Thermalization of a Color Glas Condensate in a partonic cascade

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Thermal equilibration of a quark-gluonic system produced in HIC occurs on a short time scale (ideal hydro to describe $v_2$).

Possible initial conditions (Color Glass Condensate) are anisotropic.

At early stage, coherent quantum effects may play a role, but...

Bremsstrahlung processes play a crucial role for momentum isotropisation ("Bottom Up" thermalisation).

elastic collisions only are not sufficient to achieve thermal equilibrium.

as the system becomes more delute, pQCD becomes applicable and important.

We investigate thermalization of a CGC in the partonic cascade BAMPS with $2 \leftrightarrow 2$ and $2 \leftrightarrow 3$ pQCD processes.
Bottom-Up Scenario of Thermalization

Baier, Mueller, Schiff, Son, PL B 51(2001) 502

1. \(Q_s^{-1} \ll t \ll \alpha^{-3/2} Q_s^{-1}\)

   Initial system is dominated by „hard“ gluons with \(p_t \sim Q_s\).

2. \(\alpha^{-3/2} Q_s^{-1} \ll t \ll \alpha^{-5/2} Q_s^{-1}\)

   In inelastic collisions soft gluons are produced. Production of hard suppressed by the Landau-Pomeranchuk-Effect

   Number of soft gluons increases.

   Soft gluons thermalize among themselves and build up a thermal bath.

3. \(\alpha^{-5/2} Q_s^{-1} \ll t < \alpha^{-13/5} Q_s^{-1}\)

   Hard gluons lose their entire energy to the thermal bath, built up by soft gluons. The system thermalizes.

Estimated timescale for thermalization \(\tau_{th} \sim \alpha_s^{-13/5} Q_s^{-1}\)
Solving the Boltzmann equation for on-shell gluons (and quarks) using Monte Carlo techniques

- 2<->3 processes included (detailed balance)
- stochastic interpretation of collision rates

----- in particular for this work
>> Boltzmann gas of gluons (only)
>> 1-Dim Bjorken expanding geometry
>> pQCD calculated cross sections
1-Dim expansion: \( v_z = \frac{Z}{\tau} = \tanh \eta \)

Boost-invariant „tube geometry“

> Within a time step collisions only with particles from the same cell

> In transverse direction: reflection on cylindrical wall with \( R=5 \text{ fm} \). One wall reflection per time step

> Test particle method applied, at least 11 particles/cell

> Bins are constructed equidistant in longitudinal \( \eta \)-Space

> Stochastical Method applied to calculate collision probabilities
Collision Terms in the Boltzmann Equation

\[
\left( \frac{\partial}{\partial t} + \frac{\vec{p}_1}{E_1} \vec{V} \right) f_1(\vec{r}, \vec{p}, t) = C_{22} + C_{23}
\]

No Bose Enhancement factors in collision terms

\[
P_{22,23} = v_{\text{rel}} \frac{\sigma_{22,23} \Delta t}{N_{\text{test}} \Delta^3 x}
\]

\[
\sigma_{23} = \frac{1}{2} s \frac{1}{(2\pi)^9} \int \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} \frac{d^3 p_3}{2E_3} |M_{1'2\rightarrow123}|^2 \cdot (2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2 - p_3)
\]

Matrix element 2->3: Gunion-Bertsch formula

\[
|M_{gg\rightarrow ggg}|^2 = \frac{9g^4}{2} \frac{s^2}{(q_t^2 + m_D^2)^2} \cdot \frac{12g^2 q_t^2}{k_t^2 [(k_t - q_t)^2 + m_D^2]} \cdot \Theta(k_t \Lambda_g - \cosh y)
\]

LPM Effect: (formation time < mean free pass $\Lambda_g$)

\[
P_{32} = \frac{1}{8E_1E_2E_3} \frac{I_{32}}{N_{\text{test}}^2 (\Delta^3 x)^2} \Delta t
\]

\[
|I_{32}|^2 = \frac{1}{16} |M_{123\rightarrow1'2'}|^2
\]

\[
I_{32} = \frac{1}{2} \frac{1}{(2\pi)^6} \int \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} |M_{1'2'\rightarrow12}|^2 \cdot (2\pi)^4 \delta^{(4)}(p_1' + p_2' + p_3' - p_1 - p_2)
\]

s. also: Talk by Zhe Xu today afternoon
A simple, "idealistic" initial parton distribution

\[ f(x, p) = \frac{c}{\alpha_s N_c t_{\text{ini}}} \frac{1}{\delta(p_z) \Theta(Q_s^2 - p_t^2)} \]

with \( t_{\text{ini}} = \frac{c}{\alpha_s N_c Q_s} \)

\[ \frac{dN}{d\eta} = c \pi R^2 \frac{N_c^2 - 1}{4 \pi^2 \alpha_s N_c} Q_s^2 \]

Saturation for \( p_T < Q_s \), high occupation number;
for gluons with higher transverse momenta occupation number = 0.

\( c = 0.4 \)

T.Lappi, PRC bf 67(2003) 054903)
The initial conditions

Color Glass Condensate

\[
f(x, p) = \frac{c}{\alpha_s N_c} \frac{1}{t_{\text{ini}}} \delta(p_z) \Theta(Q_s^2 - p_t^2)\]

\[
\frac{dN}{d\eta} = c \pi R^2 \frac{N_c^2 - 1}{4 \pi^2 \alpha_s N_c} Q_s^2
\]

RHIC: \( Q_s = 2 \) GeV (mean value of saturation momentum, small x)
LHC: \( Q_s \approx 4 \) GeV

Simulations with
\( Q_s = 2, 3, 4 \) GeV, \( \alpha_s = 0.3 \)
\( Q_s = 3 \) GeV, \( \alpha_s = 0.1, 0.2, 0.3 \)

Sampling:
\( \eta \in [-3; 3] \)
\( y = \eta \)
\( p_t^2 \in [0; Q_s^2] \)
\( p_z = p_t \sinh y \)
Scaled Temperature

\[ T = \frac{e}{3n} \]

\[ e = 3p \]

\[ t_{\text{th}}(\alpha_s=0.3, Q_S=2\,\text{GeV}) = 1.2\,\text{fm/c} \]
\[ t_{\text{th}}(\alpha_s=0.3, Q_S=3\,\text{GeV}) = 0.75\,\text{fm/c} \]
\[ t_{\text{th}}(\alpha_s=0.3, Q_S=4\,\text{GeV}) = 0.55\,\text{fm/c} \]

\[ t_{\text{th}}(\alpha_s=0.1, Q_S=3\,\text{GeV}) = 1.75\,\text{fm/c} \]
\[ t_{\text{th}}(\alpha_s=0.2, Q_S=3\,\text{GeV}) = 1.0\,\text{fm/c} \]
\[ t_{\text{th}}(\alpha_s=0.3, Q_S=3\,\text{GeV}) = 0.75\,\text{fm/c} \]

Comparison with Bottom-Up:

\[ t_{\text{th}} \sim \frac{1}{Q_S} \quad \text{fulfilled} \]
\[ t_{\text{th}} \sim \alpha_s^{-\frac{13}{5}} \quad \text{wrong} \]

BAMPS:

\[ t_{\text{th}} \sim \alpha_s^{-2}(\ln \alpha_s)^{-2} Q_S^{-1} \]
Momentum Isotropy

\[ Q(t) = \frac{1}{3} + \left( \frac{p_z^2}{E^2}(t_0) - \frac{1}{3} \right) \exp \left( -\frac{t-t_0}{\theta_{rel}(t)} \right) \]

Fits with \( t_0 = 0.5 \text{ fm} \)
\( \theta_{rel} = 0.85, 0.52, 0.42 \text{ fm} \)

Isotropisation completed before full thermalisation

Complete isotropization not possible due to viscous effects.
Value of viscosity and \( \eta/s \)?
Momentum Spectra

Particle annihilation and refilling of high momentum region at early times

Produced “hard” gluons are immediately thermal

“Soft” and “hard” spectra thermal on same time scale

(Q_s = 3 GeV, \alpha_s = 0.3)
Considering a more “realistic” initial condition

KLN Approach:

\[
\frac{dN_{\text{gluon}}}{d^2 r \, dy} = \frac{4 N_c}{N_{c}^{2} - 1} \int_{p_{\perp}^{\text{max}}}^{p_{\perp}} \frac{d^2 p_{\perp}}{p_{\perp}^2} \int_{q_{\perp}}^{p_{\perp}} \frac{d^2 k_{\perp}}{4} \alpha_s \phi_A \left( x_1, \frac{(p_{\perp} + k_{\perp})^2}{4} \right) \phi_B \left( x_2, \frac{(p_{\perp} - k_{\perp})^2}{4} \right)
\]

\[
\phi(x, k_{\perp}^2 ; r_{\perp}) \sim \frac{1}{\alpha_s(Q_s^2) \max(Q_s^2, k_{\perp}^2)} \frac{Q_s^2}{x^2 Q_s^2 \max(Q_s^2, k_{\perp}^2)}
\]


Provides: x, y, p_T, y
We assume \( \eta = y \), \( \tau_0 = \frac{1}{<Q_s>} \)

For Au+Au at 200 AGeV:

\[
\frac{dN}{d\eta} \left( \text{initial} \right) = 700 \quad y = \eta \in (-3; 3)
\]
\[
\tau_0 = 0.1 \text{ fm}
\]
\[
\alpha_s = 0.1, 0.3, 0.6
\]
Temperature, Multiplicity

\[ \frac{1}{\text{gluon}} \Leftrightarrow \frac{1}{\text{meson}} \]

\[ \frac{dN}{d\eta_{\text{charged}}} = \frac{2}{3} \frac{dN}{d\eta_{\text{gluons}}} \]

Final gluon multiplicity \( \sim 1000 \) (charged 700) @ PHOBOS

Thermal by approx.

0.5 \( fm \), 0.75 \( fm \), 1.5 \( fm \)
Spectra

Fit with Boltzmann distribution

\[ f(x, p) = \frac{16}{(2\pi)^3} e^{-E/T} \]
What’s different

CGCkt spectrum is considerably harder. Particle production processes dominate, “cooling” the hard region.

CGCkt: $\langle Q_s \rangle \approx 2 \text{GeV}$

With CGCkt ‘Bottom Up’ is realized much better than with the simplified of CGC
Hydrodynamic evolution in BAMPS

Fitting transverse spectra with

\[ f(x, p) = \frac{16}{(2\pi)^3} \frac{1}{e^{pt \cosh y}} \left( 1 + C_0 \cdot \pi_{\mu \nu} p^\mu p^\nu \right) \]

In case of (0+1) Dim expansion, in local rest frame:

\[ \pi_{\mu \nu} = \text{diag}(0, -\pi/2, -\pi/2, \pi) \]

(shear tensor)

with \( \pi = \frac{1}{3} e^{-T_{33}} \)

and \( C_0 = \frac{\pi^2}{128\lambda T^6} \)

\[ f(x, p) = \frac{16}{(2\pi)^3} \frac{1}{e^{pt \cosh y}} \left( 1 + \frac{\pi^2}{128\lambda T^6} \cdot \pi (p_Z^2 - 1/2 p_T^2) \right) \]
Using the ansatz
\[ f(p) = \frac{16}{(2\pi)^3} e^{\frac{p_T}{T} \cosh y} \left( 1 + C_0 \cdot \pi^{\mu \nu} p^\mu p^\nu \right) \]
we can calculate the shear viscosity

From
\[ S^\mu(x) = -\frac{g}{(2\pi)^3} \int \frac{d^3p}{E} p^\mu f(x, p) \ln[g^{-1}f(x, p) - 1] \]
follows
\[ \partial_\mu S^\mu = -\frac{g}{(2\pi)^3} \int dw p^\mu \partial_\mu f(x, p) \ln[g^{-1}f(x, p)] \]

On the other hand
\[ \partial_\mu S^\mu = (2\eta)^{-1} \pi^{\alpha \beta} \pi_{\alpha \beta} \]

\( C[f] \)

(collision term in the Boltzmann Eq.)

\[ \eta = -\frac{\pi^{\mu \nu} \pi^{\mu \nu}}{2 \cdot C_0 \pi^{\mu \nu} P^{\mu \nu}} \]

with
\[ P_{\alpha \beta} = \frac{1}{(2\pi)^3} \int \frac{d^3p}{E} p_\alpha p_\beta C[f] \]

Comparison between hydro (IS) and BAMPS to be reported soon
Calculating $\eta/s$ with pQCD crossections, including 2<->3 processes

1. Solve Israel-Stewart:  
\[ \frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} - \frac{\bar{\pi}}{\tau} = 0 \]  
\[ \frac{d\bar{\pi}}{d\tau} = \frac{-\bar{\pi}}{2\beta_2 \eta} - \frac{\bar{\pi}}{2} \left( \frac{1}{\tau} + \frac{1}{\beta_2 T} \right) \frac{d}{d\tau} \left( \frac{\beta_2}{T} \right) + \frac{2}{3} \frac{1}{\beta_2 \tau} \]  
\[ \frac{dn}{d\tau} + \frac{n}{\tau} = \text{[Source Term]} \]

2. Model $f(x,p)$ using $T,\pi$  
\[ f(x,p) = \frac{16}{(2\pi)^3} e^{-\frac{p_T}{T} \tanh y} \left( 1 + \frac{\pi^2}{128 \lambda T^6} \cdot \bar{\pi} \left( p_z^2 - 1/2 p_T^2 \right) \right) \]

e.g. G.Baym et al, PRL 64 (1990) 1867

3. Calculate $\eta$ using  
\[ \eta = -\frac{\pi_{\mu \nu} \pi_{\mu \nu}}{2 C_0 \pi_{\mu \nu} P_{\mu \nu}} \]

s. also: Talk by Zhe Xu today afternoon
Equilibration of quark and gluon matter at RHIC and LHC on a shorter time scale than the one suggested by “Bottom-Up” scenario is observed in BAMPS.

“Hard” and “soft” gluons thermalize almost simultaneously.

Thermalization times are ~1.2 fm/c for RHIC and ~0.6 fm/c for LHC regimes.

Thermalization even quicker with KLN CGC.

Thermalization time is proportional to $\alpha_s^{-2} (\ln \alpha_s)^{-2} Q_s^{-1}$, thus the dependence on $\alpha_s$ is weaker than the “Bottom-Up” result.

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Gluon Number

> during the first 0.3-0.75 fm/c: gluon annihilation (~10% of initial number).
3->2 processes dominant at early times

> parametric enhancement of total gluon number (“Bottom-Up”) not observed

> the initial CGC is oversaturated.