Eigenvalue repulsion in an effective theory of SU(2) Wilson lines in three dimensions

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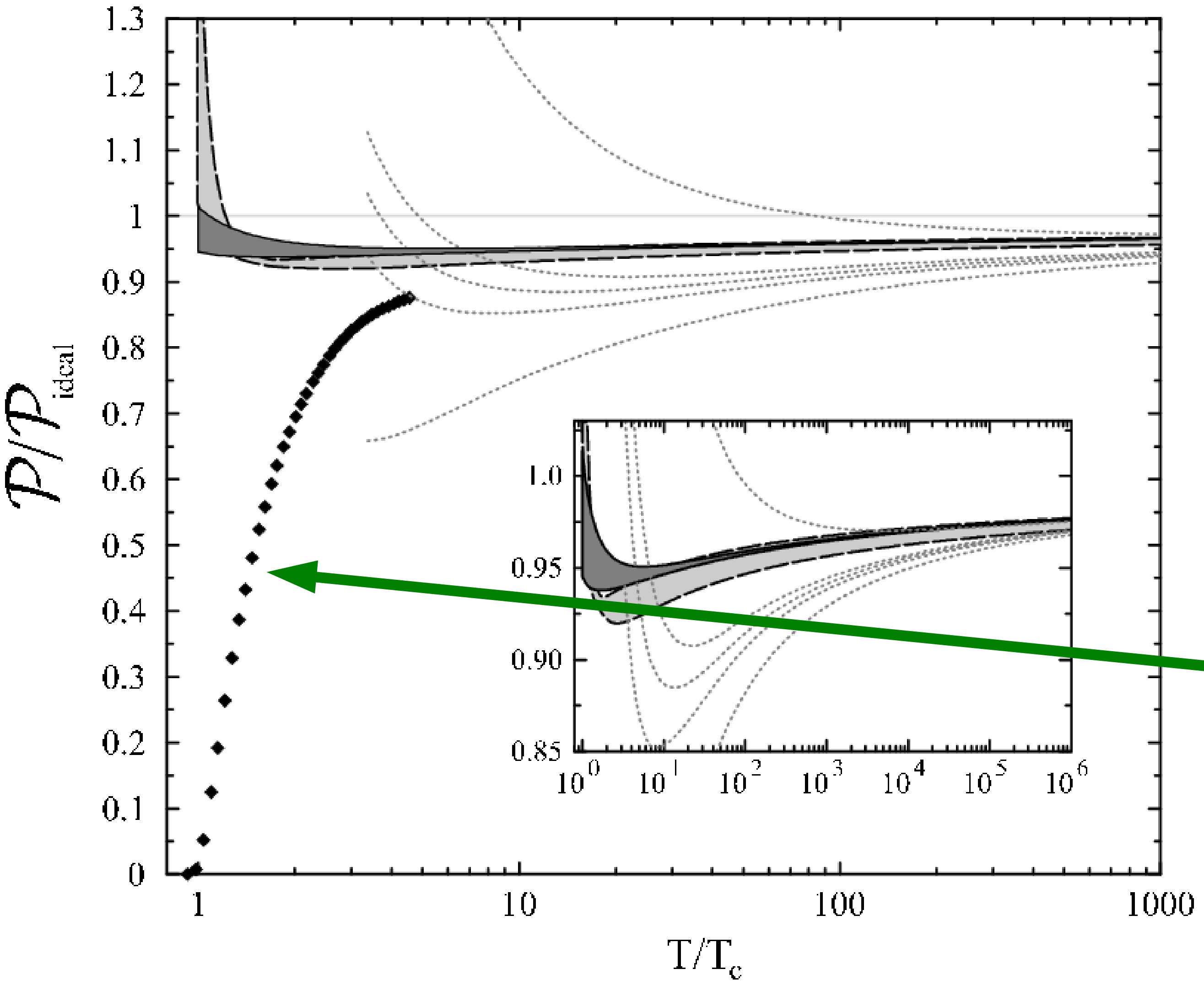
Quark Matter Studies

Outline:

- Introduction
- Monte Carlo Simulation
- Results
- Summary

Introduction

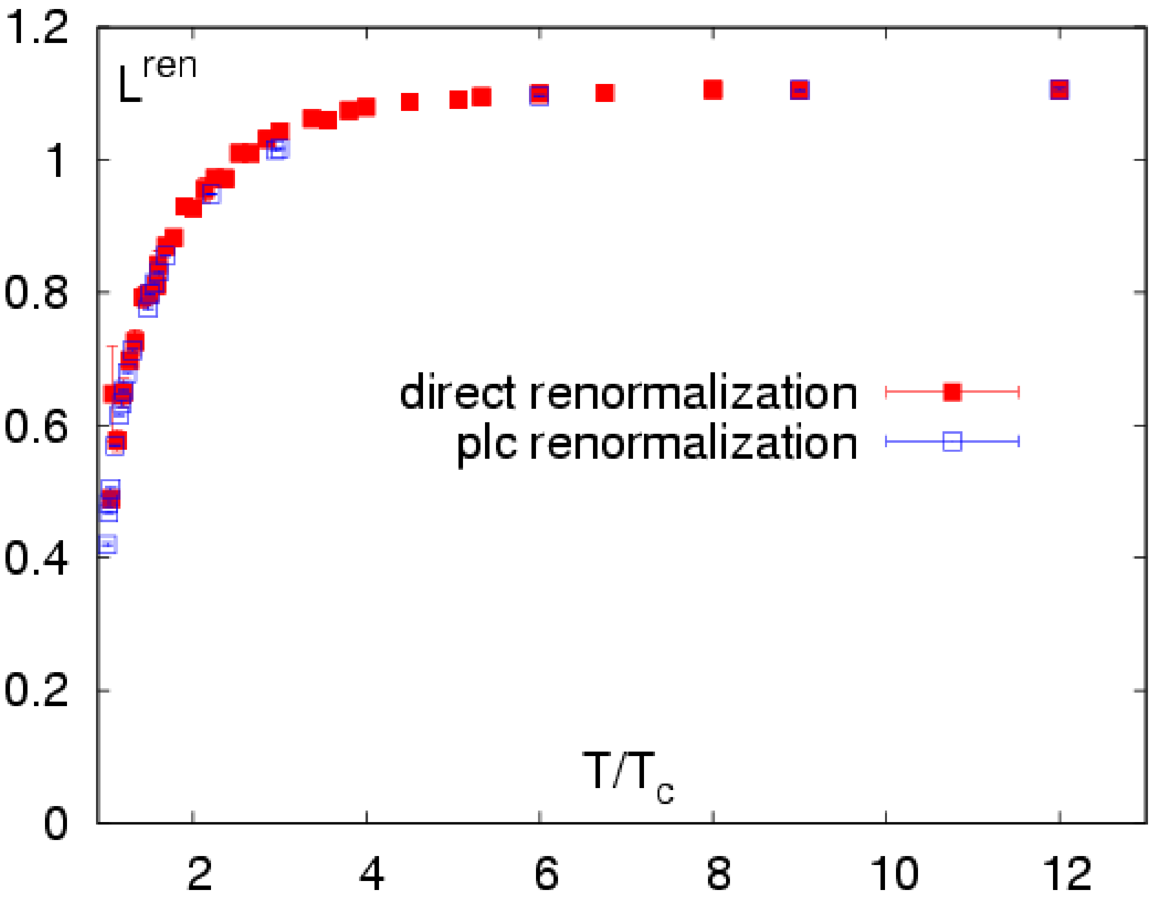


Anderson,
Strickland,
hep-ph/0404164

4d SU(3) Lattice data

- Resummed Perturbation Theory fails below $\sim 4\,T_C$!
- WHY ?

- HTL-P.T. expands about trivial vacuum,
  $A_0/T=0 \quad \leftrightarrow \quad \mathbf{L \sim \exp(ig\, A_0/T) \sim 1}$



Kaczmarek,
Gupta,
Hübner,
hep-lat/0710.2277

Need different vacuum: repulsion of eigenvalues of **L** near T_c ?

Does one see repulsion of eigenvalues of Wilson line in the deconfined phase

(at non-asymptotic T) ?

- Potential in powers of fundamental Wilson Line to one loop order:

$$V^{eff}_{1-loop} = \frac{-2}{\pi^2} T^4 \sum_{n\geq 1} \frac{1}{n^4} |tr\, L^n|^2$$

Gross, Pisarski,
Yaffe,
Rev. Mod. Phys. 53
43 (1981)

- Minimized by perturbative vacuum: **< L > = 1**

→ Eigenvalue attraction, for any T

- Idea: add non-perturbative “fuzzy bag” contribution:

$$V^{eff}_{non-pert.} = B_f T^2 |tr\, L|^2$$

Pisarski,
hep-ph/0608242

- We performed Monte Carlo Simulations of SU(2) matrix models on cubic lattices with periodic boundary conditions.

- Lattice action: Kinetic nearest neighbor interaction and mass term (*i* labels sites, <*ij*> labels links)

$$S=-\frac{1}{2}\beta\sum_{\langle ij\rangle} Tr(L_i^{+}L_j+h.c.)-m^2\sum_i |TrL_i|^2$$

- Partition Function involves integral over SU(2) measure at each site:

$$Z=\int\prod_n dL_n\; e^{-S}$$

- Kinetic term has global SU(2) x SU(2) symmetry. Mass term is locally SU(2) invariant.

- Mean field analysis of $m^2=0$ case yields phase transition at $\beta_C \approx 1/3$

Kogut, Stone, Snow
Nucl. Phys. B 200 211 (1982)

- We generalized to $m^2 \neq 0$

$$Z_{ss} = \int [dL] \exp\{d\,\beta\ Tr(LU^+ + UL^+)\} + m^2 |Tr\,L|^2$$
$$= \ \exp\{-F_{ss}(U, U^+)\}$$

(Consider single site problem and replace nearest neighbours by fixed matrix)

- Self-consistency condition $\langle L \rangle = U / z$ leads to minimization criterium for mean field free energy:

(z = 2*d)

$$\frac{\partial}{\partial U} F_{mf}(U) = 0$$

$$F_{mf}(U) = F_{ss}(U) + d\,\beta\, Tr(U^{\dagger} U)$$

- Assuming U to be proportional to unit matrix reduces action to function of trace:

$$\exp\{-F_{SS}(u)\} = \int [dL] \exp\{2\,d\,\beta\, u\, Tr(L + L^{+})\} + m^{2} |Tr\, L|^{2}$$

- Diagonalize the matrix L under the trace

$$L = \exp\ diag(i\phi, -i\phi + 2i\pi n)$$

- Use parametrisation by Weyl and Vandermonde:

$$[dL] \sim d\phi |\Delta(\phi)|^2 = d\phi \sin^2\phi$$

- Up to a constant we get:

$$\exp\{-F_{SS}(u)\} = \int_{-1}^{1} d\cos\phi \ldots$$

$$\ldots \exp\left\{4 d\beta\, u \cos\phi + 4\, m^2 \cos^2\phi + \frac{1}{2}\ln(1 - \cos^2\phi)\right\}$$

- Solve integral numerically. Look for value of u which minimizes mean field free energy.

$$F_{mf}(u) = F_{ss}(u) + 2\,d\,\beta\,u^2$$

- A potential emerges from the integration measure, which contributes to the action (Vandermonde Potential).

$$V = -\ln(1 - \cos^2\phi)$$

- Compare to Monte Carlo.

Monte Carlo Simulation

- Use quaternionic representation:

$$L = a_0 I + a_1 i\sigma_1 + a_2 i\sigma_2 + a_3 i\sigma_3 \qquad \sum_i |a_i|^2 = 1$$

- Standard Metropolis Method with sequential updating to generate configurations:

- Trial steps are taken by rotating single matrices. Acceptance depends on ratio of Boltzmann weights. $w = e^{-S}$

- Accept if $r = \frac{w_T}{w} > 1$ or larger than random number $\in [0,1]$

- Consider Autocorrelation Function: $\Gamma(j) = \dfrac{\langle E_i E_{i+j} \rangle - \langle E \rangle^2}{\langle E^2 \rangle - \langle E \rangle^2}$

- Integrated autocorrelation time obtained through the binning method:

$$\tau_{\mathrm{int}} = \frac{\sigma_{binned}}{\sigma_{original}}$$

(Fraction of variance of binned series and original series)

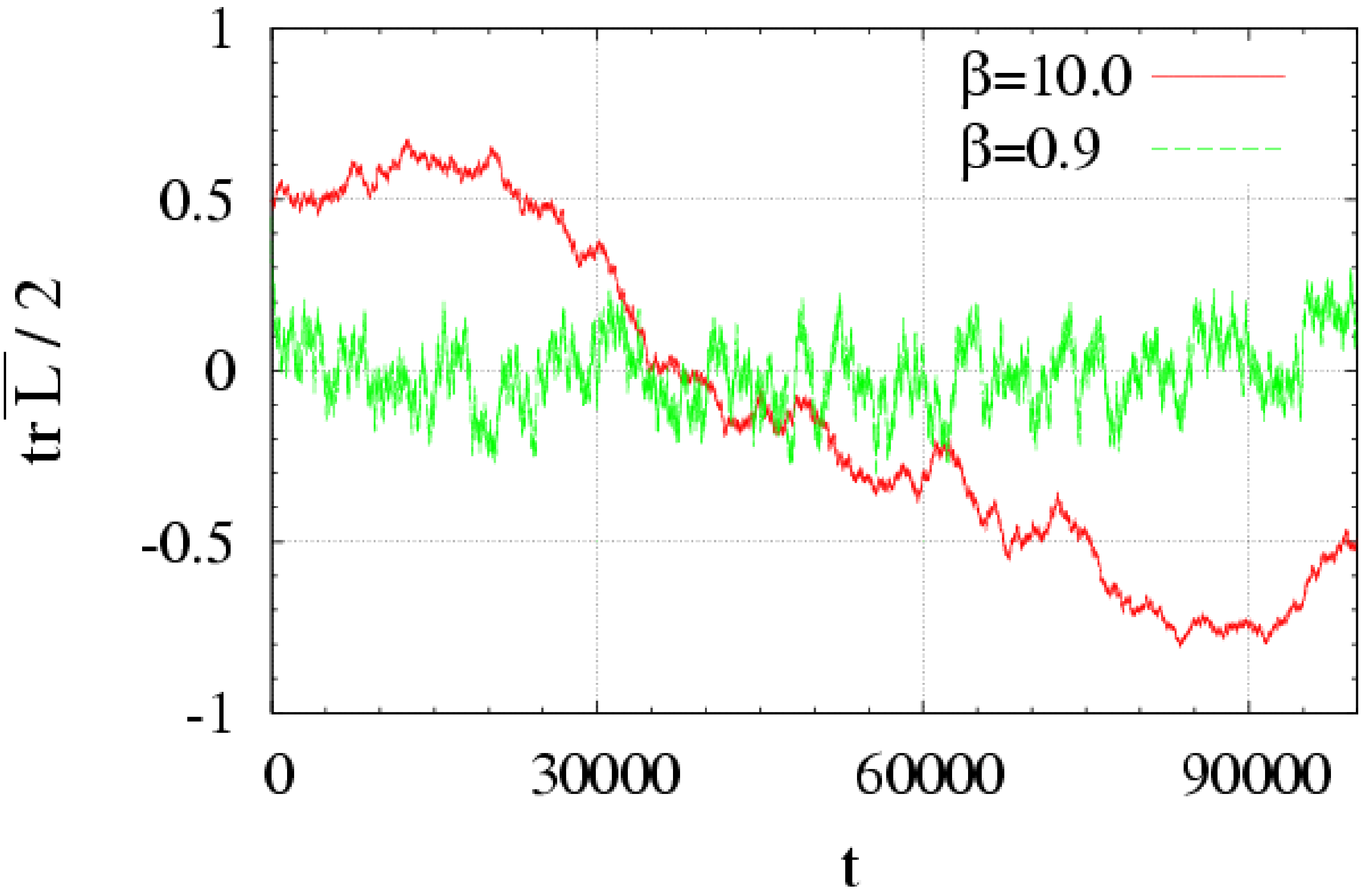
- Best results when looking at dependence on binsize:

$$\tau_{\mathrm{int}}(N_b) = \frac{\sigma_{binned}(N_b)}{\sigma_{original}}$$

Results

- First study the $m^2=0$ case: $S=-\frac{1}{2}\beta\sum_{\langle ij\rangle} Tr(L_i^+ L_j + h.c.)$

- Ensemble average of trace vanishes: $\langle \frac{1}{V}\sum_i Tr\, L_i\rangle = 0$



- Rotation in group space clearly visible !

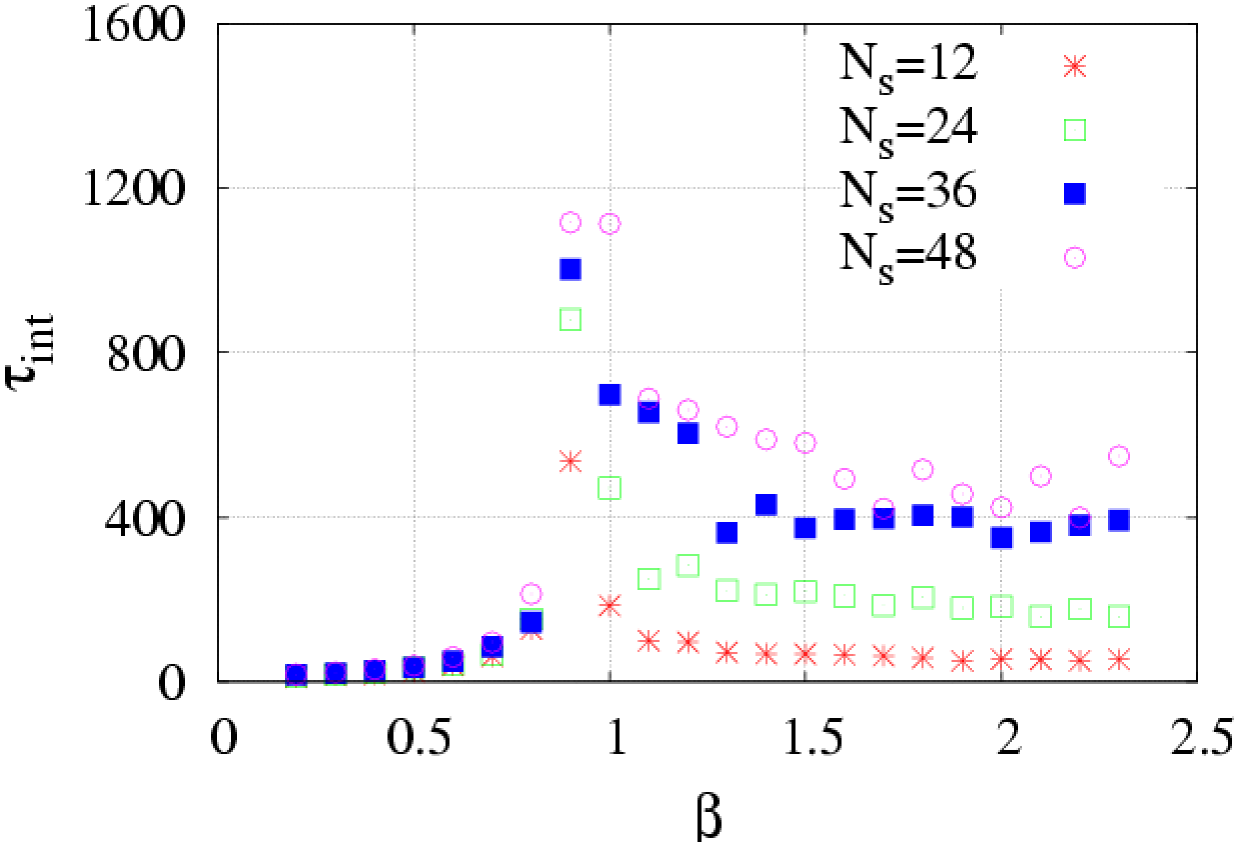
- Note proportionality of volume averaged field to SU(N) matrix after each Monte Carlo step:

$$\bar{L} = u_0 L$$

- Isolate the “length” u and use it as order parameter.

$$Tr(u_0^* L^+)(u_0 L) = |u_0|^2 \, Tr L^+ L = N \; |u_0|^2$$

- Measurements were done for Lattice sizes $N_s = \{12^3, 24^3, 36^3, 48^3\}$
- More than 1000 independent data points for each value of β.

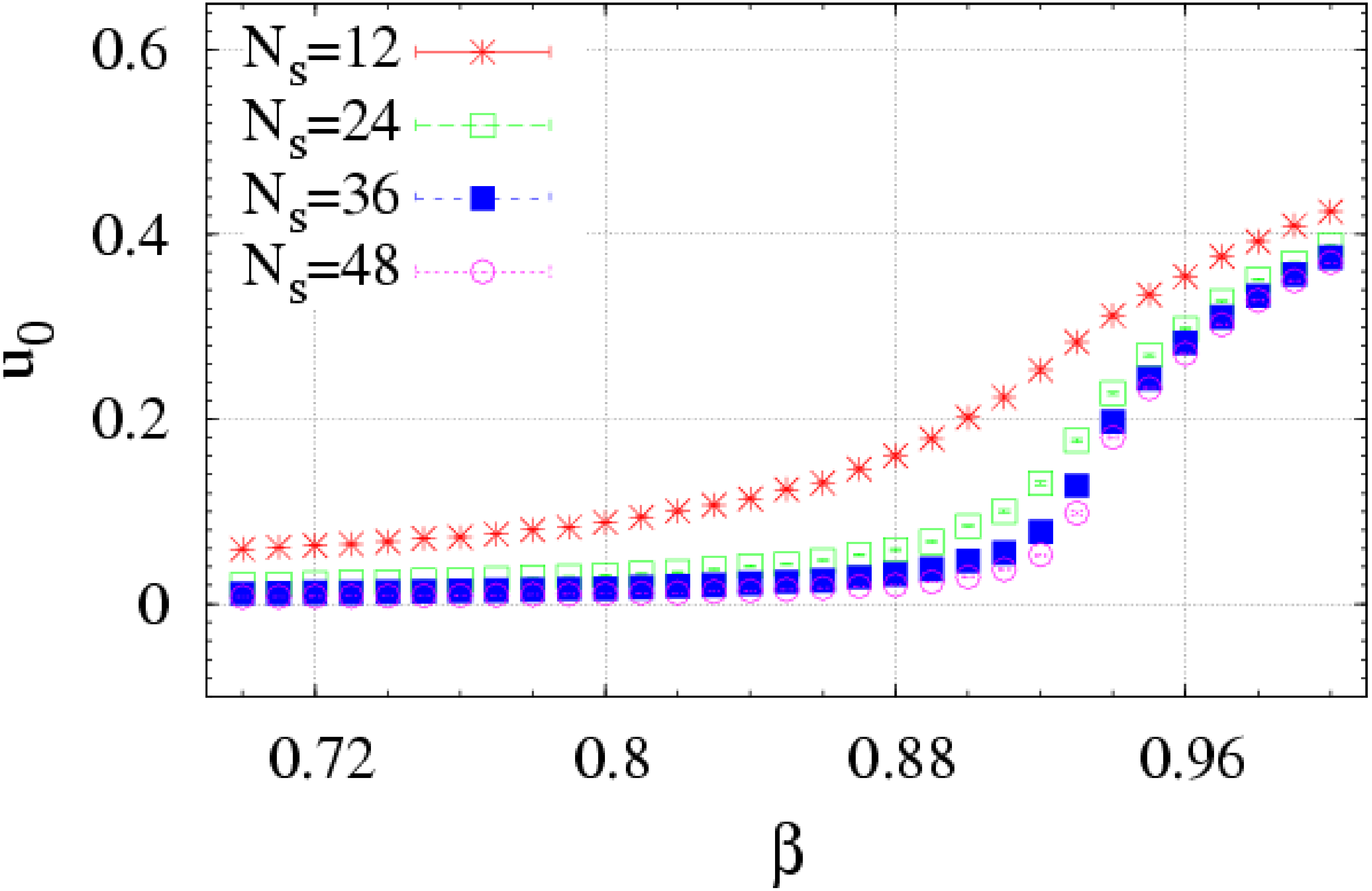


Critical slowing down!

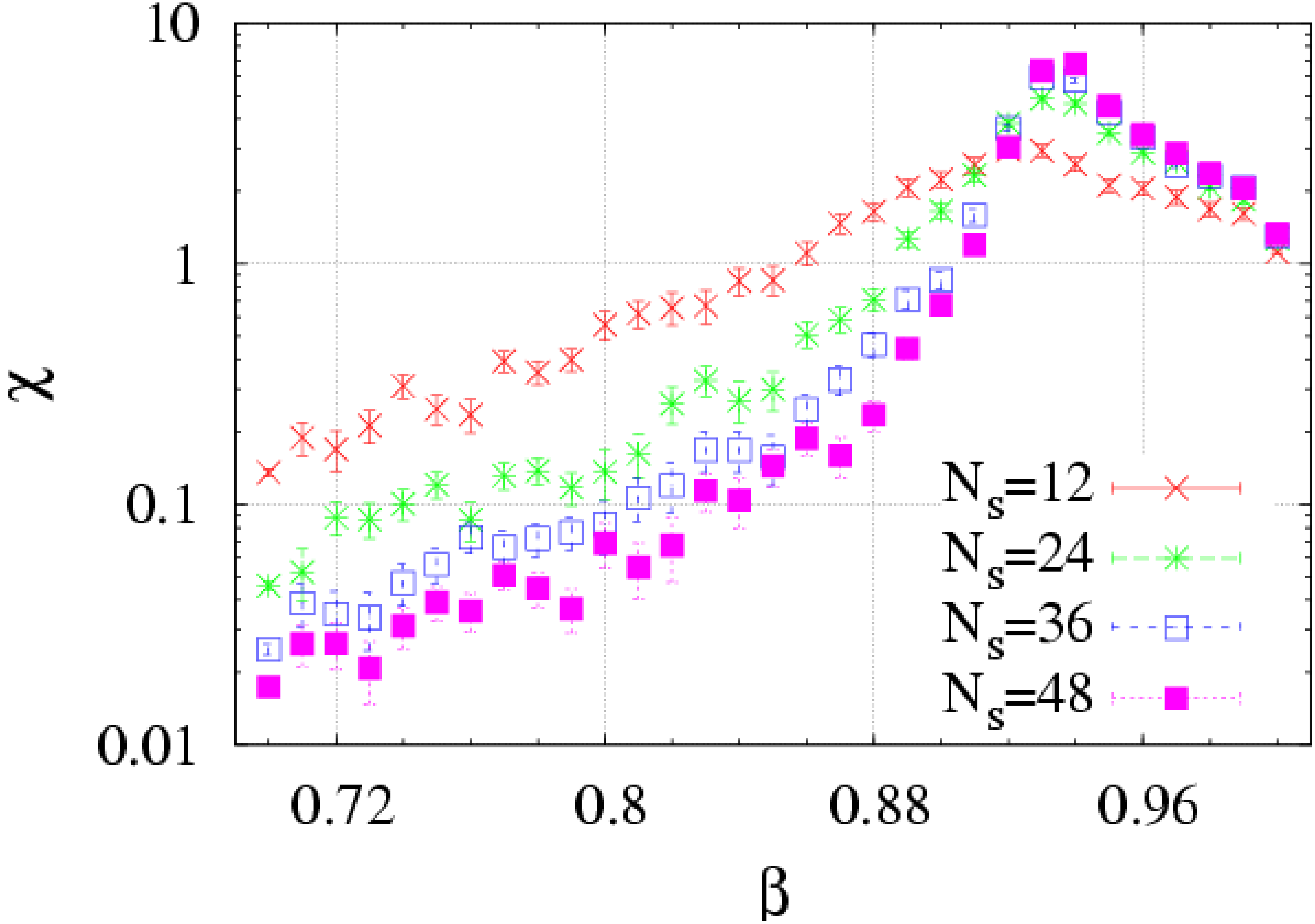
- Scaling at β_C : $\tau_{\text{int}} \sim N_s^{1/\nu}$ $\nu = 0.72(4)$

- Second Order Phase Transition (in infinite volume) is apparent !

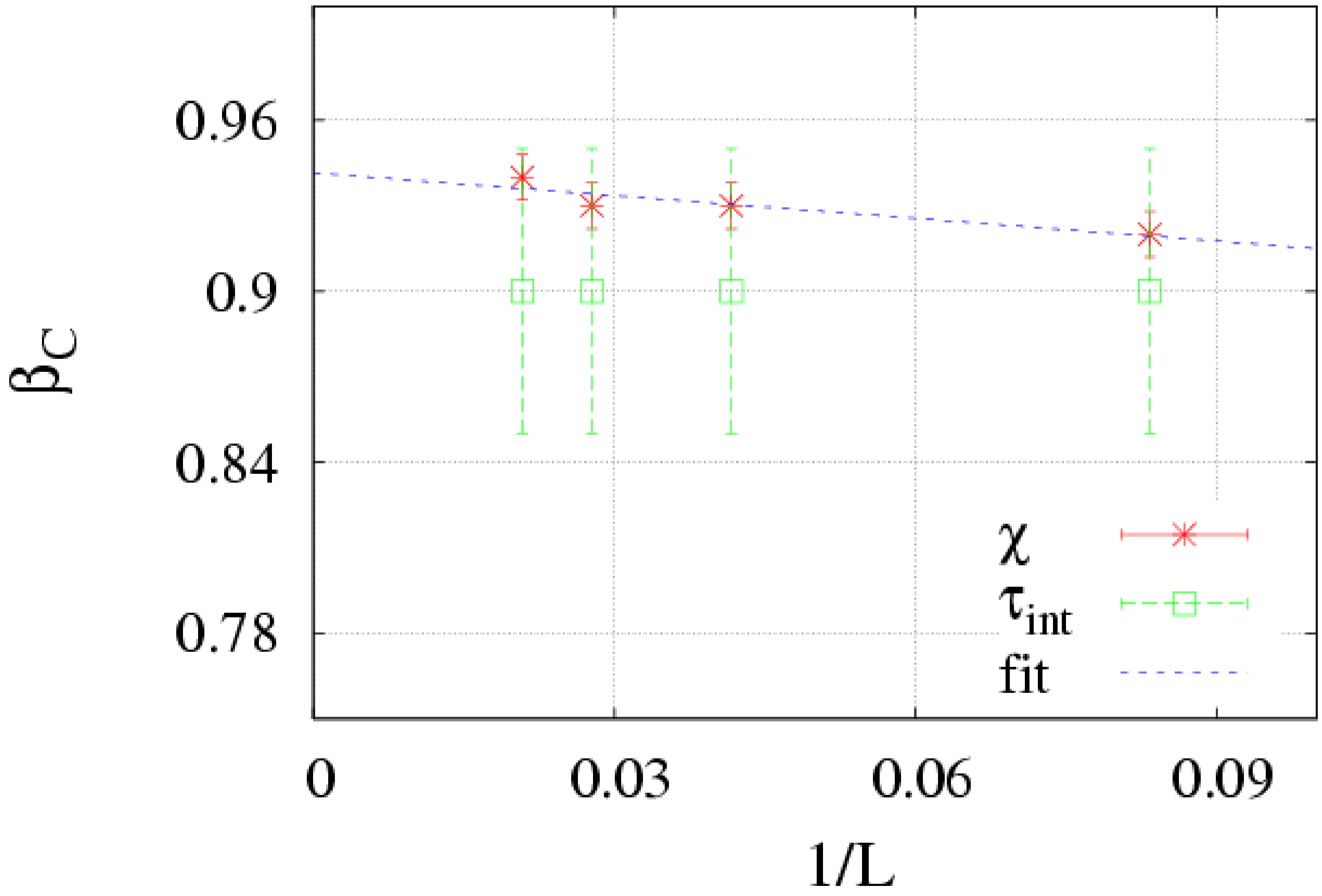
$$\frac{1}{V}\langle \sum_i L_i \rangle = u_0 L$$



- To find critical coupling, extrapolate peak of susceptibility to infinite volume limit !



- Scaling with inverse lattice size:



- Phase Transition at $\beta_C \approx 0.941$ differs from mean field result! $(\beta_C \approx 1/3)$

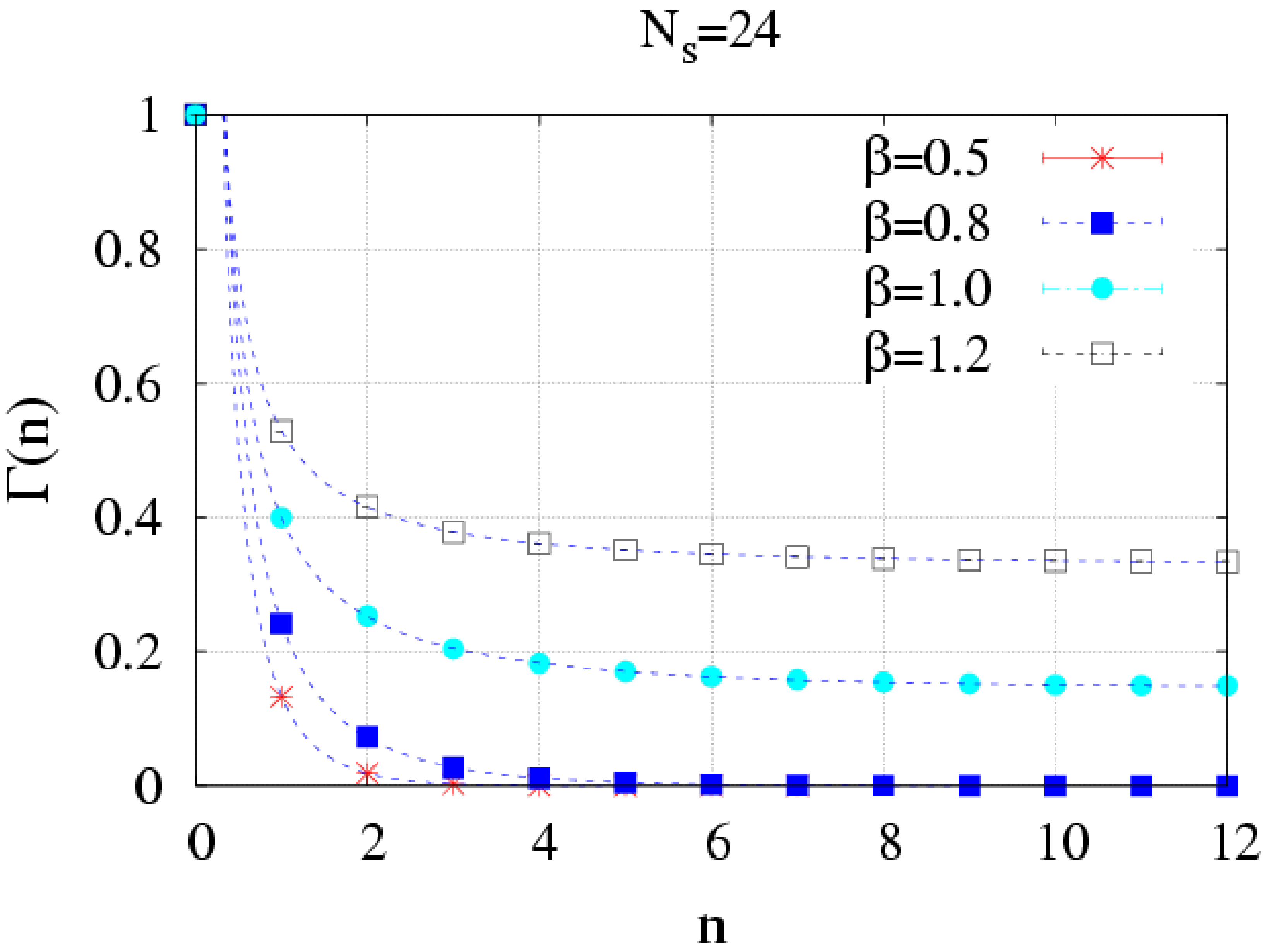
- Measure correlation function:

$$\Gamma(r) = \frac{1}{2} \langle tr\, L^{+}(r_0) \cdot L(r_0 + r) \rangle$$

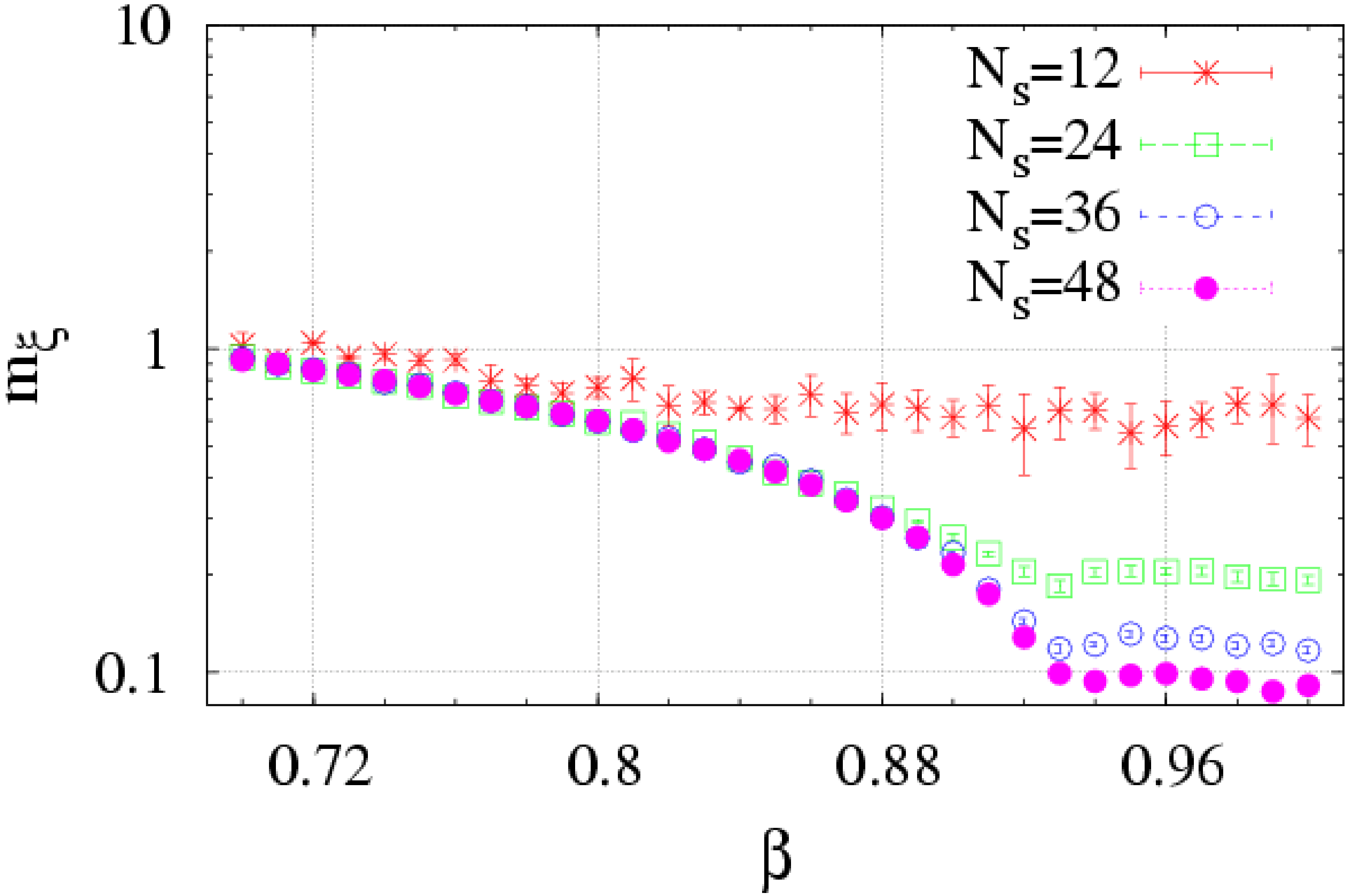
- Fit with Yukawa-like form, obtained from Klein-Gordon, to extract the effective mass:

$$\Gamma(r) = \frac{a}{m \cdot r} \exp(-m \cdot r) + b$$

- Results for $N_s = \{12^3, 24^3, 36^3, 48^3\}$
- Mass drops to zero at phase transition point!



- Fit with: $\Gamma(r) = \frac{a}{m \cdot r} \exp(-m \cdot r) + b$



- Lattice size scaling at phase transition:

$$m(\beta_c) \sim N_s^{-1/\nu} \qquad \nu = 0.938(5)$$

- Singlet channel and “pionic” correlation function yield similar results!

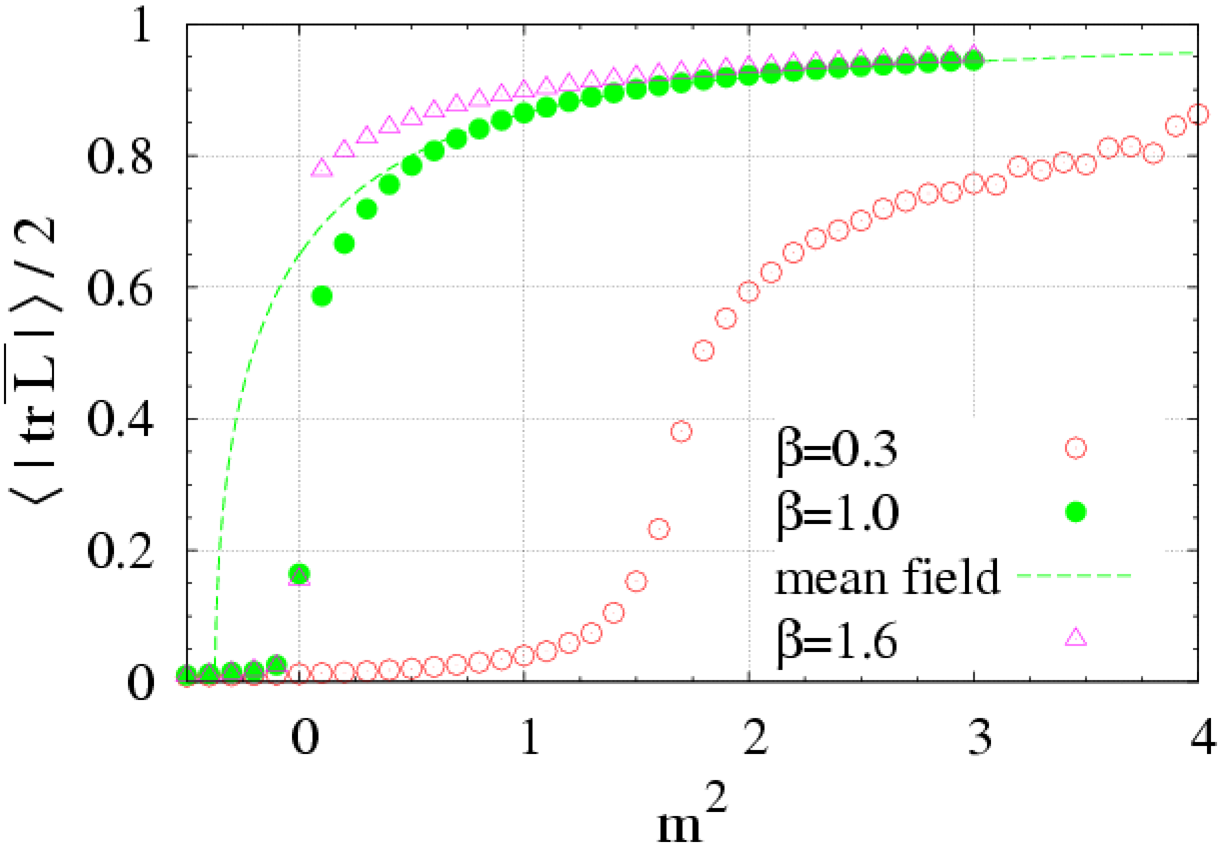
- Singlet: $$\Gamma(r) = \frac{1}{2}\langle tr\, L^{+}(r_0)\cdot tr\, L(r_0+r)\rangle = \frac{1}{2}\langle a_0(r_0)\cdot a_0(r_0+r)\rangle$$

- Adjoint: $$\Gamma(r) = \frac{1}{2}\langle a_i(r_0)\cdot a_i(r_0+r)\rangle$$ (i=1,2,3)

$$L=a_0 I+a_1 i\sigma_1+a_2 i\sigma_2+a_3 i\sigma_3$$

- Next study $m^2 \neq 0$ case: $S = -\frac{1}{2}\beta \sum_{\langle ij \rangle} Tr(L_i^+ L_j + h.c.) - m^2 \sum_i |Tr L_i|^2$

- Now $\langle | \frac{1}{V} \sum_i tr L_i | \rangle$ is good order parameter !

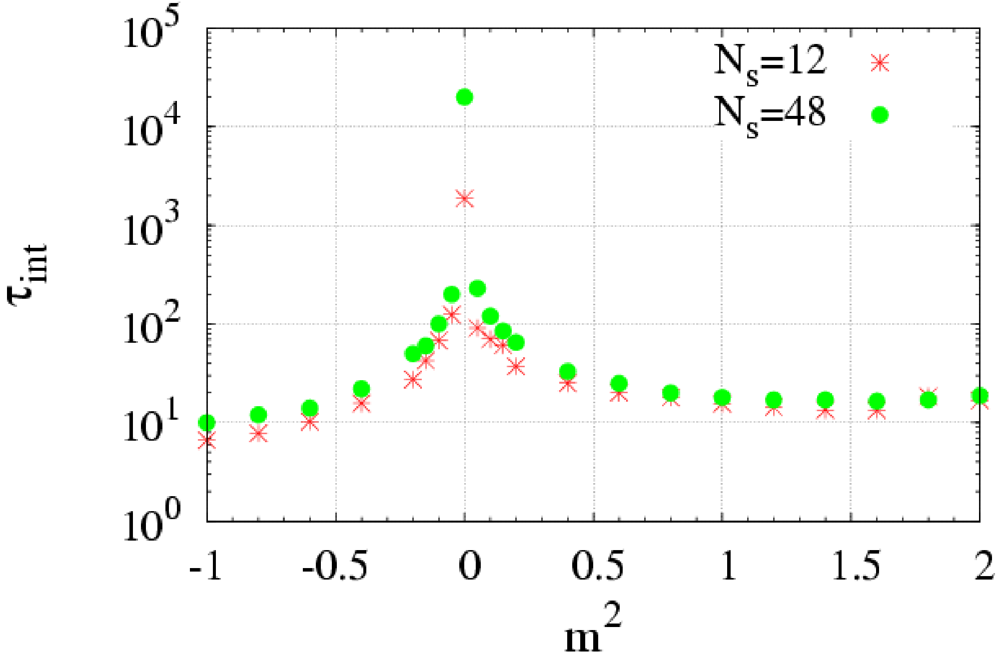


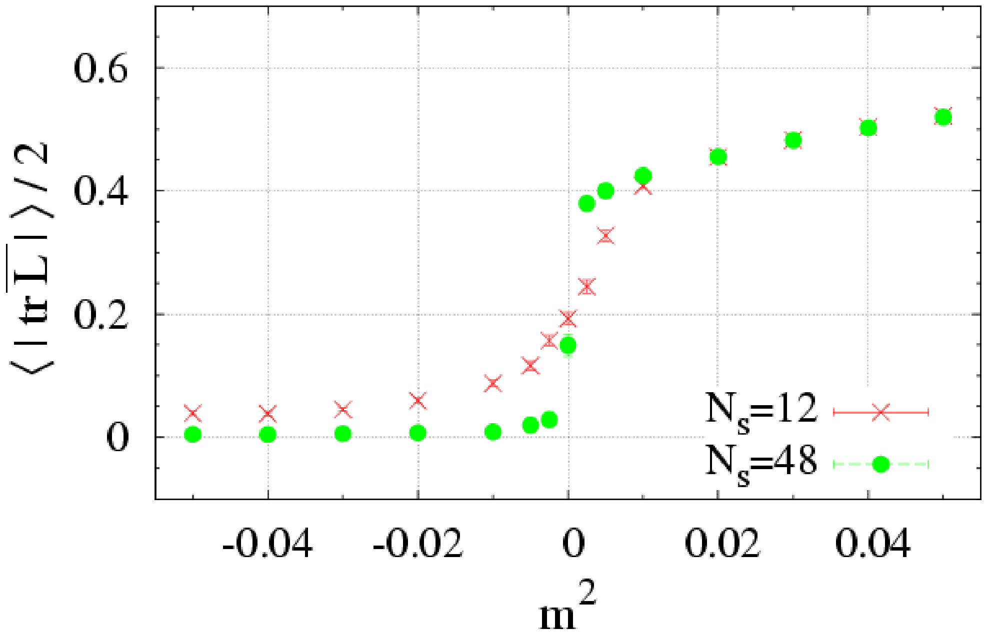
Compare to mean field:
shift along x-axis by +0.94

(12x12 lattice)

- In addition to shift, there is difference in functional dependence!

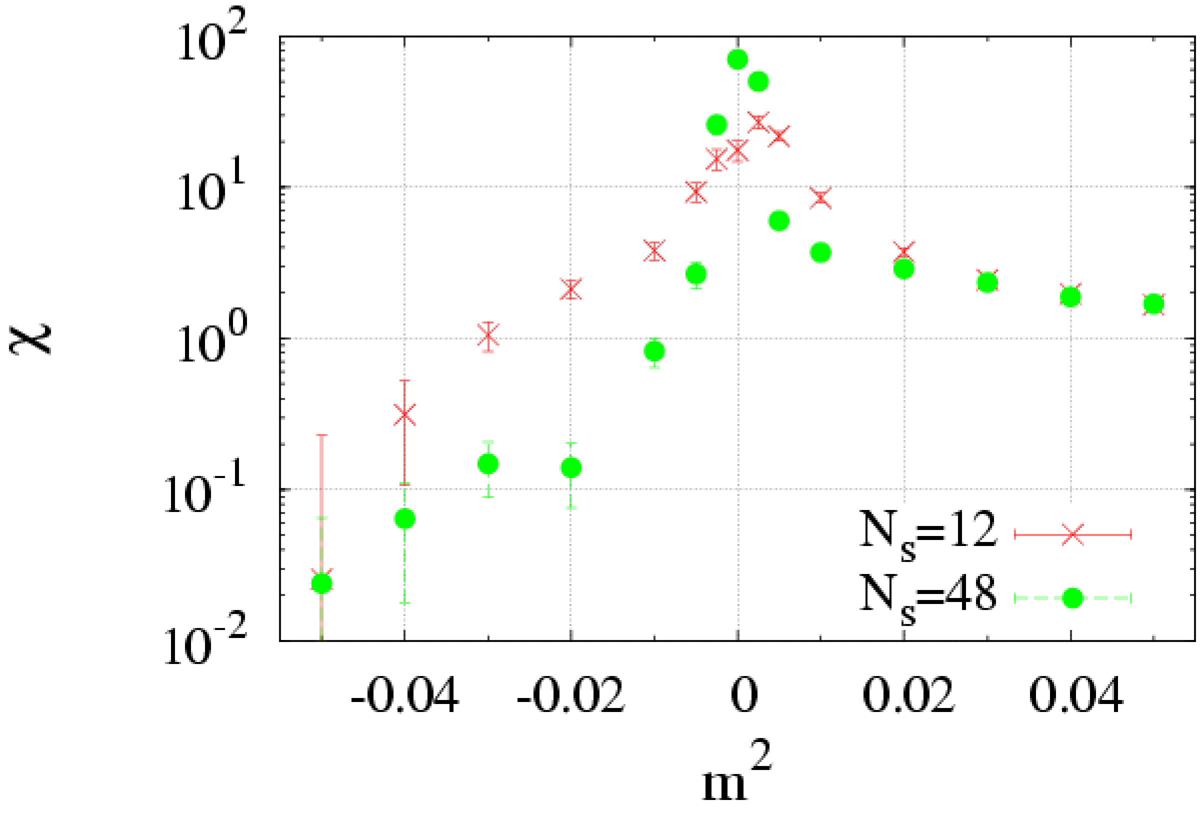
- Detailed investigation of the phase transition region for $\beta = 1.0$





- Transition is very sharp but definitely of second order !

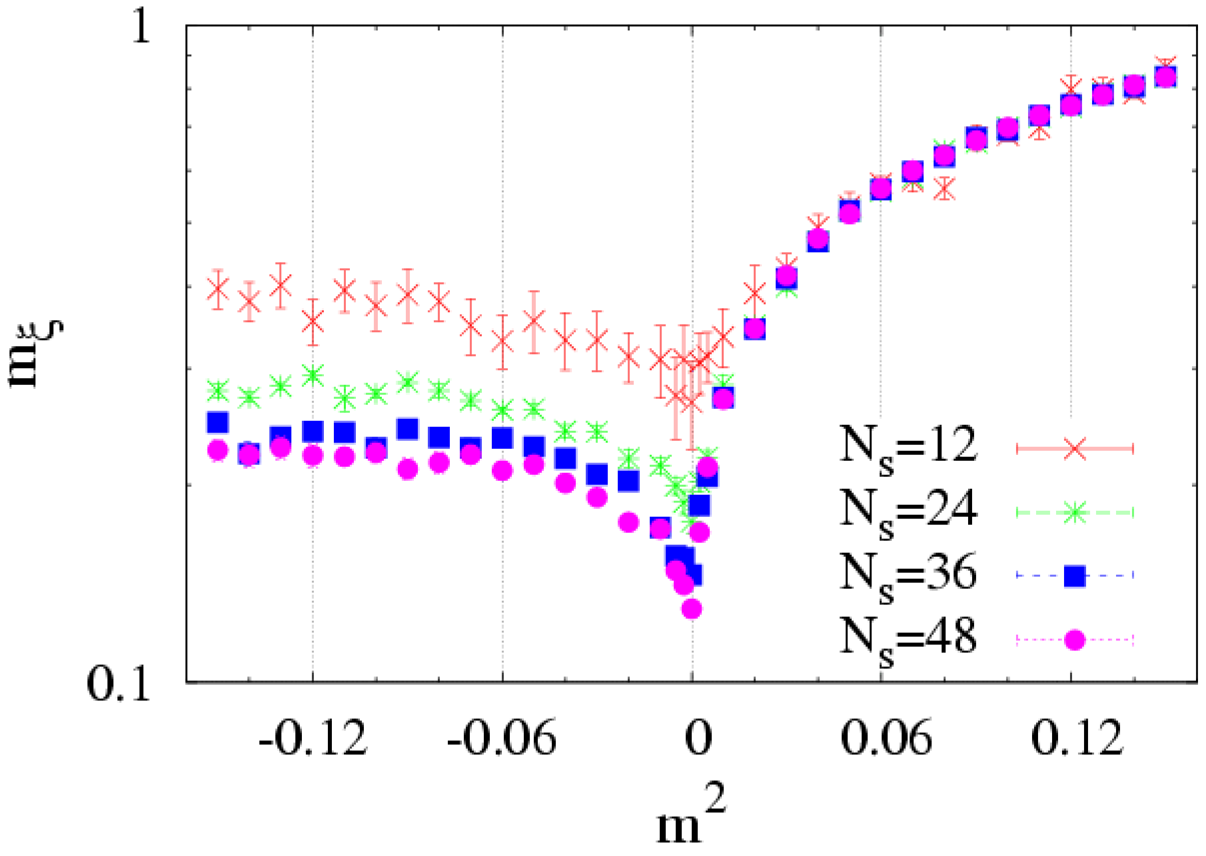
$$\tau_{\mathrm{int}} \sim N_s^{1/\nu} \qquad \nu = 1.4(4)$$



- Infinite volume limit of susceptibility used for m_C^2 .
- Phase transition occurs at $m^2=0.0$ independent of β !

(Can be solved by adding background field)

- Again the correlation function: $\Gamma(r) = \frac{1}{2}\langle tr L^{+}(r_0) L(r_0+r)\rangle$
- Fit with: $\Gamma(r) = \frac{a}{m\cdot r}\exp(-m\cdot r)+b$



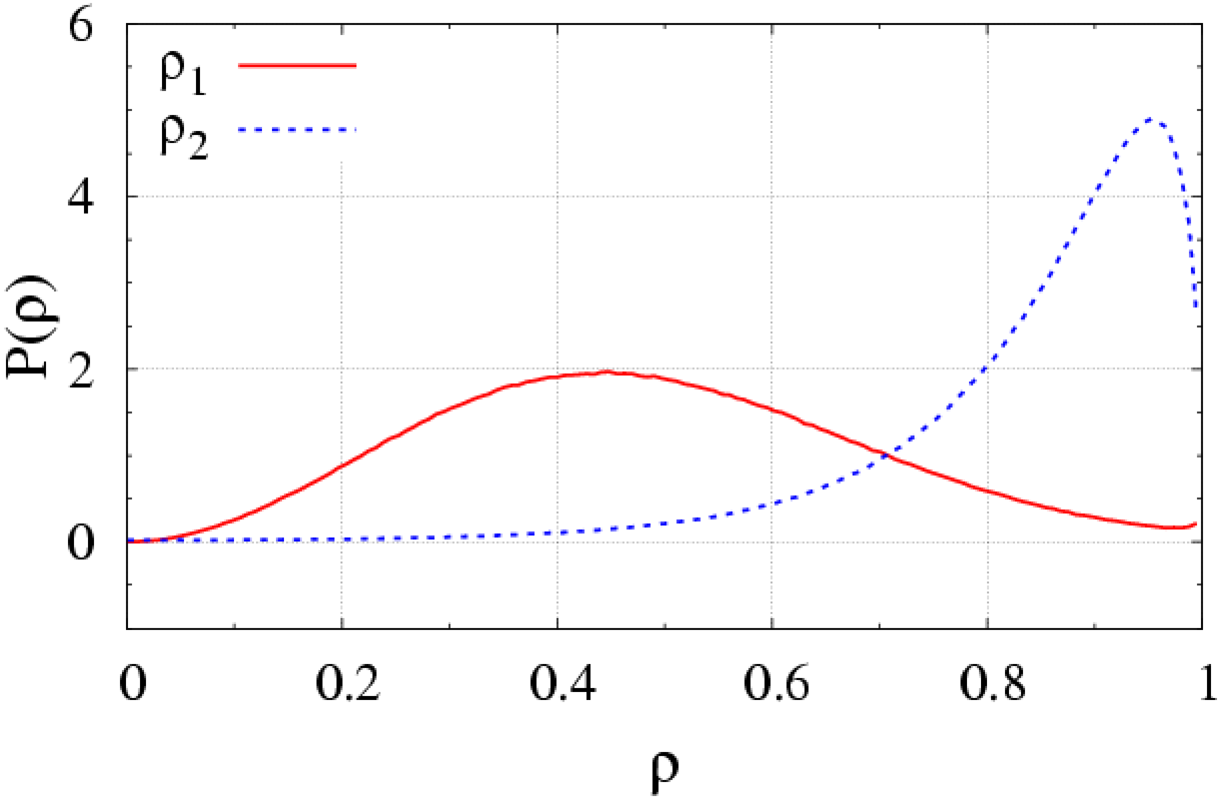
- Mass drops like: $m(\beta_c) \sim N_s^{-1/\nu}$ $\nu = 2.28(8)$

- Look at eigenvalue distribution:

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$$\beta = 1.0 \qquad m^2 = 0.8$$



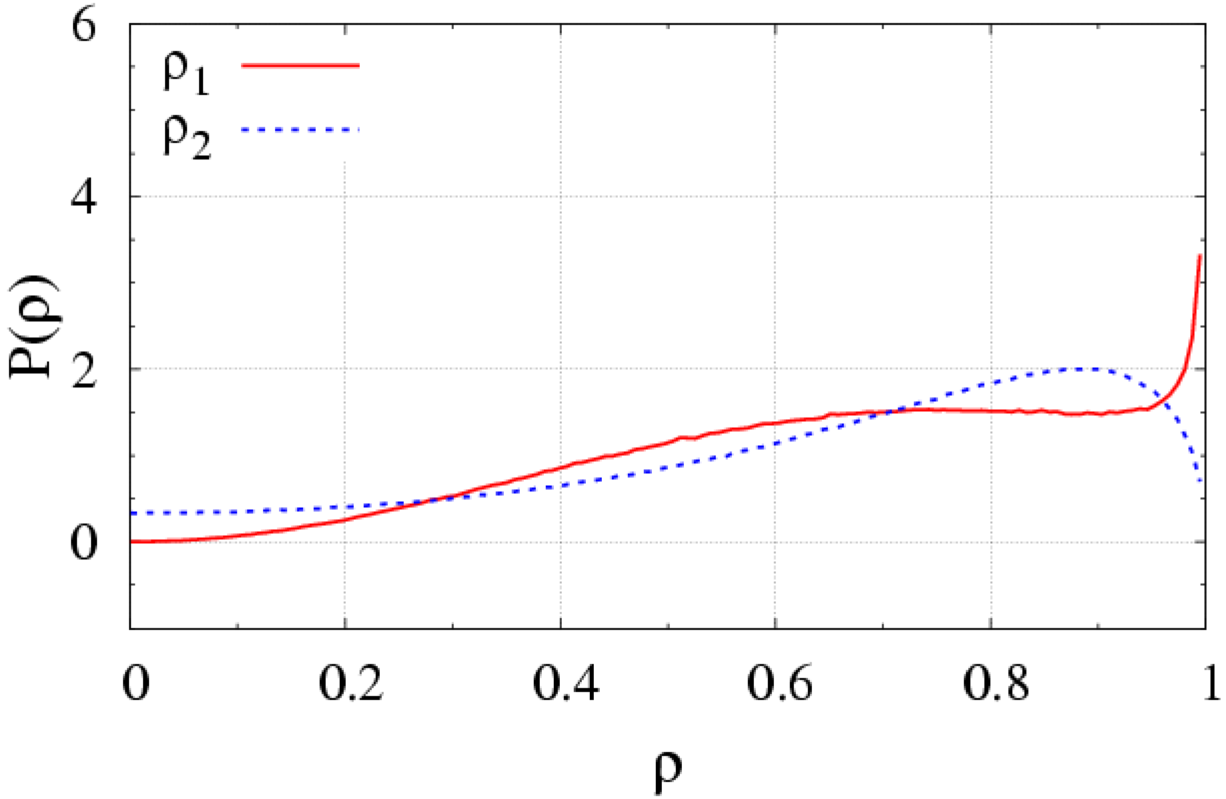
- Strongly peaked distribution for large value of m^2 !

  (Eigenvalue “attraction”)

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$\beta = 1.0 \qquad m^2 = 0.15$

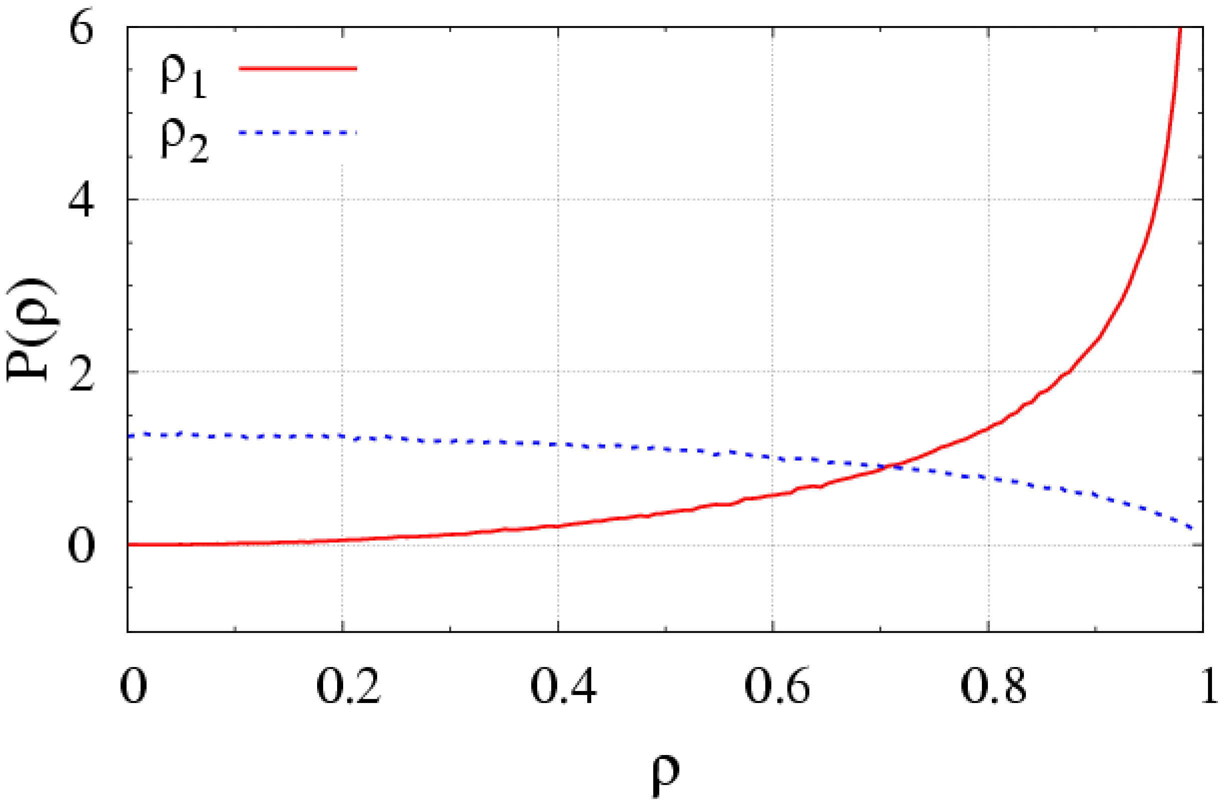


- Lowering m^2 evens out the distribution: “repulsion” of eigenvalues sets in !

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$\beta = 1.0 \qquad m^2 = 0.0$

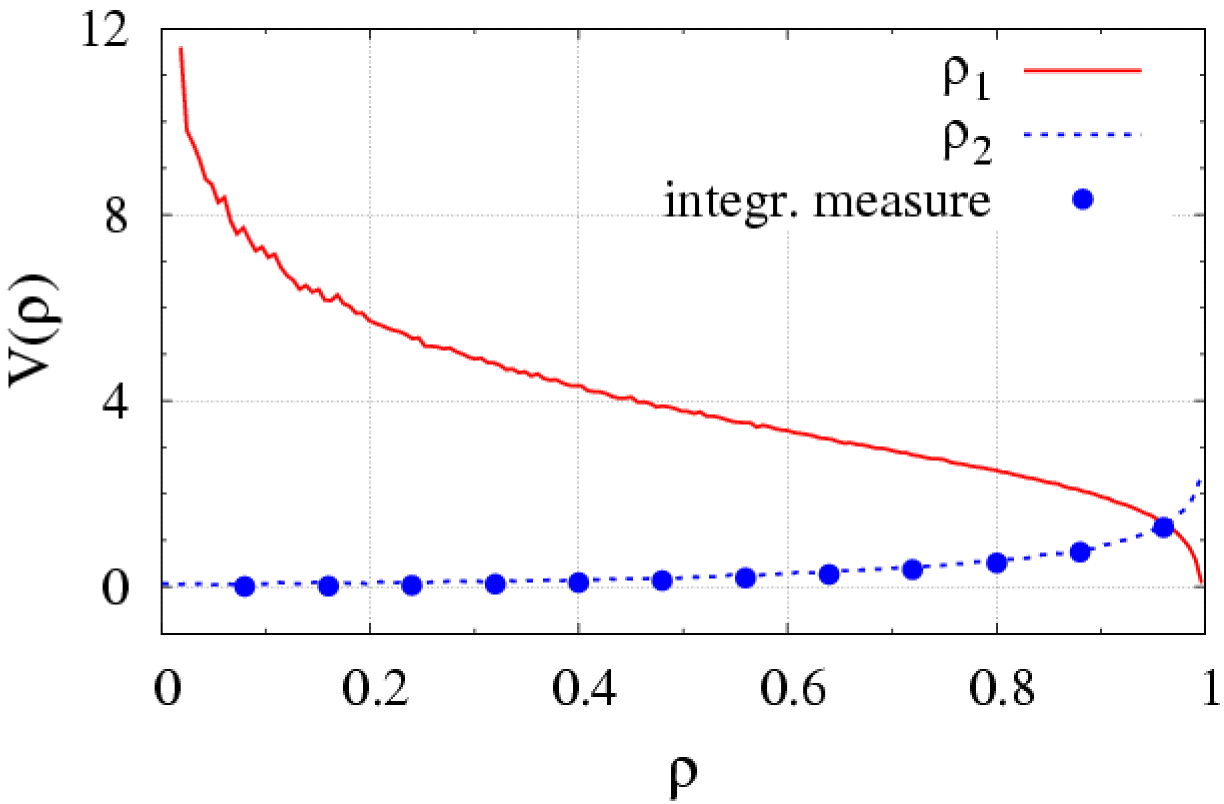


(Divergence from integration measure)

- Distribution of eigenvalue sum is almost flat !

- Computing the negative logarithm and shifting along the y-axis, gives the “potential” for the eigenvalues:

$$V(\rho)=-\log P(\rho)$$



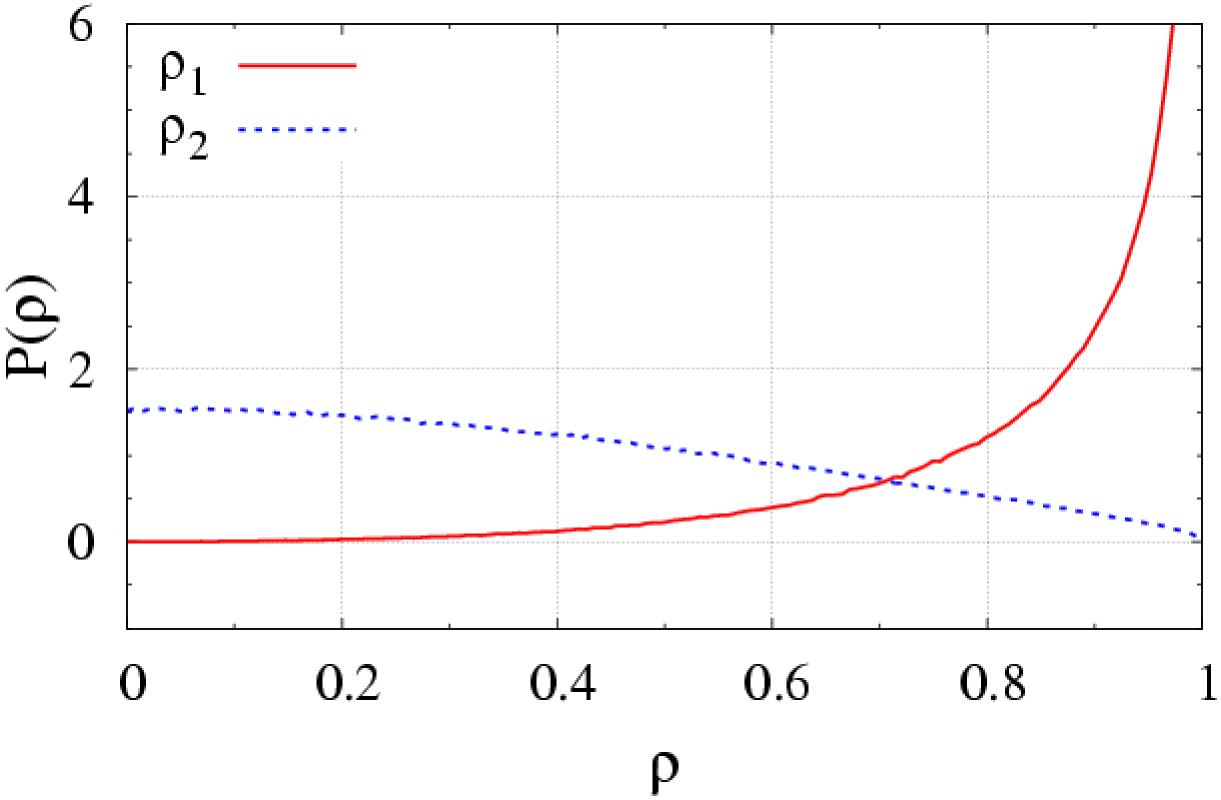
- Potential is dominated by the integration measure !

Remember Vandermonde: $V=-\ln(1-\cos^2\phi)$

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$$\beta = 1.0 \qquad m^2 = -0.15$$



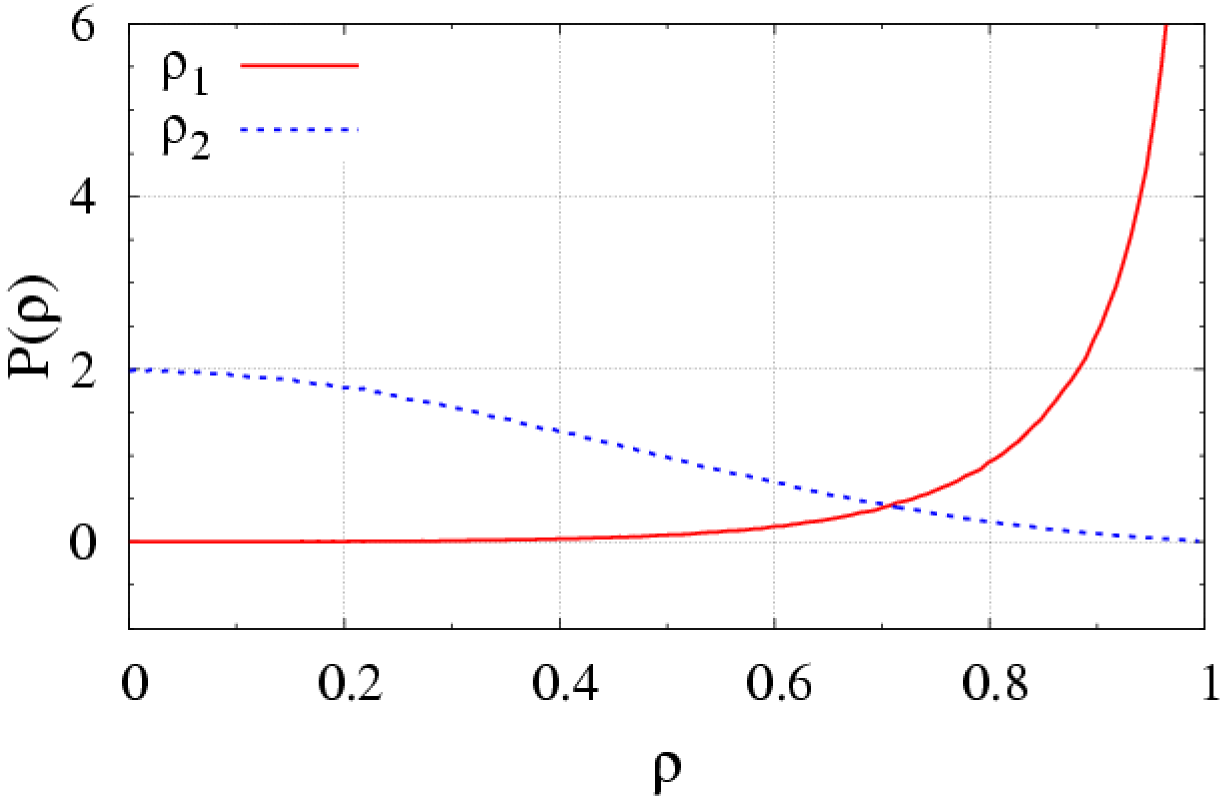
- Negative value for m^2 pushes distribution to the left.

  (Eigenvalue “repulsion”)

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

$$\beta = 1.5 \qquad m^2 = -0.15$$

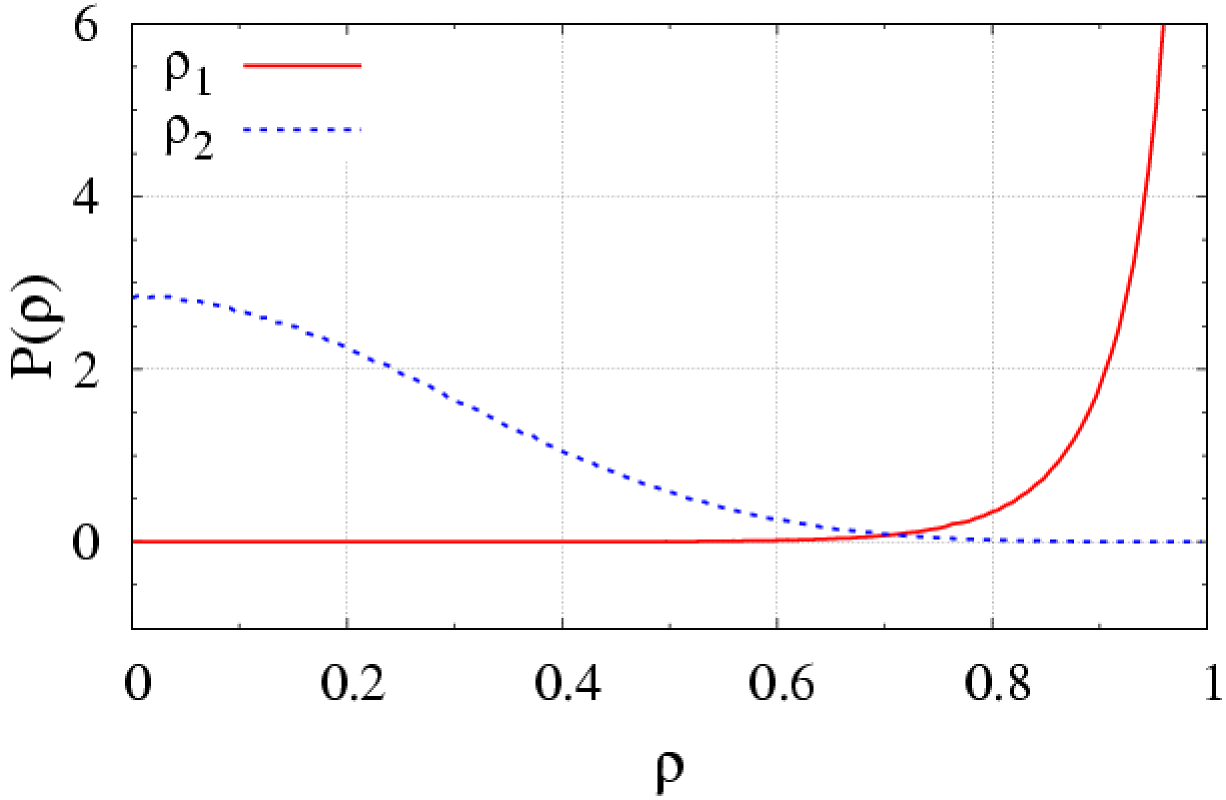


- Increasing β suppresses fluctuations.

$$\rho_1 = \frac{1}{2}|\lambda_1 - \lambda_2|$$

$$\rho_2 = \frac{1}{2}|\lambda_1 + \lambda_2|$$

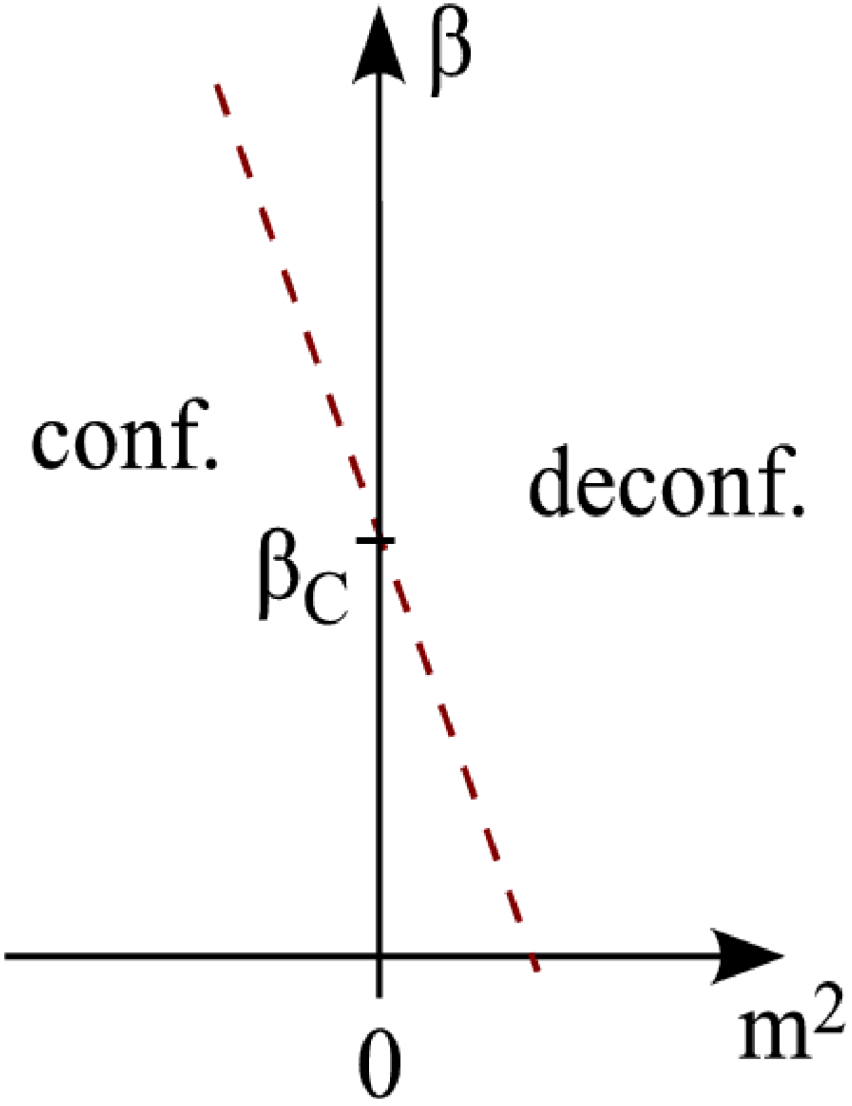
$\beta = 3.0 \qquad m^2 = -0.15$



- Strong eigenvalue repulsion is observed !

Phase Diagram

- For small β and small m^2 confinement is realized in a trivial way: $\bar{L} \rightarrow 0$
- Raising m^2 aligns Wilson Lines with unit Matrix.
- "Length" of $\bar{L}$ becomes finite for $\beta > \beta_C$. Direction of alignment is determined by potential. (in or out of Z(N) center)
- Small background field will tilt phase boundary to the left above β_C.



Summary and Outlook

- We performed Monte Carlo simulations of an effective theory of SU(2) Wilson Lines in three dimensions.
- We found a deconfining phase transition of second order.
- We showed that Mean Field does not describe the situation accurately.
- We observed eigenvalue repulsion in deconfined phase of 3d effective theory at $\beta>\beta_C$ (corresponding to moderately weak coupling in 4d theory). Disappears in extreme $\beta\rightarrow\infty$ limit.
- Non-trivial Z(N)-symmetric confined vacuum emerges at $\beta>\beta_C$:

  $\mathbf{L}_c \sim \mathrm{diag}(1, z, z^2, \ldots , z^{N-1})$, $z=\exp(2\pi i/N)$

  (while $\mathbf{L}_c \sim 0$ at small β)

Future:

- Include spatial components of the gauge fields (magnetic sector).
- Match couplings to real physical values.
- Extend to SU(3).

Thanks for coming !!!

This work was in part based on the MILC collaboration's public lattice gauge theory code.
See http://physics.utah.edu/~detar/milc.html

Reference:

Dumitru, Smith
Eigenvalue repulsion in an effective theory of SU(2) Wilson lines in three dimensions
arXiv:hep-lat/0711.0868