A Statistical Model for a Complete Supernova Equation of State

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Introduction

- EOS essential for any simulation in astrophysics
- plenty of EOSs for cold neutron stars (uniform, catalyzed matter)
- difficulties for supernova EOS:
  - finite temperature $T = 0 - 100$ MeV
  - no weak equilibrium $Y_p = 0 - 0.6$
  - broad density range $\rho = 10^4 - 10^{15}$ g/cm$^3$

→ huge parameter region
Liquid-Gas Phase Transition in Supernovae

- first order phase transition below $T_c \sim 15$ MeV
- mixed phase with formation of nuclei
- at $\rho \sim 10^{11} - 10^{13}$ g/cm$^3$: neutrino spheres
- stall of the shock front

$\rightarrow$ special role of EOS below saturation density

[Liebendörfer, Fischer et al.]
Commonly Used EOSs

• Lattimer, Swesty: non-relativistic liquid drop model
  [Lattimer, Swesty; 1991NPA535]

• Shen, Toki, et al.: relativistic mean-field, Thomas-Fermi approximation
  [Shen et al.; 1998NPA637]

Limitations
• one representative nucleus: “single nucleus approximation”
• only $\alpha$-particles for light clusters
• no shell effects
Advanced Models

Focusing on single aspects / restricted in parameter range, e.g.:

• nuclear statistical equilibrium models applied in multifragmentation experiments and astrophysics, e.g. “SMM”  
  [Botvina, Mishustin; 2004PLB233] [Blinnikov e al.; arXiv:0904.3849]

• quantum many-body approach for medium effects on light clusters  
  [Typel, et al.; 2010PRC81] [Röpke; 2009PRC79]

• impact of light clusters & nuclear distributions on dynamical processes in supernovae (e.g. neutrino and electron emission, absorption & scattering)  
  [Arcones et al.; 2008PRC78] [Caballero, Horowitz et al.; 2006PRC74]
Excluded Volume NSE Model with Interactions

interacting nucleons and nuclei with excluded volume corrections in nuclear statistical equilibrium

new model for the SN EOS, which
• is applicable for all conditions in SN
• allows “fast” calculation of new SN EOS tables
• contains new nuclear physics aspects

→ phenomenological model
→ has to be confronted with theoretical and experimental constraints
Nucleons

• relativistic mean-field model
• interactions mediated via exchange of mesons and meson (self-) interactions

\[ \mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - M) \psi \\
\quad + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - g_\sigma \bar{\psi} \sigma \psi \\
\quad - \frac{1}{4} \Omega_{\mu \nu} \Omega^{\mu \nu} - \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} g_4 (\omega_\mu \omega^\mu)^2 - g_\omega \bar{\psi} \gamma^\mu \psi \omega_\mu \\
\quad - \frac{1}{4} R^a_{\mu \nu} R^{a \mu \nu} + \frac{1}{2} m_\rho^2 \rho^a_\mu \rho^{a \mu} - g_\rho \bar{\psi} \gamma_\mu \tau^a \psi \rho^{a \mu} - \Lambda \omega_\mu \omega^\mu \rho^a_\nu \rho^{a \nu} \]

• coupling constants and meson masses fitted to experimental data
Nucleons – Parameter Sets

- TM1: fitted to masses of heavy nuclei & EOS of relativistic BHF; used in Shen EOS
  \[\text{[Sugahara, Toki; 1994NPA579]}\]

- TMA: interpolation between TM1 and TM2 (fit to light nuclei)
  \[\text{[Geng et al.; 2005PTP113] [Gong et al.; 2001CoPhC136]}\]

- FSUgold: including coupling of the isovector $\rho$ to the isoscalar $\omega$ meson
  \[\text{[Todd-Rutel, Piekarewicz.; 2005PRL95]}\]

<table>
<thead>
<tr>
<th></th>
<th>$a_{\text{sym}}$ [MeV]</th>
<th>$K$ [MeV]</th>
<th>$M_{\text{max}}$ [$M_{\odot}$]</th>
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<tbody>
<tr>
<td>TM1</td>
<td>36.9</td>
<td>281</td>
<td>2.2</td>
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<tr>
<td>TMA</td>
<td>30.7</td>
<td>318</td>
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<tr>
<td>FSUgold</td>
<td>32.6</td>
<td>230</td>
<td>1.7</td>
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</table>
EOS of Pure Neutron Matter

- affects symmetry energy
- agreement of various models, e.g.:
  - S&P: model independent EFT
  - APR: variational approach based on NN potentials
  - Vlowk: low-momentum effective interaction
- important constraint for the low-density EOS
- implications for large densities

- relativistic mean-field: $\omega$-$\rho$-coupling required (only included in FSUgold)

[Tolos et al.; 2008NPA806]
Nuclei

- experimental data of AWT (2003):
  over 2000 precisely measured binding energies
  [Audi et al.; 2003NPA729]

unknown nuclei: mass table generated with theoretical nuclear model

- **TMA:** [Geng et al.; 2005PTP113]
  - uses the same relativistic mean-field description
  - BCS $\delta$-force pairing, axial deformations, mass-number dependent coupling constants

- **FRDM:** [Möller et al.; 1995ADNDT59]
  - finite range droplet model
  - precise reproduction of known masses
Nuclei

• semi-empirical formula for intrinsic partition function of hot nuclei

\[ g_i(T) = g_i^0 + \frac{c_1}{A_i^{5/3}} \int_0^{B_i} dE e^{-E/T} \exp(2\sqrt{a_i E}) \]  

[Fai, Randrup; 1982NPA381]

• Coulomb energies in Wigner-Seitz approximation

\[ E_{Coul} = \frac{3}{5} \frac{Z_i^2 \alpha}{R_i} \left( \frac{3}{2} x - \frac{1}{2} x^3 \right) \]
Excluded Volume

model assumptions
- each baryon fills a volume $1/n_B^0$, volume of a nucleus $V_{A,Z} = A/n_B^0$
- nucleons must not be inside of nuclei

• filling factor of the nucleons
\[ \xi = \frac{V'}{V} = 1 - \sum_{A,Z} A \frac{n_{A,Z}}{n_B^0} \]

• free volume fraction
\[ \kappa = 1 - \frac{n_B}{n_B^0} \]

• total baryon number density
\[ n_B = \xi (n'_n + n'_p) + \sum_{A,Z} A \cdot n_{A,Z} \]
\[ n'_n, n'_p \text{: local number densities} \]
Thermodynamics

- thermodynamic potential: free energy density

\[ f = f_e^0(T, n_e) + \sum_{A,Z} f_{A,Z}^0(T, n_{A,Z}) + f_{\text{Coul}}(n_e, \{n_{A,Z}\}) \]

\[ + \xi f_{\text{RMF}}^0(T, n'_n, n'_p) - T \sum_{A,Z} n_{A,Z} \ln(\kappa) , \]

ideal gas expressions, Coulomb contribution, filling factor for nucleons, direct excluded volume contribution

- medium effects on the nuclei are mimicked by excluded volume approach:
  - low fraction of nuclei, \( \xi \sim 1 \): unmodified RMF model
  - \( \kappa \to 0 \) for \( n_B \to n^0_B \) \( \Rightarrow f \to \infty \) if nuclei are present
    \( \Rightarrow \) no nuclei above saturation density

- thermodynamic fully consistent
Composition for TMA

- general isothermal density dependence: nucleons → light → (heavies) → uniform nuclear matter
- stepwise change for T=1 MeV: narrow distributions, almost discrete change of nucleus
- continuous change for larger T

- distinction of light and heavy nuclei by charge Z=6:
  \[ X_a = \sum_{A,Z \leq 5} A n_{A,Z} / n_B \]
  \[ X_A = \sum_{A,Z \geq 6} A n_{A,Z} / n_B \]
Nuclear Distributions

• $T=0.1$ MeV: only one nucleus
• peaks caused by neutron shell effects
• neutron magic numbers (40), 50, 82, 126, 184
• $T=1$ MeV: shell effects leading to narrow distributions with multiple peaks
• $T \geq 5$ MeV:
  - broad distributions
  - exponential $\rightarrow$ power-law $\rightarrow$ u-shape
  - as manifestation of the onset of the liquid-gas phase transition
Heavy Nuclei – Shell Effects

- comparison to $T=0$ calculation in $\beta$-equilibrium with explicit lattice energy
- discrete changes
- up to $10^{-4}$ fm$^{-3}$ based only on experimental data

$\rightarrow$ excellent description at low temperatures and densities

[Rüster, MH, Schaffner-Bielich; 2006PRC73]
Light Clusters

- **T = 1 MeV:**
  light clusters well represented by \( \alpha \)s
- **T \geq 5 MeV:**
  large Deuteron contribution at low \( n_B \),
  differences for \( \alpha \)s at large \( n_B \)
- **\( n_B \sim 10^{-2} \text{ fm}^{-3} \):**
  whole distribution of light clusters important

- behaviour not representable by alphas alone
Equation of State I

- excellent agreement at low $T$ across all densities
- $T \geq 5$ MeV: thermal distributions lead to additional binding around $n_B \sim 5 \times 10^{-3}$ fm$^{-3}$
- at $n_B > 10^{-2}$ fm$^{-3}$: restricted mass table, strong excluded volume: increased free energy

T = 5 MeV

$Y_p = 0.3$
Equation of State II

- increased entropy and energy from the light cluster distribution
- deuterons modify the EOS at low densities
Symmetry energy – Comparison with Experimental data

- symmetry energy extracted from low-energy heavy ion collisions
- $v_{\text{surf}}$: velocity of particles at ejection, characterizes emission time
- corresponding to different $T=3-7$ MeV and $n_B=1/100-1/20$ $n_B^0$
- cluster formation leads to increased symmetry energy

<table>
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<th>$T$ [MeV]</th>
<th>3.3</th>
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<th>3.6</th>
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<th>5.3</th>
<th>6.2</th>
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<tr>
<td>$n_B$ [$10^{-3}$ fm$^{-3}$]</td>
<td>2.1</td>
<td>1.7</td>
<td>2.3</td>
<td>3.8</td>
<td>4.7</td>
<td>4.9</td>
<td>5.5</td>
<td>6.4</td>
</tr>
</tbody>
</table>

[Kowalski et al.; 2007PRC75]
[Natowitz et al.; arXiv:1001.1102]
Supernova simulations
collaboration with Tobias Fischer, Basel

- exploratory first core-collapse SN simulations
- all light clusters treated as alpha-particles, only average heavy nucleus
- slightly more compact PNS core

→ change of the low-density model more important than change TM1↔TMA
Adiabatic Decompression without Weak Reactions

\[ Y_p = 0.3 \quad s/n_B \sim 1 \quad FSU\text{gold} + \text{FRDM} \]

\[ T = 35.48 \text{ MeV} \]
\[ n_B = 1.0 \times 10^0 \text{ fm}^{-3} \]
Adiabatic Decompression without Weak Reactions

- possible freeze-out scenario
- formation of heavy elements beyond iron
- possible seed nuclei for r-process
Conclusions & Outlook

• phenomenological, thermodynamic consistent model for an ensemble of nuclei in equilibrium with an interacting nucleon gas
  → provide EOS tables with different nuclear interactions

• excellent description at low T and $n_B$; strong shell effects

• deviations to commonly used EOSs at large temperatures due to thermal distributions; important role of light clusters

• good agreement with experimental measurements of symmetry energy

• first SN simulations: model for the low-density EOS as important as the nuclear interactions → study implications further

→ take distributions of nuclei in the reaction rates into account

→ implement phase transition to quark matter