Plasticity and Learning

- Hebbian plasticity
- Unsupervised Learning (Hebbian and beyond)
- Learning to see
A Taxonomoy of Learning

- L. of representations, models, behaviors, facts, …
  - Unsupervised L.
  - Reinforcement L.
  - Imitation L.
  - Instruction-based L.
  - Supervised L.

- In addition, there’s evolutionary “learning”
spike-timing dependent plasticity

- long-term depression
- long-term potentiation

synaptic scaling

intrinsic plasticity

structural plasticity

neurogenesis

short-term facilitation

- short-term depression

neuronal adaptation

neuromodulation

how do they interact?
how do they shape neural circuits?
how do they shape neural codes?
Hebb, Correlation and Causation

- Hebb (1949): “When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased.” (causation: A drives B)

- Hebb (1949): "The general idea is an old one, that any two cells or systems of cells that are repeatedly active at the same time will tend to become 'associated', so that activity in one facilitates activity in the other." (correlation: fire together, wire together)
Long-term Potentiation (LTP) & Long-term Depression (LTD)

- Potentiated level
- Depressed, partially depotentiated level
- Control level
Linear Firing-Rate Model

- if weights change slowly, we can assume that firing rates have reached their steady state before weights can change much:

\[ \tau_r \frac{dv}{dt} = -v + w^T u = -v + \sum_{b=1}^{N_u} w_b u_b \]

- Synaptic modification is included by specifying how \( w \) changes as a function of pre- and postsynaptic activity.
Basic Hebb Rule

\[ \tau_w \frac{dw}{dt} = \nu u \]

- this has the spirit of „fire together - wire together“
- represents a form of correlation-based learning:

  if weights change very slowly, we can average over the effects of many stimuli that are shown:

\[ \tau_w \frac{dw}{dt} = \langle \nu u \rangle \]
• In unsupervised learning setting, we can assume:  \( v = w^T u \)

• From  \( \tau_w \frac{dw}{dt} = \langle vu \rangle \)

follows:  \( \tau_w \frac{dw}{dt} = Qw \)  or:  \( \tau_w \frac{dw_b}{dt} = \sum_{b'=1}^{N_u} Q_{bb'} w_{b'} \)

• \( Q \) is the input correlation matrix given by:

• \( Q = \langle uu^T \rangle \)  or:  \( Q_{bb'} = \langle u_b u_{b'} \rangle \)

• Problem with this simplest rule: it’s unstable. Even when allowing for negative firing rates \( v \) the weights will grow without bound

• discrete version of learning rule:  \( w \rightarrow w + \epsilon Qw \)
Problems: Stability and Competition

- Hebbian Learning without appropriate adjustments of plasticity rules or introduction of constraints tends to produce uncontrolled growth of synaptic strengths.

- Possible solution: add constraint in plasticity rule that synapse cannot grow stronger than some maximum value.

- Also: for excitatory synapse add constraint that strength cannot become negative (i.e., synapse turns inhibitory)

- If all (or at least many) synapses grow all the way to the maximum value, the neuron is no longer selective to certain inputs. To avoid this situation, plasticity rules can introduce competition between synapses
8.3 Unsupervised Learning
Hebbian learning with single postsynaptic neuron

- simple Hebb rule for linear unit gives (see earlier):
  \[
  \tau_w \frac{dw}{dt} = Qw
  \]

- to solve this equation, we use matrix diagonalization, i.e., we express \( w \) in terms of the eigenvectors of \( Q \)
  \[
  Qe_\mu = \lambda_\mu e_\mu
  \]

- Since \( Q \) is symmetric eigenvectors are orthogonal
  \[
  e_\mu \cdot e_\nu = \delta_{\mu \nu}
  \]

- Since \( Q \) is positive semi-definite, all eigenvalues are real and non-negative; we put them in order such that:
  \[
  \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N_a}
  \]
now express $w(t)$ in terms of the Eigenvectors of $Q$:

$$w(t) = \sum_{\mu=1}^{N_u} c_\mu(t) e_\mu$$

this leads to the solution:

$$w(t) = \sum_{\mu=1}^{N_u} \exp \left( \frac{\lambda_\mu t}{\tau_w} \right) \left( w^T(0)e_\mu \right) e_\mu$$

The projection of $w(t)$ onto $e_1$ grows fastest. Thus, for large $t$, $w$ will be in the direction of $e_1$

For the response to an arbitrary input, we find:

$$v \propto u^T e_1$$
• Note: weights grow without bounds in this scheme, which is unrealistic

• What’s the effect of hard saturation, i.e., each weight can only grow to some maximum strength?

Example for negatively correlated inputs:

$$e_1 = (1, -1)/\sqrt{2}$$

If started outside of dashed lines, weight vector aligns with one of the axes. If started between dashed lines, weight vector aligns with diagonal.
Plain Hebbian learning short summary

Averaged Hebb rule governing dynamics of weight vector

\[ \tau_w \frac{d\mathbf{w}}{dt} = \langle \mathbf{v}\mathbf{u} \rangle \quad \mathbf{v} = \mathbf{w} \cdot \mathbf{u} \]

Correlation based learning

\[ \tau_w \frac{d\mathbf{w}}{dt} = \mathbf{Q} \cdot \mathbf{w} \quad \text{or} \quad \tau_w \frac{d\mathbf{w}_b}{dt} = \sum_{b'=1}^{N_u} Q_{bb'} \mathbf{w}_{b'} \]

with input correlation matrix \( \mathbf{Q} = \langle \mathbf{u}\mathbf{u} \rangle \) or \( Q_{bb'} = \langle u_b u_{b'} \rangle \)

Represent \( \mathbf{w} \) in basis of eigenvectors:

\[ \mathbf{w}(t) = \sum_{\mu=1}^{N_u} \exp \left( \frac{\lambda_{\mu} t}{\tau_w} \right) \left( \mathbf{w}(0) \cdot \mathbf{e}_{\mu} \right) \mathbf{e}_{\mu} \]

Thus, after long time \( \mathbf{w} \) is parallel to principle eigenvector of corrections matrix.