Multiplicity Fluctuations
Past - Present - Future

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VI-SIM workshop in Bad Liebenzell
1. Elementary Particle Collisions
   - Koba-Nielsen-Olesen Scaling
   - Wroblewski Relations

2. Heavy Ion Collisions
   - Centrality Dependence
   - Energy Dependence
   - Phase Space Dependence

3. Conclusion
Evidence for the Systematic Behavior of Charged-Prong Multiplicity Distributions in High-Energy Proton-Proton Collisions*

P. Slattery

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 2 October 1972)

Evidence is presented to support the onset in the 50–303-GeV/c region of incident momentum of the asymptotic prediction of Koba, Nielsen, and Olesen regarding the scaling behavior of the charged-prong multiplicity distribution in proton-proton collisions. Lower-energy data, at 19 and 28.5 GeV/c, are found not to demonstrate this behavior. The impact of this observation on the Mueller-Regge viewpoint is discussed.

One of the simplest and most direct measurements which can be made with a bubble chamber is the determination of charged-particle multiplicities. The present availability of bubble-chamber facilities at Serpukhov, USSR, and at Batavia, USA, has consequently made available for the first time accurate measurements of topological cross sections for very-high-energy proton-proton collisions (50–303 GeV/c). In this article we wish to examine the energy variation of these partial cross sections, and to compare the experimental data with two contrasting asymptotic predictions,
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One of the simplest and most direct measurements which can be made with a bubble chamber is the determination of charged-particle multiplicities. The present availability of bubble-chamber
A Bubble Chamber Event

Michigan - Rochester Collaboration
405 GeV Proton - Proton Interaction
in the 30-inch Hydrogen Bubble Chamber
at National Accelerator Laboratory

\[ P + P \rightarrow \pi^+ (19 \text{ charged particles}) \]

\[ \pi^- + \Lambda^0 \]

\[ P \rightarrow \pi^- \]

May, 1973

\[ X = -0.38 \]

\[ P_T = 0.58 \text{ GeV/c} \]
### Table I: Experimental values of $\langle n \rangle$ and of $\langle n \rangle$ ($\sigma_{n}/\sigma_{\text{inel}}$) for the reaction $pp \rightarrow n$ charged particles at incident momenta of 19, 50, 69, 102, 205, and 303 GeV/c.

<table>
<thead>
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<th>$p_{\text{lab}}$ (GeV/c)</th>
<th>19</th>
<th>50</th>
<th>69</th>
<th>102</th>
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<tr>
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</tbody>
</table>

*Not plotted in Fig. 1.

P. Slattery, Phys. Rev. D 7, 2073 (1973)
Multiplicity Distributions in elementary collisions

\[
\frac{\sigma_n}{\sigma_{\text{inel}}} \rightarrow \frac{1}{s} \psi\left(\frac{n}{\langle n \rangle}\right). \tag{1}
\]

Here, \( \sigma_n \) is the partial cross section for the reaction \( pp \rightarrow n \) charged particles, \( \sigma_{\text{inel}} \) is the total inelastic \( pp \) cross section (throughout this paper \( \sigma_2 \) does not include the elastic channel), \( \langle n \rangle \) is the average number of charged particles produced at a particular value of squared center-of-mass energy \( s \), and \( \psi \) is an energy-independent function.

In Fig. 1 we examine this prediction in the 50–303-GeV/c range of incident momenta by plotting \( \langle n \rangle (\sigma_n/\sigma_{\text{inel}}) \) versus \( n/\langle n \rangle \) for these data. The fact that a smooth curve may be drawn through all of the data points with a satisfactory \( \chi^2 \) is a dramatic indication that the semi-inclusive scaling concept is experimentally valid in this range of incident momentum. The particular parametrization of the function \( \psi(\langle n \rangle/n) \) which yields the curve shown in the figure is given by the formula

\[
\psi(z = n/\langle n \rangle) = (3.79z + 33.7z^3
- 6.64z^5 + 0.332z^7)e^{-3.04z}. \tag{2}
\]
The Concept of Similarity of Distributions

Fig. 1. Definition of the concept of similarity of continuous functions (KNO scaling). Normalized functions (a) are similar if upon a linear contraction of each of them along the horizontal direction in proportion to some of its horizontal dimensions—for example, \( \langle n \rangle \) (b)—and a linear extension along the vertical direction in the same proportion (c) they coincide at each point.

A. I. Golokhvastov, Phys. Atom. Nucl. 64 (2001) 84
Scaling of Dispersions

KNO scaling

\[ P(n) = \frac{\sigma_n}{\sigma_{\text{inel.}}} \to \frac{1}{\langle n \rangle} \psi \left( \frac{n}{\langle n \rangle} \right) \]

implies scaling of moments

\[ \langle n^q \rangle \to c_q \langle n \rangle^q, \quad q = 2, 3, 4, \ldots \]

which implies scaling of Dispersions

\[ D_q \equiv \left( \langle n^q \rangle - \langle n \rangle^q \right)^{1/q} = \langle n \rangle \times \text{const} \]

Scaled Variance

\[ \omega \equiv \frac{(D_2)^2}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} \]

Figure actually found in:
**KNO-G scaling**

\[ P(n) = \int_{\bar{n}}^{\bar{n}+1} P(\bar{n}) , \quad \text{where} \]

\[ P(\bar{n}) = \frac{1}{\langle \bar{n} \rangle} \psi(\frac{\bar{n}}{\langle \bar{n} \rangle}) \quad \text{and} \quad \langle \bar{n} \rangle \approx \langle n \rangle - 1 \]


**Wroblewski relation**

follows from KNO-G scaling

\[ D_2 = 0.576 (\langle n \rangle - 1) \]


**Log-Normal Scaling Function**

\[ \psi(z) = \frac{N}{\sqrt{2\pi}\sigma} \frac{1}{z+c} \exp \left[ -\frac{\ln(z+c)-\mu}{2\sigma^2} \right] \]

Enhanced fluctuations are one of the main proposed signals for a possible phase transition of QGP matter to hadronic matter or even a possible critical point.

Unlike for pp multiplicity distributions, data is only available for limited geometric acceptance.

Unlike in pp collision, the number of interacting nucleons fluctuates in A+A collisions.

Possibly there are a few ‘unexpected‘ things to learn?
Fixed Target experiments

In this context some important features are:

- mostly forward acceptance
- calorimeter to measure projectile spectators
- however one cannot measure target spectators
Centrality Dependence and Acceptance

- fixed number of projectile participants
- target participants cannot be measured
- acceptance $1.1 < y_{c.m.} < 2.6$

Apparently unexpected:

- Strong increase towards peripheral collisions seen in Pb+Pb data
- Not reproduced by models!

average number of target and projectile participants should be equal, \( \langle N_{part}^{targ} \rangle \approx \langle N_{part}^{proj} \rangle \)

BUT: only number of projectile participants is fixed experimentaly, and number of target participant fluctuates considerable (in transport simulations)

This ‘trivial‘ contribution can be minimized only for the sample of most central events.

Why is similar behaviour not seen in transport simulations with NA49 acceptance?
Transport Models are too ‘transparent’

Fluctuations in the target hemispher do not move across to the projectile hemispher.

There seems to be a significant amount of ‘mixing’ of target and projectile matter in data.

In this context some important features are:

- mostly acceptance around mid-rapidity
- they have calorimeters to measure spectator nucleons of both colliding ions
- however, since one cannot measure beam fragments, the measurement of $N_{\text{part}}$ is rather unprecise
Centrality definition

Beam-Beam-Counters (BBC) measure charged particle multiplicity in the pseudo-rapidity range $3.0 < |\eta| < 3.9$.

Participant fluctuations

- cannot be neglected!
- but are also not necessarily the same in data and HSD simulations!

PHENIX Multiplicity Fluctuation Data

Independent source model

\[ \omega = \omega^{NN} + n \omega_P \]

- \( \omega^{NN} \) mult. fluc. of N+N collisions
- \( n \) average multiplicity from one source
- \( \omega_P \) fluctuation of number of sources

‘acceptance scaling’ with \( q \)

\[ \omega^{acc} = 1 - q + q \omega^{NN} + q n \omega_P \]

Centrality dependence

is different from that at SPS!

Possibly only due to different methods for centrality determination ??
A Short Summary on Centrality

- $N_{\text{part}}$ fluctuations are a dominant source of multiplicity fluctuations!

- Even at fixed number of projectile participants - the number of target participants can still vary considerably.

- For the study of multiplicity fluctuations only the sample of most central collisions (about 1%) should be used.

- For purely technical reasons centrality selection is done in different ways in fixed target and collider experiments.
Resonance Gas Multiplicity Fluctuations


GCE : no conservation laws enforced
CE : only charge (B,S,Q) conservation
MCE : energy and charge fixed

Fluctuations are different in different ensembles


Comparison of Resonance Gas to NA49 Data

Agreement with Data is surprising!

Especially since we made some strong assumptions/approximations
do we see the effect of conservation laws on fluctuations?

Experimental Acceptance

- changes from 4% at 20AGeV to about 16% at 158AGeV
- has been taken into account via an ‘uncorrelated particle’ approximation

Other Choices for Parameters

For reviews see also:

- lead to VERY similar results
- with the notable exception of $\gamma_q$ models, however due to energy conservation still $\omega < 1$

In both transport models the scaled variance is similar in A+A and p+p collisions.

In particular $\omega \propto \langle N \rangle$ (Wroblewski relation)

$\omega$ increases monotonically with $\sqrt{s_{NN}}$

In the SPS energy range both relativistic microscopic transport models and MCE HRG are below $\omega < 1$.

However for RHIC energies they differ by a factor of 10.

Fluctuations in transport models do not ‘thermalize’?!
Both HSD and MCE HRG are in good agreement with NA49 multiplicity fluctuation data for (1%) most central Pb+Pb collisions.

Larger experimental acceptance should allow to distinguish equilibrium and non-equilibrium models.

A Short Summary on Energy Dependence

- Transport models show similar behavior of $\omega$ in A+A and p+p collisions.

- In comparison to the above the thermal model shows a rather flat dependence of the scaled variance on collision energy.

- Both transport models and MCE formulation of HRG are in good agreement with NA49 data.

- Present NA49 data does not allow for a conclusive distinction between models.
Boltzmann pion gas at $T = 160\text{MeV}$ and zero charge density.

- Each bin contains same fraction of total yield
- Bars indicate size of the bin

Energy and momentum conservation lead to suppressed multiplicity fluctuations at high $|y|$ and $p_T$. 
Momentum Cuts in UrQMD

Construction of bins is the same as before.

MCE suppression of fluctuations also in non-equilibrium systems?

Momentum Cuts in NA49 Data

UrQMD vs. NA49 158AGeV Pb+Pb data

Rapidity and transverse momentum dependence also seen in data!

MCE effects are of similar magnitude as proposed enhancement due to a phase transition / critical point!

B. Lungwitz, talk given at Workshop on Critical Point and Onset Deconfinement and private communication
Multiplicity Distribution of negatively charged particles for most central (1%) Pb+Pb collision at 158 AGeV, both Data and UrQMD simulation, acceptance $1 < y_\pi < y_{beam}$.

B. Lungwitz and M. Bleicher, private communication

UrQMD overpredicts yields here by 33% but $\omega$ agrees within 1%

Both UrQMD and data well fitted by Gaussians!
### Comparison of HRG to NA49 Distribution Data

<table>
<thead>
<tr>
<th>Energy (AGeV)</th>
<th>Data</th>
</tr>
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<tbody>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
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<tr>
<td>80</td>
<td></td>
</tr>
<tr>
<td>158</td>
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</table>


A lot of good data is now available and has been studied.

More effort is needed in order to understand trivial and not so trivial effects in data.

Data in larger acceptance, for different ion sizes, and at different energies would be very helpful.

A systematic study of momentum space dependence should be carried out.

A more detailed description of phase transitions and their effect on fluctuations is needed.

Fluctuation data carries quite a lot of information about dynamics!
Conclusion

For supplying plots and references

Thanks to

- Benjamin
- Marcus
- Marek
- Giorgio, and
- Volodya
Many More Open Questions

Fluctuations and Interaction

Van der Waals Gas

- Model repulsive interactions between hadrons

Suppression of densities can be removed by rescaling the system volume

- Suppression of fluctuations is qualitatively different

- Could be a first step towards a simple model with a phase transition


Maybe only of academical interest, but would that hold true in transport theory?


Many More Open Questions

Fluctuations and the QGP

Does the signal survive hadronization?

The qMD model

- treats quarks and anti-quarks as classical point-like objects
- interaction via long-range color potential


Baryon-strangeness correlations

Different degrees of freedom in QGP and HRG

\[ C_{BS} = \frac{\langle B \cdot S \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} \]

- QGP: \( C_{BS} \approx 1 \)
- HRG: \( C_{BS} \approx 0.66 \)


The KNO scaling function

Koba-Nielsen-Olesen Scaling Function

$$\Psi(z) = \frac{N}{\sqrt{2\pi}\sigma} \frac{1}{z+c} \exp \left[ -\frac{\ln(z+c)-\mu}{2\sigma^2} \right]$$


And as soon as it was found
Already scaling violation!