Simulating strongly coupled quantum fluids

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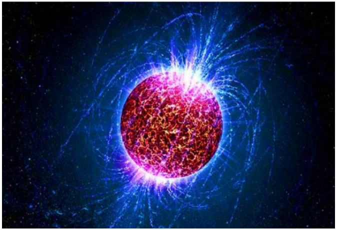
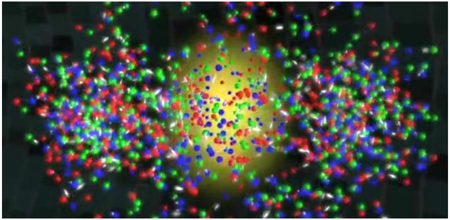
Non-equilibrium Dynamics (NeD-2015), August 31, 2015, Giardini Naxos, Italy

Examples for SCQFs



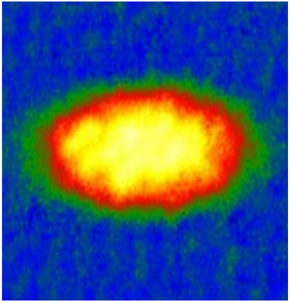
Neutron Star



Quark Gluon Plasma

Examples for SCQFs

high-T superconductor



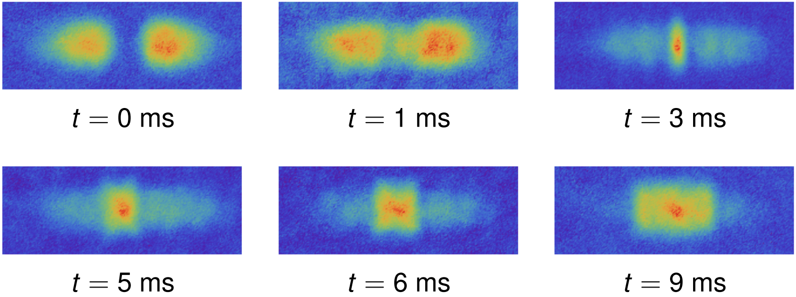
ultracold Fermi gas

$\rightarrow$ interested in:

- thermodynamic properties
- non-equilibrium dynamics (transport coefficients, relaxation processes, ...)

Fermi gas cloud collision

- repulsive optical potential divides trapped gas into two clouds
- once switched off, the two clouds accelerate and collide

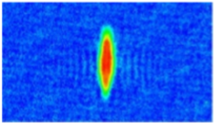


$\rightarrow$ shock wave formation (2 shock fronts)

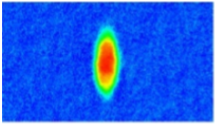
Anisotropic expansion

- ultracold Fermi gas released from deformed trapping potential

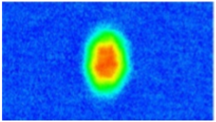
  K.M. O'Hara et al., Science **298** (2002)



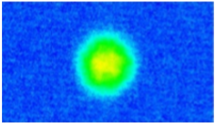
$t = 100\ \mu s$



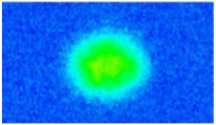
$t = 200\ \mu s$



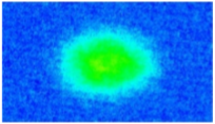
$t = 400\ \mu s$



$t = 600\ \mu s$



$t = 800\ \mu s$



$t = 1000\ \mu s$

$\rightarrow$ observe fluid dynamical behavior (elliptic flow)

- coordinate-space anisotropy converted to momentum-space anisotropy
- perfect to study shear viscosity η and bulk viscosity ζ

Unitary Fermi gas - primer

ultracold atomic Fermi gas studied in trap-experiments:

- macroscopically occupied mixture of atoms of half-integer total spin, e.g. ^{6}Li
- at low density n and low temperature T: details of atomic interaction and atomic structure not resolved
- atoms describable as the two components of point-like non-relativistic spin-1/2 fermions with ($T > T_F$)

$$\mathcal{L}_{eff} = \psi^\dagger \left(\imath\partial_t + \frac{\nabla^2}{2m} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

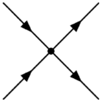
- coupling constant $C_0 \sim$ s-wave scattering length a

at low n and T:

$\to$ higher partial waves and range corrections unimportant
$\to$ 2-body s-wave scattering amplitude:

$$\mathcal{M} = \frac{4\pi}{m} \frac{1}{1/a + \imath q}$$

$\to$ in dimensional regularization $C_0 = 4\pi a/m$

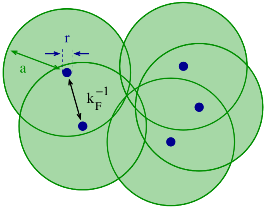


q: relative momentum

Unitary Fermi gas - primer

unitary limit $a \to \infty$:

- no intrinsic dimensionful parameters; theory scale invariant
- universal s-wave collision cross-section: $\sigma = 4\pi / q^2$
- extremely strong interactions; strong correlations even if n is low
- properties of the gas are universal functions of n and T only, e.g. $\eta = \hbar n \cdot \tilde{\eta}(T/\mu)$



dilute regime: $r \cdot n^{1/3} \ll 1$

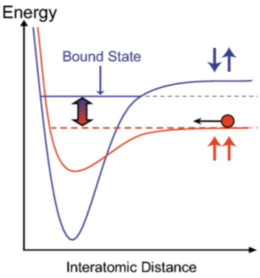
strongly correlated regime: $a \cdot n^{1/3} \gg 1$

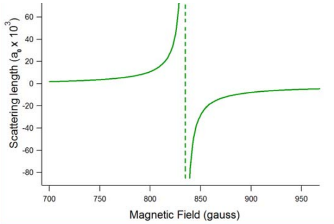
degeneracy occurs for $T \ll T_F \to$ inter-particle spacing $\sim k_F^{-1}$

for $T \gg T_F$ typical momenta are thermal: $p_T = \sqrt{2mTk_B}$

Unitary Fermi gas - primer

$\rightarrow$ unitary limit achieved by magnetically tuning atoms to a Feshbach resonance



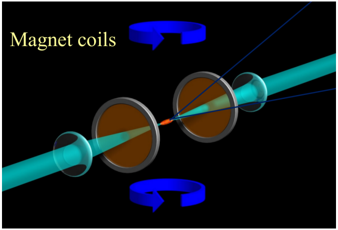


broad Feshbach resonance of ^{6}Li at $B = 832$ G

resonances occur as bound states if interatomic potential (*hyperfine interaction*) is tuned into resonance with the energy of the two colliding atoms $\leftarrow$ magnetic tuning (possible if magnetic moments of bound state and the 2 unbound atoms different)

$\rightarrow$ interaction strength can widely be varied

Elliptic flow measurement



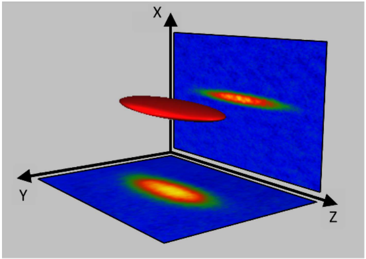
experimental set-up

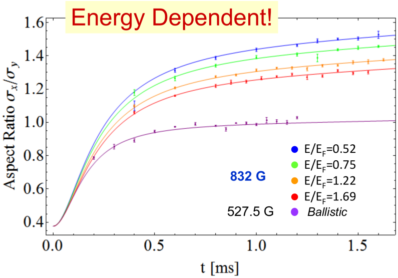
- 50-50 mixture of 2 lowest ^{6}Li hyperfine states $|F = \frac{1}{2}, M = \pm\frac{1}{2}\rangle$
- $N \simeq 10^5$ atoms in the cloud
- optical trapping with ultrastable CO_2-laser
- magnetic coils: broad Feshbach resonance at 832 G
- resonant scattering maintained in expanding gas
- cooling by forced, rapid evaporation in optical trap
- ultracold: μK-range ($T = 0.1$ neV)
- barrier height for p-wave scattering in ^{6}Li is 8 mK $\rightarrow$ negligible

$\rightarrow$ excellent exp. system: widely tunable interaction strengths, densities and temperatures

Elliptic flow measurement

→ after abrupt release of the cloud, study time-evolution of all 3 cloud radii
(rapid transverse expansion; nearly stationary in axial direction)





- harmonic confinement potential:

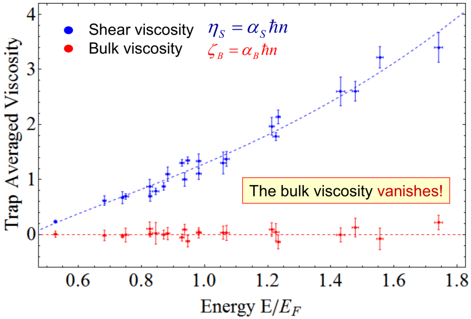
$$V = \frac{1}{2} m \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right)$$

  with $\omega_x > \omega_y \gg \omega_z$
- absorption pictures: 2 cameras

→ time-evolution of transverse aspect ratio σ_x/σ_y

- ballistic, free streaming
- energy-dependent: E (energy per particle)
- η counteracts differential acceleration
  → expansion rate decreases w/ increasing η

Determination of transport coefficients



E. Elliott et al., PRL **112** (2014)

determine transport coefficients from expansion data, e.g. by simulating evolution within fluid dynamics (parameters)

- need to know EoS and initial conditions (Gaussian density profile at large T)

caveat: only trap-averaged parameters

$$\alpha_S = \frac{1}{N\hbar} \int d^3\vec{x}\, \eta(\vec{x}, t)$$

$\rightarrow$ must assume $\eta \rightarrow 0$ at low n

$\rightarrow \langle \eta \rangle / \langle s \rangle \lesssim 0.4$ in normal phase (close to holographic bound); $\langle \zeta \rangle = 0$

Reminder - standard fluid dynamics

fluid dynamics = **the** *universal* effective description (*theory*) of non-equilibrium many-body systems (low energy/frequency, long time, *long* wavelength/*distance*) for the dynamics of conserved and/or spontaneously broken symmetry variables in classical and quantum liquids, gases and plasmas

requirement: system relaxes to approximate local thermodynamic equilibrium on the time scale of the observation

competition of time scales:

- τ_{fluid} (microscopic) rate of disturbance relaxation
- τ_{diff} conserved charge relaxation (diffusion, collective motion)

$\rightarrow$ fluid dynamics valid when clear scale separation $\tau_{fluid} \ll \tau_{diff}$

breakdown scale: $\omega_{fluid} = 1/\tau_{fluid} \sim Ts/\eta$ $\rightarrow$ fluid dynamics most effective for (almost) perfect fluids

Reminder - standard fluid dynamics

simple, non-relativistic fluid:

conserved charges are mass density ρ, momentum density $\vec{\pi}$, and energy density $\mathcal{E} = \mathcal{E}_0 + \frac{1}{2}\rho\vec{u}^2$

$$
\begin{aligned}
\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{\pi} &= 0 \\
\frac{\partial\pi_i}{\partial t} + \nabla_j\Pi_{ij} &= 0 \\
\frac{\partial\mathcal{E}}{\partial t} + \vec{\nabla}\cdot\vec{\jmath}^{\mathcal{E}} &= 0
\end{aligned}
$$

constitutive relations: currents must be systematically expandable in gradients of fluid dynamical variables

- $\vec{\pi} = \rho\vec{u} \;\rightarrow$ momentum density = mass current (Ward identity; defines $\vec{u}$)
- stress tensor $\Pi_{ij} = \Pi_{ij}^{(0)} + \delta\Pi_{ij} = \rho u_i u_j + P\delta_{ij} + \delta\Pi_{ij}$ (rotational and Galilean invariance)
- energy current $\vec{\jmath}^{\mathcal{E}} = \vec{u}(\mathcal{E}_0 + P) + \delta\vec{\jmath}^{\mathcal{E}}$
- ideal fluid dynamics: $\delta\Pi_{ij} = \delta\vec{\jmath}^{\mathcal{E}} = 0$ (time reversal invariance; entropy conservation)
- Navier-Stokes fluid dynamics: $\delta\Pi_{ij}^{(1)} = -\eta\sigma_{ij} - \zeta\delta_{ij}\langle\sigma\rangle$ with $\sigma_{ij} = \nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\langle\sigma\rangle$, $\langle\sigma\rangle = \vec{\nabla}\cdot\vec{u}$ and $(\delta\vec{\jmath}^{\mathcal{E}})^{(1)} = u_j\delta\Pi_{ij} - \kappa\nabla_i T$
- transport coefficients: η, ζ, thermal conductivity κ are parameters

expansion only meaningful if $\Pi_{ij}^{(0)} \gg \delta\Pi_{ij}^{(1)} \gg \delta\Pi_{ij}^{(2)} \ldots$

Regime of applicability of standard fluid dynamics

for compressible fluids ($Ma = u/c_s \sim 1$) $\rightarrow$ suitable expansion parameter is Re^{-1}:

$$Re^{-1} = \frac{\eta}{\hbar n} \times \frac{\hbar}{muL} \ll 1$$

fluid property   flow property

for a flow factor $muL \sim \hbar$ $\rightarrow$ hydrodynamics applicable if $\eta/(\hbar n) \lesssim 1$

nearly perfect fluids exhibit fluid dynamical behavior on length scales comparable to microscopic length scales

$\rightarrow$ kinetic theory estimate: $\eta \sim n\langle p\rangle l_{mfp}$

$$Re^{-1} = Ma \cdot Kn \Rightarrow Kn = l_{mfp}/L \ll 1$$

Kinetic theory (at unitarity)

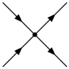
$\rightarrow$ provides simplest microscopic description of a fluid

fluid dynamical equations derivable (long-distance/time limit of kinetics is fluid dynamics)

Boltzmann-equation for the single-particle distribution function $f_p(\vec{x}, t)$:

$$\left(\partial_t + (\vec{\nabla}_p E_p) \cdot \vec{\nabla}_x + (\vec{\nabla}_x E_p) \cdot \vec{\nabla}_p\right) f_p(\vec{x}, t) = \mathcal{C}[f_p]$$

$$\mathcal{C}[f_p] =$$



Chapman-Enskog expansion: $f_p = f_p^0 + \delta f_p^1 \cdots = f_p^0(1 + \chi_p/T) \ldots \rightarrow \mathcal{C} = \frac{f_p^0}{T}\mathcal{C}[\chi_p]$

gradient expansion $\delta f_p^n = \mathcal{O}(\nabla^n) \equiv$ Knudsen expansion $\delta f_p^n = \mathcal{O}(Kn^n)$

$\rightarrow$ solve order-by-order in the Knudsen number Kn:

- first-order: $\delta\Pi_{ij}^{(1)} = -\eta\sigma_{ij}$ with $\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2}$; Bruun, Smith (2007)
- second-order: $\delta\Pi_{ij}^{(2)} \rightarrow$ relaxation time $\tau_\pi = \eta/P$; Schäfer (2014)

$\rightarrow$ breakdown of kinetic theory for $\omega > \omega_{micro} \sim T$

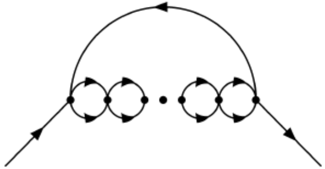
Bulk viscosity and conformal symmetry breaking

at unitarity (conformal symmetry): $P = \frac{2}{3}\mathcal{E}_0$; $\zeta = 0$; Ho (2004), Son (2007)

conformal symmetry breaking in thermodynamics:

$$1 - \frac{2}{3}\frac{\mathcal{E}_0}{P} = \frac{\langle \mathcal{O}_C \rangle}{12\pi maP} \sim \frac{1}{3\pi}\left(z\frac{\lambda}{a}\right) \quad ; \quad \mathcal{O}_C = C_0^2 \psi\psi\psi^\dagger\psi^\dagger \text{ contact density}$$

$\rightarrow$ impact on ζ:



Dusling, Schäfer (2013)

excitations aquire a momentum-dependent effective mass $\leftrightarrow$ self-energy (LO in z):

$$Re\Sigma(p) \sim \left(z\frac{\lambda}{a}\right) T\sqrt{\frac{T}{E_p}}\, F_D\left(\sqrt{\frac{E_p}{T}}\right)$$

$$Im\Sigma(p) \sim zT\sqrt{\frac{T}{E_p}}\, Erf\left(\sqrt{\frac{E_p}{T}}\right)$$

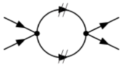
- bulk viscosity $\zeta = \frac{\lambda^{-3}}{24\sqrt{2}\pi}\left(z\frac{\lambda}{a}\right)^2 \Rightarrow \zeta/\eta \sim \left(1 - \frac{2}{3}\frac{\mathcal{E}_0}{P}\right)^2$

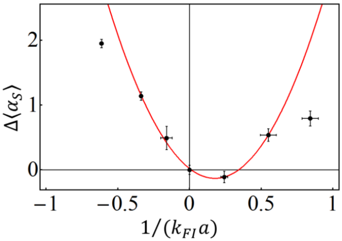
Shear viscosity and conformal symmetry breaking

consider η for $a \neq \infty$:

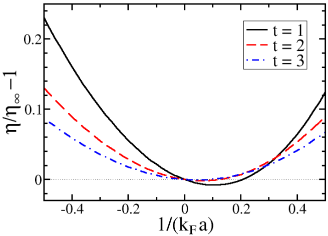
$$\eta = \eta_\infty \left(1 + \mathcal{O}\left(\frac{\lambda^2}{a^2}\right) + \mathcal{O}\left(\frac{z\lambda}{a}\right) + \dots\right)$$

medium-effects at $\mathcal{O}\left(z\lambda/a\right)$ originate from self-energy, in-medium scattering

$\Pi(P, q) =$ 



exp.: Elliott et al. (2014)

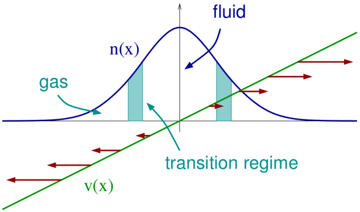


kin.th.: MB, Schäfer (2014)

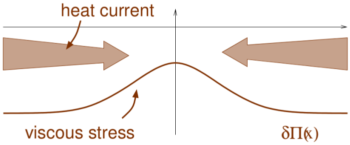
Local shear viscosity determination - $\eta(n, T)$

$\rightarrow$ deduce dependence on local n and T from global observables

- very first exp. approaches (inversion of trap-averaged data) Joseph et al. (2014)



whole cloud not a fluid $\rightarrow$ α_S ill defined at high T



paradoxical fluid dynamical behavior: constant amount of heating by current from spatial infinity although no dissipative force exists for Hubble-flow

$\rightarrow$ fluid dynamics breakdown in dilute regime

$\rightarrow$ need a reliable treatment of low density corona (”graceful” exit):

- solve full Boltzmann equation; Lattice Boltzmann simulation; Quantum Monte Carlo; combine fluid dynamics & Boltzmann equation; Anisotropic fluid dynamics MB, Schäfer (2015)

Anisotropic fluid dynamics

combines a fluid dynamical treatment of the core with a ballistic description of the corona smoothly

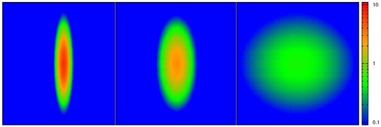
consider $f_p = f_p^{an} + \delta f_p^1 + \ldots$ with

$$f_p^{an} = \exp\left(\frac{\mu}{T_{le}} - \sum_a \frac{mc_a^2}{2T_a}\right), \qquad T_{le} = \prod_a T_a^{1/3}.$$

conservation laws & evolution of *non-hydrodynamic* d.o.f. $\mathcal{E}_{a=x,y,z}$ (in Lagrangian form):

$$-n\mathcal{D}_0\left(\frac{\mathcal{E}_a}{n}\right) = \delta_{ia}\nabla_i\left(u_i P + u_j \delta\Pi_{ij}\right) + \frac{\Delta P_a}{2\tau}$$

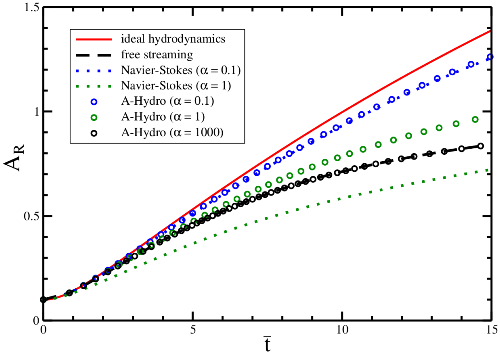
$$\Delta P_a = P_a - P = n(T_a - T)\ , \ \tau = \eta / P$$



AVH1 hydro code: MB, Schäfer (2015)

- small τ: fast relaxation to Navier-Stokes theory
- large τ: additional conservation laws $\rightarrow$ ballistic expansion

Anisotropic expansion in AVH1



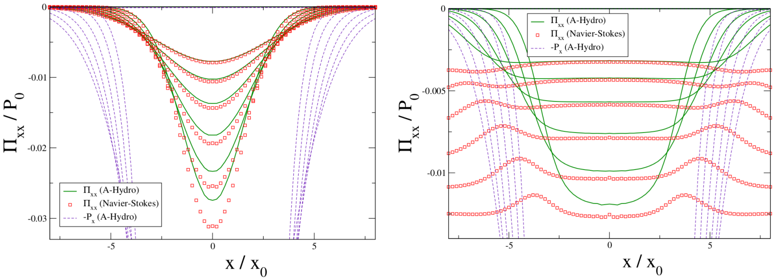
Consider $\eta = \alpha n$ and $\alpha \in [0, \infty)$

Navier-Stokes: Ideal hydro $\to$ very viscous hydro.

A-hydro: Ideal hydro $\to$ ballistic expansion.

Dissipative corrections in AVH1

$\eta = \alpha_n n$ $\eta = \alpha_T (mT)^{3/2}$



Π_{xx}(Navier-Stokes) Π_{xx}(A-Hydro)

Conclusions and Outlook

- ultracold Fermi gases provide an excellent playground for studying strongly coupled/correlated quantum fluids
- at unitarity: nearly perfect fluid
- fluid dynamics provides its "most effective" description
- (non)elusive goal: unfold local n and T dependence of η
- need a description that "gracefully" exits fluid dynamics $\rightarrow$ AVH1
- spin diffusion can also be studied
- for apples-to-apples comparison: extension to two-fluid system, restoration of rotational invariance

many thanks to

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J. A. Joseph
T. Schäfer