A Statistical Model for a Complete Supernova Equation of State

Nucl. Phys. A, in press; arXiv:0911.4073

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15th Workshop on Nuclear Astrophysics, Castle Ringberg, Kreuth
March 22, 2010

HGIR Graduate Program Hadron and Ion Research

comp star

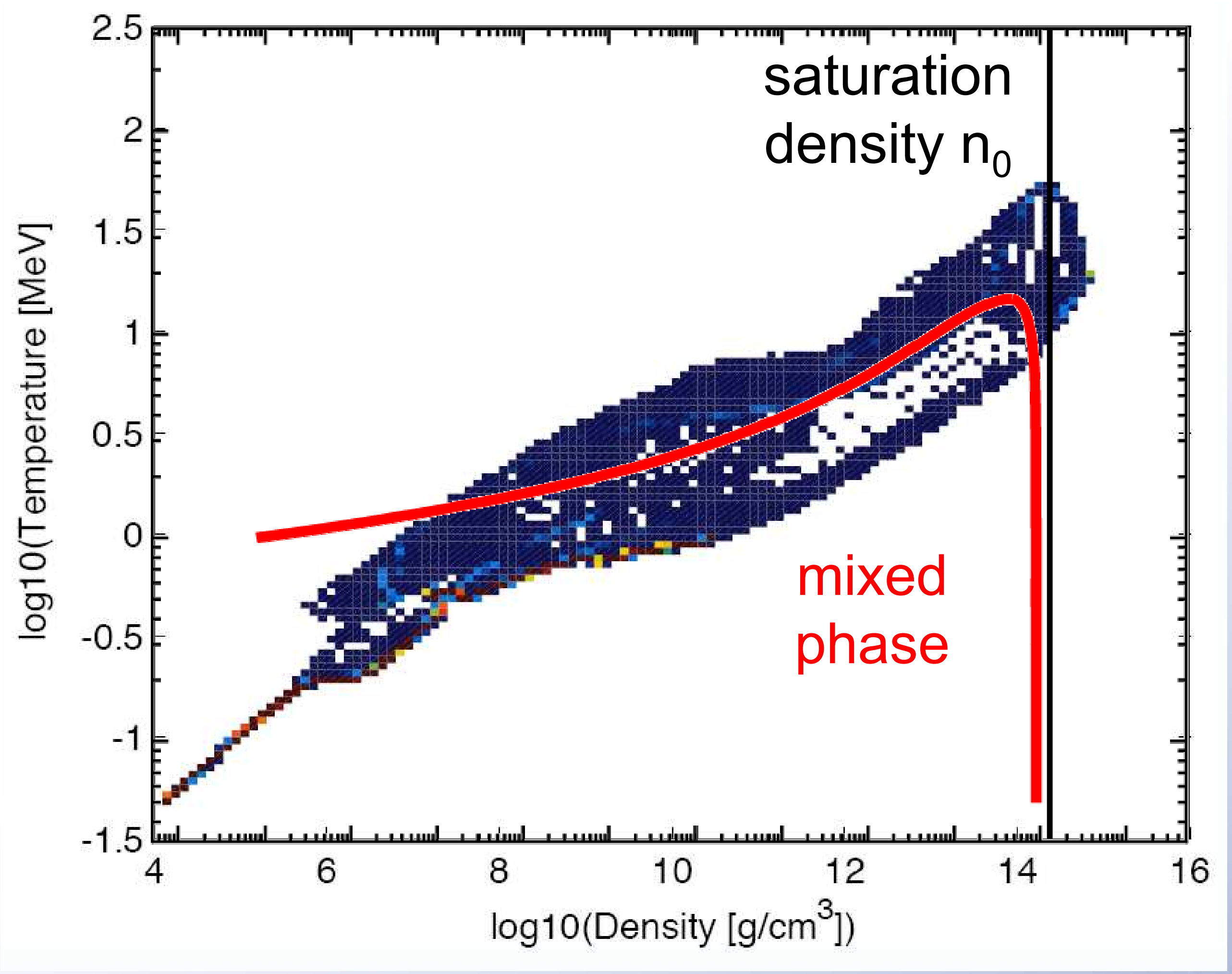
Introduction

- EOS essential for any simulation in astrophysics
- plenty of EOSs for cold neutron stars (uniform, catalyzed matter)
- difficulties for supernova EOS:
    - finite temperature T = 0 – 100 MeV
    - no weak equilibrium $Y_p = 0 - 0.6$
    - broad density range $\rho = 10^4 - 10^{15}$ g/cm^3

→ huge parameter region

Liquid-Gas Phase Transition in Supernovae

EoS calls during SN simulation



[Liebendörfer, Fischer et al.]

- first order phase transition below T_c~15 MeV
- mixed phase with formation of nuclei
- at $\rho \sim 10^{11} – 10^{13}$ g/cm³: neutrino spheres
- stall of the shock front

→ special role of EOS below saturation density

Commonly Used EOSs

- Lattimer, Swesty: non-relativistic liquid drop model
  [Lattimer, Swesty; 1991NPA535]
- Shen, Toki, et al.: relativistic mean-field, Thomas-Fermi approximation
  [Shen et al.; 1998NPA637]

Limitations

- one representative nucleus: “single nucleus approximation”
- only α-particles for light clusters
- no shell effects

Advanced Models

Focusing on single aspects / restricted in parameter range, e.g.:

- nuclear statistical equilibrium models applied in multifragmentation experiments and astrophysics, e.g. “SMM”
  [Botvina, Mishustin; 2004PLB233] [Blinnikov e al.; arXiv:0904.3849]
- quantum many-body approach for medium effects on light clusters
  [Typel, et al.; 2010PRC81] [Röpke; 2009PRC79]
- impact of light clusters & nuclear distributions on dynamical processes in supernovae (e.g. neutrino and electron emission, absorption & scattering)
  [Arcones et al.; 2008PRC78] [Caballero, Horowitz et al.; 2006PRC74]

Excluded Volume NSE Model with Interactions

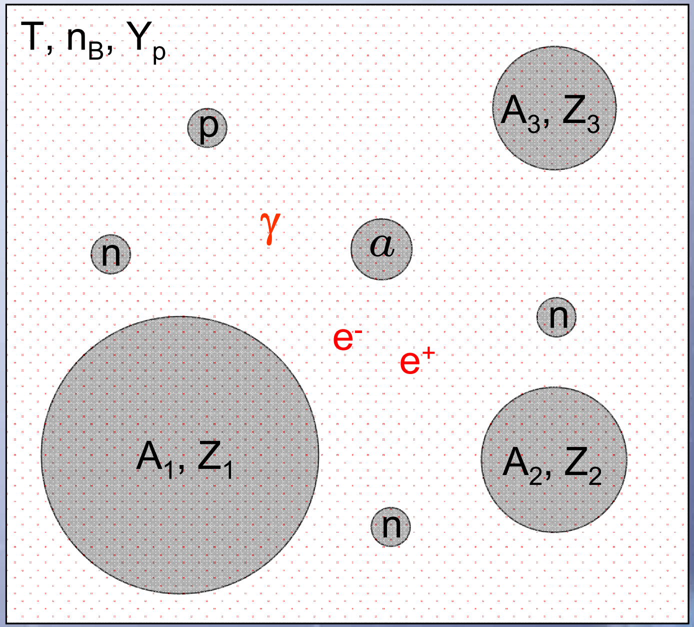
interacting nucleons and nuclei with excluded volume corrections in nuclear statistical equilibrium

new model for the SN EOS, which

- is applicable for all conditions in SN
- allows “fast” calculation of new SN EOS tables
- contains new nuclear physics aspects

→ phenomenological model

→ has to be confronted with theoretical and experimental constraints



Nucleons

- relativistic mean-field model
- interactions mediated via exchange of mesons and meson (self-) interactions

$$\begin{aligned}
\mathcal{L} \;=\;& \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi \\
&+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_\sigma\bar{\psi}\sigma\psi \\
&-\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}g_4(\omega_\mu\omega^\mu)^2 - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu \\
&-\frac{1}{4}R^a{}_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} - g_\rho\bar{\psi}\gamma_\mu\tau^a\psi\rho^{\mu a} - \Lambda\omega_\mu\omega^\mu\rho_\nu^a\rho^{a\nu}
\end{aligned}$$

- coupling constants and meson masses fitted to experimental data

Nucleons – Parameter Sets

- TM1: fitted to masses of heavy nuclei & EOS of relativistic BHF; used in Shen EOS

  [Sugahara, Toki; 1994NPA579]

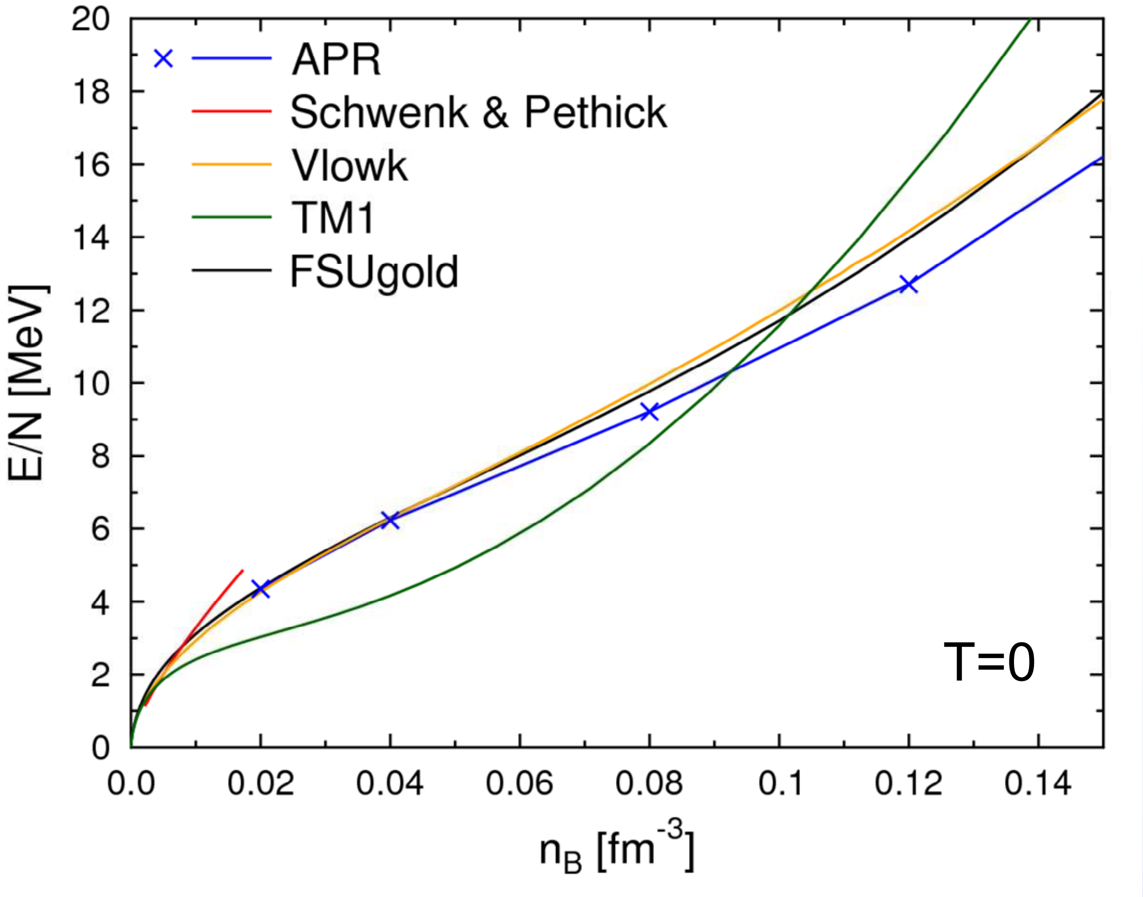
- TMA: interpolation between TM1 and TM2 (fit to light nuclei)

  [Geng et al.; 2005PTP113] *[Gong et al.; 2001CoPhC136]*

- FSUgold: including coupling of the isovector ρ to the isoscalar ω meson *[Todd-Rutel, Piekarewicz.; 2005PRL95]*

	a_{sym} [MeV]	K [MeV]	M_{max} [M_{sun}]
TM1	36.9	281	2.2
TMA	30.7	318	2.0
FSUgold	32.6	230	1.7

EOS of Pure Neutron Matter

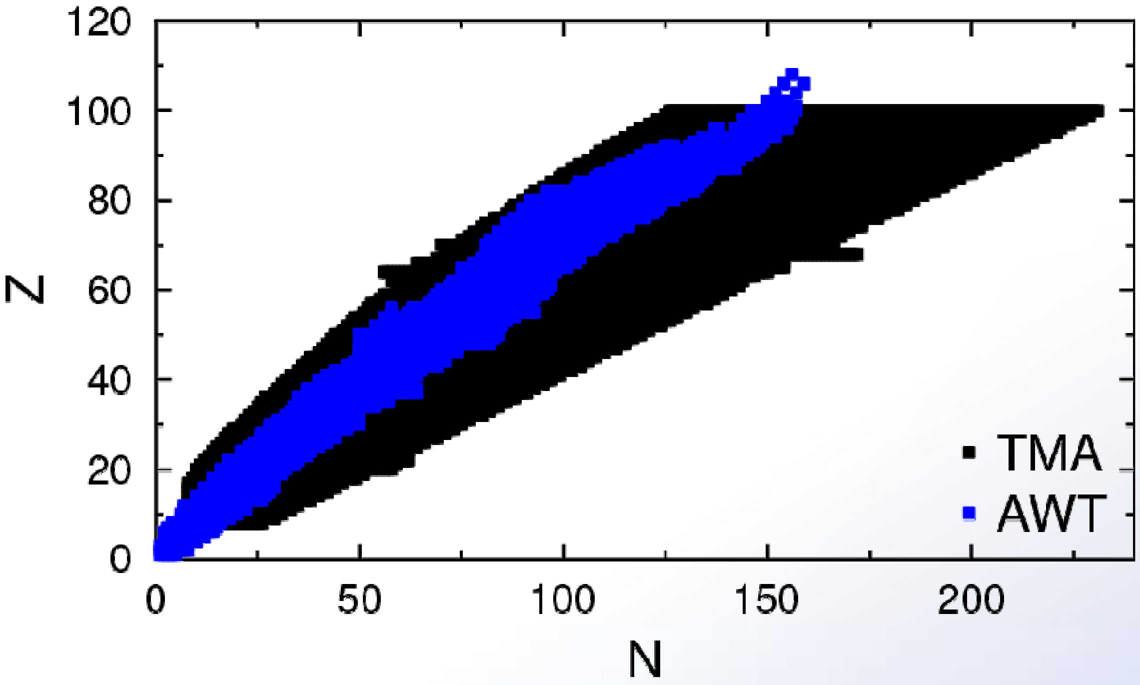


- affects symmetry energy
- agreement of various models, e.g.:
  - S&P: model independent EFT
  - APR: variational approach based on NN potentials
  - Vlowk: low-momentum effective interaction
- important constraint for the low-density EOS
- implications for large densities

- relativistic mean-field: ω-ρ-coupling required (only included in FSUgold)

[Tolos et al.; 2008NPA806]

Nuclei



- experimental data of AWT (2003):

  over 2000 precisely measured binding energies

  [Audi et al.; 2003NPA729]

unknown nuclei: mass table generated with theoretical nuclear model

- TMA: *[Geng et al.; 2005PTP113]*
  - uses the same relativistic mean-field description
  - BCS δ-force pairing, axial deformations, mass-number dependent coupling constants
- FRDM: *[Möller et al.; 1995ADNDT59]*
  - finite range droplet model
  - precise reproduction of known masses

Nuclei

- semi-empirical formula for intrinsic partition function of hot nuclei

$$g_i(T) = g_i^0 + \frac{c_1}{A_i^{5/3}} \int_0^{B_i} dE e^{-E/T} \exp(2\sqrt{a_i E})$$

[Fai, Randrup; 1982NPA381]

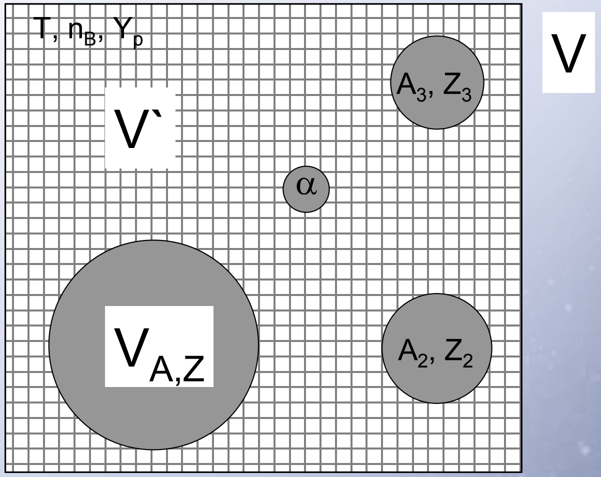
- Coulomb energies in Wigner-Seitz approximation

$$E_{Coul} = -\frac{3}{5}\frac{Z_i^2 \alpha}{R_i}\left(\frac{3}{2}x - \frac{1}{2}x^3\right)$$

Excluded Volume

model assumptions

- each baryon fills a volume $1/n^0_B$, volume of a nucleus $V_{A,Z} = A/n^0_B$
- nucleons must not be inside of nuclei



- filling factor of the nucleons

$$\begin{aligned} \xi &= V'/V \\ &= 1 - \sum_{A,Z} A\, n_{A,Z}/n^0_B \end{aligned}$$

- free volume fraction

$$\kappa = 1 - n_B/n^0_B$$

- total baryon number density

$$n_B = \xi(n'_n + n'_p) + \textstyle\sum_{A,Z} A\, n_{A,Z}$$

n'_n, n'_p: local number densities

Thermodynamics

- thermodynamic potential: free energy density

$$\begin{aligned} f &= f_e^0(T,n_e)+\sum_{A,Z} f_{A,Z}^0(T,n_{A,Z})+f_{Coul}(n_e,\{n_{A,Z}\}) \\ &\quad +\xi f_{RMF}^0(T,n_n',n_p')-T\sum_{A,Z} n_{A,Z}\ln(\kappa)\ , \end{aligned}$$

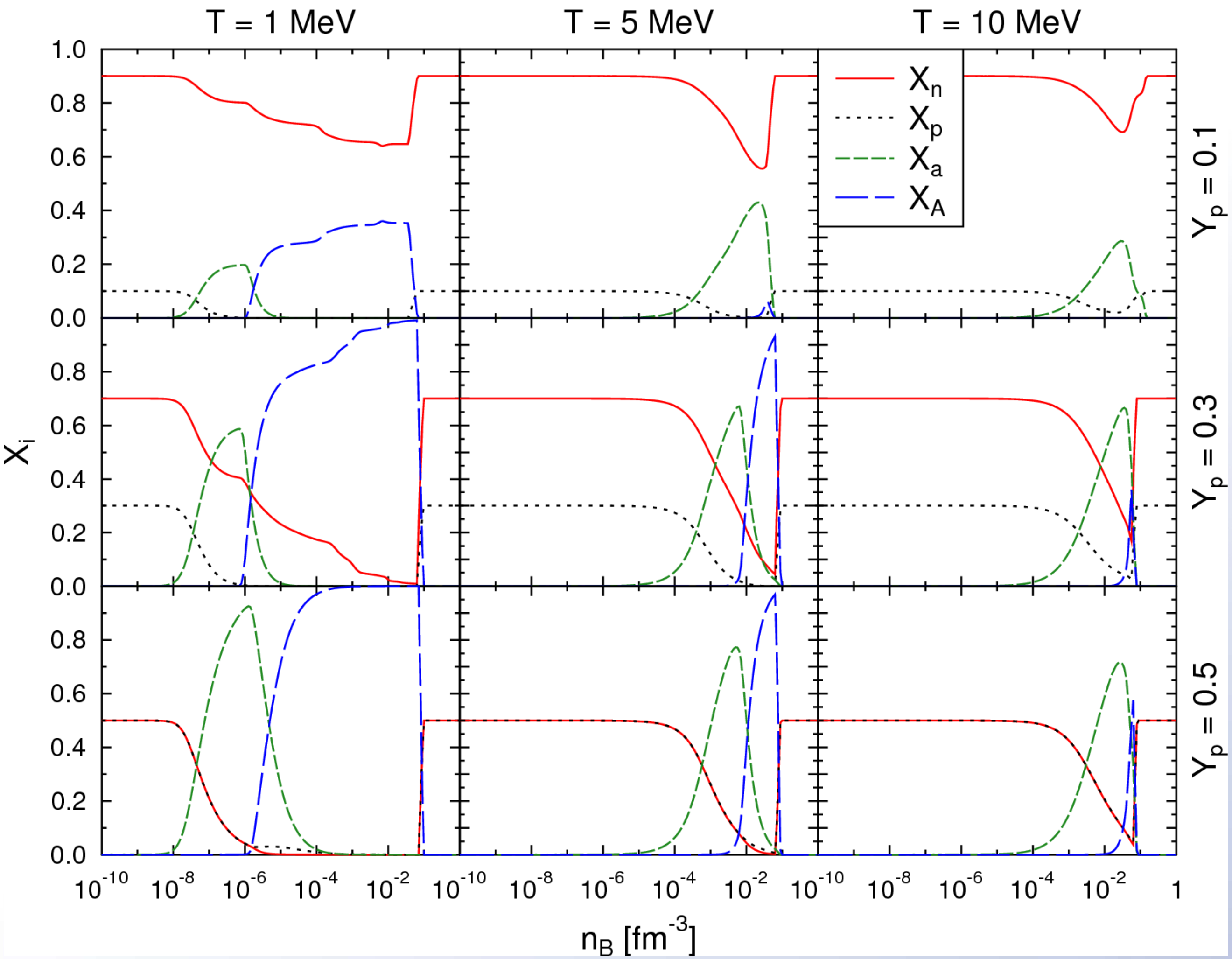
  ideal gas expressions, Coulomb contribution, filling factor for nucleons, direct excluded volume contribution

- medium effects on the nuclei are mimicked by excluded volume approach:
  - low fraction of nuclei, $\xi \sim 1$: unmodified RMF model ✓
  - $\kappa \rightarrow 0$ for $n_B \rightarrow n_B^0$ $\Rightarrow f \rightarrow \infty$ if nuclei are present

    → no nuclei above saturation density ✓

- thermodynamic fully consistent

Composition for TMA



- general isothermal density dependence:

  nucleons → light → (heavies) → uniform nuclear matter
- stepwise change for T=1 MeV:

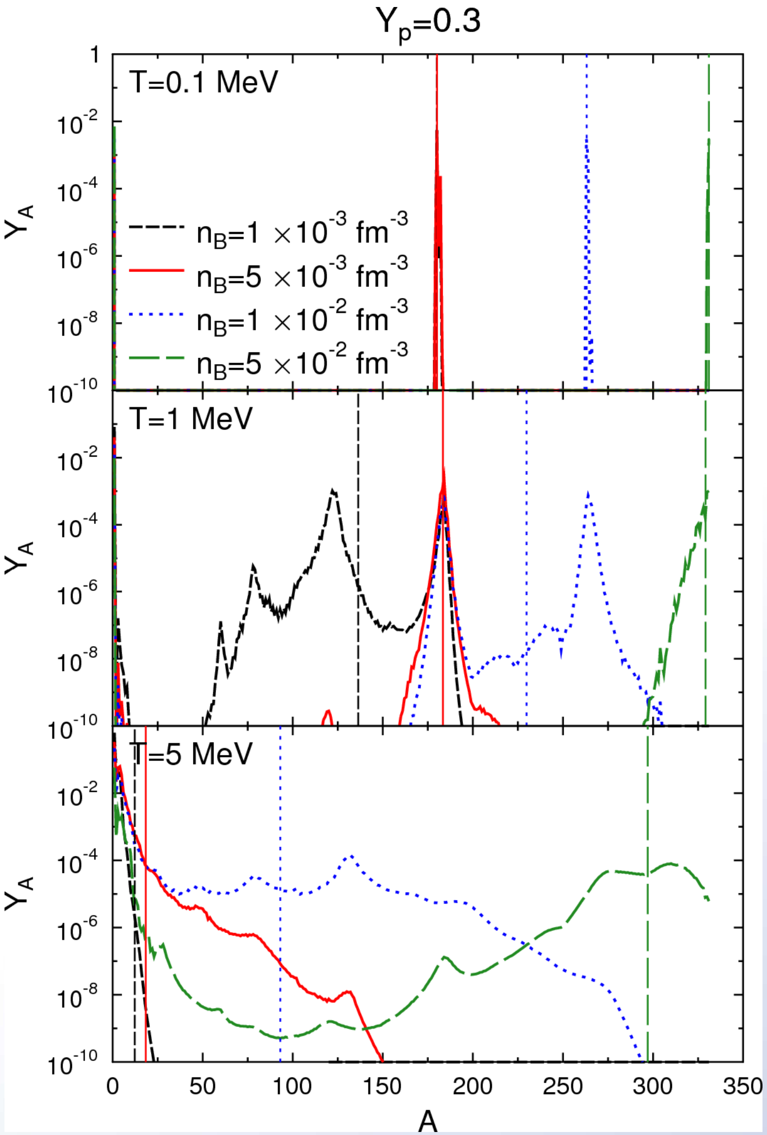
  narrow distributions, almost discrete change of nucleus
- continuous change for larger T

- distinction of light and heavy nuclei by charge Z=6:

$$X_a = \sum_{A,Z\leq 5} A n_{A,Z}/n_B$$

$$X_A = \sum_{A,Z\geq 6} A n_{A,Z}/n_B$$

Nuclear Distributions



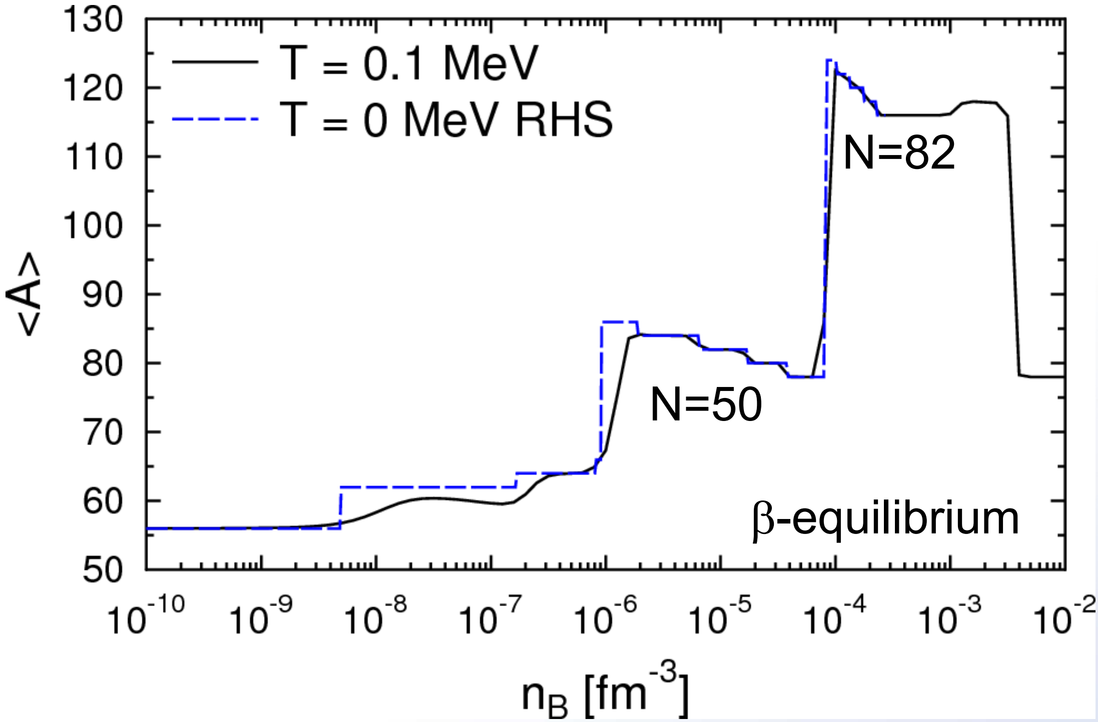
- T=0.1 MeV: only one nucleus
- peaks caused by neutron shell effects
- neutron magic numbers (40), 50, 82, 126, 184
- T=1 MeV: shell effects leading to narrow distributions with multiple peaks
- T ≥ 5 MeV:

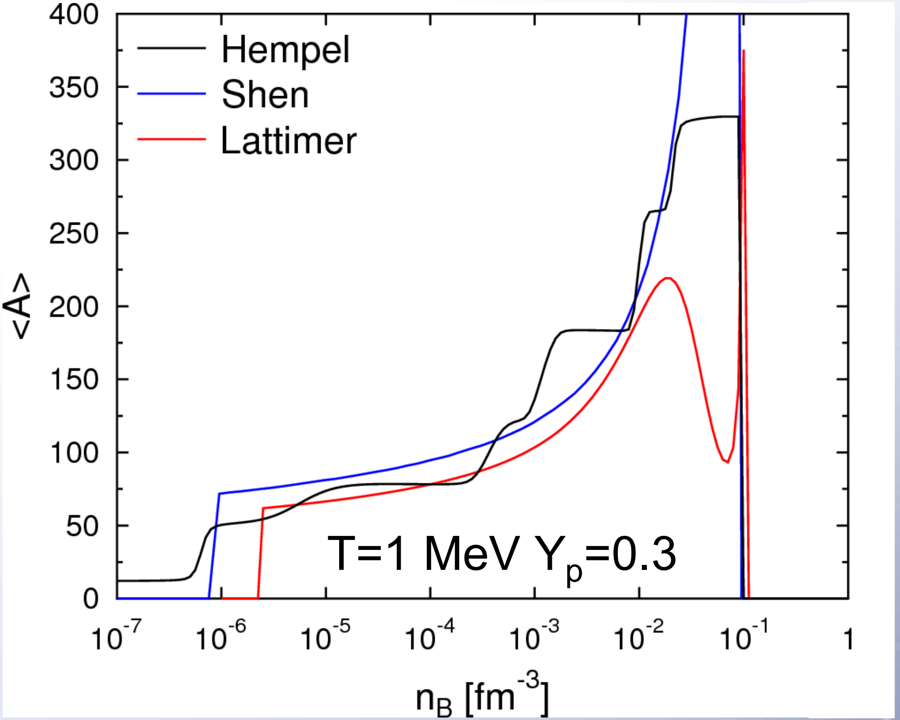
  broad distributions

  exponential → power-law → u-shape

  as manifestation of the onset of the liquid-gas phase transition

Heavy Nuclei – Shell Effects



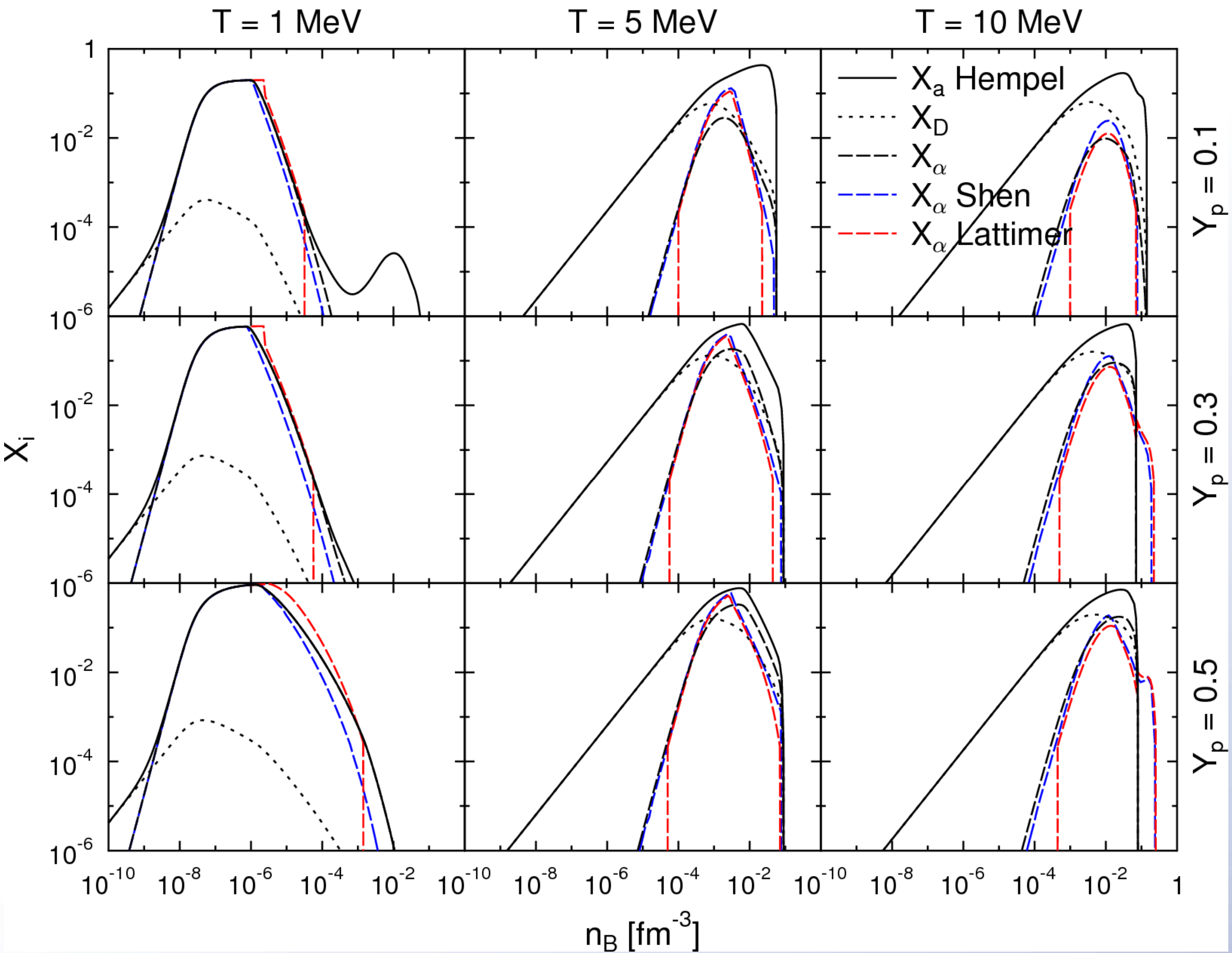


- comparison to T=0 calculation in β-equilibrium with explicit lattice energy
- discrete changes
- up to 10^{-4} fm^{-3} based only on experimental data

→ excellent description at low temperatures and densities

[Rüster, MH, Schaffner-Bielich; 2006PRC73]

Light Clusters



- T=1 MeV:

  light clusters well represented by αs
- T ≥ 5 MeV:

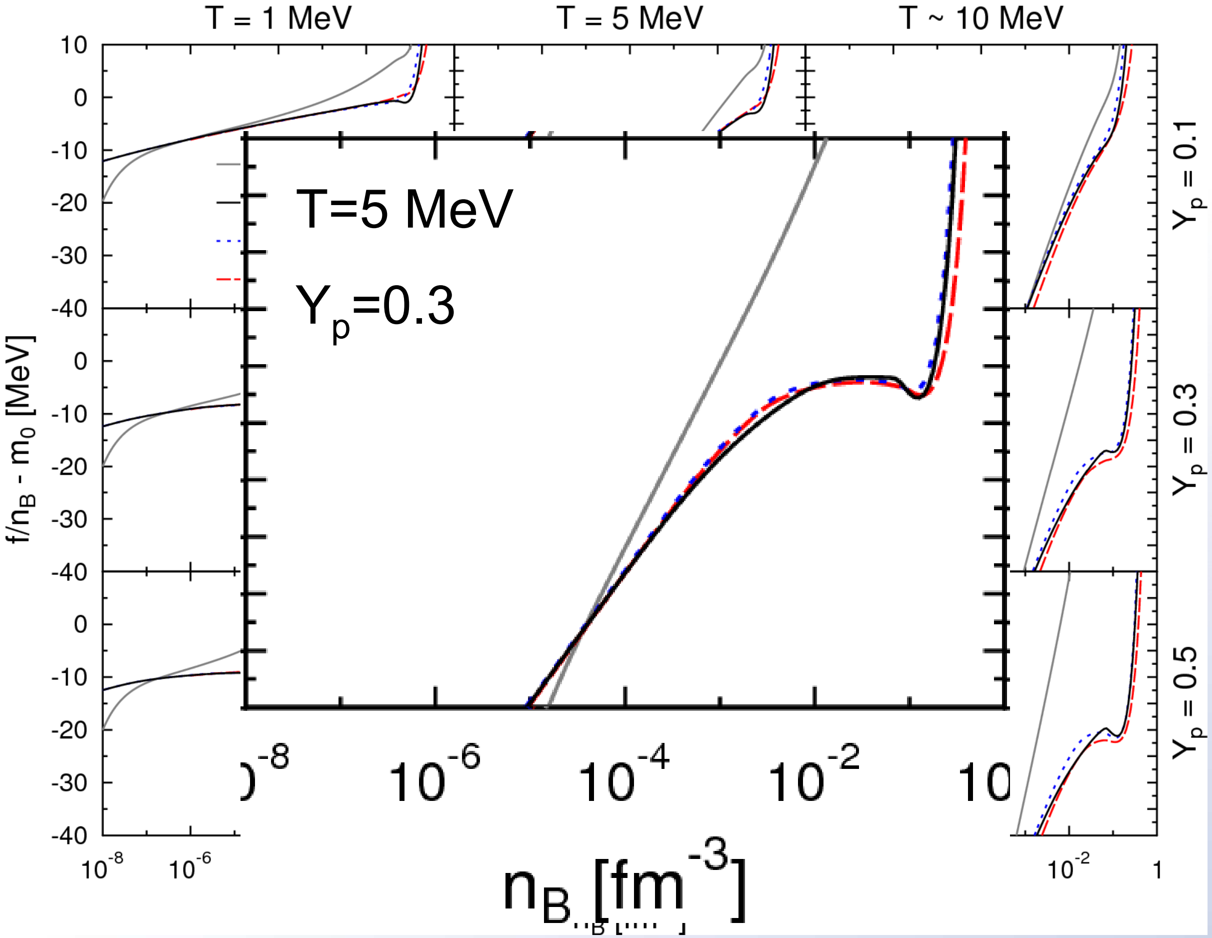
  large Deuteron contribution at low n_B,

  differences for αs at large n_B
- $n_B \sim 10^{-2}$ fm^{-3}:

  whole distribution of light clusters important

- behaviour not representable by alphas alone

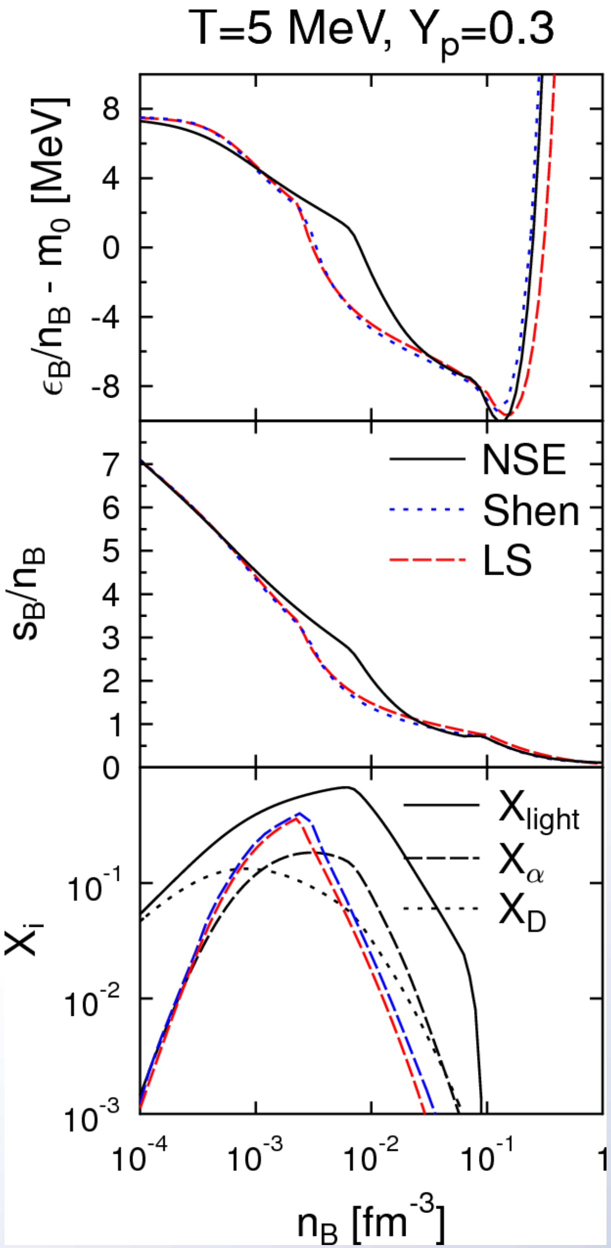
Equation of State I



- excellent agreement at low T across all densities
- T ≥ 5 MeV:

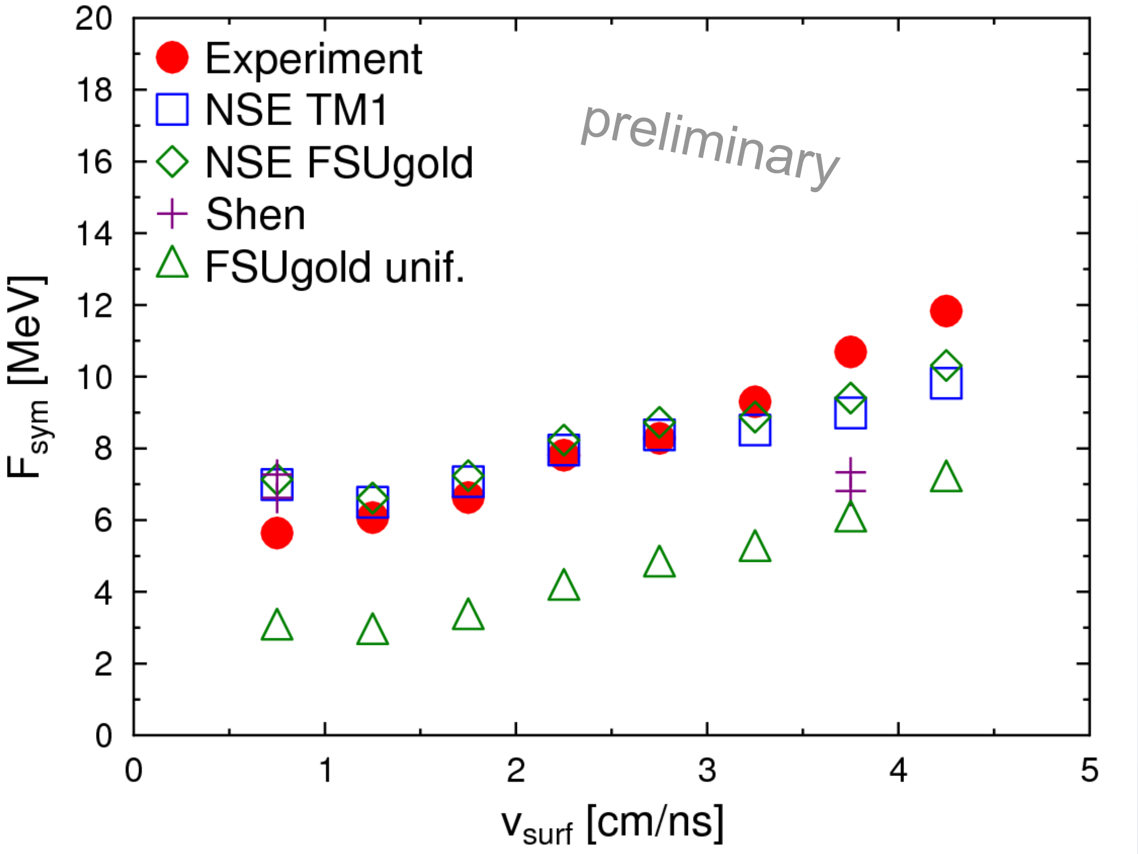
  thermal distributions lead to additional binding around n_B~5×10^{-3} fm^{-3}
- at $n_B > 10^{-2}$ fm^{-3}: restricted mass table, strong excluded volume: increased free energy

Equation of State II



- increased entropy and energy from the light cluster distribution
- deuterons modify the EOS at low densities

Symmetry energy – Comparison with Experimental data



- symmetry energy extracted from low-energy heavy ion collisions
- v_{surf}: velocity of particles at ejection, characterizes emission time
- corresponding to different T=3 – 7 MeV and n_B=1/100 – 1/20 n_B^0
- cluster formation leads to increased symmetry energy

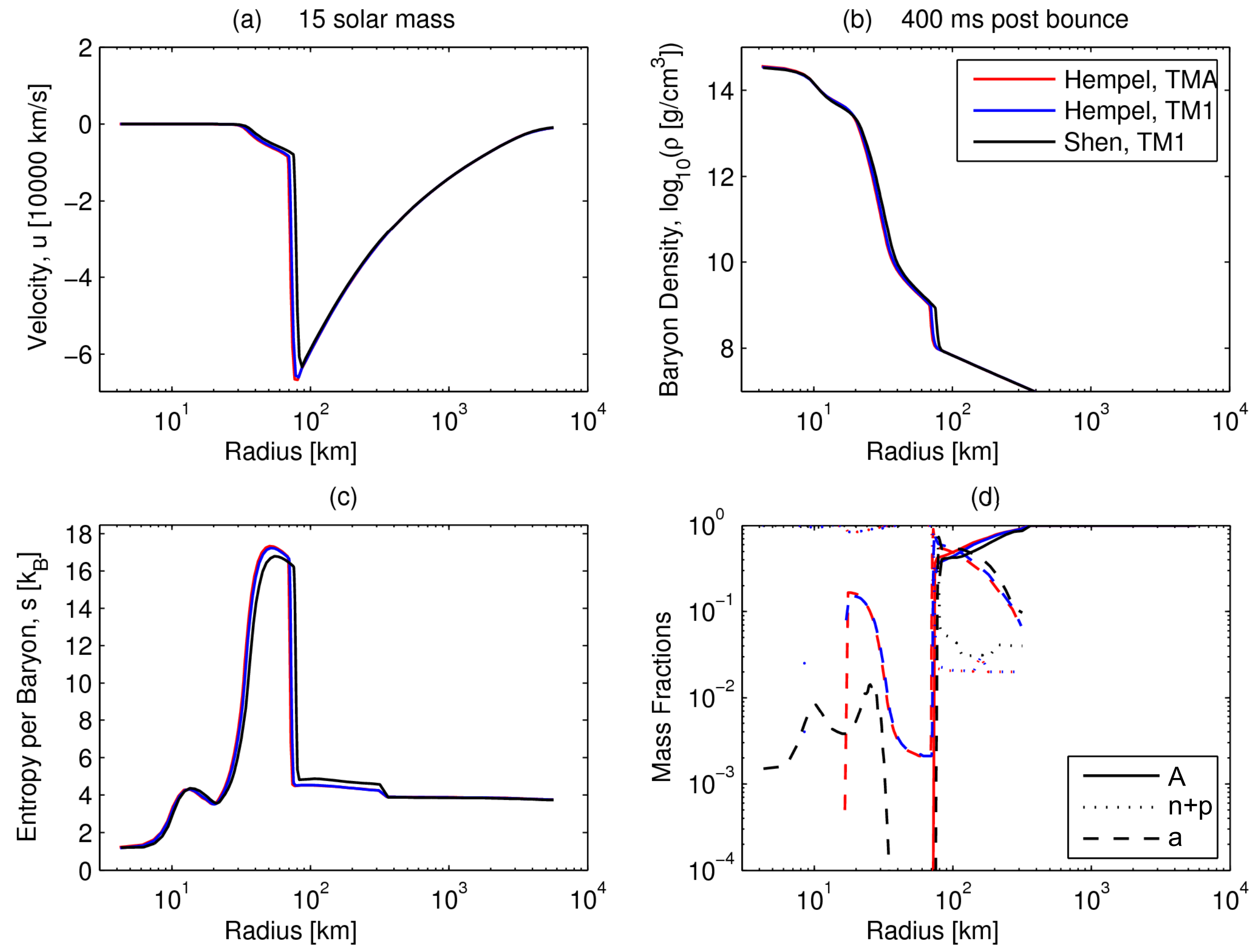
T [MeV]	3.3	3.3	3.6	4.2	4.7	5.3	6.2	7.5
n_B [10^{-3} fm^{-3}]	2.1	1.7	2.3	3.8	4.7	4.9	5.5	6.4

[Kowalski et al.; 2007PRC75]

[Natowitz et al.; arXiv:1001.1102]

Supernova simulations

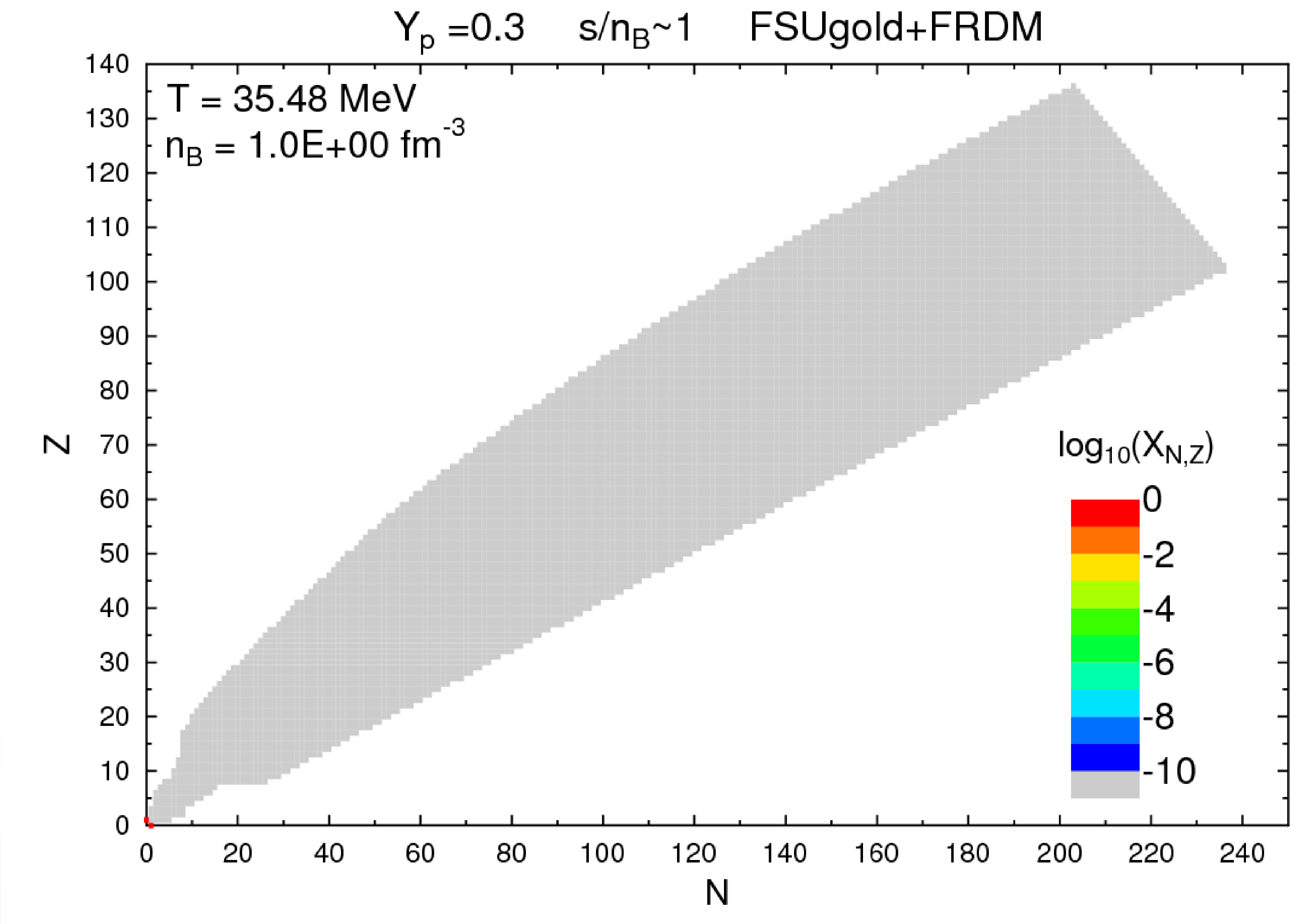
collaboration with Tobias Fischer, Basel



- exploratory first core-collapse SN simulations
- all light clusters treated as alpha-particles, only average heavy nucleus
- slightly more compact PNS core

→ change of the low-density model more important than change TM1↔TMA

Adiabatic Decompression without Weak Reactions



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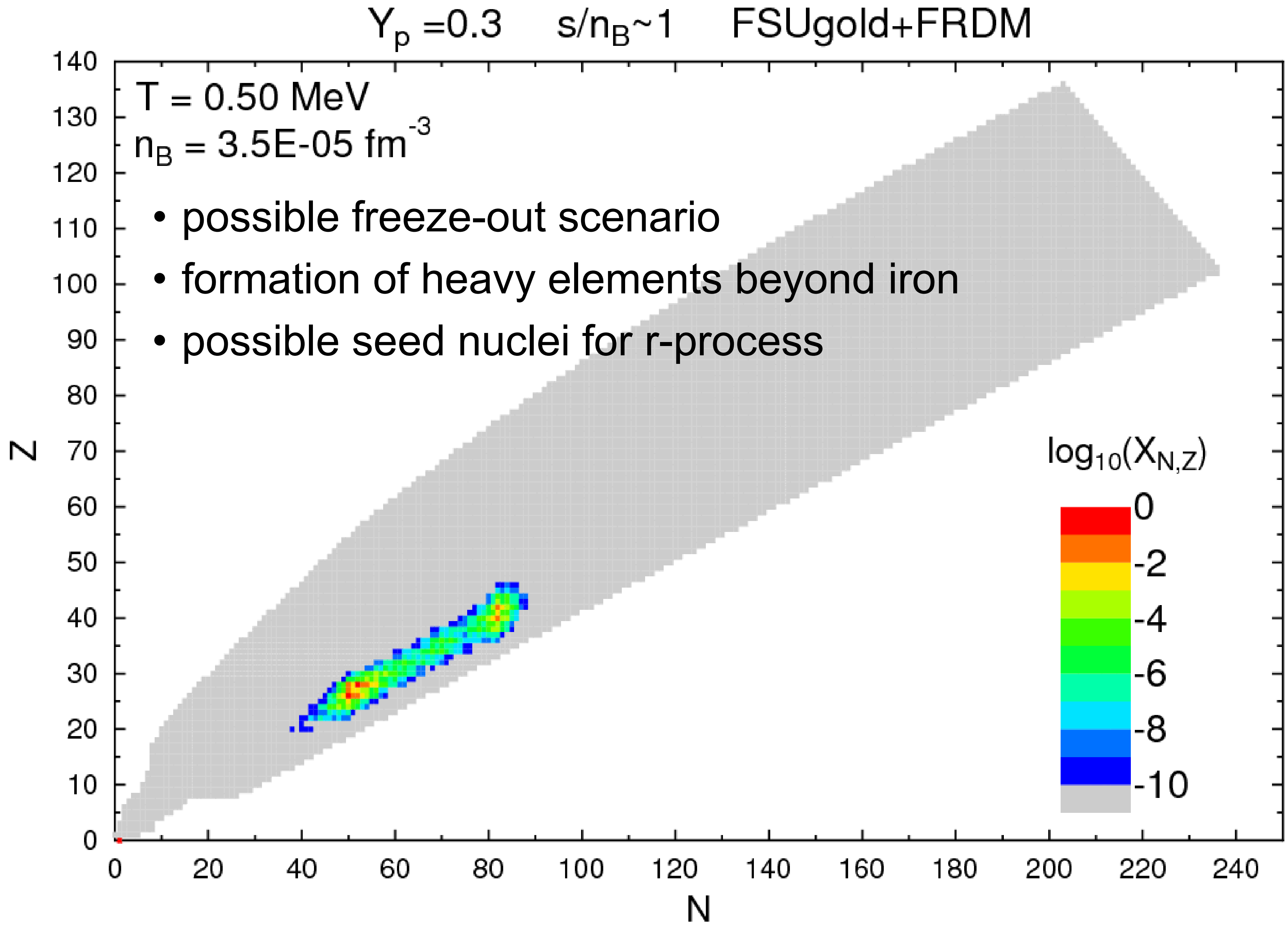

- possible freeze-out scenario
- formation of heavy elements beyond iron
- possible seed nuclei for r-process

Conclusions & Outlook

- phenomenological, thermodynamic consistent model for an ensemble of nuclei in equilibrium with an interacting nucleon gas

  → provide EOS tables with different nuclear interactions

- excellent description at low T and n_B; strong shell effects

- deviations to commonly used EOSs at large temperatures due to thermal distributions; important role of light clusters

- good agreement with experimental measurements of symmetry energy

- first SN simulations: model for the low-density EOS as important as the nuclear interactions → study implications further

→ take distributions of nuclei in the reaction rates into account

→ implement phase transition to quark matter