Eigenvalue repulsion in an effective theory of SU(2) Wilson lines in three dimensions

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Quark Matter Studies

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- Lattice Results
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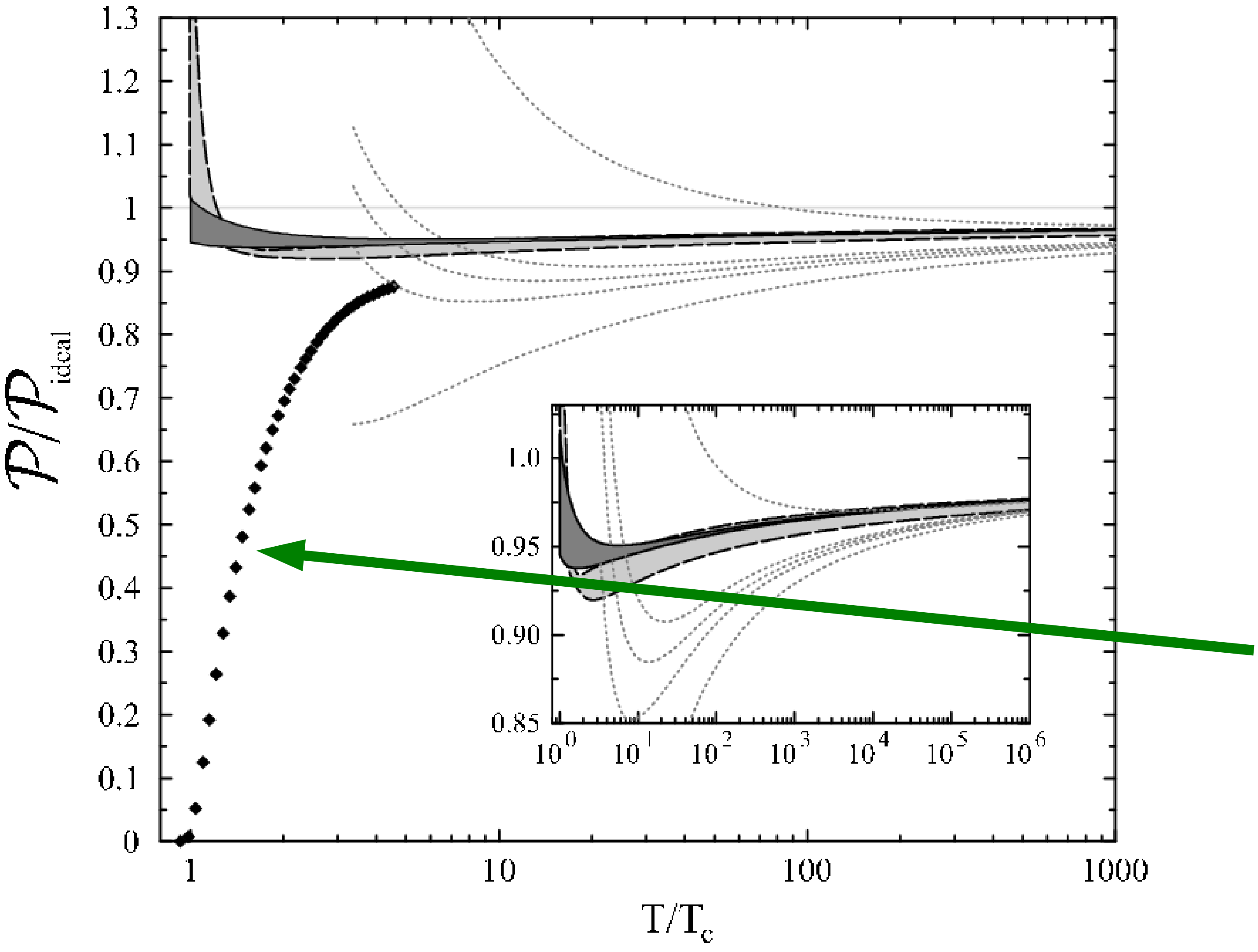
Introduction

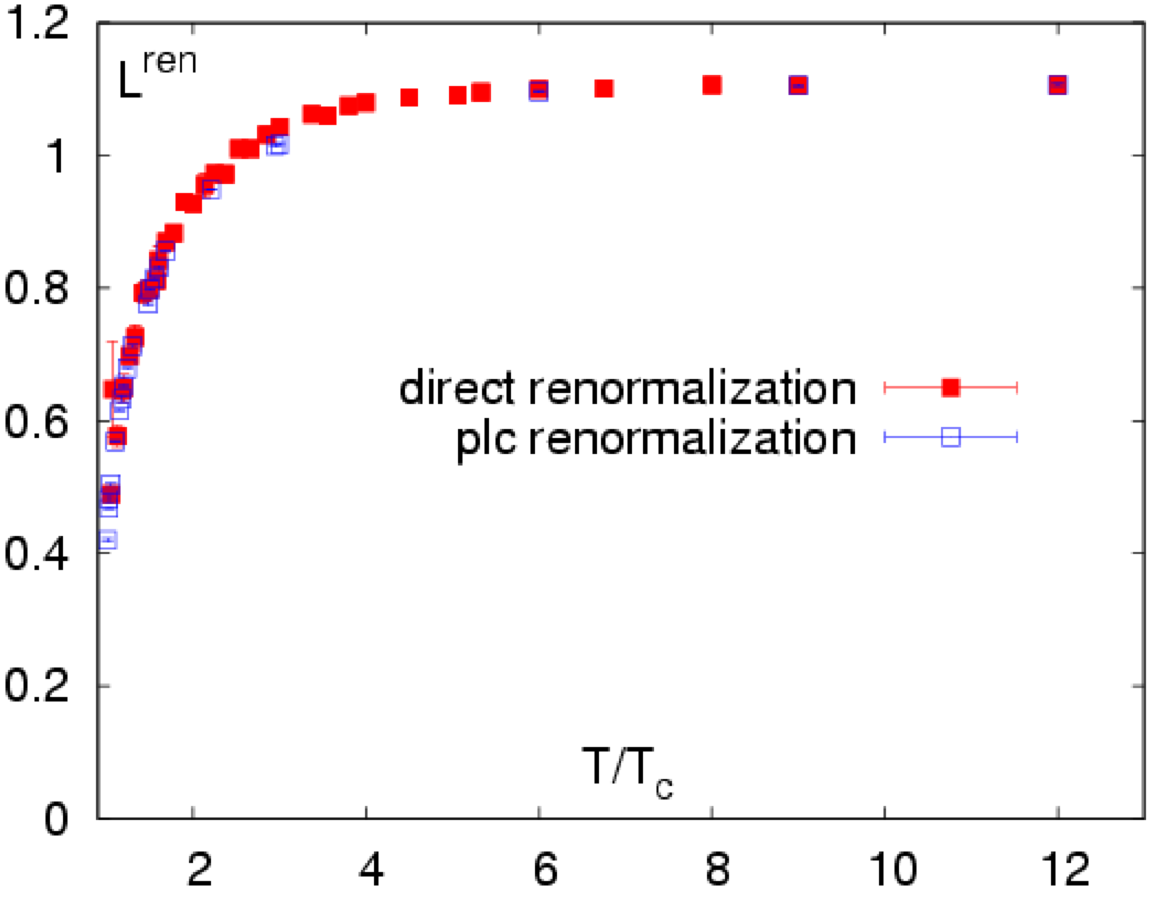


Anderson,
Strickland,
hep-ph/0404164

4d SU(3) Lattice data

- Resummed Perturbation Theory fails below $\sim 4\, T_C$!
- WHY ?

- HTL-P.T. expands about trivial vacuum,
  $A_0/T=0 \quad \leftrightarrow \quad \mathbf{L \sim \exp(ig\, A_0/T) \sim 1}$



Kaczmarek,
Gupta,
Hübner,
hep-lat/0710.2277

Need different vacuum: “repulsion of eigenvalues” of **L** near T_c

(**L** move away from unit matrix in group space)

Does one see repulsion of eigenvalues of Wilson line in the deconfined phase (at non-asymptotic T) ?

- Potential in powers of fundamental Wilson Line to one loop order:

$$V^{eff}_{1-loop} = \frac{-2}{\pi^2} T^4 \sum_{n \geq 1} \frac{1}{n^4} |tr\, L^n|^2$$

Gross, Pisarski,
Yaffe,
Rev. Mod. Phys. 53
43 (1981)

- Minimized by perturbative vacuum: **< L > = 1**

→ Eigenvalue attraction, for any T

- Idea: add non-perturbative "fuzzy bag" contribution:

$$V^{eff}_{non-pert.} = B_f T^2 |tr\, L|^2$$

Pisarski,
hep-ph/0608242

- Lattice action: Kinetic nearest neighbor interaction and mass term (i labels sites, $<ij>$ labels links)

$$S = -\frac{1}{2}\beta \sum_{\langle ij \rangle} Tr(L_i^{+} L_j + h.c.) - m^2 \sum_i |Tr L_i|^2$$

- Used quaternionic representation and Metropolis updating

Monte Carlo Results

- First study the $m^2=0$ case: $S=-\frac{1}{2}\beta\sum_{\langle ij\rangle} Tr(L_i^+ L_j+h.c.)$

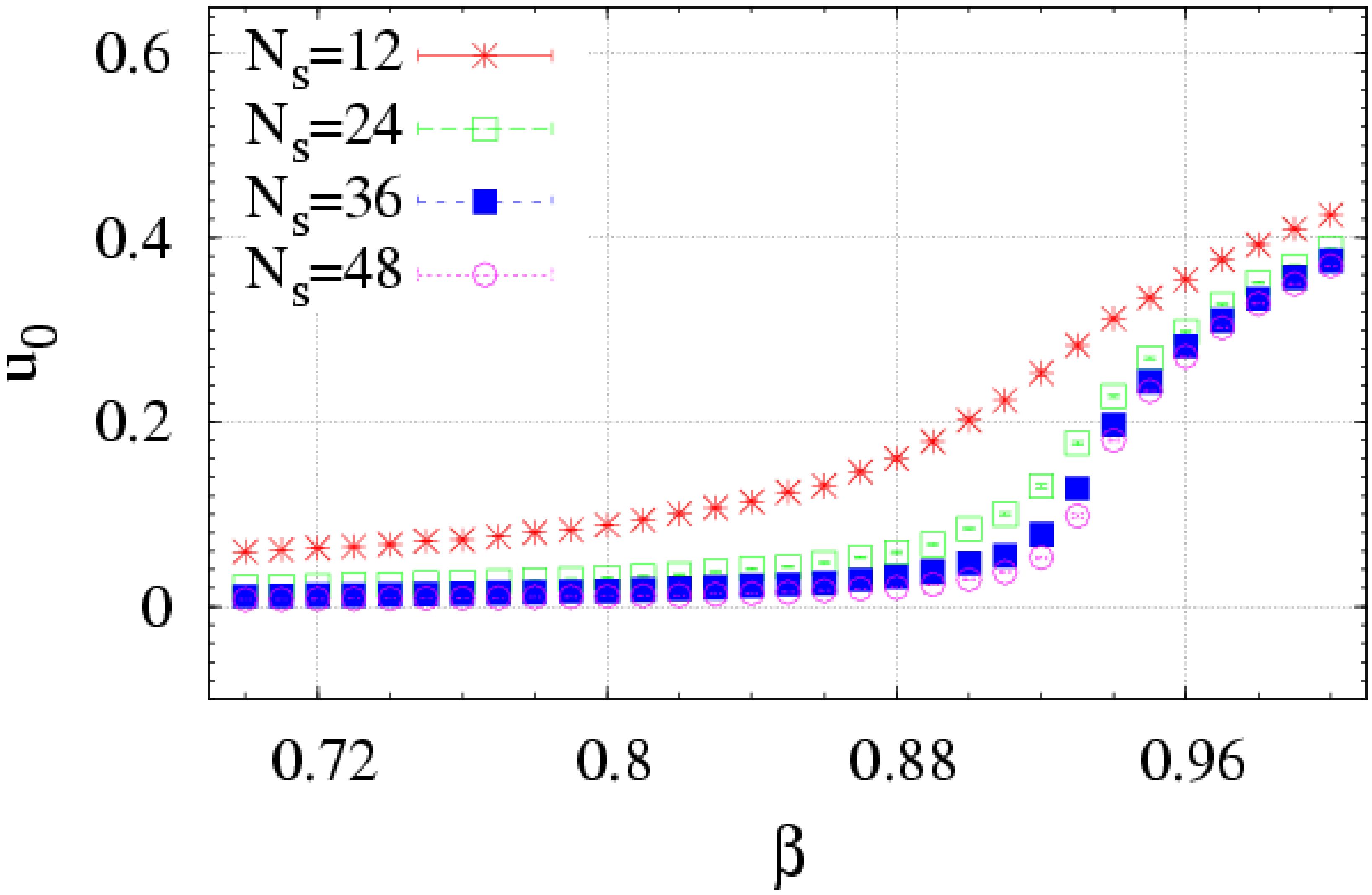
- Ensemble average of trace vanishes: $\langle\frac{1}{V}\sum_i Tr\, L_i\rangle=0$

- Note proportionality of volume averaged field to SU(2) matrix after each Monte Carlo step:

$$\overline{L}=u_0 L$$

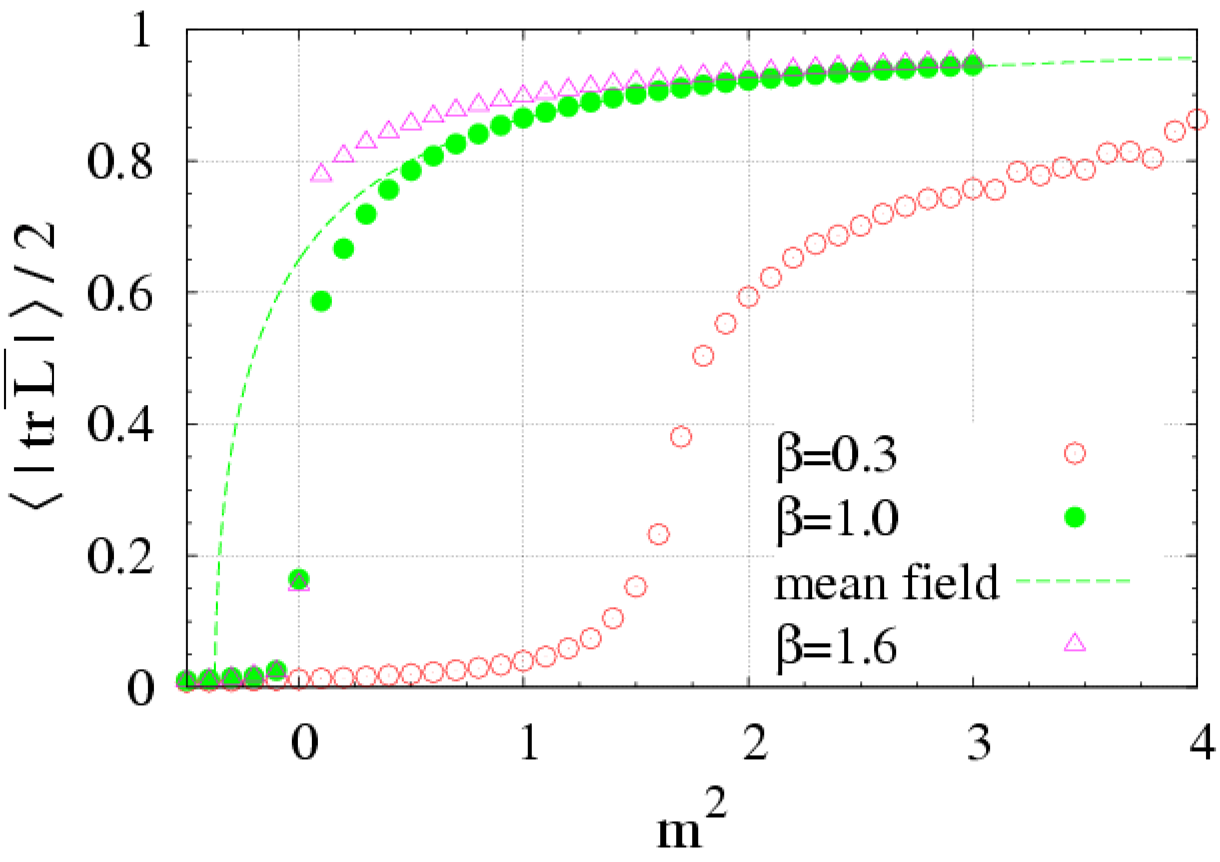
- Isolate the "length" u and use it as order parameter.

$$Tr(u_0^* L^+)(u_0 L)=|u_0|^2\, Tr\, L^+ L=N\ |u_0|^2$$



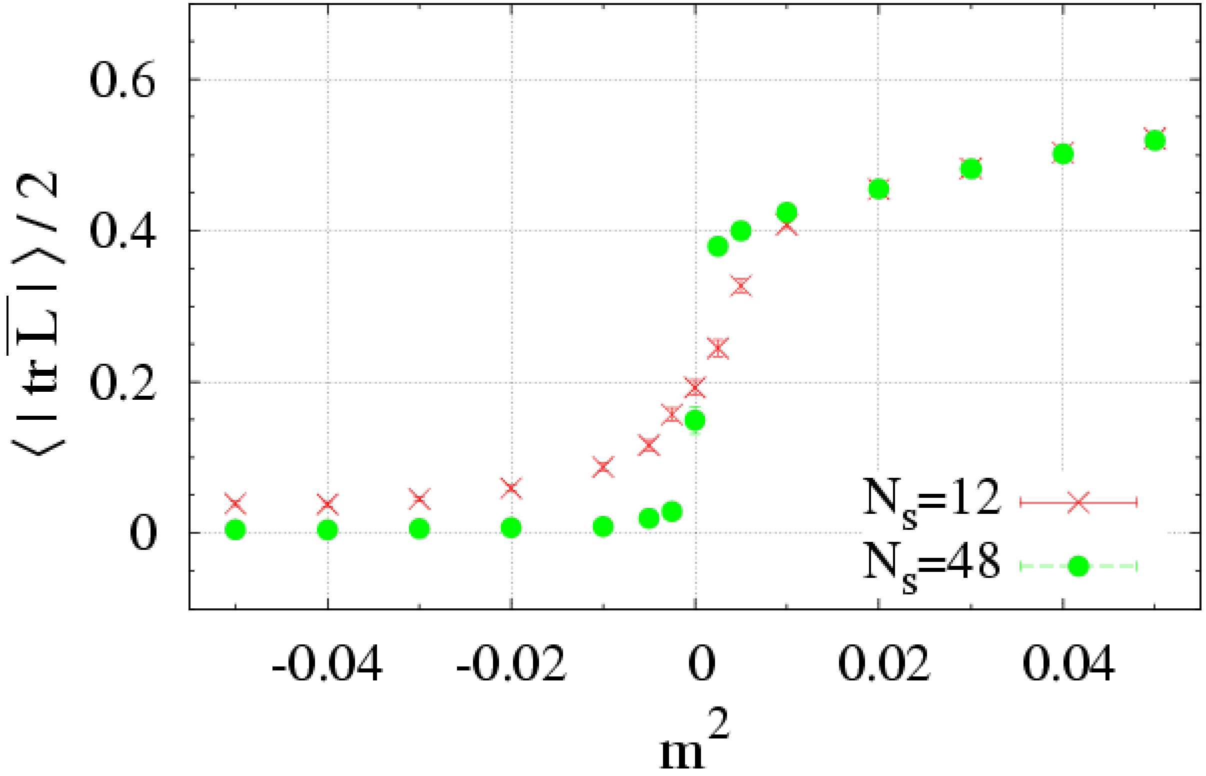
- Phase Transition in infinite volume at $\beta_C \approx 0.941$!

- Next study $m^2 \neq 0$ case: $S = -\frac{1}{2}\beta \sum_{\langle ij \rangle} Tr(L_i^+ L_j + h.c.) - m^2 \sum_i |Tr L_i|^2$
- Now $\langle | \frac{1}{V} \sum_i tr L_i | \rangle$ is good order parameter !



(12x12 lattice)

- Detailed investigation of the phase transition region for $\beta = 1.0$



- Transition is very sharp but definitely of second order !
- Phase transition occurs at $m^2 = 0.0$ independent of β !

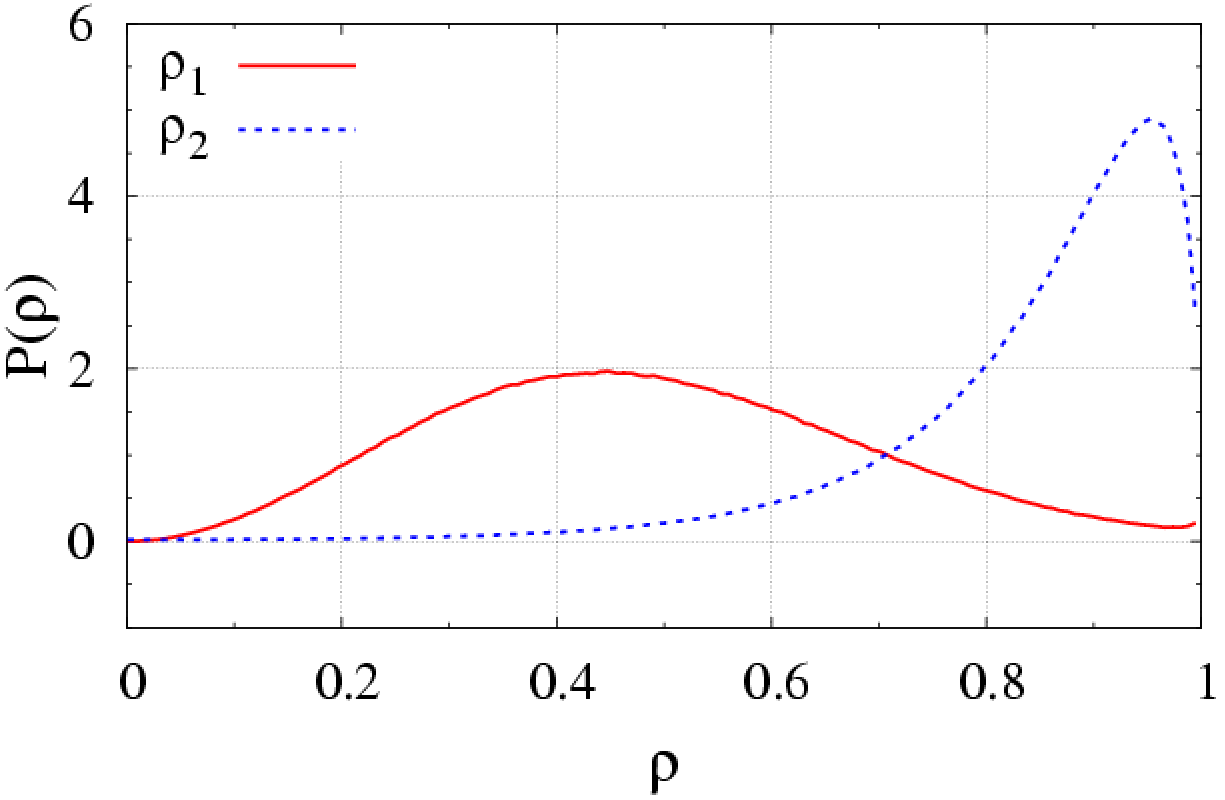
(Can be solved by adding background field)

- Look at eigenvalue distribution:

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$$\beta = 1.0 \qquad m^2 = 0.8$$



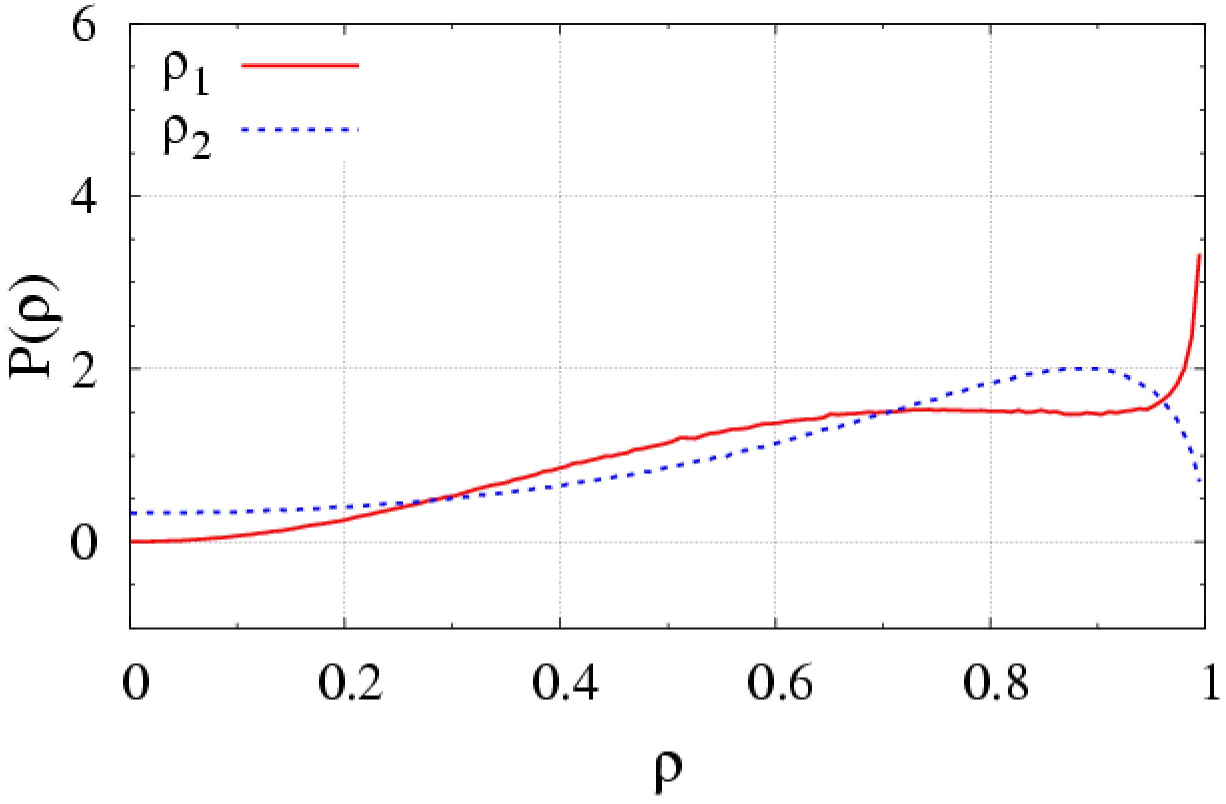
- Strongly peaked distribution for large value of m^2 !

  (Eigenvalue “attraction”)

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$$\beta = 1.0 \qquad m^2 = 0.15$$

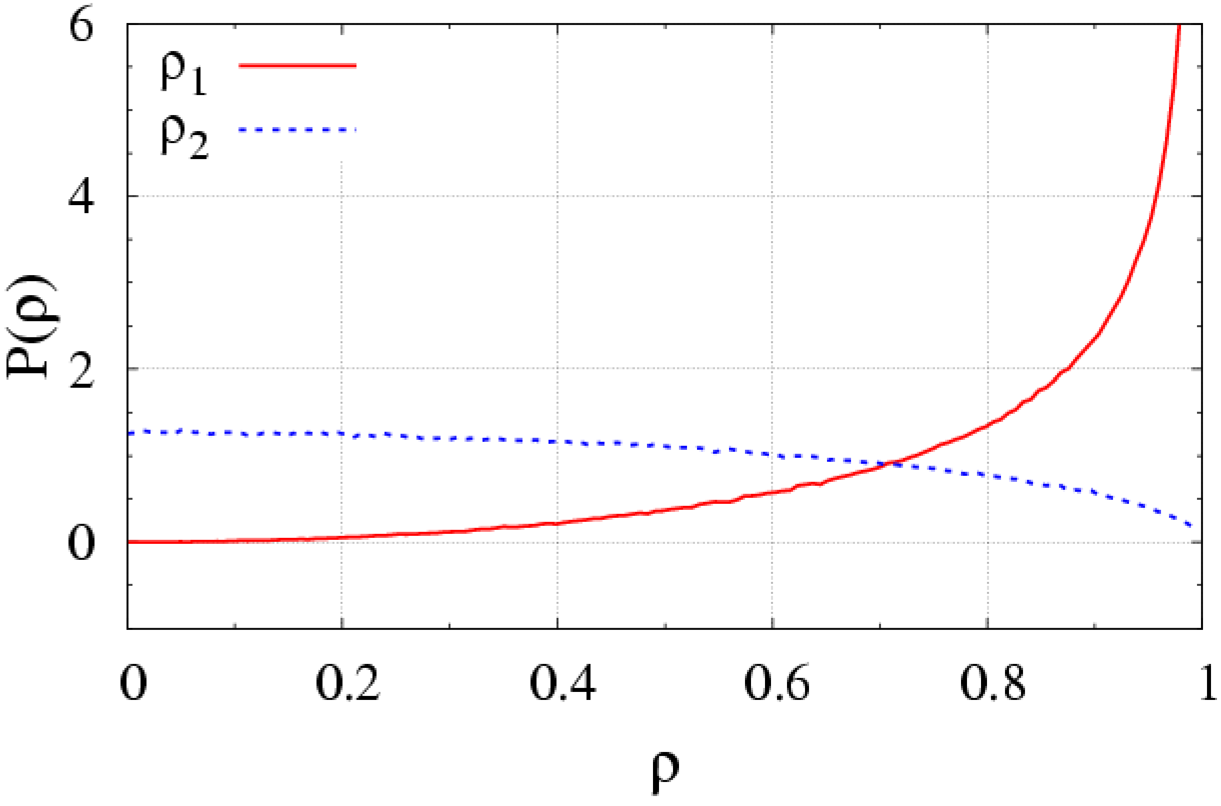


- Lowering m^2 evens out the distribution: “repulsion” of eigenvalues sets in !

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$$\beta = 1.0 \qquad m^2 = 0.0$$



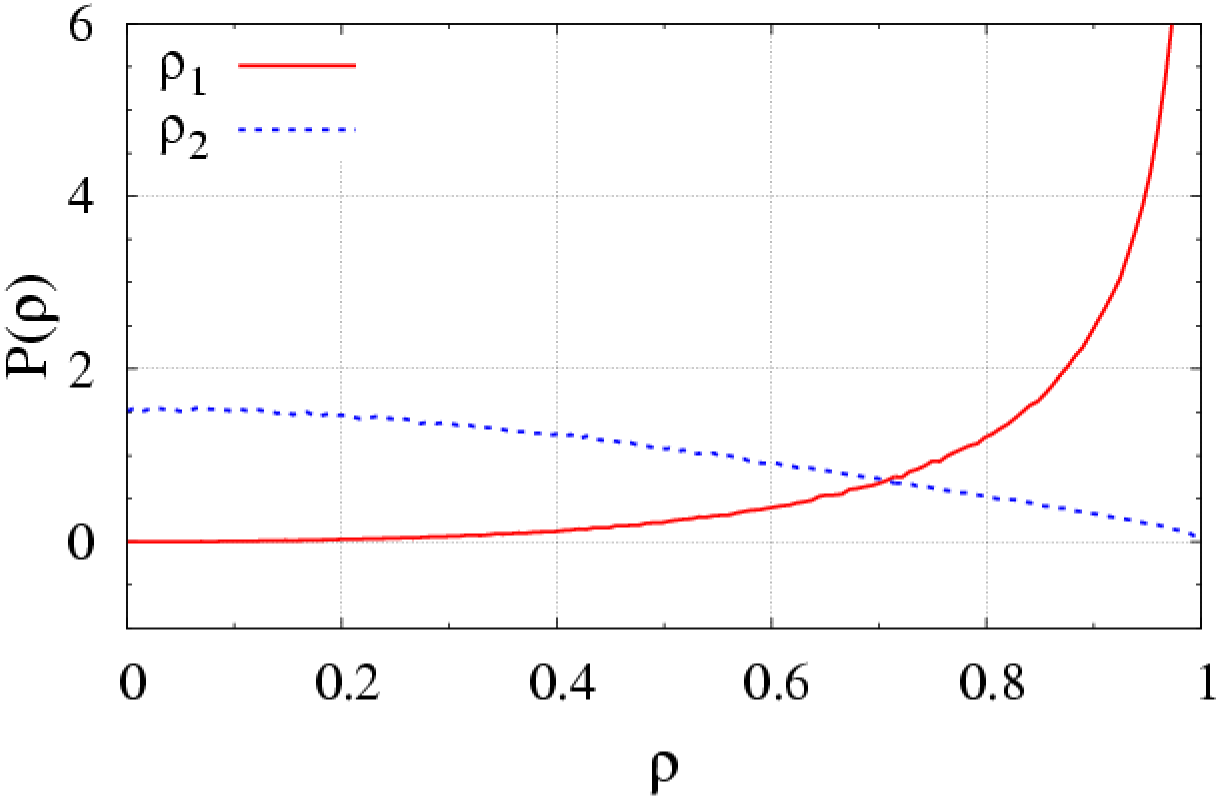
(Divergence from integration measure)

- Distribution of eigenvalue sum is almost flat !

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$\beta = 1.0 \qquad m^2 = -0.15$



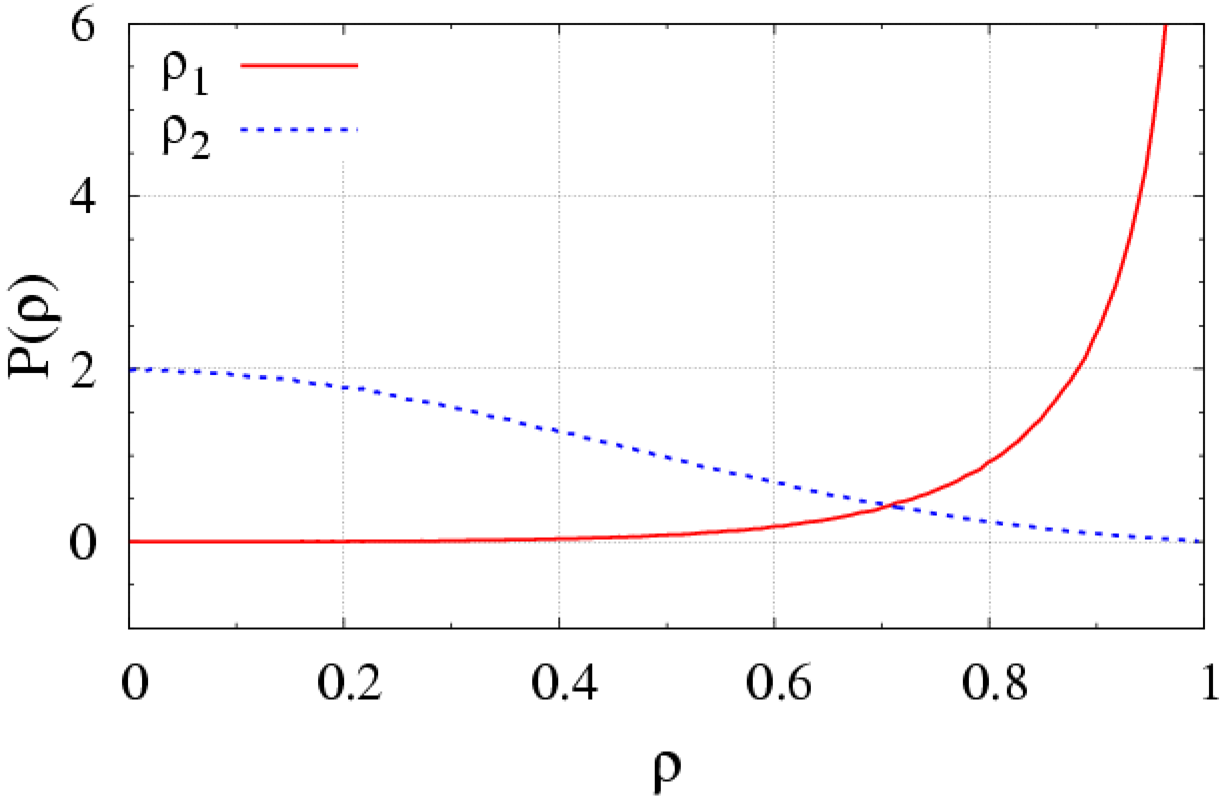
- Negative value for m^2 pushes distribution to the left.

  (Eigenvalue "repulsion")

$$\beta = 1.5 \qquad m^2 = -0.15$$

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

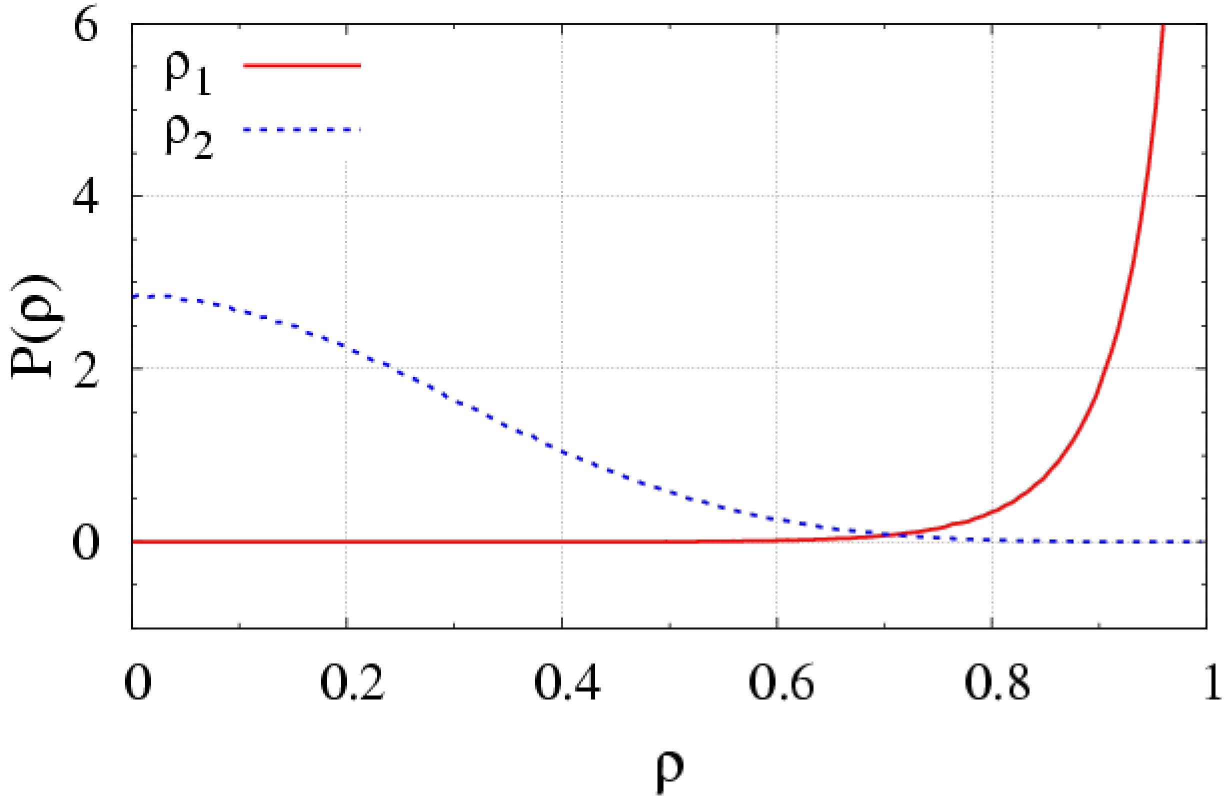


- Increasing β suppresses fluctuations.

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

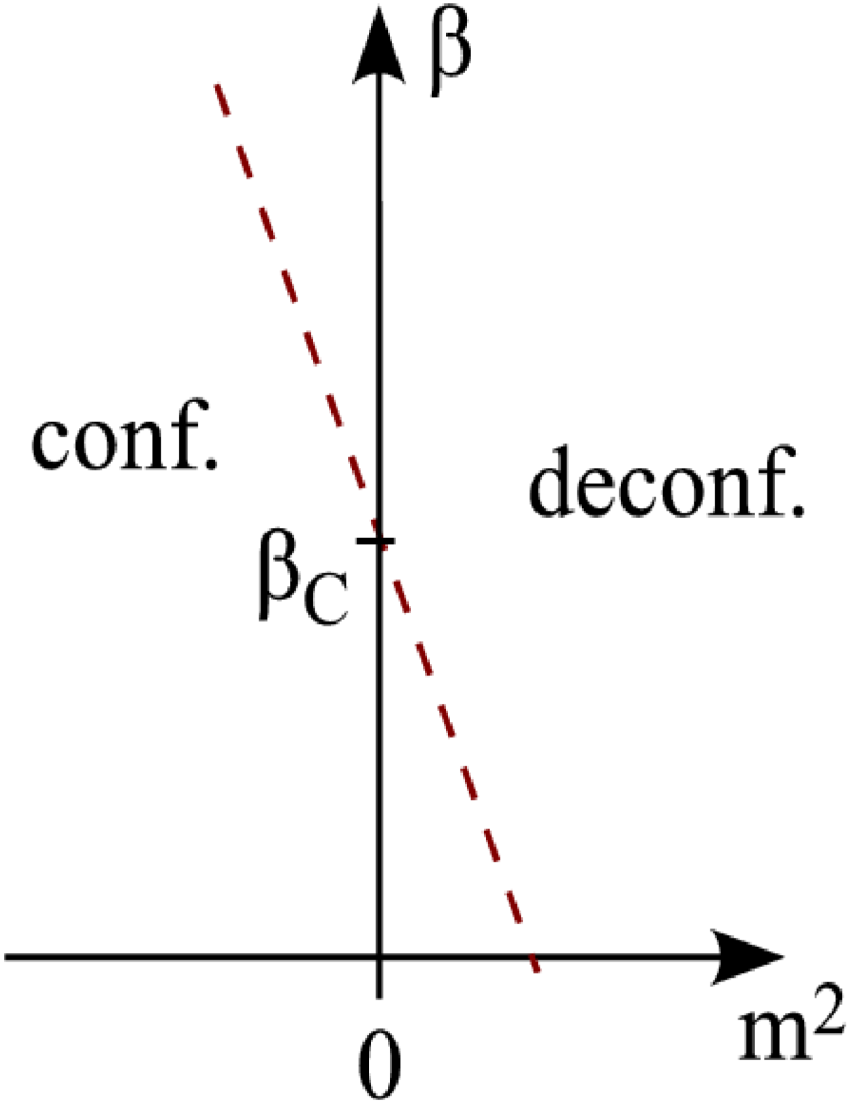
$$\beta = 3.0 \qquad m^2 = -0.15$$



- Strong eigenvalue repulsion is observed !

Phase Diagram

- For small β and small m^2 confinement is realized in a trivial way: $\bar{L} \to 0$
- Raising m^2 aligns Wilson Lines with unit Matrix.
- "Length" of $\bar{L}$ becomes finite for $\beta > \beta_C$. Direction of alignment is determined by potential. (in or out of Z(2) center)
- Small background field will tilt phase boundary to the left above β_C .



Summary and Outlook

- We performed Monte Carlo simulations of an effective theory of SU(2) Wilson Lines in three dimensions.
- We found a deconfining phase transition of second order.
- We observed eigenvalue repulsion in deconfined phase of 3d effective theory at $\beta>\beta_C$ (corresponding to moderately weak coupling in 4d theory).
- Non-trivial Z(N)-symmetric confined vacuum emerges at $\beta>\beta_C$:

  $\mathbf{L}_c \sim \mathrm{diag}(1, z, z^2, \ldots , z^{N-1})$, $z=\exp(2\pi i/N)$

  (while $\mathbf{L}_c \sim 0$ at small β)

Future:

- Include spatial components of the gauge fields (magnetic sector).
- Match couplings to real physical values.
- Extend to SU(3).

Thanks for coming !!!

This work was in part based on the MILC collaboration's public lattice gauge theory code.
See http://physics.utah.edu/~detar/milc.html

Reference:

Dumitru, Smith
Eigenvalue repulsion in an effective theory of SU(2) Wilson lines in three dimensions
arXiv:hep-lat/0711.0868