Soft EoS from heavy-ion data and the implications for compact stars

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Physics of Neutron Stars
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Compressibility and Mass-Radius relation of Compact stars

**Punch Line:** Apply a soft nuclear equation of state \((K_0 \leq 200\text{ MeV}, \text{deduced from heavy ion data})\) to study the implications on masses and radii of compact stars by calculating:

1. Maximum masses of neutron stars as a probe for nuclear matter compressibility using a phenomenological equation of state \((Prakash et al. PRL(1988))\)
2. Low mass neutron star radii as a probe for symmetry energy \((Horowitz et al., APJ (2003), \text{Correlation between neutron radius of } ^{208}\text{Pb and radii of low mass neutron stars})\)
3. Highest possible mass of compact stars \((Rhoades and Ruffini, PRL (1974))\)
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3. Highest possible mass of compact stars (Rhoades and Ruffini, PRL (1974))
Motivation: Compressibility of nuclear matter by KaoS Collaboration: Direct production of $K^+$ by:

$NN \rightarrow N \Delta K^+ \ (E_{\text{thr}} = 1.58 \text{ GeV})$

$NN \rightarrow NNK^- K^+ \ (E_{\text{thr}} = 2.5 \text{ GeV})$

Additional multi-step processes as channels at subthreshold energies, e.g.:

$NN \rightarrow N \Delta$

- $\Delta N \rightarrow N K^+ Y$
- $\Delta N \rightarrow K^- K^+ NN$
- $\pi N \rightarrow K^+ Y$

Probability increases with higher density

Trend of Kaon production up to a density of $n \sim 2.5 - 3 \ n_0$ is described by a soft equation of state ($K_0 \leq 200 \text{MeV}$)

Ch. Fuchs et al., PRL 86 (2001)
Phenomenological EoS

Energy per particle (Prakash et al. 1988):

\[ E/A = m_n (1 - x_p) + m_p x_p + E_0 u^{2/3} + \frac{A}{2} u + \frac{B}{\sigma + 1} u^\sigma + (1 - 2x_p)^2 S(u) \]

\[ S(u) = \left( 2^{2/3} - 1 \right) E_0 \left( u^{2/3} - F(u) \right) + S_0 F(u), \ u = n_b/n_0 \]

\[ E_0 = \text{average kinetic energy of symmetric matter at } n_0 \]

\[ Au/2 + Bu^\sigma / (\sigma + 1): \text{ describe interaction} \]

Input Parameters:

- **Binding energy** at saturation density: - 16 MeV at \( n_0 = 0.16 \text{fm}^{-3} \)
- **Asymmetry energy** \( S_0 \): Energy difference between pure neutron matter and normal symmetric matter at saturation density
- **Density dependence of asymmetry energy**
  \( \rightarrow \) Assume power law dependence \( F(u) = u^\alpha \).
- **Compressibility** \( K_0 \) for symmetric nuclear matter at saturation density
In the following

Mass-radius calculations using

- Core: Phenomenological EoS:
  \[ \frac{E}{A} = m_n \left(1 - x_p\right) + m_p x_p + E_0 u^{2/3} + \frac{A}{2} u + \frac{B}{\sigma+1} u^\sigma + (1 - 2x_p)^2 S(u) \]
- Inner crust: Negele and Vautherin, NPA, 1973

Variation of parameters:

- \(K_0 = 160\) MeV - 200 MeV (C.Sturm et al.(2001), KaoS collaboration)
- \(S_0 = 28\) MeV - 32 MeV (e.g. Steiner (2007))
- \(\alpha = 0.7 - 1.1\) (e.g. L.-W Chen et al.(2005), B.-A Li and A.W.Steiner (2006))

In addition: Compare to frequently used EoS

<table>
<thead>
<tr>
<th>EoS</th>
<th>(BE) [MeV]</th>
<th>(\rho_0) [fm(^{-3})]</th>
<th>(S_0) [MeV]</th>
<th>(K_0) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bsk8</td>
<td>-15.824</td>
<td>0.1589</td>
<td>28.0</td>
<td>230.2</td>
</tr>
<tr>
<td>Sly4</td>
<td>-15.969</td>
<td>0.160</td>
<td>32.0</td>
<td>229.9</td>
</tr>
<tr>
<td>TM1</td>
<td>-16.3</td>
<td>0.145</td>
<td>36.9</td>
<td>281</td>
</tr>
<tr>
<td>(Goriely et al., NPA 750 (2005))</td>
<td>(Chabanat et al., NPA 635 (1998))</td>
<td>(Y.Sugahara and H.Toki, NPA 579 (1994))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Maximum Mass - Motivation

- KaoS Experiment: $K_0 \leq 200$ MeV up to 2.5 - 3 $n_0$
- Compressibility of nuclear matter controls the maximum mass of neutron stars and its central density
- What are the consequences of a soft EoS for the maximum mass configuration?
Maximum Masses

Maximum masses $K_0 = 200$ MeV (soft EoS, compatible with KaoS) $\sim 1.9 M_\odot$, central densities of maximum mass configuration $\sim 8n_0$

Maximum masses for $K_0 = 160$ MeV $\rightarrow \sim 1.65 M_\odot$, central density $\sim 9n_0$

Note: Radii of low mass neutron stars are sensitive to symmetry energy
Maximum Mass - Skyrme and RMF EoS

Maximum masses for SLy4 Skyrme EoS: $\sim 2.05 \, M_\odot$ (central density: $\sim 8n_0$)

Maximum masses for TM1 RMF EoS: $\sim 2.17 \, M_\odot$ (central density: $\sim 6.4n_0$)
Maximum masses for a soft Skyrme type EoS (compatible with KaoS) up to $\sim 2 M_\odot$

**But:** Central densities in maximum mass configurations are between $8 - 9 n_0$

Requirement for stiffening of EoS for higher densities or onset of exotic physics

Relativistic mean field EoS: TM1 $\rightarrow$ central densities at maximum mass $\sim 6.4 n_0$
Low mass neutron stars - Motivation

- Smallest pulsar mass know so far from observation: $1.18 \pm 0.02 \, M_\odot$ (J1756-2251)
- Central densities in low-mass neutron stars might be $\leq 3 \, n_0$ → within the density region explored by KaoS
- As seen: Radius of low mass neutron stars sensitive to the asymmetry term
Low mass neutron stars - $M_{psr} = 1.18 \pm 0.02 \, M_\odot$

<table>
<thead>
<tr>
<th>$K_0$ [MeV]</th>
<th>$S_0$ [MeV]</th>
<th>$\alpha$</th>
<th>$n$ [$n_0$]</th>
<th>$R$ [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>32</td>
<td>1.1</td>
<td>2.4</td>
<td>$\sim 13.2$</td>
</tr>
<tr>
<td>200</td>
<td>28</td>
<td>0.7</td>
<td>2.9</td>
<td>$\sim 12.1$</td>
</tr>
<tr>
<td>160</td>
<td>32</td>
<td>1.1</td>
<td>2.8</td>
<td>$\sim 12.9$</td>
</tr>
<tr>
<td>160</td>
<td>28</td>
<td>0.7</td>
<td>3.7</td>
<td>$\sim 11.5$</td>
</tr>
</tbody>
</table>
Low mass neutron stars - $M_{psr} = 1.18 \pm 0.02 \, M_\odot$

- For $K_0=200$ MeV → densities are in the upper range of the probed densities by KaoS.
- Radius variation due to symmetry energy term: $\sim 1$ km
Results

Highest possible mass

**Motivation:** *(Rhoades and Ruffini, RRL (1974))*

- For densities below a critical value use known equation of state: Harrison - Wheeler EoS
- For higher densities: stiffest possible (i.e. causal) equation of state with

\[ p(\epsilon) = \epsilon - \epsilon_0 \]

\( \epsilon_0 \): parameter to fit stiffest EoS on soft EoS
- Result: Largest possible masses for neutron stars
- \( \rightarrow \) for \( \rho_{\text{crit}} = 4.6 \cdot 10^{14} \text{ g/cm}^3 \) highest possible mass of 3.2 \( M_\odot \)

**Update:** *(Kalogera and Baym, ApJ 470 (1996))*

- For densities below critical value: EoS by Wiringa et al. (Phys.Rev.C, 38, 1988), for \( \rho < 2.5 \cdot 10^{14} \text{ g/cm}^3 \) equation of state of Baym, Pethick and Sutherland (1971)
- Result:

\[ M_{\text{max}} = 6.7M_\odot \left( \frac{\rho_c}{10^{14} \text{ g/cm}^3} \right)^{-1/2} \]

for critical density of \( 2 \rho_0 \rightarrow M_{\text{max}} = 2.9 M_\odot \)

**Here:**

- Here: For low densities \( (n \leq 3n_0) \): EoS should fullfil results from KaoS
Results

Highest possible mass - Phenomenological EoS

For $n_0 = 0.16 \text{ fm}^{-3}$ and assuming that the “phase transition” takes place for $n > 2.5n_0$

$\rightarrow M_{\text{limit}} \lesssim 2.7 \text{ M}_\odot$
Similar result for SLy4 Skyrme equation of state with $K_0 = 230$ MeV, $S_0 = 32$ MeV
Results

Highest possible mass - Skyrme Bsk8

Similar result for Bsk8 Skyrme equation of state with $K_0 = 230$ MeV, $S_0 = 28$ MeV
Slightly higher masses for TM1
Summary

- Heavy Ion Data (KaoS Collaboration) → soft nuclear equation of state with $K_0 \leq 200$ MeV up to $n \sim 3 \, n_0$
- Applying soft equation of state gives maximal masses $M \lesssim 2 \, M_\odot$ with large central densities → onset of exotic matter?
- For low mass neutron stars: central densities are in the range of $2.5 - 3 \, n_0$ (within the density range probed by KaoS), Neutron star radii show strong dependence on symmetry energy term
- Applying the stiffest equation of state in combination with the KaoS data can restricts the highest possible pulsar mass ($M \lesssim 2.7$ for $n_{\text{crit}} \geq 2.5 \, n_0$; for TM1 (relativistic mean field EoS): $M \lesssim 2.84$ for $n_{\text{crit}} \geq 2.5 \, n_0$)
Thank You for your Attention