Modeling the hadronization processes in HIC (based on the Nambu Jona-Lasinio Lagrangian)

in collaboration with
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Motivation:
How to study phase transitions at finite chemical potential (NICA,FAIR)

Lattice results only reliable for μ/T << 1 :

- ❑ assumptions about continuation to finite μ
- ❑ effective theories which allow for such an extension intrinsically

The Nambu Jona Lasinio Lagrangian is such an effective field theory

Nambu

- ❑ allows for predictions for finite T and μ
- ❑ needs as input only vacuum values + YM Polyakov loop
- ❑ shares the symmetries with the QCD Lagrangian
- ❑ can be « derived » from QCD Lagrangian

Jona-Lasinio

- How one does obtain the NJL Lagrangian?
- How to construct mesons Mesons and Baryons?
- Cross section for elastic scattering and hadronisation
- Expanding plasma: How quarks hadronize?
- Realistic simulations

Conserving the QCD symmetries in an effective Langrangian

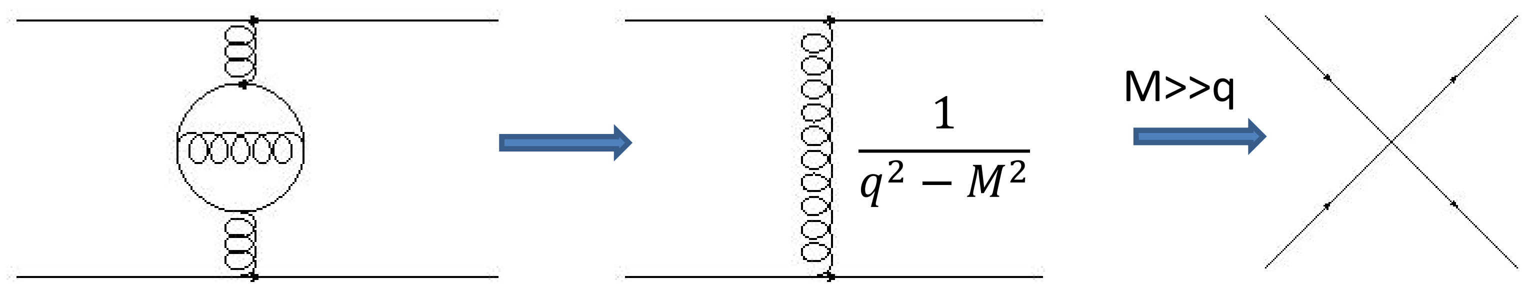
1) local *SUc*(3) color gauge transformation (by construction)

2) global SU_f (3) flavor symmetry

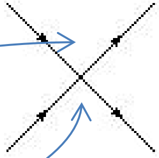
3) for massless quarks ONLY:
   chiral invariance of QCD Lagrangian: $\boldsymbol{SU_f(3)_V \times SU_f(3)_A}$

However, chiral symmetry is a spontaneously broken since quarks have non-zero masses.

$\Rightarrow$ An *effective Lagrangian* with the same symmetries for the quark degrees of freedom can be obtained by discarding the gluon dynamics completely.

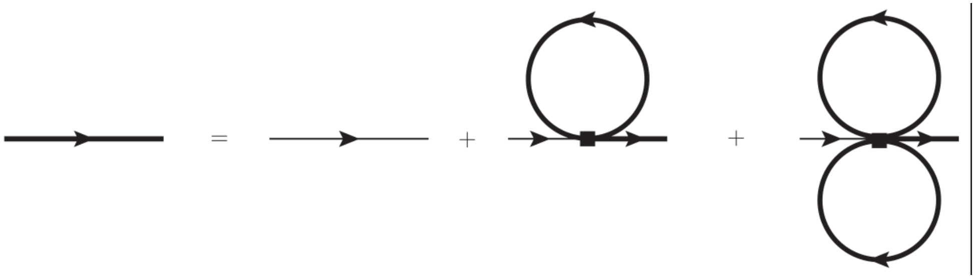


NJL Lagrangian



$$\mathscr{L}_{NJL} = \bar{\Psi}_i(i\gamma_\mu\partial^\mu - \hat{M}_0)\Psi_i - G_c^2\ [\bar{\Psi}_i\gamma^\mu T^a\delta_{ij}\Psi_j]\ [\bar{\Psi}_k\gamma_\mu T^a\delta_{kl}\Psi_l]$$
$$+ H \det_{ij} [\bar{\Psi}_i(1-\gamma_5)\Psi_j] - H \det_{ij} [\bar{\psi}_i(1+\gamma_5)\psi_j]$$

$\mathscr{L}_{NJL}$: Shares the symmetries with the QCD Lagrangian (color we discuss later)
Allows for calculating effective quark masses:



$$M = \hat{M}_0 - 4G < \bar{\psi}\psi > +2H < \bar{\psi}'\psi' >< \bar{\psi}''\psi'' >$$

But contains only quarks

no gluons and

no hadrons

So not very obvious how of use for hadronisation.

Polyakov NJL: gluons on a static level

Eur.Phys.J. C49 (2007) 213-217

It is not possible to introduce gluons as dynamical degrees of freedom without spoiling the simplicity of the NJL Lagrangian which allows for real calculations

but

one can introduce gluons through an effective potential for the Polyakov loop

$$\frac{U(T,\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}(\bar{\Phi}\Phi)^3$$

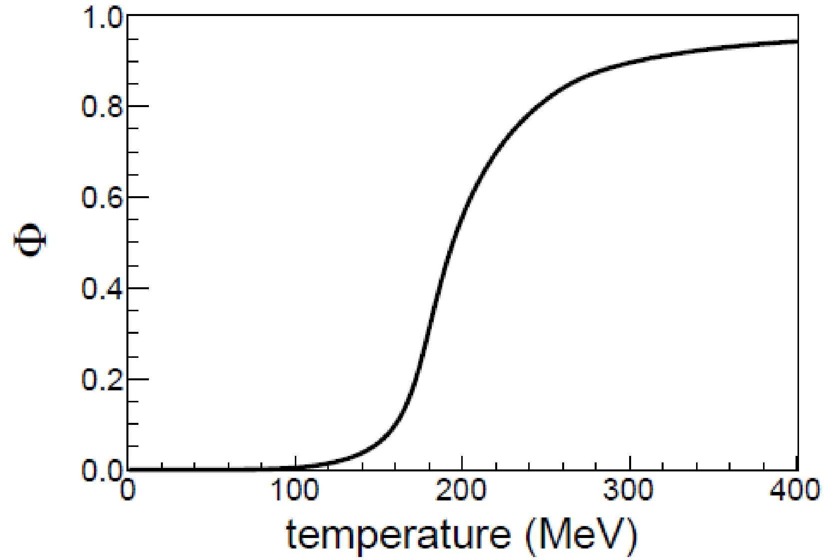
$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

$$a_0 = 6.75, a_1 = -1.95, a_2 = 2.625, a_3 = -7.44, b_3 = 0.75, b_4 = 7.5$$

Parameters-> right pressure in the SB limit

Φ is the order parameter of the deconfinement transition

$$\Phi = \frac{1}{N_c}\,\mathrm{Tr}_c\,\left\langle P\exp\left(-\int_0^\beta d\tau A_0(x,\tau)\right)\right\rangle$$

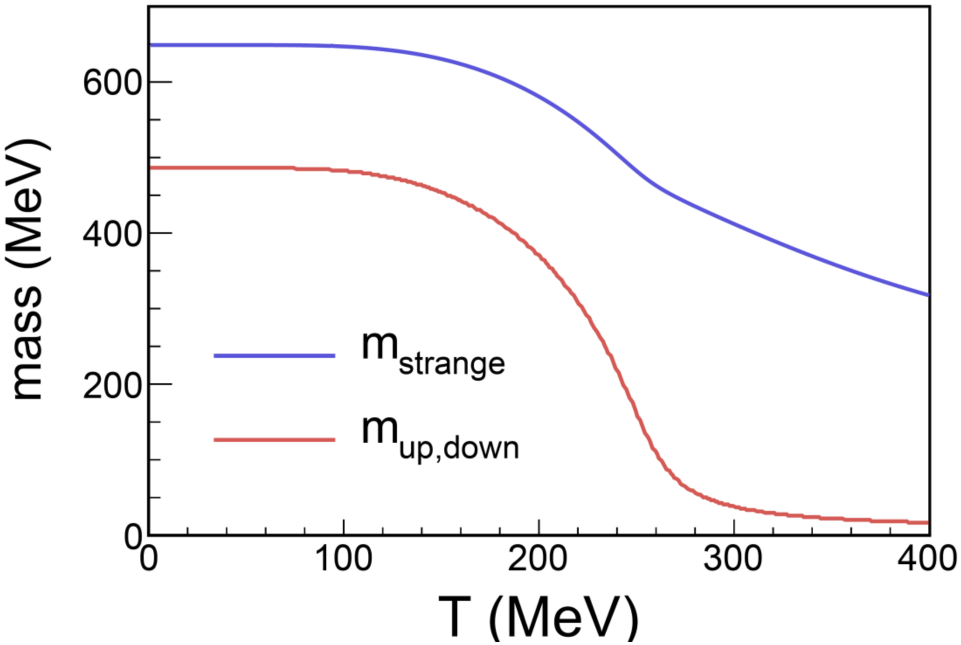


Quark Masses in NJL and PNJL

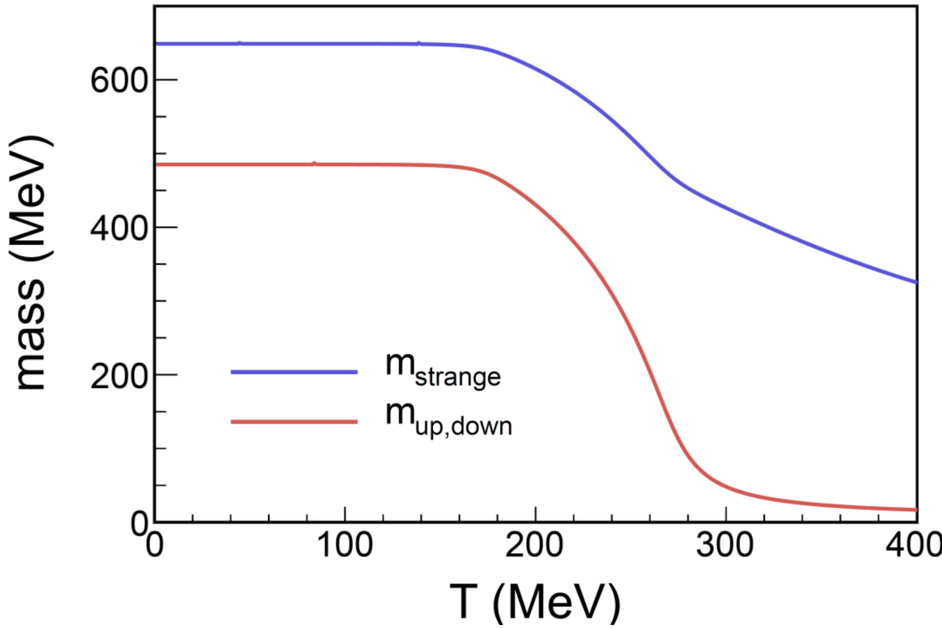
Quark masses are obtained by minimizing the grand canonical potential

$$\mathbf{M} = \hat{\mathbf{M}}_0 - 4\mathbf{G} < \bar{\psi}\psi > +2\mathbf{H} < \bar{\psi}'\psi' >< \bar{\psi}''\psi'' >$$

NJL



PNJL



In PNJL the transition is steeper than in NJL

How can we get mesons?

Quarks are the degrees of freedom of the Lagrangian

To study the phase transition we need mesons

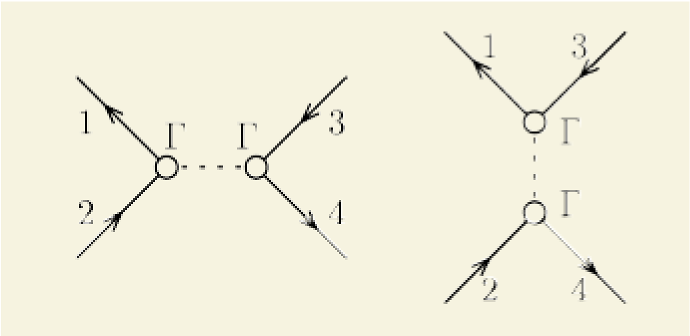
Use a Trick : Fierz transformation of the original Lagrangian

Fierz Transformation allows for a reordering of the field operators in 4 point contact interactions. It is simultaneously applied in Dirac, colour and flavour space

Example in Dirac space:

$$(\bar{\chi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\chi) = (\bar{\chi}\chi)(\bar{\psi}\psi) - \frac{1}{2}(\bar{\chi}\gamma^\mu\chi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}(\bar{\chi}\gamma^\mu\gamma_5\chi)(\bar{\psi}\gamma_\mu\gamma_5\psi) - (\bar{\chi}\gamma_5\chi)(\bar{\psi}\gamma_5\psi)$$

Scalar vector peudovector pseudoscalar



How can we get mesons? II

$$\mathscr{L}_{int} = -\mathrm{G}_c^2 \;\; [\bar{\Psi}_i \gamma^\mu T^a \delta_{ij} \Psi_j] \;\; [\bar{\Psi}_k \gamma_\mu T^a \delta_{kl} \Psi_l]$$

Fierz transformation transforms original Lagrangian to one for mesons

$$\mathcal{L}_{\mathrm{Pseudo\ scalar}} = \mathrm{G}\; (\Psi_i \; \tau^a_{il} \; \mathbb{1}_c i\gamma_5 \; \Psi_l)\; (\Psi_k \; \tau^a_{kj} \; \mathbb{1}_c i\gamma_5 \; \Psi_j)\; ; \qquad \mathrm{G} = \frac{N_c^2 - 1}{N_c^2} \mathrm{G}_c$$

$\mathscr{K}$

Singulet in color mixing of flavour

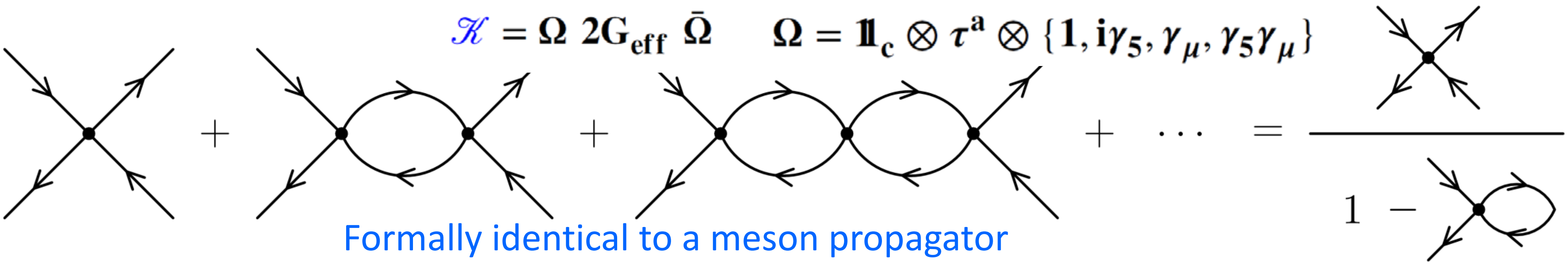
Similar terms can be obtained for
Vector mesons γ_μ
Scalar Mesons 1
Pseudovector mesons $\gamma_\mu \gamma_5$

How can we get mesons? III

We use $\mathscr{K}$ as a kernel for a Bethe-Salpeter equation (relativistic Lippmann-Schwinger eq.)

$$\mathbf{T(p)} = \mathscr{K} + \mathrm{i} \int \frac{d^4k}{(2\pi)^4}\, \mathscr{K}\, S\left(k + \frac{p}{2}\right) S\left(k - \frac{p}{2}\right) \mathbf{T(p)}$$

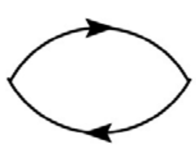
$$\mathscr{K} = \Omega\ 2G_{eff}\ \bar{\Omega} \qquad \Omega = \mathbb{1}_c \otimes \tau^a \otimes \{1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu\}$$



Formally identical to a meson propagator

In (P)NJL one can sum up this series analytically:

$$\mathbf{T(p)} = \frac{2G_{eff}}{1 - 2G_{eff}\,\Pi(p)}\,, \qquad \Pi(p_0, p) = -\frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\, \Omega\ S\left(k + \frac{p}{2}\right) \Omega\ S\left(k - \frac{p}{2}\right)$$

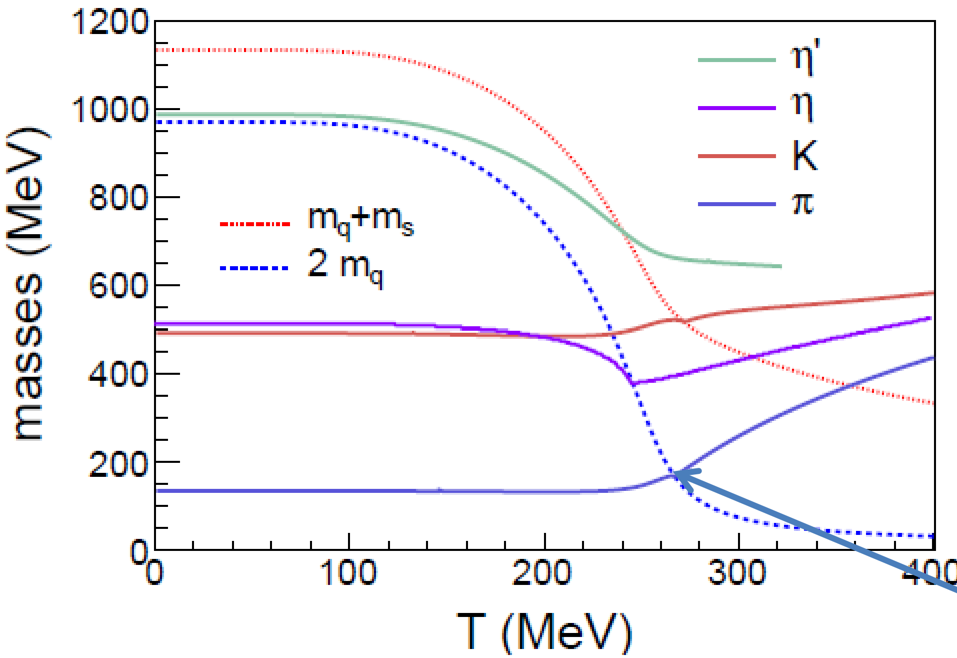


$\equiv \Pi$

How to get mesons? IV

The meson pole mass and the width one obtains by solving:

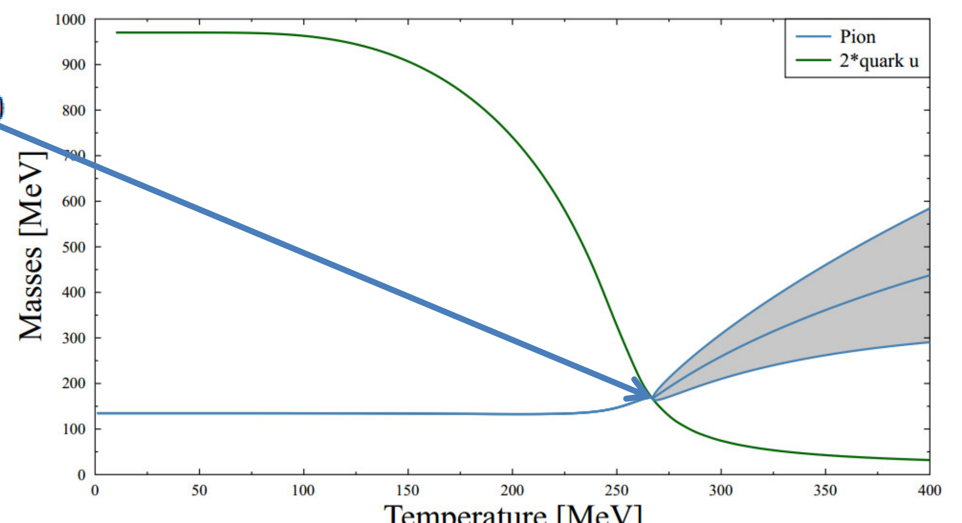
$$1 - 2G_{eff}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, p = 0) = 0$$



masses of pseudoscalar mesons and of quarks at μ = 0

At T=0 physical and calculated mass agree quite well

When mesons become unstable they develop a width



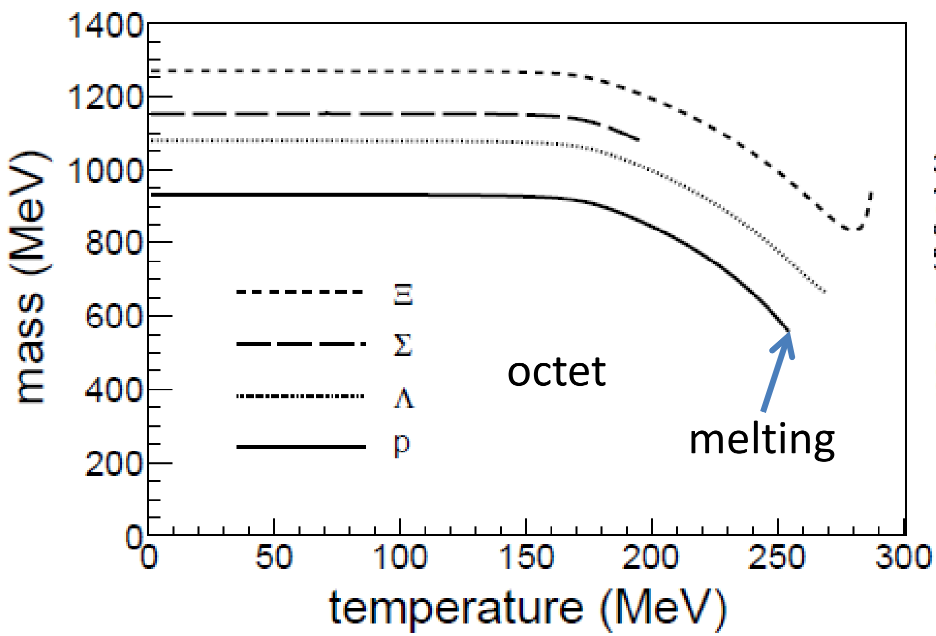
Baryons

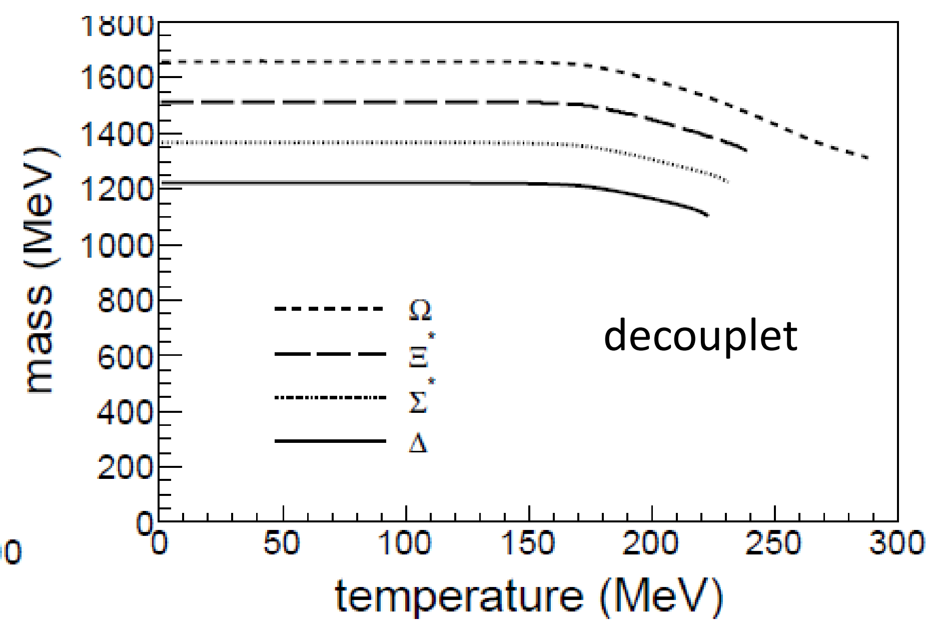
Omitting Dirac and flavour structure :

$$\left[1-\frac{2}{m_{quark}}\frac{1}{\beta}\sum_{n}\int\frac{d^3q}{(2\pi)^3}S_q(i\omega_n,q)\ t_D(i\nu_l-i\omega_n,-q)\right]\Bigg|_{i\nu_l\to P_0+i\epsilon=M_{Baryon}}=0$$

where we approximated the quark propagator for the exchanged quark by:

$$S_q(q)=\frac{1}{\not{q}-m_{quark}}\to-\frac{\mathbb{1}_{Dirac}}{m_{quark}}$$ 5% error (Buck et al. (92))





The more strange quarks the higher the melting temperature

Looking back

We have seen that the NJL model describes quite well meson properties
For this one has to fix the 5 parameters of the model

Λ = upper cut off of the internal momentum loops
G_c = coupling constant
M_0 = bare mass of u,d and s quarks
H= coupling constant ‘t Hooft term

These parameters have been adjusted to reproduce

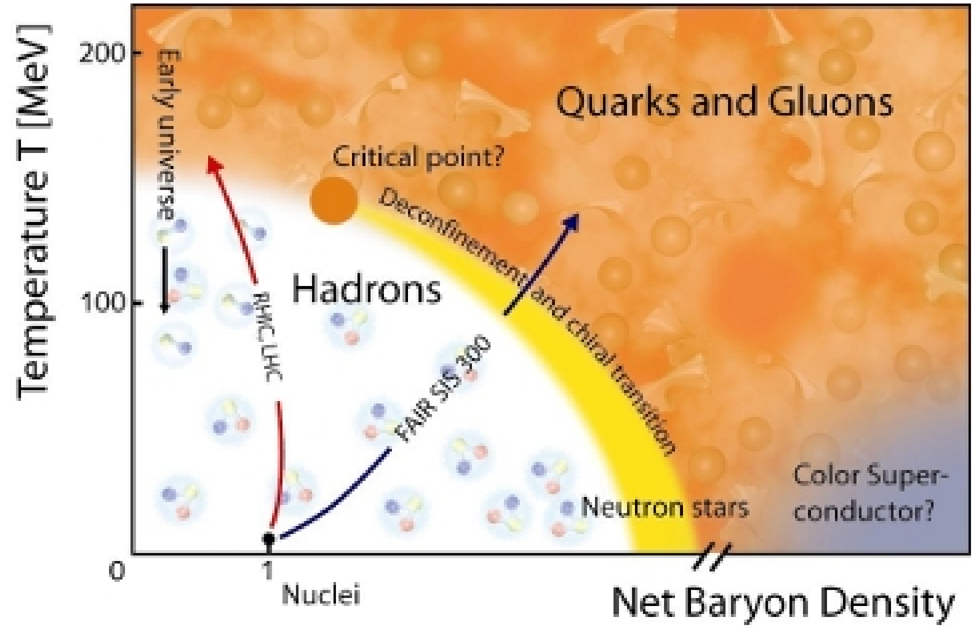
Masses of π and K in the vacuum , as well as the η-η’ mass splitting
π decay constant, $q\bar{q}$ condensate (-241 MeV)^3

Therefore:
All masses, cross sections etc. at finite μ and T
follow without any new parameters from ground state observables.

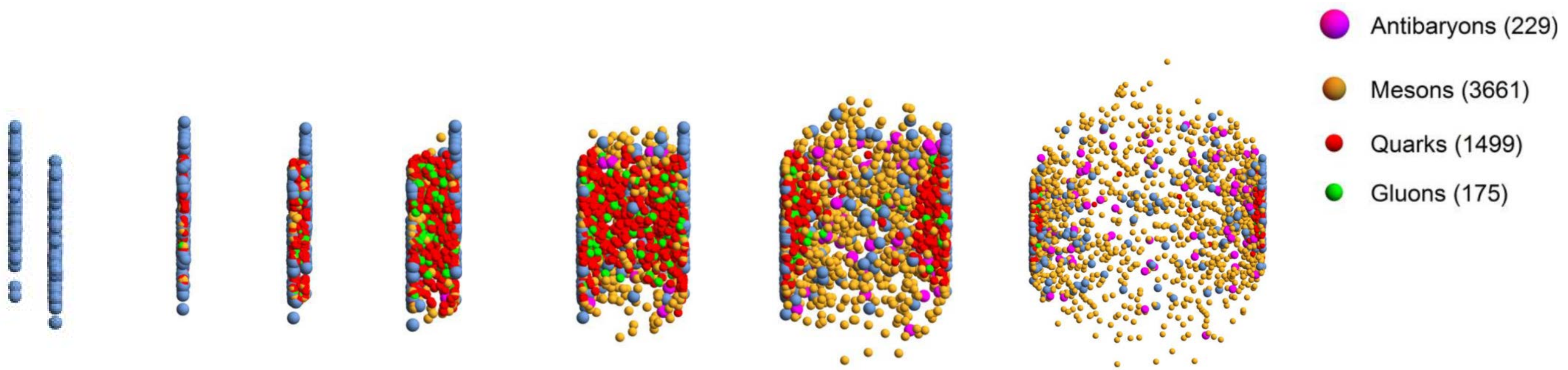
Baryons can be calculated in an analogous way.

circumstantial evidence:

For beam energies > ≈ 100 AGeV
a plasma of quark and gluons (QGP)
is formed



The challenge:
How to come from quarks to hadrons



As PHSD calculations see a heavy ion reaction
is there local equilibrium?

Courtesy:
P. Moreau 2015

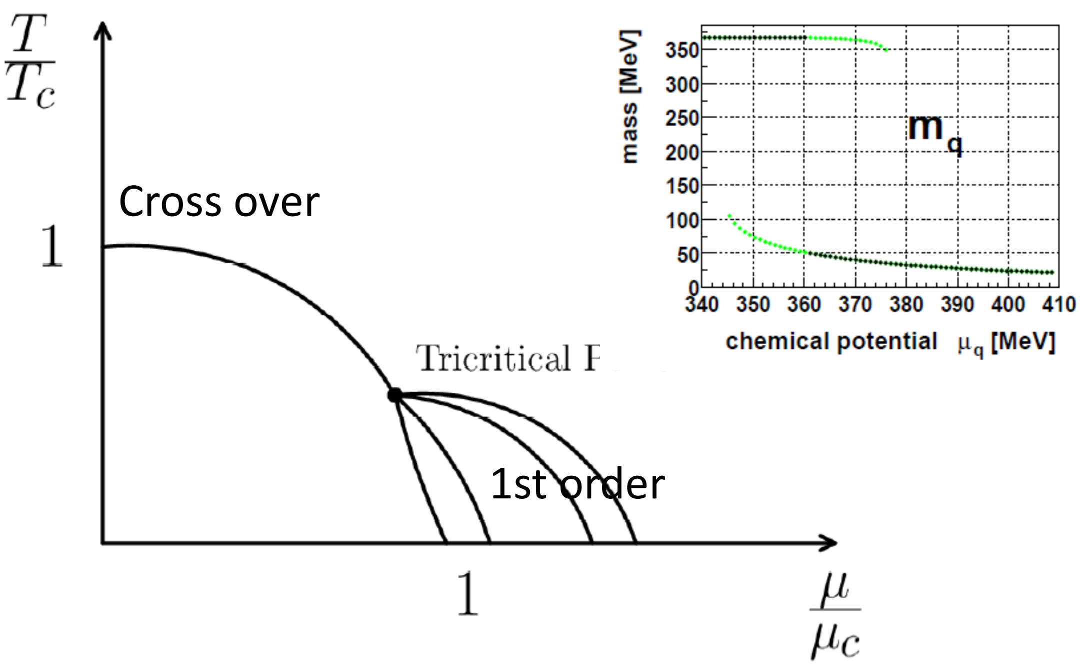
Masses close to the tricritical point

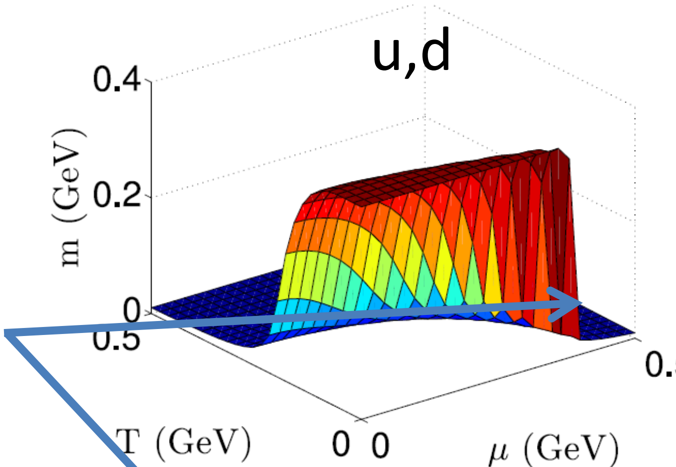
NJL Lagrangian:
transition between quarks and hadrons

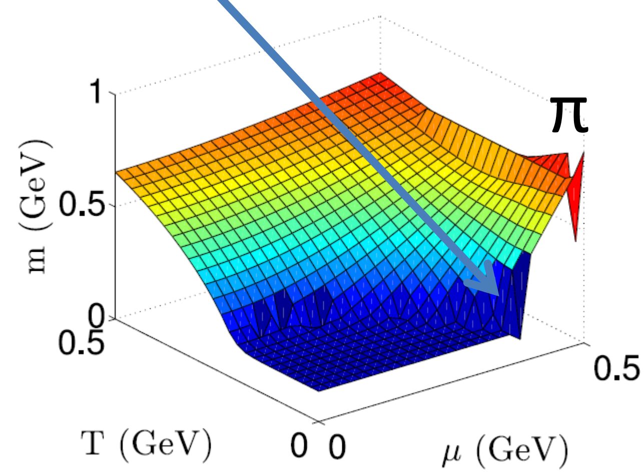
Cross over at μ = 0

1st order transition μ >>0

sudden change of q and meson mass





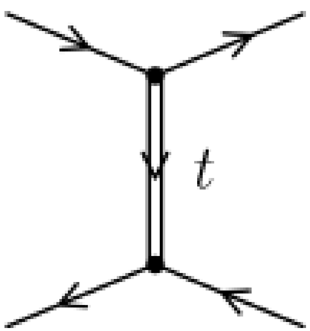


Details have not been explored yet

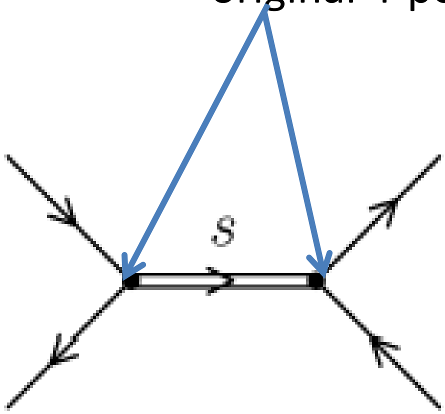
Cross sections

Having the Lagrangian we can derive in the usual way the Feynman rules and can calculate cross setions

Example: $u\bar{u} \rightarrow u\bar{u}$ matrixelements



Related to the original 4-point interaction



But also

elastic cross sections like $uu \rightarrow uu$

hadronization cross sections $q\bar{q} \rightarrow M\,M$ M=π,K,η,η',ρ …

hadronization cross sections Diq Diq -> baryons +q etc

$u\bar{u} \to u\bar{u}$ Cross sections

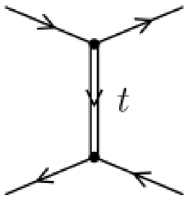
Phys.Rev. C53 (1996) 410-429

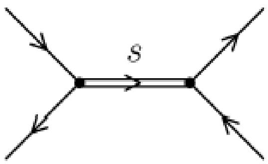
$$-i\mathcal{M}_s = \delta_{c1,c2}\delta_{c3,c4}\bar{v}(p_2)Tu(p_1)\left[i\mathcal{D}_s^S(p_1+p_2)\right]\bar{u}(p_3)Tv(p_4) + \delta_{c1,c2}\delta_{c3,c4}\bar{v}(p_2)(i\gamma_5 T)u(p_1)\left[i\mathcal{D}_s^P(p_1+p_2)\right]\bar{u}(p_3)(i\gamma_5 T)v(p_4)$$

$$-i\mathcal{M}_t = \delta_{c1,c3}\delta_{c2,c4}\bar{u}(p_3)Tu(p_1)\left[i\mathcal{D}_t^S(p_1-p_3)\right]\bar{v}(p_2)Tv(p_4) + \delta_{c1,c3}\delta_{c2,c4}\bar{u}(p_3)(i\gamma_5 T)u(p_1)\left[i\mathcal{D}_t^P(p_1-p_3)\right]\bar{v}(p_2)(i\gamma_5 T)v(p_4) \quad .$$

D= meson propagator

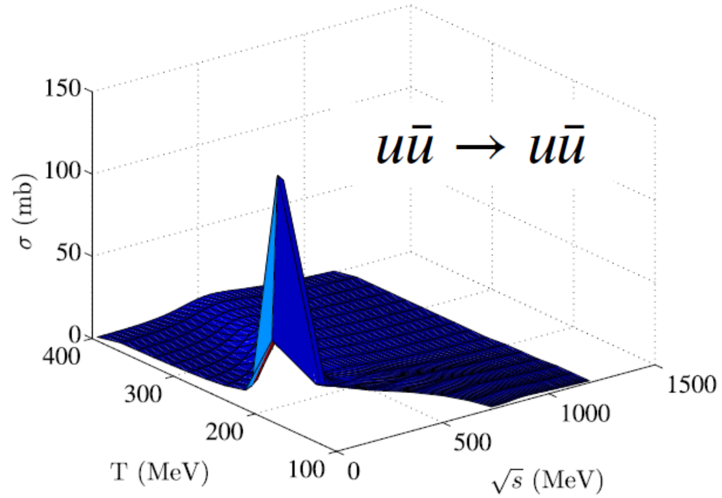
$$D(p_0,\vec{p}) \propto \frac{2G}{1-2G\Pi(p_0,\vec{p})}$$





Cross section up to 100 mb
close to cross over
due to resonant s-channel

otherwise small (5-10 mb)



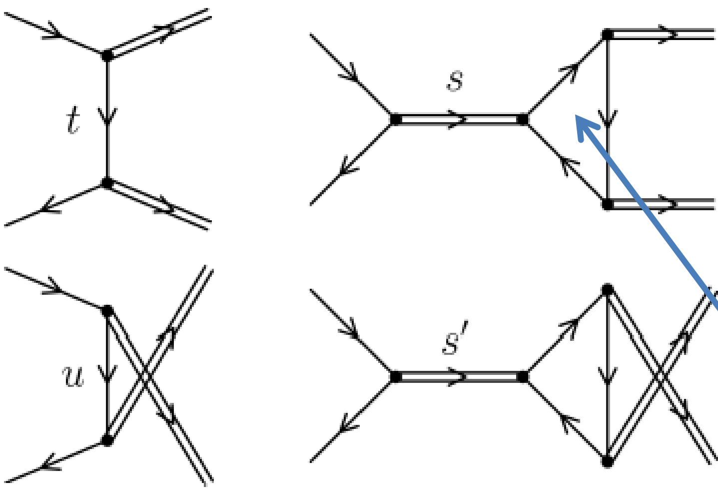
Hadronization cross sections

$$q\bar{q} \rightarrow M\,M$$

$$-iM_s = g^2_{Mqq'} f_s \bar{v}_2 u_1 \Gamma_\nu (i\mathscr{D}^{\mathcal{S}}{}_M) \Gamma^\nu_{q_1 q_2 q_3} + ..$$

$$-iM_t = g^2_{Mqq'} f_t \bar{v}_2 \Gamma_\nu \frac{i(\not{p}_1 - \not{p}_3 + m_t)}{(p_1 - p_3)^2 - m_t^2} \Gamma^\nu u_1$$

$$-iM_u = g^2_{Mqq'} f_u \bar{v}_2 \Gamma_\nu \frac{i(\not{p}_1 - \not{p}_4 + m_t)}{(p_1 - p_4)^2 - m_t^2} \Gamma^\nu u_1$$

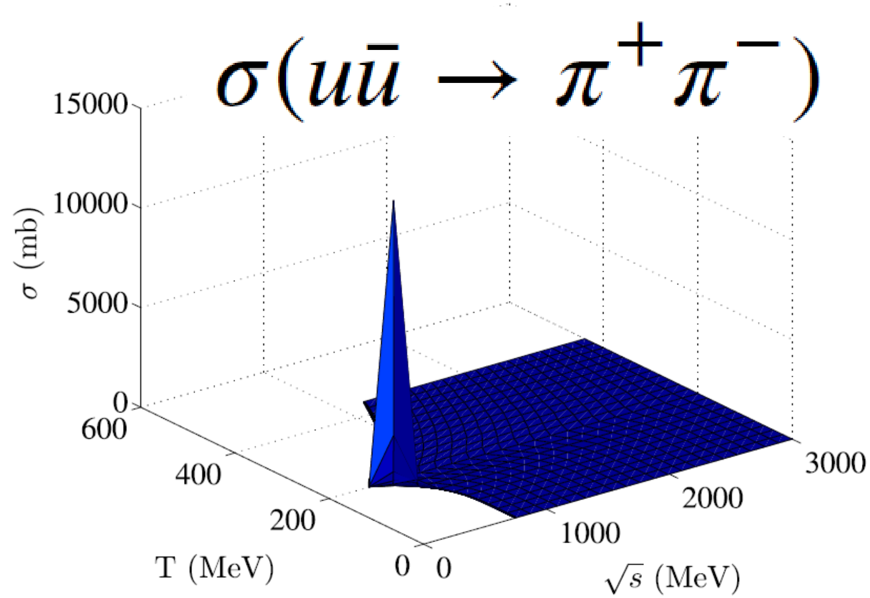


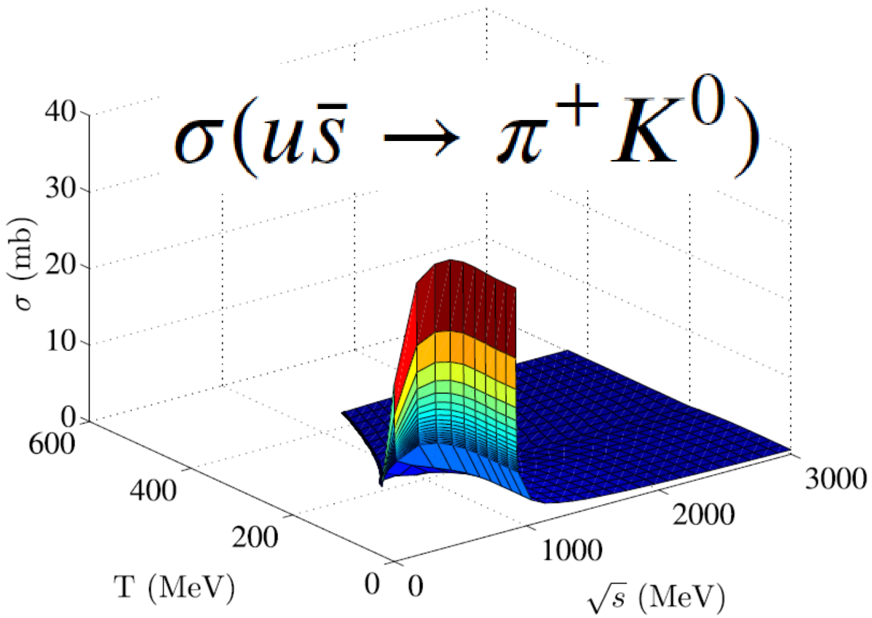
$\Gamma_{q_1 q_2 q_3}$ triangle vertex

Γ_ν appropriate γ matrix

Hadronization cross sections

These s-channel resonances create as well very large hadronization cross section close to T_c





Consequence:
If an expanding plasma comes to T_c quarks are converted into hadrons

despite of the NJL Lagragian does not contain confinement

How to make a transport theory out of NJL

Using 7 parameters fitted to ground state properties of mesons and baryons

the NJL model allows for calculating

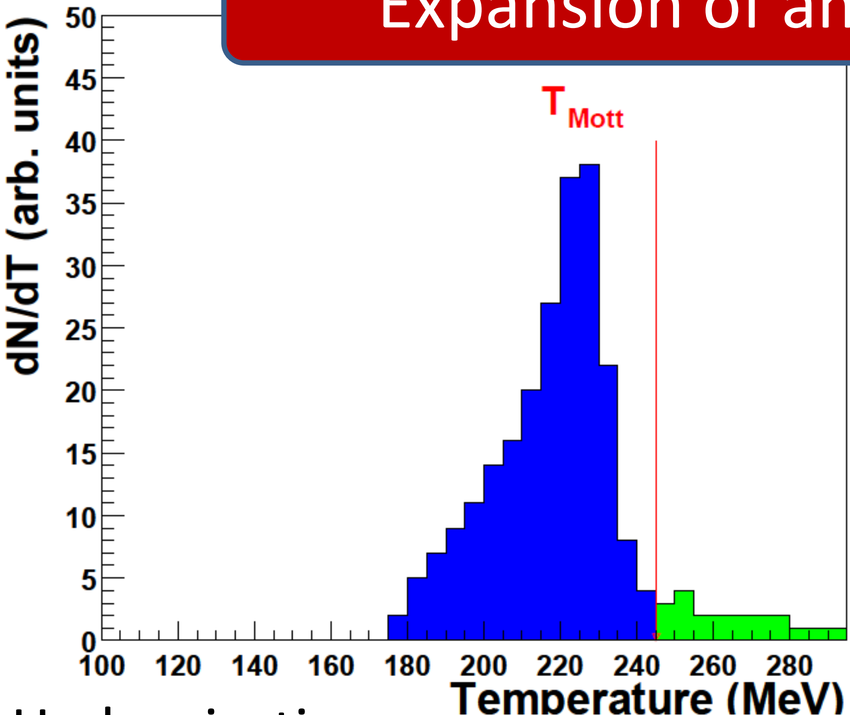
Quark masses (T,μ)
Hadron masses (T,μ)
Elastic cross sections (T,μ)
Hadronization cross sections (T,μ)

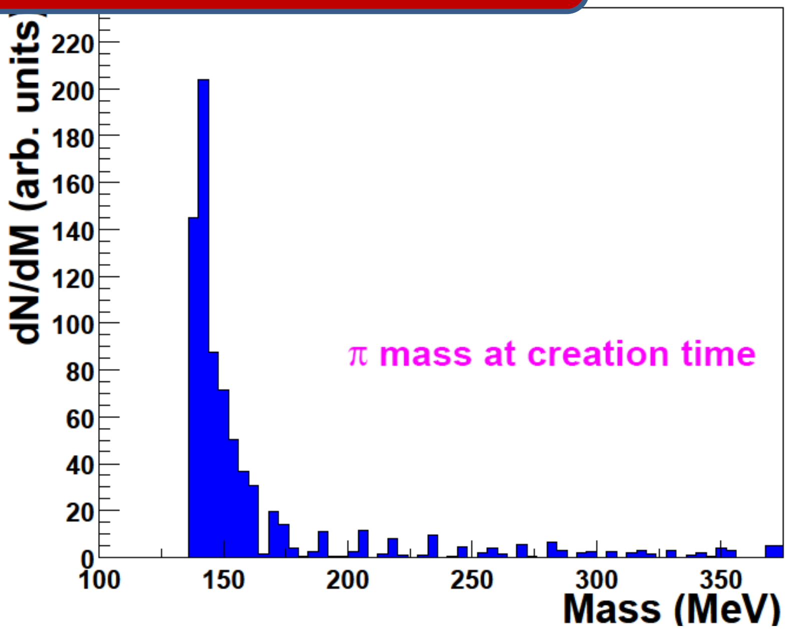
So we have all ingredients for a transport theory

Problem:
With a mass of 2 MeV and temperatures $>$ 200 MeV the quarks move practically with the speed of light.

So we have to construct a fully relativistic transport theory (all details in **Phys.Rev. C87 (2013) , 034912)**

Expansion of an equilibrated plasma



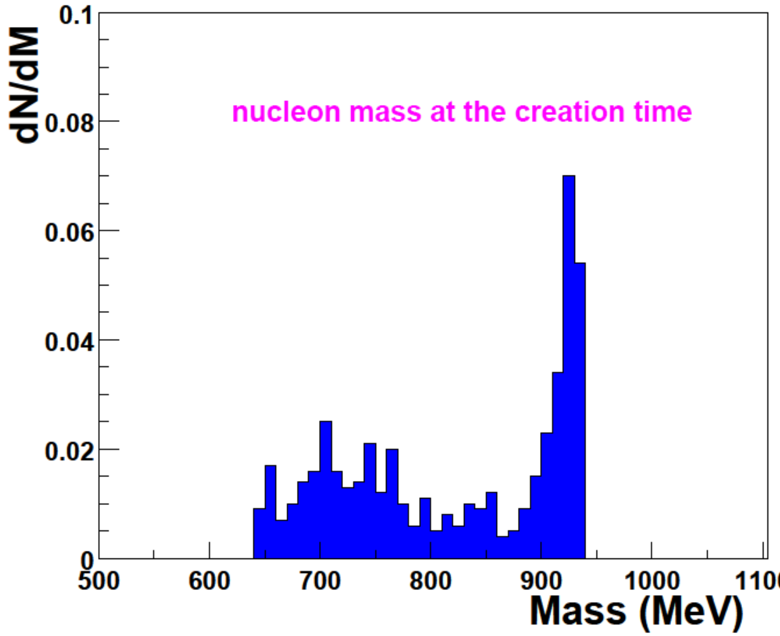


Hadronization:

Not at a fixed T but broad T distribution

Particles are produced over a wide mass range

Come to vacuum mass during expansion

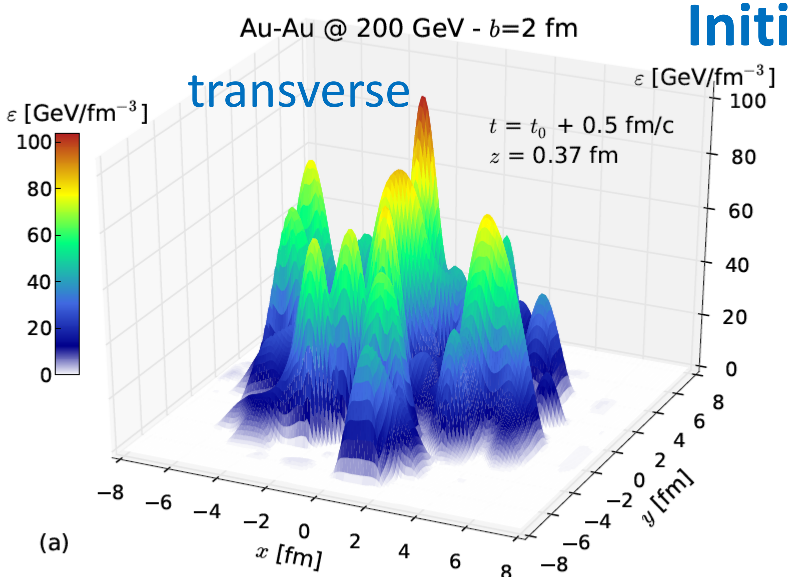


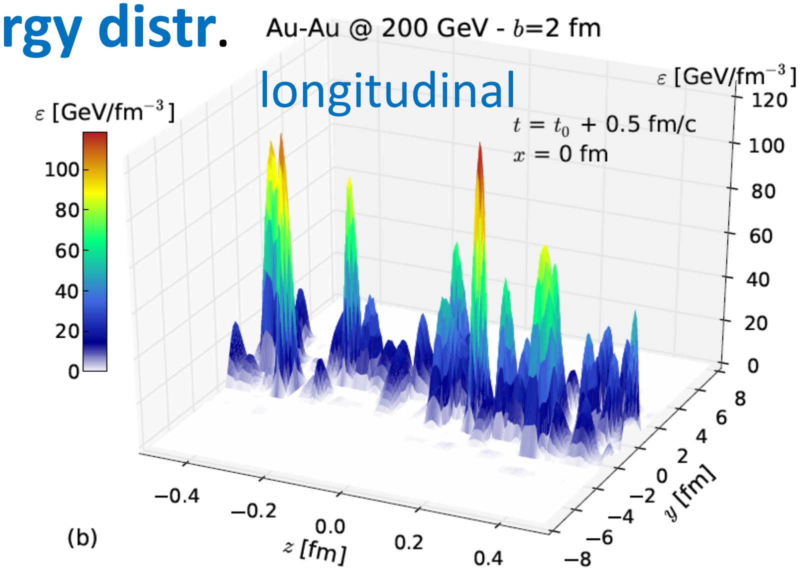
Expansion of a plasma

For realistic calculations we use the initial configuration of the PHSD approach and compare NJL with PHSD calculations

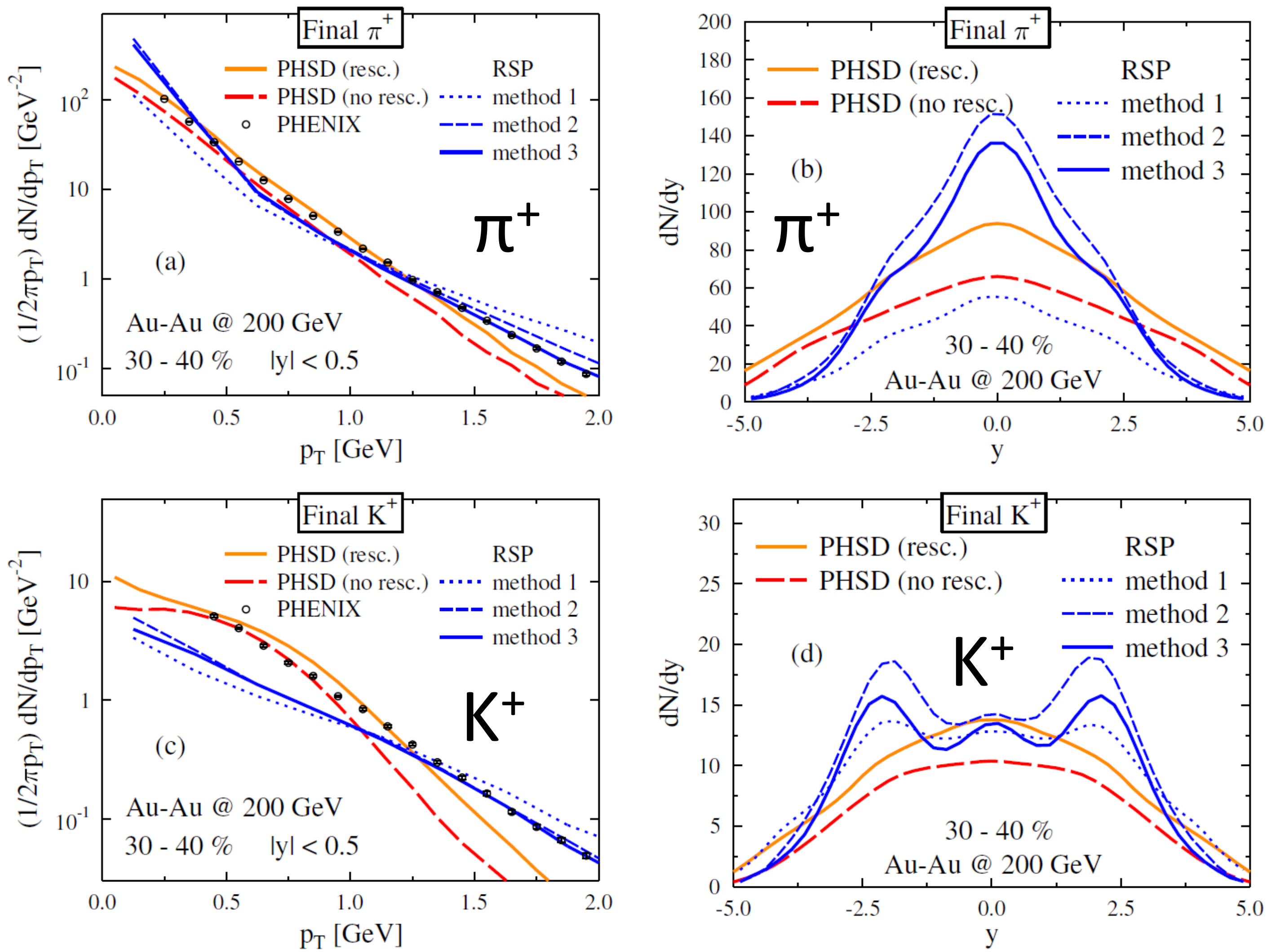
NJL	PHSD
$m_q \approx 5$ MeV	400 MeV $\leq m_q \leq$ 800 MeV
no gluons	gluons
g fix	g running (T/T_c)
Hadronization by cross section $q\bar{q} \rightarrow m_1 + m_2$	$q\bar{q} \rightarrow m$ (or "string"); $qqq \rightarrow b$ (or "string")

Initial energy distr.

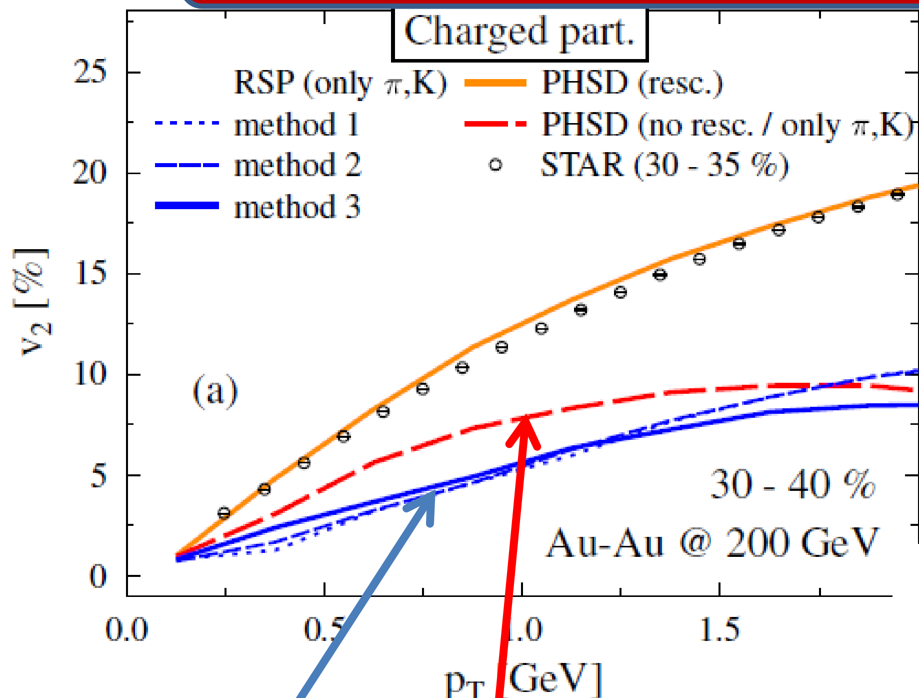




Expansion of a plasma with PHSD initial cond. I



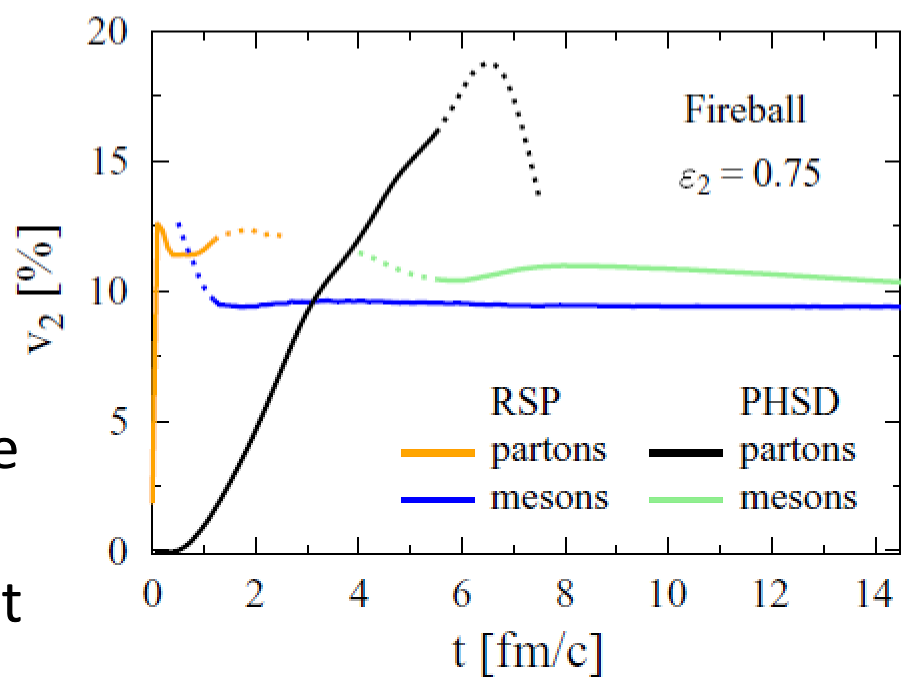
Expansion of a plasma with PHSD initial cond. II



Expanding almond shaped fireball as initial condition

NJL (RSP) has no hadronic rescattering

without rescattering
NJL (RSP) and PHSD have about the same v_2
Time evolution completely different



Summary of our long way

Starting point: NJL Lagrangian which shares the symmetries with QCD

Fierz transformation -> color less meson channel and qq channels

Bethe Salpeter equation in $q\bar{q}$ → mesons as pole masses
Bethe Salpeter equation in qq → diquarks as pole masses
(diquark-quark Bethe Salpeter equation → baryons as pole masses)
All masses described (10% precision) by 7 parameters fitted to ground state properties
(PNJL needs additional parameteres to fix the Polyakov loop)
Extension of all masses to finite T and μ without any new parameter
cross section (elastic and hadronisation) without any new parameter

Relativistic molecular dynamics approach based on constraints gives
time evolution equations of particles in a 6+1 dim. phase space

Studies of hadronization in realistic plasmas:
No sudden transition between quarks and hadrons
experimental results reasonably well reproduced (quite astonishing)
Almost all ready to study first order phase transition