Anomalous transport model study of chiral magnetic and vortical effects in HIC

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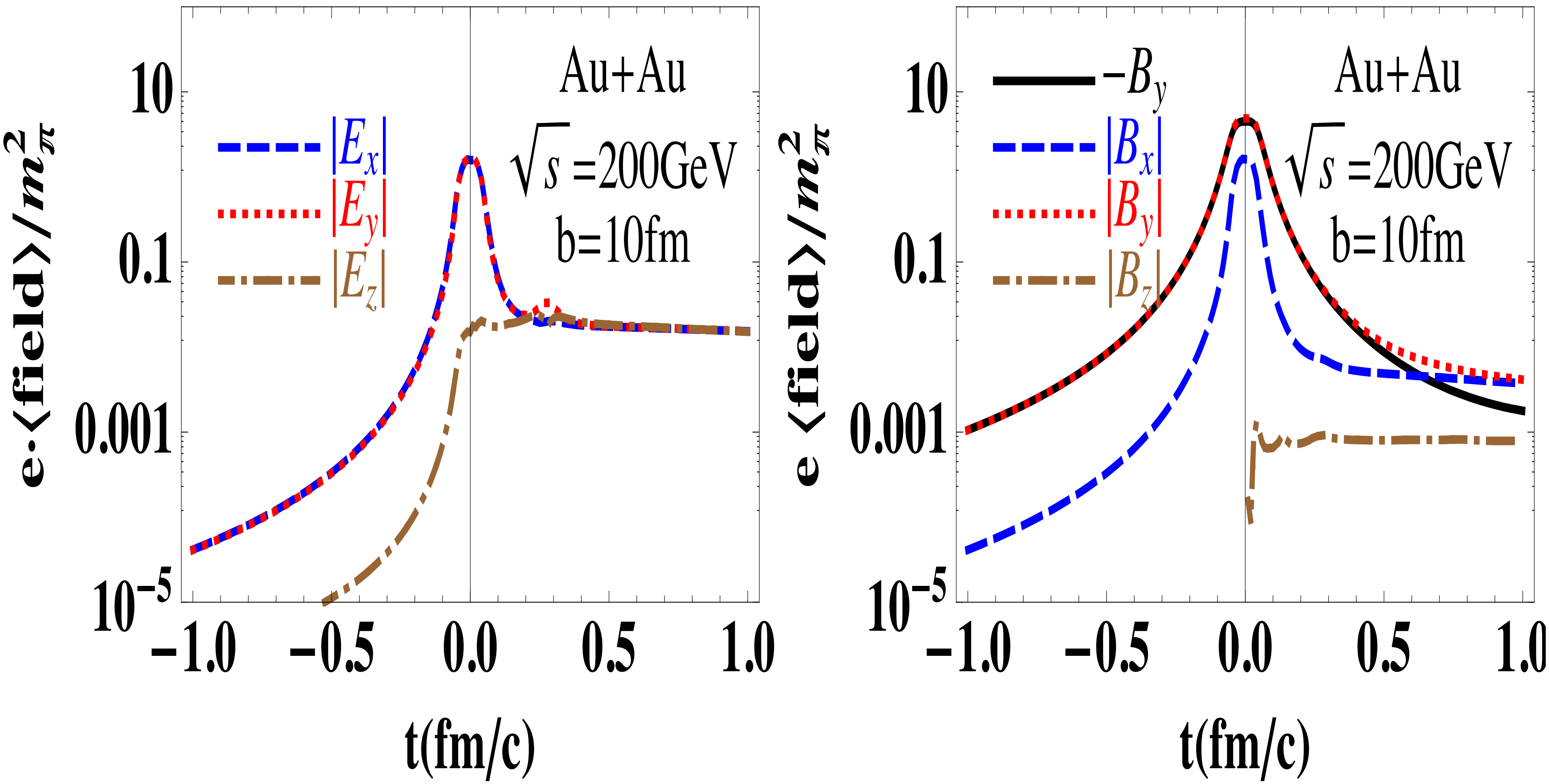
- Introduction
  - Electromagnetic field in HIC
  - Anomalous chiral effects
- Anomalous kinetic equation
- Chirality changing quark-antiquark scattering
- Chiral magnetic effect in HIC
- Chiral vortical effect in HIC
- Summary

Based on work [PRC 94, 045204 (2016)] in collaboration with Yifeng Sun and Feng Li

Supported by US Department of Energy and the Welch Foundation

Electromagnetic field in relativistic HIC

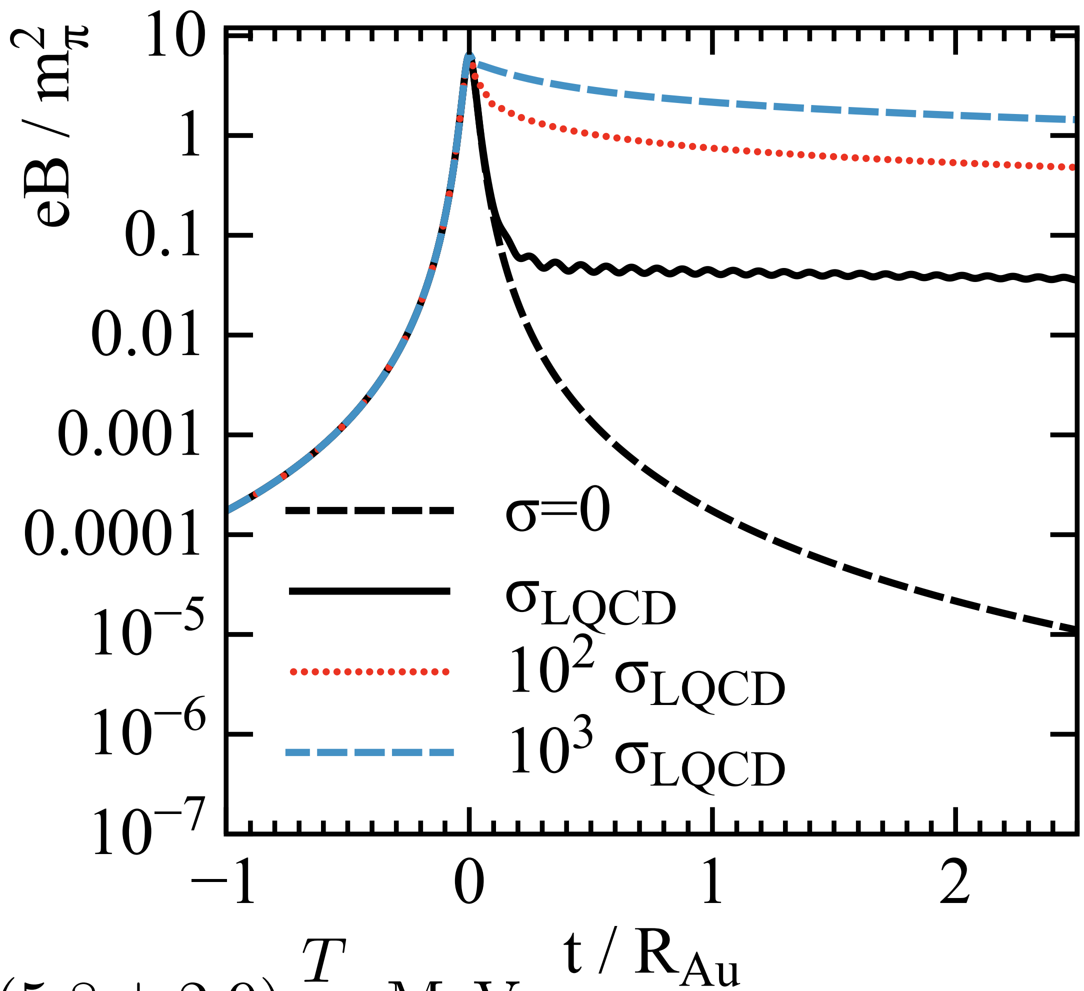
W. T. Deng and X. G. Huang, PRC 85, 044907 (2012)



- Based on HIJING. Similar results from AMPT

Effect of QGP conductivity on magnetic field

L. Mclerran & V. Skokov, NPA 929, 184 (2014)

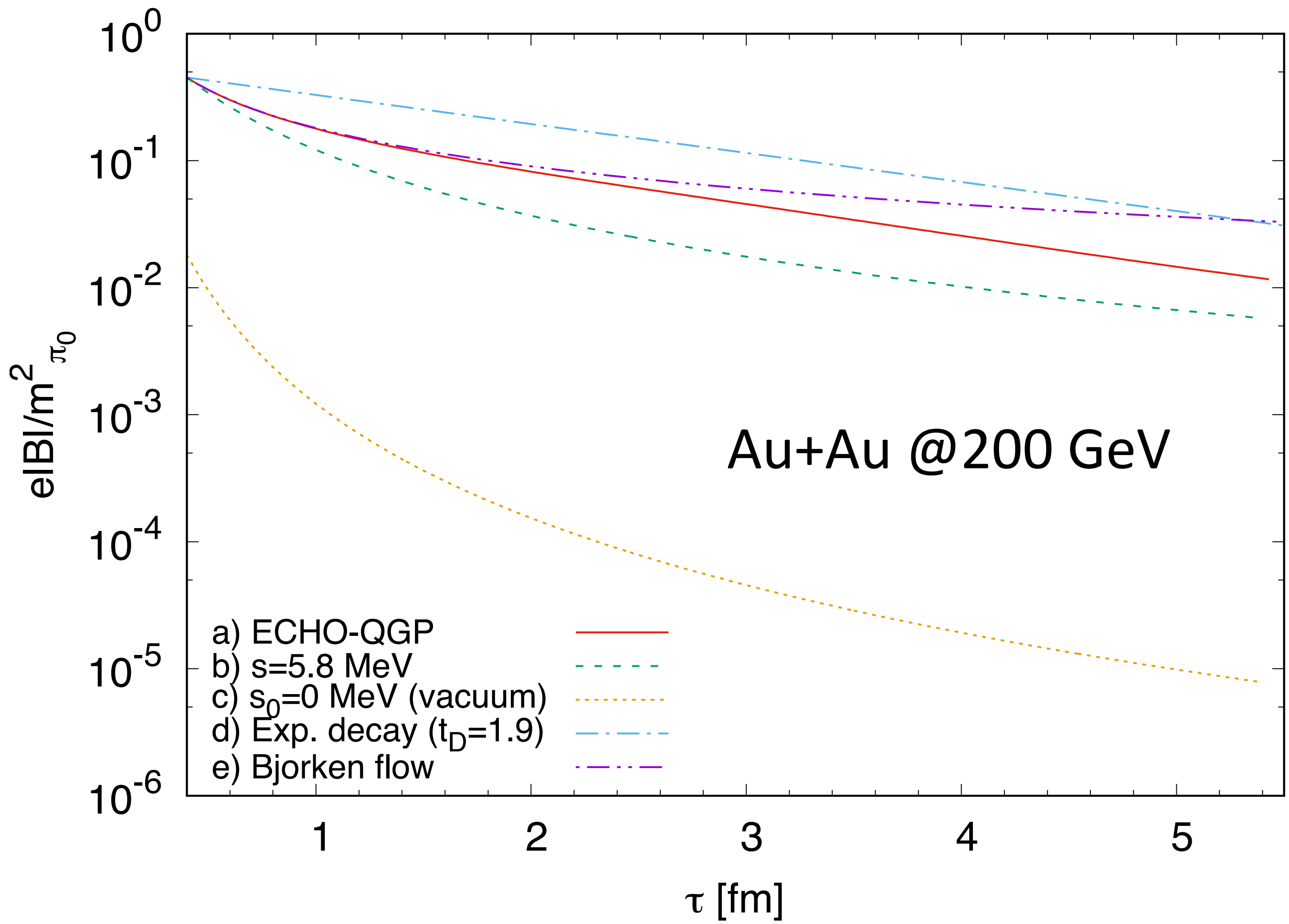


$$\sigma_{\rm Ohm}^{\rm LQCD} = (5.8 \pm 2.9)\frac{T}{T_0}\ \text{MeV}$$

- Lifetime of magnetic field is long only if QGP is a perfect conductor.

Magnetic field from magneto-hydrodynamics

Inghirami, Zanna,Beraudo, Maghaddam, Becattini & Bleicher, arXiav:1609.03042v2 [hep-ph]



Anomalous chiral effects

Vector current $J^\mu = \langle \bar{\Psi}\gamma^\mu \Psi \rangle = J_R^\mu + J_L^\mu$

Axial vector current $J_5^\mu = \langle \bar{\Psi}\gamma^\mu \gamma_5 \Psi \rangle = J_R^\mu - J_L^\mu$

Axial anomaly $\partial_\mu J_5^\mu = \frac{Qe^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B}$

$$\begin{pmatrix} \mathbf{J} \\ \mathbf{J_5} \end{pmatrix} = \begin{pmatrix} \sigma & \sigma_5 \\ \sigma_{\chi e} & \sigma_S \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}$$

Ohm's law $\mathbf{J} = \sigma \mathbf{E}$

Chiral magnetic effect $\mathbf{J} = \sigma_\mathbf{5} \mathbf{B}$

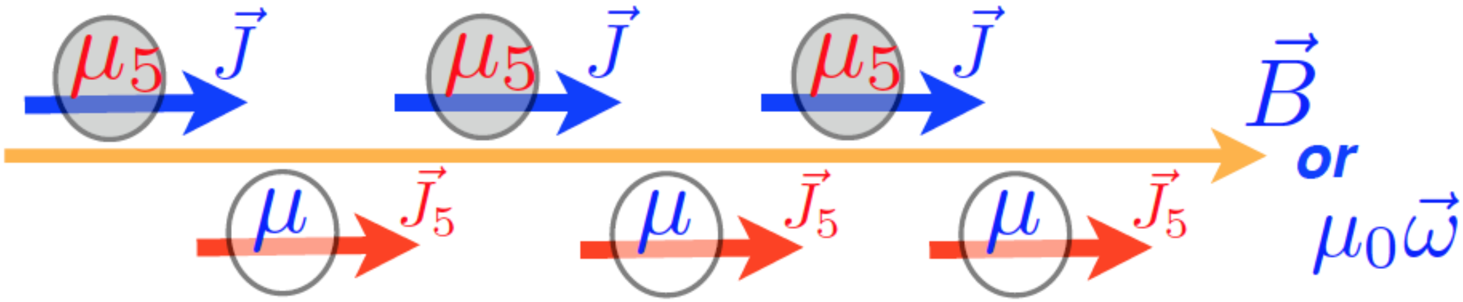
Chiral electric separation effect $\mathbf{J_5} = \sigma_{\chi \mathbf{e}} \mathbf{E}$

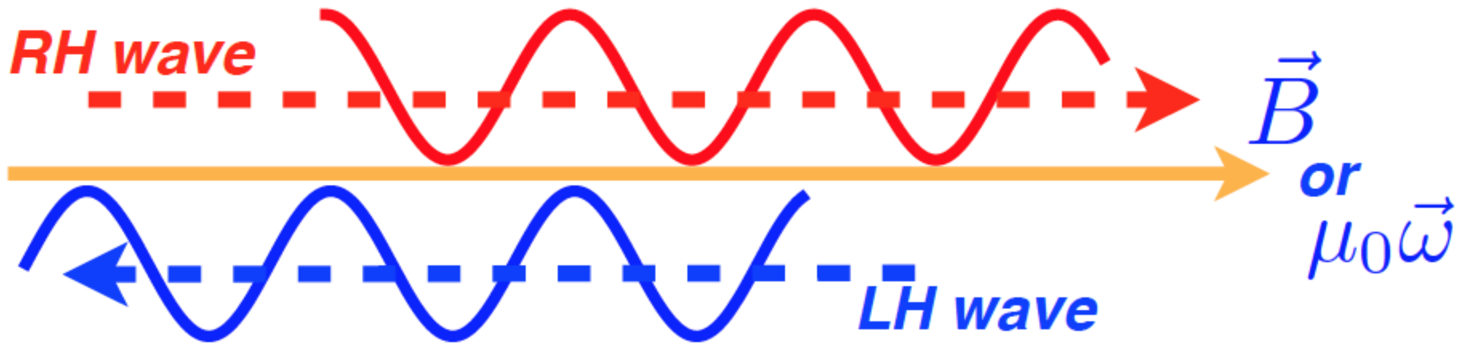
Chiral separation effect $\mathbf{J_5} = \sigma_\mathbf{S} \mathbf{B}$

Interplay between chiral separation and magnetic effects leads to the chiral magnetic wave.

The chiral magnetic wave

Kharzeev, Liao, Voloshin & Wang, Prog. Part. Nucl. Phys. 88, 1 (2016)



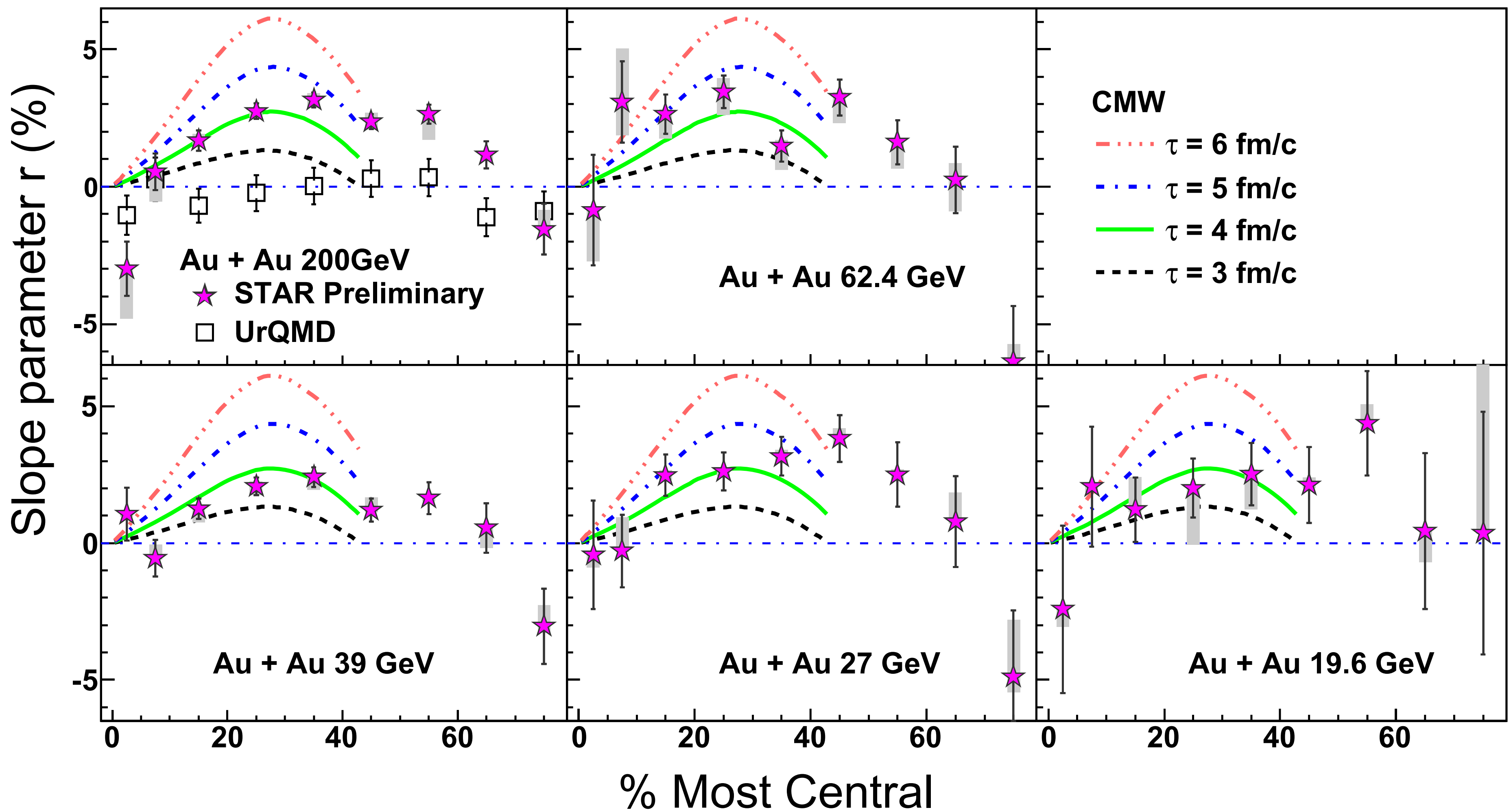


$$\left(\partial_0 \pm \frac{(Qe)}{(4\pi^2)\chi}\vec{\mathbf{B}}\cdot\nabla\right)\delta J^0_{R/L} = \left(\partial_0 \pm v_B \partial_{\hat{\mathbf{B}}}\right)\delta J^0_{R/L} = 0$$

$$v_B \equiv \frac{(Qe)B}{(4\pi^2)\chi}$$

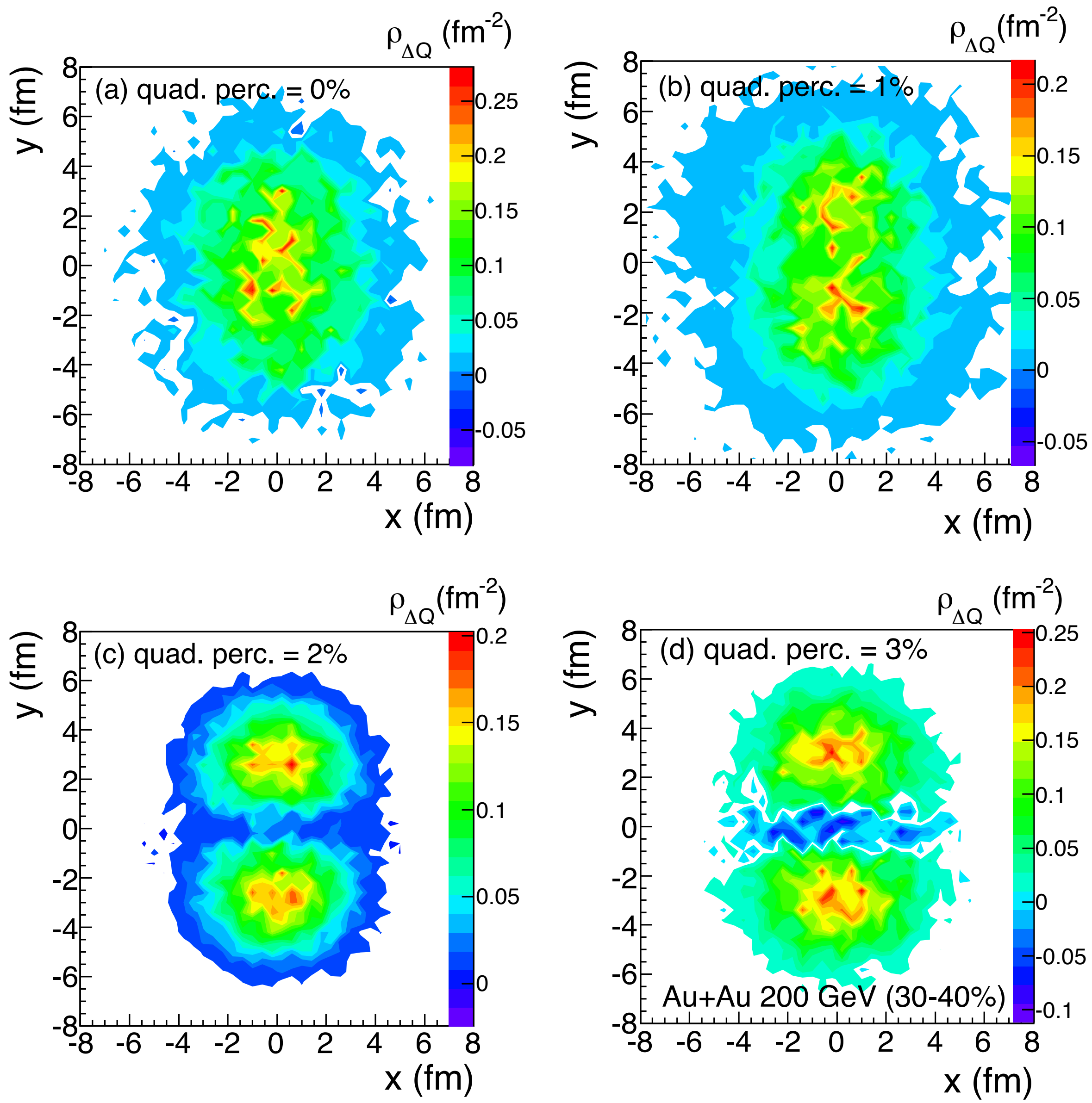
Chiral magnetic wave and elliptic flow splitting

G. Wang et al., NPA 904-905, 248c (2013)



$$\Delta v_2 = v_2(-) - v_2(+), \quad A_\pm = \frac{N_+ - N_-}{N_+ + N_-}, \quad \text{slope parameter} = \frac{\Delta v_2}{A_\pm}$$

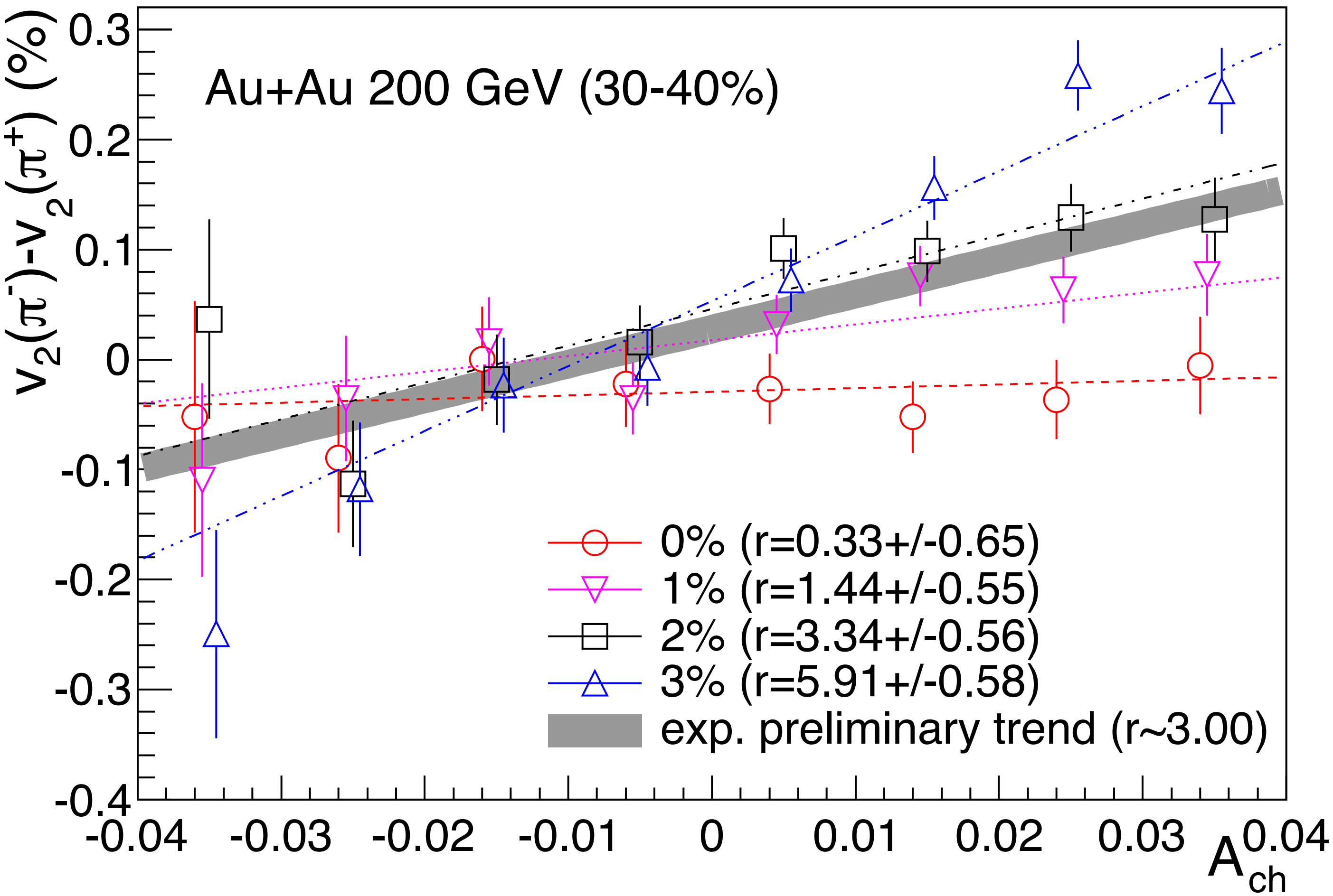
Final-state effect on charge asymmetry dependence of pion elliptic flow



G. L. Ma, PLB 735, 383 (2014)

Modified initial distributions in the transverse plane of a collision described by AMPT

Charge asymmetry dependence of pion elliptic flow splitting



- Both intersection at A_{ch} = 0 and slope parameter are sensitive to initial quadrupole moment in transverse plane.

Chiral kinetic equation

- Path integral: Stephanov & Yin, PRL 109, 162001 (2012)
- Poisson brackets: Son & Yamamoto, PRD 87, 085016 (2013)
- Covariant Wigner function: Chen, Pu, Wang & Wang, PRL 110, 262301 (2013)

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0$$

$$\frac{dt}{d\tau} = 1 \pm Q\mathbf{b}\cdot\mathbf{B}$$

Plus: positive helicity
Minus: negative helicity

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B} \pm Q(\mathbf{E}\times\mathbf{b})$$

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{b} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\mathbf{b})\mathbf{b}$$

$$\mathrm{Three-dimensional\ Berry\ curvature} \quad \mathbf{b} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

For E = 0,

$$\frac{d\mathbf{x}}{dt} = \frac{\hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B}}{1 \pm Q\mathbf{b}\cdot\mathbf{B}}$$

$$\frac{d\mathbf{p}}{dt} = \frac{Q\hat{\mathbf{p}} \times \mathbf{B}}{1 \pm Q\mathbf{b}\cdot\mathbf{B}}$$

Heuristic derivation

$$\begin{aligned} \frac{d\mathbf{J}}{dt} &= \frac{d\left(\mathbf{x}\times\mathbf{p} \pm \frac{\hat{\mathbf{p}}}{2}\right)}{dt} \\ &= \dot{\mathbf{x}}\times\mathbf{p} + \mathbf{x}\times\dot{\mathbf{p}} \pm \left[\frac{\dot{\mathbf{p}}}{\mathbf{2}|\mathbf{p}|} - \mathbf{p}\left(\frac{\mathbf{p}}{\mathbf{2}|\mathbf{p}|^{\mathbf{3}}}\cdot\dot{\mathbf{p}}\right)\right] \\ &= \left(\dot{\mathbf{x}} \mp \dot{\mathbf{p}}\times\frac{\mathbf{p}}{2|\mathbf{p}|^3}\right)\times\mathbf{p} + \mathbf{x}\times\dot{\mathbf{p}} \end{aligned}$$

Using $\dot{\mathbf{p}} = \mathbf{F}$ and $\dot{\mathbf{J}} = \mathbf{x}\times\mathbf{F}$, then $\dot{\mathbf{x}} \mp \dot{\mathbf{p}}\times\mathbf{b} = f(|\mathbf{p}|)\hat{\mathbf{p}}$

Since $\dot{\mathbf{x}} = \hat{\mathbf{p}}$ when $\mathbf{F} = 0$, $f(|\mathbf{p}|) = 1$, so $\dot{\mathbf{x}} = \hat{\mathbf{p}} \pm \dot{\mathbf{p}}\times\mathbf{b}$

Including Lorentz force $\dot{\mathbf{p}} = Q\dot{\mathbf{x}} \times \mathbf{B}$, then

$$
\begin{aligned}
\dot{\mathbf{x}} &= \hat{\mathbf{p}} \pm \dot{\mathbf{p}} \times \mathbf{b} \\
&= \hat{\mathbf{p}} \pm Q(\dot{\mathbf{x}} \times \mathbf{B}) \times \mathbf{b} \\
&= \hat{\mathbf{p}} \pm Q\mathbf{B}(\mathbf{b} \cdot \dot{\mathbf{x}}) \mp Q\dot{\mathbf{x}}(\mathbf{b} \cdot \mathbf{B}) \\
&= \hat{\mathbf{p}} \pm Q\mathbf{B}(\mathbf{b} \cdot \hat{\mathbf{p}}) \mp Q\dot{\mathbf{x}}(\mathbf{b} \cdot \mathbf{B}) \\
&= \frac{\hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}} \cdot \mathbf{b})\mathbf{B}}{1 \pm Q\mathbf{b} \cdot \mathbf{B}}
\end{aligned}
$$

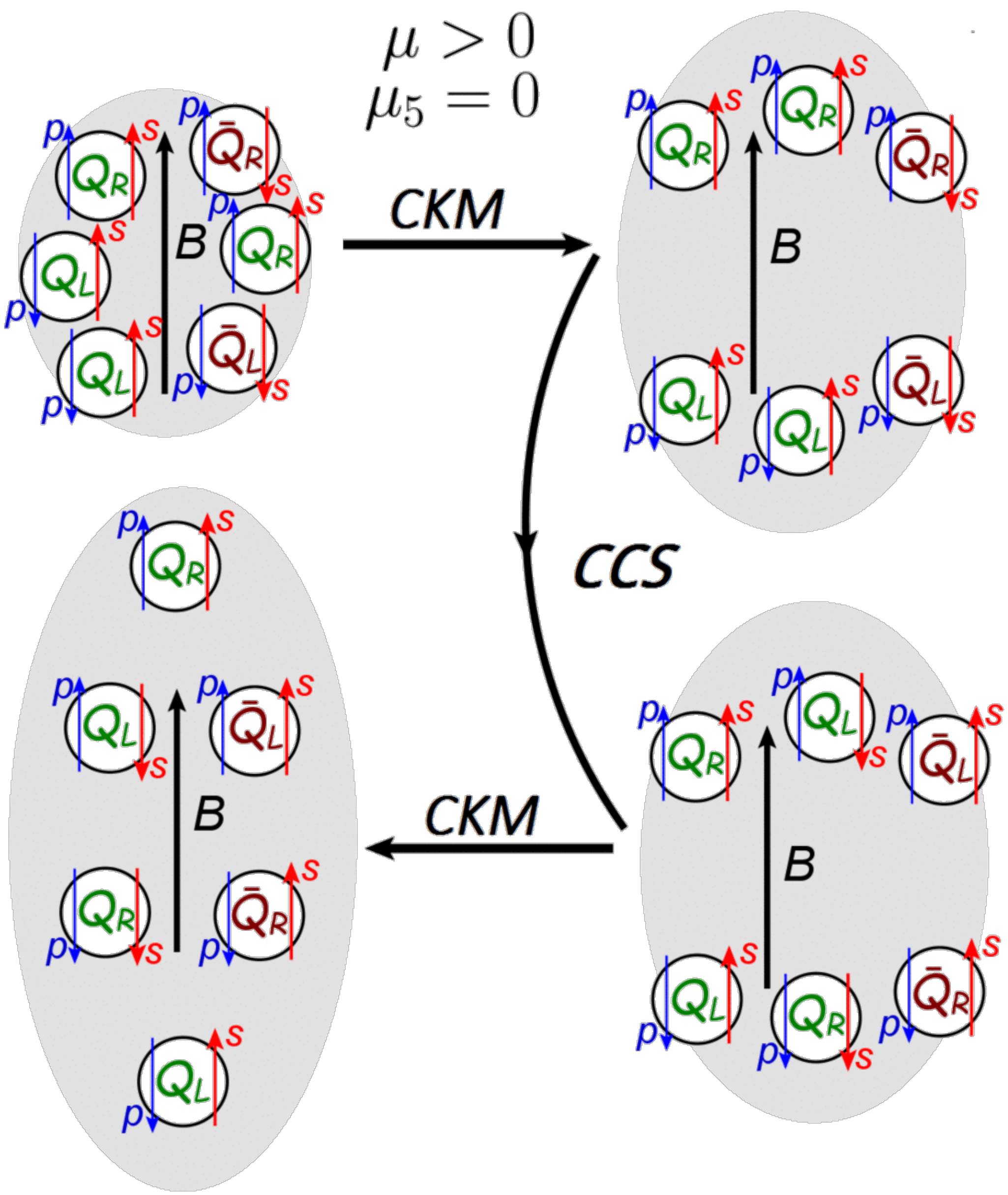
$$
\begin{aligned}
\dot{\mathbf{p}} &= Q\dot{\mathbf{x}} \times \mathbf{B} \\
&= \frac{\hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}} \cdot \mathbf{b})\mathbf{B}}{1 \pm Q\mathbf{b} \cdot \mathbf{B}} \times \mathbf{B} \\
&= \frac{Q\hat{\mathbf{p}} \times \mathbf{B}}{1 \pm Q\mathbf{b} \cdot \mathbf{B}}
\end{aligned}
$$

Chiral kinetic motion

$$
\begin{aligned}
\dot{\mathbf{x}} &= \hat{\mathbf{p}} \\
\dot{\mathbf{p}} &= Q\hat{\mathbf{p}} \times \mathbf{B}
\end{aligned}
$$

Normal kinetic motion

Chirality changing scattering (CCS)



- CKE leads to the separation of particles of right chirality and left chirality.

- CCS ($R\overline{R} \leftrightharpoons L\overline{L}$) resulting in more positively charged particles moving in y-direction.

$$\dot{\mathbf{r}} = \frac{\hat{\mathbf{p}} + Qh(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B}}{1 + Qh\mathbf{B}\cdot\mathbf{b}} \qquad \mathbf{b} = \frac{\mathbf{p}}{2p^3}$$

Anomalous transport model

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = C(f_{R/L})$$

Application to non-central HIC with initial conditions

$$T(x,y) = \frac{T_0}{\left(1 + e^{\frac{\sqrt{x^2+y^2/c^2}-R}{a}}\right)^{1/3}}$$

$$eB_y = \frac{eB_0}{1+(t/\tau)^2}$$

Longitudinal distribution

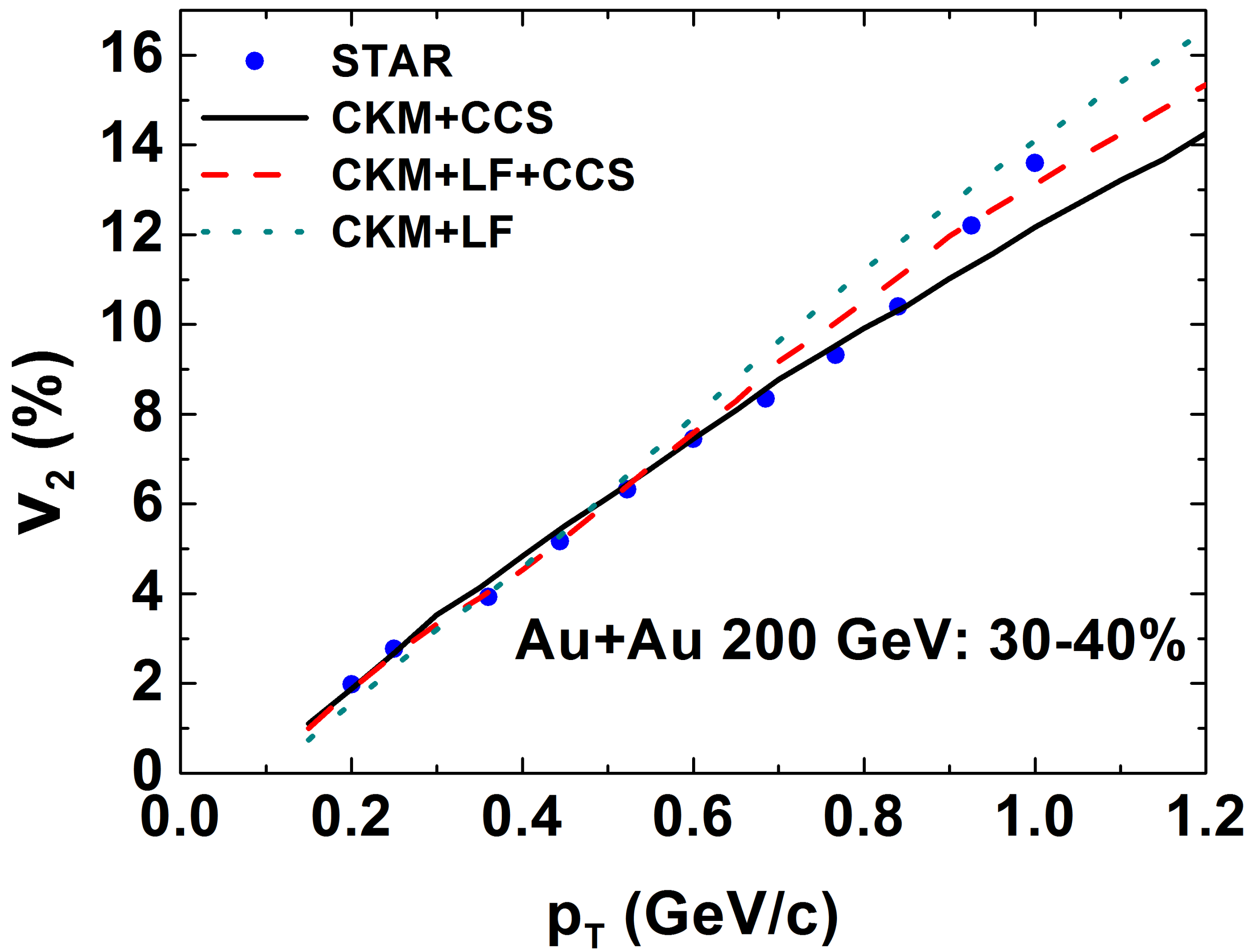
$z = \tau_0 \sinh y$, $p_z = m_T \cosh y$

$\tau_0 = 0.4$ fm/c

$$T_0 = 300\ \text{MeV},\ R = 3.5\ \text{fm},\ a = 0.5\ \text{fm},\ c = 1.5\ \text{fm}$$
$$eB_0 = 7\ m_\pi^2,\ \tau_0 = 6\ \text{fm}/c$$

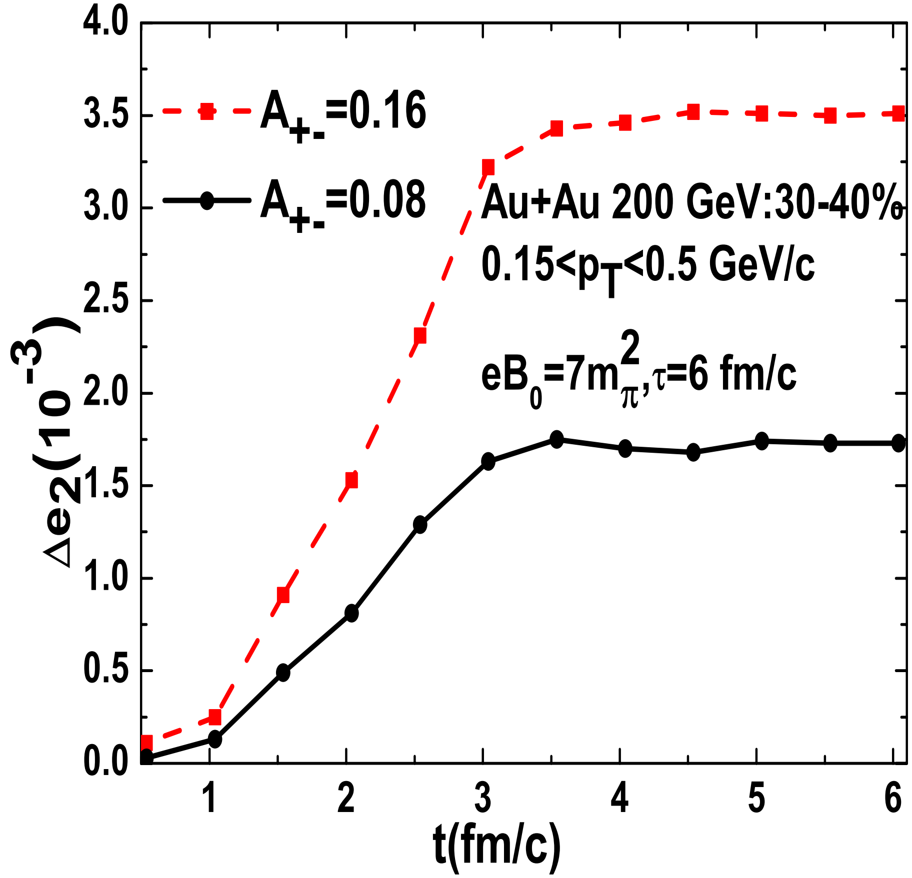
$\sigma = \sigma_0(T_0/T)^3$ with $\sigma_0 = 13 \sim 15$ mb by fitting to measured v_2

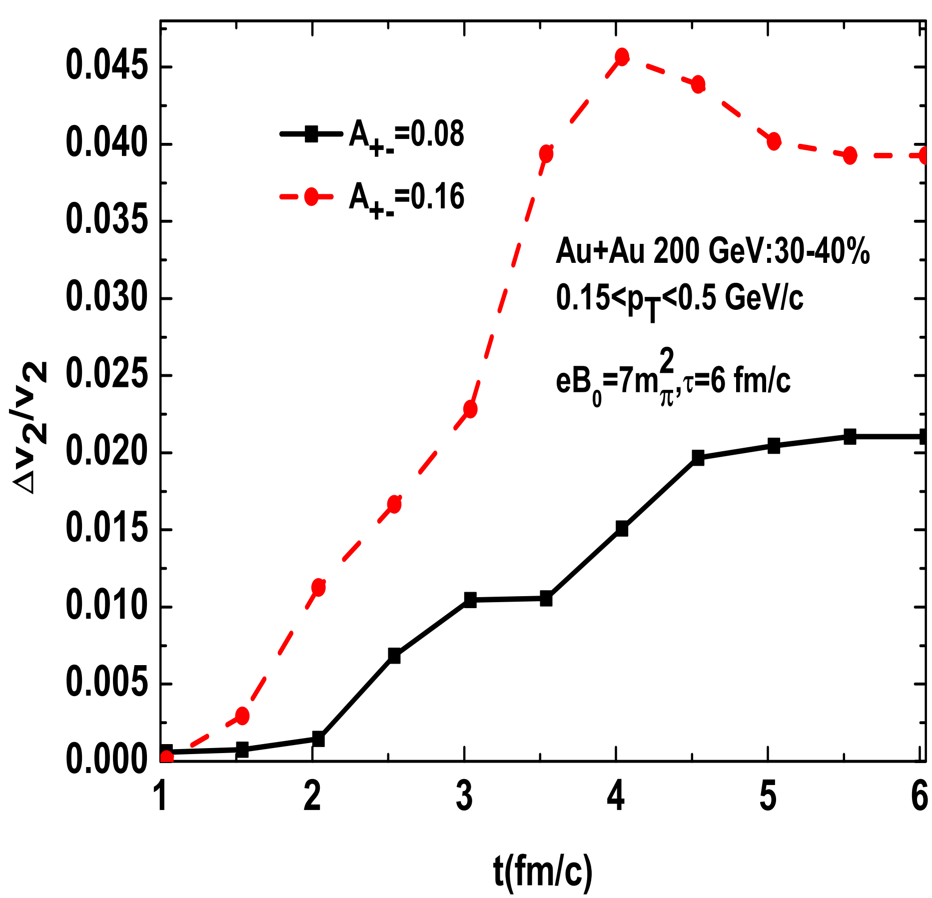
Differential elliptic flow



Data from Adams *et al.* (STAR Collaboration), PRC 72, 014904 (2005)

Time evolution of eccentricity and v2 splittings

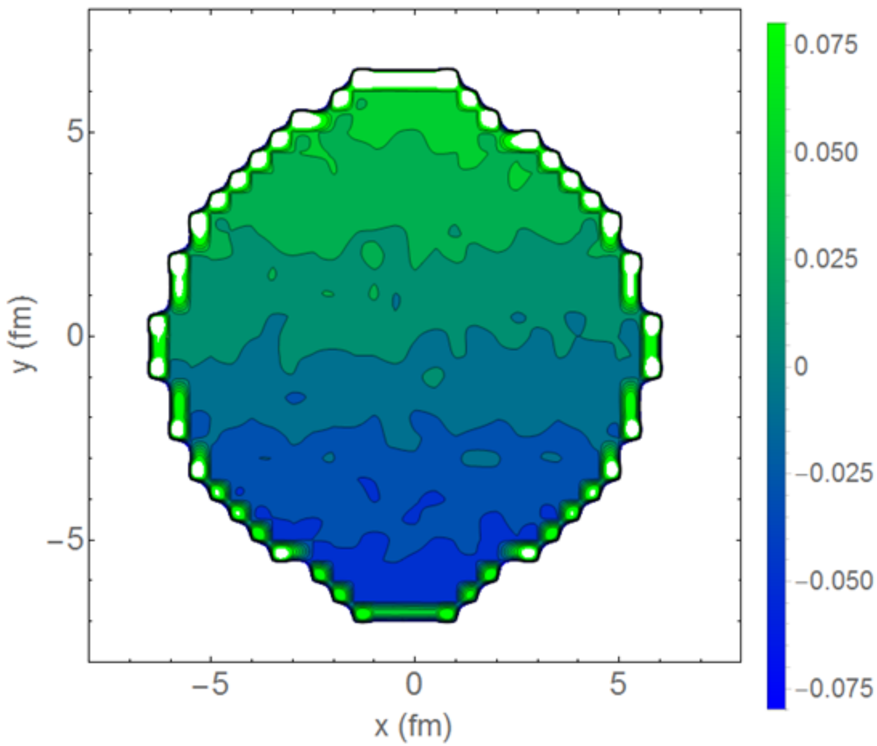




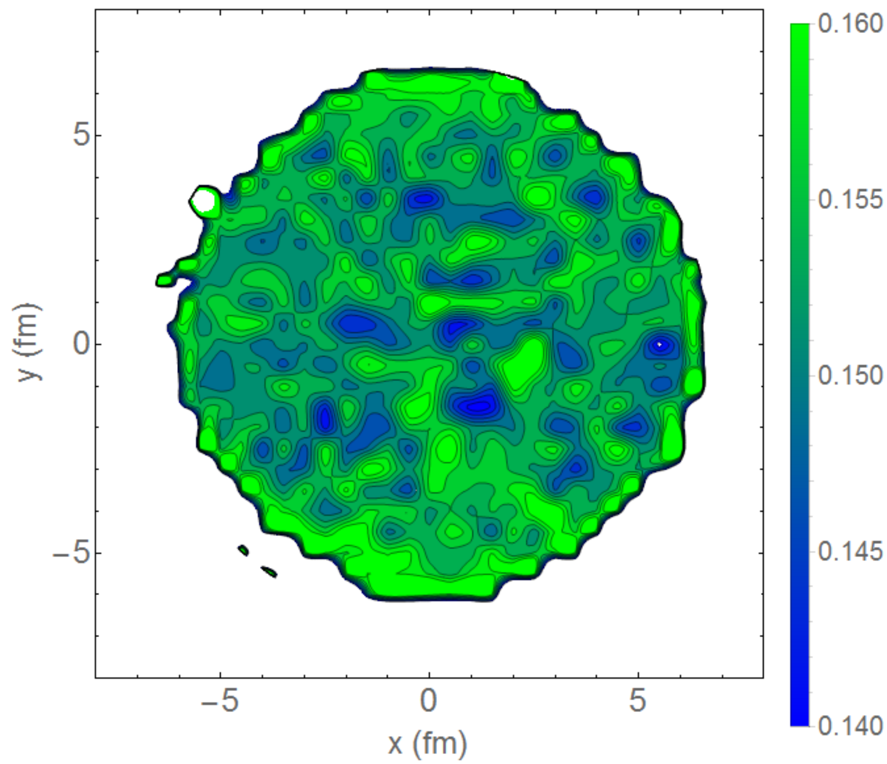
- Including only chiral kinetic motion (CKM) and chirality changing quark-antiquark scattering (CCS) and neglecting the Lorentz force.

Vector and axial vector charge distributions

@ z = 0 & $A_{\pm}$ = 0.16

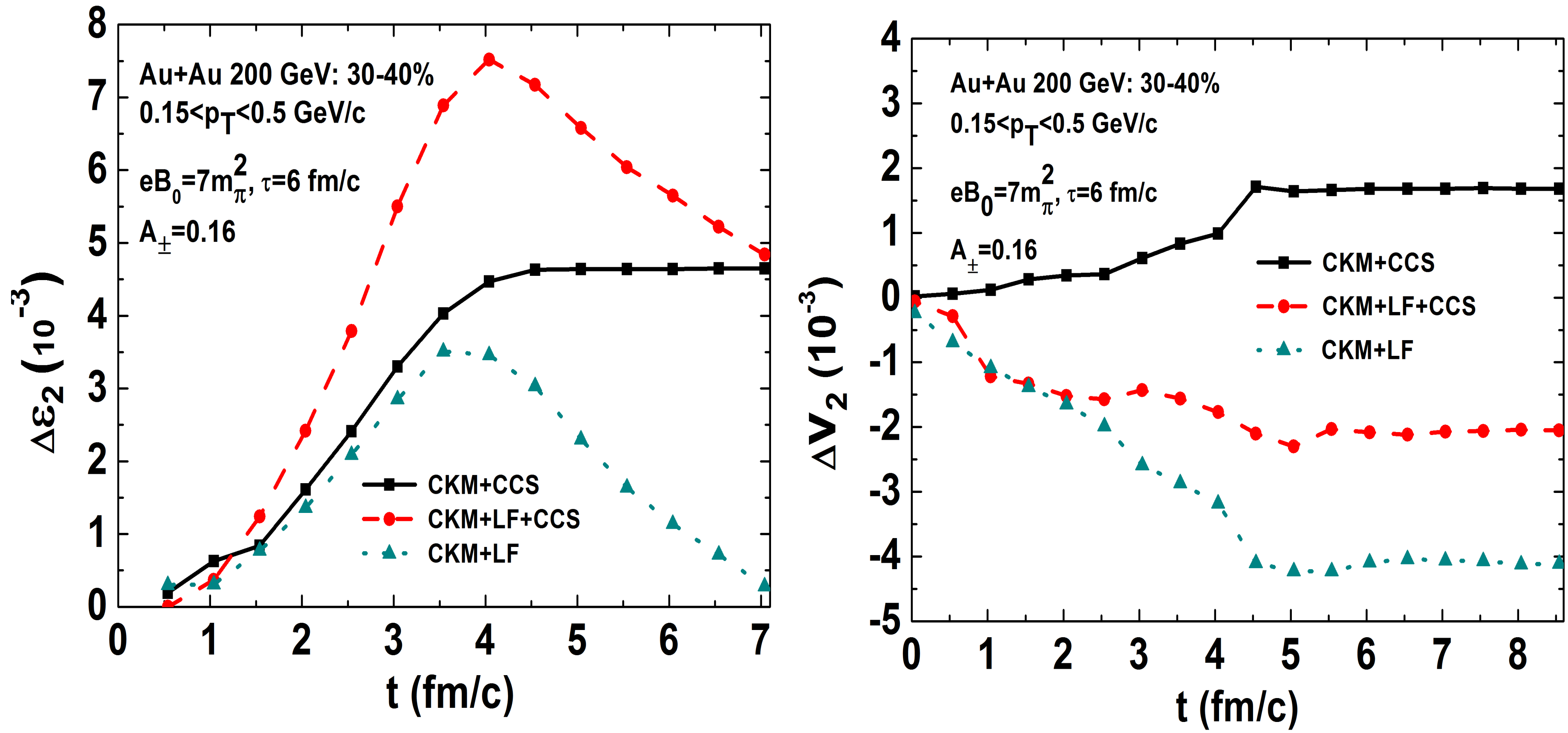


Axil charge distribution
(dipole moment)



Charge distribution
(quadrupole moment)

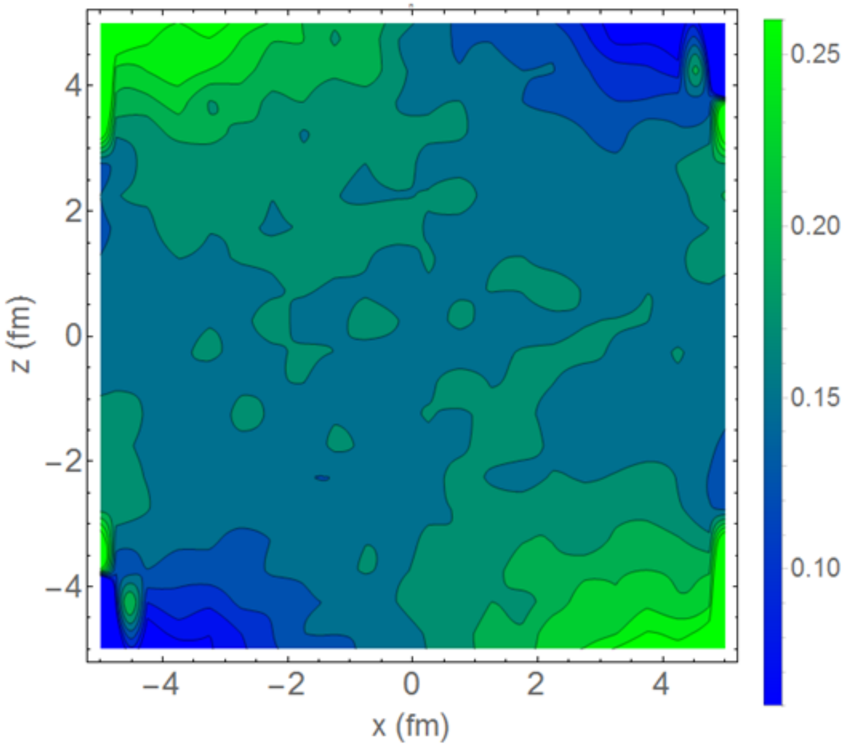
Effect of Lorentz force

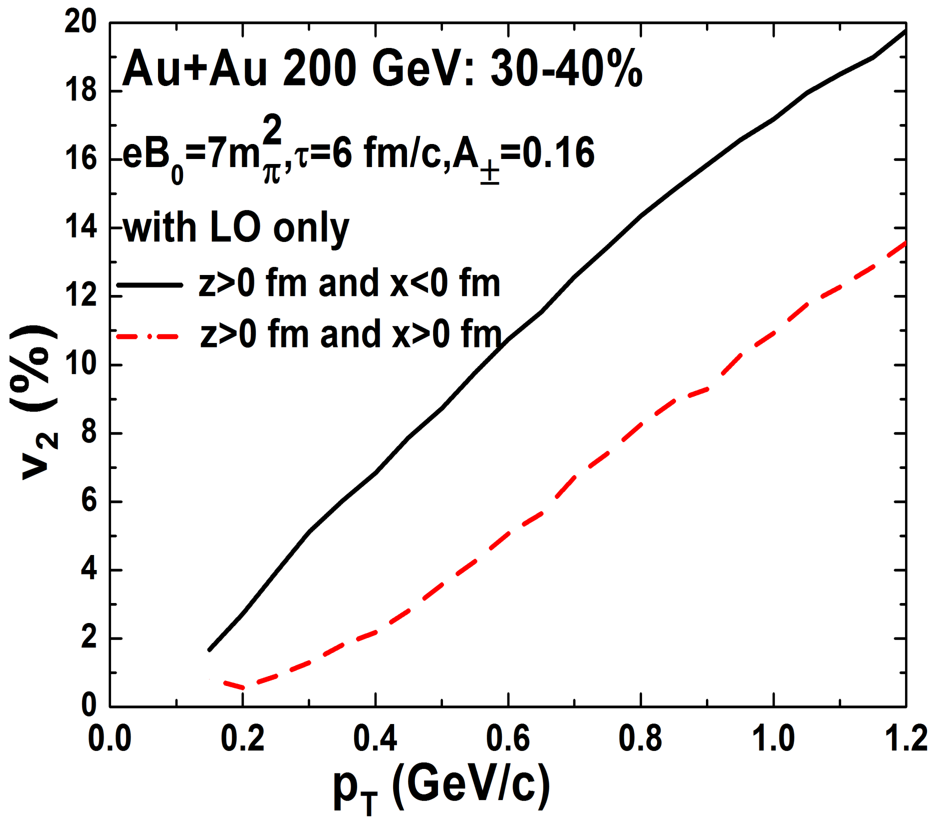


- Not included before [Y. Burnier et al., PRL 107 (2011); M. Hongo *et al*., arXiv 1309.2823 (2013); Yee & Yin, PRC 89 (2014)].
- Larger elliptic flow for positively charged than for negatively charged particles, leading to negative v_2 splitting.

Effect of Lorentz force on charge distribution

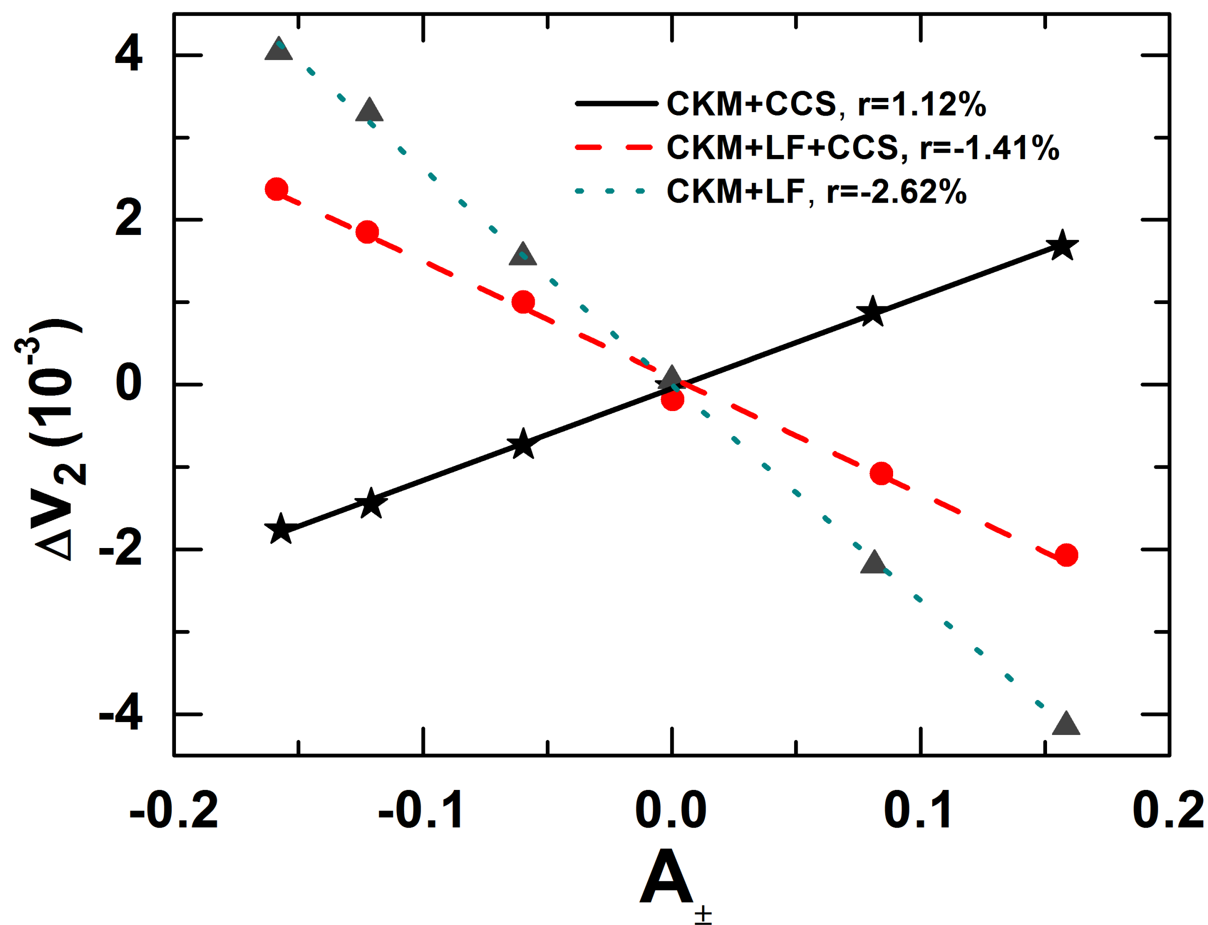
$$\frac{\mu}{T} = \frac{N_R - N_{\bar{R}} + N_L - N_{\bar{L}}}{N_R + N_{\bar{R}} + N_L + N_{\bar{L}}}$$





- Flow is larger in z-direction because of initial narrow size in z-direction.
- Lorentz force leads to different v_1 for positively and negatively charged particles.
- Elliptic flow is larger for particles in upper left and lower right quadrants.

Charge asymmetry dependence of v_2 splitting



- Lorentz force leads to negative slope parameter.
- The positive slope parameter (r = 1%) without LF is smaller than experiment data (r = 3%).

Chiral kinetic equation with vorticity

- Path integral: Stephanov & Yin, PRL 109, 162001 (2012)
- Poisson brackets: Son & Yamamoto, PRD 87, 085016 (2013)
- Covariant Wigner function: Chen, Pu, Wang & Wang, PRL 110, 262301 (2013)

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0$$

$$\frac{dt}{d\tau} = 1 \pm Q\mathbf{b}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\mathbf{b}\cdot\omega)$$

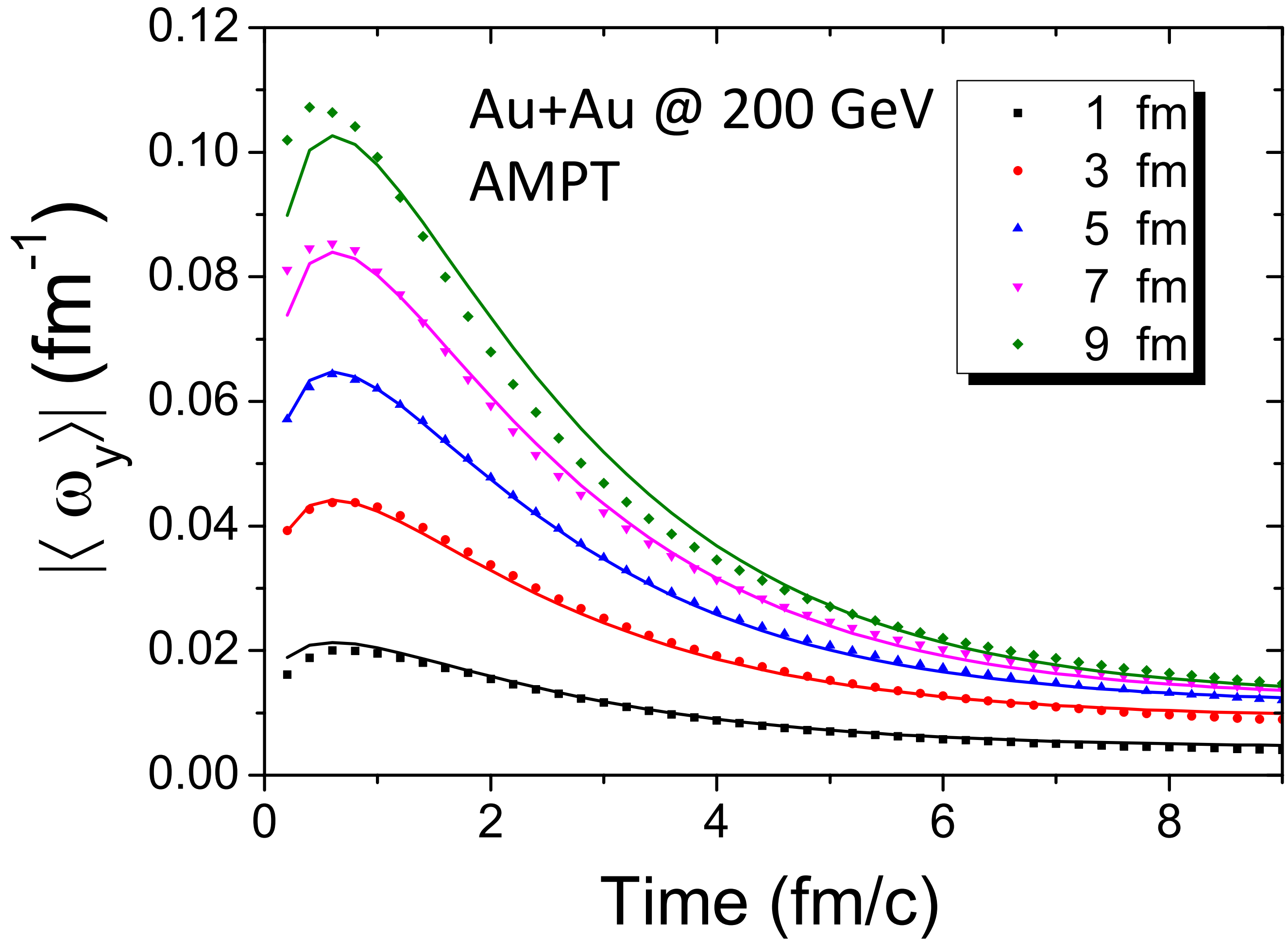
$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{b})\mathbf{B} \pm Q(\mathbf{E}\times\mathbf{b}) \pm \frac{1}{|\mathbf{p}|}\omega$$

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{b} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\mathbf{b})\mathbf{b} \pm 3Q(\mathbf{b}\cdot\omega)(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}}$$

Three – dimensional Berry curvature $\quad \mathbf{b} = \dfrac{\mathbf{p}}{2|\mathbf{p}|^3}$

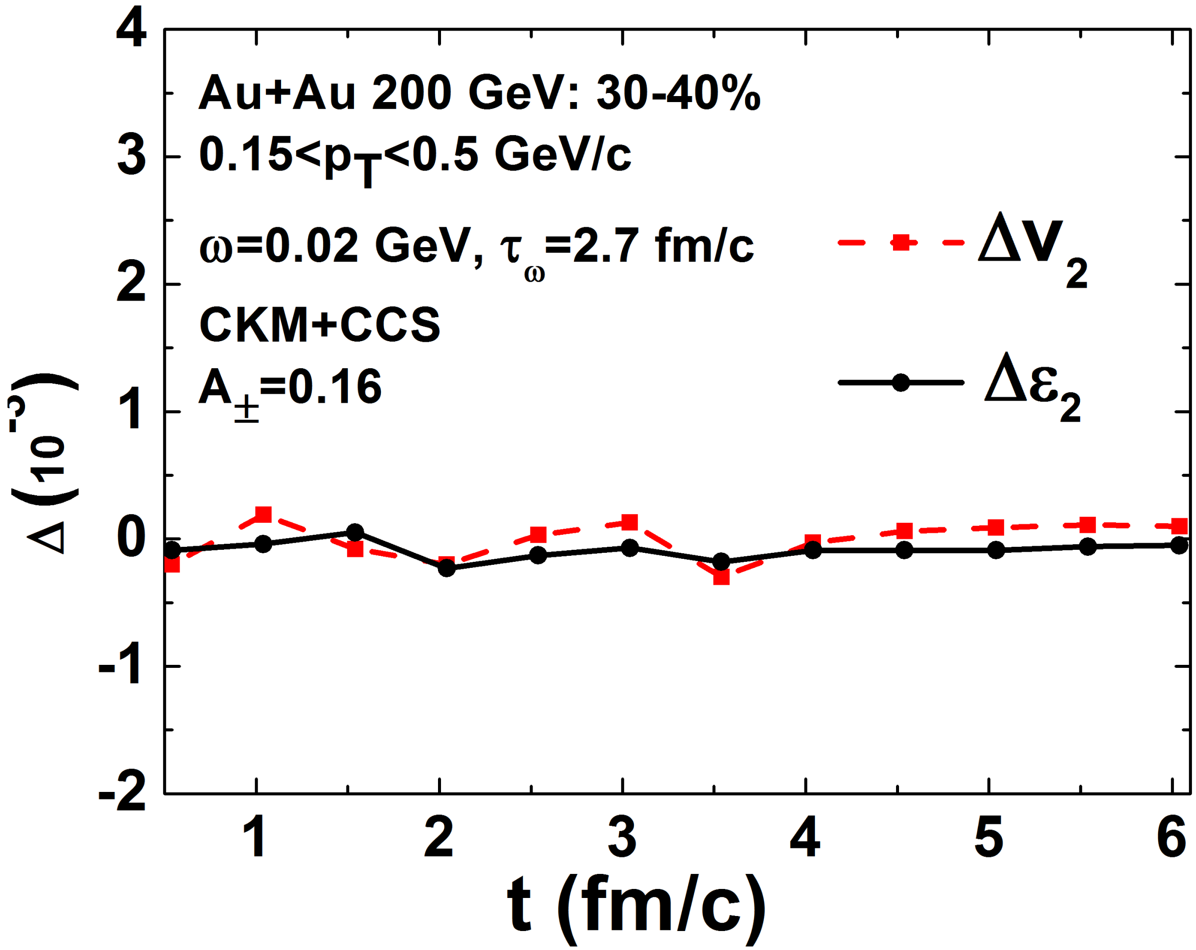
Vorticity in relativistic heavy ion collisions

Jiang, Lin & Liao, ar Xiv:1602.0658 [nucl-th]



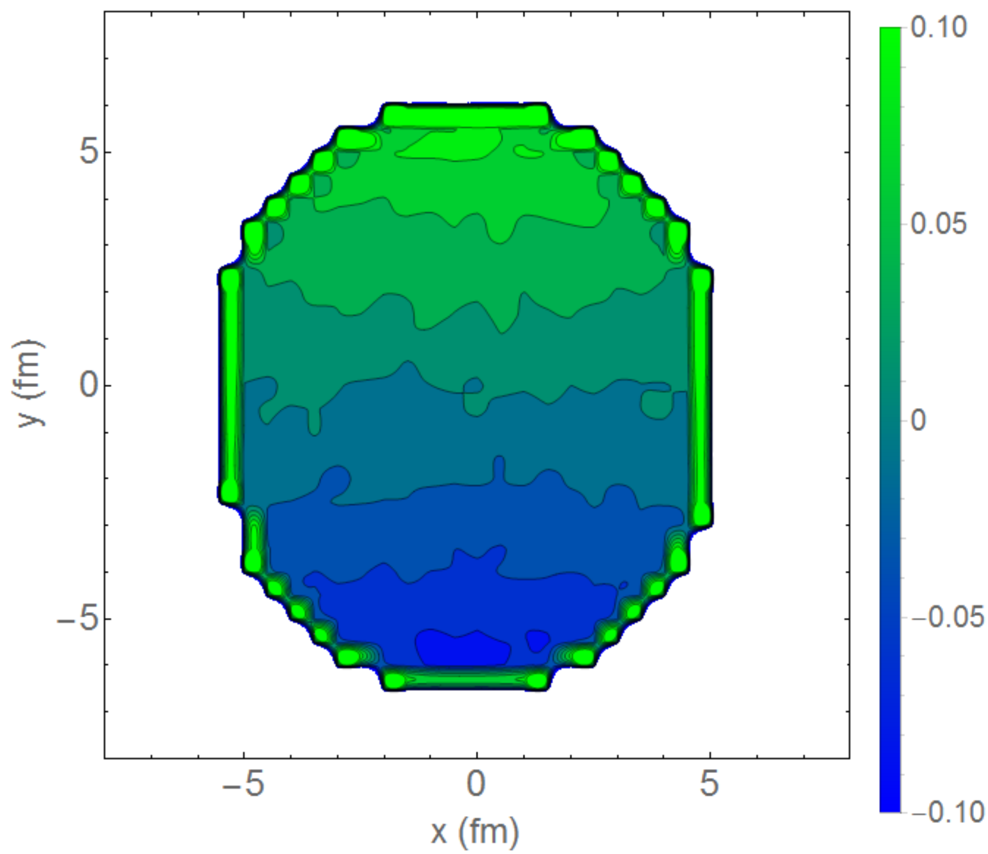
$$\vec{\omega} \quad = \quad \frac{1}{2}\nabla \times \vec{v}, \quad \langle \omega_y \rangle = \frac{\int d^3\vec{r}\ [\mathcal{W}(\vec{r})]\, \omega_y(\vec{r})}{\int d^3\vec{r}\ [\mathcal{W}(\vec{r})]}$$

Time evolution of eccentricity and v_2 splittings with voritical effect only (B = 0)



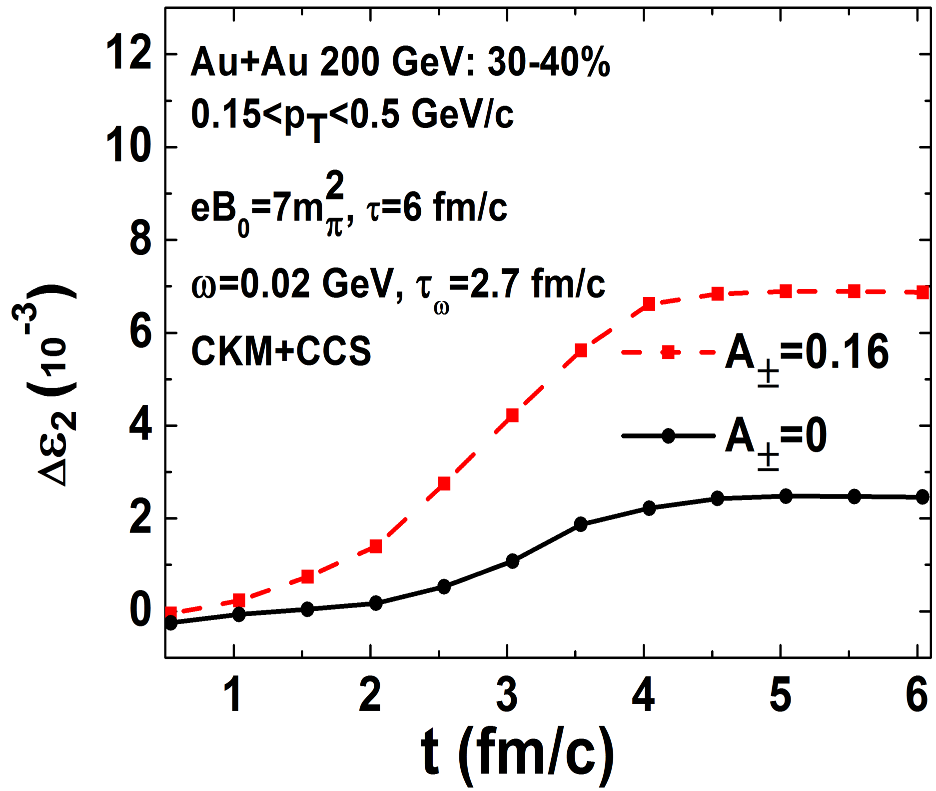
- Similar effects on positively and negatively charged particles with chiral kinetic motion (CKM) and chirality changing quark-antiquark scattering (CCS).

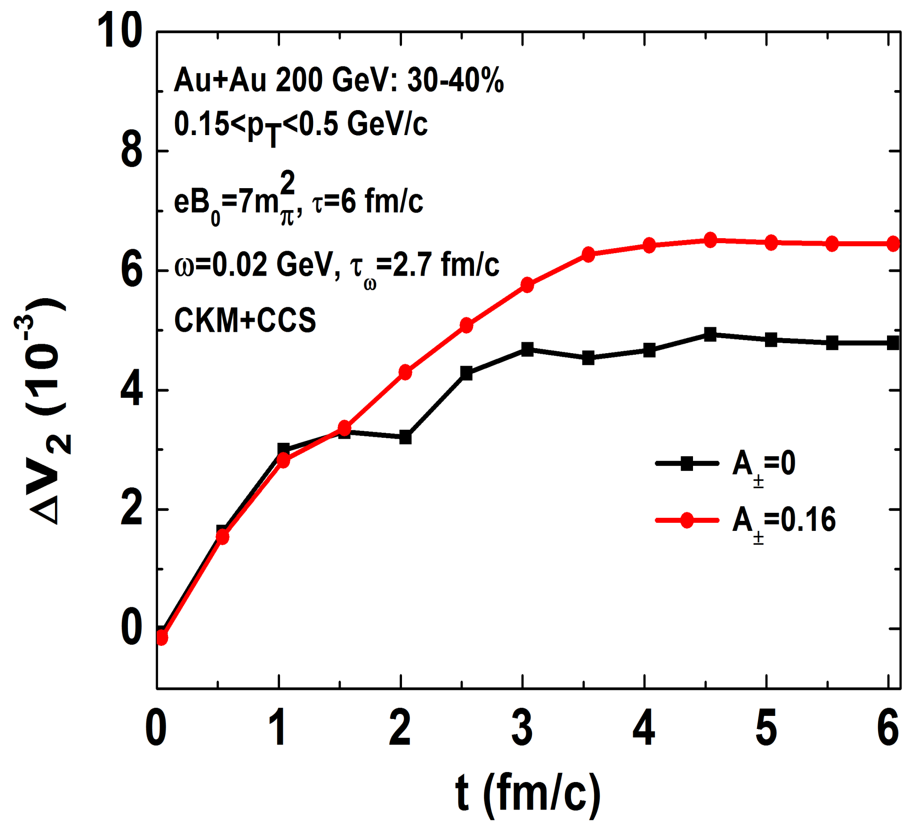
Axial charge distribution for zero charge asymmetry with vortical effect



- Large axial charge dipole moment than the case when the charge asymmetry is 0.16 in the presence of a strong and long-lived magnetic field.

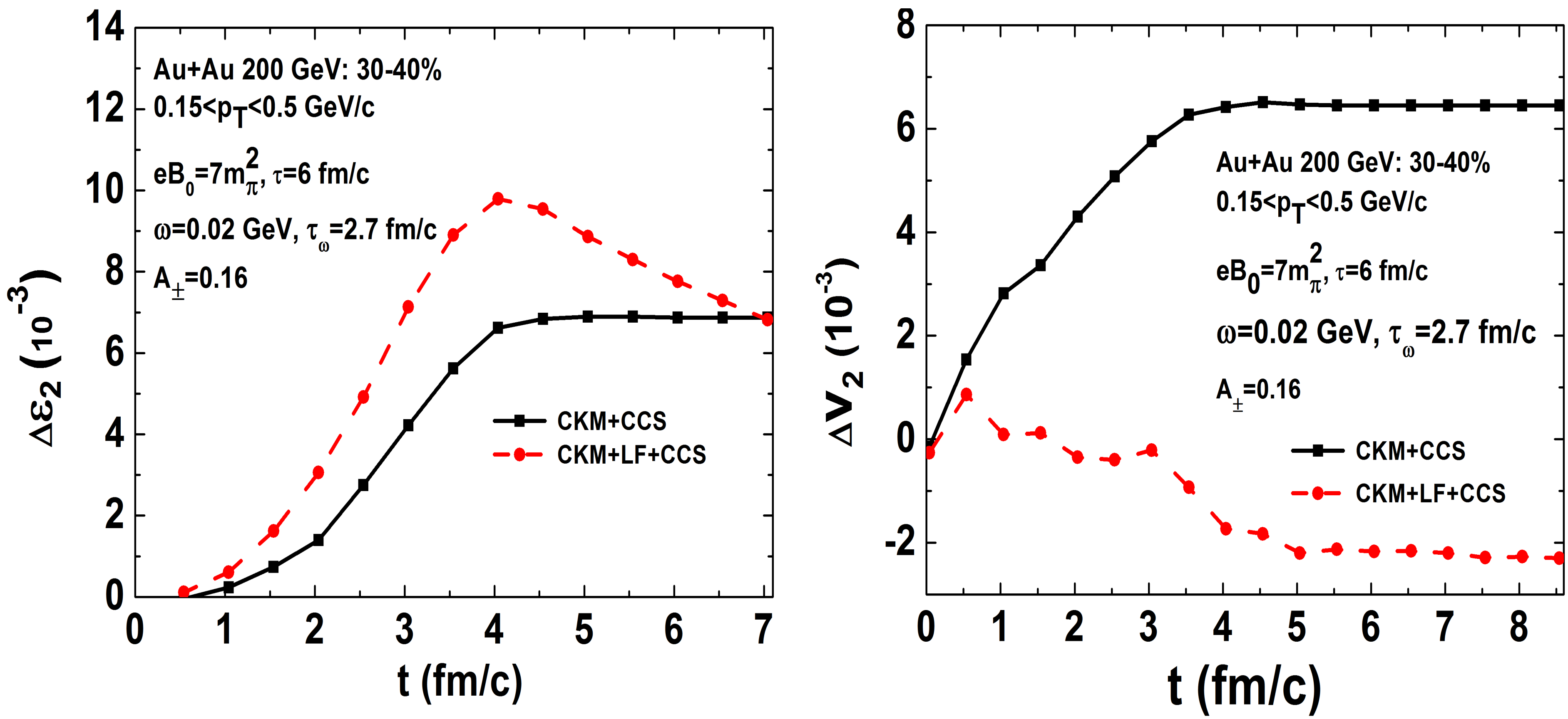
Time evolution of eccentricity and v_2 splittings with both vorticity and magetic field effects





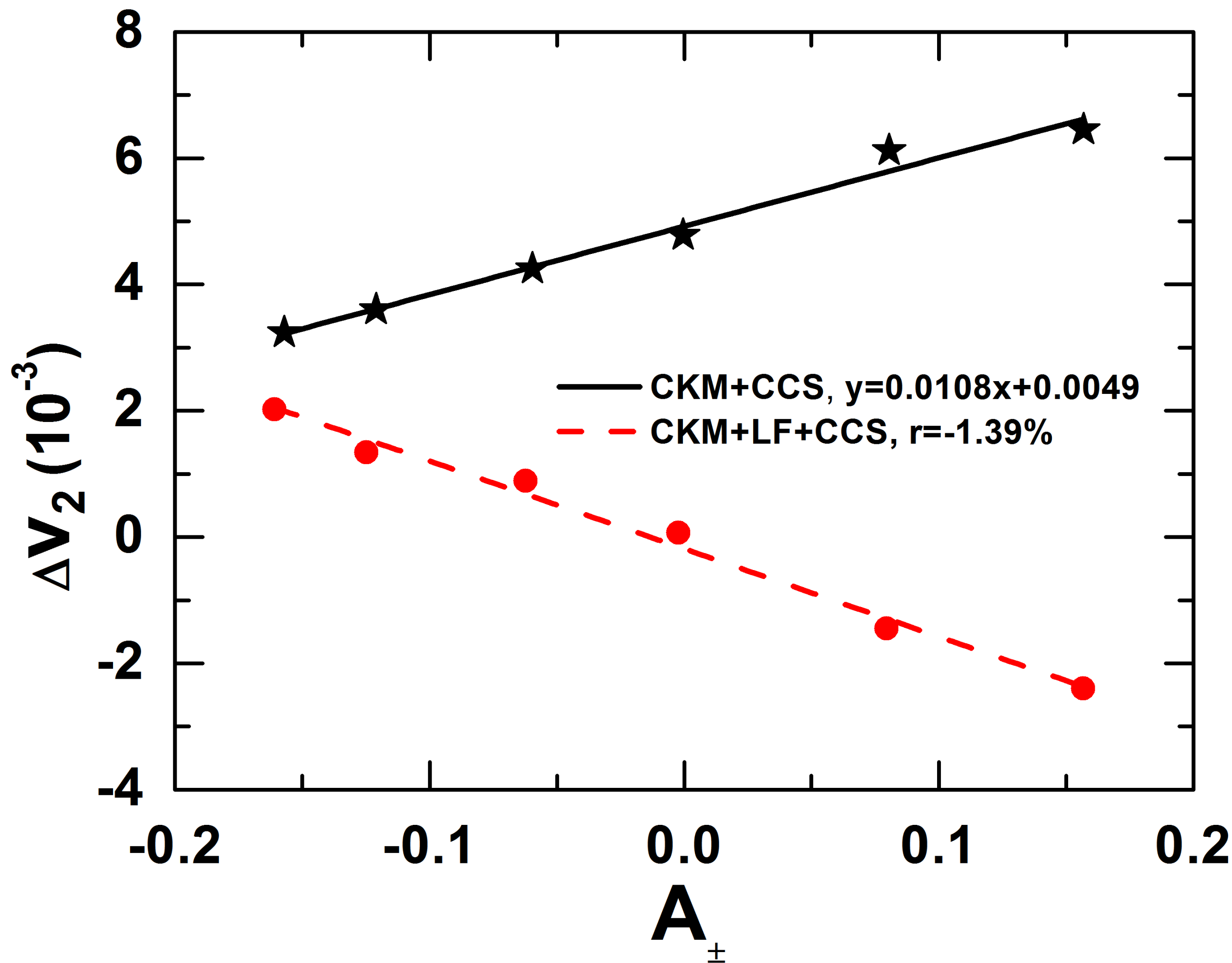
- Finite eccentricity and elliptic flow splittings even when charge asymmetry is zero.
- Elliptic flow splitting develops faster than in the presence of magnetic field only.

Effect of Lorentz force on time evolution of eccentricity and v2 splittings



- Lorentz force leads to larger elliptic flow for positively charged than negatively charged particles.

Charge asymmetry dependence of v_2 splitting



- Large elliptic flow splitting when charge asymmetry is zero.
- Lorentz force destroys chiral effects.

Summary

- Magnetic field and vorticity generated in non-central relativistic heavy ion collisions are large but short-lived.
- In the presence of strong magnetic field and large vorticity that last sufficiently long, anomalous transport study shows that
  - Chirality changing scattering is essential for generating eccentricity and elliptic flow splittings.
  - CMW enhances v_2 of negatively charged particles and leads to a positive slope parameter. Including also CVW leads to nonzero v_2 splitting at zero charge asymmetry.
  - Lorentz force enhances v_2 of positively charged particles and leads to a negative slope parameter or destroys the chiral magnetic and vortical effects.
- Long-lived magnetic field and fast rotating QGP in relativistic heavy ion collisions is not supported by microscopic calculations.
- It remains a challenge to find the mechanisms for extending the lifetime of strong magnetic field in relativistic heavy ion collisions.