Jet Propagation and Mach Cones in Heavy-Ion Collisions

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Jets and Mach Cones in HIC

- Why do we study jets and Mach Cones in heavy-ion collisions?
  - What do they tell us about the medium created?

- Experiment:
  - Present results

- Theoretical approaches:
  - Jets & hydrodynamics
  - Jets in AdS/CFT

- Conclusions and Outlook
Explore The Matter Created in HIC

- Medium similar to the one created after Big Bang
- Insights into theory of strong interactions (QCD)
- Explore the phase diagram of QCD

S. Bass, Talk QM’08
RHIC Results

- Jets are suppressed
- System is dense and opaque
- Medium behaves like a fluid
What we know about the Medium

From first principles, it is unclear if medium is ...

- **Particles don’t interact, expansion independent of initial shape**

- **Particles interact, expansion determined by density gradient**

- Quantitative distinguished by mean free path, viscosity
Elliptic Flow

Hydrodynamics: eccentricity of flow as function of numbers of particles, parametrized by $v_2$

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\phi) \right]$$

Data described by hydrodynamics $\rightarrow$ „Ideal Fluid“?
Jet - Studies in HIC I

- Jet moving through dense matter, depositing its energy
  → should eventually disappear

- Jet suppression: signal for creation of an opaque matter (Quark-Gluon Plasma)

→ Is Quark-Gluon Plasma an (ideal) fluid?
Jet - Studies in HIC II

Can energy lost by jets tell us something about medium properties?

→ Idea: Propagation of fast parton generates a Mach cone pattern

→ reflect interaction of jet with medium

Mach cone angle sensitive to EoS

\[
\cos \phi_M = \frac{c_s}{v}
\]

By observation:
- Confirm fast thermalization
- Study EoS of the fluid
Jet - Studies in HIC III

- Generation of Mach cone pattern
- Redistribution of energy to lower $p_T$-particles
- Re-appearance of the away-side for low and intermediate $p_T^{assoc}$

$\sqrt{s} = 200$ GeV
$4 < p_T^{trigger} < 6$ GeV/c
$0.15 < p_T^{assoc} < 4$ eV/c

PHENIX arXiv:0705.3238 [nucl-ex]
The Caveat

- Assumption:
  Correlations from flow anisotropy and jets are uncorrelated

ZYAM (Zero Yield At Minimum)

- Subtraction of:
  Estimated elliptic flow modulated background

- Leads to:
  Double peaked structure

Background:
Particle correlation from elliptic flow

Jet - Studies at SPS Energies

- High-\(p_T\) correlations at SPS energies show conical structure?

\[\rightarrow\] Conical structure even before subtracting the effect of elliptic flow - ZYAM

**Correlation function:**

\[
C(\Delta\phi) = \frac{N_{pair}^{same}(\Delta\phi)}{N_{pair}^{mixed}(\Delta\phi)} \times \frac{\int N_{pair}^{mixed}(\Delta\phi)}{\int N_{pair}^{same}}
\]

**Conditional yield:**

\[
J_2(\Delta\phi) = \frac{1}{N^T} \frac{dN^{TA}}{d(\Delta\phi)} = \frac{N^{TA}}{N^T} \int C_2^{jet}(\Delta\phi) d(\Delta\phi)
\]

Flow contribution subtracted using ZYAM method
Jet – Studies: 3-Particle Correlations

3-Particle Correlations seem to corroborate the Mach Cone picture

Deflected jet

Mach Cone

$\Delta \phi = \Delta \phi_{q} - \phi_{\text{trig}}$


$\rightarrow$ Experimental data show 2 off-diagonal peaks
The Theory Part
Modelling of Jets

- Jet deposition mechanism unclear
  - GLV, BDMPS, ASW, AMY, Higher Twist

Jets can be modelled using hydrodynamics:
\[ \partial_{\mu} T^{\mu\nu} = S^{\nu} \]

not the source term of the jet

- residue of energy and momentum given by the jet
  - this residue is unknown

Hydro: medium has to be strongly coupled, jet-medium can be anything
First Jet – Energy Loss Studies

- Jet deposits energy and momentum along a trajectory similar to

\[ S' \sim \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp \left( -\frac{(r - \bar{r})^2}{2\sigma^2} \right) \left( \frac{dE}{dx}, -\frac{dM}{dx}, 0, 0 \right) \]

- Applying linearized hydrodynamics


\[ \frac{dE}{dx} = 12.6 \text{ GeV/fm} \]

\[ \frac{dE}{dx} = 2 \text{ GeV/fm} \]

\[ \Delta \phi (\text{rad}) \]

Mach cone for sound waves
Why linearized Hydro is not good

- Mach Cone
  - Linear hydrodynamics
  - Connected to EoS

- Head wave pile-up
  - Non-linear hydrodynamics
  - Signal not well understood
  - Non-Mach cone angle

- Source
  - Non-linear hydrodynamics
  - Non-thermalized

- Diffusion Wake
  - Proportional to source
  - Not seen experimentally

→ Something more sophisticated is needed
Study Jets in (3+1)d Ideal Hydro

- Hydrodynamics represents (local) conservation of energy-momentum
  \[ \partial_\mu T^{\mu\nu} = 0 \rightarrow \partial_\mu T^{\mu\nu} = S^\nu \]

- (local) charge
  \[ \partial_\mu N^\mu = 0 \]

- If matter is in local thermodynamical equilibrium
  \[ T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu} \]
  \[ u^\nu = \gamma(1, \vec{v}) \quad \gamma = (1-v^2)^{-1/2} \]
  \[ N^\mu = n u^\mu \]
  \[ g^{\mu\nu} = (+1,-1,-1,-1) \]

- EoS
  \[ p = p(e) = e/3 \]
The Caveat: Freeze-out Prescription

Cooper-Frye Freeze-out:

\[
\frac{dN}{p_T dp_T d\phi} = C \int dV \exp[-\gamma(E - p_i v_i)/T] \sim e^{+p \cdot v}
\]

→ mainly flow driven

- Assumption of isochronous/isothermal freeze-out
- No interaction afterwards
The Punch-Through Jet
Punch – Through Jet I

Applying a static medium and an ideal gas EoS for massless gluons → Maximal fluid response
Assume: Near-side jet is not modified by medium

\[ S^\nu = \int_{\tau_i}^{\tau_f} \frac{dP^\nu}{d\tau} \delta^{(4)}(x^\mu - x^\mu(\tau)) \, d\tau \]

\[ P^\nu = (E, \vec{M}) \]

\[ x^\mu(\tau) = x^\mu_0 + u^\mu_{\text{jet}} \tau \]

\[ \tau_0 = 200 \text{ MeV}, \nu = 0.999 \]

\[ \frac{dE}{dt} = 1.5 \text{ GeV/fm}, \quad \frac{dM}{dt} = 0 \text{ GeV/fm} \]

\[ \nu = 0.999 \]

\[ \frac{dM}{dt} = \frac{dE}{dt} = 1.5 \text{ GeV/fm} \]

\[ t = 4.5/\nu \text{ fm} \]
Punch – Through Jet II

Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[
\frac{dN}{p_T dp_T dy d\phi}
\bigg|_{y=0} = \int d\Sigma \mu \rho(\mu) [f_{\text{Boltzmann}}(u^\mu, p^\mu, T) - f_{\text{eq}}]
\]

Energy Flow Distribution

\[
\frac{dE}{d\phi dy}
\bigg|_{y=0} = \int d^3 \vec{x} \ E(\vec{x}) \ \delta(\phi - \Phi(\vec{x}))
\]

\[
\Phi(\vec{x}) = \tan^{-1}\left( \frac{m_y(\vec{x})}{m_x(\vec{x})} \right)
\]

Diffusion wake causes peak in jet direction

Assuming: Particles in subvolume will be emitted into the same direction
Punch – Through Jet III

Does the jet-pattern reproduce the features of a Mach cone?

\[ \cos \phi_M = c_s / v \]

→ Velocity dependence of the emission angle

Creation of Bow Shock for smaller \( v \) strengthens peak in jet direction
The Stopped Jet
Stopped Jet I

Jet decelerating from $v=0.999$ according to Bethe-Bloch formalism

Known and applied, e.g., in tumor therapy

$$\frac{dE}{dx}(t) = a \frac{1}{v^2(t)}$$

$a=1.4 \text{ GeV/fm}$ adjusts path length

Simplest back-reaction from the medium

$t_0 = 200 \text{ MeV}, v=0.999$

$$\frac{dE}{dt}(0) = 1.5 \frac{\text{GeV}}{\text{fm}} \quad \frac{dM}{dt}(0) = 0 \frac{\text{GeV}}{\text{fm}}$$

$t=4.5/v \text{ fm}$

Barbara Betz
Seminar in Hadron Physics, Montreal, Canada
Stopped Jet II

Bragg Peak

\[ \frac{dM}{dt} (t) = \frac{dE}{dx} (t) \]

\[ \frac{dE}{dt} (t) = v(t) \frac{dM}{dt} (t) \rightarrow \frac{dM}{dt} (t) > \frac{dE}{dt} (t) \]

Dominance of diffusion wake
Summary so far (Theory Part)

- Diffusion wake & Freeze-out procedure \{ critically influences correlation pattern \}
- Small impact of jet-stopping
Modelling Jets with pQCD I


Considering a static medium and linearized hydrodynamics for a punch-through jet

\[
\frac{\left| \vec{x} \right| \epsilon(\vec{x})}{m_\pi^2 T} \quad \frac{\eta}{s} = 0.13
\]

\[
\frac{\left| \vec{x} \right| g(\vec{x})}{m_\pi^2 T} \quad \frac{\eta}{s} = 0.13
\]


→ Obvious to conclude existence of Mach cones?
Modelling Jets with pQCD II

Contour plots of magnitude of perturbed momentum density

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]

Strong diffusion wake

→ Strong flow in jet-direction

→ Freeze-out pattern needed to draw conclusions
Modelling Jets with pQCD IV

Comparing different freeze-out scenarios, neglecting the diffusion wake:

Cooper Frye Brick Freezeout Scenario

Cooper Frye Radial Freezeout Scenario
Radial Flow = 0.6 c


→ Is the diffusion wake really an artefact???
The Diffusion Wake

→ The diffusion wake exists!

G. Burau, Genua Harbour, September 2008
Jets in AdS/CFT
Jets in AdS/CFT I

Analogues: Heavy Quark $\rightarrow$ String
N=4 SYM Thermal Background $\rightarrow$ Black hole in AdS space

Fries et al, PRD 75: 106003 (2007)
S. Gubser et al., PRL 100 012301 (2008)

$\rightarrow$ Mach cone in coordinate space
Jets in AdS/CFT II

Jet travelling at $v=0.75$

Pattern similar to pQCD
Jets in AdS/CFT III

Non-Mach correlations caused by Neck region

\[ \cos \phi_M = c_s / v \]
Jets in pQCD vs AdS/CFT
Heavy Quark Jets in pQCD vs AdS/CFT

Idea: Compare weakly and strongly coupled models

Using heavy quark punch-through jet
Applying ideal hydrodynamics for a static medium and an ideal gas EoS of massless gluons
Assume that the near-side jet is not modified by the medium

pQCD: Neufeld et al. source for a heavy quark
AdS/CFT: Stress tables provided by S. Gubser, A. Yarom and S. Pufu with $\eta/s = 1/(4\pi)$

$\Delta T/T_0 \ v=0.90$

$t=4.5/v$ fm

The Neck Zone in pQCD vs AdS/CFT

AdS/CFT

\[ 100 \times \Delta \varepsilon_{ij}/\varepsilon(T_0) \ (\nu=0.9, \lambda=5.5, N_c=3) \]

\[ (z-vt)\pi T \]

Noronha et al., arXiv:0807.1038

Strong transverse flow

pQCD

\[ \Delta T/T_0 \ \nu=0.90 \]

\[ x_i = x - v t \ (1/\pi T_0) \]

Betz et al., arXiv: 0807.4525 [hep-ph]

No strong transverse flow
Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[
\frac{dN}{p_T dp_T dy d\phi} \bigg|_{y=0} = \int d\sum_{\mu} p_{\mu}^{z} \left[ f_{\text{Boltzmann}}(u^\mu, p^\mu, T) - f_{eq} \right] 
\]

Isochronous freezeout needed to compare pQCD and AdS/CFT

→ No Mach-like peaks

\[ p_T = 2.5 \text{ GeV} \]
Heavy Quark Jets in pQCD vs AdS/CFT

Momentum Flow Distribution

\[
\frac{dS}{d\cos \theta} = \int d^3 \vec{x} \left| M(\vec{x}) \right| \\
\times \delta(\cos \theta - \frac{M_x(\vec{x})}{|M(\vec{x})|})
\]

Assuming: Particles in subvolume will be emitted into the same direction

→ Mach-like peaks & Strong impact of diffusion wake

Independent of \( p_T \)-cut
Conclusions pQCD vs AdS/CFT

- Isochronous Cooper-Frye freeze-out for the Neufeld et al. pQCD model does not lead to a conical structure

- Influence of the Neck region is different in pQCD vs. AdS/CFT

- Measurement of identified heavy quark jet velocity dependence will provide a constraint on pQCD vs. AdS/CFT models

- More realistic simulations require an expanding medium
An Expanding Medium

- Consequences of expansion?
  - Radial flow
  - Elliptic flow

- Predictions:
  - Transverse flow causes disentangling
  - Expansion broadens Mach cone angle

Satarov et al, PLB 627:64 (2005)
Synopsis of different jet energy loss models
Other Jet-Medium Models I

Determines angular correlation pattern

- Fireball model
- Lattice QCD EoS
- BDMPS-like energy loss

\( f: \) "fraction ... of energy lost to the medium
[that] excites a collective mode"

\((1-f): \) "remaining energy fraction ... [that] in essence heats
the medium and leads to some amount of the collective
drift along the jet axis..."

\( \rightarrow \text{Mach cones only if } dM/dx << dE/dx \)

\( \rightarrow \text{sound wave} \)

\( \rightarrow \text{diffusion} \)

Other Jet-Medium Models II

Calculates the azimuthal distribution

- Expanding (2+1)d hydro (AZHYDRO)
- First-order phase transition
- Isothermal Freeze-out
- Source term:

\[ S^\nu(x) = S(x)(+1,-1,0,0) \]

\[ S(x) = \frac{dE}{dx}(x) \left| \frac{dx_{\text{jet}}}{dt} \right| \delta^3(\vec{r} - \vec{r}_{\text{jet}}(t)) \]

\[ \frac{dE}{dx} = \left. s(x) \frac{dE}{dx} \right|_{0} \]

→ Quenched jet

Chaudhuri and Heinz, PRL 97:062301 (2006)
Other Jet-Medium Models II

Effect from diffusion wake

Hottest region at the head of the jet

→ No Mach cone-like correlation if \( \frac{dE}{dx} = \frac{dM}{dx} \)

Chaudhuri and Heinz, PRL 97:062301 (2006)
Other Jet-Medium Models

All jet-hydro simulations for

- linearized to full (3+1)d hydro
- a static/expanding system
- isochronous/isothermal freeze-out

get the **same** result:

- diffusion wake is created if $dM/L/dx > \text{threshold}$
- this diffusion wake destroys the Mach cone signal in a Cooper-Frye freeze-out
Conclusions

- Jet propagation is connected to medium properties

- Experimentally something like a Mach Cone is found
  - Unclear how to interpret

- Mach cone-like pattern may arise in hydro-like models
  - Influence of the diffusion wake has to be clarified
What is needed

• Systematic study of deposition mechanism
  ➔ Consistent picture of the system, interpolating between unthermalized jet and thermalized (strongly coupled) medium

• Investigation of expanding medium:
  Study effects of diffusion wake
  ➔ Seems not to be seen experimentally

• Reliable freeze-out:
  Resonance effects
Backup
Evolution of the Universe

Excellence Cluster Universe – Research Areas:

A: Planck Scale & Strings
B: Matter & Forces
C: Particle Hierarchy
D: Early Universe
E: Dark Universe
F: Black Holes
G: Heavy Elements

Time (sec, years)
Temperature (Kelvin)
Energy (GeV)

\begin{tabular}{lcccccccc}
& \text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} & \text{G} & \text{T}oday \\
\text{Time (sec, years)} & 10^{-44} & 10^{-38} & 10^{-18} & 10^{-12} & 4 \times 10^{17} & 10^{6} & 1.37 \times 10^{18} \\
\text{Temperature (Kelvin)} & 10^{12} & 10^{29} & 10^{16} & 10^{12} & 3000 & 15 & 2.7 \\
\text{Energy (GeV)} & 10^{19} & 10^{16} & 1000 & 10^{9} & 3 \times 10^{10} & 10^{12} & 2.3 \times 10^{-13} \\
\end{tabular}
Energy-Momentum Relation

\[
\frac{dM}{dt} = \frac{dM}{dx} \frac{dx}{dt} = \beta \frac{dM}{dx} = \beta \left( \frac{dE}{dx} \beta + \frac{d\beta}{dx} \right)
\]

\[
E = \gamma m
\]

\[
M = \beta \gamma m = \beta E
\]

\[
\beta^2 = 1 - \frac{1}{\gamma^2}
\]

\[
2\beta \frac{d\beta}{dx} = 2 \frac{1}{\gamma^3} \frac{d\gamma}{dx}
\]

\[
\frac{d\beta}{dx} = \frac{1}{\beta \gamma^3} \frac{dE}{dx} \frac{1}{m}
\]

\[
\frac{dM}{dt} = \frac{dE}{dx}
\]
Modelling Jets with pQCD III

Neufeld et al. pQCD Source shows for a punch-through

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]

\[ \frac{\eta}{s} = \frac{3}{4\pi} \]

\[ \frac{\eta}{s} = \frac{6}{4\pi} \]

For larger viscosity:

- Mach cone contribution gets weaker
- Diffusion wake contribution gets stronger
The Question Left

Is the diffusion wake an artefact?
Transverse Momentum Deposition

- Considering a punch-through jet with $v=0.999$

$$S^\nu = \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp \left( -\frac{(\vec{r} - \vec{x})^2}{2\sigma^2} \right) \left( \frac{dE}{dt} - \frac{dM_L}{dt}, - \frac{dM_T^L}{dt} \sin \theta, - \frac{dM_T^T}{dt} \cos \theta \right)$$

- For a static medium and an ideal Gas EoS for massless gluons

Still influence of diffusion wake

from explosion of matter

$t=4.5/v$ fm

$T_0 = 200$ MeV, $v=0.999$
Near-side Jet

- Considering a punch-through jet with $\nu = 0.999$

$$S^\nu = \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp\left(-\frac{\left(\vec{r} - \vec{x}\right)^2}{2\sigma^2}\right) \left(\frac{dE}{dt}, -\frac{dM}{dt}, 0, 0\right)$$

- Assuming energy-momentum conversation and the disappearance of the near-side jet after $t = 0.5\text{fm}$

Reduction of diffusion wake

Not strong enough to be seen in the freeze-out pattern

$T_0 = 200\text{ MeV}, \nu = 0.999 \quad t = 4.5/\nu \text{ fm}$
Peaked away-side structure modelled by UrQMD

→ Effect of global momentum conservation?
→ Medium response???

Pb+Pb (0-5%) 158AGeV
2.5 < p_t^{trig} < 4.0 GeV
1.0 < p_t^{trig} < 2.5 GeV