Multiplicity Distributions in Statistical Ensembles

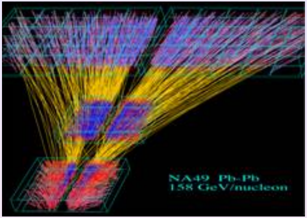
Michael Hauer[1]

[1]Helmholtz Research School, University of Frankfurt, Frankfurt, Germany

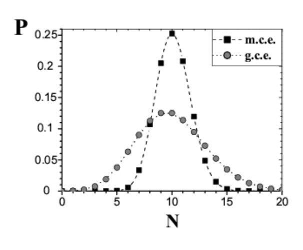
NEW TRENDS IN HIGH-ENERGY PHYSICS
Yalta, Crimea, Ukraine, September 15 - 22, 2007

H-QM Helmholtz Research School
Quark Matter Studies

Outline



NA49 Pb-Pb
158 GeV/nucleon



P
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
N
m.c.e.
g.c.e.

Motivation

1. Enhanced fluctuations are expected near a critical point of strongly interacting matter, or as a result of the onset of deconfinement
2. Correlations, i.e. susceptibilities, are different in HRG and QGP
3. Good experimental data are becoming available
4. Calculate baseline (or statistical) fluctuations
5. Can we understand properties of non-equilibrium models? (c.f. Volodya's talk)
6. Test the statistical hadronization model

Statistical Ensembles

GCE	CE	MCE
$Z^{GCE}(V,T,\mu)$	$Z^{CE}(V,T,Q)$	$Z^{MCE}(V,E,Q)$

Grand Potential	Helmholtz Free Energy	Entropy
$\Omega = -T \ln Z^{GCE}$	$F = -T \ln Z^{CE}$	$S = \ln Z^{MCE}$

$$\mu_{CE} = \left(\frac{\partial F}{\partial Q}\right)_{V,T}$$

$$\frac{1}{T_{MCE}} = \left(\frac{\partial S}{\partial E}\right)_{V,Q}$$

Thermodynamic Limit: $V, E, Q \to \infty$

$$\frac{E}{V} = const, \; \frac{Q}{V} = const, \quad \Longrightarrow \quad T_{MCE} \to T, \quad \mu_{CE} \to \mu$$

Ultrarelativistic Gas of Neutral Particles

GCE Partition Function

$$Z^{GCE}(V,T) = \exp\left[Vg \int \frac{d^3p}{(2\pi)^3} e^{-|p|/T}\right] = \exp\left[Vg \frac{T^3}{\pi^2}\right]$$

All Micro-states with N Particles

$$\mathcal{Z}^N(V,T) = \int_{-\pi}^{\pi} \frac{d\phi_N}{2\pi}\, e^{-iN\phi_N}\, \exp\left[Vg \frac{T^3}{\pi^2} e^{i\phi_N}\right] = \frac{\left(Vg\frac{T^3}{\pi^2}\right)^N}{N!}$$

GCE Multiplicity Distribution is **Poissonian**

$$P_{GCE}(N) = \frac{\mathcal{Z}^N(V,T)}{Z^{GCE}(V,T)} = \frac{\left(Vg\frac{T^3}{\pi^2}\right)^N}{N!} \exp\left(-Vg\frac{T^3}{\pi^2}\right)$$

Ultrarelativistic Gas of Neutral Particles

All Micro-states with N Particles and Energy E

$$\mathcal{Z}^{N,E}(V,T) = \int_{-\pi}^{\pi} \frac{d\phi_N}{2\pi} \int_{-\infty}^{\infty} \frac{d\phi_E}{2\pi}\, e^{-iN\phi_N}\, e^{-iE\phi_E} \exp\left[Vg \int \frac{d^3p}{(2\pi)^3} e^{-|p|/T} e^{i|p|\phi_E} e^{i\phi_N} \right]$$

$$= \left(\frac{gV}{\pi^2}\right)^N \frac{E^{3N-1}}{N!(3N-1)!}\, e^{-E/T} = Z^{MCE}(V,E,N)\, e^{-E/T}$$

All Micro-states with Energy E

$$\mathcal{Z}^{E}(V,T) = \sum_{N=1}^{\infty} \mathcal{Z}^{E,N}(V,T) \quad \text{or} \quad Z^{MCE}(V,E) = \sum_{N=1}^{\infty} Z^{MCE}(V,E,N)$$

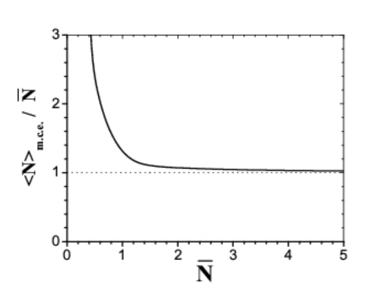
MCE Multiplicity Distribution

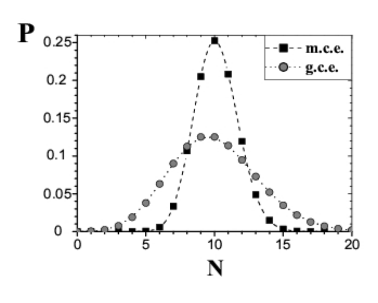
$$P_{MCE}(N) = \frac{\mathcal{Z}^{N,E}\, e^{+E/T}}{\mathcal{Z}^{E}\, e^{+E/T}} = \frac{Z^{MCE}(V,E,N)}{Z^{MCE}(V,E)}$$

MCE $P(N)$ can be expressed through conditional GCE $P(N|E)$!

V.V.Begun, M.I. Gorenstein, A.P. Kostyuk, and O.S.Zozulya, Phys.Rev C **71**, 054904 (2005)
M.H., V.V.Begun, M.I. Gorenstein, arXiv:0706.3290 [nucl-th]

Asymptotic Behavior





Mean Values

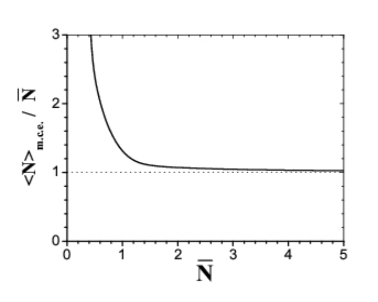
Equivalence of ensembles holds for mean values in the thermodynamic limit.

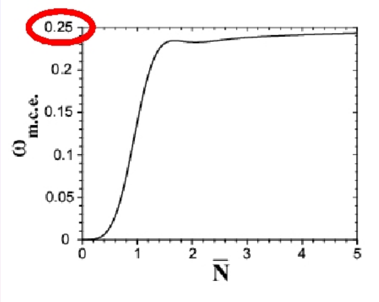
However ....

This seems not to apply to higher moments of a distribution!

V.V.Begun, M.I. Gorenstein, A.P. Kostyuk, and O.S.Zozulya, Phys.Rev C **71**, 054904 (2005)

Asymptotic Behavior





Express the Width by:

Scaled Variance

$$\omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$

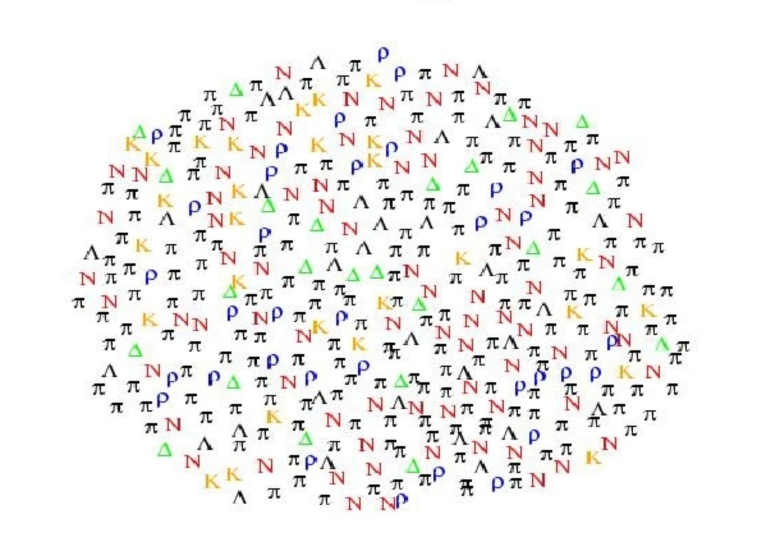
$V, E \rightarrow \infty$, and $E/V = const$

$\omega_{gce} = 1$, but

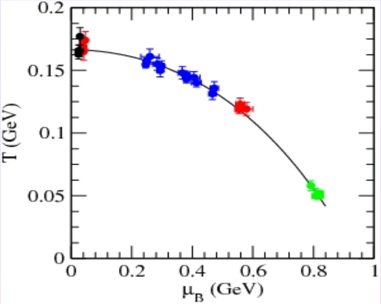
$\omega_{mce} \rightarrow 0.25$

V.V.Begun, M.I. Gorenstein, A.P. Kostyuk, and O.S.Zozulya, Phys.Rev C **71**, 054904 (2005)

Hadron Resonance Gas Model



HRG Model in Heavy Ion Collisions



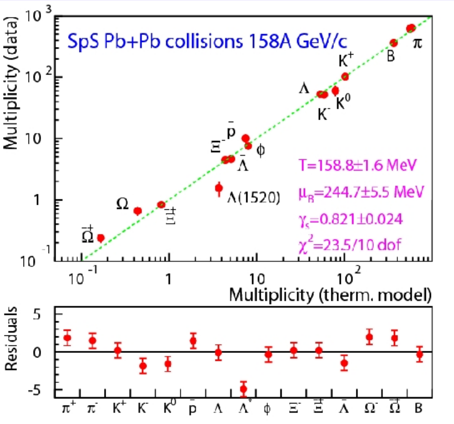
For instance:

J. Cleymans and K. Redlich, Phys. Rev. Lett. **81**, 5284 (1998)
J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C **73**, 034905 (2006)
F. Becattini, J. Manninen, and M. Gaździcki, Phys. Rev. C **73**, 044905 (2006)
A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A **772**, 167 (2006)

Hadron Resonance Gas Model

- Used as an effective model of strong interaction
- Includes all hadrons and resonances up to $\sim$ 2 GeV
- Depending on the version it assumes partial or complete chemical equilibrium
- However, thermal equilibrium is always assumed

HRG Model in Heavy in Collisions



Despite it simplicity it fits a broad range of data

- with just 3 parameters
- interpretation, however, remains ‘controversial‘

F.Becattini, private communication

General Relativistic Ideal Multi-specie Hadron Gas

Fourier Spectral Analysis of GCE Partition Function

$$\mathcal{Z}^{Q^j,E^k}(V,T,\mu_j) = \left[\prod_{j=1}^{3} \int_{-\pi}^{\pi} \frac{d\phi_j}{2\pi} e^{-iQ^j\phi_j}\right]\left[\prod_{k=1}^{4} \int_{-\infty}^{\infty} \frac{d\phi_k}{2\pi} e^{-iE^k\phi_k}\right] \exp\left[V\sum_l \psi_l(\phi_j,\phi_k)\right]$$

Single Particle Partition Function

$$\psi_l(\phi_j,\phi_k) = \frac{g_l}{(2\pi)^3}\int d^3p \ln\left(1 \pm e^{-\frac{\sqrt{m_l^2+p^2}-\mu_l}{T}} e^{iq_l^j\phi_j} e^{i\varepsilon_l^k\phi_k}\right)^{\pm 1}$$

It can be shown that generally:

$$\mathcal{Z}^{Q^j,E^k}(V,T,\mu_j) = Z^{MCE}(V,Q^j,E^k)\, e^{\frac{Q^j\mu_j}{T}}\, e^{-\frac{E}{T}}$$

Some Definitions:

$$Q^j = (B,S,Q)$$

$$E^k = (E,P_x,P_y,P_z)$$

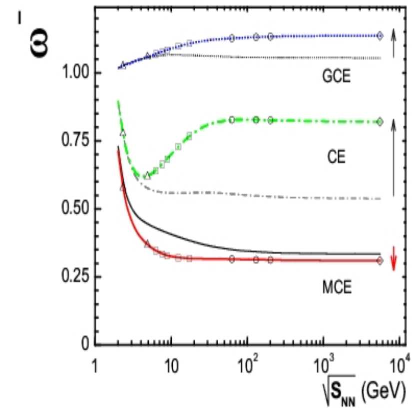
$$q_l^j = (b_l,s_l,q_l)$$

$$\varepsilon_l^k = (\varepsilon_l,p_x,p_y,p_z)$$

In the large volume limit $\mathcal{Z}^{Q^j,E^k}(V,T,\mu_j)$ converges to a Multivariate-Normal-Distribution. Finite Volume corrections are given in the form of Hermite polynomials of low order.

M.H., V.V.Begun, M.I. Gorenstein, arXiv:0706.3290 [nucl-th]

Scaled Variance, Full Acceptance



The arrows:

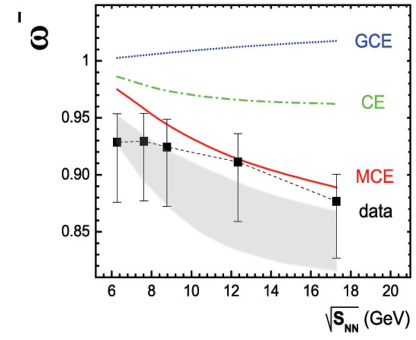
indicate the effect of resonance decay

MCE

Energy conservation leads to correlation with (neutral) particles, thus suppresses final state fluctuations.

V.V.Begun, M.Gaździcki, M.I.Gorenstein, M.H., V.P.Konchakovski, and B.Lungwitz, Phys. Rev. C **76** (2007) 024902

Scaled Variance Measured by NA49



Thick Lines

Acceptance Scaling

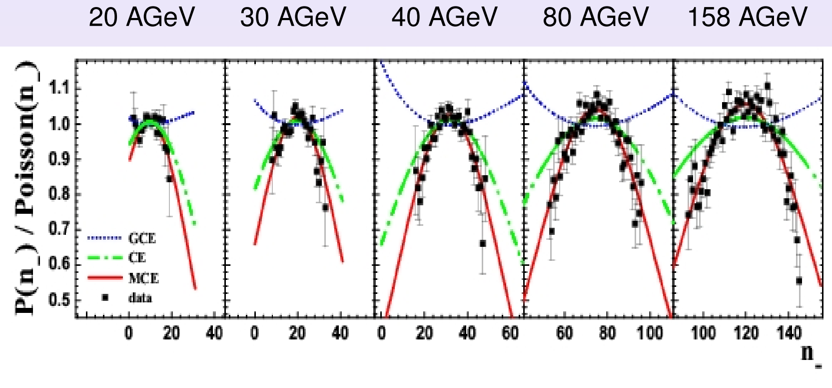
$$\omega^{acc} = 1 - q + q\omega^{4\pi}$$

Grey Area

Guess-timate of effect of acceptance in momentum space in MCE

B. Lungwitz, AIP Conf. Proc. **892**, 400 (2007)
V.V.Begun, M.Gaździcki, M.I.Gorenstein, M.H., V.P.Konchakovski, and B.Lungwitz, Phys. Rev. C **76** (2007) 024902
V.V. Begun, M.I. Gorenstein, M. H., V.P. Konchakovski, and O.S. Zozulya, Phys. Rev. C **74**, 044903 (2006)
M.H., in preparation

Comparison to NA49 Fluctuation Data

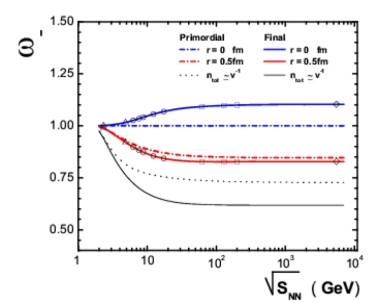


B. Lungwitz, AIP Conf. Proc. **892**, 400 (2007)
V.V.Begun, M.Gaździcki, M.I.Gorenstein, M.H., V.P.Konchakovski, and B.Lungwitz, Phys. Rev. C **76** (2007) 024902

GCE Van der Waals Gas

Has been used to:

model repulsive interactions between hadrons



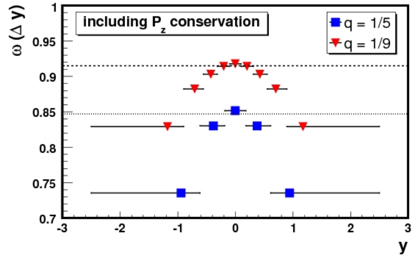
- suppression of densities can be removed by rescaling the system volume
- suppression of fluctuations is qualitatively different
- could be a first step towards a simple model with a phase transition

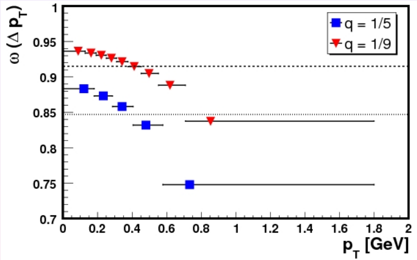
  M.I. Gorenstein, M. Gaździcki, W. Greiner, Phys. Rev. C **72**, 024909 (2005)
- can be extended to include conservation laws

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, Z.Phys.C**51** 485-490, 1991
M. I. Gorenstein, M.H. and D. O. Nikolajenko, Phys. Rev. C **76**, 024901 (2007)

Momentum Cuts in Micro-Canonical Ensemble

Boltzmann pion gas at $T = 160 MeV$ and zero charge density.





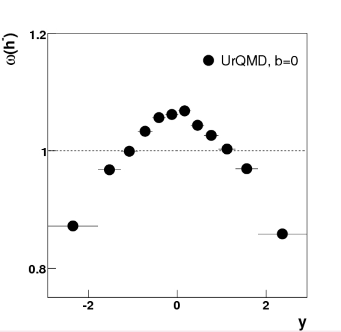
- Each bin contains same fraction of total yield
- Bars indicate size of the bin

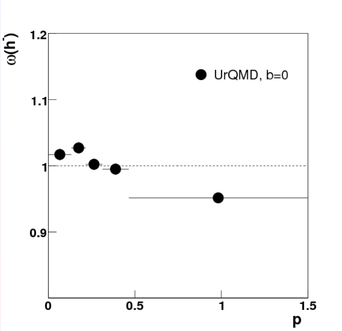
Energy and momentum conservation lead to suppressed multiplicity fluctuations at high $|y|$ and p_T.

M.H. in preparation

Momentum Cuts in UrQMD

UrQMD simulation of central Pb+Pb collision at b=0





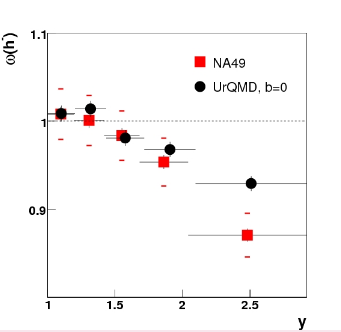
Construction of bins is the same as before.

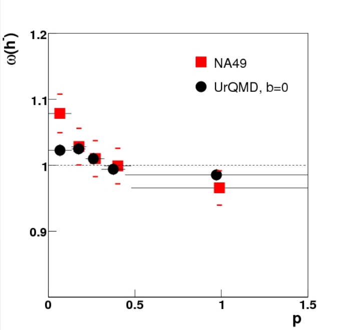
MCE suppression of fluctuations also in non-equilibrium systems?

B. Lungwitz and M. Bleicher, arXiv:0707.1788 [nucl-th], Phys. Rev. C, in print

Momentum Cuts in NA49 Data

UrQMD vs. NA49 158AGeV Pb+Pb data





Rapidity and transverse momentum dependence also seen in data!

MCE effects are of similar magnitude as proposed enhancement due to a phase transition / critical point!

B. Lungwitz, talk given at Workshop on Critical Point and Onset Deconfinement and private communication

Conclusion

1. Particle number fluctuations were discussed in different statistical ensembles and compared to NA49 data on nucleus-nucleus collisions
2. Fluctuations are different in different ensembles !
3. GCE in clear contradiction to data !
4. Data as well as CE and MCE show suppressed fluctuations (with respect to a Poissonian)
5. Fluctuations are a important test for the statistical hadronization model
6. We need to understand the role of experimental acceptance !

I Would Like to Thank

A Big ‘ThanX‘ to

- Annti
- Azwinndini
- Benjamin
- Elena
- Francesco
- Gary
- Giorgio
- Lorenzo
- Marek
- Spencer
- Victor
- Volodymyr

And Last But Not Least To MARK,

for MANY discussions and his guidance!

What is the role of T in MCE?

Partition Function

$$\mathcal{Z}^E(V,T) \equiv Z^{MCE}(V,E)\, e^{-\frac{E}{T}}$$

Entropy

$$S = \ln\left(\mathcal{Z}^E\, e^{+\frac{E}{T}}\right)$$

Determine Equilibrium Temperature

$$\left(\frac{\partial S}{\partial E}\right)_V = \frac{\frac{\partial \mathcal{Z}^E}{\partial E} e^{+\frac{E}{T}} + \frac{1}{T}\, \mathcal{Z}^E\, e^{+\frac{E}{T}}}{\mathcal{Z}^E\, e^{+\frac{E}{T}}} = \frac{1}{T_{MCE}}$$

$T = T_{MCE}$ implies:

$$\frac{\partial \mathcal{Z}^E}{\partial E} = 0$$

$\Rightarrow$ Maximize GCE Partition Function for Energy E

GCE Partition Function

$$Z^{GCE}(V,T) = 1 + \int_0^\infty dE\, Z^{MCE}(V,E) e^{-\frac{E}{T}} = 1 + \int_0^\infty dE\, \mathcal{Z}^E(V,T)$$

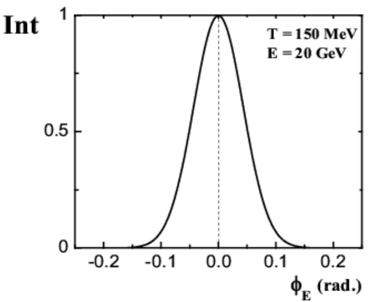
What is the advantage of introducing T in MCE?

OR:

Why is it of advantage to define MCE multiplicity distributions through joint GCE distributions ?

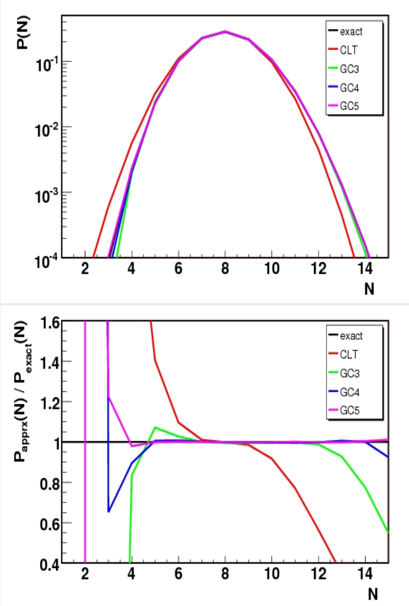
Principle Problem:

In CE and MCE calculations one has to deal with a heavily oscillating (or even irregular) integrand.



- Our version is however very smooth!
- Main contribution comes from small region around the origin.
- Analytical expansion is possible
- Numerical integration for large system becomes feasible!

Quality of Approximation



Large Volume Limit

- Multiplicity Distribution becomes Gaussian (CLT)
- $\mu \longrightarrow \mu_{GCE}$
- $T \longrightarrow T_{GCE}$

Finite Volume Correction

- Given by different order of Gram-Charlier expansion (GC3 - GC5)
- In general only applicable to ‘body‘ of distribution
- Very good description, even for small system size

M.H., V.V.Begun, M.I. Gorenstein, arXiv:0706.3290 [nucl-th]