Theoretical Neuroscience II – SoSe 2017

Problem Set #3

The problem set is due on Friday, May 19, 23:59. Send to eppler@fias.uni-frankfurt.de. Please include [theoneuro2] in the subject.

Please name your file hw_yoursurname.pdf or hw_yoursurname.zip (if you are submitting several files).

Please, provide plots with sufficient annotation.

1. (4 points) Assume the distribution of firing rates $n$ can be approximated by Gaussian densities

$$p(n|s_0) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(n-n_0)^2}{2\sigma_n^2}\right)$$

and

$$p(n|s_1) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{(n-n_1)^2}{2\sigma_n^2}\right)$$

for stimuli $s_1$ and $s_0$ and SD $\sigma_n$. Compute the log-likelihood ratio defined by

$$l = \ln \frac{p(n|s_1)}{p(n|s_0)}.$$

Discuss the result in terms of a decision threshold in firing rate. Sketch a figure of the situation.

2. (6 points + 4 bonus points) Consider an array of $N$ neurons coding for a one-dimensional stimulus. The tuning curves are distributed uniformly across the full range of possible stimulus values $s$. They are described by Gaussians

$$f_a(s) = r_{\text{max}} \exp\left(-\frac{1}{2}\left(\frac{s-s_a}{\sigma_a}\right)^2\right).$$

with preferred stimulus $s_a = 0, \pm 1, \pm 2, \ldots$ equally spaced in stimulus space $s$. Their maximum value is identical and given by $r_{\text{max}}$. For simplicity, assume an identical tuning width $\sigma_a = \sigma$. For a given neuron the probability of firing $n_a = r_a T$ spikes in a fixed time interval $T$ is Poisson distributed

$$P[r_a|s] = \frac{(f_a(s)T)^{r_a} \exp(-f_a(s)T)}{(r_aT)!}$$

and independent for the different neurons.

(a) (2 points) Simulate responses to one stimulus at $s = 0$ for a single trial in a time interval $T$ (Use a random number generator to produce the Poisson distributed firing spikes). Plot the responses as a function of $s_a$ for the neurons at $s_a = -6, -5, \ldots, 6$ for $\sigma = 1$ and $r_{\text{max}} = 5$. Do this 100 times and plot the mean and the standard deviation of responses using error bars. Repeat for $r_{\text{max}} = 1$.

(b) (2 points) Write down the probability $P[r|s]$ for observing the response vector $r$. Now suppose you are given a response $r$ and you would like to estimate the stimulus that most likely has caused it. Derive the maximum likelihood (ML) estimate for the stimulus

$$s_{\text{ML}} = \frac{\sum_a r_a s_a}{\sum_a r_a}.$$
Here you make use of the approximation that $\sum_{a} f_{a}(s)$ is independent of $s$. How good is this approximation? Plot or compute it.

(c) (1 point) Next, to reveal how accurate your estimate of the stimulus is you are plotting the histogram of your ML estimates for the experiment above using $r_{\text{max}} = 5$. This is narrower than the distribution of the responses themselves. Explain.

(d) (1 point) You start suspecting that your estimate of the stimulus could depend on the width $\sigma$ of the tuning curves. You try $\sigma = 0.5$ and $\sigma = 3.0$. What do you see?

(e) (2 bonus points) The Fisher information is defined by

$$I_{F}(s) = \left\langle -\frac{d^{2}}{ds^{2}} \ln P[r|s]\right\rangle$$

Show that for the array of tuning curves the Fisher information can be expressed as

$$I_{F}(s) = T \sum_{a=1}^{N} \left( \frac{f'_{a}(s)}{f_{a}(s)} \right)^{2}.$$  

and interpret this result.

(f) (2 bonus points) Now suppose that the response tuning curves densely cover the range of stimulus values with a density $\rho_{s}$ which is equal to the number of neurons with preferred stimulus values lying within a unit range of $s$ values. For these tuning curves derive the expression

$$I_{F}(s) \approx \frac{\sqrt{2\pi} \rho_{s} \sigma r_{\text{max}} T}{\sigma_{r}^{2}}.$$  

Compare this to the results of your simulation above and discuss.