Theoretical Neuroscience II – SoSe 2017

Problem Set #8

The problem set is due on Friday, July 7, 23:59. Send to eppler@fias.uni-frankfurt.de
Please include [theoneuro2] in the subject.

Please name your file hw_yoursurname.pdf or hw_yoursurname.zip
(if you are submitting several files).
Please, provide plots with sufficient annotation.

Figure 1: Orientation domains and ocular dominance. (A) Contour map showing iso-orientation contours (gray lines) and the boundaries of acular dominance stripes (black lines) in a 1.7 x 1.7 mm patch of macaque primary visual cortex. Iso-orientation contours are drawn at intervals of 11.25. Pinwheels are singularities in the orientation map where all the orientations meet, and linear zones are extended patches over which the iso-orientation contours are parallel. (B) Ocular dominance and orientation map produced by the elastic net model. The significance of the lines is the same as in (A), except that the darker gray lines show orientation preferences of 0. (A adapted from Obermayer & Blasdel, 1993; B from Erwin et al., 1995.)

Classical experimental work suggests that in some species orientation and ocular dominance map are statistically related to one another, showing an increased tendency of iso-orientation and iso-ocular dominance contours to cross at right angles. Being intrigued by this result you decide to explore this in a suitable surrogate data set, simple enough to be treated easily, yet sufficiently complex to capture the essence of the problem. To this end you decide to compute the histogram of intersection angles in a simple model: stripes of ocular dominance randomly superimposed on stripes of orientation preference (no pinwheels), with both stripes exhibiting the same wavenumber \( k \), but different orientations randomly drawn from the uniform distribution over the interval \([0, 2\pi)\).

1. (5 Points) Compute/estimate (choose your method of choice) the histogram of intersection angles by averaging over all intersections found per unit area \(1/k^2\) in this ensemble of pairs of stripe patterns. How do you interpret this?

2. (3 bonus points) To render this model slightly more realistic, employ a simple strategy for generating more realistic patterns by superimposing 10 plane-waves with identical \( k \) but angles randomly drawn from the same uniform distribution.
Re-compute the histogram of intersection angles between such patterns and stripe pattern with wavenumber $k$.

3. (3 points) The dynamics of the elastic net model linearized around the homogeneous state reads:

$$\partial_t y(x) = \left( \eta \Delta + \frac{\langle s^2 \rangle}{\sigma^2} - 1 \right) y(x) - \frac{\langle s^2 \rangle}{4\pi\sigma^4} \int d^2 \xi \, e^{-\frac{(x-\xi)^2}{4\sigma^2}} y(\xi)$$  \hspace{1cm} (1)

with the Laplace operator $\Delta = \sum_a \frac{\partial^2}{\partial x_a^2}$. Show that its eigenvalues are given by

$$\lambda(k) = -1 + \frac{\langle s^2 \rangle}{\sigma^2} \left( 1 - e^{-k^2\sigma^2} \right) - \eta k^2$$  \hspace{1cm} (2)

Explain which term is responsible for suppressing high-frequency components.

4. (2 points + 3 bonus points) Show that the maximum eigenvalue is positive if

$$\sigma < \sigma^* = \sqrt{1 - \eta + \eta \ln \eta}$$  \hspace{1cm} (3)

Plot the phase diagram, which shows the region in the $\sigma$-$\eta$ plane, in which the homogeneous state is stable and the region, in which it is unstable with respect to a near periodic pattern. How does the periodicity of the emerging pattern depend on the two parameters? On what time scale does it evolve? What happens with the time scale when $\sigma$ is approaching $\sigma^*$?