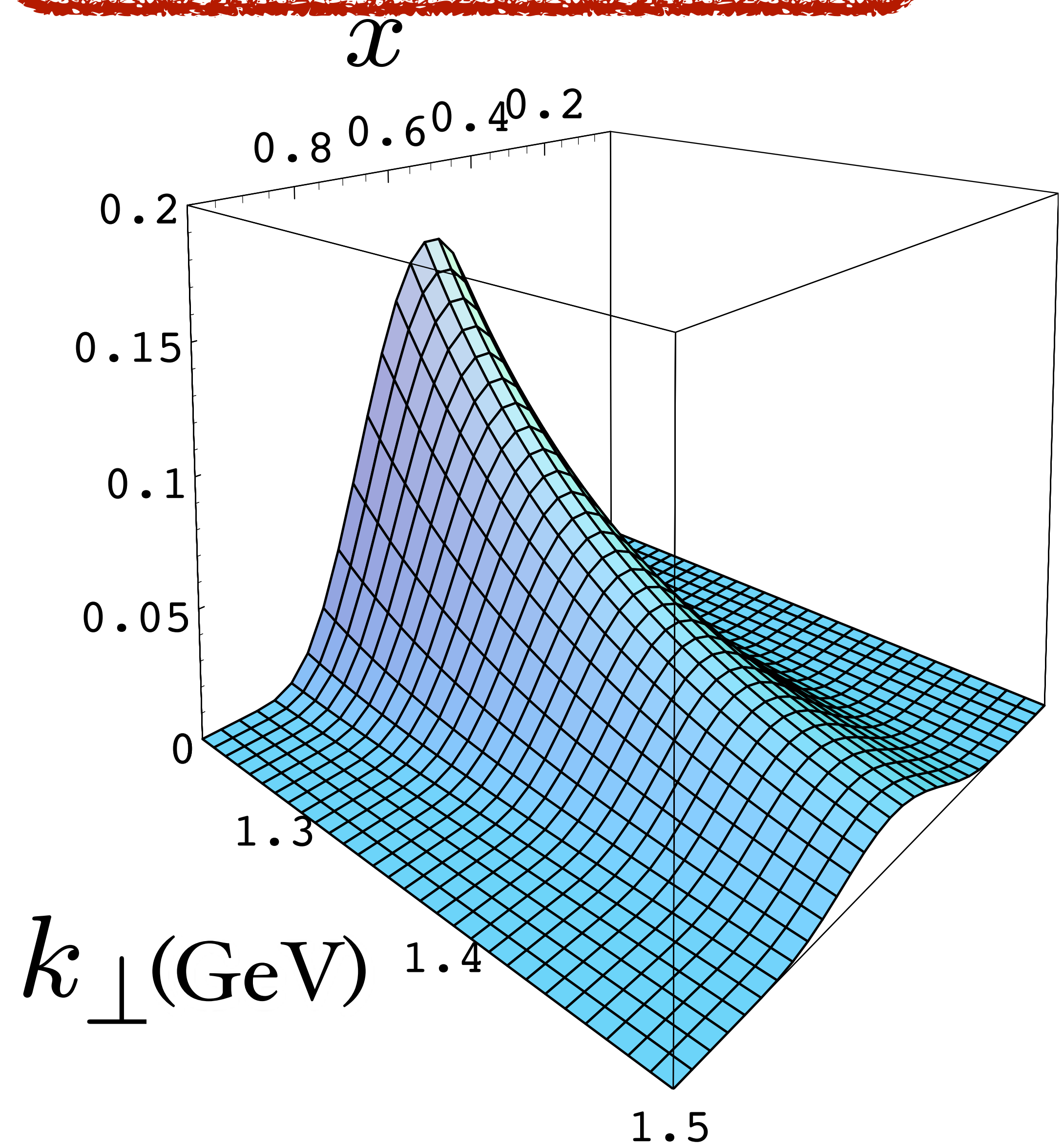
AdS5: Conformal Template for QCD

- *Light-Front Holography*

Duality of AdS₅ with LF Hamiltonian Theory



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



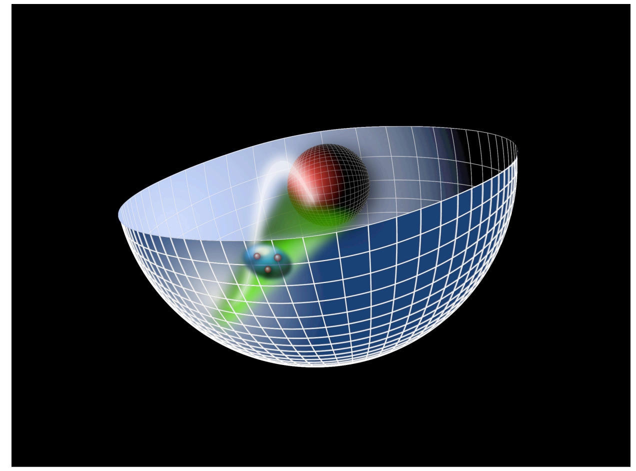
- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

Single scheme-independent fundamental mass scale κ



Light-Front Holography

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

(m_q=0)

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***