Conical Correlations, Bragg Peaks, and Transverse Flow Deflections in Jet Tomography

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Jet - Studies in HIC

- Generation of Mach cone pattern
- Redistribution of energy to lower $p_T$-particles
- Re-appearance of the away-side for low and intermediate $p_T^{assoc}$
- Mach cone angle sensitive to EoS: $\cos \phi_M = c_s / v_{jet}$

$\Delta \phi (\text{rad})$

$Y_{jet} = 1/N_A^A N_B^B dA d\Delta \phi$

3-4 $\otimes$ 2-3 GeV/c

$\sqrt{s} = 200$ GeV


$1/N_{trigger} dN/d(\Delta \phi)$

$\Delta \phi$ (radians)

$4 < p_T^{trigger} < 6$ GeV/c


Reflect interaction of jet with medium
Modelling of Jets

Medium created in a HIC can be described using hydrodynamics.

Jets can be modelled using hydrodynamics:

$$\partial_\mu T^{\mu\nu} = S^\nu$$

not the source term of the jet

- residue of energy and momentum given by the jet


Stopped Jet I

Applying a static medium and an ideal Gas EoS for massless gluons  
Maximal fluid response  
Assume: Near-side jet is not modified by medium

\[ S^\gamma = \int_{\tau_i}^{\tau_f} \frac{dP^\gamma}{d\tau} \delta^{(4)}(x^\mu - x^\mu(\tau)) \, d\tau \]

\[ P^\gamma = (E, \vec{M}) \]

\[ x^\mu(\tau) = x^\mu_0 + u^\mu_{\text{jet}} \tau \]

Jet decelerating from \( v=0.999 \)  
according to Bethe-Bloch formalism

\[ \frac{dE}{dx}(t) = a \frac{1}{v^2(t)} \quad a=-1.36 \text{ GeV/fm} \]

Simplest back-reaction from the medium

Bragg Peak

\[ \text{BB et al., Phys. Rev. C 79, 034902 (2009)} \]
Stopped Jet II

\[ T_0 = 200 \text{ MeV, } v=0.999 \]

\[ \frac{dE}{dt}(0) = 1.5 \frac{\text{GeV}}{\text{fm}} \quad \frac{dM}{dt}(0) = 0 \frac{\text{GeV}}{\text{fm}} \]

\[ \frac{dE}{dt} = v \quad \frac{dM}{dt}(0) = 1.5 \frac{\text{GeV}}{\text{fm}} \]

\[ t=4.5/v \text{ fm} \]


→ Mach cone for sound waves

→ Diffusion wake
Stopped Jet III

- Jet stops after $t = 4.5/v$ fm

\[ \frac{dE}{dt} (0) = 1.5 \text{ GeV/fm} \]
\[ \frac{dM}{dt} (0) = 0 \text{ GeV/fm} \]

\[ T_0 = 200 \text{ MeV}, \gamma_0 = 0.999 \]

$\tau_0 = 4.5/v$ fm

$\tau_0 = 6.5/v$ fm

$\tau_0 = 8.5/v$ fm

Diffusion wake still present $\rightarrow$ Vorticity conservation
**Stopped Jet IV**


Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[ \frac{dN}{p_T dp_T dy d\phi} \bigg|_{y=0} = \int d\Sigma \mu p^\mu [f_{\text{Boltzmann}}(u^\mu, p^\mu, T) - f_{eq}] \]

Energy Flow Distribution

\[ \frac{dE}{d\phi dy} \bigg|_{y=0} = \int d^3 x \ E(\vec{x}) \ \delta(\phi - \Phi(\vec{x})) \]

\[ \Phi(\vec{x}) = \tan^{-1} \left( \frac{m_y(\vec{x})}{m_x(\vec{x})} \right) \]

Diffusion wake causes peak in jet direction

Assuming: Particles in subvolume will be emitted into the same direction
Heavy Quark Jets in pQCD vs AdS/CFT

Idea: Compare weakly and strongly coupled models

Using heavy quark punch-through jet

Applying ideal hydrodynamics for a static medium and an ideal gas EoS of massless gluons

Assume that the near-side jet is not modified by the medium

pQCD: Neufeld et al. source for a heavy quark

AdS/CFT: Stress tables provided by S. Gubser, A. Yarom and S. Pufu with $\eta/s=1/(4\pi)$

$\Delta T/T_0 \ v=0.90$

$t=4.5/\nu \ fm$

BB et al., arXiv: 0807.4525 [hep-ph]
Heavy Quark Jets in pQCD vs AdS/CFT

Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[
\frac{dN}{p_T dp_T dy d\phi} \bigg|_{y=0} = \int d\Sigma \mu p^\mu [f_{\text{Boltzmann}} (u^\mu, p^\mu, T) - f_{eq}]
\]

Isochronous freezeout needed to compare pQCD and AdS/CFT

→ No Mach-like peaks: \( \cos \phi_M = c_s / v_{jet} \)

Strong influence of the Neck region

see J. Noronha’s poster
Expanding Medium I

- Consequences of expansion?
  Radial flow, Elliptic flow
- Predictions:
  Transverse flow causes distortion
  Expansion broadens Mach cone angle
  - Mach cones are sensitive to the background flow
- Qualitative, model-independent effect

Satarov et al, PLB 627:64 (2005)
Expanding Medium II

Experimental results based on many events

Consider different jet paths


Apply Glauber initial conditions and an ideal Gas EoS for massless gluons

Focus on radial flow contribution

\[ S^n = \int_{t_i}^{t_f} \frac{dP^n}{d\tau} \left( \frac{T(\tau)}{T_{\max}} \right)^3 \delta^{(4)}(x^\mu - x^\mu(\tau)) \, d\tau \]

\[ \left. \frac{dE}{dt} \right|_0 = v \left. \frac{dM}{dt} \right|_0 = 1.0 \text{ GeV/fm} \]

Jet 150

T_0 = 200 \text{ MeV}
Expanding Medium III

- Jet deposition stopped
  - $E_{\text{tot}} = 5$ or $10$ GeV
  - $p_T^{\text{trig}} = 3.5$ and $7.5$ GeV
  - $T < T_{\text{crit}} = 130$ MeV

- Isochronous Cooper-Frye Freeze-out when $T_{\text{all cells}} < T_{\text{crit}} = 130$ MeV

- Two-particle correlation:
  \[ N(\phi) = A(\phi) + \int_{0}^{2\pi} d\phi^* N(\phi - \phi^*) f(\phi^*) \]
  \[ \text{CF}(\phi) = \frac{N(\phi) - N_{\text{back}}(\phi)}{2\pi} \int_{0}^{2\pi} N_{\text{back}}(\phi) \, d\phi \]
  $f(\phi)$ represents the near-side jet
  Normalized, background-subtracted Cooper-Frye Freeze-out at mid-rapidity
Expanding Medium IV

E$_{tot}$ = 5 GeV
$p_T^{\text{trig}}$ = 3.5 GeV

broad away-side peak
due to non-central jets

double peaked structure

Expanding Medium V

$E_{\text{tot}} = 10 \text{ GeV}$

$p_T^{\text{trig}} = 7.5 \text{ GeV}$

broad away-side peak

due to non-central jets

Strong impact of the Diffusion wake

→ Causes smaller dip for $p_T = 2 \text{ GeV}$

Expanding Medium VI

$p_T=1.0 \text{ GeV}$

$E_{\text{tot}} = 5 \text{ GeV}$

$p_T^{\text{trig}} = 3.5 \text{ GeV}$

broad away-side peak

Pure energy deposition

$\rightarrow$ No conical distribution in expanding medium

$\rightarrow$ Jet 180: No peaks on away-side
Summary

- Diffusion wake dominates the freezeout distribution in a static medium
  - always created if dM/dx > threshold
- Different impact of pQCD and AdS/CFT source terms
  - effect of the Neck region
- Radial flow affects the results by deflecting jets and interacting with the Diffusion wake
- Diffusion wake contribution depends on path length of the jet
- Away-side peaks: non-central jets have to be included
- Pure energy deposition: No conical structure in expanding medium
  - clear difference between static & expanding medium
  - Non-linear Hydrodynamics is fundamental for quantitative studies of jets in medium
Backup
Punch – Through Jet I

Applying a static medium and an ideal Gas EoS for massless gluons → Maximal fluid response

Assume: Near-side jet is not modified by medium

\[ S^\nu = \int_{\tau_i}^{\tau_f} \frac{dP^\nu}{d\tau} \delta^{(4)}(x^\mu - x^\mu(\tau)) d\tau \]

\[ P^\nu = (E, \vec{M}) \]

\[ x^\mu(\tau) = x_0^\mu + u_\text{jet}^\mu \tau \]

\[ T_0 = 200 \text{ MeV}, \nu=0.999 \]

\[ \frac{dE}{dt} = 1.5 \text{ GeV/fm}, \quad \frac{dM}{dt} = 0 \text{ GeV/fm} \]

\[ \nu=0.999 \]

\[ \frac{dM}{dt} = \frac{dE}{dt} = 1.5 \text{ GeV/fm} \quad t=4.5/\nu \text{ fm} \]
Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[ \frac{dN}{p_T dp_T dy d\phi} \bigg|_{y=0} = \int d\Sigma \mu \rho' \left[ f_{\text{Boltzmann}} (u^\mu, p^\mu, T) - f_{eq} \right] \]

Energy Flow Distribution

\[ \frac{dE}{d\phi dy} \bigg|_{y=0} = \int d^3 \vec{x} \ E(\vec{x}) \ \delta(\phi - \Phi(\vec{x})) \]

\[ \Phi(\vec{x}) = \tan^{-1} \left( \frac{m_y(\vec{x})}{m_x(\vec{x})} \right) \]

Diffusion wake causes peak in jet direction

Assuming: Particles in subvolume will be emitted into the same direction
Punch – Through Jet III

Does the jet-pattern reproduce the features of a Mach cone?

\[ \cos \phi_M = \frac{c_s}{v} \]

→ Velocity dependence of the emission angle

Creation of Bow Shock for smaller \( v \) strengthens peak in jet direction
Punch – Through Jet IV

- Transverse momentum deposition:

\[ S' = \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp\left(\frac{(\bar{r} - \bar{x})^2}{2\sigma^2}\right) \left(\frac{dE}{dt}, \frac{dM_L}{dt}, \frac{dM_T}{dt}\cos\theta, \frac{dM_T}{dt}\sin\theta\right) \]

- Still influence of diffusion wake from explosion of matter

- Vorticity conservation

\[ t = 4.5/v \text{ fm} \]

\[ \frac{dE}{dt} = \frac{dM_T}{dt} \]

\[ \frac{dM_L}{dt} = \frac{1}{4} \frac{dM_T}{dt} \]

Punch – Through vs Stopped Jet


Punch-Through Jet

Stopped Jet

→ Similar freeze-out patterns
The Diffusion Wake

G. Burau, Genua Harbour, September 2008

→ The diffusion wake exists!
Stopped Jet

Diffusion wake causes peak in jet direction

Larger impact of thermal smearing


\[ t_{FO} = 4.5/v \text{ fm} \]

\[ t_{FO} = 6.5/v \text{ fm} \]

\[ t_{FO} = 8.5/v \text{ fm} \]
Different Contributions

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<th>Component</th>
<th>Contribution</th>
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<tr>
<td>$E_{\text{Mach}}$</td>
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<tr>
<td>$E_{\text{Diff}}$</td>
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</tr>
<tr>
<td>$E_{\text{Neck}}$</td>
<td>57.4%</td>
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<tr>
<td>$E_{\text{Head}}$</td>
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<tbody>
<tr>
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<td>$P^x_{\text{Diff}}$</td>
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<td>73.7%</td>
</tr>
<tr>
<td>$P^x_{\text{Head}}$</td>
<td>1.0%</td>
</tr>
</tbody>
</table>


$T_0 = 200$ MeV, $\nu = 0.999$

d\[ \frac{dE}{dt}(0) = \nu \frac{dM}{dt}(0) = 1.5 \text{ GeV fm}^{-1} \]
Expanding Medium

Jet 90

Jet 120

Jet 150

Jet 180
Expanding Medium

For $b=6$ fm

- Distortion of the conical structure
- Dependence on background flow (centrality)
The Neck Zone in pQCD vs AdS/CFT

AdS/CFT

\[ 100 \times \Delta \varepsilon_{\text{ad}}/\varepsilon(T_0) \]  
\( (v=0.9, \lambda=5.5, N_c=3) \)


Strong transverse flow

pQCD

\[ \Delta T/T_0 \quad v=0.90 \]


No strong transverse flow
Heavy Quark Jets in pQCD vs AdS/CFT

Momentum Flow Distribution

\[
\frac{dS}{d\cos \theta} = \int d^3\vec{x} \left| M(\vec{x}) \right| \\
\times \delta(\cos \theta - M_\chi(\vec{x}) / |M(\vec{x})|)
\]

Assuming: Particles in subvolume will be emitted into the same direction

- Mach-like peaks &
- Strong impact of diffusion wake

Independent of \( p_T \) - cut
The Caveat: Freeze-out Prescription

Cooper-Frye Freeze-out:

\[ \frac{dN}{p_T dp_T d\phi} = C \int dV \exp\left[-\gamma (E - p_i v^i)/T\right] \sim e^{+p \cdot v} \]

→ mainly flow driven

- Assumption of isochronous/isothermal freeze-out
- No interaction afterwards
Isothermal Freeze-out

\[ CF(\phi) \]

- Isothermal and isochronous freeze-out lead to very similar results

- Beak occurs for non-central jets
Energy-Momentum Relation

\[
\frac{dM}{dt} = \frac{dM}{dx} \frac{dx}{dt} = \beta \frac{dM}{dx} = \beta \left( \frac{dE}{dx} \beta + E \frac{d\beta}{dx} \right)
\]

\[
E = \gamma m
\]

\[
M = \beta \gamma m = \beta E
\]

\[
\beta^2 = 1 - \frac{1}{\gamma^2}
\]

\[
2 \beta \frac{d\beta}{dx} = 2 \frac{1}{\gamma^3} \frac{d\gamma}{dx}
\]

\[
\frac{d\beta}{dx} = \frac{1}{\beta \gamma^3} \frac{dE}{dx} \frac{1}{m}
\]

General:

\[
\frac{dM}{dt}(t) = \frac{dE}{dx}(t)
\]

\[
\frac{dE}{dt}(t) = v(t) \frac{dM}{dt}(t) \rightarrow \frac{dM}{dt}(t) > \frac{dE}{dt}(t)
\]
Jet – Energy Loss Studies

- Jet deposits energy and momentum along a trajectory
- Applying linearized hydrodynamics


Diffusion wake

Mach cone for sound waves
Jets in AdS/CFT


Energy density perturbation

Poynting vector perturbation

Diffusion Wake contribution

Attention: No clear Mach cone signal
Jets in AdS/CFT II

Non-Mach correlations caused by Neck region


\[
\frac{dN(\phi)}{d\phi} - \frac{dN(0)}{d\phi}
\]

\[\cos \phi_M = c_s / v\]
Jets in pQCD I

Considering a static medium and linearized hydrodynamics for a punch-though jet


Mach cone signal & Diffusion Wake
Jets in pQCD II

Contour plots of magnitude of perturbed momentum density

\[ \frac{\eta}{s} = \frac{1}{4\pi} \quad \frac{\eta}{s} = \frac{3}{4\pi} \quad \frac{\eta}{s} = \frac{6}{4\pi} \]

→ Strong flow in jet-direction

\[ \frac{dM}{dt} > \frac{dE}{dt} \]
Jet – Studies: 3-Particle Correlations

3-Particle Correlations seem to corroborate the Mach Cone picture

Deflected jet

Mach Cone

→ Experimental data show 2 off-diagonal peaks