Lattice simulation of a center symmetric three-dimensional effective theory for SU(2) Yang Mills

Dominik Smith,
Adrian Dumitru,
Stefan Schramm -
University of Frankfurt

Helmholtz Research School
Quark Matter Studies

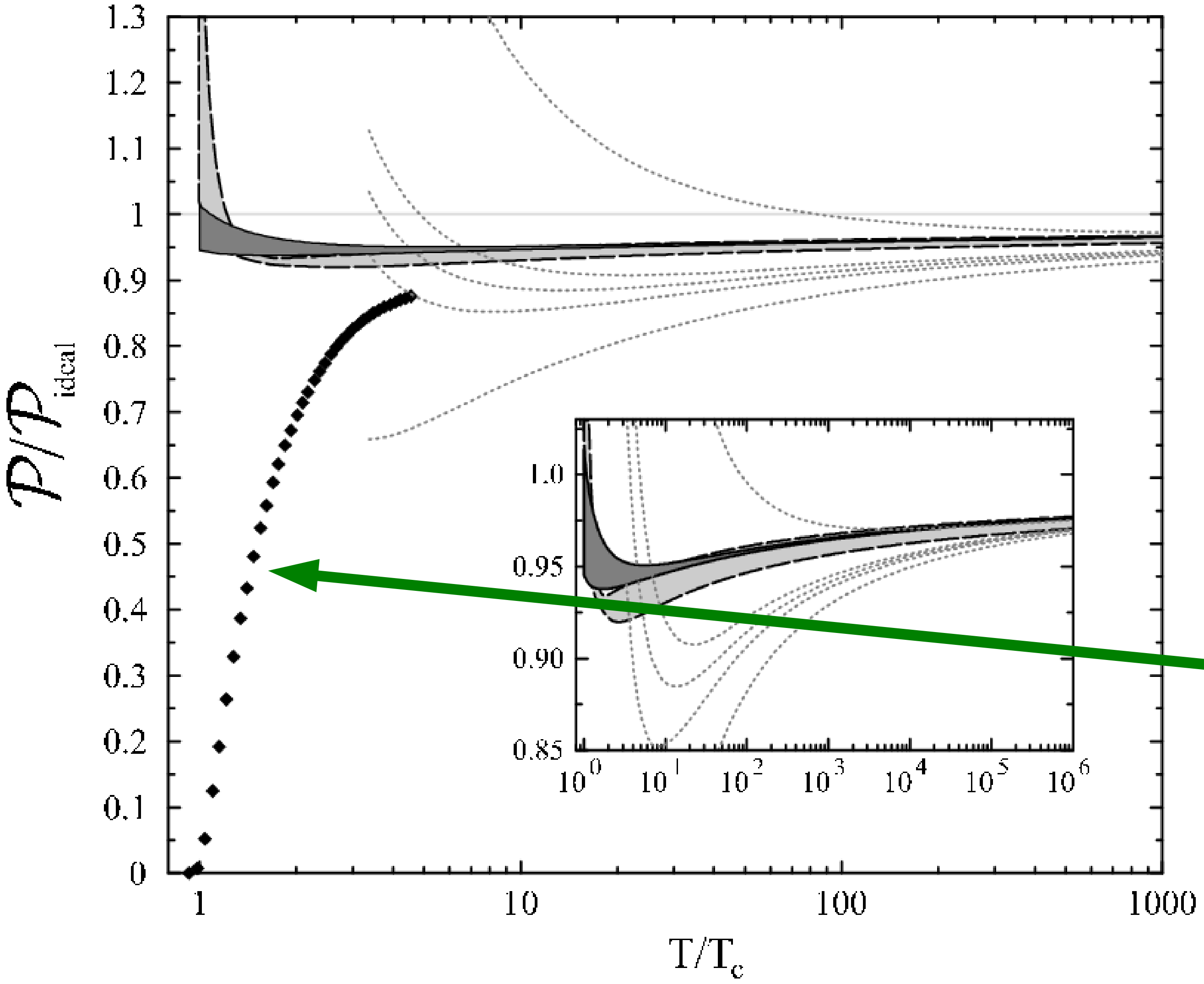
Outline:

- Introduction
- Effective Theory
- Lattice Simulation
- Summary

Introduction

- Recent results at the Relativistic Heavy Ion Collider (RHIC) demonstrate qualitatively new behavior for Heavy Ion Collisions.
- A Quark-Gluon Plasma appears to be produced at high energies.
- Experiments and simulations suggest that a strongly coupled phase exists right above the phase transition (SQGP).
- Hard-Thermal-Loop perturbation theory and other approaches fail to give correct prediction here.

- Example: pressure.



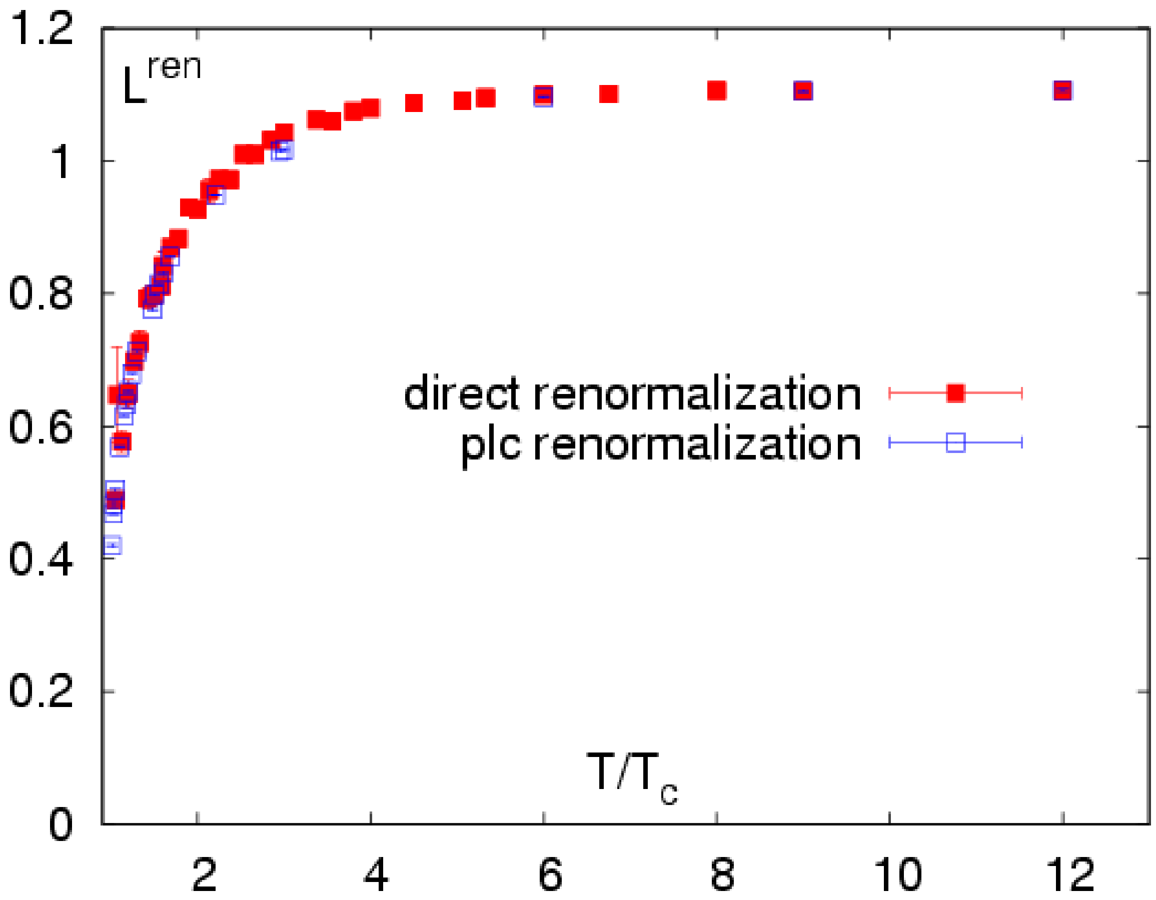
Anderson, Strickland, hep-ph/0404164

4d SU(3) Lattice data

- Resummed Perturbation Theory fails below $\sim 4T_C$!
- WHY ?

- HTL-P.T. expands about trivial vacuum,

$$A_0/T=0 \quad \leftrightarrow \quad L(\vec{x},\tau)=P\,\exp\left(ig\int_0^{1/T} A_0(\vec{x},\tau)\,d\tau\right) \approx Id$$



Kaczmarek,
Gupta,
Hübner,
hep-lat/0710.2277

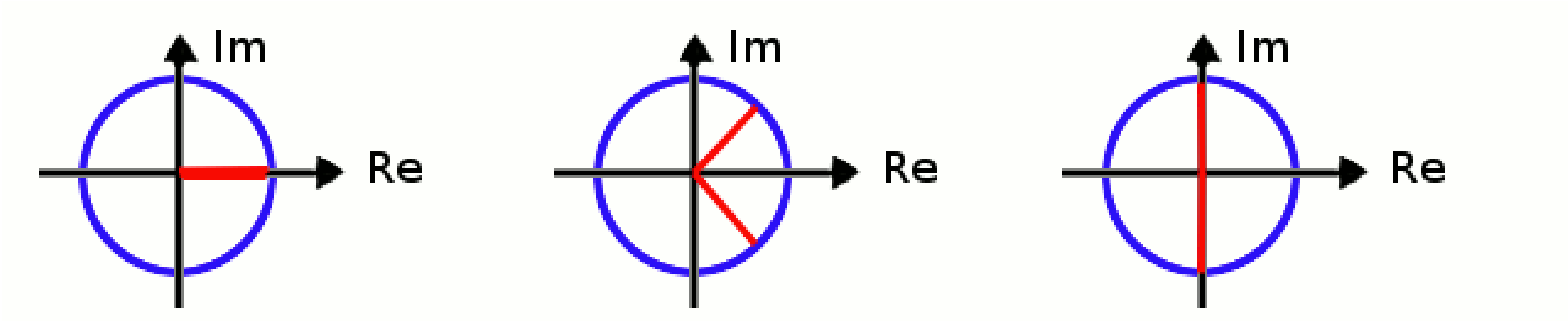
- Assumtion clearly not fullfilled close above T_C !

- This choice explicitly breaks the global Z(N) center-symmetry of the euclidean action.

$$S = \int \int_0^{1/T} d^3x\, d\tau\, \mathscr{L} \qquad \mathscr{L} = \frac{1}{4} tr\, F_{\mu\nu}^{a} F_{a}^{\mu\nu} + \overline{\psi} (i \gamma_\mu D^\mu - m) \psi$$

local SU(N) and global Z(N) symmetry
(without quarks) !

- Need different vacuum: “repulsion of eigenvalues” of **L** near T_c ?



example SU(2)

Effective theory

- Consider a 4D field theory in euclidean space.

$$\phi(\vec{x},\tau)=\sqrt{1/T}\sum_n \int_p \phi_n(p)\, e^{i\vec{p}\vec{x}+i\omega_n\tau} \qquad \Delta(\omega_n,\vec{p})=\frac{1}{p^2+\omega_n^2}$$

- Matsubara frequencies act like masses in the propagators.
- Nonstatic modes are heavy.
- Theorem: Heavy fields decouple at small coupling. Integrate out !

Appelquist, Carrazone, Phys. Rev. D11, 2856

→ theory in 3D !

- Valid on length scales >> $1/T$ and non-renormalizable.
- Degrees of freedom are Wilson Lines → Respects center symmetry !

$$\mathscr{L}^{eff} = \frac{1}{2} tr\, G_{ij}^2 + \frac{T^2}{g^2} tr\, |L^+ D_i L|^2 - \frac{2}{\pi^2} T^4 \sum_{n \geq 1} \frac{1}{n^4} |tr\, L^n|^2 + B_f T^2 |tr\, L|^2$$

- Includes perturbative potential in powers of fundamental Wilson Line to one loop order.

Gross, Pisarski, Yaffe, Rev. Mod. Phys. 53 43 (1981)

- Without last term: Minimized by perturbative vacuum.

→ Eigenvalue attraction, for any T **< L > = 1**

- Idea: add non-perturbative “fuzzy bag” contribution.

Pisarski, hep-ph/0608242

- We study SU(2) ! Qualitatively similar but technically simpler SU(3) !

Does one see repulsion of eigenvalues of Wilson line in the deconfined phase

(at non-asymptotic T) ?

Lattice Simulation

$$\mathscr{L}^{eff} = \frac{1}{2}\, tr\, G_{ij}^{2} + \frac{T^{2}}{g^{2}}\, tr\, |L^{+} D_i L|^{2} - \frac{2}{\pi^{2}}\, T^{4} \sum_{n \geq 1} \frac{1}{n^{4}} |tr\, L^{n}|^{2} + B_f T^{2} |tr\, L|^{2}$$

$$S = \beta \sum_{Pl.} \left(1 - \frac{1}{2} Re\, Tr\, U_{Pl.}\right) - \frac{1}{2} \beta \sum_{\langle ij \rangle} Tr\left(L_i U_{ij} L_j^{+} U_{ij}^{+} + h.c.\right) - m^{2} \sum_{i} |Tr\, L_i|^{2}$$

- Lattice action: Kinetic nearest neighbor interaction, mass term (i labels sites, <ij> labels links) and standard Wilson action (sum runs over Plaquettes)
- Periodic boundary conditions
- Standard Metropolis and Overrelaxation updating algorithms.
- (Only first term of perturbative potential included.)

- Initial results were obtained by leaving out the magnetic sector !

  Dumitru,
  Smith,
  arXiv:0711.0868[hep-lat]

- Second order phase transitions found along both axises.

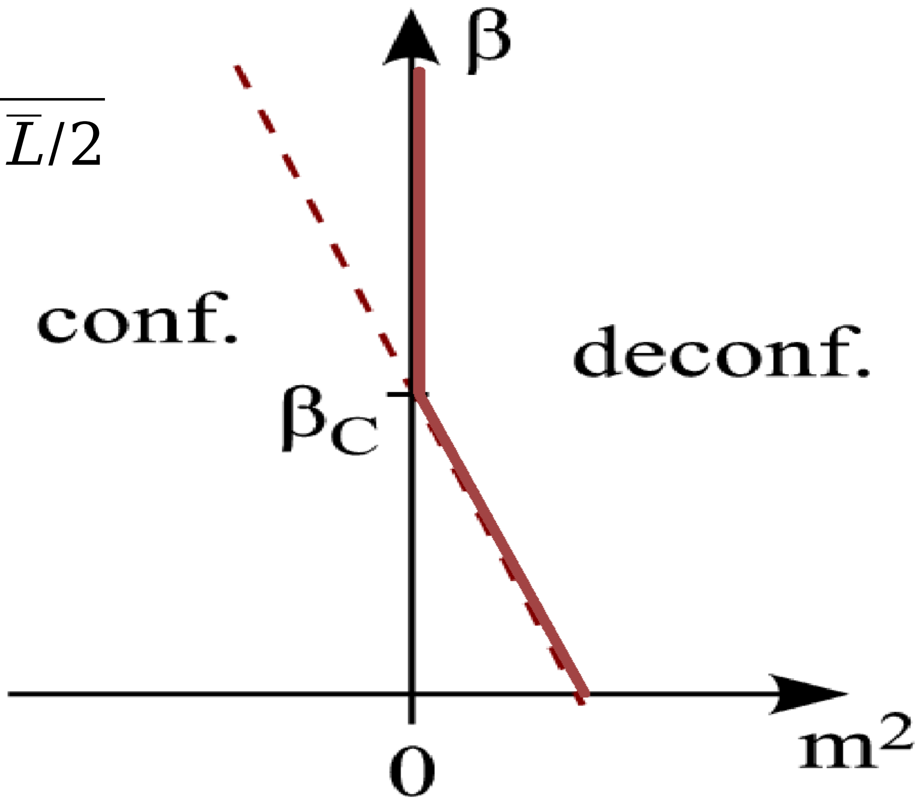
- Dashed line assumes small background field.

- Order parameters are $u = \sqrt{tr\, \overline{L}^{+} \overline{L}/2}$ and $\langle | \frac{1}{V} \sum_i tr L_i | \rangle$ respectively.

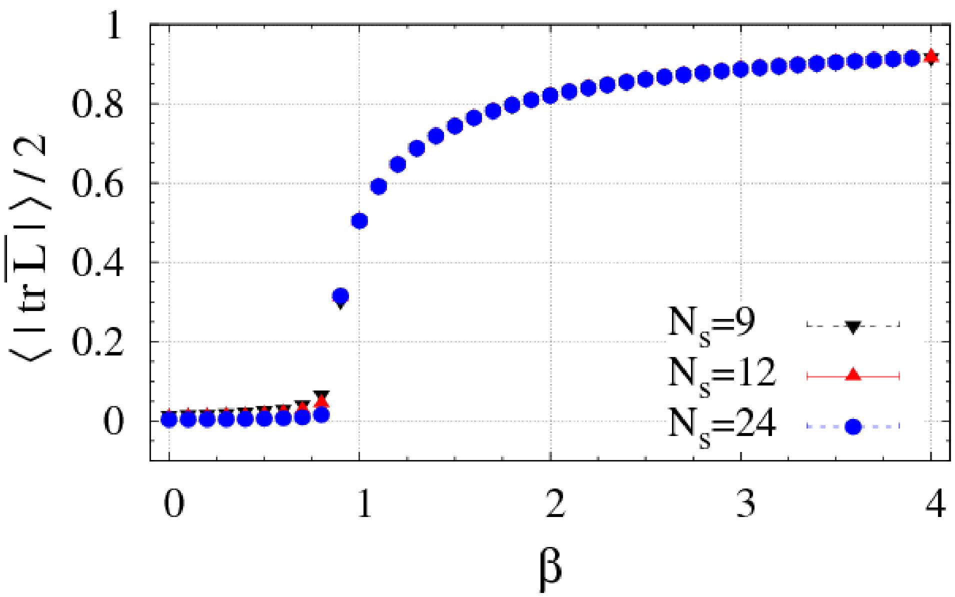
- Detailed comparison:

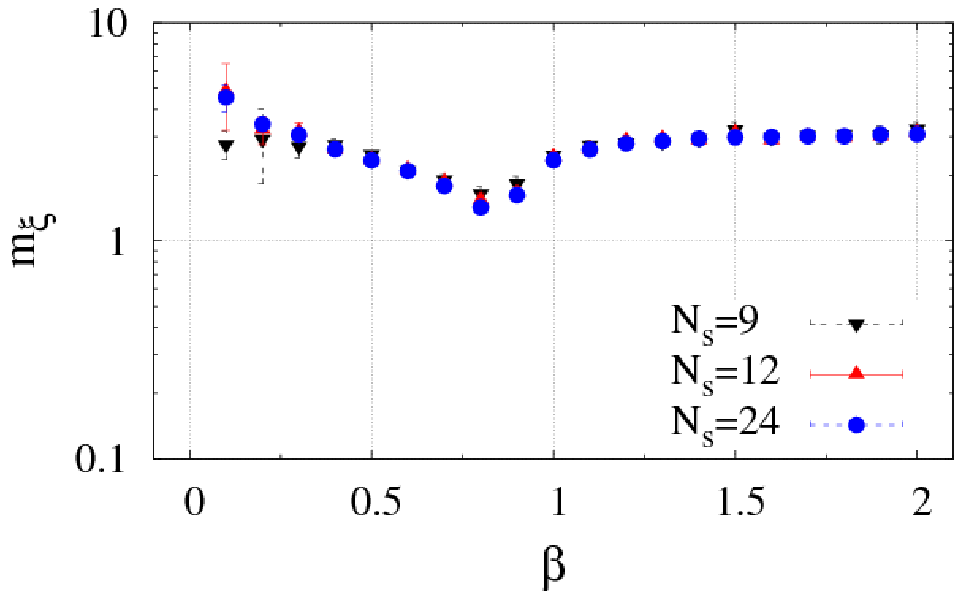
  Smith
  arXiv:0810.1129[hep-lat]

→ full theory



- Setting $m^2=0$ and varying β yields a second order phase transition.





- Polyakov loop expectation value rises from zero at $\beta \approx 0.9$.

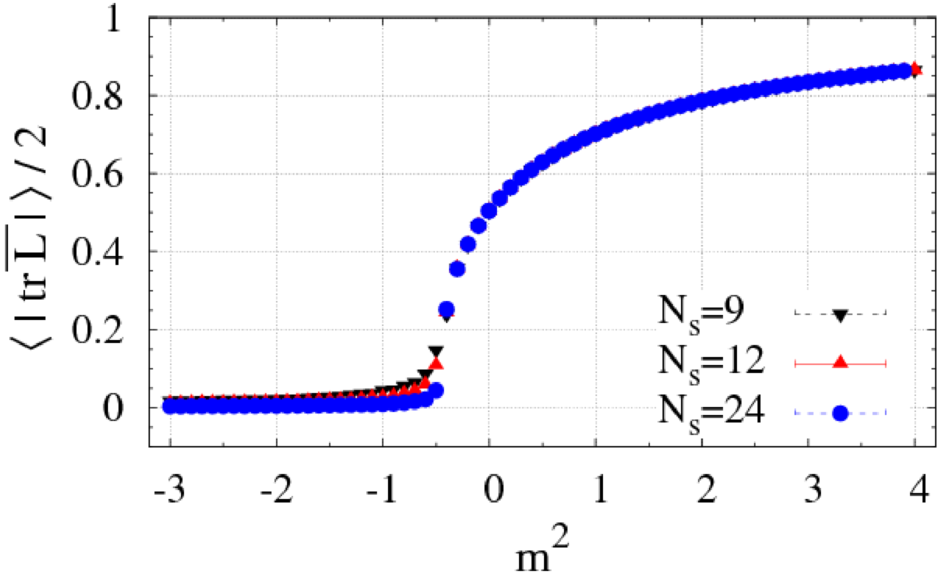
- Inverse correlation length drops at phase transition point.

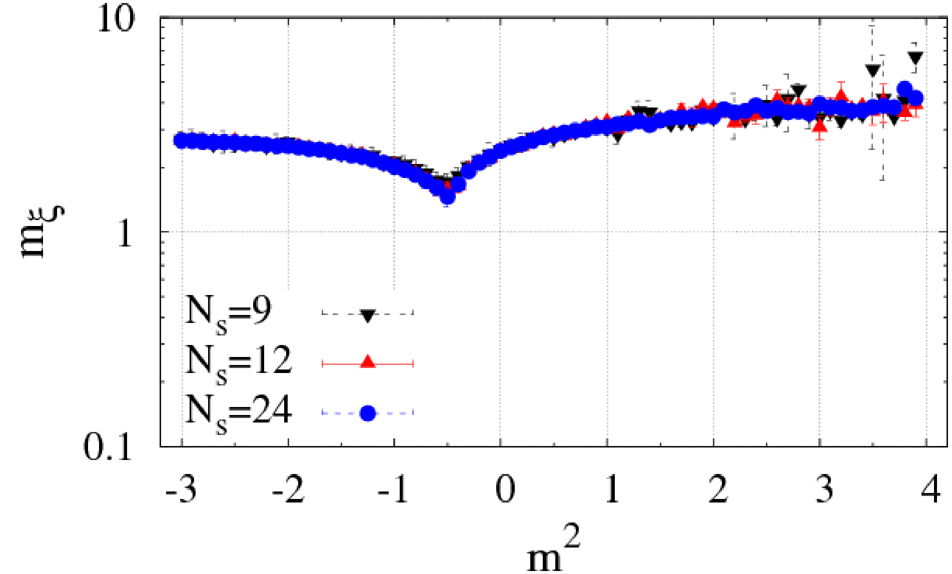
$$\Gamma(r)=\frac{1}{2}\langle tr\, L^{+}(r_0)\; U^{+}(r_0,r)\cdot L(r_0+r)\; U(r_0,r)\rangle$$

$$U(r_0,r)=\prod_{dr=0}^{r} U(r_0+dr)$$

- Fit with: $\Gamma(r)=\frac{a}{m\cdot r}\exp(-m\cdot r)+b$

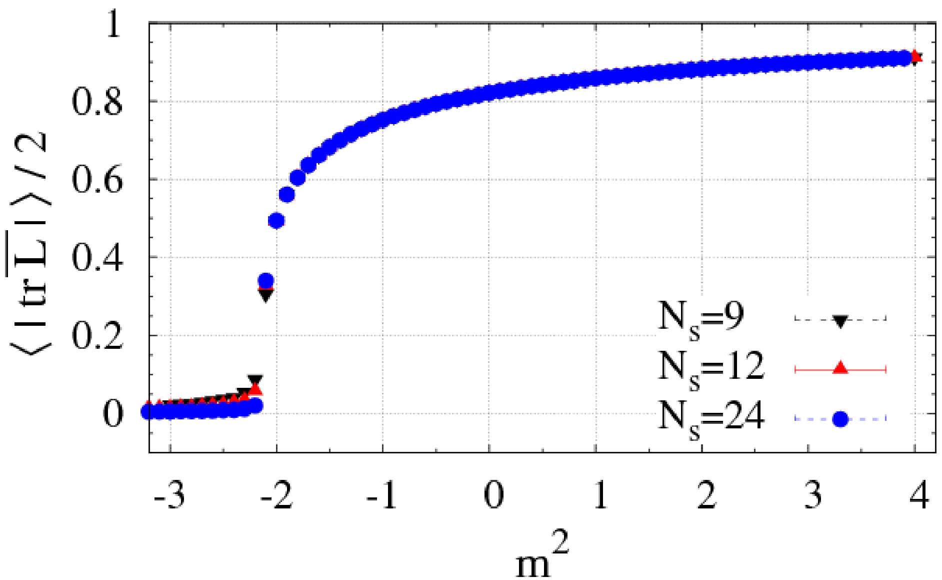
- Similar results are obtained for fixed beta.
- Example: $\beta = 1.0$

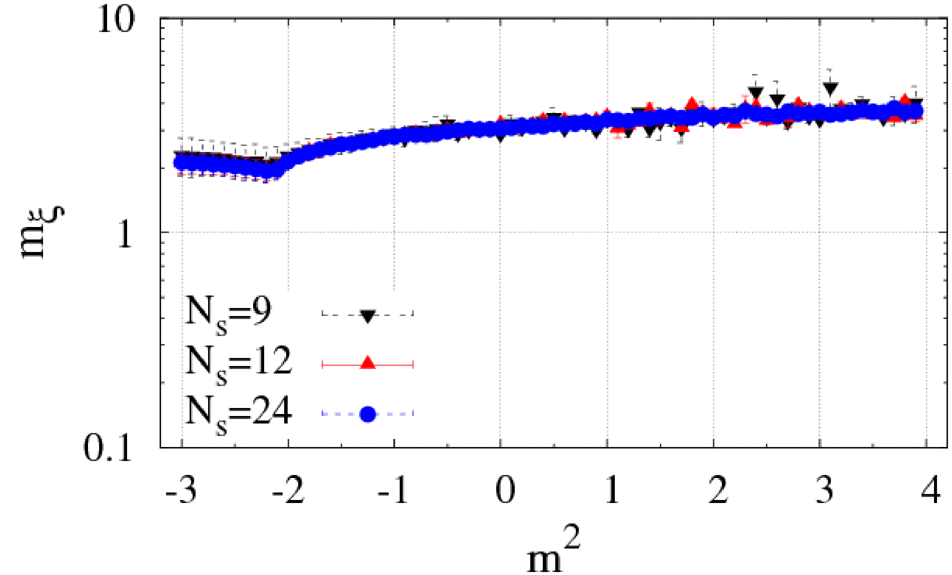




- Raising β shifts transition point to the left and makes it sharper.

$(\beta = 2.0)$

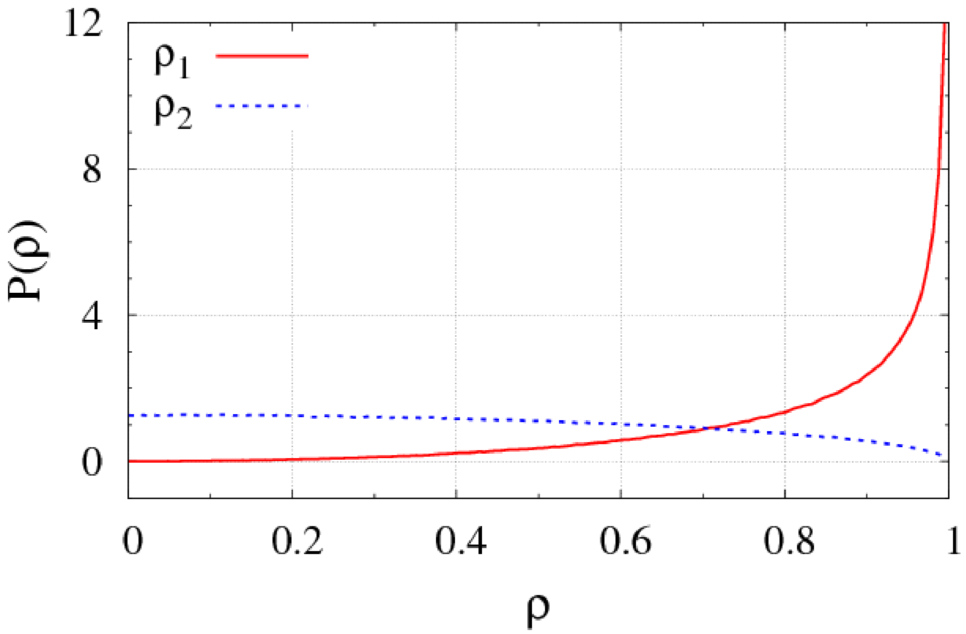




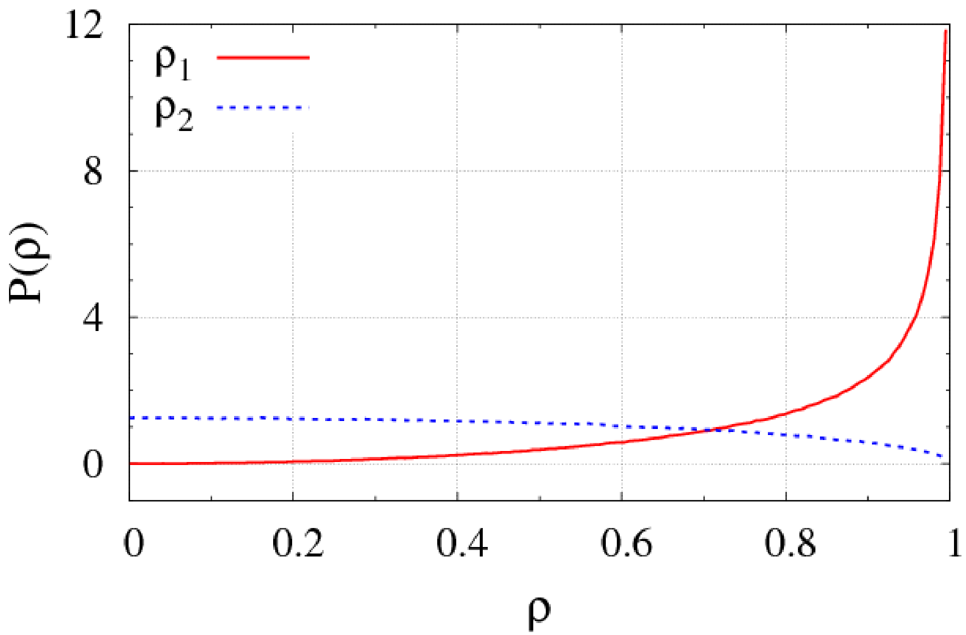
- Distribution of eigenvalues: $\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2| \qquad \rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$

- Consider only kinetic term ! $(m^2 = 0)$

$\beta = 0.1$



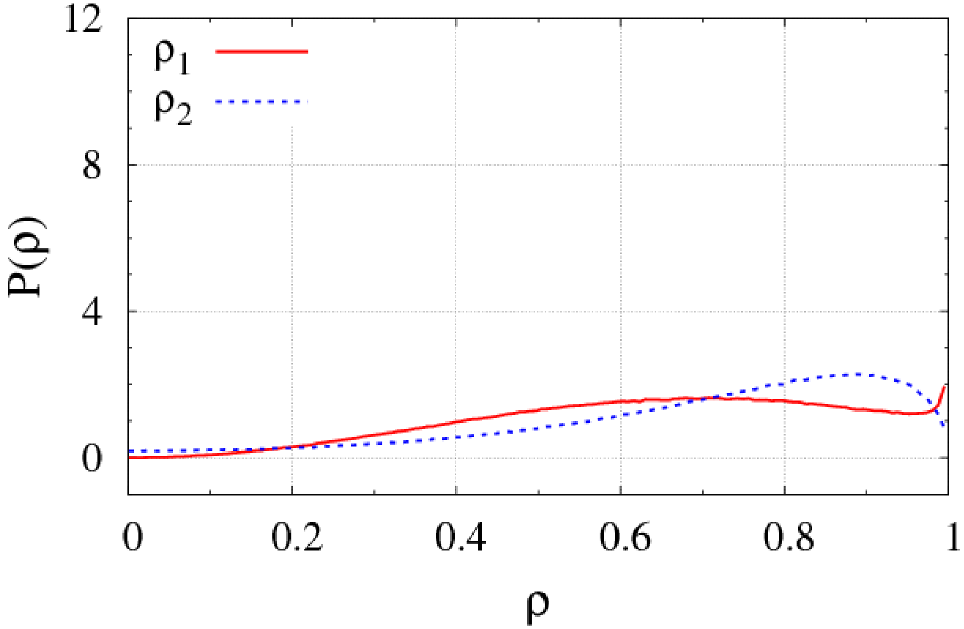
$\beta = 0.7$

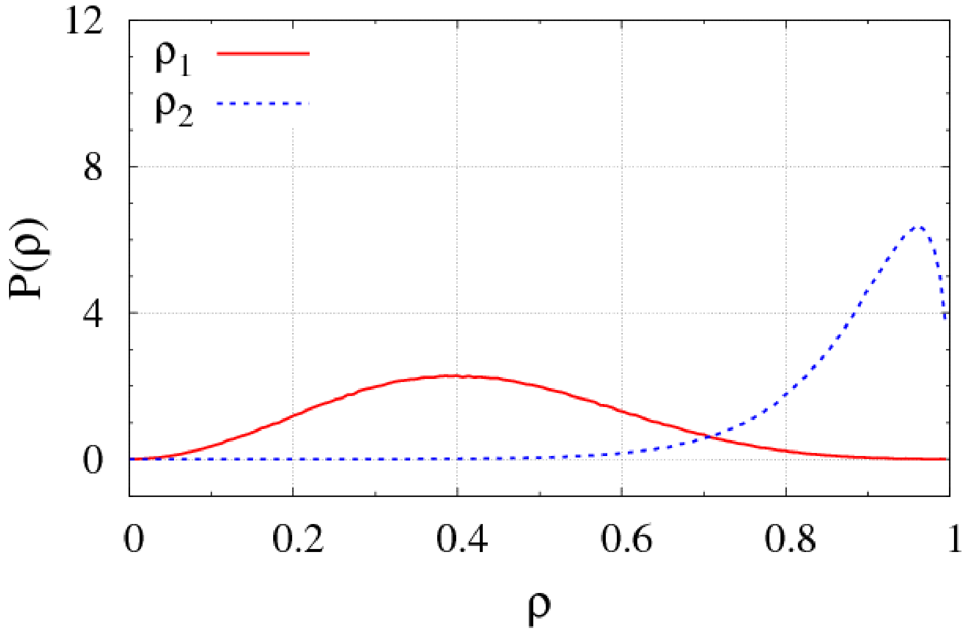


- Flat up to a divergence from group measure !

- Distribution of eigenvalues: $\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$ $\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$

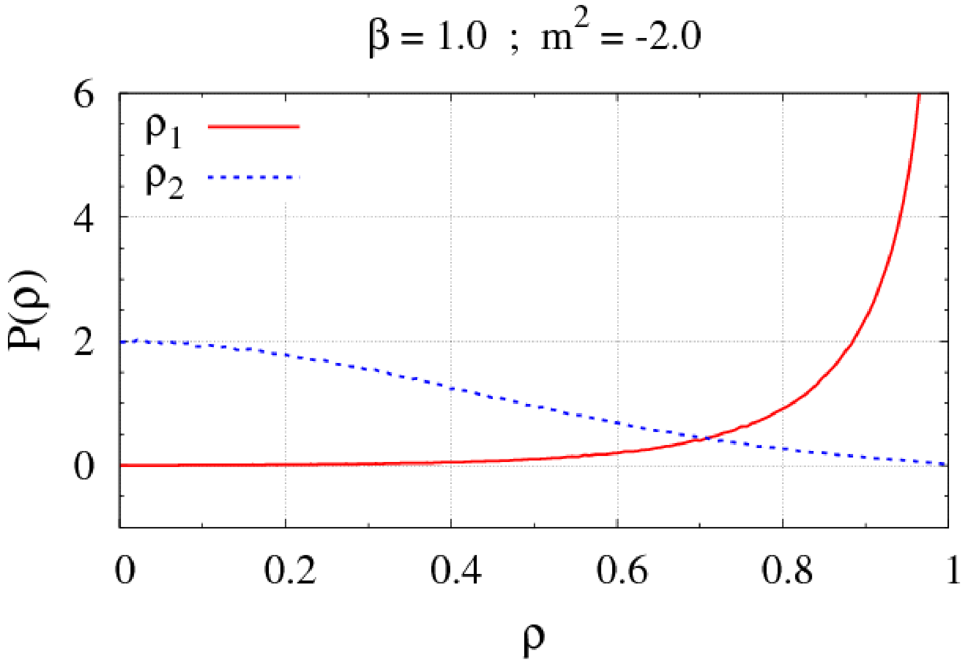
$\beta = 1.3$ $\beta = 3.0$

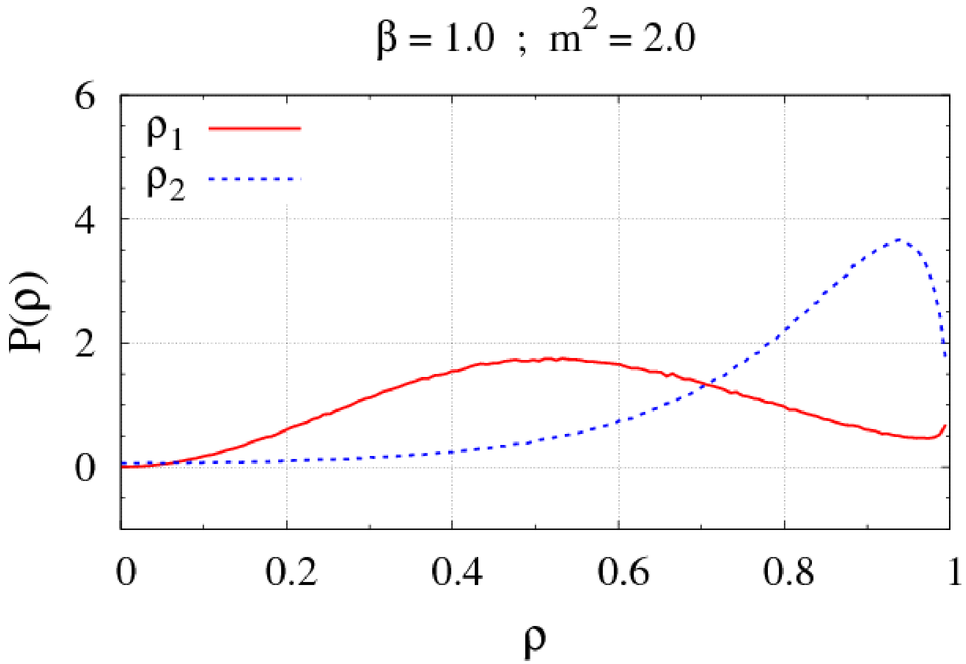




- Raising β forces alignment with unit matrix.

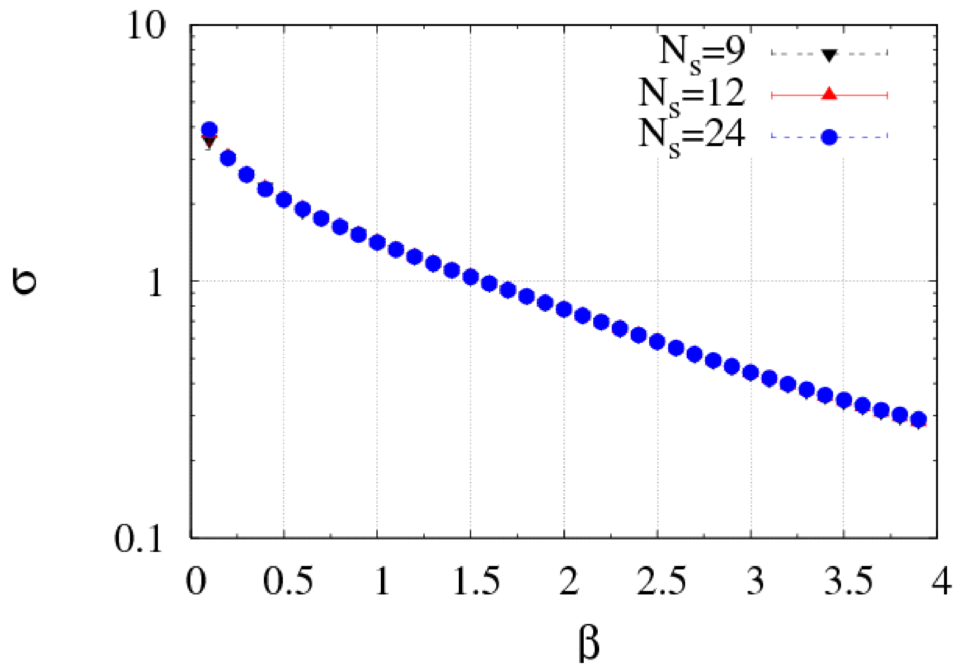
- Distribution of eigenvalues: $\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2| \qquad \rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$

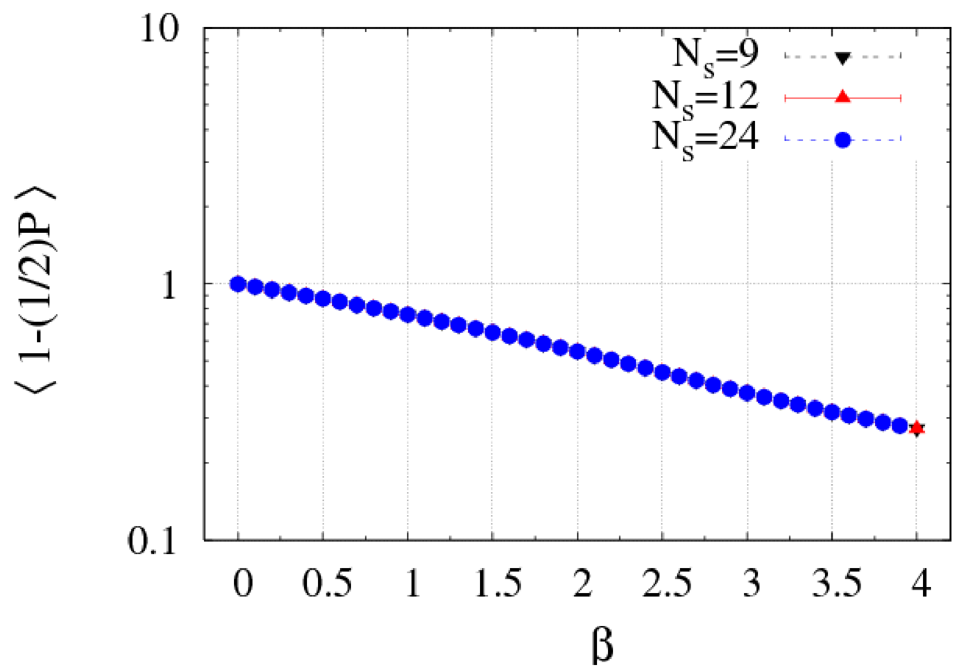




- A positive mass term pushes the system further into the perturbative vacuum.
- A negative mass term generates repulsion of eigenvalues.

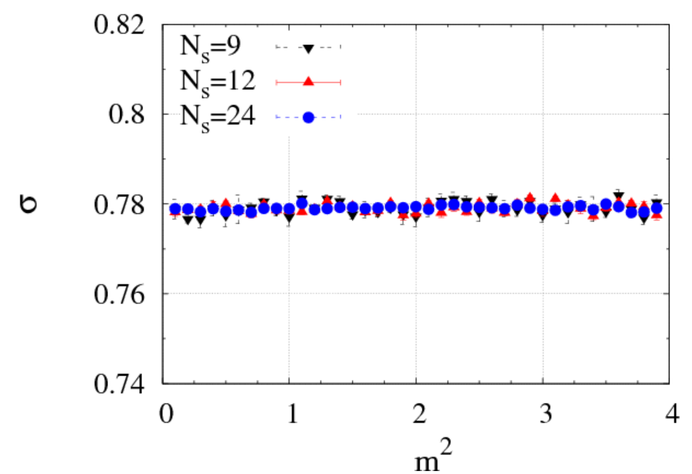
- Expectation value of Wilson action and spatial string tension.





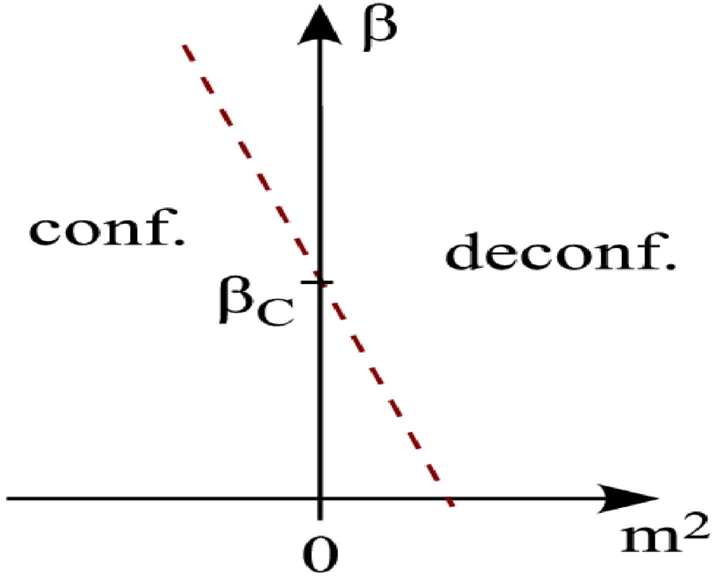
- For string tension: Measure rectangular Wilson loops $W(I,J)$.
- Obey area law: $W(I,J) \propto \exp(-\sigma A)$, $A = I * J$
- Independent of m^2 :

$(\beta = 2.0)$



Summary and Outlook

- We performed Monte Carlo simulations of an effective theory of SU(2) Wilson Lines in three dimensions.
- We found a deconfining phase transition of second order.
- We observed eigenvalue repulsion in deconfined phase of 3d effective theory at $\beta>\beta_C$ (corresponding to moderately weak coupling in 4d theory) and at negative m^2 . Disappears in extreme $\beta\to\infty$ limit.
- Non-trivial Z(N)-symmetric confined vacuum emerges at $\beta>\beta_C$:
  $\mathbf{L}_c \sim \mathrm{diag}(1, z, z^2, \ldots, z^{N-1})$, $z=\exp(2\pi i/N)$
  (while $\mathbf{L}_c \sim 0$ at small β)



Future:

- Match couplings to real physical values.
- Extend to SU(3).

Thanks for coming !!!

This work was in part based on the MILC collaboration's public lattice gauge theory code.
See http://physics.utah.edu/~detar/milc.html

References:

A.Dumitru, D.Smith
Eigenvalue repulsion in an effective theory of SU(2) Wilson lines in three dimensions
Phys. Rev. D 77, 094022 (2008), arXiv:0711.0868[hep-lat]

D.Smith
Lattice simulation of a center symmetric three-dimensional effective theory for SU(2) Yang Mills
arXiv:0810.1129[hep-lat]