Holography & quasinormal modes - tools for strongly coupled far from equilibrium dynamics

Bangtao Beach Resort & Spa, Phuket, NonEquilibrium Dynamics 2016

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My basic idea

Assume we have a hard problem that is difficult to solve in a given theory, for example the **standard model**

?

- gravity dual to QCD or standard model?
- not known yet

model

holography (gauge/gravity correspondence)

(Hard) problem in "similar" theory

Simple problem in a particular gravitational theory

➡ Holography is good at predictions that are qualitative or universal.

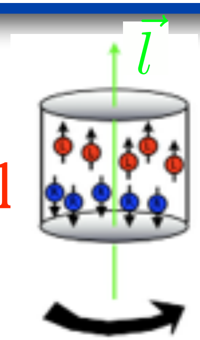
➡ Construct effective (field theory) description.

Example: Chiral hydrodynamics

Any relativistic quantum system with anomalous currents exhibits chiral transport effects at linear order in hydrodynamics.
[Son, Surowka; PRL (2009)]

time

Chiral vortical effect



[Erdmenger, Haack, MK, Yarom; JHEP (2009)]
holographic model (2008)

[Banerjee et al; JHEP (2011)]

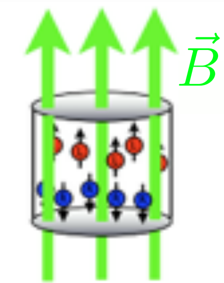
[Kharzeev, McLerran, Warringa; Nucl.Phys.A (2008)]

[Kharzeev, Zhitnitsky; Nucl.Phys.A (2007)]

[...]

[Vilenkin; ... (1979)]

Chiral magnetic effect



[...]

[Fukushima et al.; PRD (2008)]

[Kharzeev et al.; Nucl.Phys.A (2007)]

[Kharzeev; Phys.Lett.B (2006)]

[...]

Physical question for today

Which are the relevant quantities in systems far from equilibrium, and how can we predict their behavior?

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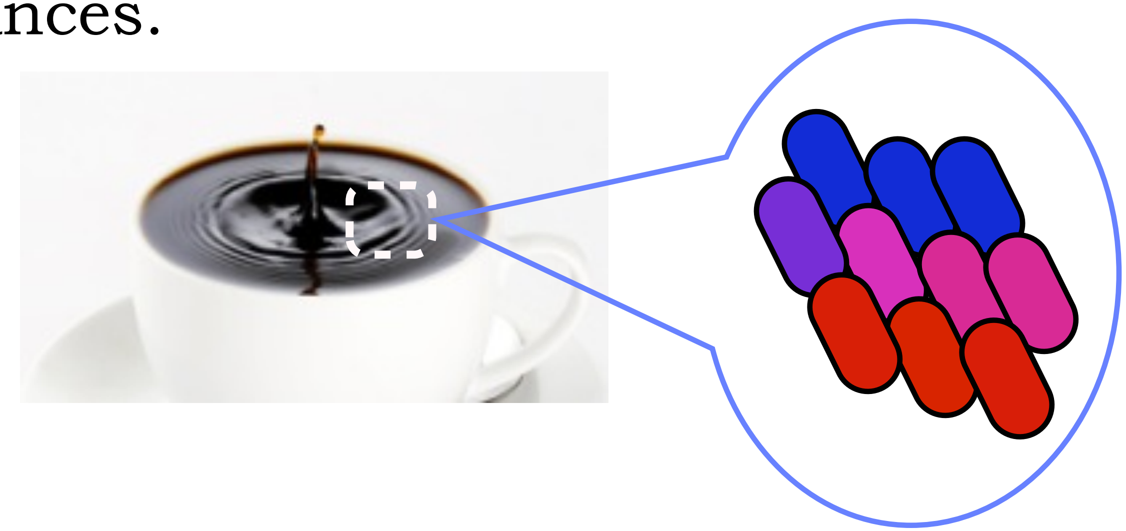
Which are the relevant quantities in systems far from equilibrium, and how can we predict their behavior?

Hydrodynamics -near equilibrium
is an effective description of systems at late times / large distances.

$$T(t,\vec{x}),\ \mu(t,\vec{x}),\ u^\nu(t,\vec{x})$$

hydrodynamic fields
-protected by symmetries,
hence they survive



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$$j^\mu = n u^\mu + \sigma E^\mu + \dots$$

conserved current is a good observable

(ideal) charge flow

conduc-tivity term

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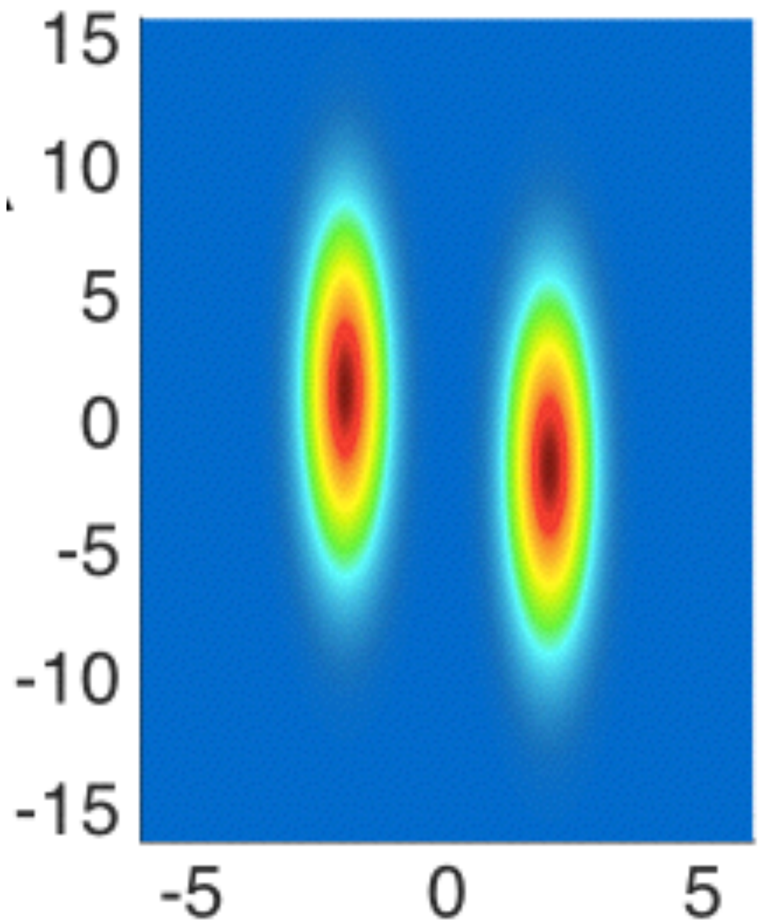
Contents

1. Near and far-from equilibrium - heavy ion collision

Example 1: Off-center collision



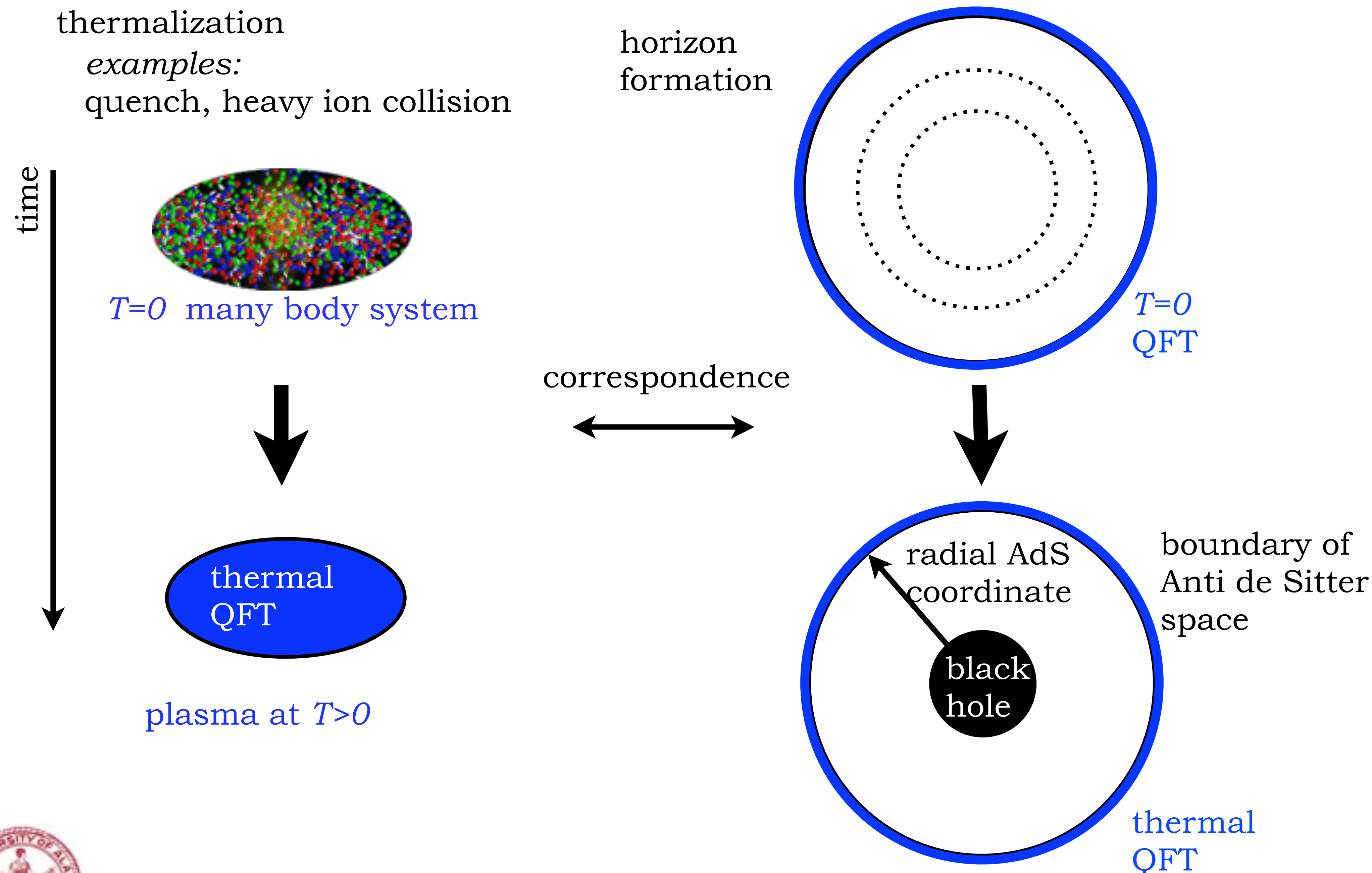
color coding:
energy density
red = high density
blue = low density

Gravitational dual

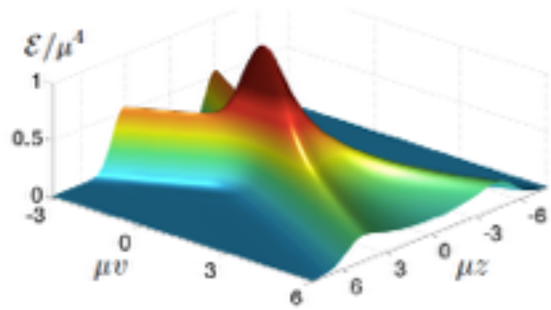
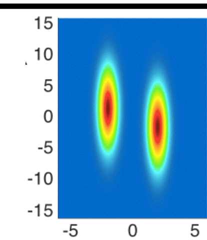
shock wave collision in
Anti de Sitter space

[Chesler, Yaffe; JHEP (2015)]

Holography far-from equilibrium



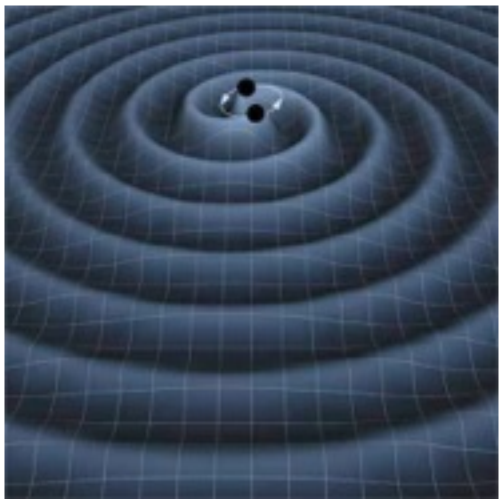
Holographic plasma thermalization results

Model	Equilibration time
Central collision of two energy lumps in $\mathcal{N}$=4 Super-Yang-Mills. *[Chesler, Yaffe; PRL (2011)]* 	~ 0.35 fm/c
Initial anisotropy in $\mathcal{N}$=4 Super-Yang-Mills, with charges / magnetic field. Confirmed by non-conformal study. *[Fuini, Yaffe; (2015)]* *[Buchel, Heller, Myers; (2015)]*	~ 0.35 fm/c largely unaffected by charges/magnetic field
Off-center collision of two energy lumps in $\mathcal{N}$=4 Super-Yang-Mills. *[Chesler, Yaffe; JHEP (2015)]* 	~ 0.25 fm/c
1/N corrections *[Schalm; conference talk]*	equilibration time increased

Equilibration happens very fast.
Hydrodynamics works way before that!

Gravitational waves in AdS

Gravitational waves are similar to waves in a pond:



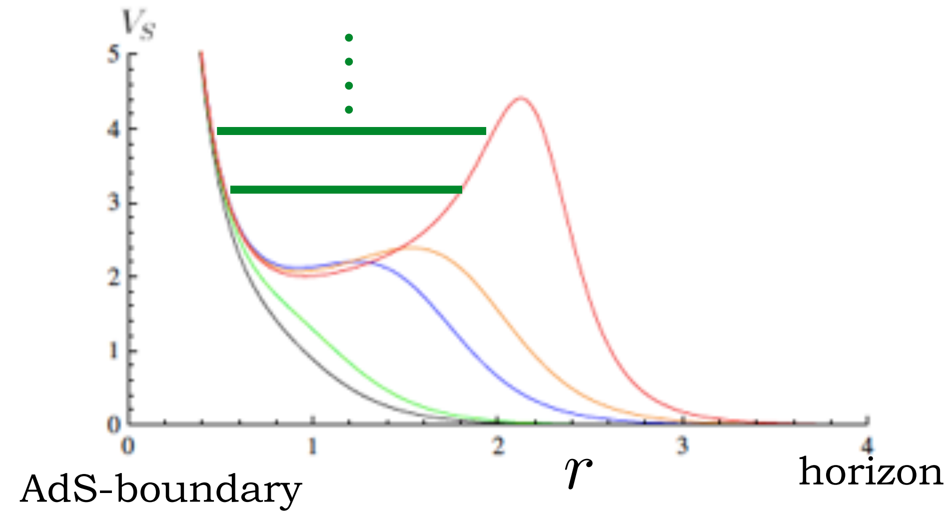
waves on spacetime
solutions to linearized Einstein equation

waves on water
solutions to wave equation

Heuristically: What are quasi-normal modes?

- heuristically: the eigenmodes of black holes or black branes

$$H\,\phi = -\partial_r{}^2\phi + V_S\phi = E\phi$$



- formal definition: (metric) fluctuations that are in-falling at horizon and vanishing at AdS-boundary

- correspond to poles of correlators in dual field theory

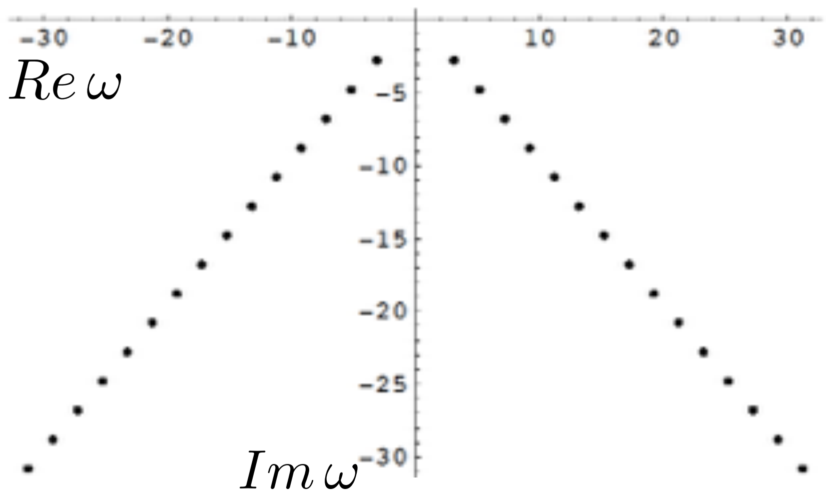
[Kovtun, Starinets; JHEP 2005]

- *example*: tensor fluctuations (known from shear viscosity bound)
  QNMs of $\phi := h_x{}^y$ are poles of $\langle T_{xy}\, T_{xy}\rangle$

[Policastro, Son, Starinets; JHEP 2002]

[Kovtun, Son, Starinets; 2004]

2. Quasi-normal modes (QNMs)



What do quasi-normal modes mean?

$$h_{xy}(t) \propto e^{-i\omega t} h_{xy}(\omega)$$

$$e^{-i\omega t} = e^{-i(\mathrm{Re}\omega)\,t}\, e^{(\mathrm{Im}\omega)\,t}$$

resonance frequency (mass of the associated quasiparticle)

damping (decay width of the quasiparticle)

What do quasi-normal modes mean?

black hole

QNMs of $\phi := {h_x}^y$ are poles of $\langle T_{xy} T_{xy} \rangle$

Fourier transformation of gravity field:

$$h_{xy}(t) \propto e^{-i\omega t} h_{xy}(\omega)$$

Resonance and decay are encoded in QNM frequency:

$$e^{-i\omega t} = e^{-i(\mathrm{Re}\omega)\, t} e^{(\mathrm{Im}\omega)\, t}$$

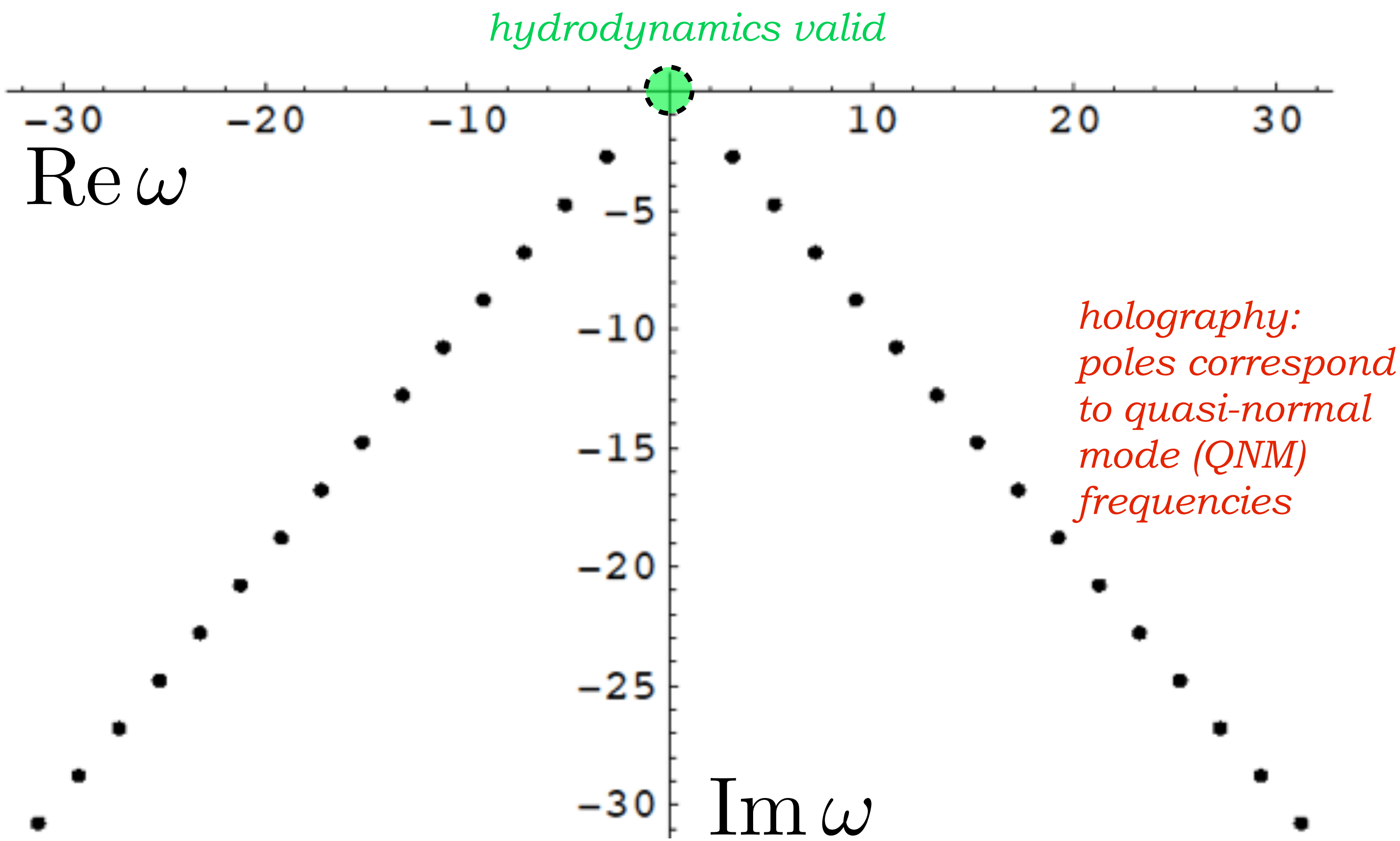
resonance frequency (mass of the associated quasiparticle)

damping (decay width of the quasiparticle)

Far beyond hydrodynamics : QNMs

Example: 3+1-dimensional N=4 Super-Yang-Mills theory; poles of

$$\langle T_{xy} T_{xy} \rangle(\omega, k) = G^R_{xy,xy}(\omega, k) = -i \int d^4x \; e^{-i\omega t + ikz} \langle [T_{xy}(z), T_{xy}(0)] \rangle$$



[Starinets; JHEP (2002)]

Method: how to compute QNMs

- start with any **gravitational background** (metric, matter content)

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t}\phi(\omega)$)

- impose **boundary conditions** that are
  in-falling at horizon:

and
  vanishing at AdS-boundary:

Method: how to compute QNMs

- start with any **gravitational background** (metric, matter content)
  Example: (charged) Reissner-Nordstrom black brane in 5-dim AdS

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2 \qquad f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

$$A_t = \mu - \frac{Q}{L r^2}$$

- choose one or more **fields to fluctuate**
  (obeying linearized Einstein equations; Fourier transformed $\phi(t) \propto e^{-i\omega t}\phi(\omega)$)

  Example: metric tensor fluctuation

$$\phi := h_x{}^y \qquad 0 = \phi'' - \frac{f(u) - u\, f'(u)}{u\, f(u)}\phi' + \frac{\omega^2 - f(u)k^2}{4 r_H^2 u\, f(u)^2}\phi$$

$$u = \left(\frac{r_H}{r}\right)^2$$

- impose **boundary conditions** that are
  in-falling at horizon:

$$\phi = (1-u)^{\pm\frac{i\tilde{\omega}}{2(2-\tilde{q}^2)}}\left[\phi^{(0)} + \phi^{(1)}(1-u) + \phi^{(2)}(1-u)^2 + \dots\right]$$

and
  vanishing at AdS-boundary: $\lim_{r\to r_{bdy}} \phi(r) = 0$

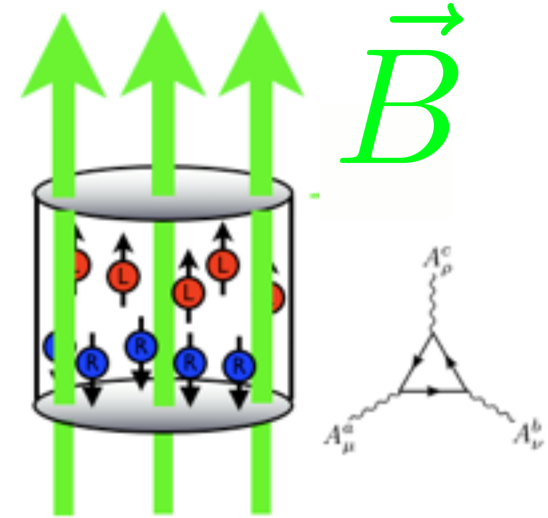
Physical question:
What is the equilibrium state of a theory with chiral current + external magnetic field ?

Hydro of theory with chiral current + external field

Constitutive relations:

$$J^\mu = J^\mu(T, \mu, u^\alpha; g_{\alpha\beta}, A_\alpha)$$

$$T^{\mu\nu} = T^{\mu\nu}(T, \mu, u^\alpha; g_{\alpha\beta}, A_\alpha)$$



Conservation equations:

$$\partial_\mu J^\mu = C\, F \wedge F$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$

Choose external fields:

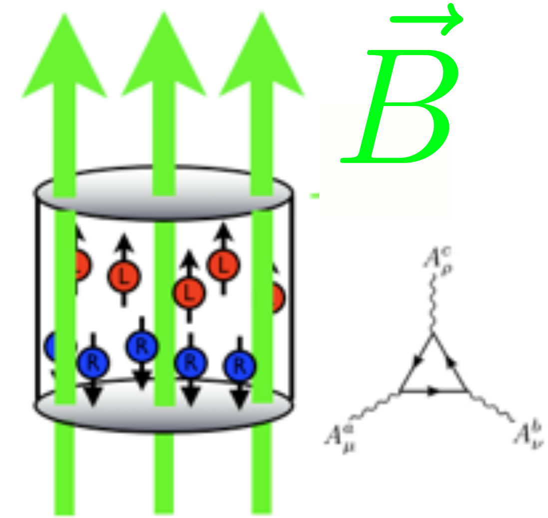
$$A_\alpha = \delta_{t,\alpha}\mu(z) + \frac{B}{2}(x\delta_{y,\alpha} - y\delta_{x,\alpha})$$

$$g_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Equilibrium with chiral anomaly + magnetic field

Trick:*

$$\partial_\mu J^\mu = C\, F \wedge F$$



$$\sim C\, \epsilon^{\alpha\beta\gamma\delta}\, F_{\alpha\beta} F_{\gamma\delta}$$

$$\partial_z J^z \sim \tilde{C}\, \epsilon^{ztxy}\, (\partial_z A_t(z))\, B$$

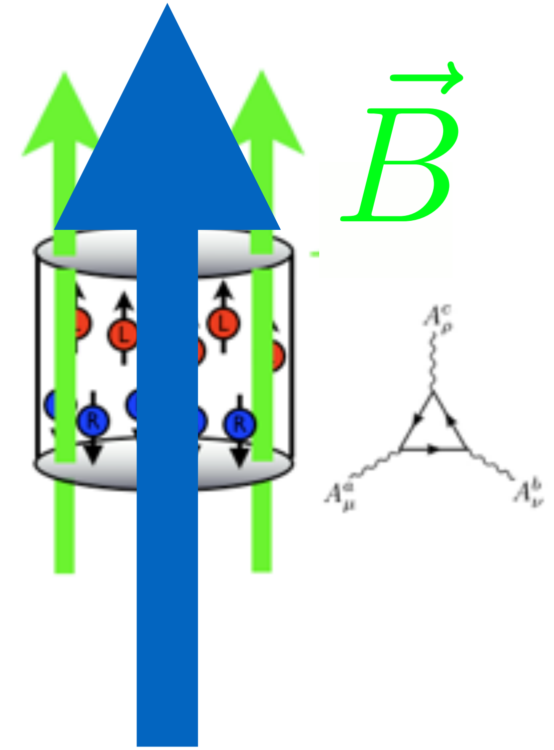
$$\cancel{ik_z} J^z \sim \tilde{C} \qquad (\cancel{ik_z} A_t(z))\, B$$

make gauge field constant in z,
and call it `chemical potential' $\Rightarrow$ $J^z \sim \tilde{C} \mu B$

"vacuum" charge current

Thanks to Martin Ammon.

Equilibrium with chiral anomaly + magnetic field



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"vacuum" charge current

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu \Rightarrow$$

$$T^{zt} \sim \tilde{C} \mu^2 B$$

"vacuum" heat current

**Thanks to Martin Ammon.*

Equilibrium with chiral anomaly + magnetic field

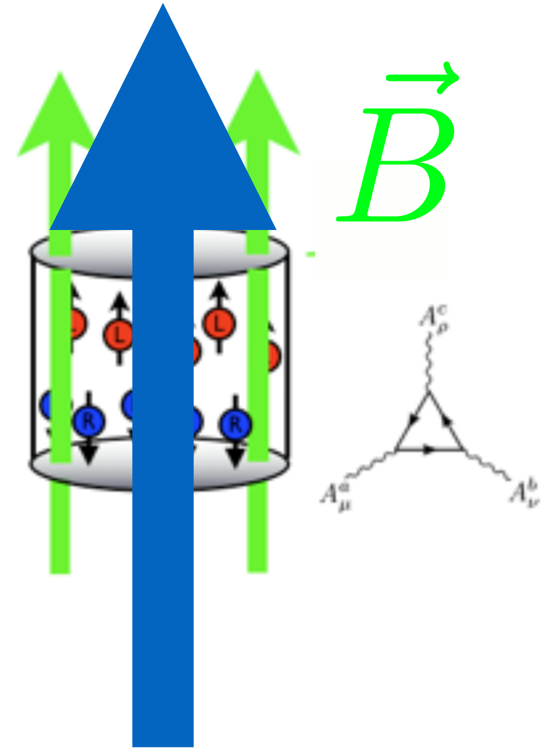
Trick:* $$\partial_\mu J^\mu = C\, F \wedge F$$

make gauge field constant in z, and call it `chemical potential'

$$= C\, \epsilon^{\alpha\beta\gamma\delta}\, F_{\alpha\beta} F_{\gamma\delta}$$

$$\partial_z J^z = \tilde{C}\, \epsilon^{ztxy}\, (\partial_z \mu)\, B$$

$$= 0$$



$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} J_\nu \;\Rightarrow$$

$$J^z = 0$$

$$T^{zt} = 0$$

same state variables, but different result !?

Discrepancy - What now?

$$J^z = 0$$

$$T^{zt} = 0$$

same state variables, but different result !?

versus

$$J^z \sim \tilde{C}\mu B$$

"vacuum" charge current

$$T^{zt} \sim \tilde{C}\mu^2 B$$

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TEST: Perform a calculation in a holographic model

$$S_{gravity} = \left[\int d^5x\sqrt{-g}(R - 2\Lambda) - \frac{1}{4}F^2) + \frac{C}{6}\int A \wedge F \wedge F\right]$$

Discrepancy - What now?

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TEST: Perform a calculation in a holographic model

$$S_{gravity} = \left[\int d^5x\sqrt{-g}(R-2\Lambda) - \frac{1}{4}F^2) + \frac{C}{6}\int A\wedge F\wedge F\right]$$

holographic result

$J^z = C\mu B$

$T^{zt} = \frac{1}{2}C\mu^2 B$

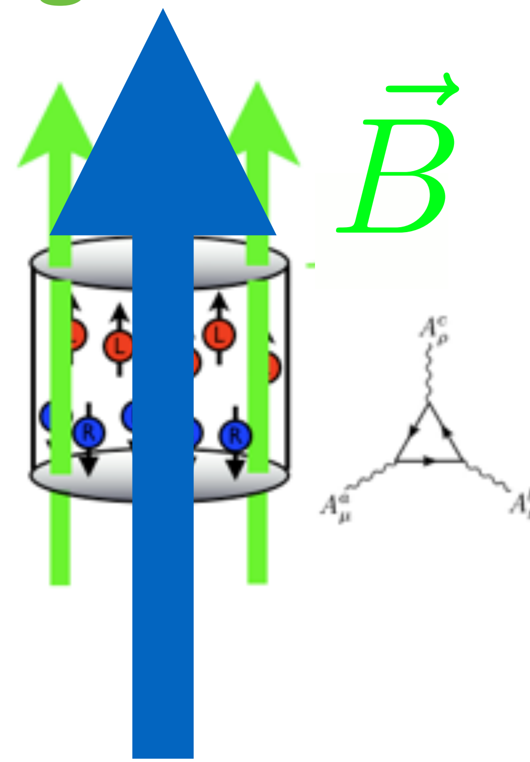
Equilibrium with chiral anomaly + magnetic field

$$J^z \sim \tilde{C}\mu B$$

"vacuum" charge current

$$T^{zt} \sim \tilde{C}\mu^2 B$$

"vacuum" heat current

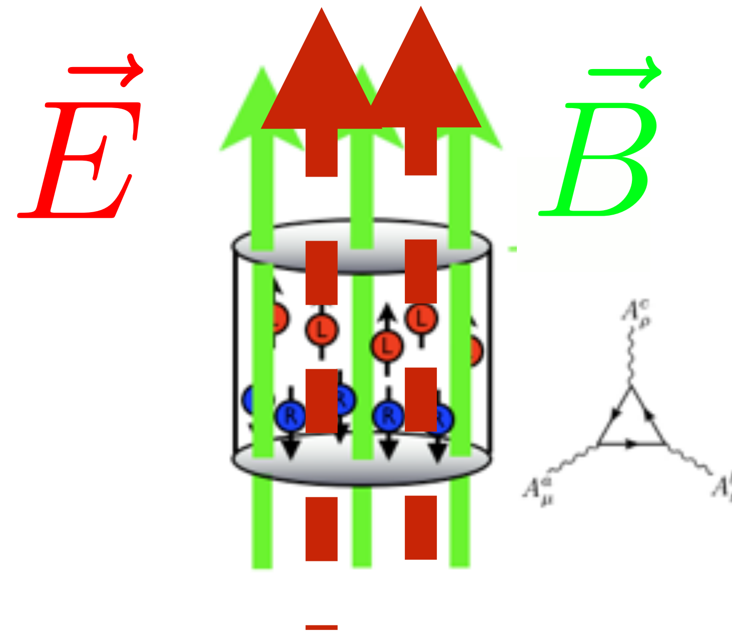


If we have a chiral anomaly and an external magnetic field, then there are two currents in equilibrium:

(i) The heat current

(ii) The (axial) charge current

So what if we put this theory in an external electric *and* external magnetic field ?



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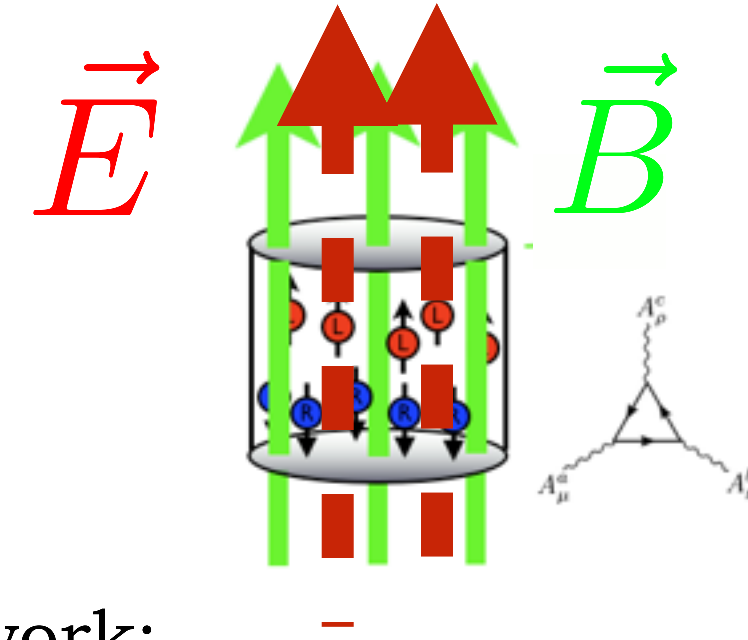

Follow Pavel Kovtun's systematic work:

Thermodynamics of polarized relativistic matter [Kovtun; JHEP (2016)]

$$T^{\mu\nu} = Pg^{\mu\nu} + (Ts + \mu\rho)u^\mu u^\nu + T^{\mu\nu}_{\text{EM}}$$

$$T^{\mu\nu}_{\text{EM}} = M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta} F^{\beta\nu} - F_{\alpha\beta} M^{\beta\nu} \right)$$

[Israel; Gen Relat Gravit (1978)]

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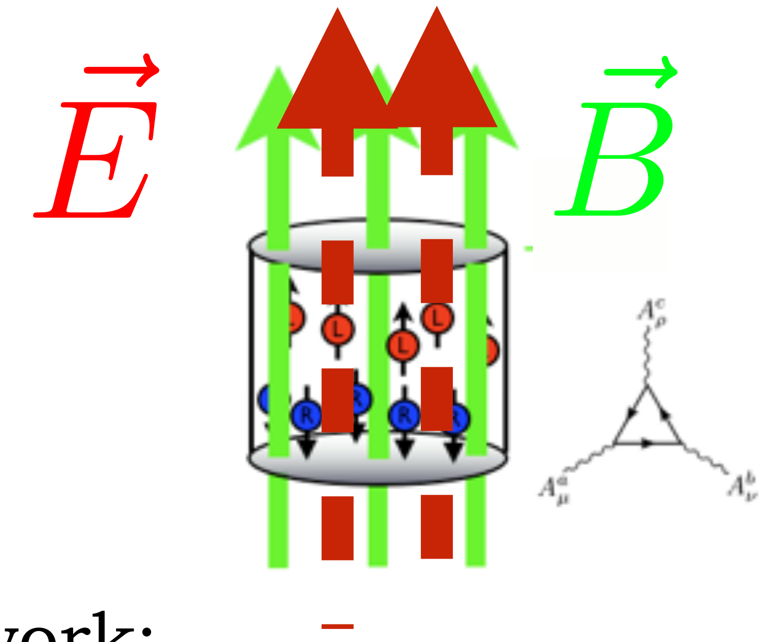

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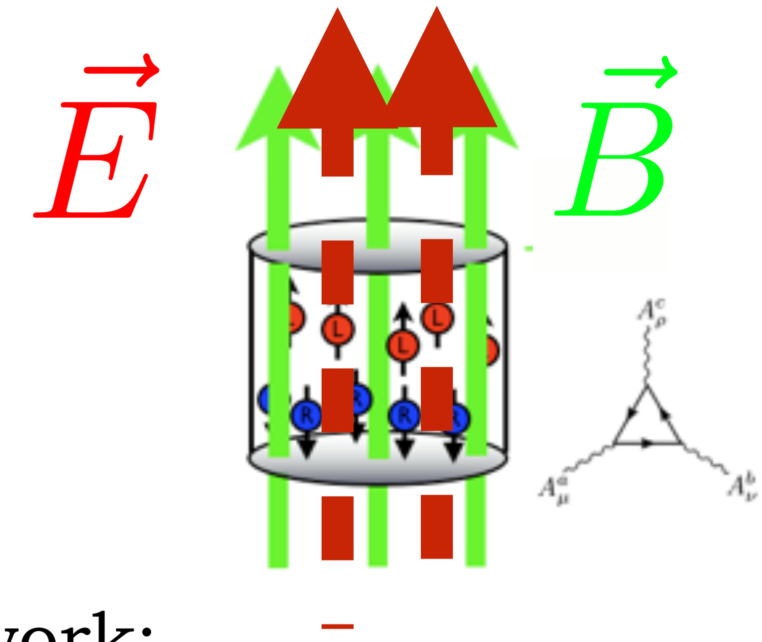
$$T^{\mu\nu}_{\rm EM} = M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta}F^{\beta\nu} - F_{\alpha\beta}M^{\beta\nu} \right)$$

[Israel; Gen Relat Gravit (1978)]

$$\begin{aligned} p^\lambda = {} & \chi_{\rm EE} E^\lambda + \chi_{\rm EB} B^\lambda + \chi_{\rm E\Omega} \Omega^\lambda + \chi_{\rm ES} S^\lambda \\ & + \gamma_1 \nabla^\lambda T + \gamma_2 \nabla^\lambda B^2 + \gamma_3 \nabla^\lambda E^2 + \gamma_4 \nabla^\lambda (E{\cdot}B) \\ & + \gamma_5 X^{\lambda\alpha}\partial_\alpha T + \gamma_6 X^{\lambda\alpha}\partial_\alpha B^2 + \gamma_7 X^{\lambda\alpha}\partial_\alpha E^2 + \gamma_8 X^{\lambda\alpha}\partial_\alpha (E{\cdot}B) + \gamma_9 X^{\lambda\alpha}\Omega_\alpha \\ & + \gamma_{10} \Delta^\lambda_\rho S^\alpha \nabla_\alpha B^\rho + \gamma_{11} \Delta^\lambda_\rho S^\alpha \nabla^\rho B_\alpha + \gamma_{12} \Delta^\lambda_\rho B^\alpha \nabla_\alpha B^\rho \\ & + \gamma_{13} X^{\lambda\rho} B^\sigma \nabla_\rho E_\sigma + \gamma_{14} X^{\lambda\rho} E^\sigma \nabla_\sigma B_\rho \,, \end{aligned}$$

$$\begin{aligned} m^\lambda = {} & \chi_{\rm BB} B^\lambda + \chi_{\rm BE} E^\lambda + \chi_{\rm B\Omega} \Omega^\lambda + \chi_{\rm BS} S^\lambda \\ & + \delta_1 \nabla^\lambda T + \delta_2 \nabla^\lambda B^2 + \delta_3 \nabla^\lambda E^2 + \delta_4 \nabla^\lambda (E{\cdot}B) \\ & + \delta_5 Y^{\lambda\alpha}\partial_\alpha T + \delta_6 Y^{\lambda\alpha}\partial_\alpha B^2 + \delta_7 Y^{\lambda\alpha}\partial_\alpha E^2 + \delta_8 Y^{\lambda\alpha}\partial_\alpha (E{\cdot}B) + \delta_9 Y^{\lambda\alpha}\Omega_\alpha \\ & + \delta_{10} \Delta^\lambda_\rho B^\mu \nabla^\rho E_\mu + \delta_{11} \Delta^\lambda_\rho B^\mu \nabla_\mu E^\rho + \delta_{12} Y^{\lambda\rho} B^\sigma \nabla_\rho E_\sigma + \delta_{13} Y^{\lambda\rho} E^\sigma \nabla_\sigma B_\rho \\ & + \delta_{14} \Delta^\lambda_\rho S^\mu \nabla_\mu E^\rho + \delta_{15} \Delta^\lambda_\rho E^\mu \nabla_\mu S^\rho + \delta_{16} \epsilon^{\lambda\mu\rho\sigma} u_\mu \nabla_\rho B_\sigma \\ & + \delta_{17} X^{\lambda\alpha}\partial_\alpha T + \delta_{18} X^{\lambda\alpha}\partial_\alpha B^2 + \delta_{19} X^{\lambda\alpha}\partial_\alpha E^2 + \delta_{20} X^{\lambda\alpha}\partial_\alpha (E{\cdot}B) \,, \end{aligned}$$

So what if we put this theory in an external electric *and* external magnetic field ?



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$$T^{\alpha\beta} = \begin{pmatrix} \epsilon & 0 & 0 & \frac{C}{2}\mu^2 B \\ 0 & P - \chi_{BB}\frac{\partial P}{\partial B} & 0 & 0 \\ 0 & 0 & P - \chi_{BB}\frac{\partial P}{\partial B} & 0 \\ \frac{C}{2}\mu^2 B & 0 & 0 & P \end{pmatrix}$$

$$J^{\alpha} = \begin{pmatrix} n \\ 0 \\ 0 \\ C\mu B \end{pmatrix}$$

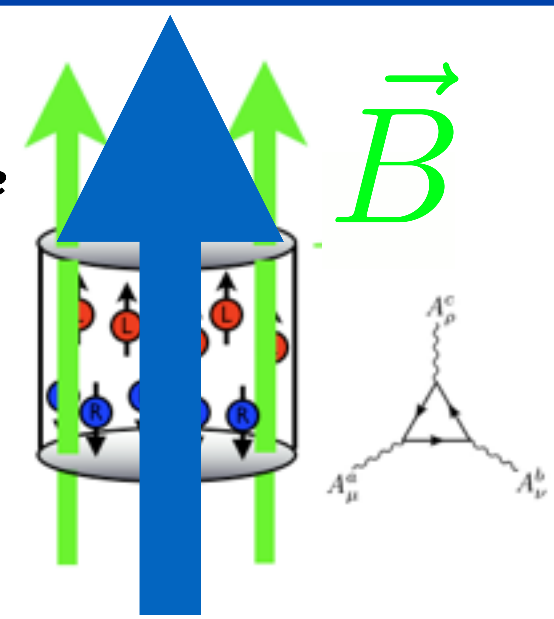
thermodynamics to
zeroth order in derivatives
with only magnetic field
(agrees with holographic model)

3. Results: QNMs of charged magnetic branes

holographic dual ...

$$S_{gravity} = \left[\int d^5x \sqrt{-g}(R - 2\Lambda) - \frac{1}{4}F^2) + \frac{C}{6} \int A \wedge F \wedge F \right]$$

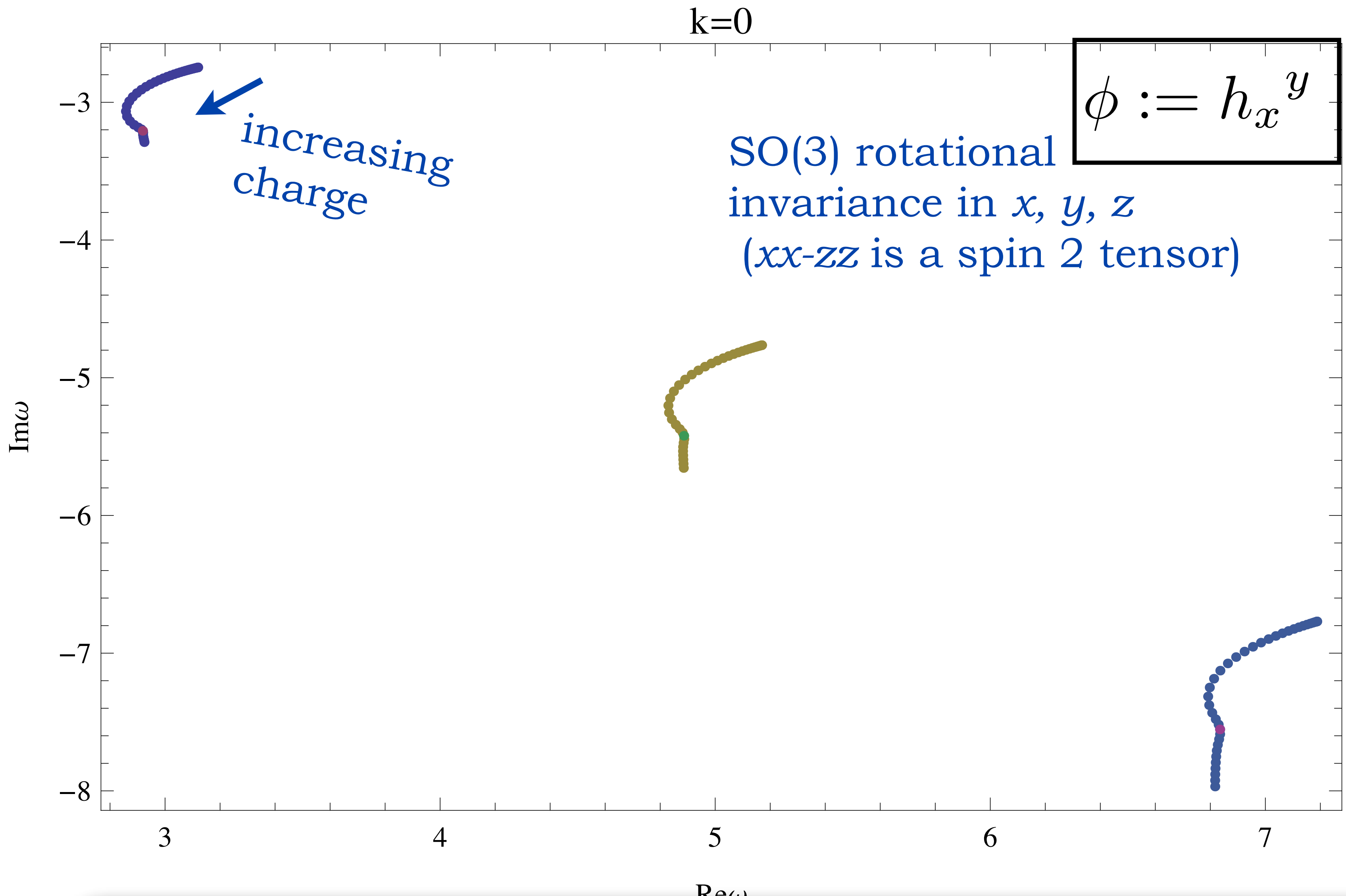
... of a particular charged magnetic plasma in presence of a chiral anomaly



Result: tensor QNMs of RN black brane

Equilibrium solution *[Janiszewski, Kaminski; PRD (2015)]*

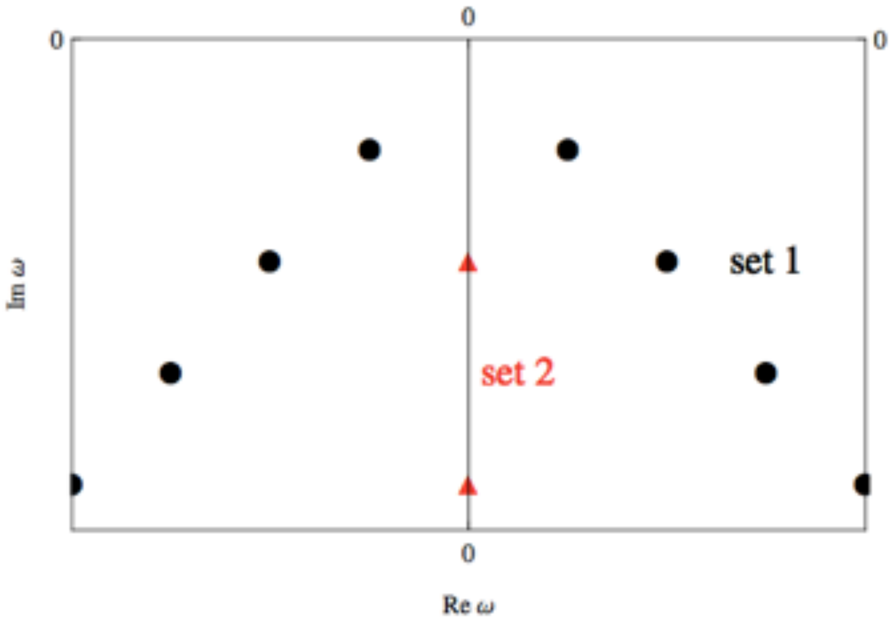
Reissner-Nordstrom (charged) black branes in 5-dim AdS



Less stable resonances at larger charges. Equilibration happens faster.
Agreement with far from equilibrium setup at late times, deviation <1%

Result: Imaginary QNMs

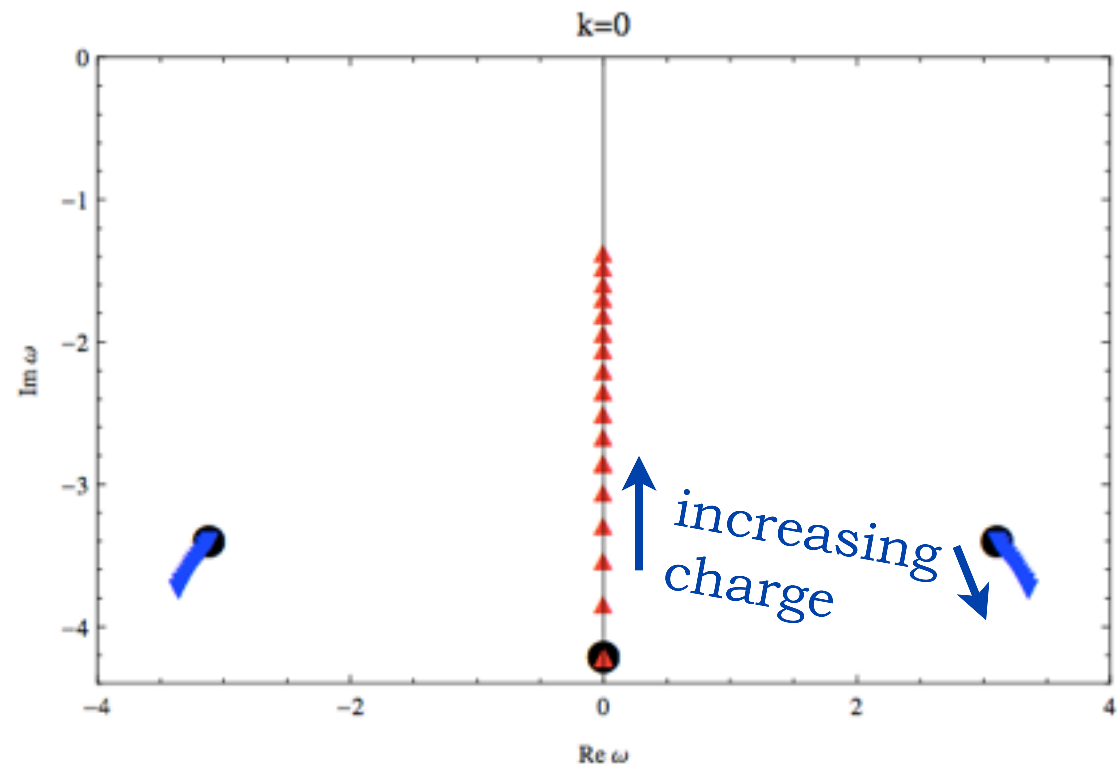
[Janiszewski, Kaminski; PRD (2015)]



$$\phi := {h_x}^y$$

two sets of QNMs

imaginary QNMs dominate late-time behavior at large charge densities



Result: tensor QNMs of magnetic black brane

Equilibrium solution

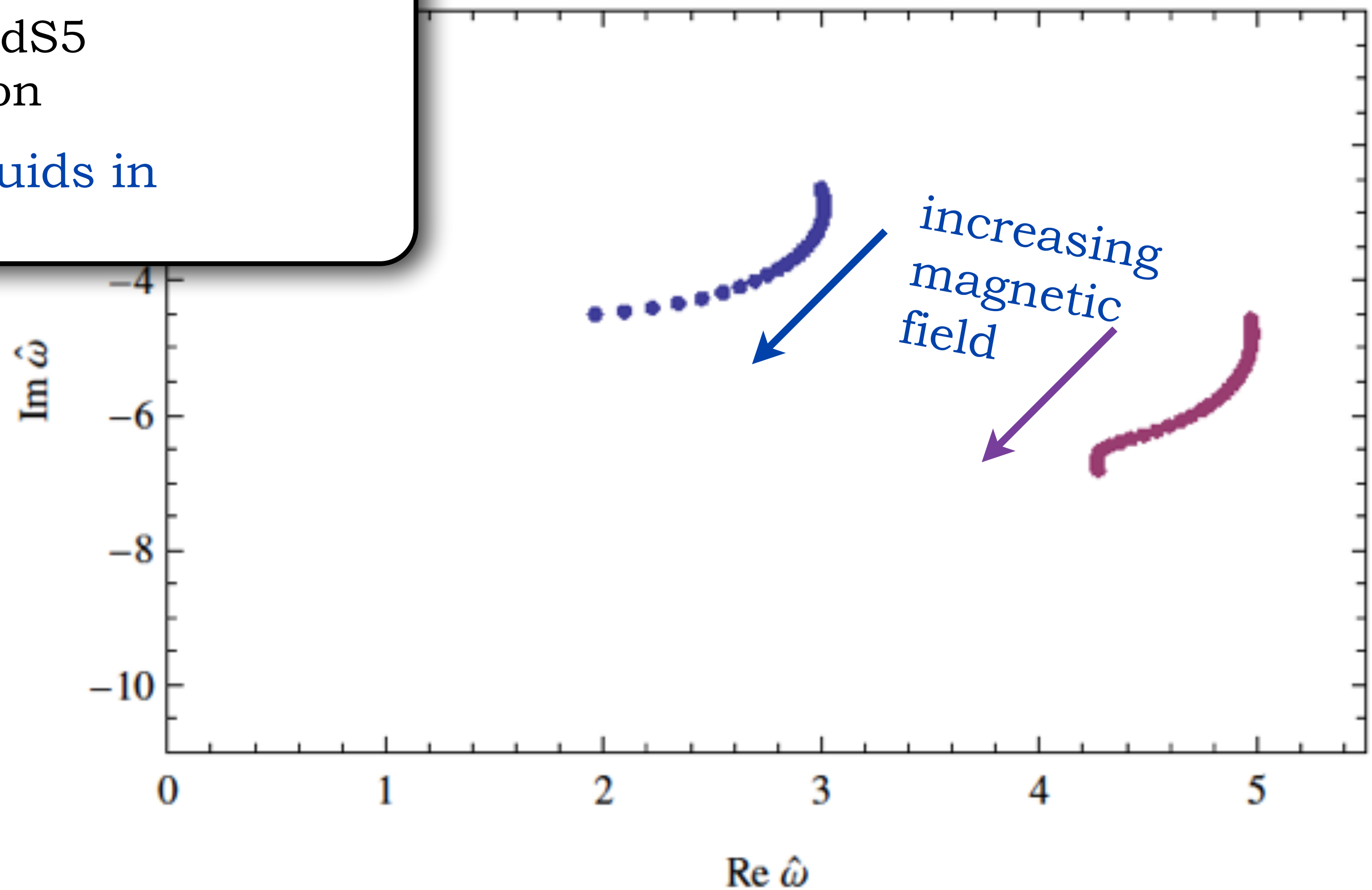
Magnetic black branes
[D'Hoker, Kraus; JHEP (2009)]

- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

Final state for fluids in magnetic field.

Quasinormal modes
[Janiszewski, Kaminski; PRD (2015)]

$$\phi := h_x{}^y$$



Result: scalar QNMs of magnetic black brane

Equilibrium solution

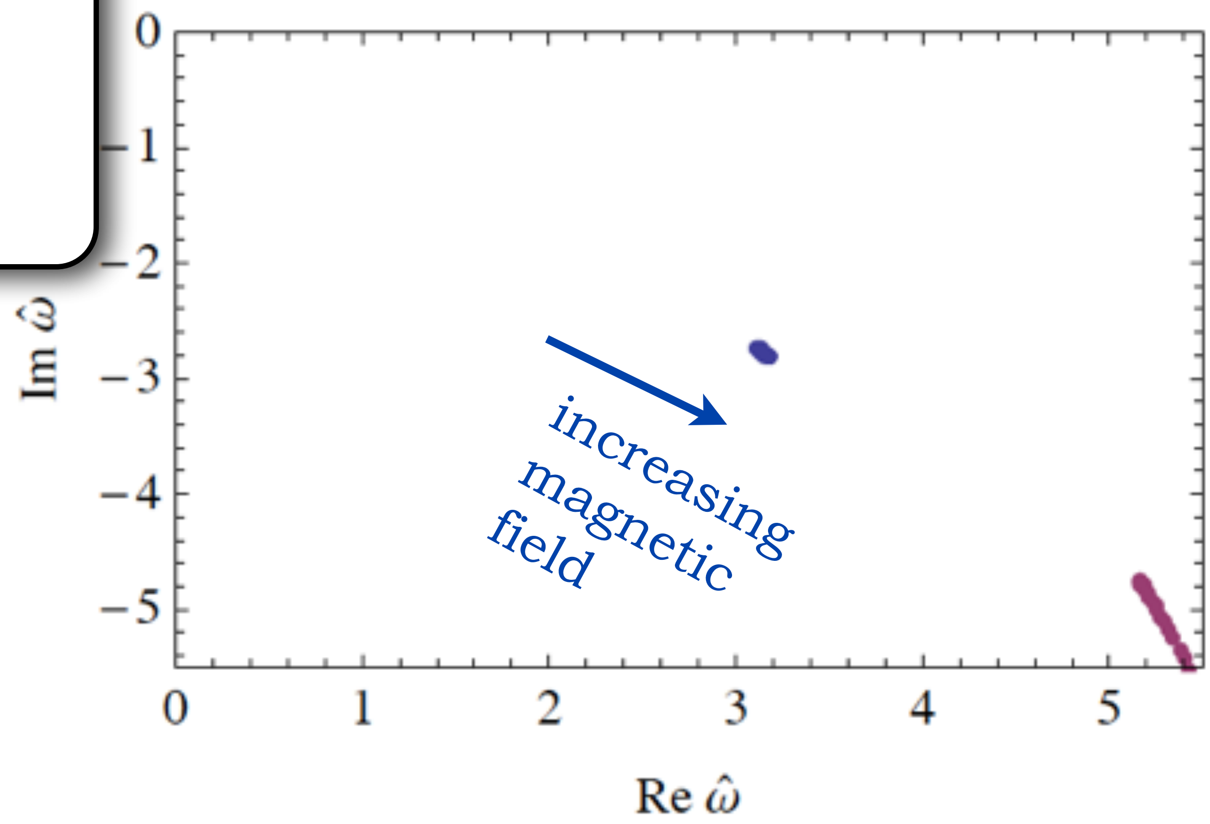
Magnetic black branes
[D'Hoker, Kraus; JHEP (2009)]

- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

Final state for fluids in magnetic field.

Quasinormal modes
[Janiszewski, Kaminski; PRD(2015)]

$$\phi = h_x{}^z$$



Agreement with far from equilibrium setup at late times: ~10%
cf. [Fuini, Yaffe; (2015)]

Hydrodynamic result: dispersion relations changed

[Ammon, Leiber, Kaminski, Koirala, Wu; in progress]

holographic calculation and hydrodynamic calculation agree that dispersion relations are changed for the hydrodynamic poles by

- chiral anomaly coefficient
- external magnetic field

[Ammon, Leiber, Macedo JHEP (2016)]

preliminary results

spin 2 *no hydrodynamic poles*

spin 1
$$\omega = \frac{1}{\epsilon + P - \chi_{BB} B^2}\left[k\,\delta v(B,c) \pm n - i\,k^2\,\eta - i\,\sigma\,B^2\right]$$

"chiral magnetic sound velocity" — *"charge gap"* — *momentum diffusion* — *"magnetic gap"*

spin 0
$$\omega_\pm = \pm v_\pm(B,C)\,k - i\,\Gamma_\pm(B,C)\,k^2$$

modified sound velocity — *modified sound attenuation*

$$\omega_0 = \delta c_0(B,C) + \delta v_0(B,C)\,k - i\,\Gamma_0(B,C)\,k^2$$

? — **?** — *modified charge diffusion*

My question to all participants: What can we do with this holographic model and with this hydrodynamics (with chiral anomaly and external fields**)?**

Summary

- translate a QFT problem into a (classical) gravity problem

- fast thermalization
- fast hydrolization

- quasinormal modes
  - “lowest QNM” describes holographic thermalization early on
  - QNM” changes with charge and magnetic field
  - hydrodynamic QNMs match hydro prediction
  - QNMs go far beyond hydrodynamic regime
- dispersion relations for hydro poles changed

- Outlook: construct effective description far from equilibrium (chiral transport in HIC)

APPENDIX

Magnetic black brane thermodynamics

[D'Hoker, Kraus; JHEP (2009)]

Magnetic black branes

- magnetic analog of RN black brane
- Asymptotically AdS5
- AdS5 near horizon

breaks rotational invariance
from SO(3) in *x*, *y*, *z*
to SO(2) in *x*, *y* plane
(*xx*-*zz* is a scalar now!)

Ansatz

$$\begin{aligned} ds^2 &= -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2V(r)}(dx^2 + dy^2) + e^{2W(r)}dz^2\,, \\ F &= b\,dx \wedge dy\,. \end{aligned}$$

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$$F = b\,dx \wedge dy\,.$$

$$\begin{aligned}
0 &= 2b^2 + 4e^{4V(u)}\big(u^3\tilde{U}'(u)(2V'(u) + W'(u)) + u^2\tilde{U}(u)(2(u(2V''(u)\\
&\quad + W''(u) + W'(u)^2) + V'(u)(2uW'(u) + 3) + 3uV'(u)^2) + 3W'(u)) - 3\big)\,,\\
0 &= 2u^2e^{4V(u)}(2u\tilde{U}''(u) + \tilde{U}'(u)(4u(V'(u) + W'(u)) + 3) + \tilde{U}(u)(4u(V''(u)\\
&\quad + W''(u) + W'(u)^2) + V'(u)(4uW'(u) + 6) + 4uV'(u)^2 + 6W'(u))) - 2(b^2 + 6e^{4V(u)})\,,\\
0 &= b^2e^{-4V(u)} + u^2(2(u\tilde{U}''(u)\\
&\quad + \tilde{U}(u)(4uV''(u) + 6V'(u)(uV'(u) + 1))) + \tilde{U}'(u)(8uV'(u) + 3)) - 6\,,\\
0 &= b^2e^{-4V(u)} + 2u^3(\tilde{U}'(u)(2V'(u) + W'(u)) + 2\tilde{U}(u)V'(u)(V'(u) + 2W'(u))) - 6\,. \qquad (24)
\end{aligned}$$

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[D'Hoker, Kraus; JHEP (2009)]

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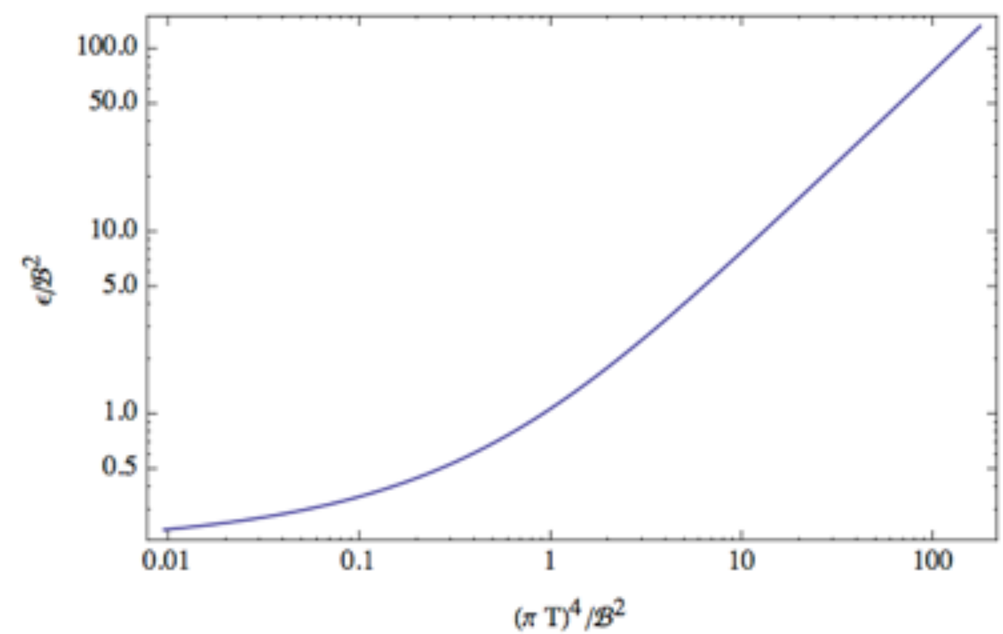
$$F = b dx \wedge dy \,.$$

Thermodynamics

$$\epsilon \equiv \langle T_{00} \rangle = \frac{2}{\kappa}\left[-\frac{3}{4}u^B_{(4)} + \frac{\mathcal{B}^2}{4}\log\mathcal{B}\right] \,,$$
$$P_1 = P_2 \equiv \langle T_{11} \rangle = \frac{2}{\kappa}\left[-\frac{1}{4}u^B_{(4)} + \frac{v^B_{(4)}}{v} - \frac{\mathcal{B}^2}{4} + \frac{\mathcal{B}^2}{4}\log\mathcal{B}\right] \,,$$
$$P_3 = \langle T_{33} \rangle = \frac{2}{\kappa}\left[-\frac{1}{4}u^B_{(4)} - 2\frac{v^B_{(4)}}{v} - \frac{\mathcal{B}^2}{4}\log\mathcal{B}\right] \,.$$



Holography concepts

Gauge/Gravity concepts

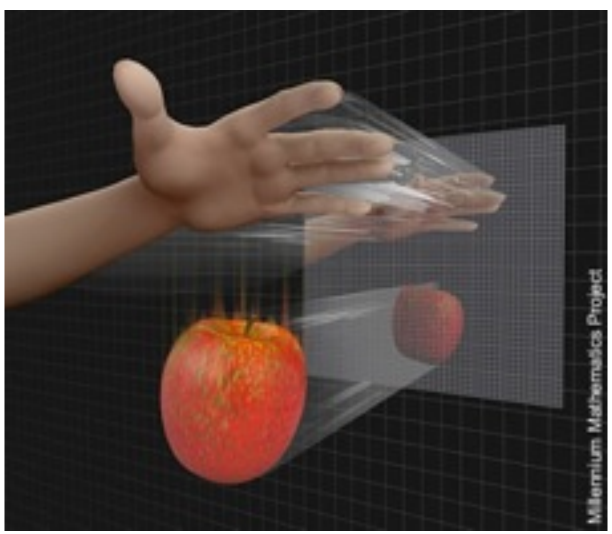
The Gauge/Gravity correspondence is based on the holographic principle. *['t Hooft (1993)]*

$$S_{max}(\text{volume}) \propto \text{surface area}$$

String theory gives one example (AdS/CFT).

N=4 Super-Yang-Mills in 3+1 dimensions (CFT)	$\Longleftrightarrow$	Typ II B Supergravity in (4+1)-dimensional Anti de Sitter space (AdS)

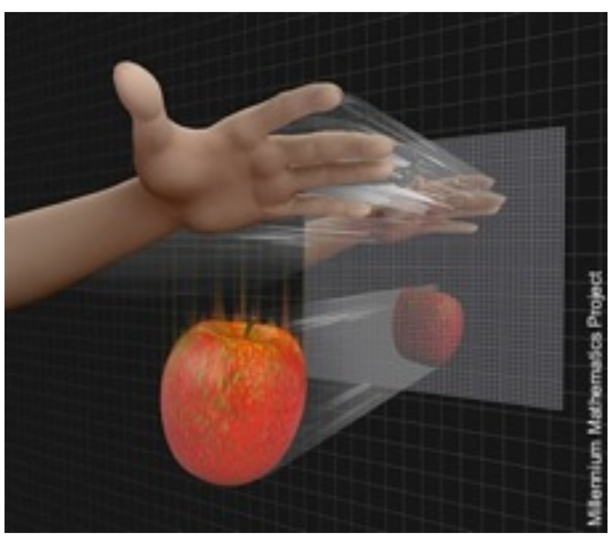
[Susskind (1995)]
[Maldacena (1997)]

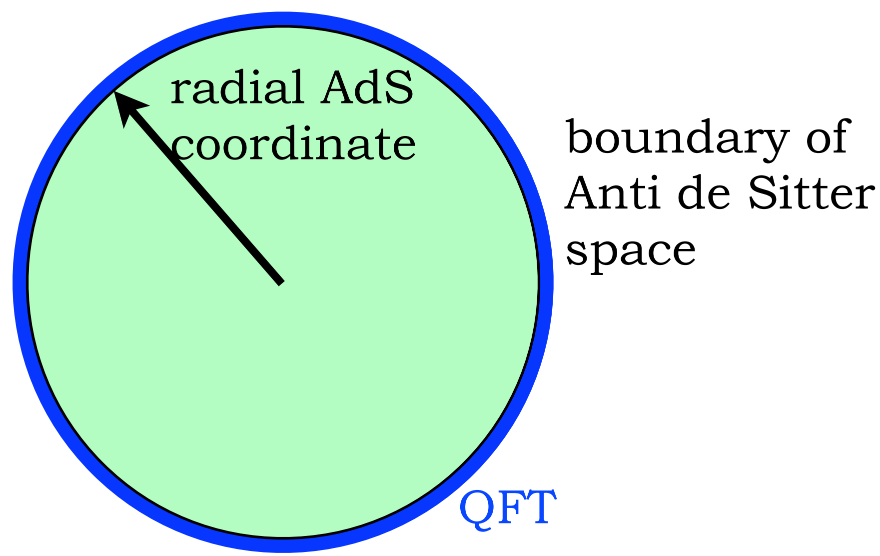


Equilibrium states

strongly coupled quantum field theory (QFT) ⟷ *correspondence* ⟷ weakly curved gravitational theory

renormalization scale ⟷ radial AdS coordinate





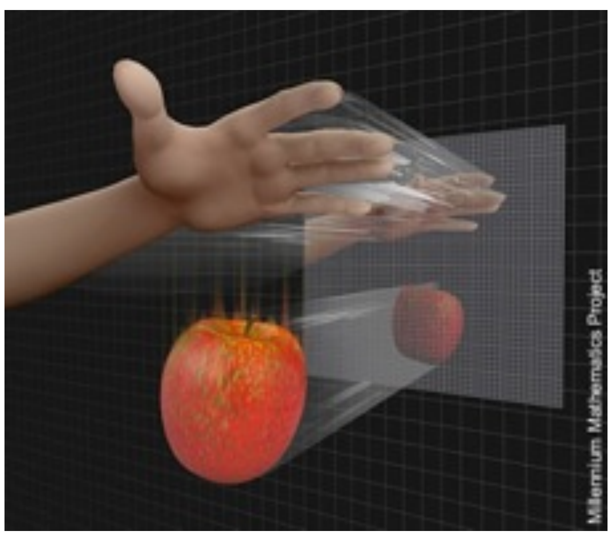
Equilibrium states

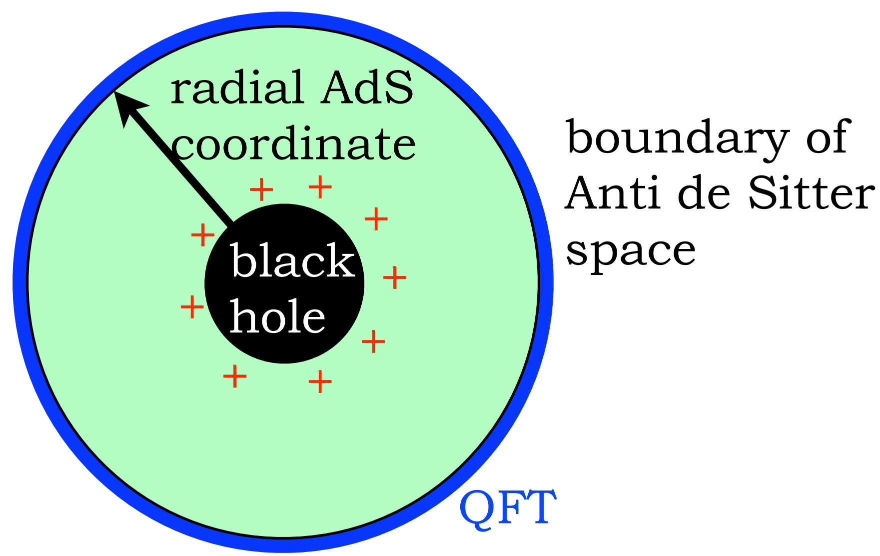
strongly coupled quantum field theory (QFT) ⟷ *correspondence* ⟷ weakly curved gravitational theory

renormalization scale ⟷ radial AdS coordinate

QFT temperature ⟷ Hawking temperature

conserved charge ⟷ charged black hole/brane





Example: Reissner-Nordström black brane

$\mathcal{N}$=4 Super-Yang-Mills theory at nonzero temperature & charge

correspondence ⟷

metric & gauge field defining a RN black brane (solve Einstein-Maxwell eq's)

metric: $ds^2 = \frac{r^2}{L^2}\left(-f\,dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2$

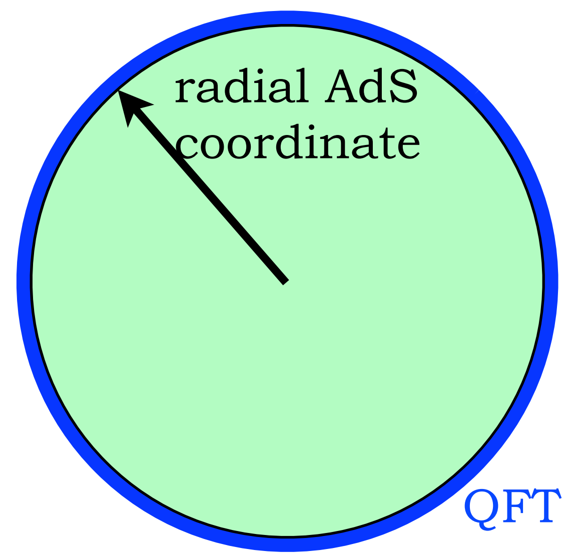
$$T = r_H^2 \frac{|f'(r_H)|}{4\pi}$$

$$f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

gauge field:

$$A_t = \mu - \frac{Q}{L r^2}$$

$$\mu = \frac{\sqrt{3}q}{2 r_H^2}$$



Example: Reissner-Nordström black brane

$\mathcal{N}$=4 Super-Yang-Mills theory at nonzero temperature & charge

correspondence ⟷

metric & gauge field defining a RN black brane (solve Einstein-Maxwell eq's)

QFT temperature: ⟷ metric: $ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f} dr^2$

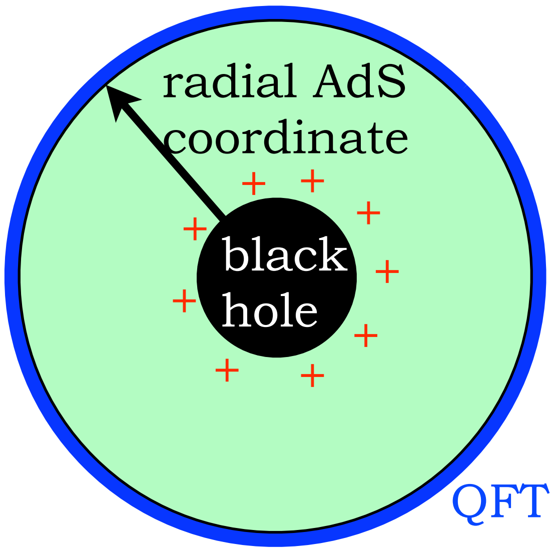
$$T = r_H^2 \frac{|f'(r_H)|}{4\pi}$$

$$f(r) = 1 - \frac{mL^2}{r^4} + \frac{q^2 L^2}{r^6}$$

conserved charge Q, thermodynamically dual to chemical potential: ⟷ gauge field:

$$A_t = \mu - \frac{Q}{Lr^2}$$

$$\mu = \frac{\sqrt{3}q}{2r_H^2}$$



Example: metric fluctuations