Inhomogeneous phases in the QCD phase diagram

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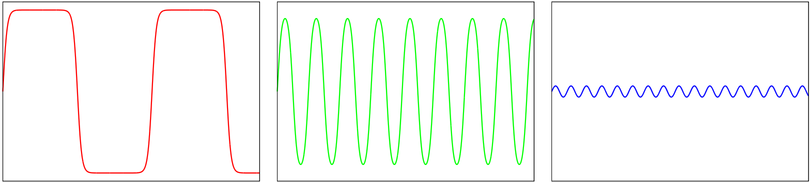

H-QM Helmholtz Research School Quark Matter Studies

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

Motivation: the QCD phase diagram (so far)

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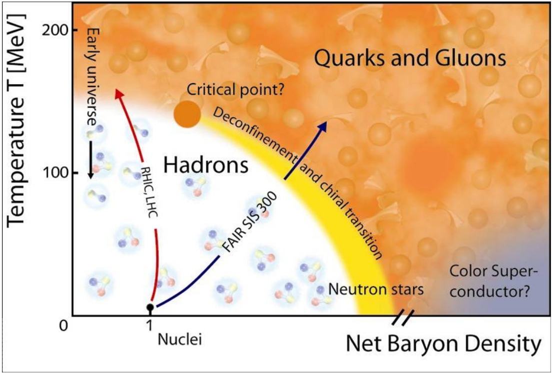

The model

- Start from the usual $N_f = 2$ NJL Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - \hat{m}\right)\psi + G_s\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau^a\psi\right)^2\right)$$

- Mean-field approximation
- Allow for spatially inhomogeneous condensates

$$\langle\bar{\psi}\psi\rangle = S(\mathbf{x})$$

$$\langle\bar{\psi}i\gamma^5\tau^a\psi\rangle = P_a(\mathbf{x})$$

Eigenvalue problem

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- Thermodynamic potential

$$\Omega(T,\mu;S(\mathbf{x}),P(\mathbf{x})) = -\frac{T}{V}\ln\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(\int_{x\in[0,\frac{1}{T}]\times V}(\mathcal{L}_{MF}+\mu\bar{\psi}\gamma^0\psi)\right) =$$

$$-\frac{TN_c}{V}\sum_n \mathrm{Tr}_{D,f,V}\,\mathrm{Log}\left(\frac{1}{T}\left(i\omega_n+\tilde{H}_{MF}-\mu\right)\right)+\frac{G_s}{V}\int_V\left(S(\mathbf{x})^2+P(\mathbf{x})^2\right)$$

- Problem: finding the eigenvalues of

$$\tilde{H}_{MF} \quad = \quad -i\gamma^0\gamma^i\partial_i+\gamma^0\left(m-2G_sS(\mathbf{x})-2iG_s\gamma^5\tau^3P(\mathbf{x})\right)$$

One-dimensional modulations

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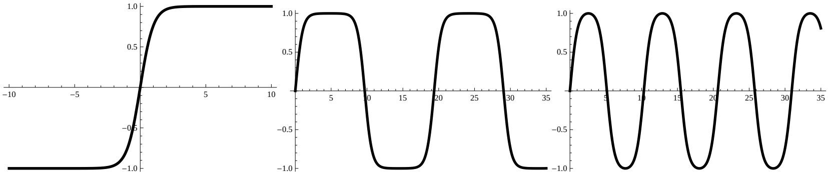
- 1D spatial modulations: analytical results from Gross-Neveu model

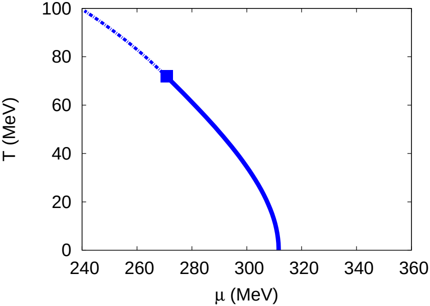
  (M.Thies et al., Annals Phys. 314 (2004) 425-447, arXiv:hep-th/0402014)

$$M(z) = \Delta\nu\, \mathrm{sn}(\Delta z | \nu)$$



- Eigenvalue spectrum well known
- Analytical expression for the Thermodynamic potential

Results: NJL (chiral limit)



- Homogeneous:
- First order phase transition
- Critical point

Results: NJL (chiral limit)

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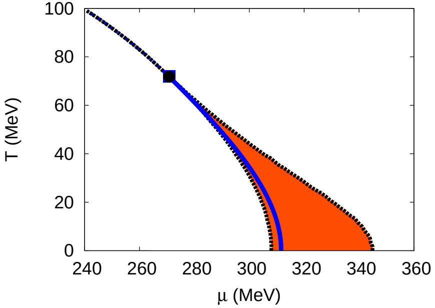
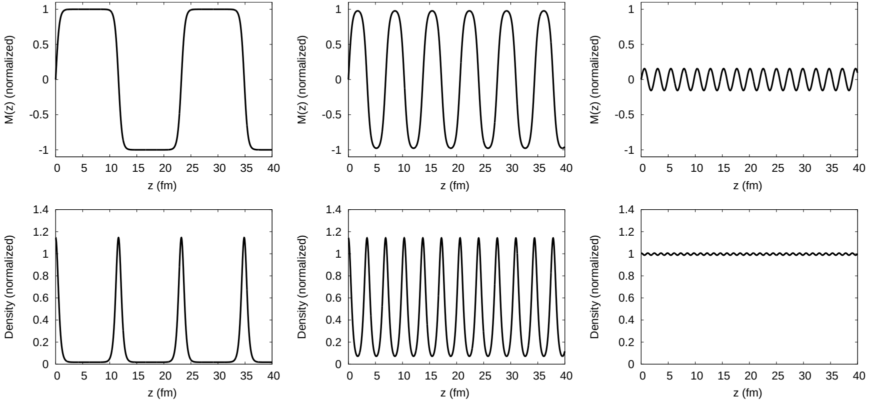


- Inhomogeneous:
- Inhomogeneous region delimited by 2nd order transition lines
- First order transition line completely covered by inhomogeneous region
- Critical point $\rightarrow$ Lifschitz point

(D. Nickel, Phys.Rev.D80 074025, 2009 - arXiv:0906.5295)

Density (T=0)

μ = 307 MeV μ = 308 MeV μ = 342 MeV



Vector interactions

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$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \hat{m}\right)\psi + G_s\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau^a\psi\right)^2\right) - G_V(\bar{\psi}\gamma^{\mu}\psi)^2$$

- The usual prescription: introduce $\tilde{\mu}$

$$\Omega(T,\mu) \to \Omega(T,\tilde{\mu}) - \frac{(\mu-\tilde{\mu})^2}{4G_V}$$

- determine $\tilde{\mu}$ by

$$\frac{\delta\Omega}{\delta\tilde{\mu}} = 0$$

- The problem: $\tilde{\mu}$ is tied to the density n

$$\tilde{\mu} = \mu - 2G_V N_c n$$

- Sacrifice complete self-consistency: pick $\tilde{\mu} \equiv \langle\tilde{\mu}\rangle_z$ instead of $\tilde{\mu}(z)$

Results: Vector interactions (Chiral limit)

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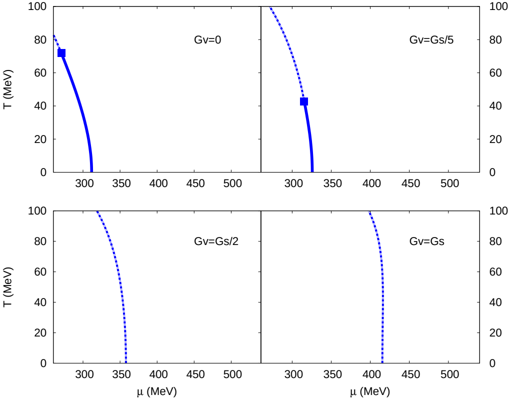
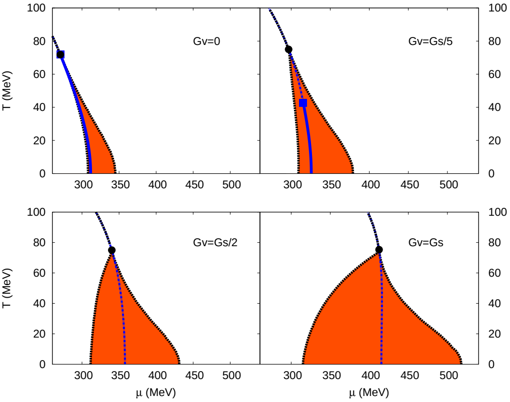


- Stretch towards higher μ
- Critical point moves towards lower T, higher μ

Results: Vector interactions (Chiral limit)



- Stretch towards higher μ
- Lifschitz point moves only in μ
- Lifschitz and critical points split

Chiral spiral

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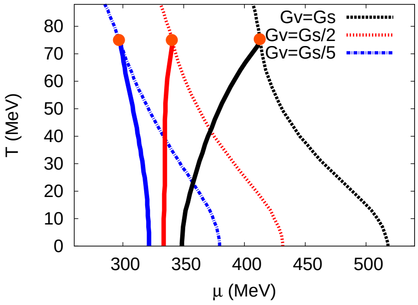
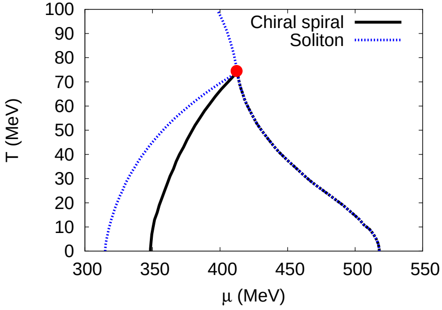
- Chiral Spiral: $M(z) = \Delta e^{iqz}$
- Density n(z) = const.





PNJL

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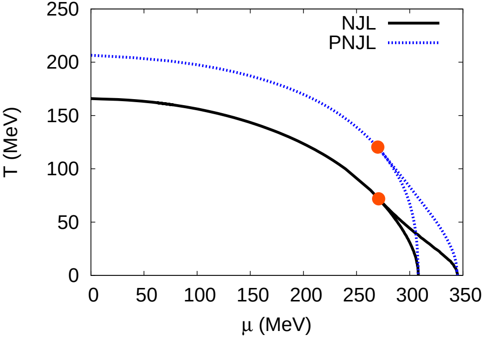
$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - \hat{m}\right)\psi + G_s\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau^a\psi\right)^2\right) - \mathcal{U}(L,\bar{L})$$



- Assumption: homogeneous $\ell, \bar{\ell}$
- Stretch towards higher T
- Quark DOF are suppressed

Conclusions

- Self-consistent lower-dimensional spatial modulations can be studied by relying on analytical results from the Gross-Neveu model
- Inhomogeneous 1D phases are favored in a region of the QCD phase diagram
- Soliton onset delimited by 2nd order phase transition
- Inclusion of the Polyakov loop $\rightarrow$ shift to higher T
- Inclusion of vector interactions $\rightarrow$ shift to higher μ
  $\rightarrow$ split of critical point / Lifschitz point

Outlook

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- Some possible next steps:
- Higher-dimensional modulations
- Self-consistent analysis for the vector interactions
- Inhomogeneous Polyakov loop
- Three flavors
- ....

Backup slides

Eigenvalue problem

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- Thermodynamic potential

$$\Omega(T,\mu;S(\mathbf{x}),P(\mathbf{x})) = -\frac{T}{V}\ln\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\exp\left(\int_{x\in[0,\frac{1}{T}]\times V}(\mathcal{L}_{MF}+\mu\bar{\psi}\gamma^0\psi)\right) =$$

$$-\frac{TN_c}{V}\sum_n \mathrm{Tr}_{D,f,V}\,\mathrm{Log}\left(\frac{1}{T}\left(i\omega_n+\tilde{H}_{MF}-\mu\right)\right)+\frac{G_s}{V}\int_V\left(S(\mathbf{x})^2+P(\mathbf{x})^2\right)$$

- Problem: finding the eigenvalues of

$$\tilde{H}_{MF} = -i\gamma^0\gamma^i\partial_i+\gamma^0\left(m-2G_sS(\mathbf{x})-2iG_s\gamma^5\tau^3P(\mathbf{x})\right)$$

$$M(\mathbf{x}) = m-2G_s(S(\mathbf{x})+iP(\mathbf{x}))$$

Lower-dimensional modulations

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- Restrict to lower-dimensional spatial modulations
- $[H, P_\perp] = 0 \rightarrow$ Full spectrum from lower-dimensional eigenvalues λ
- Boost along the transverse directions:

$$(\lambda, \mathbf{0}) \rightarrow (\sqrt{p_\perp^2 + \lambda^2}, \mathbf{p}_\perp)$$

$$\Omega(T, \mu; M(\mathbf{x})) = -\frac{2TN_c}{V_{||}} \sum_\lambda \int \frac{d\mathbf{p}_\perp}{(2\pi)^{d_\perp}} \ln\left(2\cosh\left(\frac{\lambda\sqrt{1 + \mathbf{p}_\perp^2/\lambda^2} - \mu}{2T}\right)\right) + \frac{1}{V}\int_V \frac{|M(\mathbf{x}) - m|^2}{4G_s} + \text{const.}$$

(D. Nickel, Phys.Rev.D80 074025, 2009 - arXiv:0906.5295)

1D modulations

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- Restrict to one-dimensional modulations

$$M(\mathbf{x}) \to M(z)$$

- The Hamiltonian becomes

$$\tilde{H}_{MF} \to H'_{MF;1D} = \begin{pmatrix} H_{1D}(M(z)) & \\ & H_{1D}(M(z)^*) \end{pmatrix}$$

$$H_{1D}(M(z)) = \begin{pmatrix} -i\partial_z & M(z) \\ M(z)^* & i\partial_z \end{pmatrix} \quad \text{Gross-Neveu Hamiltonian}$$

Gross-Neveu

- One of the simplest interacting fermionic field theories
- Defined in 1+1 dimensions

$$\mathcal{L}_{GN} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \frac{1}{2} g^2 (\bar{\psi}\psi)^2$$

- Hartree-Fock $\rightarrow$ recover SUSY QM-like equation
- Real self-consistent solutions of the form

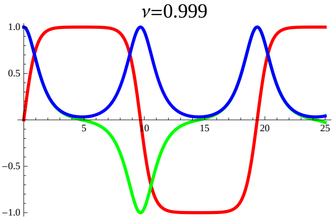
$$M(z) = \Delta \left(\nu \,\mathrm{sn}(b|\nu)\mathrm{sn}(\Delta z|\nu)\mathrm{sn}(\Delta z + b|\nu) + \frac{\mathrm{cn}(b|\nu)\mathrm{dn}(b|\nu)}{\mathrm{sn}(b|\nu)} \right)$$

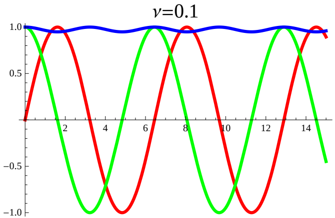
- Eigenvalue spectrum well known

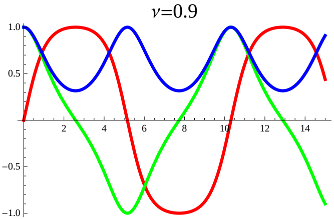
(M.Thies et al., Annals Phys. 314 (2004) 425-447, arXiv:hep-th/0402014)

Elliptic functions: sn$(z|\nu)$, cn$(z|\nu)$, dn$(z|\nu)$

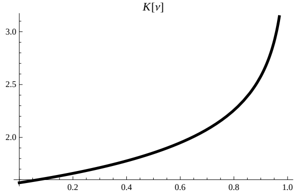
$\nu \in [0, 1]$







Period $\propto \mathbf{K}(\nu)$:



Thermodynamic potential

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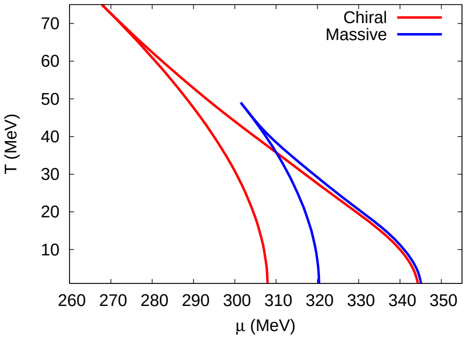
$$\Omega_{MF}^{NJL}(T, \mu; \Delta, \nu, \delta) = -2N_c \int_0^\infty dE\, \tilde{\rho}(E; \nu, \Delta) \tilde{f}_{\text{bare}}\left(\sqrt{E^2 + \delta\Delta^2}\right) + \frac{1}{4G_s P} \int_0^P dz\, |M(z) - m|^2 + \mathrm{C}$$

$$\begin{aligned} \tilde{f}_{\text{bare}}(x) &= \tilde{f}_{\text{UV}}(x) + \tilde{f}_{\text{medium}}(x) \\ \tilde{f}_{\text{UV}}(x) &= x \\ \tilde{f}_{\text{medium}}(x) &= T \ln\left(1 + \exp\left(-\frac{x-\mu}{T}\right)\right) + T \ln\left(1 + \exp\left(-\frac{x+\mu}{T}\right)\right) \end{aligned}$$

$$\tilde{f}_{\text{UV}}(x) \to \tilde{f}_{\text{PV}}(x) = \sum_{j=0}^{3} c_j \sqrt{x^2 + j\Lambda^2} \qquad (c_0 = 1, c_1 = -3, c_2 = 3, c_3 = -1)$$

Massive quarks



- Inhomogeneous region shrinks
- Crossover above the Lifschitz point
- No dramatic change of the picture

T vs Density phase diagram

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