Conical Correlations in Heavy-Ion Collisions

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Explore The Matter Created in HIC

- Medium similar to the one created after Big Bang
- Insights into theory of strong interactions (QCD)
- Explore the phase diagram of QCD

S. Bass, Talk QM’08
A Heavy-Ion Collision

UrQMD Simulation, H. Weber
The Accelerators

RHIC, BNL

SPS and future LHC, CERN

future FAIR, GSI
• Jets are suppressed
  → System is dense and opaque

• Medium behaves like a fluid
What we know about the Medium

From first principles, it is unclear if medium is ...

- Particles don’t interact, expansion independent of initial shape
- Particles interact, expansion determined by density gradient

• Quantitative distinguished by mean free path, viscosity
Elliptic Flow

Hydrodynamics: eccentricity of flow as function of numbers of particles, parametrized by $v_2$

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\phi) \right]$$

Data described by hydrodynamics $\rightarrow$ "Ideal Fluid"?
Ideal Hydrodynamics

- Hydrodynamics represents (local) conservation of
  \[ \partial_\mu T^{\mu \nu} = 0 \]
  (local) charge \[ \partial_\mu N^{\mu} = 0 \]

- If matter is in local thermodynamical equilibrium
  \[ T^{\mu \nu} = (e+p)u^\mu u^\nu - pg^{\mu \nu} \quad u^\nu = \gamma(1, \tilde{v}) \quad \gamma = (1-v^2)^{-1/2} \]
  \[ N^\mu = n u^\mu \quad g^{\mu \nu} = (+1,-1,-1,-1) \]

- EoS \[ p = p(e) = e/3 \]
Viscous Hydrodynamics I

- Including dissipative terms $\Pi, q^\mu, \pi^{\mu\nu}, \nu^\mu$

energy-momentum tensor

$$T^{\mu\nu} = \epsilon \ u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + 2 \ q^{(\mu} u^{\nu)} + \pi^{\mu\nu}$$

charge current

$$N^\mu = n u^\mu + \nu^\mu$$

with $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ and $q^{(\mu} u^{\nu)} = \frac{1}{2} (q^\mu u^\nu + q^\nu u^\mu)$

- Deriving the Equations of Motion for $\Pi, q^\mu, \pi^{\mu\nu}, \nu^\mu$

starting from the Boltzmann Equation $k^\mu \ \partial_\nu f(k,x) = C[f]$
\[ \tau_\Pi \dot{\Pi} + \Pi = \Pi_{NS} + \tau_{\Pi q} q \cdot \dot{u} - \ell_{\Pi q} \partial \cdot q - \zeta \hat{\delta}_0 \Pi \theta + \lambda_{\Pi q} q \cdot \nabla \alpha + \lambda_{\Pi \pi} \pi^{\mu \nu} \sigma_{\mu \nu} \]

\[ \tau_q \Delta^{\mu \nu} \dot{q}_\nu + q^\mu = q_{NS}^\mu - \tau_{q \Pi} \Pi \dot{u}^\mu - \tau_{q \pi} \pi^{\mu \nu} \dot{u}_\nu + \ell_{q \Pi} \nabla^\mu \Pi - \ell_{q \pi} \Delta^{\mu \nu} \partial^\lambda \pi_{\nu \lambda} + \tau_q \omega^{\mu \nu} q_\nu - \frac{\kappa}{\beta} \hat{\delta}_1 q^\mu \theta - \lambda_{qq} \sigma^{\mu \nu} q_\nu + \lambda_{q \Pi} \Pi \nabla^\mu \alpha + \lambda_{q \pi} \pi^{\mu \nu} \nabla_\nu \alpha \]

\[ \tau_\pi \dot{\pi}^{<\mu \nu>} + \pi^{\mu \nu} = \pi_{NS}^{\mu \nu} + 2 \tau_{\pi q} q^{<\mu \dot{u}^\nu>} + 2 \ell_{\pi q} \nabla^{<\mu q^\nu>} + 2 \tau_{\pi} \pi_\lambda^{<\mu \omega^{\nu}>\lambda} - 2 \eta \hat{\delta}_2 \pi^{\mu \nu} \theta - 2 \tau_{\pi} \pi_\lambda^{<\mu \sigma^{\nu}>\lambda} - 2 \lambda_{\pi q} q^{<\mu \nabla^\nu>\alpha} + 2 \lambda_{\pi \Pi} \Pi \sigma^{\mu \nu} \]

A. Muronga, PRC 76 (2007) 014909
A. Muronga, PRC 76 (2007) 014909
this work; B. Betz, D. Henkel, DHR, in preparation
Jets in Heavy-Ion Collisions
Jet - Studies in HIC I

- Jet moving through dense matter, depositing its energy
  ➔ should eventually disappear

- Jet suppression: signal for creation of an opaque matter (Quark-Gluon Plasma)

➔ Is Quark-Gluon Plasma an (ideal) fluid?
Jet - Studies in HIC II

Can energy lost by jets tell us something about medium properties?

→ Idea: Propagation of fast parton generates a Mach cone pattern

→ reflect interaction of jet with medium

Mach cone angle sensitive to EoS

$$\cos \phi_M = \cos \beta = \frac{c_s}{v_{jet}}$$

By observation:
- Confirm fast thermalization
- Study EoS of the fluid
Jet - Studies in HIC III

- Generation of Mach cone pattern
- Redistribution of energy to lower $p_T$-particles
- Re-appearance of the away-side for low and intermediate $p_T^{assoc}$

**Graphical Data:**

\[ Y_{jet} = 1/N_A dN/d\Delta\phi \]

- $3-4 \otimes 2-3$ GeV/c
- $\sqrt{s} = 200$ GeV
- \[ \Delta\phi \] (rad)

**Legend:**

- \( p+p \)
- \( Au+Au \) $0-20\%$ (preliminary)
- \( 4 < p_T^{trigger} < 6 $ GeV/c
- \( 0.15 < p_T^{assoc} < 4 $ GeV/c

**References:**

- PHENIX arXiv:0705.3238 [nucl-ex]
Jet - Studies in HIC IV: The Caveat

- Assumption:
  Correlations from flow anisotropy and jets are uncorrelated

ZYAM (Zero Yield At Minimum)

- Subtraction of:
  Estimated elliptic flow modulated background

- Leads to:
  Double peaked structure

Background:
Particle correlation from elliptic flow

Jet - Studies in HIC V

p_T-dependence (associated jet):
- double peaked structure seems to get broader (but within errorbars)

p_T-dependence (trigger jet):
- one peak structure evolves (possible punch-through)

PHENIX arXiv:0705.3238 [nucl-ex]
Jet - Studies in HIC VI

Investigation of path length dependence:

- Double-peaked structure becomes more pronounced out-of-plane

\[ \text{Au+Au} \sqrt{s_{NN}} = 200 \text{GeV, Cent}=30-40\%, \quad 1<p_{T,\text{assoc}}<2 \text{ GeV/c}, \quad 3<p_{T,\text{trig}}<4 \text{ GeV/c} \]

\[ \Delta v_2 \quad \Delta \text{resol.} \]

\[ \text{PHENIX Preliminary} \]

\[ \text{PHENIX} \]

A. Sickle, arXiv:0809.3703 [nucl-ex]
Jet – Studies: 3-Particle Correlations

3-particle correlations seem to corroborate the Mach Cone picture

Deflected jet  Mach Cone

\[ \Delta \phi_{3T} = \Delta \phi_T - \phi_{\text{Trig}} \]


Experimental data show 2 off-diagonal peaks
The Theory Part
**Modelling of Jets**

- Jet deposition mechanism unclear
  - GLV, BDMPS, ASW, AMY, Higher Twist

Jets can be modelled using hydrodynamics:

\[ \partial_\mu T^{\mu\nu} = S^\nu \]

not the source term of the jet

- residue of energy and momentum given by the jet
  - this residue is unknown

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The Caveat: Freeze-out Prescription

Cooper-Frye Freeze-out:

\[
\frac{dN}{p_T dp_T d\phi} = C \int dV \exp[-\gamma (E - p_i v^i) / T] \sim e^{p \cdot \vec{v}}
\]

→ mainly flow driven

- Assumption of isochronous/isothermal freeze-out
- No interaction afterwards
The Punch-Through Jet
Punch – Through Jet I

Applying a static medium and an ideal Gas EoS for massless gluons

Maximal fluid response

Assume: Near-side jet is not modified by medium

\[ S^\gamma = \int_{\tau_i}^{\tau_f} \frac{dP^\gamma}{d\tau} \delta^{(4)}(x^\mu - x_i^\mu(\tau)) \, d\tau \]

\[ P^\gamma = (E, \vec{p}) \]

\[ x^\mu(\tau) = x_0^\mu + u_{\text{jet}}^\mu \tau \]

\[ T_0 = 200 \text{ MeV}, \nu = 0.999 \]

\[ \frac{dE}{dt} = 1.5 \text{ GeV/fm}, \quad \frac{dM}{dt} = 0 \text{ GeV/fm} \]

\[ v = 0.999 \]

\[ \frac{dM}{dt} = \frac{dE}{dt} = 1.5 \text{ GeV/fm} \]

\[ t = 4.5 / v \text{ fm} \]
Punch – Through Jet II

\[ T_0 = 200 \text{ MeV}, \nu = 0.999 \]

\[ \frac{dE}{dt} = 1.5 \frac{\text{GeV}}{\text{fm}}, \quad \frac{dM}{dt} = 0 \frac{\text{GeV}}{\text{fm}} \]

\[ \nu = 0.999 \]

\[ \frac{dM}{dt} = \frac{dE}{dt} = 1.5 \frac{\text{GeV}}{\text{fm}} \]

\[ t = 4.5/\nu \text{ fm} \]

→ Mach cone for sound waves

→ Diffusion wake
Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[
\frac{dN}{p_T dp_T dy d\phi} \bigg|_{y=0} = \int d\Sigma \mu p \left[ f_{\text{Boltzmann}}(u^\mu, p^\mu, T) - f_{\text{eq}} \right]
\]

Energy Flow Distribution

\[
\frac{dE}{d\phi dy} \bigg|_{y=0} = \int d^3 \vec{x} \ E(\vec{x}) \ \delta(\phi - \Phi(\vec{x}))
\]

\[
\Phi(\vec{x}) = \tan^{-1}\left( \frac{m_y(\vec{x})}{m_x(\vec{x})} \right)
\]

Diffusion wake causes peak in jet direction

Assuming: Particles in subvolume will be emitted into the same direction
The Stopped Jet
Stopped Jet I

Jet decelerating from $v=0.999$ according to Bethe-Bloch formalism
Known and applied in tumor therapy

$$\frac{dE}{dx}(t) = a \frac{1}{v^2(t)}$$
$a=-1.36 \text{ GeV/fm}$ adjusts path length

Simplest back-reaction from the medium

$t_0 = 200 \text{ MeV}$, $v=0.999$

$$\frac{dE}{dt}(0) = 1.5 \text{ GeV/fm} \quad \frac{dM}{dt}(0) = 0 \text{ GeV/fm}$$

$t=4.5/v \text{ fm}$
Stopped Jet II

→ Bragg Peak

\[
\frac{dM}{dt}(t) = \frac{dE}{dx}(t)
\]

general:

→ \[\frac{dE}{dt}(t) = v(t) \frac{dM}{dt}(t) \rightarrow \frac{dM}{dt}(t) > \frac{dE}{dt}(t)\]

Dominance of diffusion wake
Stopped Jet III

- Jet stops after $t=4.5/v$ fm

\[
\begin{align*}
\frac{dE}{dt}(0) &= 1.5 \text{ GeV/fm} \\
\frac{dM}{dt}(0) &= 0 \text{ GeV/fm}
\end{align*}
\]

$t_{FO}=4.5/v$ fm  $t_{FO}=6.5/v$ fm  $t_{FO}=8.5/v$ fm

Diffusion wake still present $\rightarrow$ Vorticity conservation
Stopped Jet IV

Diffusion wake causes peak in jet direction

Larger impact of thermal smearing

arXiv:0812.4401 [nucl-th]
The Diffusion Wake
The Diffusion Wake

G. Burau, Genua Harbour, September 2008

→ The diffusion wake exists!
Jet – Energy Loss Studies

- Jet deposits energy and momentum along a trajectory
- Applying linearized hydrodynamics

\[
\frac{dE}{dx} = 12.6 \ \text{GeV/fm}
\]

\[
\frac{dE}{dx} = 2 \ \text{GeV/fm}
\]


Diffusion wake

Mach cone for sound waves
Other Jet Prescriptions
Jets in AdS/CFT I


Pointing vector perturbation

Energy density perturbation

Diffusion wake contribution

Attention: No clear Mach cone signal
Jets in AdS/CFT II

\[ 100 \times \Delta \epsilon_{\text{inel}}/\epsilon(T_0) \ (v=0.9, \lambda=5.5, N_c=3) \]

Noronha et al, arXiv:0807.1038

Non-Mach correlations caused by Neck region

\[ \cos \phi_M = c_s / v \]
Jets in pQCD I

Considering a static medium and linearized hydrodynamics for a punch-though jet


→ Mach cone signal & Diffusion Wake
Jets in pQCD II

Contour plots of magnitude of perturbed momentum density

\[ \frac{\eta}{s} = \frac{1}{4\pi} \]

\[ \frac{\eta}{s} = \frac{3}{4\pi} \]

\[ \frac{\eta}{s} = \frac{6}{4\pi} \]

→ Strong flow in jet-direction

\[ \frac{dM}{dt} > \frac{dE}{dt} \]
Jets in pQCD vs AdS/CFT
Heavy Quark Jets in pQCD vs AdS/CFT

Idea: Compare weakly and strongly coupled models

Using heavy quark punch-through jet
Applying ideal hydrodynamics for a static medium and an ideal gas EoS of massless gluons
Assume that the near-side jet is not modified by the medium

pQCD: Neufeld et al. source for a heavy quark
AdS/CFT: Stress tables provided by S. Gubser, A. Yarom and S. Pufu with $\eta/s=1/(4\pi)$

$t=4.5/\nu$ fm

Heavy Quark Jets in pQCD vs AdS/CFT

Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

\[
\frac{dN}{p_T dp_T dy d\phi} \bigg|_{y=0} = \sum_{\Sigma} d\Sigma \mu p^\mu [f_{Boltzmann}(u^\mu, p^\mu, T) - f_{eq}]
\]

Isochronous freezeout needed to compare pQCD and AdS/CFT

\[\rightarrow \text{No Mach-like peaks}\]

\[p_T = 2.5 \text{ GeV}\]
Heavy Quark Jets in pQCD vs AdS/CFT

Momentum Flow Distribution

\[ \frac{dS}{d \cos \theta} = \int d^3 \vec{x} \ |M(\vec{x})| \]

\[ \times \delta(\cos \theta - \frac{M_x(\vec{x})}{|M(\vec{x})|}) \]

Assuming: Particles in subvolume will be emitted into the same direction

→ Mach-like peaks &

Strong impact of diffusion wake

Independent of \( p_T \) - cut
The Neck Zone in pQCD vs AdS/CFT

AdS/CFT

$100 \times \Delta \epsilon_{\text{eff}}/\epsilon(T_0)$ (v=0.9, $\lambda=5.5$, $N_c=3$)

Strong transverse flow

Noronha et al, arXiv:0807.1038

pQCD

$\Delta T/T_0$ v=0.90

No strong transverse flow

Conclusions Theory Part

- Diffusion wake is universal and dominates the freezeout distribution
  - always created if dM/dx > threshold

- Freeze-out procedure critically influences correlation pattern
  - Energy flow distribution: peak around Mach cone angle & diffusion wake

- Diffusion wake exists for different jet prescriptions (pQCD and AdS/CFT)

- How will expansion (radial flow) alter the correlation?
An Expanding Medium

- Consequences of expansion?
  
  Radial flow
  
  Elliptic flow

- Predictions:
  
  Transverse flow causes disentangling
  
  Expansion broadens Mach cone angle

Satarov et al, PLB 627:64 (2005)
Conclusions

- Jet propagation is connected to medium properties

- Experimentally something like a Mach cone is found
  - Unclear how to interpret

- Mach cone-like pattern may arise in hydro-like models
  - Influence of the diffusion wake has to be clarified
Where we are from

University Frankfurt

FIAS, Frankfurt

Thank you very much for your invitation and hospitality!