Thermalization of a Color Glas Condensate in a partonic cascade

Andrej El, Zhe Xu, Carsten Greiner

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Motivation

> Thermal equilibration of a quark-gluonic system produced in HIC occurs on a short time scale (ideal hydro to describe v_2)

> Possible initial condtions (Color Glass Condensate) are anisotropic

> At early stage, coherent quantum effects may play a role, but...

> Bremsstrahlung processes play a crucial role for momentum isotropisation

("Bottom Up" thermalisation)

>> elastic collisions only are not sufficient to achieve thermal equilibrium

>> as the system becomes more delute, pQCD becomes applicable and important

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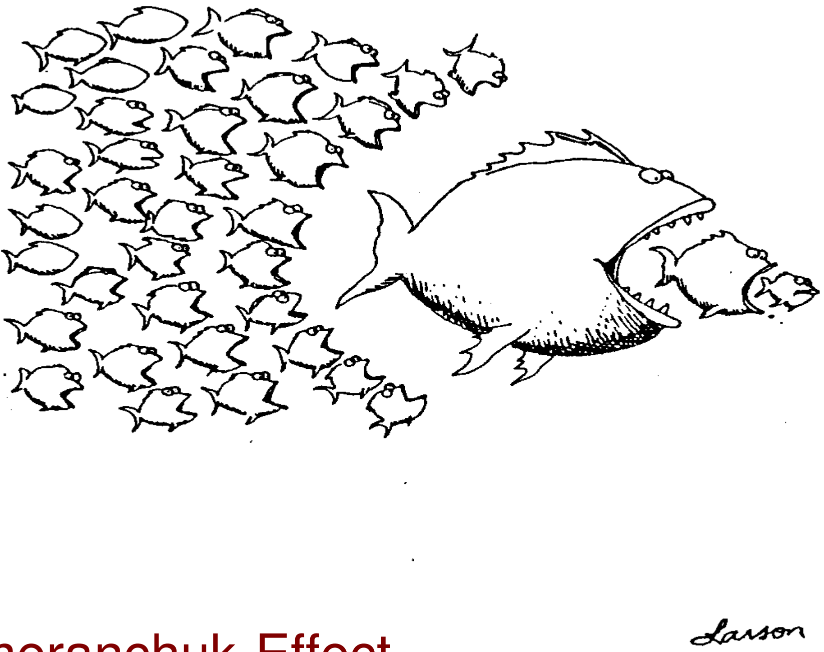
We investigate thermalization of a CGC in the partonic cascade BAMPS with 2↔2 and 2↔3 pQCD processes

Bottom-Up Scenario of Thermalization

Baier, Mueller, Schiff, Son, PL B 51(2001) 502



1. $Q_S^{-1} \ll t \ll \alpha^{-3/2} Q_S^{-1}$

Initial system is dominated by „hard“ gluons with $p_t \sim Q_S$.

2. $\alpha^{-3/2} Q_S^{-1} \ll t \ll \alpha^{-5/2} Q_S^{-1}$

In **inelastic collisions** soft gluons are produced.
Production of hard suppressed by the Landau-Pomeranchuk-Effect
Number of soft gluons increases.
Soft gluons thermalize among themselves and build up a **thermal bath**.

3. $\alpha^{-5/2} Q_S^{-1} \ll t < \alpha^{-13/5} Q_S^{-1}$

Hard gluons lose their entire energy to the thermal bath, built up by soft gluons.
The system thermalizes.

Estimated timescale for thermalization $\tau_{th} \sim \alpha_s^{-13/5} Q_S^{-1}$

BAMPS

Z.Xu, C.Greiner, A.El, O.Fochler , I.Bouras

[Xu, Greiner, PR C 71 (2005) 064901]

> Solving the Boltzmann equation for on-shell gluons (and quarks) using Monte Carlo techniques

> 2<->3 processes included (detailed balance)
> stochastic interpretation of collision rates

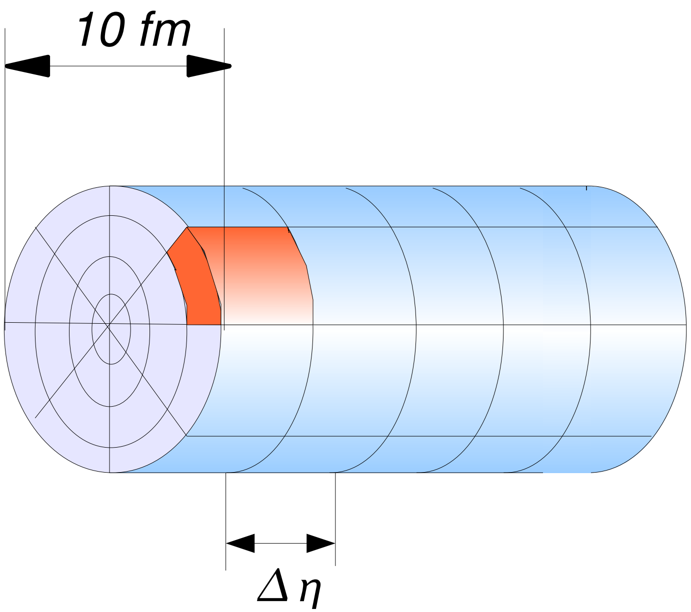
----- in particular for this work
>> Boltzmann gas of gluons (only)
>> 1-Dim Bjorken expanding geometry
>> pQCD calculated cross sections

Boltzmann
Approach for
Multi-
Parton
Scatterings

BAMPS: Construction

1-Dim expansion: $v_z = \frac{z}{\tau} = \tanh\eta$

Boost-invariant „tube geometry“



> Within a time step collisions only with particles from the same cell

> In transverse direction: reflection on cylindrical wall with R=*5 fm*. One wall reflection per time step

> Test particle method applied, at least 11 particles/cell

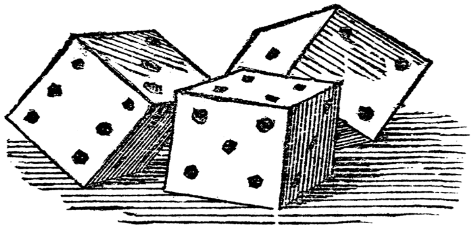
> Bins are constructed equidistant in longitudinal *η*-Space

> Stochastical Method applied to calculate collision probabilities

Collision Terms in the Boltzmann Equation

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}_1}{E_1}\vec{\nabla}\right)f_1(\vec{r},\vec{p},t)=C_{22}+C_{23}$$

No Bose Enhancement factors in collision terms



$$P_{22,23}=v_{rel}\frac{\sigma_{22,23}}{N_{test}}\frac{\Delta t}{\Delta^3 x}$$

$$\sigma_{23}=\frac{1}{2s}\frac{1}{(2\pi)^9}\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3}|M_{1'2'\rightarrow 123}|^2\cdot(2\pi)^4\delta^{(4)}(p'_1+p'_2-p_1-p_2-p_3)$$

Matrix element 2->3: Gunion-Bertsch formula

$$|M_{gg\rightarrow ggg}|^2=\frac{9g^4}{2}\frac{s^2}{(\vec{q}_t^{\,2}+m_D^2)^2}\cdot\frac{12g^2\vec{q}_t^{\,2}}{\vec{k}_t^{\,2}[(\vec{k}_t-\vec{q}_t)^2+m_D^2]}\cdot\Theta(k_t\Lambda_g-\cosh y)$$

LPM Effect: (formation time < mean free pass Λ_g)

$$P_{32}=\frac{1}{8E_1E_2E_3}\frac{I_{32}}{N_{test}^2}\frac{\Delta t}{(\Delta^3 x)^2}$$

$$|M_{123\rightarrow 1'2'}|^2=\frac{1}{16}|M_{1'2'\rightarrow 123}|^2$$

$$I_{32}=\frac{1}{2}\frac{1}{(2\pi)^6}\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}|M_{1'2'3'\rightarrow 12}|^2\cdot(2\pi)^4\delta^{(4)}(p'_1+p'_2+p'_3-p_1-p_2)$$

s. also: Talk by Zhe Xu today afternoon

Color Glass Condensate
[McLerran, Venugopalan , PR D 49 (1994) 2233]

A simple, „idealistic“ initial parton distribution

$$f(x,p) = \frac{c}{\alpha_s N_c} \frac{1}{t_{ini}} \delta(p_z) \Theta(Q_s^2 - p_t^2) \qquad \text{with} \quad t_{ini} = \frac{c}{\alpha_s N_c} \frac{1}{Q_S}$$

$$\frac{dN}{d\eta} = c \quad \pi R^2 \frac{N_c{}^2 - 1}{4\pi^2 \alpha_s N_c} Q_s{}^2$$

Saturation for $p_T < Q_S$, high occupation number;
for gluons with higher transverse momenta occupation number = 0.

c=0.4
(A.Krasnitz, Y.Nara, R.Venugopalan, PRL 87 (2001) 192302 NPA 727(2003) 427-436
T.Lappi, PRC bf 67(2003) 054903)

The initial conditions

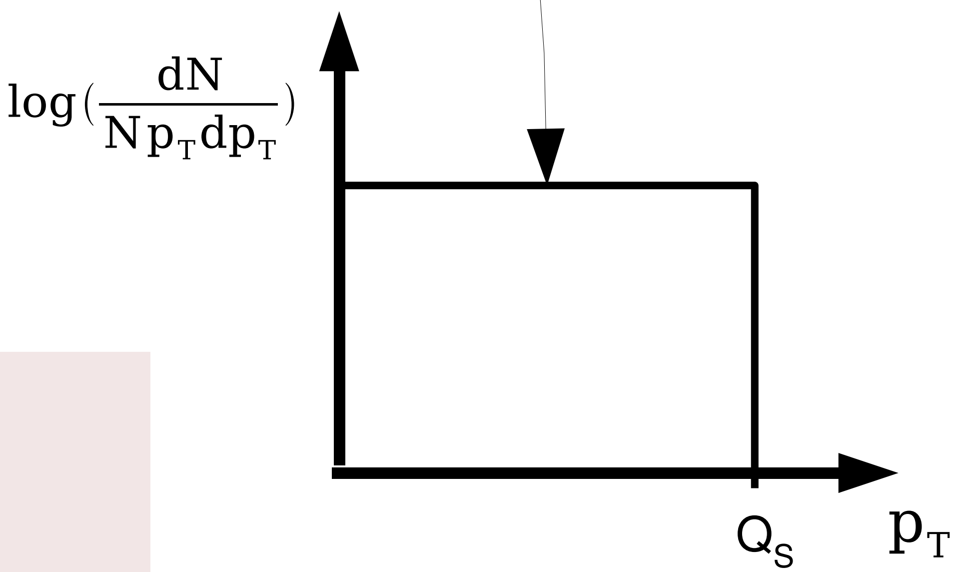
Color Glass Condensate

$$f(x,p)=\frac{c}{\alpha_s N_c}\frac{1}{t_{ini}}\delta(p_z)\Theta(Q_s^2-p_t^2)$$ with $$t_{ini}=\frac{c}{\alpha_s N_c}\frac{1}{Q_S}$$

$$\frac{dN}{d\eta}=c\quad \pi R^2\frac{N_c^2-1}{4\pi^2\alpha_s N_c}Q_s^2$$

RHIC: Q_S=2 GeV (mean value of saturation momentum, small x)

LHC: Q_S ~4 GeV



Sampling:

$$\eta\in[-3;\ 3]$$
$$y=\eta$$
$$p_t^2\in[0;Q_s^2]$$
$$p_z=p_t\sinh\ y$$

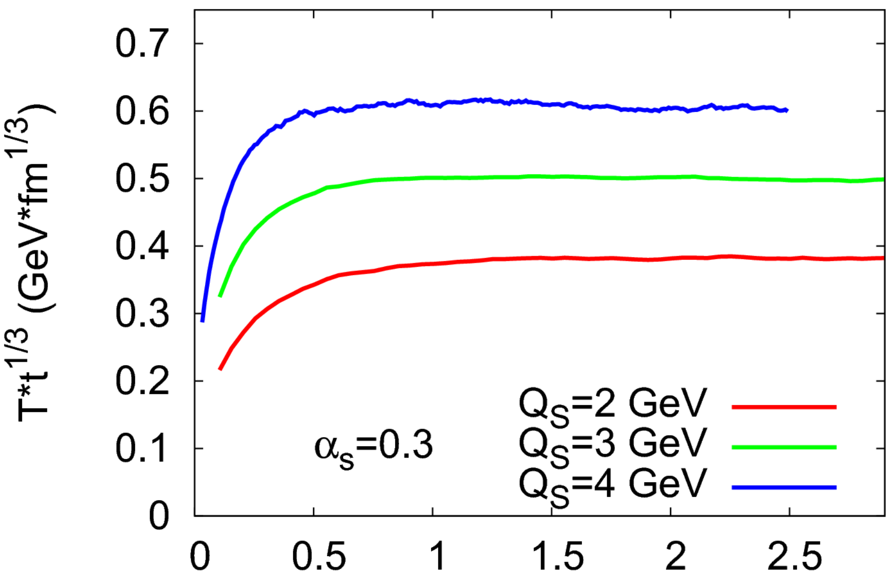
Simulations with

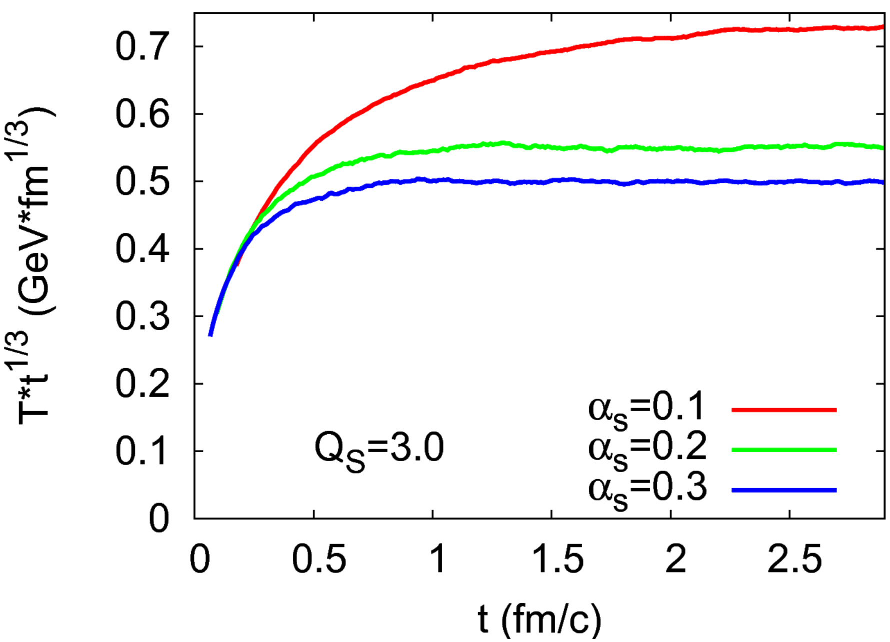
Q_S=2, 3, 4 GeV, α_s=0.3

Q_S=3 GeV, α_s=0.1, 0.2, 0.3

Scaled Temperature

$T(t) \cdot t^{\frac{1}{3}}$





$$T = \frac{e}{3n}$$

$$e = 3p$$

$$t_{th}(\alpha_s = 0.3, Q_S = 2\,\mathrm{GeV}) = 1.2\,\mathrm{fm/c}$$
$$t_{th}(\alpha_s = 0.3, Q_S = 3\,\mathrm{GeV}) = 0.75\,\mathrm{fm/c}$$
$$t_{th}(\alpha_s = 0.3, Q_S = 4\,\mathrm{GeV}) = 0.55\,\mathrm{fm/c}$$

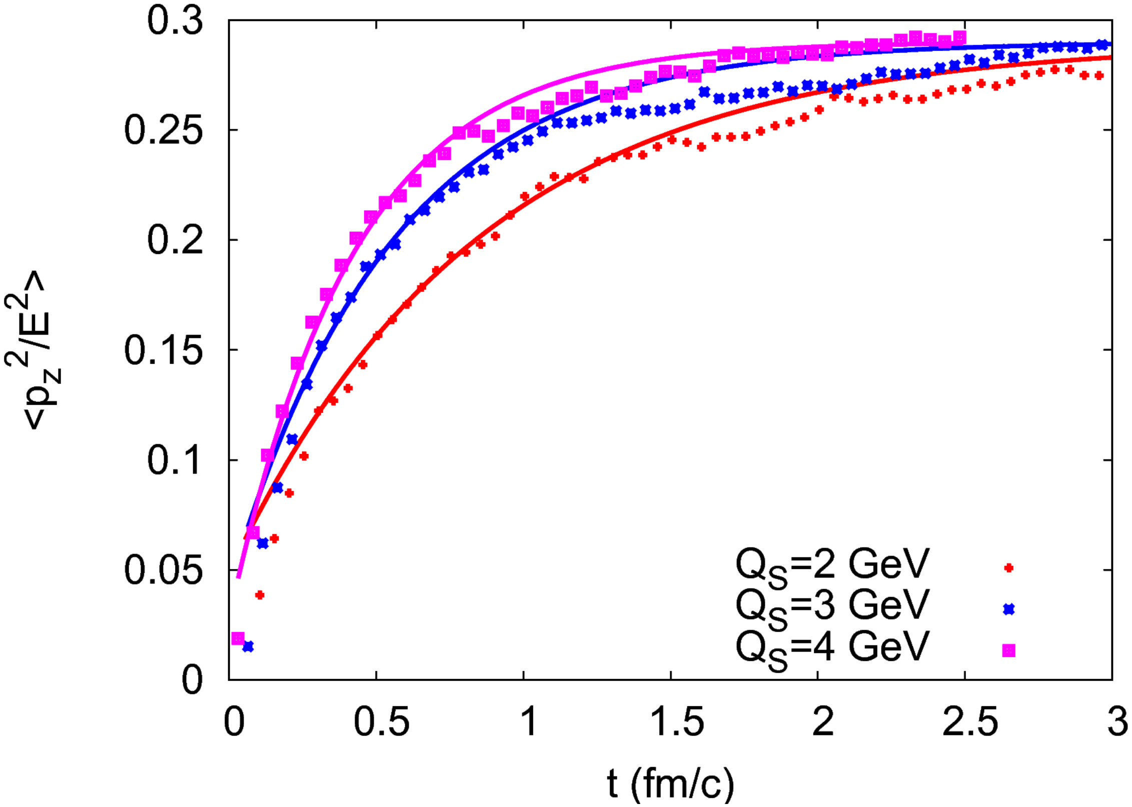
$$t_{th}(\alpha_s = 0.1, Q_S = 3\,\mathrm{GeV}) = 1.75\,\mathrm{fm/c}$$
$$t_{th}(\alpha_s = 0.2, Q_S = 3\,\mathrm{GeV}) = 1.0\,\mathrm{fm/c}$$
$$t_{th}(\alpha_s = 0.3, Q_S = 3\,\mathrm{GeV}) = 0.75\,\mathrm{fm/c}$$

Comparison with Bottom-Up:

$t_{th} \sim \frac{1}{Q_S}$ **fullfilled** $t_{th} \sim \alpha_s^{-\frac{13}{5}}$ **wrong**

BAMPS: $t_{th} \sim \alpha_s^{-2} (\ln \alpha_s)^{-2} Q_S^{-1}$

Momentum Isotropy



$$Q(t)=\frac{1}{3}+\left(\left\langle\frac{p_z^2}{E^2}\right\rangle(t_0)-\frac{1}{3}\right)\exp\left(-\frac{t-t_0}{\theta_{rel}(t)}\right)$$

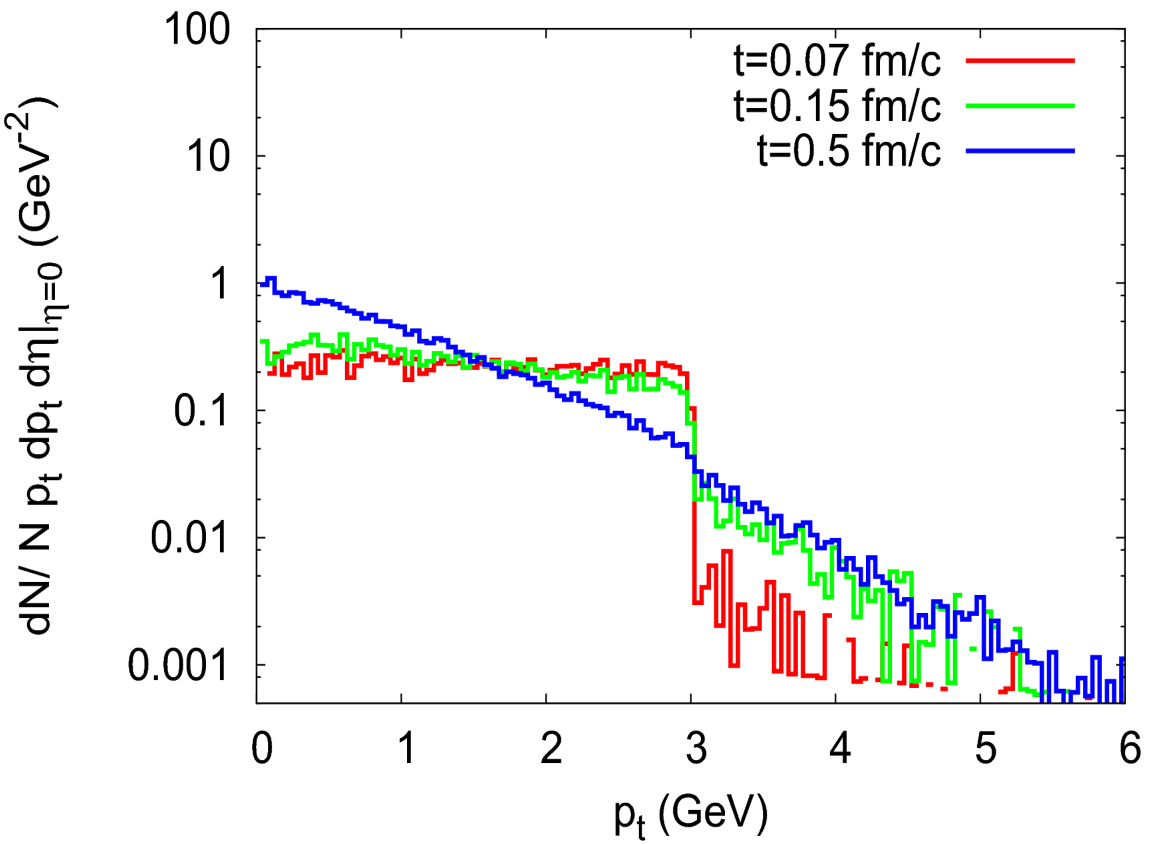
Fits with $t_0=0.5\,\mathrm{fm}$

$\theta_{rel}=0.85, 0.52, 0.42\,\mathrm{fm}$

Isotropisation completed before full thermalisation

Complete isotropization not possible due to viscous effects.
Value of viscosity and η/s ?

Momentum Spectra



(Q_s=3 GeV, α_s=0.3)

> Particle annihilation and refilling of high momentum region at early times

> Produced “hard” gluons are immediately thermal

>> “Soft” and “hard” spectra thermal on same time scale

Considering a more “realistic” initial condition

Drescher, Nara
Phys.Rev. C75 (2007) 034905

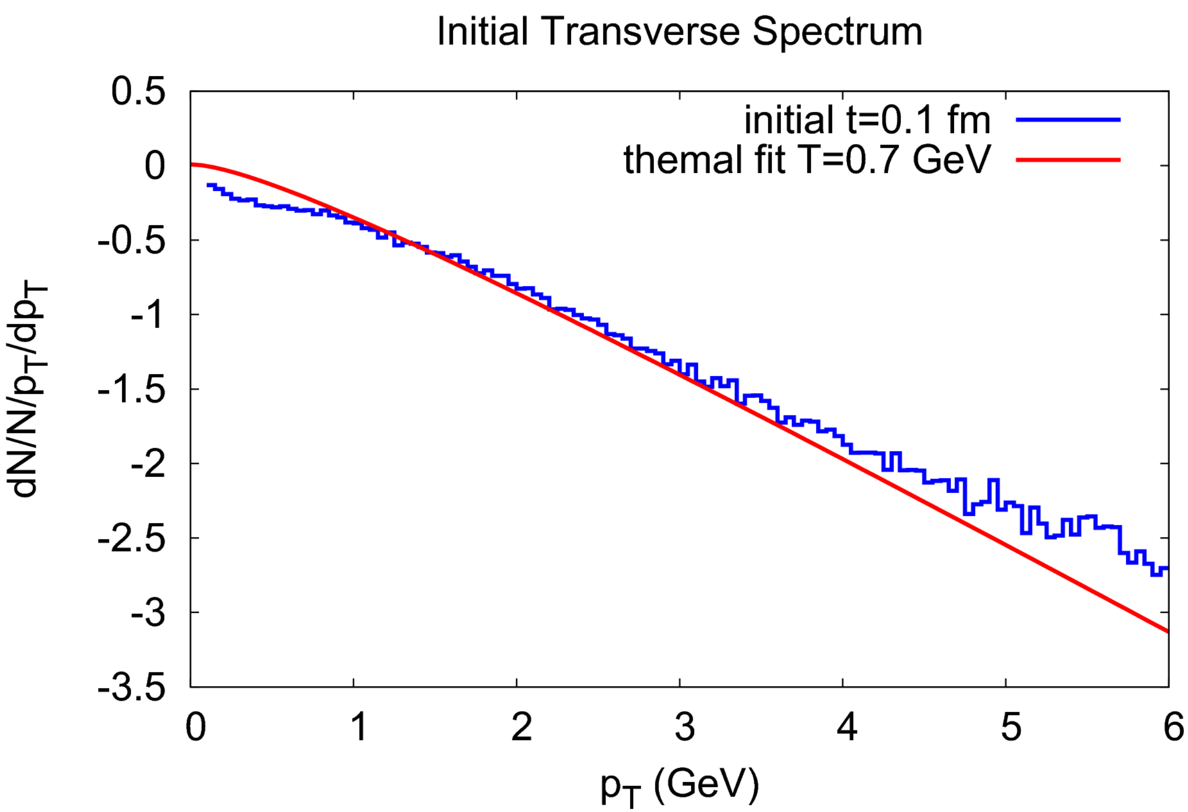
<u>KLN Approach:</u>

$$\frac{dN_{gluon}}{d^2 r_\perp dy} = \frac{4 N_c}{N_c^2 - 1} \int^{p_\perp^{max}} \frac{d^2 p_\perp}{p_\perp^2} \int^{p_\perp} \frac{d^2 k_\perp}{4} \alpha_s \phi_A\left(x_1, \frac{(p_\perp + k_\perp)^2}{4}\right) \phi_B\left(x_2, \frac{(p_\perp - k_\perp)^2}{4}\right)$$

$$\left(x_{1,2} = p_\perp \exp(\pm y)/\sqrt{s}\right)$$

$$\phi(x, k_\perp^2; r_\perp) \sim \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{\max(Q_s^2, k_\perp^2)}$$

D. Kharzeev and M. Nardi, Phys. Lett. B507, 121 (2001)



Provides: x, y, p_T, y

We assume $\eta = y\ ,\ \ \tau_0 = \frac{1}{\langle Q_s \rangle}$

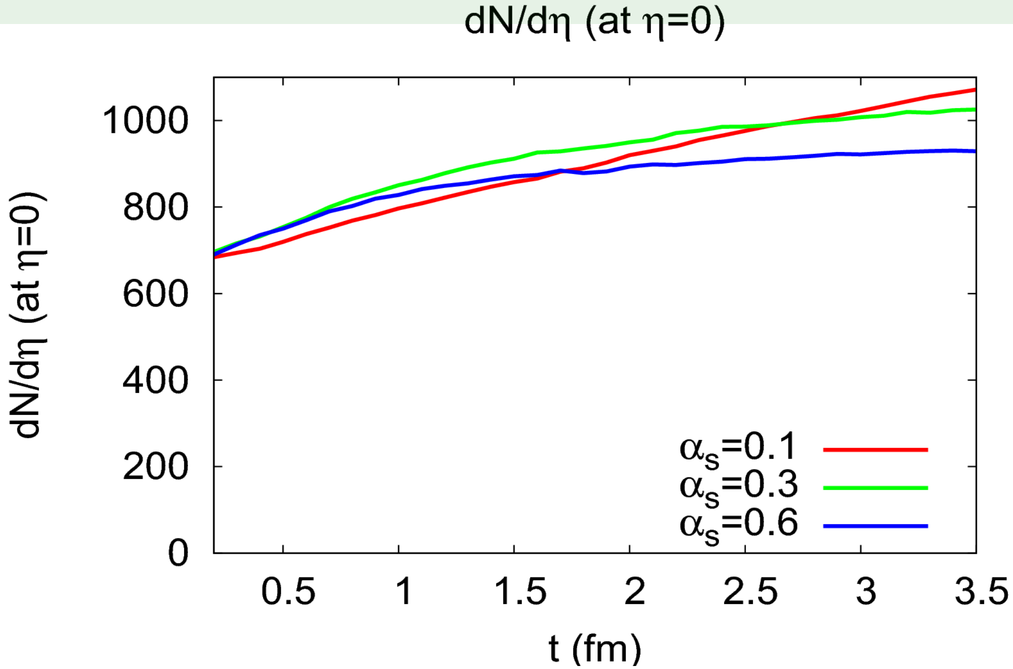
For Au+Au at 200 AGeV:

$$\frac{dN}{d\eta}(\text{initial}) = 700 \quad y = \eta \in (-3; 3)$$

$$\tau_0 = 0.1\,\text{fm}$$

$$\alpha_s = 0.1,\ 0.3,\ 0.6$$

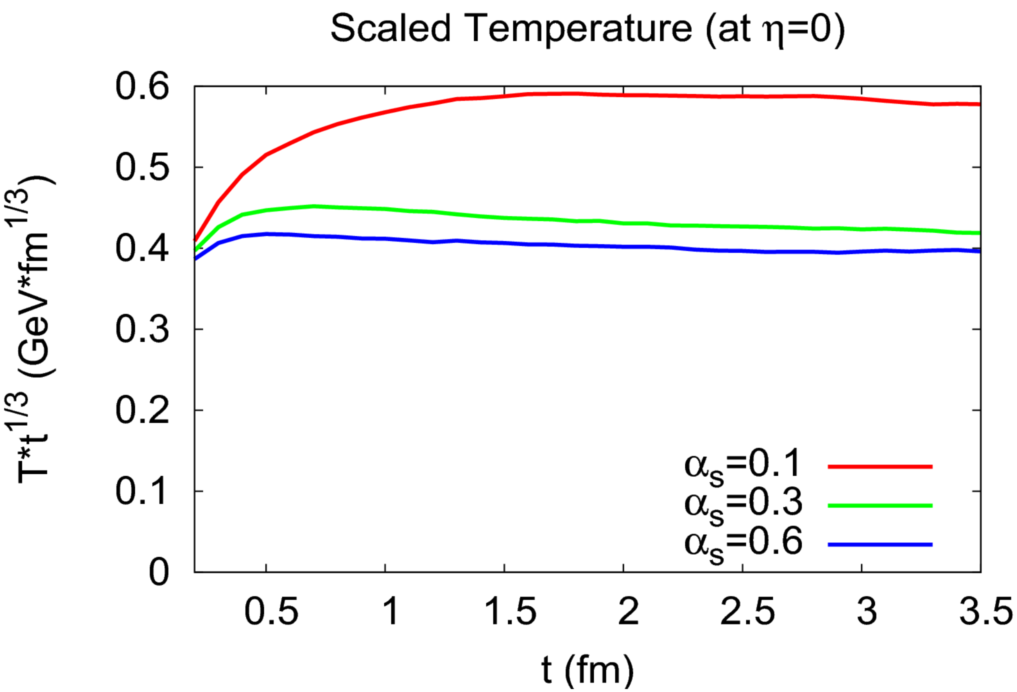
Temperature, Multiplicity



$$1\,\mathrm{gluon} \Leftrightarrow 1\,\mathrm{meson}$$

$$\frac{dN}{d\eta}_{\mathrm{charged}} = \frac{2}{3}\frac{dN}{d\eta}_{\mathrm{gluons}}$$

Final gluon multiplicity ~ 1000 (charged 700) @ PHOBOS

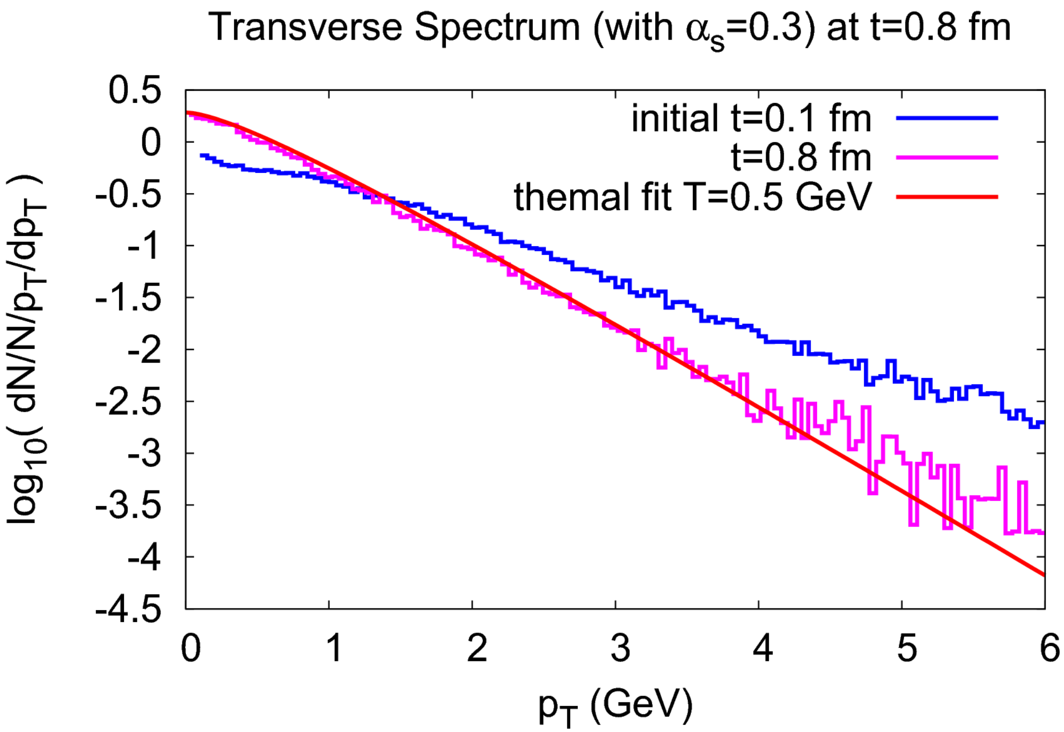


Thermal by approx.

0.5 *fm*, 0.75 *fm*, 1.5 *fm*

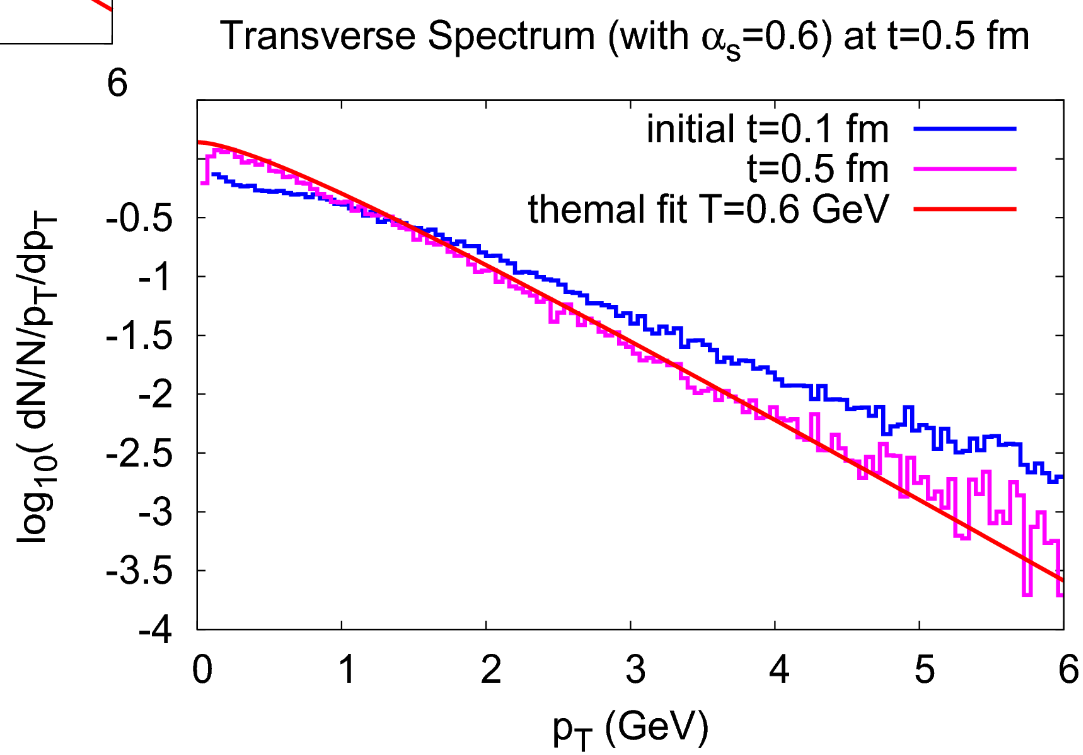
Spectra

Thermal shape by 0.5 fm (α_S =0.6)
resp. 0.8 fm (α_S=0.3)

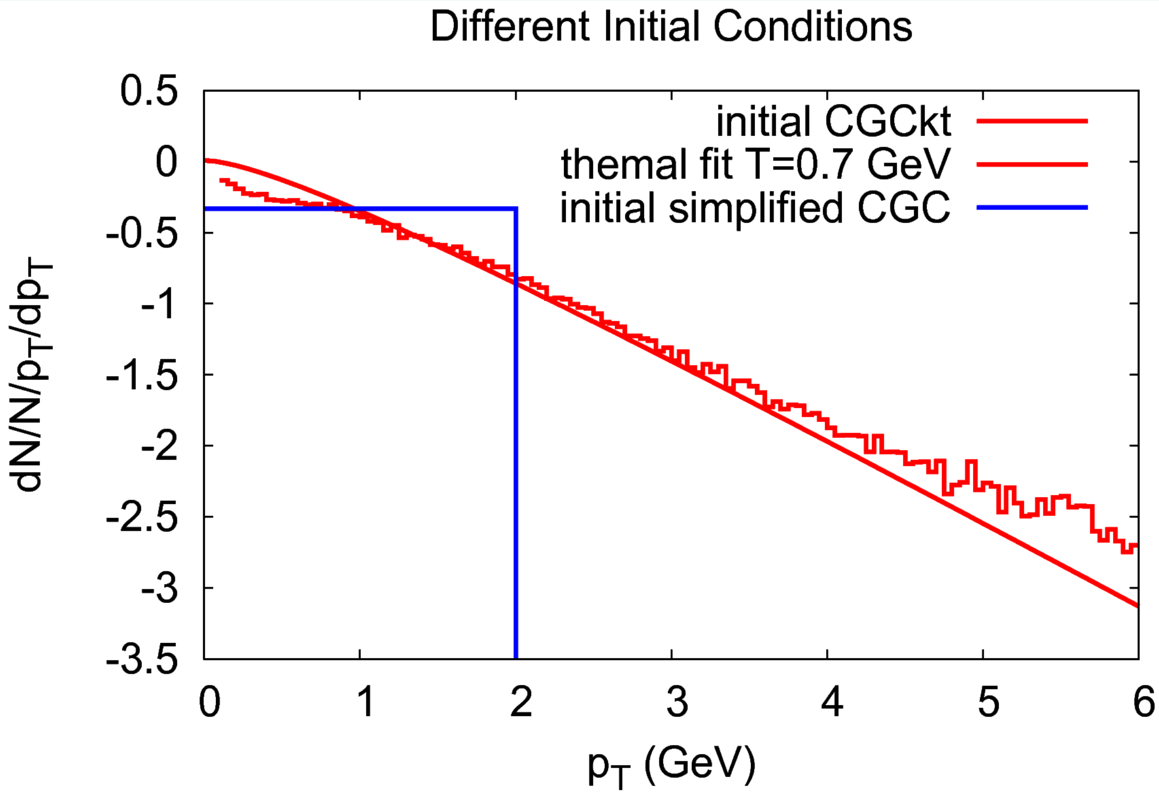


Fit with Boltzmann distribution

$$f(x,p)=\frac{16}{(2\pi)^3}e^{-E/T}$$



What's different



CGCkt:
$< Q_s > \simeq 2\,\mathrm{GeV}$

> CGCkt spectrum is considerably harder. Particle production processes dominate, "cooling" the hard region.

> With CGCkt 'Bottom Up' is realized much better than with the simplified of CGC

Hydrodynamic evolution in BAMPS

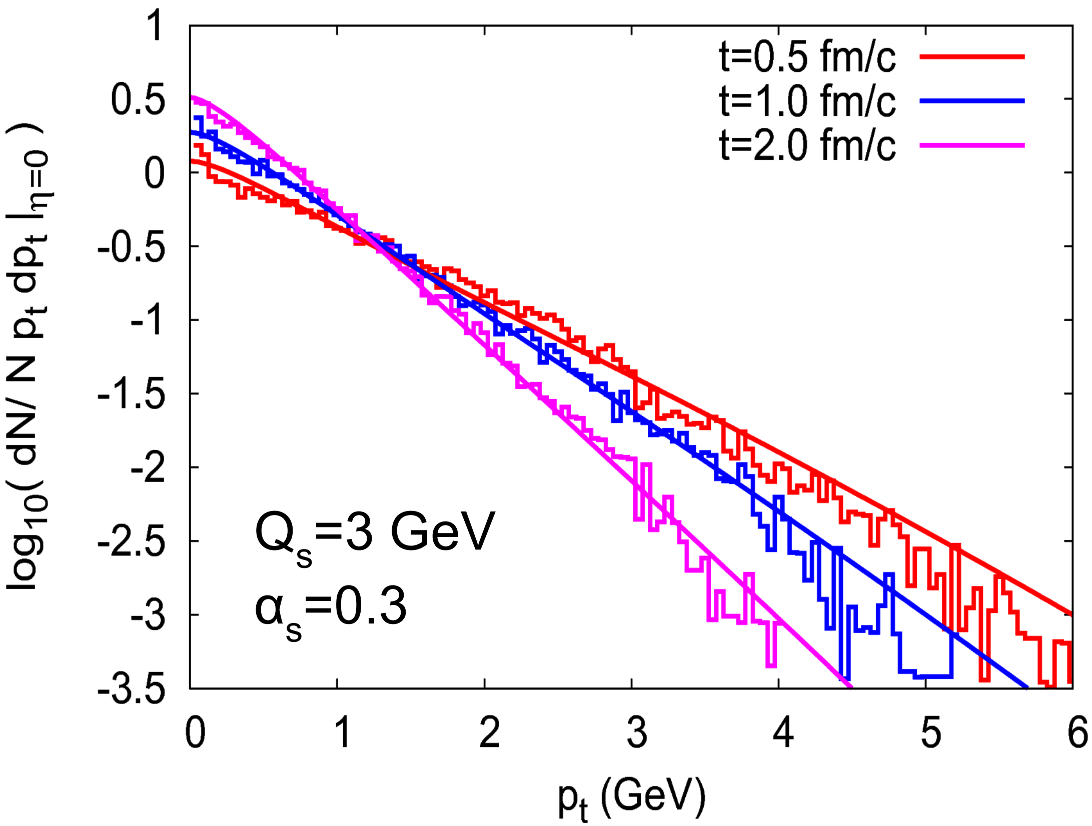
Fitting transverse spectra with

$$f(x,p)=\frac{16}{(2\pi)^3}e^{-\frac{p_T}{T}\cosh y}\left(1+C_0\cdot\pi_{\mu\nu}p^\mu p^\nu\right)$$

In case of (0+1) Dim expansion,
in local rest frame:

$$\pi_{\mu\nu}=\mathrm{diag}(0,-\bar{\pi}/2,-\bar{\pi}/2,\bar{\pi})$$

(shear tensor)



with $\bar{\pi}=\frac{1}{3}e-T_{33}$

and $C_0=\frac{\pi^2}{128\lambda T^6}$

$$f(x,p)=\frac{16}{(2\pi)^3}e^{-\frac{p_T}{T}\cosh y}\left(1+\frac{\pi^2}{128\lambda T^6}\cdot\bar{\pi}\left(p_Z^2-1/2\,p_T^2\right)\right)$$

Shear viscosity from BAMPS

Using the ansatz

$$f(p)=\frac{16}{(2\pi)^3}e^{-\frac{p_T}{T}\cosh y}\left(1+C_0\cdot\pi_{\mu\nu}p^{\mu}p^{\nu}\right)$$

we can calculate the shear viscosity

From

$$S^{\mu}(x)=-\frac{g}{(2\pi)^3}\int\frac{d^3p}{E}p^{\mu}f(x,p)\ln[g^{-1}f(x,p)-1]$$

follows

$$\partial_\mu S^{\mu}=-\frac{g}{(2\pi)^3}\int dw\, p^{\mu}\partial_\mu f(x,p)\ln[g^{-1}f(x,p)]$$

$$C[f]$$

(collision term in the Boltzmann Eq.)

On the other hand

$$\partial_\mu S^{\mu}=(2\eta)^{-1}\pi^{\alpha\beta}\pi_{\alpha\beta}$$

$$\eta=-\frac{\pi_{\mu\nu}\pi^{\mu\nu}}{2\cdot C_0\pi_{\mu\nu}P^{\mu\nu}}$$

with $P_{\alpha\beta}=\frac{1}{(2\pi)^3}\int\frac{d^3p}{E}p_\alpha p_\beta C[f]$

Comparison between hydro (IS) and BAMPS to be reported soon

Calculating η/s with pQCD crossections, including 2<->3 processes

1. Solve Israel-Stewart: (e.g. 0+1 case)

$$\frac{d\epsilon}{d\tau}+\frac{\epsilon+p}{\tau}-\frac{\bar{\pi}}{\tau}=0$$

$$\frac{d\bar{\pi}}{d\tau}=\frac{-\bar{\pi}}{2\beta_2\eta}-\frac{\bar{\pi}}{2}\left(\frac{1}{\tau}+\frac{1}{\beta_2}T\frac{d}{d\tau}\left(\frac{\beta_2}{T}\right)\right)+\frac{2}{3}\frac{1}{\beta_2\tau}$$

$$\frac{dn}{d\tau}+\frac{n}{\tau}=[\text{Source Term}]$$

with

$$T=\frac{\epsilon}{3n}$$

$$\beta_2=3/(4p)$$

$$\epsilon=3p$$

e.g. G.Baym et al, PRL 64 (1990) 1867

2. Model f(x,p) using T,π

$$f(x,p)=\frac{16}{(2\pi)^3}e^{-\frac{p_T}{T}\cosh y}\left(1+\frac{\pi^2}{128\lambda T^6}\cdot\bar{\pi}(p_Z^2-1/2\,p_T^2)\right)$$

3. Calculate η using

$$\eta=-\frac{\pi_{\mu\nu}\pi^{\mu\nu}}{2C_0\pi_{\mu\nu}P^{\mu\nu}}$$

with

$$P_{\alpha\beta}=\frac{1}{(2\pi)^3}\int\frac{d^3p}{E}p_\alpha p_\beta C[f]$$

s. also: Talk by Zhe Xu today afternoon

Summary

> Equilibration of quark and gluon matter at RHIC and LHC on a shorter time scale than the one suggested by "Bottom-Up" scenario is observed in BAMPS

> "hard" and "soft" gluons thermalize almost simultaneously

> thermalization times are ~1.2 fm/c for RHIC and ~0.6 fm/c for LHC regimes

> Thermalization even quicker with KLN CGC

> thermalization time is proportional to $\alpha_s^{-2}(\ln\alpha_s)^{-2}Q_S^{-1}$, thus the dependence on α_s is weaker than the "Bottom-Up" result

Thanks to the organizers,

Zhe Xu, Carsten Greiner

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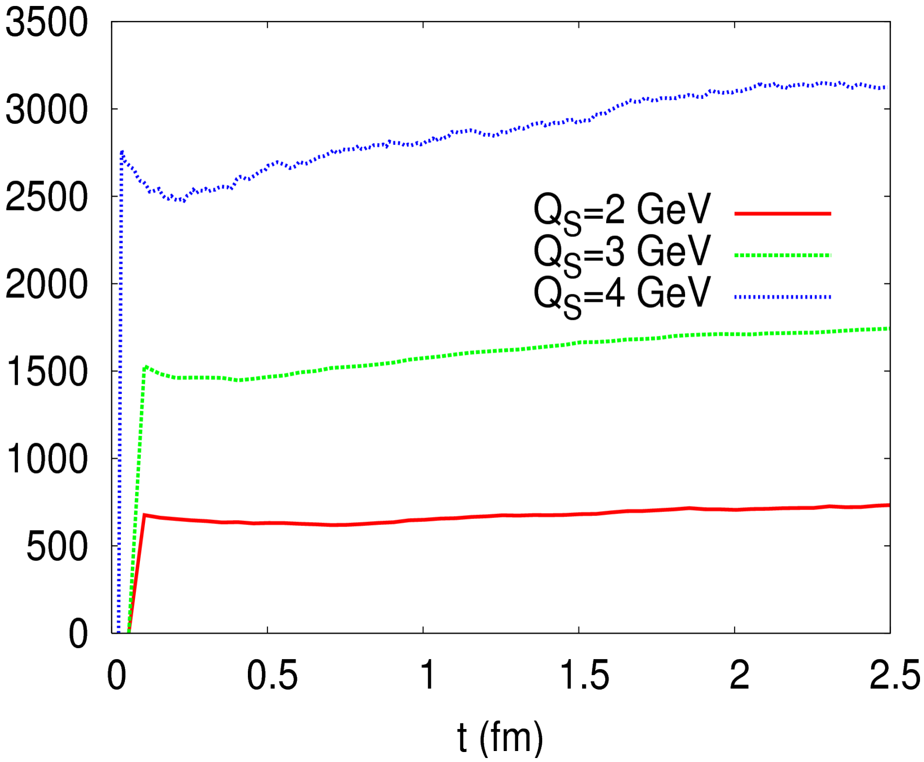
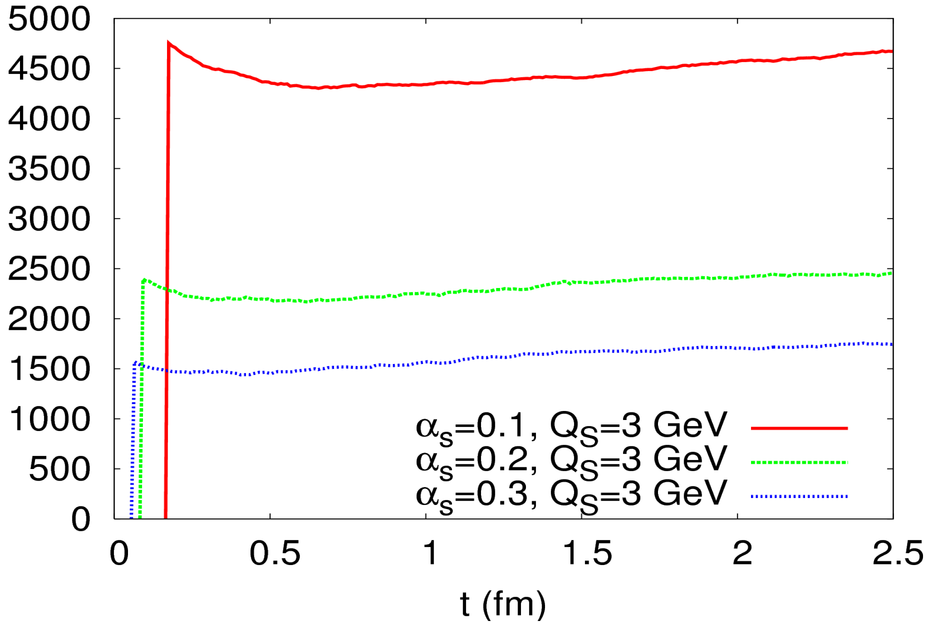
H-QM Helmholtz Research School Quark Matter Studies

Gluon Number





> during the first 0.3-0.75 fm/c: gluon annihilation (~10% of initial number).
3->2 processes dominant at early times

> parametric enhancement of total gluon number (“Bottom-Up”) not observed

> the initial CGC is oversaturated.