

# The evolution of the net-proton kurtosis in the QCD critical region

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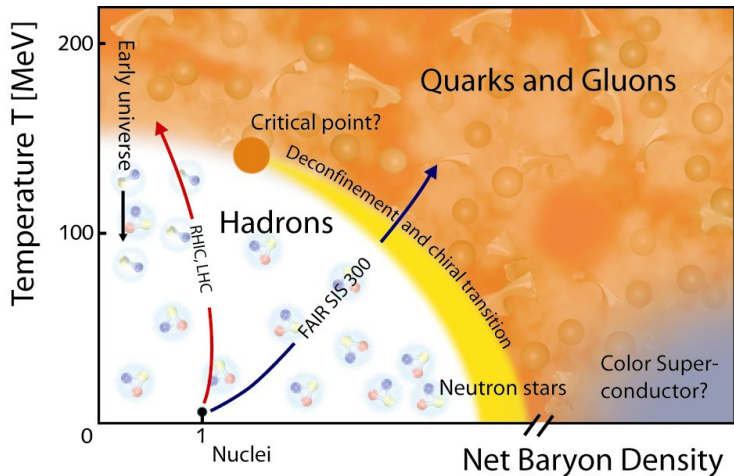
# SUT







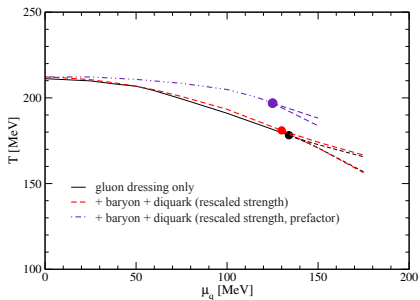
# The QCD phase diagram



# Finding the critical point - I

## 1. From the QCD Lagrangian

- Solve partition function  $\mathcal{Z}$  on a lattice (sign problem for finite  $\mu$ )
- Solve Dyson-Schwinger equations



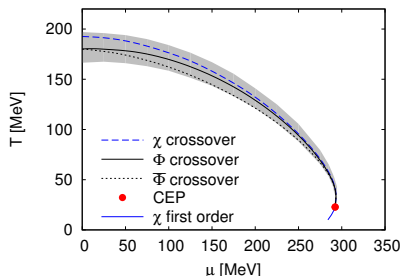
$$\begin{aligned} \text{---} \circ \text{---}^{-1} &= \text{---}^{-1} + \text{---} \circ \text{---}^{-1} \\ \text{---} \circ \text{---}^{-1} &= \text{---} \circ \text{---}^{-1} + \text{---} \circ \text{---}^{-1} \end{aligned}$$

(Eichmann, Fischer, Welzbacher, Phys. Rev. D **93** (2016))

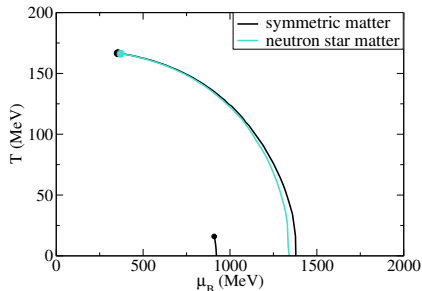
# Finding the critical point - II

## 2. From effective models

- Respect chiral symmetry (Sigma model, NJL model, ...)
- Existence/location of CP not universal!



(Herbst, Pawłowski, Schaefer, Phys. Lett. B **696** (2011) 58-67)



(Dexheimer, Schramm, Phys. Rev. C **81** (2010) 045201)

# Finding the critical point - III

## 3. From experiment

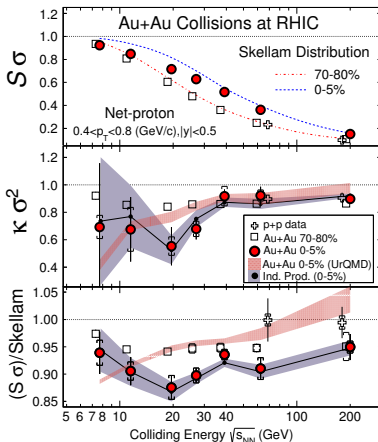
- Fluctuations sensitive to critical region

$$\sigma^2 = \langle \delta N^2 \rangle \sim \xi^2$$

$$S\sigma = \frac{\langle \delta N^3 \rangle}{\langle \delta N^2 \rangle} \sim \xi^{2.5}$$

$$\kappa\sigma^2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3\langle \delta N^2 \rangle \sim \xi^5$$

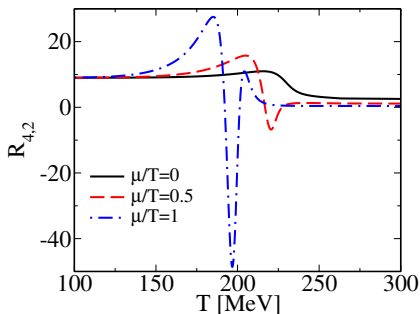
(Stephanov, Phys. Rev. Lett. **102** (2009))



(STAR collaboration, Phys. Rev. Lett. **112** (2014))



# From Susceptibilities to cumulants I - Baryon number



(Skokov, Friman, Redlich, Phys. Rev. C. **83** (2011))

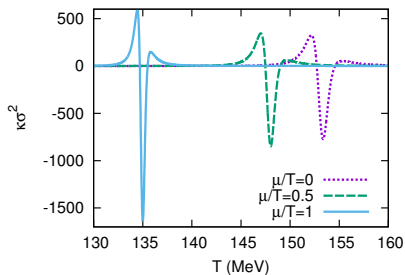
- Generalized quark number susceptibilities:

$$c_2 = \frac{\partial^2(p/T^4)}{\partial(\mu/T)^2} = \frac{1}{VT^3} \langle \delta N^2 \rangle$$

$$c_4 = \frac{\partial^4(p/T^4)}{\partial(\mu/T)^4} = \frac{1}{VT^3} \left[ \langle \delta N^4 \rangle - 3 \langle \delta N^2 \rangle^2 \right]$$

$$\kappa\sigma^2 = c_4/c_2 = \frac{\langle \delta N^4 \rangle}{\langle \delta N^2 \rangle} - 3 \langle \delta N^2 \rangle$$

# From Susceptibilities to cumulants II - Sigma field



- Generalized sigma susceptibilities ( $\tilde{\sigma} = \sigma/T$ ):

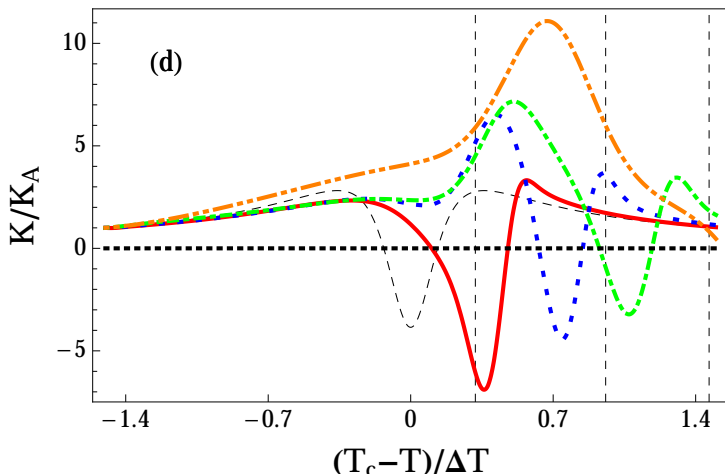
$$c_2 = \left( \frac{\delta^2 \Gamma}{\delta \tilde{\sigma}^2} \right)^{-1}$$

$$c_4 = -\frac{\delta^4 \Gamma}{\delta \tilde{\sigma}^4} \left( \frac{\delta^2 \Gamma}{\delta \tilde{\sigma}^2} \right)^{-1} + 3 \left( \frac{\delta^3 \Gamma}{\delta \tilde{\sigma}^3} \right)^2 \left( \frac{\delta^4 \Gamma}{\delta \tilde{\sigma}^4} \right)^{-5}$$

(CH, Nahrgang, *in preparation*)

$$\kappa\sigma^2 = c_4/c_2 = \frac{\langle \delta \tilde{\sigma}^4 \rangle}{\langle \delta \tilde{\sigma}^2 \rangle} - 3 \langle \delta \tilde{\sigma}^2 \rangle$$

# The kurtosis in heavy-ion collisions II



(Mukherjee, Venugopalan, Yin, Phys. Rev. C **92**, (2015))

# The $N_\chi$ FD model - I

## Ingredients for $N_\chi$ FD model

- Fluctuations (chiral fields)
- Fluid (quarks)

$$\frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi$$
$$\partial_\mu T_q^{\mu\nu} = S_\sigma^\nu$$

(Nahrgang, Leupold, CH, Bleicher, Phys. Rev. C 84 (2011))

- Potential  $\Omega$  and equation of state from effective QCD models
- Successfully describes: critical fluctuations, spinodal decomposition

# The $N_\chi$ FD model - II

Effective Lagrangian from quark-meson model  
(plus possible extensions, e.g. Polyakov loop  $\ell$ , dilaton field  $\chi$ )

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - g\sigma) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)$$
$$U(\sigma) = \frac{\lambda}{4}(\sigma^2 - \nu^2)^2 - h\sigma - U_0$$

One-loop thermodynamic potential reads

$$\Omega(T, \mu; \sigma) = U(\sigma) - d_q T \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 + e^{-\frac{E \pm \mu}{T}} \right)$$

Pressure, energy density are given by

$$p(T, \mu; \sigma) = -\Omega(T, \mu; \sigma)$$
$$e(T, \mu; \sigma) = Ts - p + \mu n$$

# The $N_\chi$ FD model - III

Two possible evolutions:

- Mean-field, local thermal equilibrium without fluctuations

$$\left. \frac{\partial \Omega(T, \mu; \sigma)}{\partial \sigma} \right|_{\sigma=\sigma_{\text{eq}}} = 0$$
$$\rho(T, \mu; \sigma) = -\Omega(T, \mu; \sigma), \quad \partial_\mu T^{\mu\nu} = 0$$

- Full nonequilibrium dynamics with **damping** and **stochastic fluctuations**

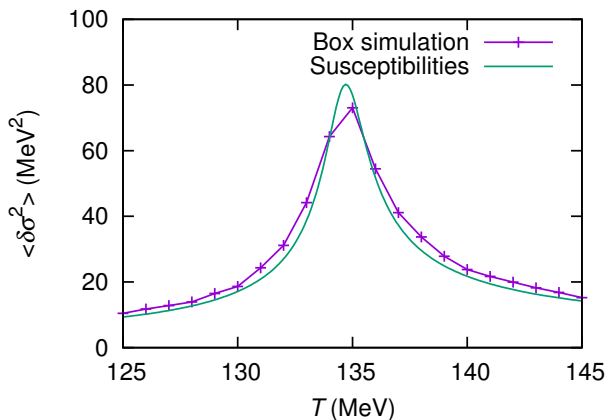
$$\frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi$$
$$\rho(T, \mu; \sigma) = -\Omega_{\bar{q}q}(T, \mu; \sigma), \quad \partial_\mu T^{\mu\nu} = S^\nu$$

In both cases quark densities are propagated via

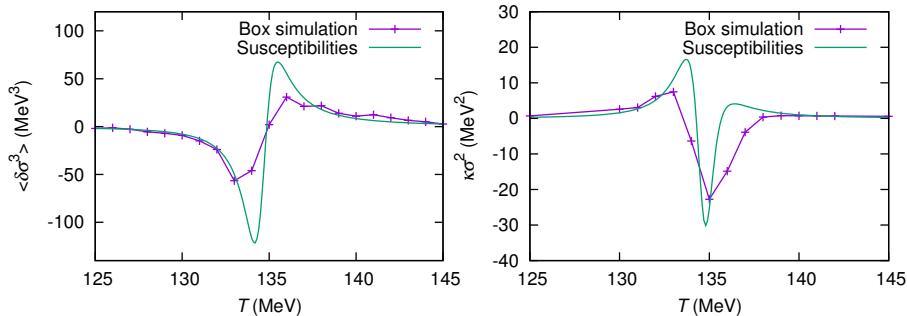
$$\partial_\mu n^\mu = 0$$

# Moments of dynamical fluctuations in a box - I

- Isothermal box with periodic boundary conditions
- Evolution of the sigma field with coarse-grained noise,  $\xi_{\text{corr}} = 1/m_\sigma$



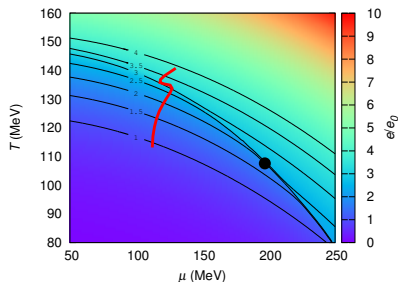
## Moments of dynamical fluctuations in a box - II



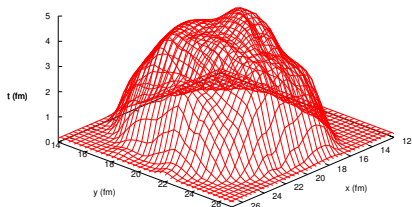
- Higher moments follow trend of susceptibilities
- Is the same true for fluctuations of  $n_q$ ? (work in progress)



# The kurtosis on different freeze-out surfaces

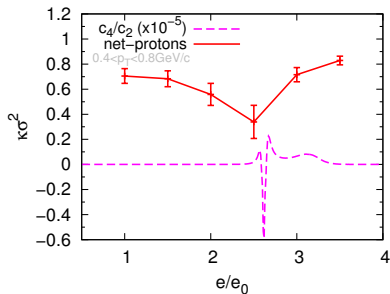
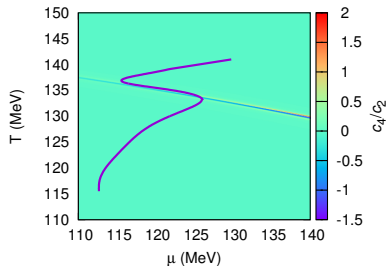


(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))



- Study crossover evolution left of CP
- Determine net-proton kurtosis on energy hypersurfaces
- Smooth hypersurfaces at crossover

# The kurtosis - net-proton vs. susceptibilities



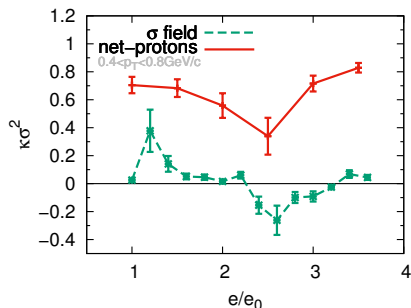
(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))

- Comparison of net-proton kurtosis to equilibrium fluctuations
- Characteristic dip imprints signal on net-proton kurtosis

# The kurtosis - net-proton vs. sigma

- Net-proton kurtosis follows kurtosis of sigma field

(CH, Nahrgang, Yan, Kobdaj, PRC 93 (2016))



- Net-proton number

$$\kappa\sigma^2 = \frac{\langle\delta N^4\rangle}{\langle\delta N^2\rangle} - 3\langle\delta N^2\rangle$$

- Sigma field

$$\kappa\sigma^2 = \frac{\langle\delta\sigma^4\rangle}{\langle\delta\sigma^2\rangle} - 3\langle\delta\sigma^2\rangle$$

# The kurtosis - net-proton dynamical vs. mean-field

- Net-proton kurtosis follows kurtosis of sigma field
- In contrast: Mean-field kurtosis remains flat
- In mean-field (hydro/eos): critical fluctuations do not build up

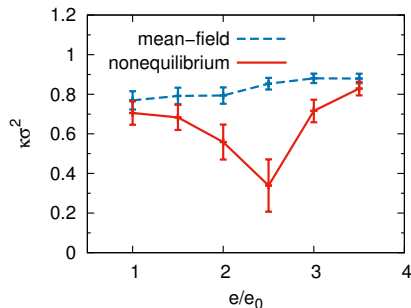
(CH, Nahrgang, Yan, Kobdaj, PRC **93** (2016))

- Mean-field

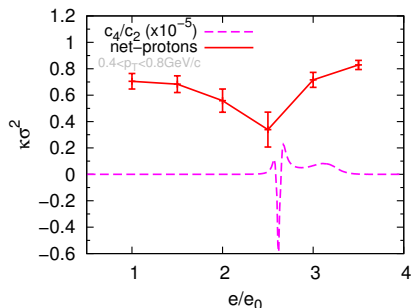
$$\left. \frac{\partial \Omega}{\partial \sigma} \right|_{\sigma=\langle \sigma \rangle} = 0$$

- Nonequilibrium

$$\frac{\partial^2 \sigma}{\partial t^2} - \nabla^2 \sigma + \eta \frac{\partial \sigma}{\partial t} + \frac{\delta \Omega}{\delta \sigma} = \xi$$



# Summary



- Modeling phase transitions in HICs with  **$N_{\chi}$ FD**
- Criticality visible in nonmonotonic net-proton kurtosis
- Hydro plus eos not sufficient

## Outlook

- Compare net-proton with net-baryon, acceptance range
- Study beam energy dependence of kurtosis
- Include baryonic degrees of freedom