Asymmetric neutrino emission in quark matter and Pulsar kicks

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Observations of large pulsar velocities

- Pulsar space velocities between 100 - 1600 km/s (Hobbs et al., 2005)
- Highest velocity directly measured for a pulsar (B1508+55) : 1083(+103/90)km/s (Chatterjee et al., 2005)
- Indications for an alignment between the velocity vector and the rotational axis (Johnston et al., 2005; Brisken et al., 2005)
Pulsar kicks by anisotropic neutrino emission

- Within seconds after the supernova explosion a proto-neutron star cools down from temperatures of $T \sim 50$ MeV to $T < 1$ MeV by neutrino emission.
- Could an asymmetry in neutrino emission accelerate a neutron star to high velocities?
- Pulsars are measured to have surface magnetic fields up to $10^{12}$ Gauss (for magnetars even up to $10^{15}$ Gauss).
- Strong magnetic fields as a source for the neutrino anisotropy?
Pulsar kicks by anisotropic neutrino emission

• **Idea:**
  
  • Neutrino emissivity from the direct quark Urca process during the proto-neutron star cooling
    
    \[ d \rightarrow u + e^- + \bar{\nu}_e \quad u + e^- \rightarrow \nu_e + d \]
  
  • The asymmetry in the neutrino emission arises due to a strong magnetic field which can align the electron spin opposite to the magnetic field direction.
  
  • The polarization of the electron spin will fix the neutrino emission in one direction along the magnetic field.
  
  • The spin-polarized neutrinos propagate along the magnetic field axis in one direction and work as a propulsion mechanism for the neutron star.
Pulsar kicks by anisotropic neutrino emission

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Pulsar kicks by anisotropic neutrino emission

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  • Is the magnetic field strong enough to polarize the electron spin?
Pulsar kicks by anisotropic neutrino emission

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Pulsar kicks by anisotropic neutrino emission

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  • Is the neutrino energy high enough to accelerate the star?
  • Is the neutrino mean free path large enough for free streaming neutrinos?
Polarization of electrons – Landau levels

• Electrons which are moving in a magnetic field larger than $4.4 \cdot 10^{13}$ Gauss are situated in Landau levels perpendicular to the magnetic field axis.

• Their energy is quantized:

$$E_n^2 = m^2 + p_z^2 + 2eB \eta = m^2 + p_z^2 + 2eB \left( \nu + \frac{1}{2} + s \right)$$

$\nu$ ... quantum number of the Landau levels $\eta$

$s$ ... electron spin with $s = \frac{1}{2}$ or $s = -\frac{1}{2}$

• Notice: Electrons which are in the lowest Landau level with $\eta=0$ are spin polarized with $s = -\frac{1}{2}$

• The electron spin polarization can be written as:

$$\chi = \frac{n(\eta=0)}{n_{s=\frac{1}{2}} + n_{s=\frac{-1}{2}}}$$
Polarization $\chi$

![Graph with polarization $\chi$ as a function of $\mu/(eB)^{1/2}$ and $T/(eB)^{1/2}$](image-url)
Polarization $\chi = 1$

Constraints on the electron chemical potential, the temperature and the magnetic field strength for $\chi = 1$

Notice: The high magnetic fields should be present in the quark matter phase, not at the surface of the star.
Kick velocity

- The velocity of the neutron star depends on the energy density $\varepsilon$ in the following way:

$$ v = \frac{4}{3} \pi R^3 \frac{\chi}{M_{\text{ns}}} \left( \varepsilon(t_0) - \varepsilon(t_f) \right) = \frac{4}{3} \pi R^3 \frac{\chi}{M_{\text{ns}}} \Delta \varepsilon $$
Kick velocity

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$$v \approx 700 \frac{\text{km}}{\text{s}} \left( \Delta \varepsilon \frac{\text{fm}^3}{\text{MeV}} \right) \left( \frac{R}{10 \text{km}} \right)^3 \left( \frac{M_{\text{solar}}}{M_{ns}} \right)$$
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• For $\Delta \epsilon$ stemming from temperature decay:

$$v = \frac{2}{3} \pi R^3 \frac{\chi}{M_{ns}} A \mu_q^2 (T_0^2 - T_f^2), \text{ with } c_q = 9 \left( 1 - \frac{2 \alpha_s}{\pi} \right) \mu_q^2 T = A \mu_q^2 T$$
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  $$\text{with } c_q = 9 \left( 1 - \frac{2 \alpha_s}{\pi} \right) \mu_q^2 T = A \mu_q^2 T$$

  $$\rightarrow v \sim 40 \text{ km/s} \left( \frac{\mu_q}{400 \text{ MeV}} \right)^2 \left( \frac{T_0}{10^{10} \text{ K}} \right)^2 \left( \frac{R}{10 \text{ km}} \right)^3 \left( \frac{1.4 M_{\text{solar}}}{M_{\text{ns}}} \right)$$
Kick velocities of ~1000km/s for a realistic quark phase radius $R < 10$km can be reached with $T > 5 \cdot 10^{10}$ Kelvin for magnetic field strengths $B \geq 1 \cdot 10^{17}$ Gauss.

$\mu_q = 400$ MeV, $M_{ns} = 1.4 M_{\text{solar}}$
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The large magnetic field is supposed to be in the interior not at the surface.
Neutrino mean free paths

Neutrino mean free paths in the interior of the neutron star are defined by absorption and scattering in quark as well as neutron matter

\[ d + \nu_e \rightarrow u + e^- , \quad q + \nu \rightarrow q + \nu \]
\[ n + n + \nu_e \rightarrow n + p + e^- , \quad n + \nu \rightarrow n + \nu \]

<table>
<thead>
<tr>
<th>Medium</th>
<th>Process</th>
<th>( E_\nu = 3T, \mu_q = 400 \text{ MeV}, \alpha_s = 0.5 )</th>
<th>( T = 5 \text{MeV}, \mu_e = 10 \text{MeV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark matter</td>
<td>Absorption</td>
<td>( l_{abs} \approx 25 \text{km} \left( \frac{T^2 \mu_e}{\text{MeV}^3} \right)^{-1} )</td>
<td>\approx 100m</td>
</tr>
<tr>
<td>Quark matter</td>
<td>Scattering</td>
<td>( l_{scat} \approx 92 \text{km} \left( \frac{T}{\text{MeV}} \right)^{-3} )</td>
<td>\approx 800m</td>
</tr>
<tr>
<td>Neutron matter</td>
<td>Absorption</td>
<td>( l_{abs} \approx 17 \text{km} \left( \frac{T}{\text{MeV}} \right)^{-3} )</td>
<td>\approx 150m</td>
</tr>
<tr>
<td>Neutron matter</td>
<td>Scattering</td>
<td>( l_{scat} \approx 244 \text{km} \left( \frac{T}{\text{MeV}} \right)^{-4} )</td>
<td>\approx 390m</td>
</tr>
</tbody>
</table>

for non-degenerate neutrinos, non-relativistic neutrons and degenerate electrons (Iwamoto, 1982)
Neutrino mean free path & Kick velocity

\[ l_i = \left( \frac{1}{l_{\text{abs}}} + \frac{1}{l_{\text{scat}}} \right)^{-1}, \quad i = n, q \]

\( l_q \) neutrino mean free path in quark matter
\( l_n \) neutrino mean free path in neutron matter

Problem: neutrino mean free paths are too small causing them to interact with the medium on their way to the surface → neutrinos isotropize → loss of polarization

\( \mu_q = 400 \text{ MeV}, \quad \mu_e = 10 \text{ MeV}, \quad M_{\text{ns}} = 1.4 M_{\odot} \)
Colour superconducting quark matter

- The neutrino quark interaction is exponentially suppressed by the pairing energy of the quarks
  
  \[ e^{-\Delta/T}, \text{ for } T < T_c \]

- The quark heat capacity is suppressed by the same factor
  
  \[ c_q = A \, \mu_q^2 \, T \, e^{-\Delta/T} \]

- The electron heat capacity
  
  \[ c_e = \frac{\mu_e^2 \, T}{2} + \frac{7 \pi^2 \, T^3}{30} \]

  becomes important for the pulsar velocity

Notice:
Though Reddy et al. (1998) find a large neutrino mean free path > 10 km in the CFL quark matter phase for a temperature of 5MeV, the interaction with massless H-bosons leads to a mean free path similar to the one for normal quark matter

14.06.2007
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EXOCT 2007
Summary

• We discussed a propulsion mechanism for pulsars based on asymmetric neutrino emission in quark matter caused by a strong magnetic field.

• We find that kick velocities $\sim 1000$ km/s require magnetic field strengths of $B \geq 10^{17}$ G for $T \geq 5 \cdot 10^{10}$ K and $R \leq 10$ km.

• **Problem:** neutrino mean free paths are too small $\rightarrow$ the asymmetry in neutrino emission is lost due to interactions with the medium.

• **Outlook:**
  - Add effects from quark pairing as well as from the magnetic field.
  - During a phase transition from ungapped to gapped quark matter an energy of $\mu_q^2 \Delta^2$ can be released $\rightarrow$ contribution to the quark star acceleration mechanism.
Literature

- N. Iwamoto, “Neutrino Emissivities and Mean Free Paths of Degenerate Quark Matter”, Annals of Physics, 1982
- W.F. Brisken, R.W. Romani and C.Y. Ng, X-ray Spins and Radio Speeds: Probing Pulsar Birth Kinematics with CXO and the VLBA. In X-Ray and Radio Connections, April 2005
Attempts for explanation of pulsar kicks

  But: Still no explosion in 2D models, triggering of explosion assuming suitable neutrino luminosities from NS core

  But: Mechanism turns on for temperatures $\leq T_c \sim 0.1$ MeV (too low to produce kick)

  But: Such vortices are typically not produced in the CFL phase

- Li, G., Horowitz, C.J., “Cumulative Parity Violation in Supernovae”, Physical Review Letters, Vol. 80, Issue 17, 1998. The neutrino transport inside a slightly polarized proto neutron star which has a strong magnetic field is examined. The parity violation asymmetry in the neutrino flux is large due to a large number of interactions till the neutrinos diffuse out of the proto-neutron star.
  But: Neutrinos produced in an approximate thermal and statistical equilibrium diffuse out of the star isotropically, even if the cross sections and scattering amplitudes are anisotropic. Need of magnetic fields at the surface in the range of $10^{16}$ G.
Polarization for different constrains – analytical and numerical results

\[ 1 \text{MeV} = 1.1605 \times 10^{10} K \]
Neutrino mean free paths

- Neutrino mean free paths for absorption and scattering in quark as well as neutron matter

- Quark matter (Iwamoto for $E_\nu = 3T$, $\mu_q = 400 \, MeV$, $\alpha_s = 0.5$):

$$l_{\text{scat}} = 5\pi^3 \left(1 - \frac{2\alpha_s}{3\pi}\right)^2 \left(\frac{2^{2/3} (C_{\nu d}^2 + C_{\nu d}^2 + C_{\nu u}^2 + C_{\nu u}^2)\mu_q^2 G_F^2 E_\nu^3}{E_\nu^3 + (\pi T)^2} \right) \sim 92 \, km \left(\frac{MeV}{T}\right)^3$$

$$l_{\text{abs}}(E_\nu, T) = 16 G_F^2 \cos \theta_c^2 2^{1/3} \mu_q^2 \mu_e \left(1 - \frac{2\alpha_s}{3\pi}\right)^2 \left(\frac{E_\nu^2 + (\pi T)^2}{1 + e^{-E_\nu/T}}\right) \sim 25 \, km \frac{MeV^3}{T^2 \mu_e}$$

- Neutron matter (Iwamoto, Sawyer et al):

$$l_{\text{abs}} = \left(\frac{3n_n}{\pi^7}\right)^{1/3} \left(4 \left(1 + 3 g_A^2\right) G_F^2 m_n E_\nu^2 T\right)^{-1} \sim 17 \, km \left(\frac{MeV}{T}\right)^3$$

$$l_{\text{scat}} = 4.5 \times 10^6 \left(\frac{n_0}{n_b}\right)^{2/3} \left(\frac{10 \, MeV}{T}\right)^4 \left(\frac{E_\nu}{T}\right)^4 + 10 \pi^2 \left(\frac{E_\nu}{T}\right)^2 + 9 \pi^4 \right)^{-1} \sim 244 \, km \left(\frac{MeV}{T}\right)^4 \, cm$$

with $C_{\nu i}$ Vector and axial current coupling constants, $g_A = 1.25$ and the weak coupling constant $G_F = 1.435 \times 10^{-49} \, erg \, cm^3$
Temperature decrease

- The PNS is becomes transparent for neutrinos for t~50s
- Pons et al:

Temperature drops very quickly to \( \sim 2\times10^9 \) K and doesn't contribute to the kick