Fast Chemical Equilibration of Hadrons in an Expanding Fireball

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Motivation

- SPS found enhancement of anti-hyperons, multi-strange baryons, and kaons to pp-data
  - Used binary collisions
    - Binary strangeness production reactions: $\pi + \bar{p} \leftrightarrow \bar{K} + \bar{\Lambda}$
    - Binary strangeness exchange reactions: $K + \bar{p} \leftrightarrow \pi + \bar{\Lambda}$
    - Gave small cross-sections → QGP! Because strange quarks produced more efficiently by gluon fusion, strangeness enhancement was considered a good signal for QGP. [1]
  - Used multi-mesonic reactions (for SPS)
    - For anti-protons [2]: $\bar{p} + N \leftrightarrow n\pi$
    - For anti-hyperons [3]: $\bar{Y} + N \leftrightarrow n\pi + n_{\bar{Y}}K$
    - Assuming $\sigma_{\rho\bar{Y}} \approx \sigma_{\rho\bar{p}} \approx 50$ mb, $\rho_B \approx 0.16 - 0.32\frac{1}{\text{fm}^3}$, and $v \approx 0.5 - 0.6\ c$ (typical for SPS),

$$\tau_{\bar{Y}} := \frac{1}{\Gamma_{\bar{Y}}} = \frac{1}{\langle\langle \sigma_{N\bar{Y}\to n\pi + n_{\bar{Y}}K} v_{\bar{Y}N}\rangle\rangle \rho_B} \approx 1 - 3\ \frac{\text{fm}}{c} \quad (1)$$

    - Fits within typical lifetime of fireball of 5-10 $\frac{\text{fm}}{c}$!
- At RHIC, however, such multi-mesonic reactions take too long to equilibrate.
  - At $T = 170$ MeV, $\rho_B^{eq} = \rho_{\bar{B}}^{eq} \approx 0.04\ \text{fm}^{-3}$ and $\tau_{\bar{B}} \approx 10\ \frac{\text{fm}}{c}$, which is too large, considering that the fireball's lifetime is about $\tau \le 4\ \frac{\text{fm}}{c}$ (see also [4]).

Solution: Hagedorn Resonances

- Heavy, highly unstable resonances, which considerably increase the phase space for multi-particle decays when the mass is increased, near the critical temperature
- Baryon anti-baryon and kaon anti-kaon production [5,6]

$$n\pi \leftrightarrow HS \leftrightarrow n_{i,b}\pi + B\bar{B}$$

$$n\pi \leftrightarrow HS \leftrightarrow n_{i,k}\pi + K\bar{K}$$

- Truncated Hagedorn mass spectrum with the Hagedorn temperature $T_H \approx 180$ MeV

$$g(m) = \int_{M_0}^{M} \frac{A e^{\frac{m}{T_H}}}{[m^2 + (m_0)^2]^{\frac{5}{4}}} dm$$

($A = 0.5\ \text{MeV}^{\frac{3}{2}}$, $m_0 = 0.5$ GeV, $M_0 = 2$ GeV, and $M = 7$ GeV)

Rate Equations for Chemical Equilibration

- The Hagedorn states, π's, and $B\bar{B}$'s are described by

$$\dot{\lambda}_i = \Gamma_{i,\pi}\left(\sum_n^{\infty} B_{i,n}\lambda_\pi^n - \lambda_i\right) + \Gamma_{i,B\bar{B}}\left(\lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2 - \lambda_i\right),$$

$$\dot{\lambda}_\pi = \sum_i \Gamma_{i,\pi}\frac{N_i^{eq}}{N_\pi^{eq}}\left(\lambda_i\langle n_i\rangle - \sum_n^{\infty} B_{i,n} n\lambda_\pi^n\right) + \sum_i \Gamma_{i,B\bar{B}}\langle n_{i,b}\rangle\frac{N_i^{eq}}{N_\pi^{eq}}\left(\lambda_i - \lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2\right),$$

$$\dot{\lambda}_{B\bar{B}} = \sum_i \Gamma_{i,B\bar{B}}\frac{N_i^{eq}}{N_{B\bar{B}}^{eq}}\left(\lambda_i - \lambda_\pi^{\langle n_{i,b}\rangle}\lambda_{B\bar{B}}^2\right) \quad (2)$$

$\lambda = \frac{N}{N^{eq}}$, N is the total number of each particle, its equilibrium value is N^{eq}, and i is the i^{th} Hagedorn resonance bin.

- Kaon anti-kaon pairs follow the same rate equations.

Parameters: Branching Ratios

- Branching ratios for $n\pi \leftrightarrow HS$ are described by a Gaussian distribution

$$B_{i,n} \approx \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(n - \langle n_i\rangle)^2}{2\sigma_i^2}}$$

where $\langle n_i\rangle = 0.9 + 1.2\frac{m_i}{m_p}$ and $\sigma_i^2 = (0.5\frac{m_i}{m_p})^2$

- After cutoff $n \ge 2$, $\langle n_i\rangle \approx 2$ to 9 and $\sigma_i^2 \approx 0.8$ to 11

Parameters: Decay Widths

- Linear fit (PDG) $\Gamma_i = 0.58 m_i - 58 = 250$ to 1000 MeV
- Baryon anti-baryon decay [7]

$$\Gamma_{i,B\bar{B}} = \langle B\rangle\Gamma_i$$

$$\Gamma_{i,\pi} = \Gamma_i - \Gamma_{i,B\bar{B}}$$

where the average baryon number is $\langle B\rangle \approx 0.06$ to 0.4.

- Analogously, for kaons where the average kaon number is $\langle K\rangle \approx 0.4$ to 0.5.

Estimated Time Scale

Naively, we would assume $N_\pi \approx N_\pi^{eq}$ and $N_i \approx N_i^{eq}$ where $\phi := \lambda_{B\bar{B}}(0)$, then

$$\dot{\lambda}_{B\bar{B}} = \sum_i \Gamma_{i,B\bar{B}}\frac{N_i^{eq}}{N_{B\bar{B}}^{eq}}\left(1 - \lambda_{B\bar{B}}^2\right) \to \lambda_{B\bar{B}} = \frac{\left(\frac{\phi+1}{\phi-1}\right)\exp\left(\frac{2t}{\tau_{B\bar{B}}}\right) + 1}{\left(\frac{\phi+1}{\phi-1}\right)\exp\left(\frac{2t}{\tau_{B\bar{B}}}\right) - 1} \quad (3)$$

$$\tau_{B\bar{B}} := \frac{N_{B\bar{B}}^{eq}}{\sum_i \Gamma_{i,B\bar{B}} N_i^{eq}} = 0.2 - 0.7\frac{fm}{c} \quad (4)$$

Parameters: Isentropic Fireball Expansion

- Find T(t) for the 5% most central collisions for $S_\pi/N_\pi \approx 5.5$

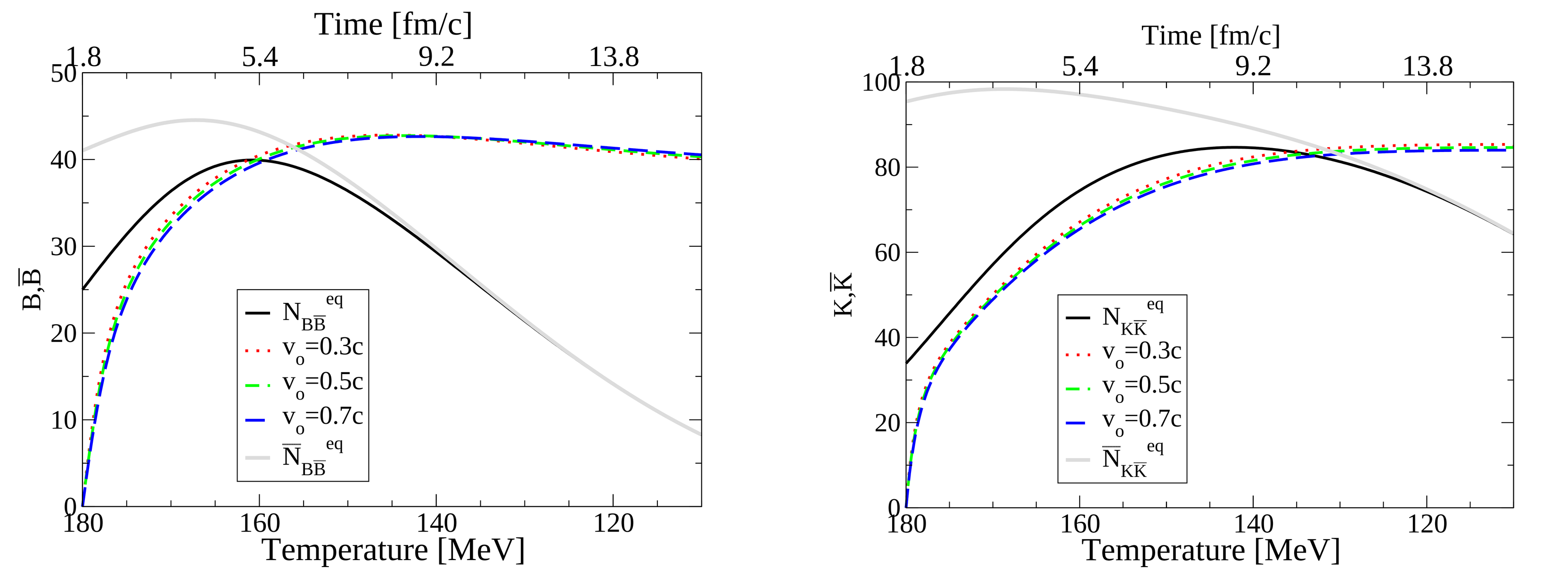
$$S^{tot} \approx s(T)V(t) = \frac{S_\pi}{N_\pi}\int\frac{dN_\pi}{dy}dy = const\ . \quad (5)$$

- Ansatz for expansion

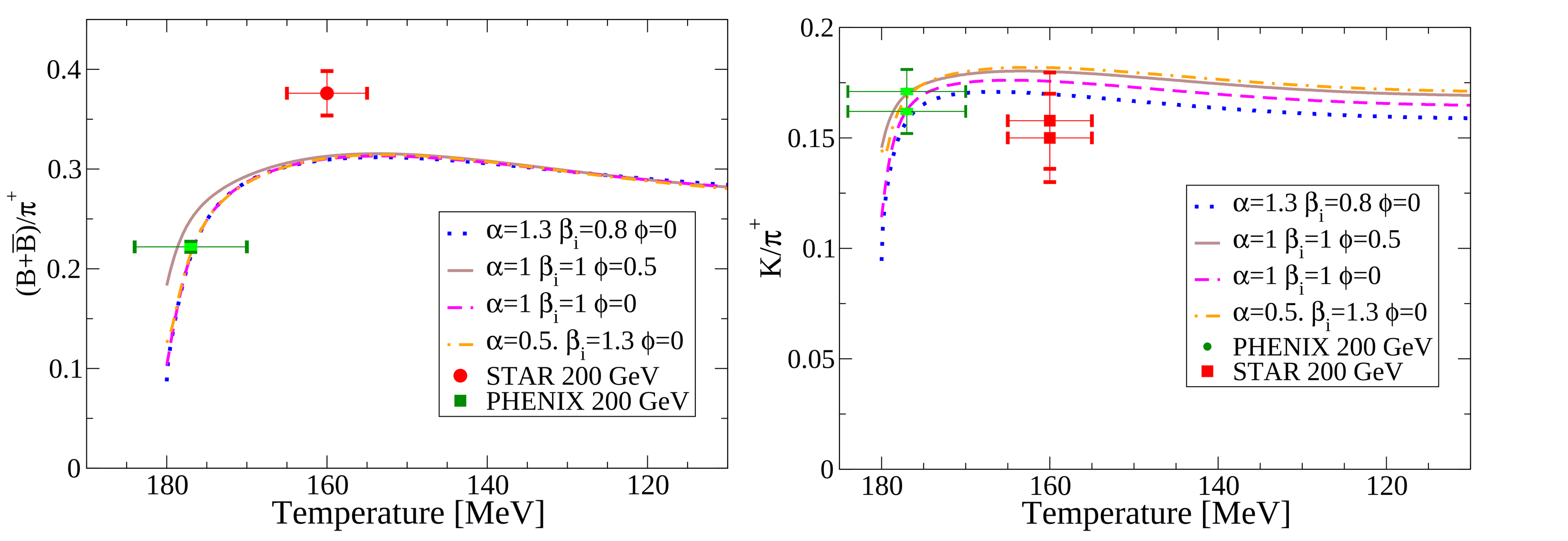
$$V_{eff}(t \ge t_0) = \pi\ ct\left(r_0 + v_0(t - t_0) + .5a_0(t - t_0)^2\right)^2 \quad (6)$$

($r_0 = 7.1$ fm, $v_0 = 0.3c, 0.5c, 0.7c$, and $a_0 = 0.035, 0.025, 0.015$)

Results: Total Number Equilibration



Results: Particle Ratios



Conclusions, discussion and outlook

- Baryon anti-baryon pairs reach equilibrium at $T \approx 165$ MeV ($\Delta t \approx 2 - 3\frac{fm}{c}$).
- Kaon anti-kaon pairs reach equilibrium at $T \approx 160 - 140$ MeV.
- $\frac{B+\bar{B}}{\pi^+}$ and $\frac{K}{\pi^+}$ ratios match experimental results for various initial conditions.
- Consider non-zero strangeness... in $B\bar{B}$
- Using a canonical model, improve branching ratios

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