Dynamical equilibration of strongly-interacting ‘infinite’ parton matter

Vitalii Ozvenchuk,
in collaboration with
E. Bratkovskaya, O. Linnyk, M. Gorenstein, W. Cassing

CPOD, Wuhan, China
11 November 2011
Motivation

- ‘big-bang’ of the universe
  - steps with kinetic and chemical equilibrium

- laboratory ‘tiny bangs’
  - phase space configurations
  - far from an equilibrium
  - fast expansion

- local thermodynamic equilibrium
Crucial question

How and on what timescales a global thermodynamic equilibrium can be achieved in heavy-ion collisions?
From hadrons to partons

In order to study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma – we need a consistent non-equilibrium (transport) model with

- explicit parton-parton interactions (i.e. between quarks and gluons) beyond strings!
- explicit phase transition from hadronic to partonic degrees of freedom
- IQCD EoS for partonic phase

Transport theory: off-shell Kadanoff-Baym equations for the Green-functions $S^<(x,p)$ in phase-space representation for the partonic and hadronic phase

QGP phase described by Dynamical QuasiParticle Model (DQPM)

Parton-Hadron-String-Dynamics (PHSD)


The Dynamical QuasiParticle Model (DQPM)

**Basic idea:** Interacting quasiparticles
- massive quarks and gluons \((g, q, q_{\bar{q}})\) with **spectral functions**:

- fit to lattice (lQCD) results (e.g., entropy density)

**Quasiparticle properties:**
large width and mass for gluons and quarks

\[
\rho(\omega) = \frac{\gamma}{E} \frac{1}{(\omega - E)^2 + \gamma^2} - \frac{1}{(\omega + E)^2 + \gamma^2}
\]

- **DQPM** matches well lattice QCD
- **DQPM** provides **mean-fields** (1PI) for gluons and quarks
  as well as effective 2-body interactions (2PI)
- **DQPM** gives transition rates for the formation of hadrons \(\rightarrow\) **PHSD**

![Graph showing quasiparticle properties](image)

DQPM thermodynamics ($N_f=3$)

entropy: $s = \frac{\partial P}{dT}$ → pressure $P$

energy density: $\epsilon = Ts - P$

interaction measure: $W(T) := \epsilon(T) - 3P(T) = Ts - 4P$

**IQCD:** Wuppertal-Budapest group  

DQPM gives a good description of IQCD results!
PHSD: Transverse mass spectra

Central Pb + Pb at SPS energies

- PHSD gives harder $m_T$ spectra and works better than HSD at high energies
  - RHIC, SPS (and top FAIR, NICA)
- however, at low SPS (and low FAIR, NICA) energies the effect of the partonic phase decreases due to the decrease of the partonic fraction

E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk, NPA856 (2011) 162
Partonic phase at SPS/FAIR/NICA energies

Partonic energy fraction vs centrality and energy

Dramatic decrease of partonic phase with decreasing energy and/or centrality!

Cassing & Bratkovskaya, NPA 831 (2009) 215
The scaling of $v_2$ with the number of constituent quarks $n_q$ is roughly in line with the data.

E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk, NPA856 (2011) 162
Goal

- study of the dynamical equilibration of strongly interacting parton matter within the PHSD
PHSD in a box

Goal
- study of the dynamical equilibration of strongly interacting parton matter within the PHSD

Realization
- a cubic box with periodic boundary conditions
- various values for chemical potential and energy density
- the size of the box is fixed to $9^3 \, \text{fm}^3$
Initialization

- light and strange quarks, antiquarks and gluons with random space positions
- initial number of partons is given

\[ N_{q(g)} = d_{q(g)} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} 2\omega \rho_{q(g)}(\omega, p) n_{F(B)} \]

- ratios between the different quark flavors are

\[ N_u \div N_d \div N_s = 3 \div 3 \div 1 \]

- Four-momenta are distributed according to the \( F_1(\omega, p) \) distribution by Monte Carlo simulations

\[ F_1(\omega, p) = \frac{d_{q(g)}}{4\pi^3} p^2 \omega \rho_{q(g)}(\omega, p) n_{F(B)} \]
A sign of chemical equilibrium is the stabilization of the numbers of partons of the different species in time.

The final abundancies vary with energy density.
Detailed balance

- The reactions rates are practically constant and obey detailed balance for
  - gluon splitting
  - quark + antiquark fusion

- The elastic collisions lead to the eventual thermalization of all particle species (e.g. u, d, s quarks and antiquarks and gluons)

- The numbers of partons dynamically reach the equilibrium values through the inelastic collisions
  - $q + q \rightarrow q + q$
  - $q + \bar{q} \rightarrow q + \bar{q}$
  - $\bar{q} + \bar{q} \rightarrow \bar{q} + \bar{q}$
  - $q + g \rightarrow q + g$
  - $\bar{q} + g \rightarrow \bar{q} + g$
  - $q + \bar{q} \rightarrow g$
  - $g \rightarrow q + \bar{q}$
Thermal equilibration

- Comparison between PHSD simulation in the box and the DQPM model

- DQPM predictions can be evaluated:

\[
\frac{d^2 N}{d\omega dp} = \frac{V d_u}{2\pi^3} p_{mid}^2 \rho_u(\omega, p_{mid}) e^{-\omega/T}
\]

- Dynamical calculations are in a good agreement with the DQPM model

- The system is in a dynamical equilibrium
PHSD provides a consistent description of off-shell parton dynamics in line with a lattice QCD equation of state and incorporates dynamical hadronization in line with conservation laws.

PHSD gives harder $m_T$ spectra and works better than HSD at RHIC and at high SPS energies.

The quark-number scaling of $v_2$ holds fairly well in PHSD at RHIC.

PHSD within a box allows to study the dynamical equilibration of strongly interacting parton matter.
Back up
The Dynamical QuasiParticle Model (DQPM)

**Basic idea:** Interacting quasiparticles
- massive quarks and gluons \((g, q, q_{\text{bar}})\) with spectral functions:

\[
\rho(\omega) = \frac{\gamma}{E} \left( \frac{1}{(\omega - E)^2 + \gamma^2} - \frac{1}{(\omega + E)^2 + \gamma^2} \right)
\]

- \(\omega^2 = p^2 + M^2 - \gamma^2\)

**Quarks**
- mass:
  \[
m^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left( T^2 + \frac{\mu_q^2}{\pi^2} \right)
\]
- width:
  \[
\gamma_q(T) = \frac{N_c^2 - 1}{2N_c} \frac{g^2 T}{4\pi} \ln \frac{c}{g^2}
\]

**Gluons**
- mass:
  \[
M^2(T) = \frac{g^2}{6} \left( (N_c + \frac{1}{2}N_f) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)
\]
- width:
  \[
\gamma_g(T) = N_c \frac{g^2 T}{4\pi} \ln \frac{c}{g^2}
\]

- \(N_c = 3, N_f = 3\)

- running coupling:
  \[
\alpha_s(T) = \frac{g^2(T)}{4\pi}
\]

\[g^2(T/T_c) = \frac{48\pi^2}{(11N_c - 2N_f) \ln(\lambda^2(T/T_c - T_s/T_c)^2)}
\]

- 3 parameters:
  - \(T_s/T_c = 0.46\)
  - \(c = 28.8\)
  - \(\lambda = 2.42\)

- fit to lattice (IQCD) results (e.g. entropy density)

- quasiparticle properties

**References:**
- DQPM: Peshier, Cassing, PRL 94 (2005) 172301;
- IQCD: O. Kaczmarek et, PRD 72 (2005) 059903
PHSD: Hadronization details

Local covariant off-shell transition rate for q+qbar fusion

\[ \frac{dN_m(x, p)}{d^4x d^4p} = \sum_j \int d^4x_j d^4p_j / (2\pi)^4 \]

\[ \times N_j(x, p) \rho_q(p_q) \rho_{\bar{q}}(p_{\bar{q}}) |v_{q\bar{q}}|^2 W_m(x_q - x_{\bar{q}}, p_q - p_{\bar{q}}) \]

\[ \times N_q(x_q, p_q) N_{\bar{q}}(x_{\bar{q}}, p_{\bar{q}}) \delta(\text{flavor, color}). \]

- \( N_j(x, p) \) is the phase-space density of parton \( j \) at space-time position \( x \) and 4-momentum \( p \)
- \( W_m \) is the phase-space distribution of the formed, pre-hadrons: (Gaussian in phase space, \( \sqrt{<r^2>} = 0.66 \) fm
- \( v_{q\bar{q}} \) is the effective quark-antiquark interaction from the DQPM

Cassing, Bratkovskaya, PRC 78 (2008) 034919; Cassing, EPJ ST 168 (2009)
PHSD: hadronization of a partonic fireball

E.g. time evolution of the partonic fireball at initial temperature $1.7 \, T_c$ at $\mu_q=0$

Consequences:

- **Hadronization**: $q+q_{\bar{q}}$ or $3q$ or $3q_{\bar{q}}$ fuse to color neutral hadrons (or strings) which subsequently decay into hadrons in a microcanonical fashion, i.e. obeying all conservation laws (i.e. 4-momentum conservation, flavor current conservation) in each event!

- **Hadronization yields** an increase in total entropy $S$ (i.e. more hadrons in the final state than initial partons) and not a decrease as in the simple recombination models!

- **Off-shell parton transport** roughly leads a hydrodynamic evolution of the partonic system

---

The mass splitting at low $p_T$ is approximately reproduced as well as the meson-baryon splitting for $p_T > 2$ GeV/c!

The scaling of $v_2$ with the number of constituent quarks $n_q$ is roughly in line with the data.

E. Bratkovskaya, W. Cassing, V. Konchakovski, O. Linnyk, NPA856 (2011) 162
Elliptic flow versus centrality in PHSD

Enhancement of $v_2$ due to the partonic interactions $v_2$ from PHSD is larger relative to HSD (in line with the data from PHOBOS)

Anisotropic flows $v_2$, $v_3$, $v_4$ vs. centrality

$v_3$, $v_4$ are only weakly sensitive to centrality (the impact parameter $b$).

$v_2$ increases strongly with $b$ up to peripheral collisions.
Initialization

- light and strange quarks, antiquarks and gluons with random space positions
- initial number of partons is given

\[
N_{q(g)} = d_{q(g)} \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} 2\omega \rho_{q(g)}(\omega, p)n_{F(B)}
\]

- ratios between different quark flavors are

\[
N_u \div N_d \div N_s = 3 \div 3 \div 1
\]

initial energy distribution

initial invariant momentum distribution