



Event by Event Analysis of the Anisotropy of
the Low Energy Direct Photons
in
Relativistic Heavy Ion Collisions

Classical Bremsstrahlung Revisited

T. Kodama^{1,3}, T. Koide¹, Y. Nara^{2,3} and H. Stöcker³

¹ Federal University of Rio de Janeiro

² Akita International University

³ FIAS, Frankfurt

Direct Photons in Heavy Ion Collisions

The electromagnetic probes carry the information from the early stage of the collision dynamics. Many theoretical Works since the very early stage of the heavy ion program.

J. Kapsta (1977); J. D. Bjorken and L. McLerran (1985); J. Thiel, T. Lippert, N. Grün (1989); V. Koch, B. Blättel, W. Cassing, U. Mosel (1990); P. A. Ruuskanen, (1992), A. Dumitru, L. McLerran, H. Stoecker (1993); D. Srivastava (1994); N. Arbex, U. Ornik, M. Plumer, R. Weiner (1995); A. Dumitru, J. A. Maruhn, D. H. Rischke (1995), T. Hirano, S. Muroya, M. Namiki (1995); J. Alam, S. Raha and B. Sinha (1996). S. Jeon, A. Chikanian, J. Kapusta, S. M. H. Wong (1999); U. Eichmann, C. Ernst, L.M. Satarov. W. Greiner (2000); R. Chatterjee, H. Holopainen, T. Renk, K. J. Eskola (2011); A. K. Chaudhuri and B. Sinha (2011), T.S. Biro, M. Gyulassi, Z. Schram (2012); G. Basar, D. Kharzeev, V. Skokov,.....

And more recently, using the state-of-art hydro or transport theories,

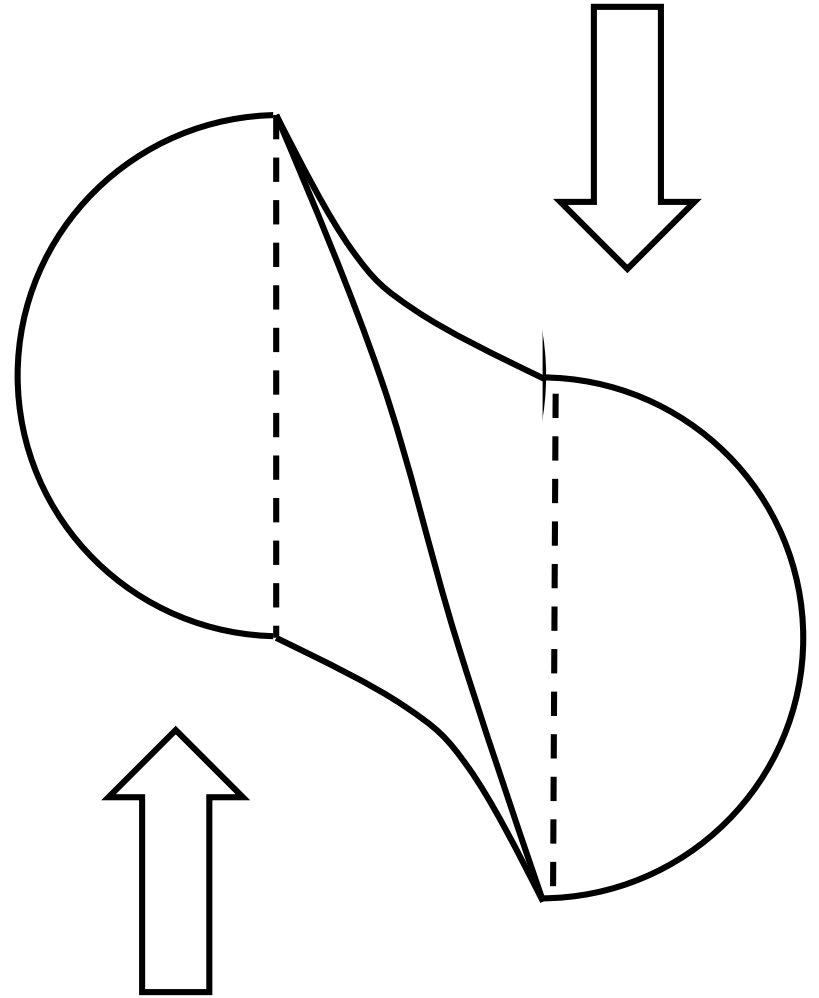
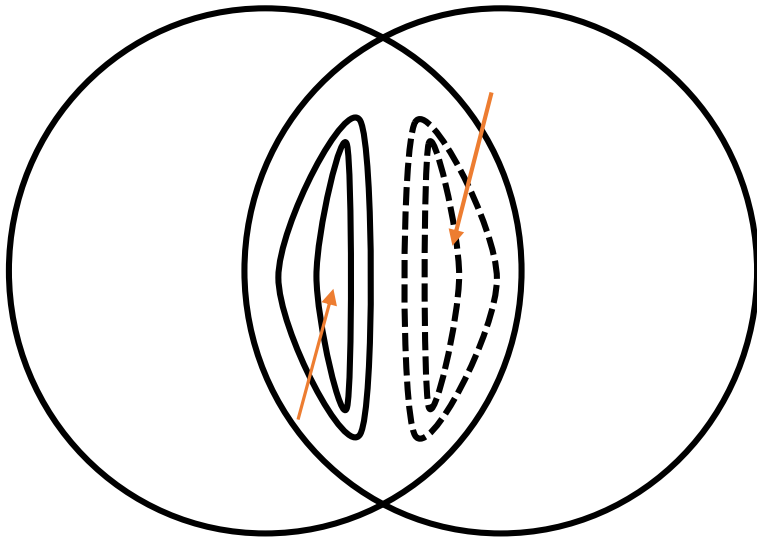
C. Shen, Ulrich Heinz, J-F. Paquet, I. Kozlov, C. Gale, C. Shen, G. S. Denicol, M. Luzum, B. Schenke, S. Jeon, Y. Hidaka, S. Lin, R. D. Pisarski, D. Satow, V. V. Skokov, G. Vujanovic, O. Linnyk, E. L. Bratkovskaya, W. Cassing, C. Greiner, M. Greif, S. Endres, H. v. Hess, J. Weil, M. Bleicher

Early days model of photon emission

Coherent Bremsstrahlung Emission due to Deceleration of Incident Nuclei

J. Kapsta, Phys. Rev. C15, 1580 (1977), J. D. Bjorken and L. McLerran, Phys. Rev. D31, 63 (1985), J. Thiel et. al., Nucl. Phys. A504, 864 (1989). V. Koch et. al., Phys. Lett. B236, 135 (1990), T. Lippert et. al., Int. J. Mod. Phys. A29, 5249 (1991), A. Dumitru et. al., Phys. Lett. B318, 583 (1993), U. Eichmann and W. Greiner, J. Phys. G23, L65 (1997), S. Jeon et. al., Phys. Rev. C58, 1666 (1998), J. Kapusta and S. M. H. Wong, Phys. Rev. C59, 3317 (1999), U. Eichmann, C. Ernst, L.M. Satarov and W. Greiner, Phys.Rev. C62, 044902 (2000) ..

Deceleration of incidente nuclei



Classical Electromagnetic Radiation by an accelerated point charge is given by (Liénard-Wiechert Potential),

$$\vec{E}(\vec{x}, t) = \frac{e}{4\pi} \frac{1}{|\vec{x} - \vec{\xi}(t')|} \frac{1}{1 - \vec{\beta}(t') \cdot \vec{n}} \left(1 - \vec{n}\vec{n}^T\right) \frac{d\vec{\beta}(t')}{dt'},$$

$$\vec{B}(\vec{x}, t) = \vec{n} \times \vec{E}(\vec{x}, t),$$

where $\vec{\xi}(t)$ is the trajectory of the point charge, and $\vec{\beta}(t) = d\vec{\xi}/dt$ is the velocity. t' is the emission time, defined by $t' = t - |\vec{x} - \vec{\xi}(t')|$

and
$$\vec{n} = \frac{\vec{x} - \vec{\xi}(t')}{|\vec{x} - \vec{\xi}(t')|}.$$

Continuum Sources (Integrate over moving charges)

For $r = |\vec{x}| \gg |\vec{\xi}|$,

$$\vec{E}(\vec{x}, t) = -\frac{e}{4\pi r} \left(1 - \vec{n}\vec{n}^T\right) \int d^3\xi \frac{\partial}{\partial \tau} \vec{J}(\vec{\xi}, \tau) \Big|_{\tau=t-|\vec{x}-\vec{\xi}|},$$

$$\vec{B}(\vec{x}, t) = \vec{n} \times \vec{E}(\vec{x}, t),$$

with $\vec{n} = \frac{1}{r} \vec{x}$.

How to calculate the photon spectrum?

Equivalent Photon Method - Jackson's book

How to calculate the photon spectrum?

Equivalent Photon Method - Jackson's book

Or introduce the Wigner function,

$$f_W(\vec{x}, \vec{p}; t) = \int d^3\vec{u} \left\{ e^{i\vec{p}\cdot\vec{u}/\hbar} \psi^\dagger(\vec{x} - \vec{u}/2, t) \psi(\vec{x} + \vec{u}/2, t) \right\}$$

where $\psi(\vec{x}, t)$ is the wave function.

Identify the Classical Electromagnetic fields as the vector type wavefunction similar to the Dirac form*.

$$\vec{\psi}(\vec{x}, t) \equiv (\vec{E} + i\vec{B}) / \sqrt{2},$$

then the Maxwell equations are equivalent to

$$i(\partial_t + \vec{\Sigma} \cdot \nabla) \vec{\psi} = \vec{j} \quad \text{and} \quad \nabla \cdot \vec{\psi} = \rho,$$

with $\vec{\Sigma} = \left\{ \frac{1}{i} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \frac{1}{i} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \frac{1}{i} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\}$ generators of $O(3)$
for spin 1

- ❖ L. Silberstein, Ann. d. Phys. 22, 579 (1907); 24, 783 (1907);
- ❖ H. Bateman, Cambridge 1915, Dover, New York, 1955).
- ❖ I. Bialynicki-Birula, Act. Phys. Pol. A86 97 (1994).
- ❖ P. Holland, Proc. R. Soc. A461, 359 (2005).

The energy density and Poyinting Vector are expressed as

$$\varepsilon = \vec{\psi}^\dagger \vec{\psi},$$

$$\vec{P} = -i \left(\vec{\psi}^\dagger \cdot \vec{\Sigma} \right) \vec{\psi},$$

where the product $\left(\vec{\psi}^\dagger \cdot \vec{\Sigma} \right)$ is defined as $\left(\vec{\psi}^\dagger \cdot \vec{\Sigma} \right) = \sum_{i=x,y,z} \psi_i^\dagger \vec{\Sigma}_i$.

The time evolution after “freeze-out” ($\rho, \vec{j} = 0$) we need only

$$i \left(\partial_t + \vec{\Sigma} \cdot \nabla \right) \vec{\psi} = 0,$$

Then the corresponding Wigner function (scalar) decomposes into the electric and magnetic contributions,

$$f_W(\vec{x}, \vec{p}; t) = f_E(\vec{x}, \vec{p}; t) + f_B(\vec{x}, \vec{p}; t)$$

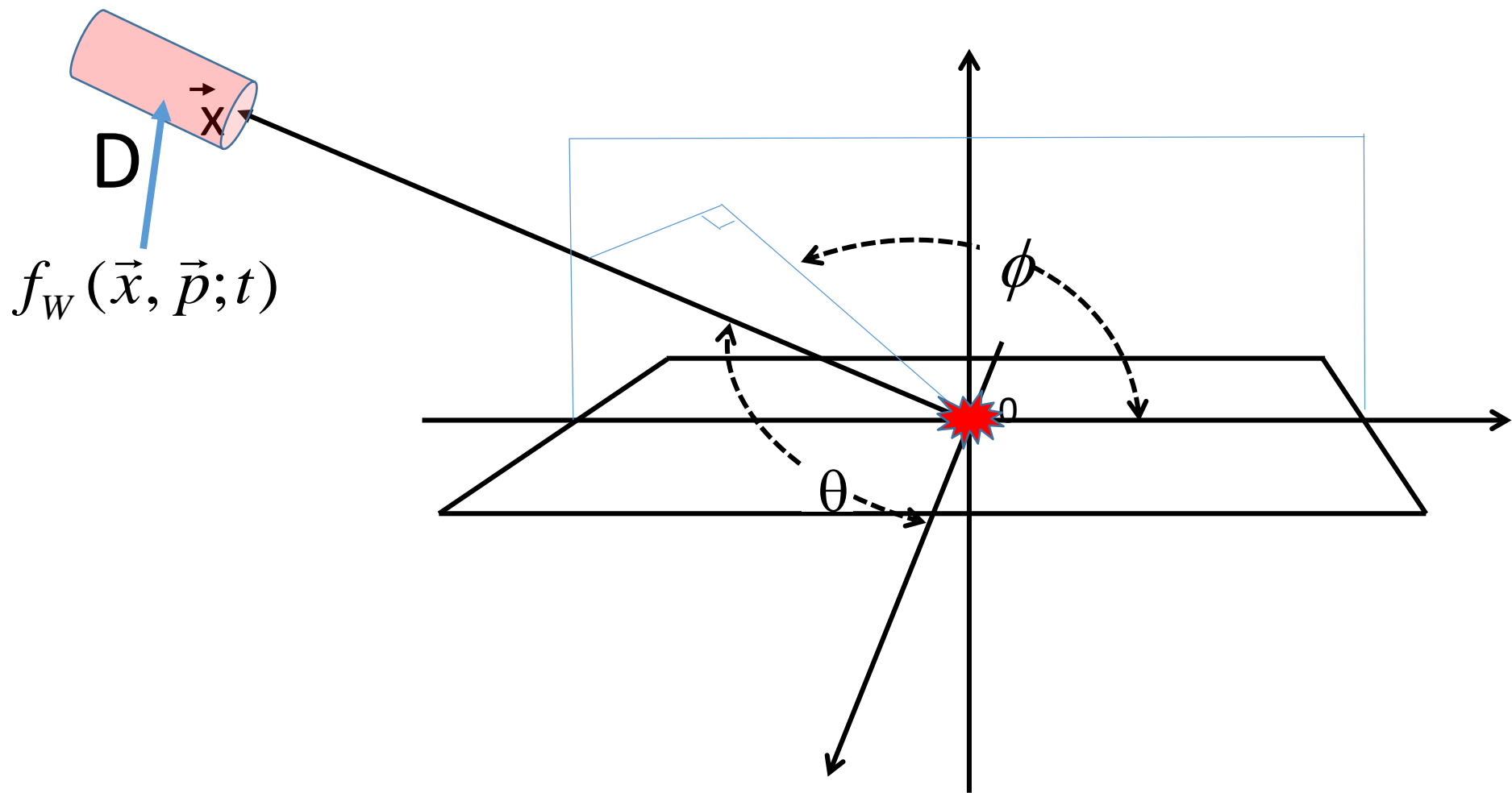
From the definition, we know that

$$\frac{d^3 n}{d^3 \vec{p}} = \int d^3 \vec{r} f_W(\vec{x}, \vec{p}; t_{FO}) = |\psi(p)|^2,$$

where t_{FO} is the freezeout time. But it is instructive to calculate the spectrum as the energy that enters into the detector from the large distance behavior of the Wigner function as;

$$E_{Detector} = \int_{\vec{r} \in D} d^3 \vec{r} f_W(\vec{x}, \vec{p}; t),$$

where D is the domain of the detector positioned at a solid angle $d^2\Omega$.



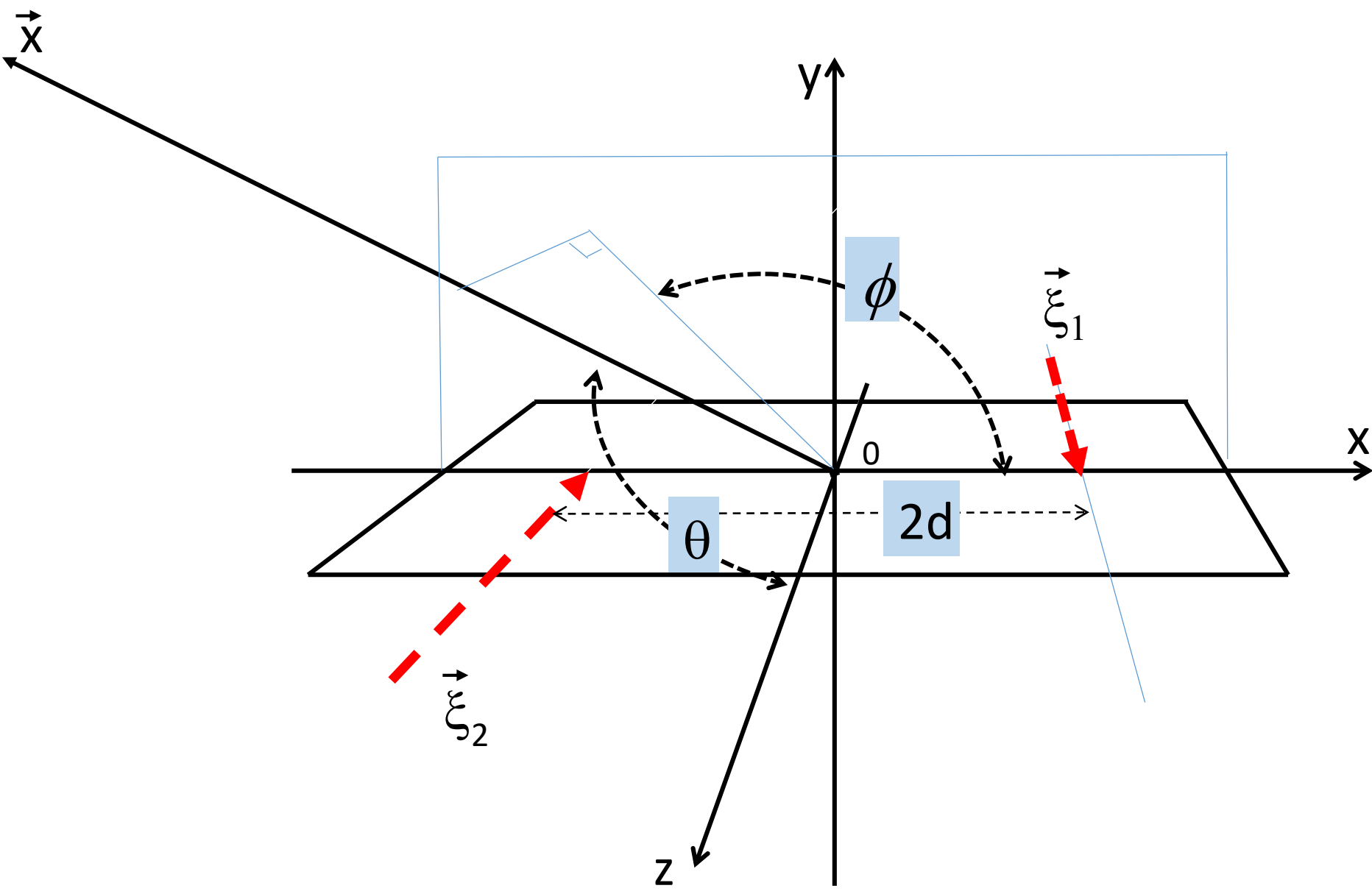
Minimal Model as Extreme Coherence

- Each set of participant protons of the two incident nuclei are considered as **point-like charges** with the effective charge Z_{eff} .
- These two effective charges **stop instantaneously** as the two nuclei collide.

Minimal Model as Extreme Coh



- Each set of participant protons of the two incident nuclei are considered as **point-like charges** with the effective charge Z_{eff} .
- These two effective charges **stop instantaneously** as the two nuclei collide.



Taking the trajectories as $\vec{\xi}_1(t) = \begin{pmatrix} d \\ 0 \\ tV_0\theta(-t) \end{pmatrix}$, $\vec{\xi}_2(t) = \begin{pmatrix} -d \\ 0 \\ -tV_0\theta(-t) \end{pmatrix}$

we get for the fields created by the trajectory-1 as

$$\vec{E}_1(\vec{x}, t) = \frac{eV_0 Z_{eff}}{4\pi} \frac{1}{r_-^3} \begin{pmatrix} -(x-d)z \\ -xy \\ (x-d)^2 + y^2 \end{pmatrix} \delta(t-r_-), \quad \vec{B}_1(\vec{x}, t) = \frac{eV_0 Z_{eff}}{4\pi} \frac{1}{r_-^2} \begin{pmatrix} y \\ -(x-d) \\ 0 \end{pmatrix} \delta(t-r_-),$$

where $r_- = \sqrt{(x-d)^2 + y^2 + z^2}$ and for the trajectory-2, we get simply by the following replacement,

$$\begin{aligned} \vec{E}_1(\vec{x}, t), \vec{B}_1(\vec{x}, t) &\Rightarrow \vec{E}_2(\vec{x}, t), \vec{B}_2(\vec{x}, t) \\ (d, V_0) &\Rightarrow (-d, -V_0) \end{aligned}$$

From these, we have

$$f_{E,B}(\vec{x}, \vec{p}; t) = f_{E,B}^{(0)}(\vec{x} - \vec{d}, \vec{p}; t) + f_{E,B}^{(0)}(\vec{x} + \vec{d}, \vec{p}; t) \\ - 2 \cos(2\vec{p} \cdot \vec{d}) f_{E,B}^{(0)}(\vec{x}, \vec{p}; t)$$

where $f_{E,B}^{(0)}(\vec{x}, \vec{p}; t) = f_{E,B}(\vec{x}, \vec{p}; t) \Big|_{d=0}$

and $f_E(\vec{x}, \vec{p}; t) = \int d^3\vec{u} \left\{ e^{i\vec{p} \cdot \vec{q} / \hbar} \vec{E}(\vec{x} - \vec{q} / 2, t) \cdot \vec{E}(\vec{x} + \vec{q} / 2, t) \right\} ,$

and analogously for f_B .

These integrals contain the product of two delta functions

$$\delta\left(t - \sqrt{(\vec{x} + \vec{q}/2)^2}\right) \delta\left(t - \sqrt{(\vec{x} - \vec{q}/2)^2}\right)$$

which reduces for large distance (i.e., $r, t \gg 1/p, d$) to

$$\delta\left(t - \sqrt{(\vec{x} + \vec{q}/2)^2}\right) \delta\left(t - \sqrt{(\vec{x} - \vec{q}/2)^2}\right) \approx \delta(t - x) \delta(q_{||})$$

for arbitrary vector \vec{q} with $|\vec{q}| \approx d$, and $q_{||}$ is the parallel component of \vec{q} with respect to \vec{x}

Finally we get

$$f_W(\vec{x}, \vec{p}; t) \cong 4\pi^2 \alpha_{EM} Z_{eff}^2 V_0^2 \frac{1}{p^2 r^2} \left\{ 1 - \cos(\vec{p} \cdot \vec{d}) \right\} \delta^2(\vec{\Omega}_{\vec{p}} - \vec{\Omega}_{\vec{x}})$$

The delta function (the last term) means that at a large (macroscopic) distance, the observational direction coincides with that of the momentum detected.

The energy spectrum is given by the integral within the volume of the detector D , which is equivalent to

$$\frac{d^3 \mathcal{E}}{d\vec{p}^3} = \frac{1}{(2\pi)^3} \int dt \int_{\Omega \in D} d^2 \Omega_{\vec{x}} R_D^2 f_W(\vec{R}_D, \vec{p}; t)$$

where R_D is the position of the detector D .

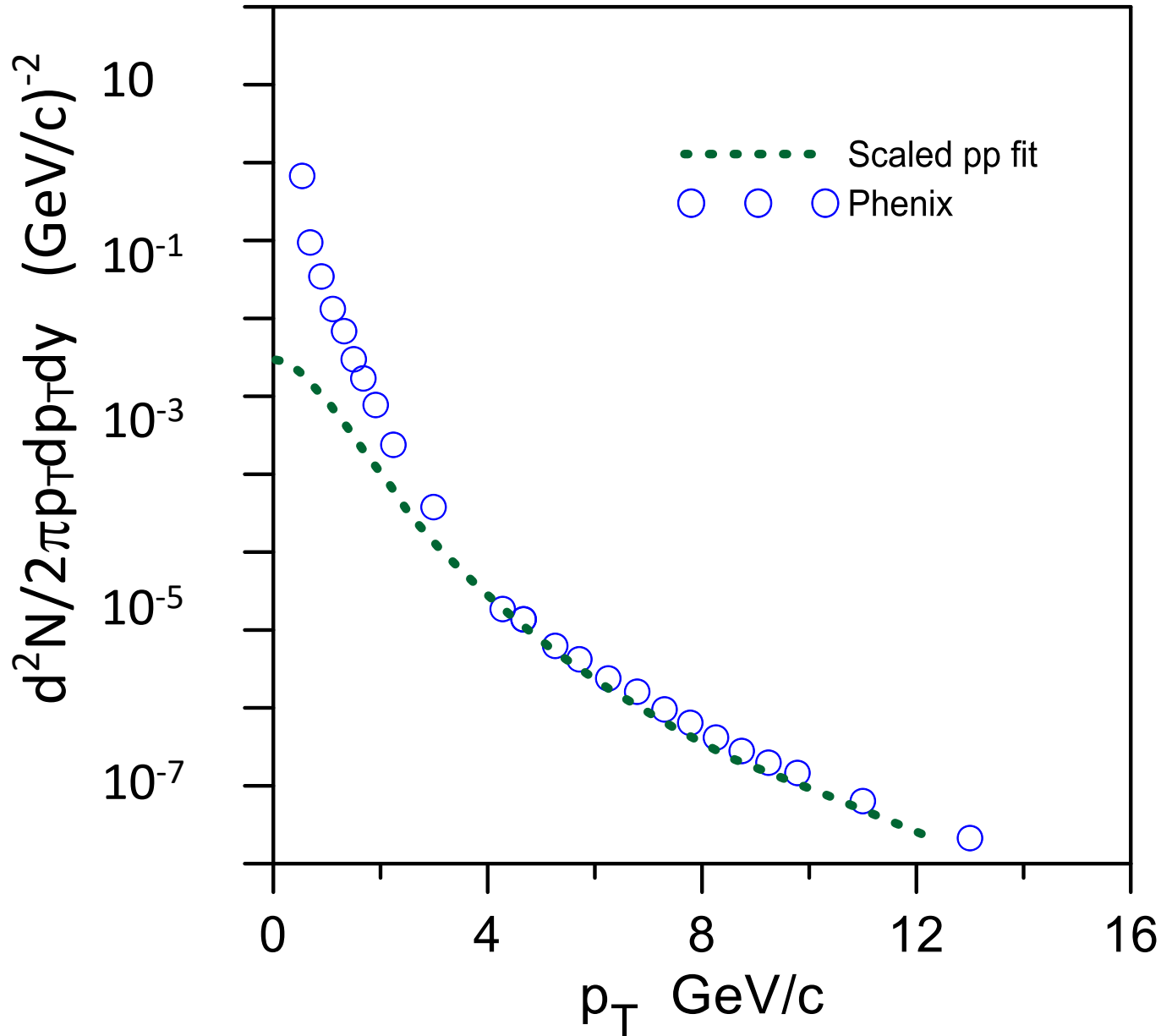
Photon number distribution is

$$\frac{d^3 N}{d\vec{p}_T^2 dy} = \frac{1}{2\pi^2} \frac{\alpha_{EM} Z_{eff}^2 V_0^2}{p^2 \cosh^2 y} \left\{ 1 - \cos(2\vec{p} \cdot \vec{d}) \right\}$$

Angle integrated p_T spectrum is given by ($V_0 \sim 1$)

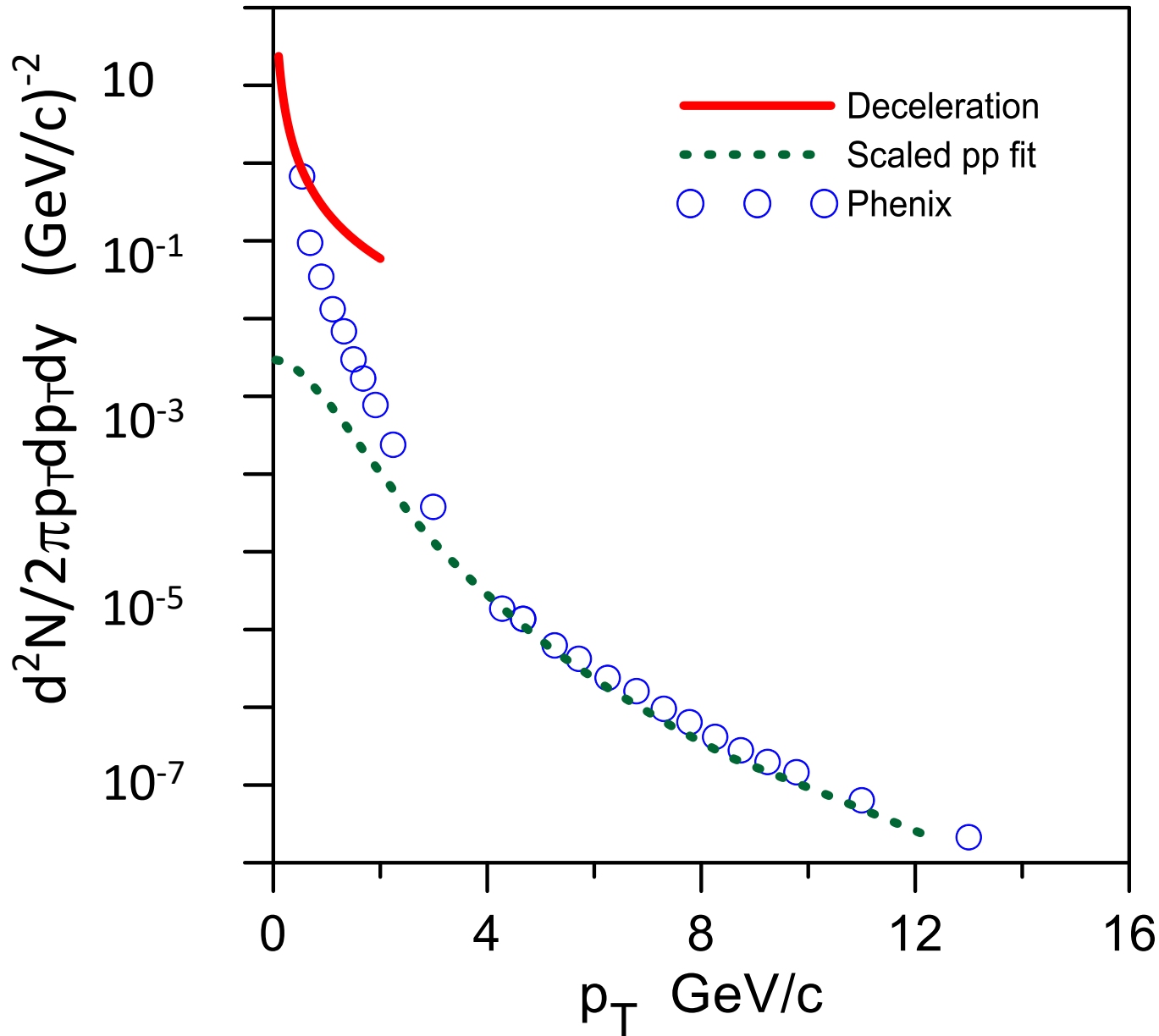
$$\left. \frac{d^3 N}{2\pi p_T dp_T dy} \right|_{y=0} \simeq 0.37 \times 10^{-3} \frac{Z_{eff}^2}{p_T^2} \left\{ 1 - J_0(2pd) \right\}$$

p_T Spectrum



$$V_0 \approx 1,$$
$$Z_{eff} \approx 80,$$
$$d \approx 1 \text{ fm}$$

p_T Spectrum

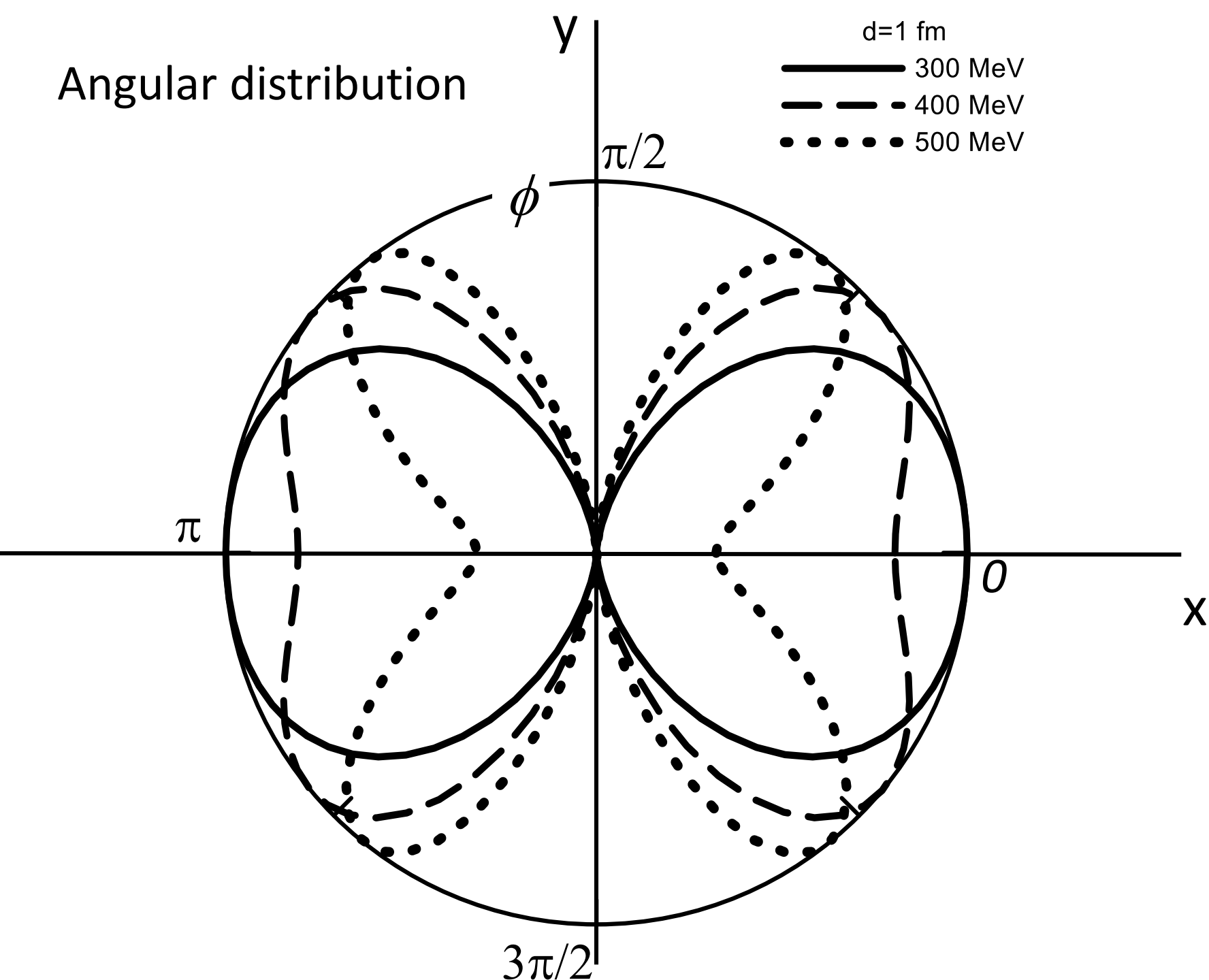


$$V_0 \approx 1,$$

$$Z_{eff} \approx 80,$$

$$d \approx 1 \text{ fm}$$

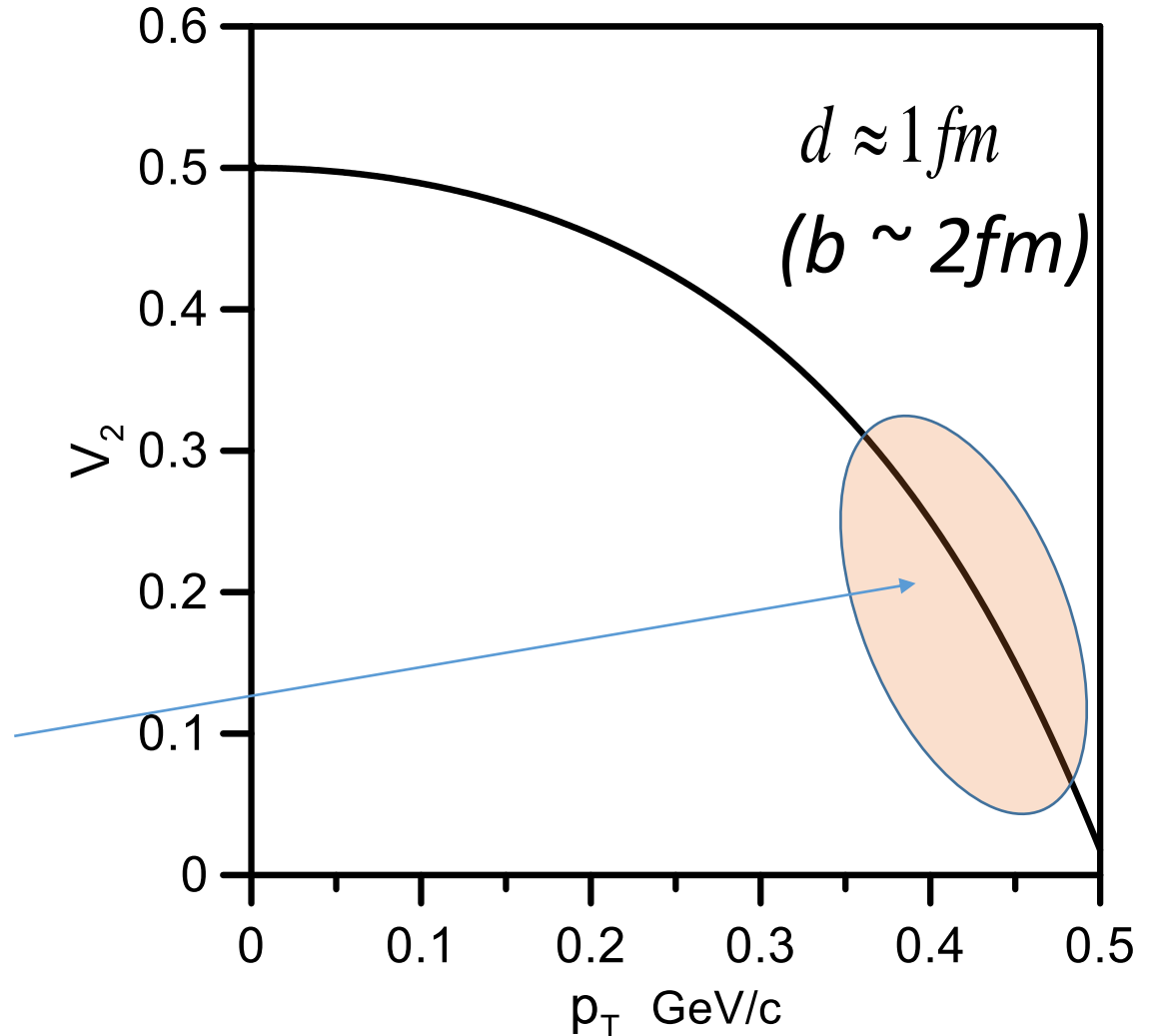
Angular distribution



Anisotropy Parameter v_2

$$v_2 = \frac{J_2(2p_T d)}{1 - J_0(2p_T d)}$$

Large increase of v_2 for lower p_T !



If we suppose incoherence occurs between electromagnetic fields generated by the incidente nuclei as exponentially with p_T , we may expect*

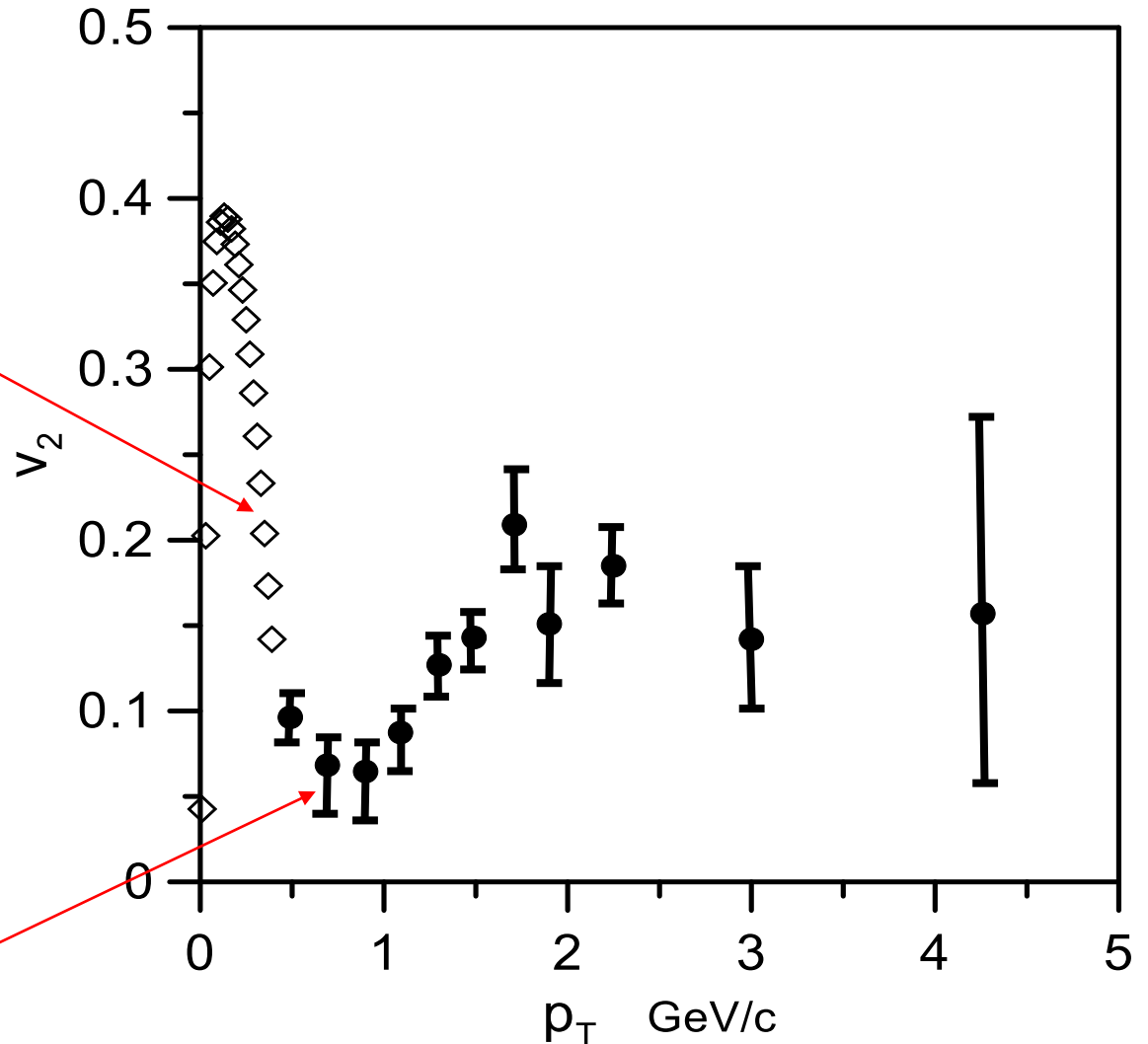
$$v_2 \approx \frac{J_2(2p_T d)}{1 + 2e^{2p_T / \Delta p} / Z_{eff} - J_0(2p_T d)}$$

* T.S. Biró, Z. Szendi and Z. Schram, Euro. Phys.J A50, 62 (2014)

Curiously,...

$V_0 \approx 1$, $Z_{eff} \approx 80$,
 $d \approx 1 \text{ fm}$,
 $\Delta p = 0.2 \text{ GeV}$,

PHENIX data
Centrality 20-40%
arXiv:1509.07758



Can the Classical Bremsstrahlung of Electromagnetic Fields be useful to understand

- Event by Event Initial Geometry
- Event by Event inhomogeneity and fluctuations
- Baryon Stopping

?

Can the Classical Bremsstrahlung of Electromagnetic Fields be useful to understand

- Event by Event Initial Geometry
- Event by Event inhomogeneity and fluctuations
- Baryon Stopping

?

More realistic simulations !

Can the O Electromagneti

- Event by Event In
- Event by Event in
- Baryon Stopping



Can the Classical Bremsstrahlung of
Electromagnetic Fields be useful to understand



June 30

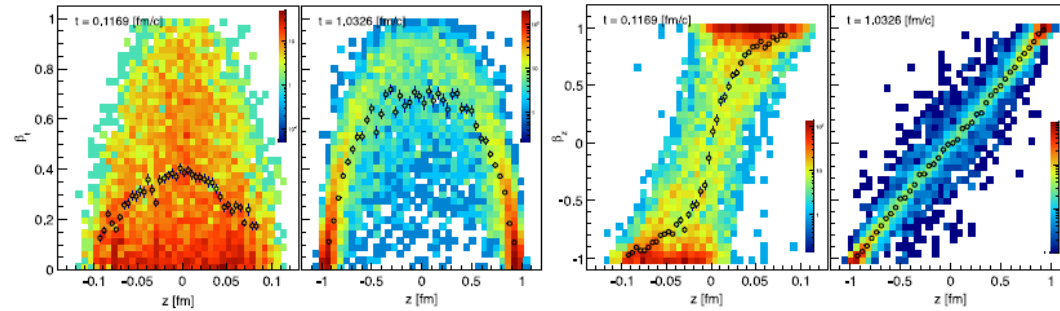


Fig. 12. Snapshots of the longitudinal profile of the particle velocity in PHSD for two different time-steps of the evolution for a central Au-Au collision at $\sqrt{s_{NN}} = 200$ GeV. The color scale in each panel represents the number of particles. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

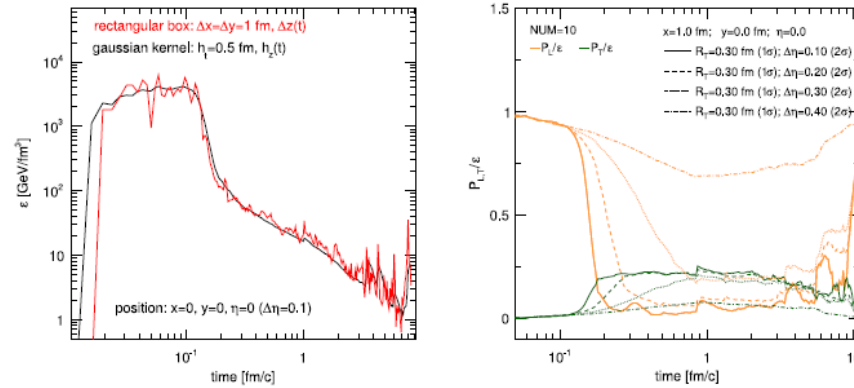
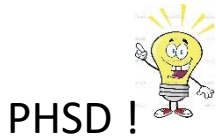


Fig. 13. Dependence of the local energy density ϵ with the time evolution for two different types of kernel functions (left plot); dependence of the longitudinal and transverse pressure components with the system evolution considering different characteristic lengths in the longitudinal direction (right plot). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



PHSD !

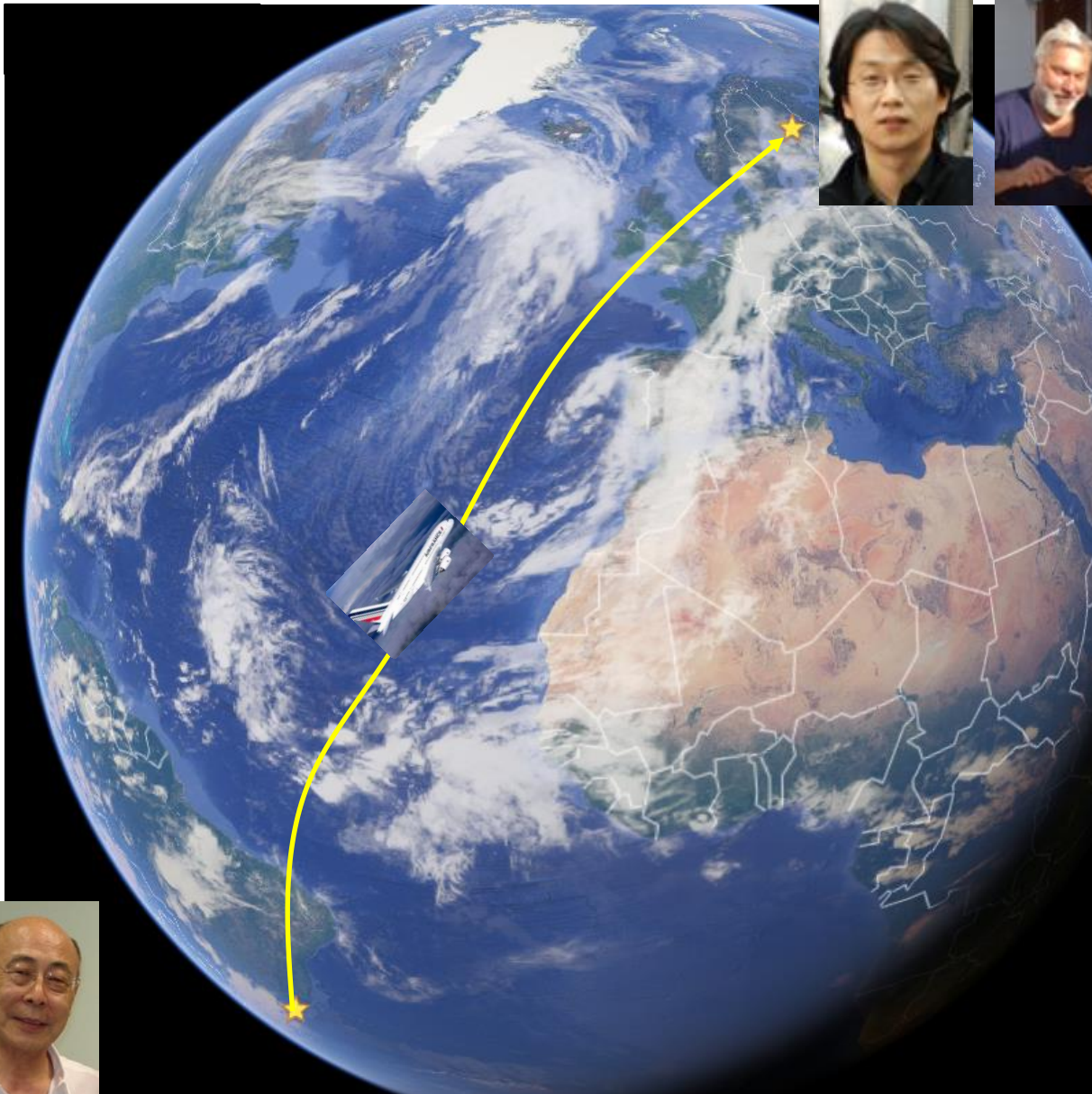
June 30, I tried to Frankfurt



PHSD !



Yasushi Horst



PHSD ! 



Yasushi Horst



PHSD !



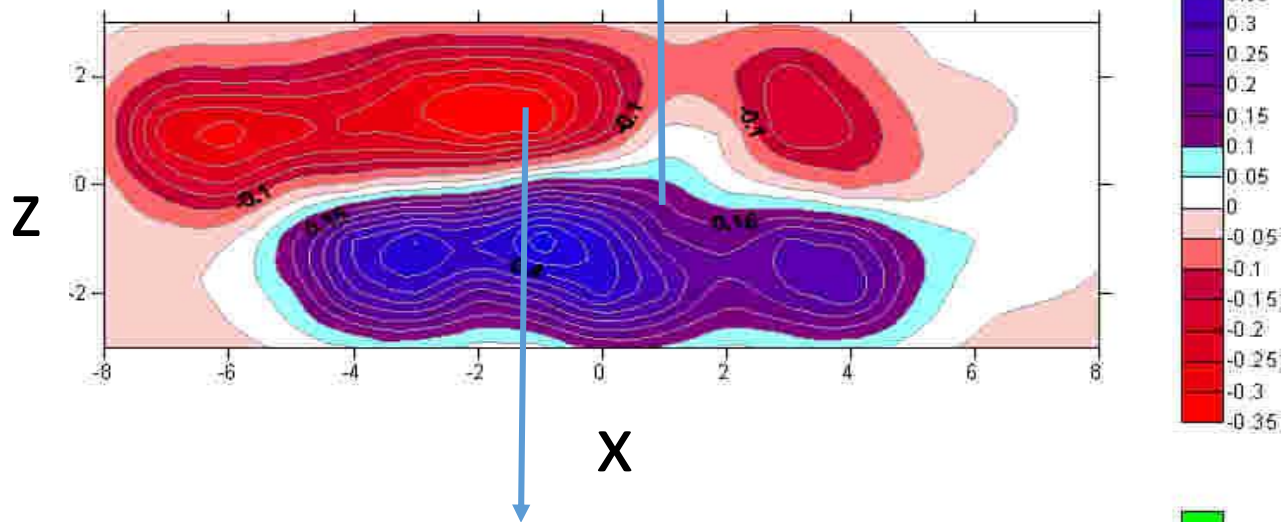
PHSD -> JAM

the time evolution of electric charge current density of a few events
for

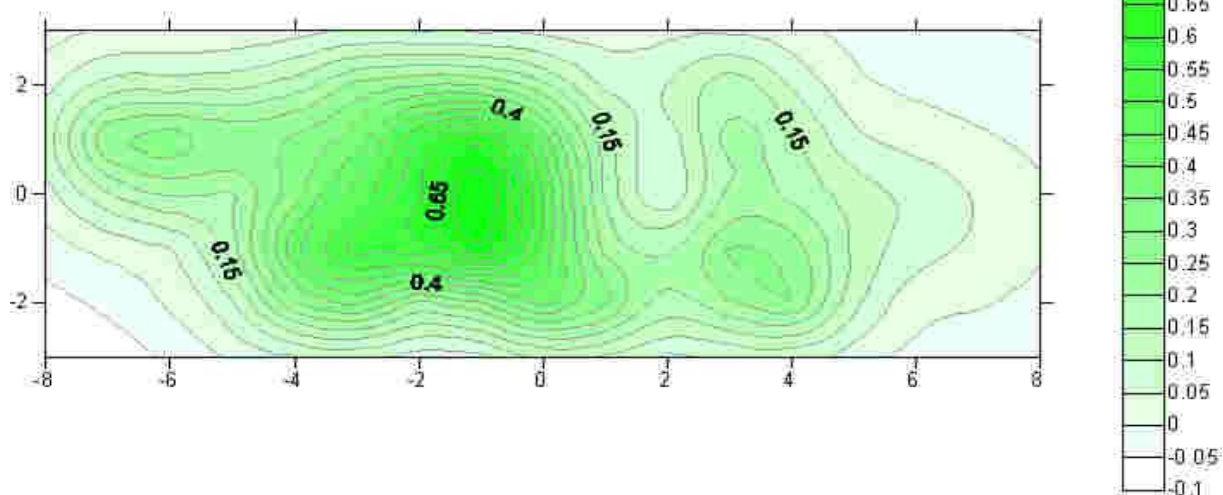
Au+Au $b=2\text{fm}$ $S_{qs} = 5\text{GeV}$

Example of the time evolution in $y=0$ plane $t=1.8\text{fm}$

Charge current density



Charge density

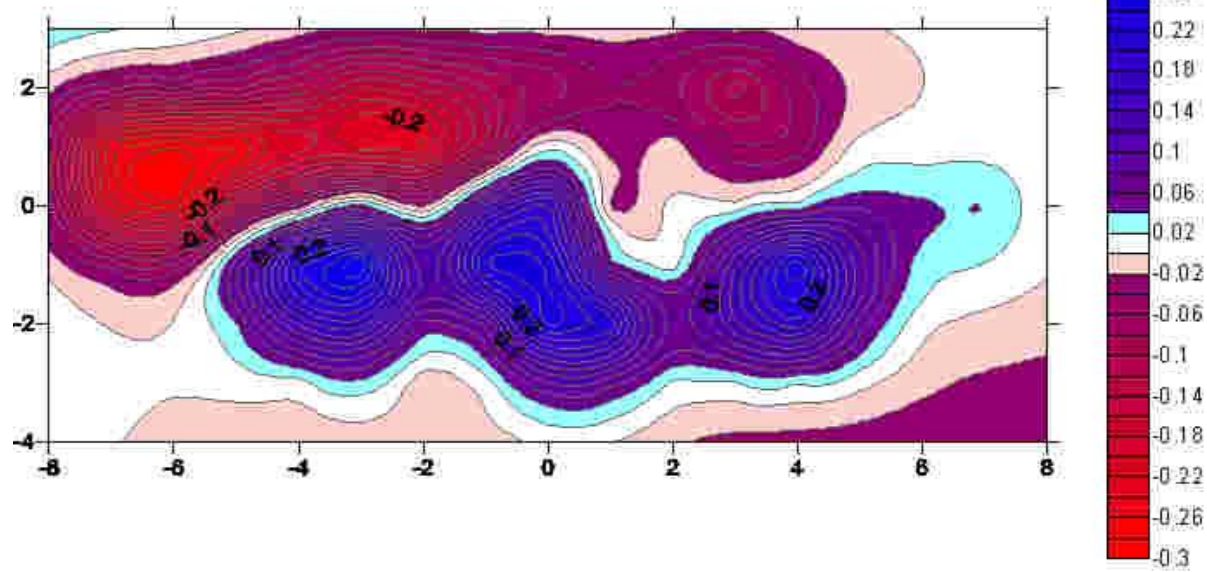


Au+Au, SQS=5GeV, $b=2\text{fm}$

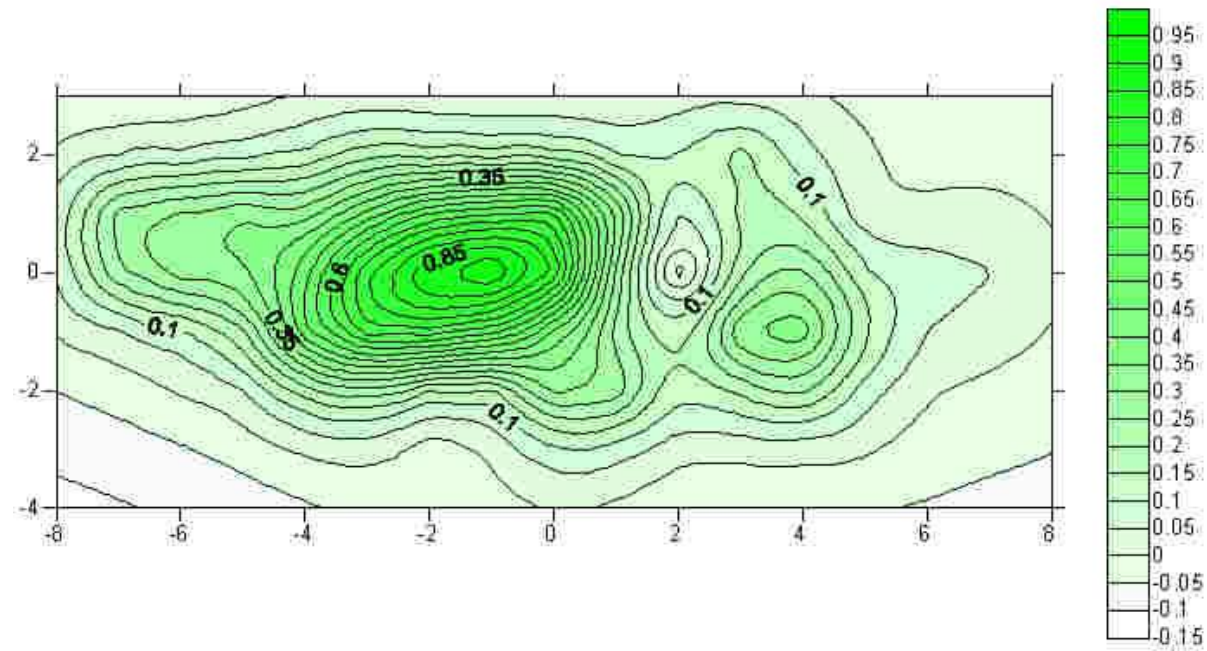
Example of the time evolution of current density

t=2.2fm

Charge current density



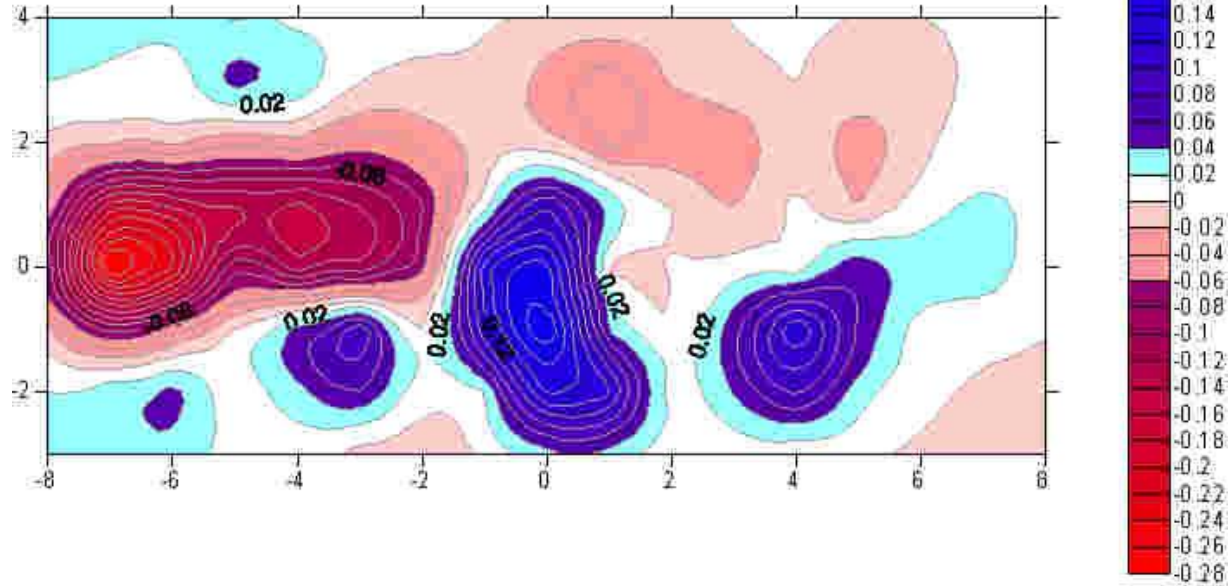
Charge density



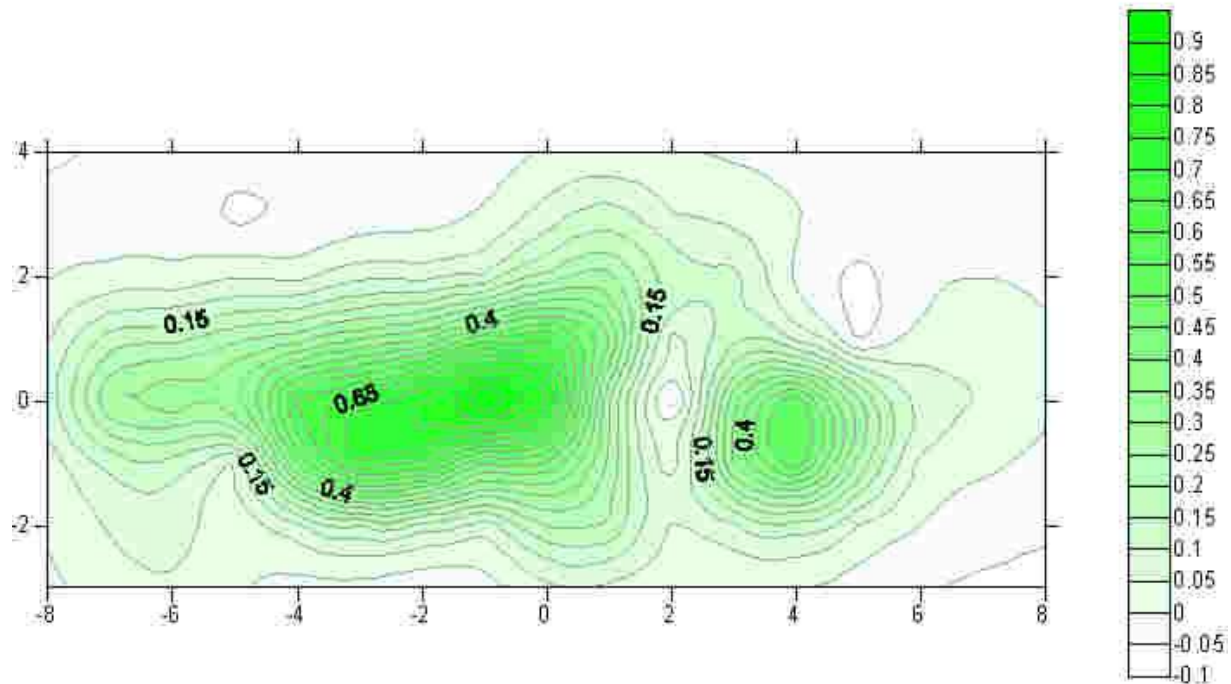
Example of the time evolution of current density

$t=2.6\text{fm}$

Charge current density



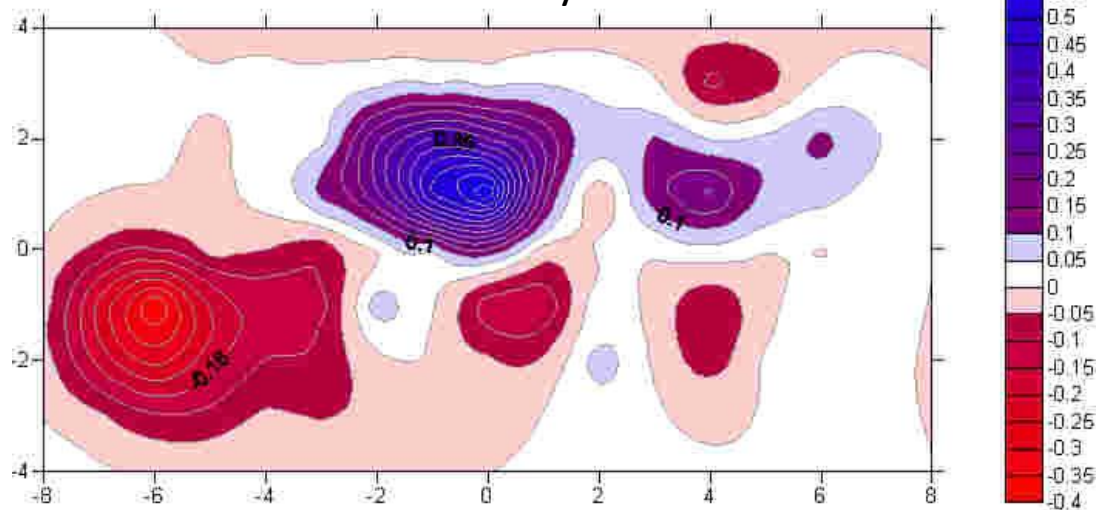
Charge density



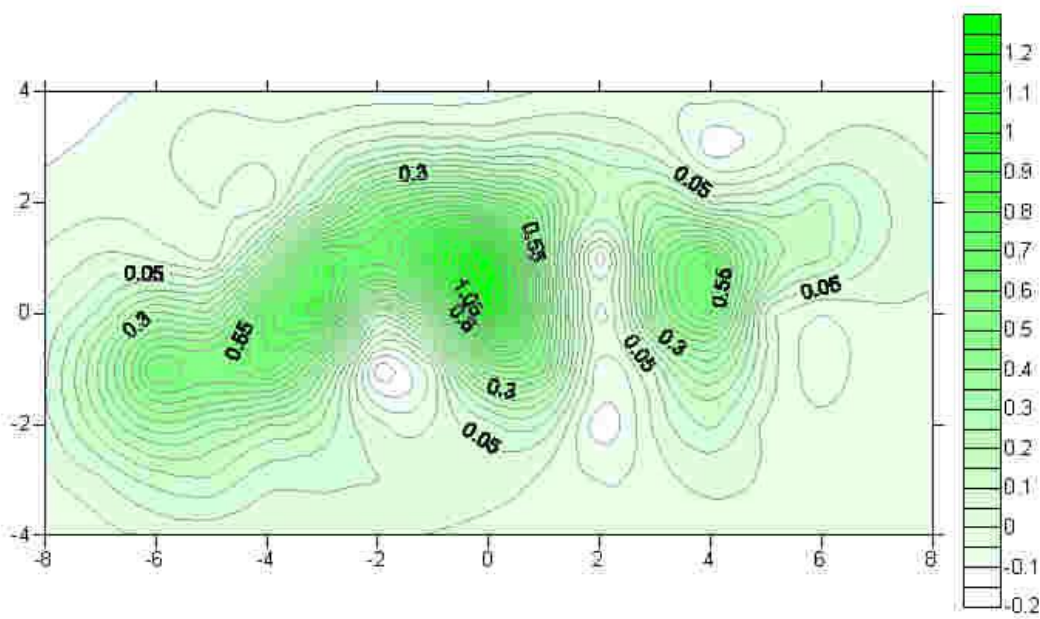
Example of the time evolution of current density

$t=4.2\text{fm}$

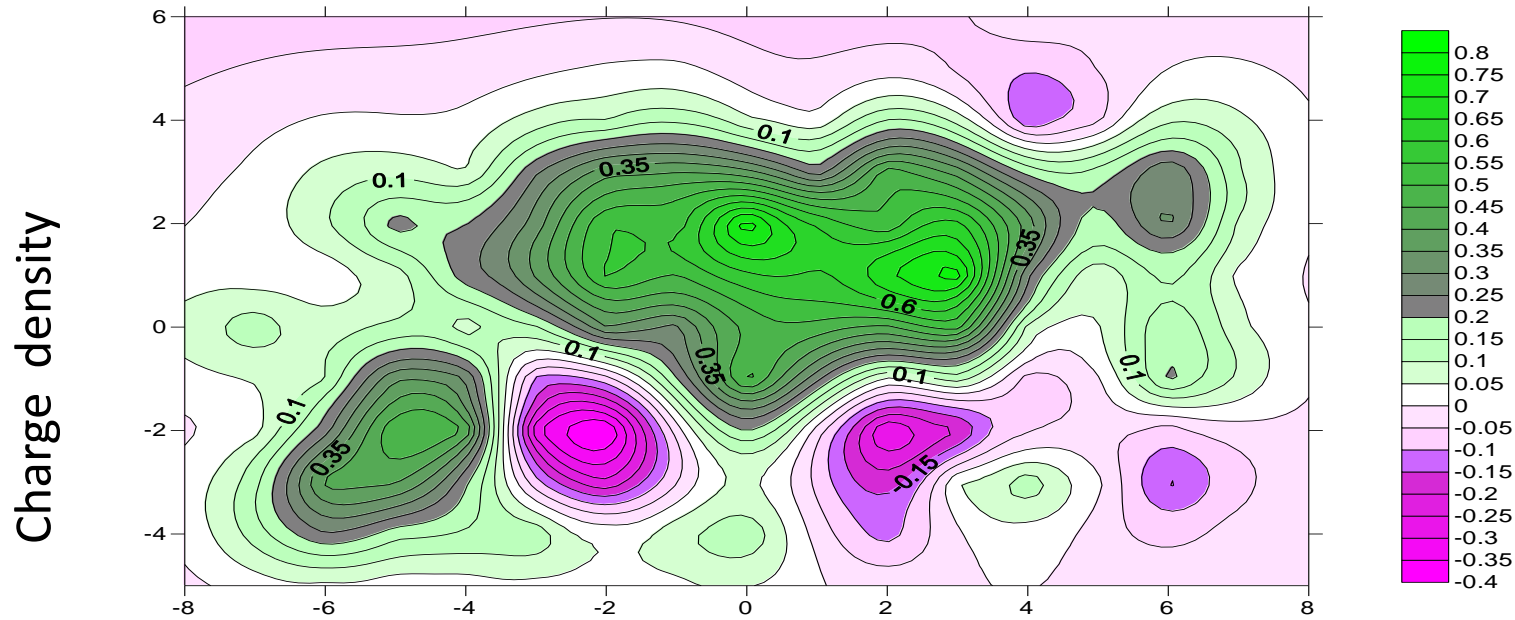
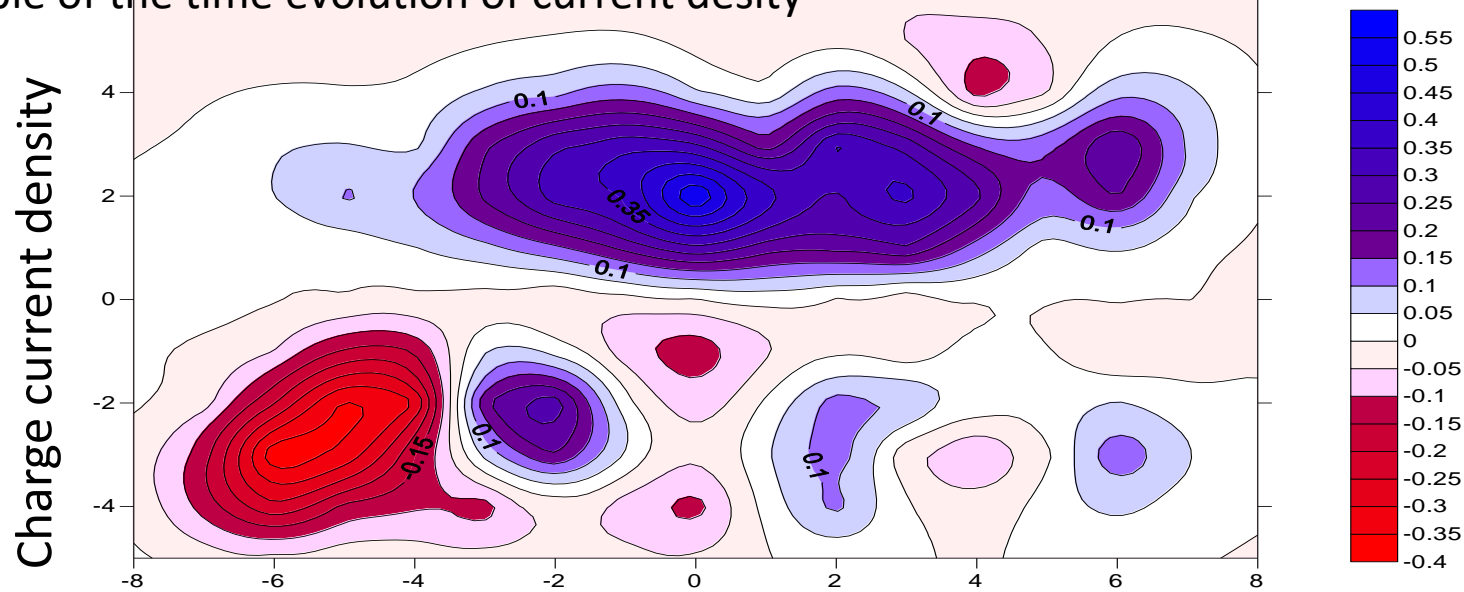
Charge current density



Charge density

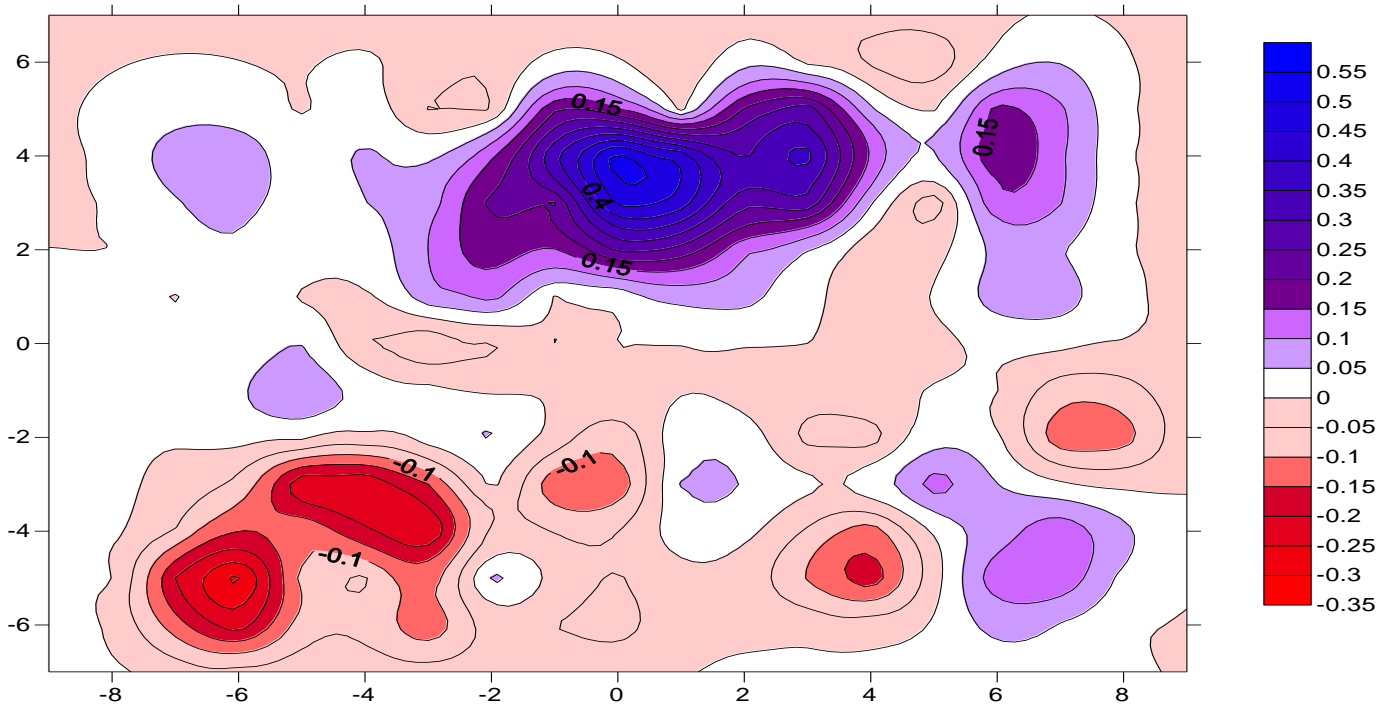


Example of the time evolution of current density t=6.0fm

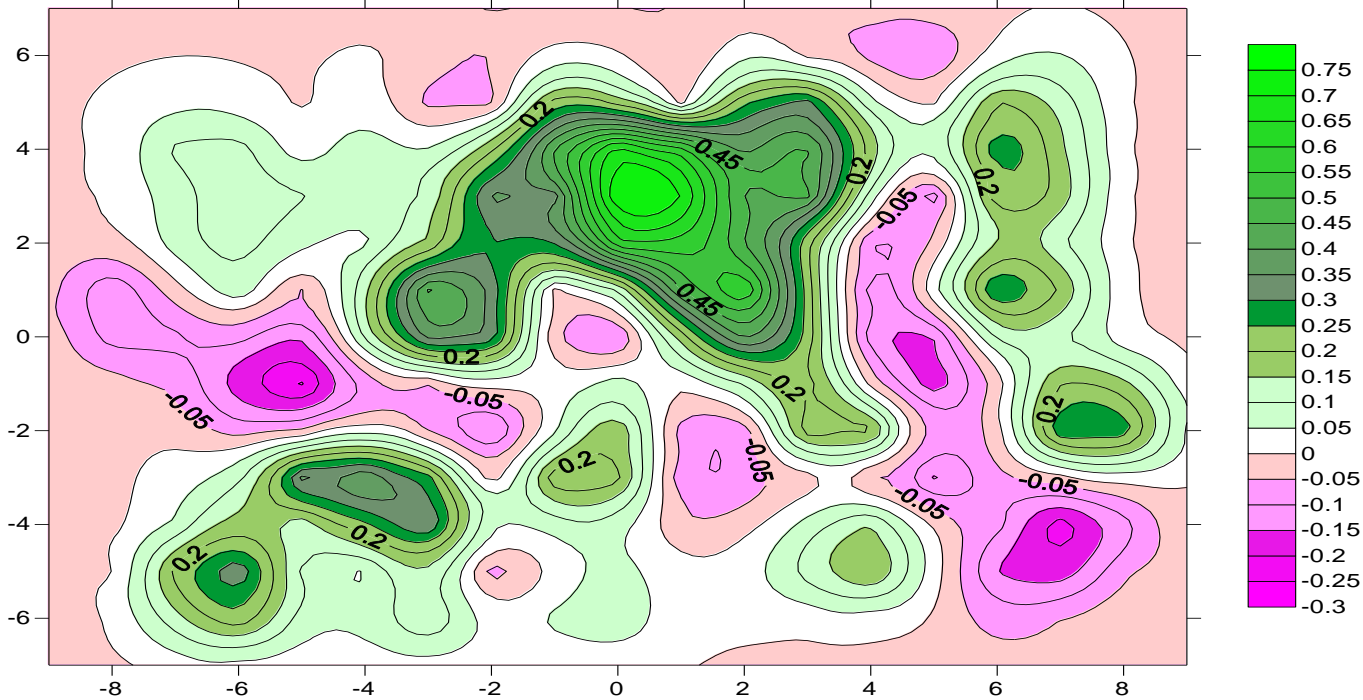


t=8.0fm

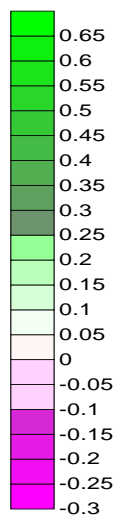
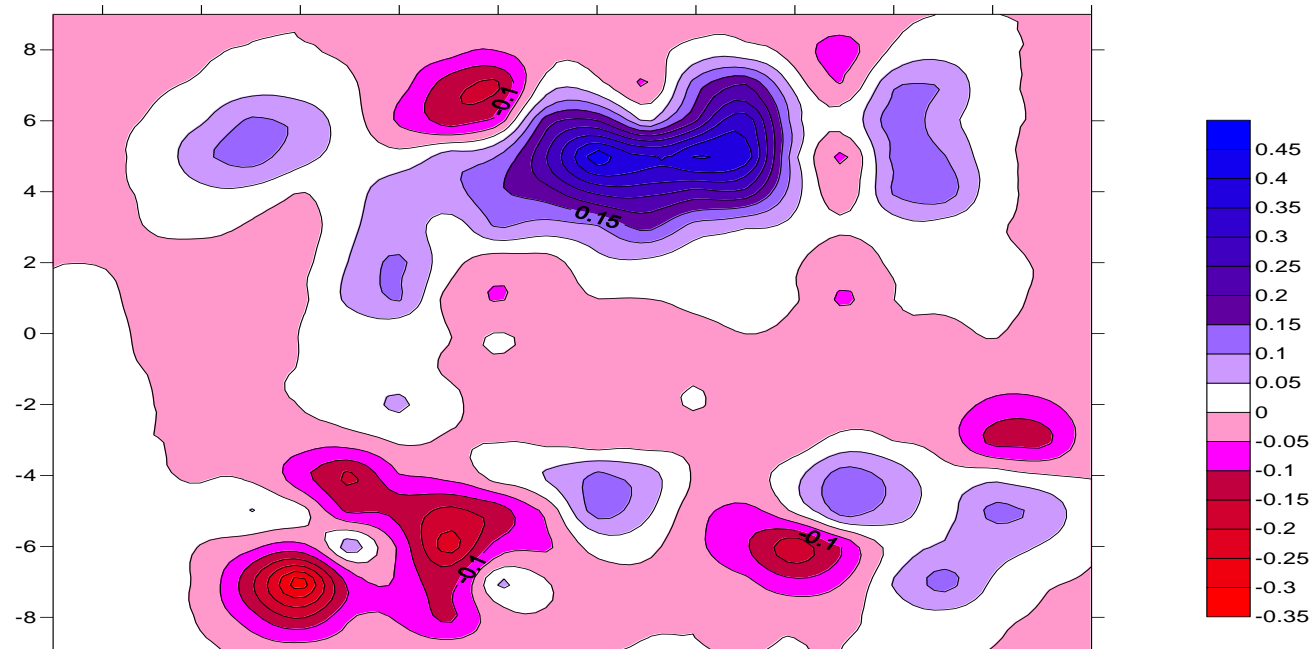
Charge current density

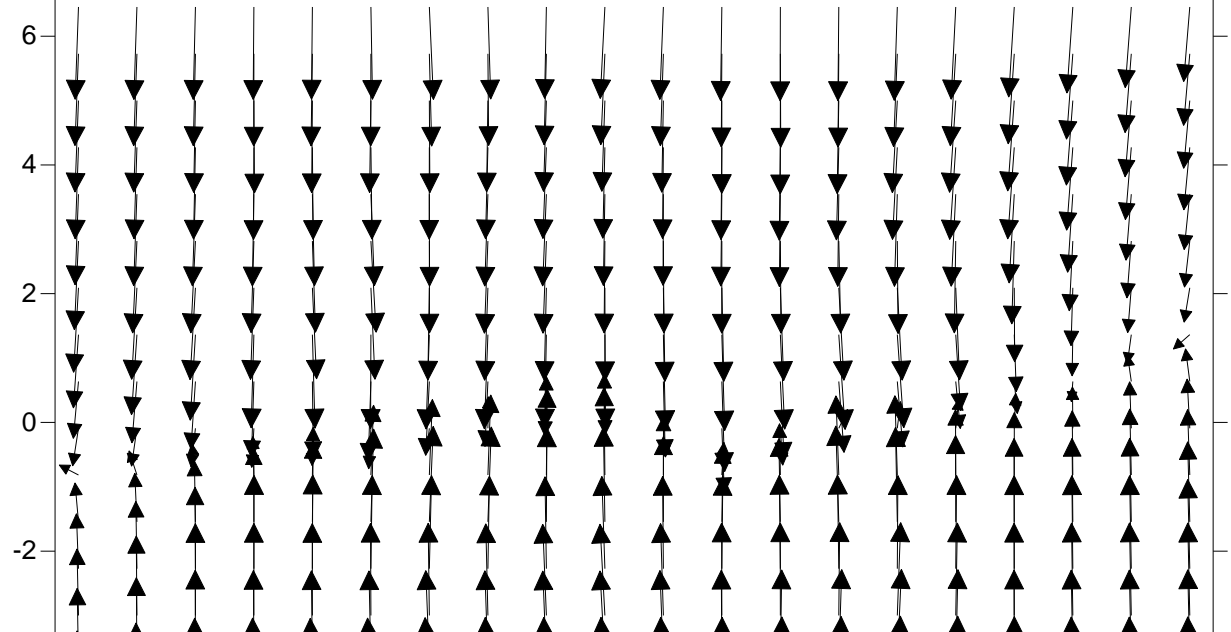
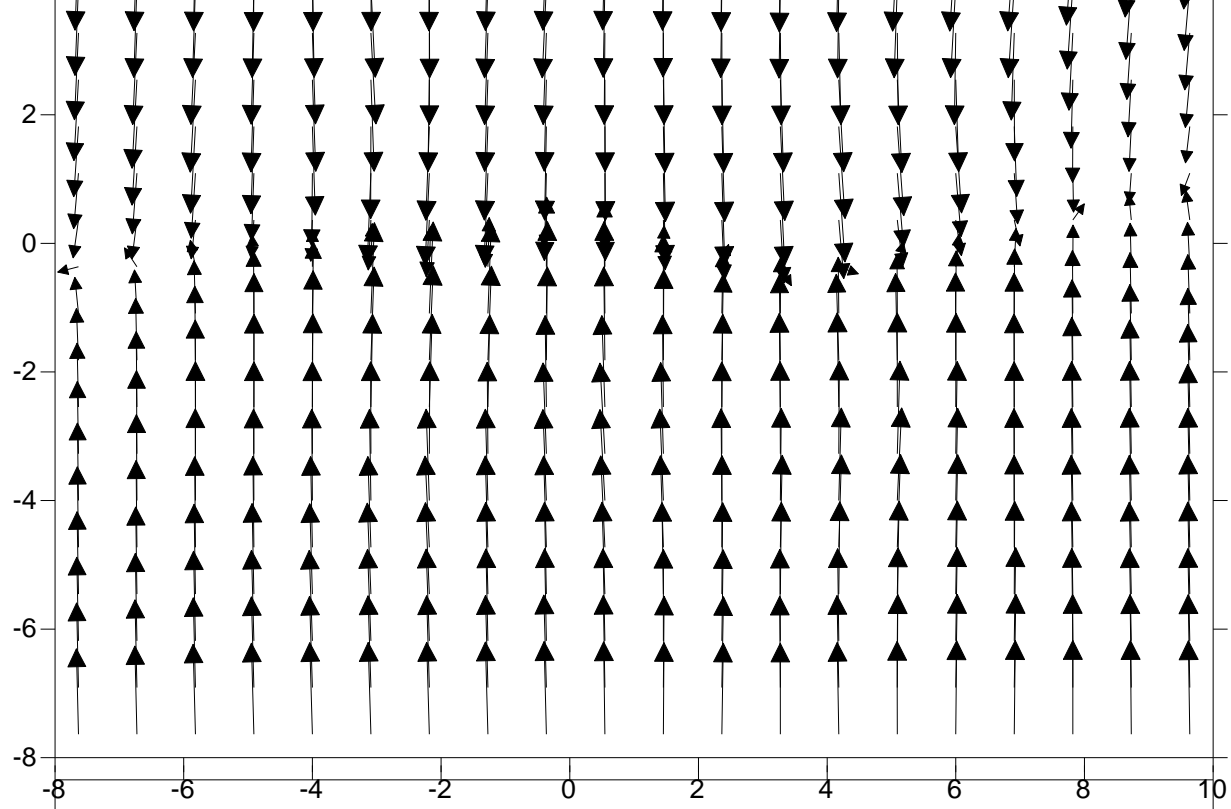


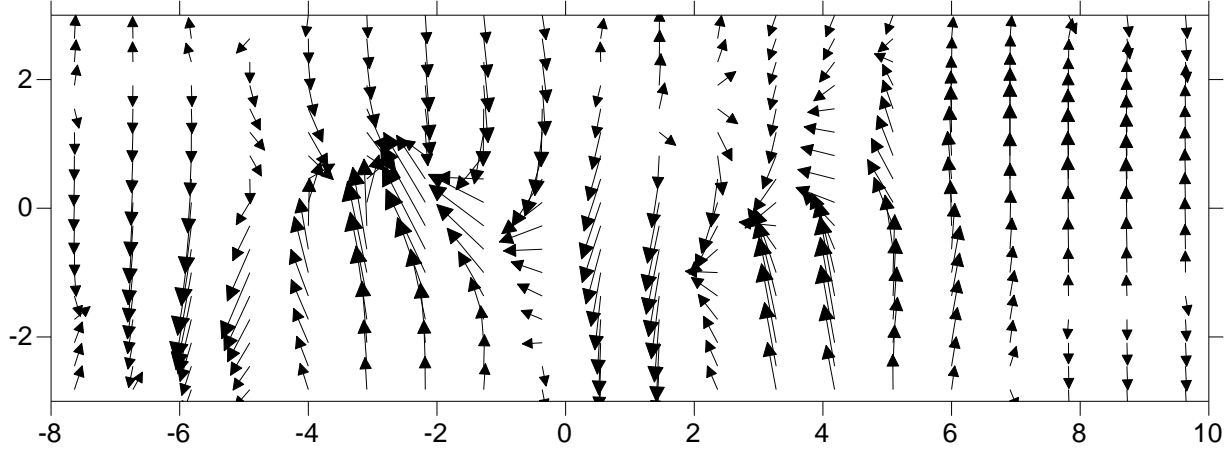
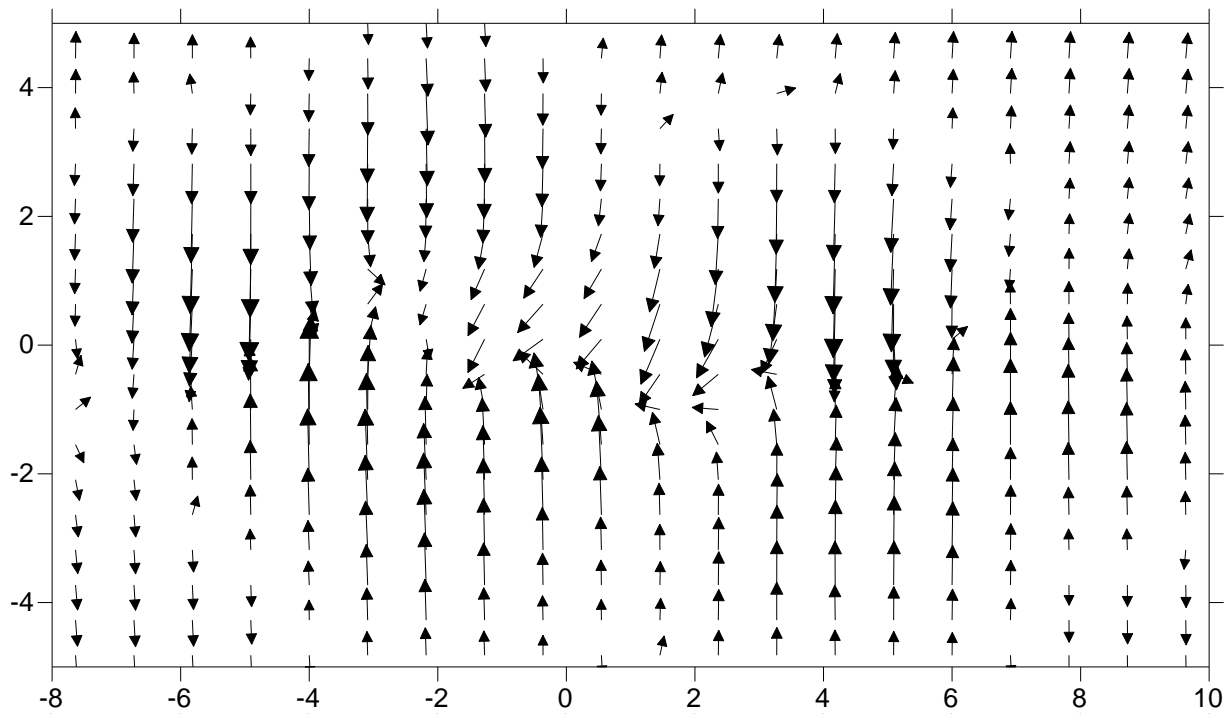
Charge density



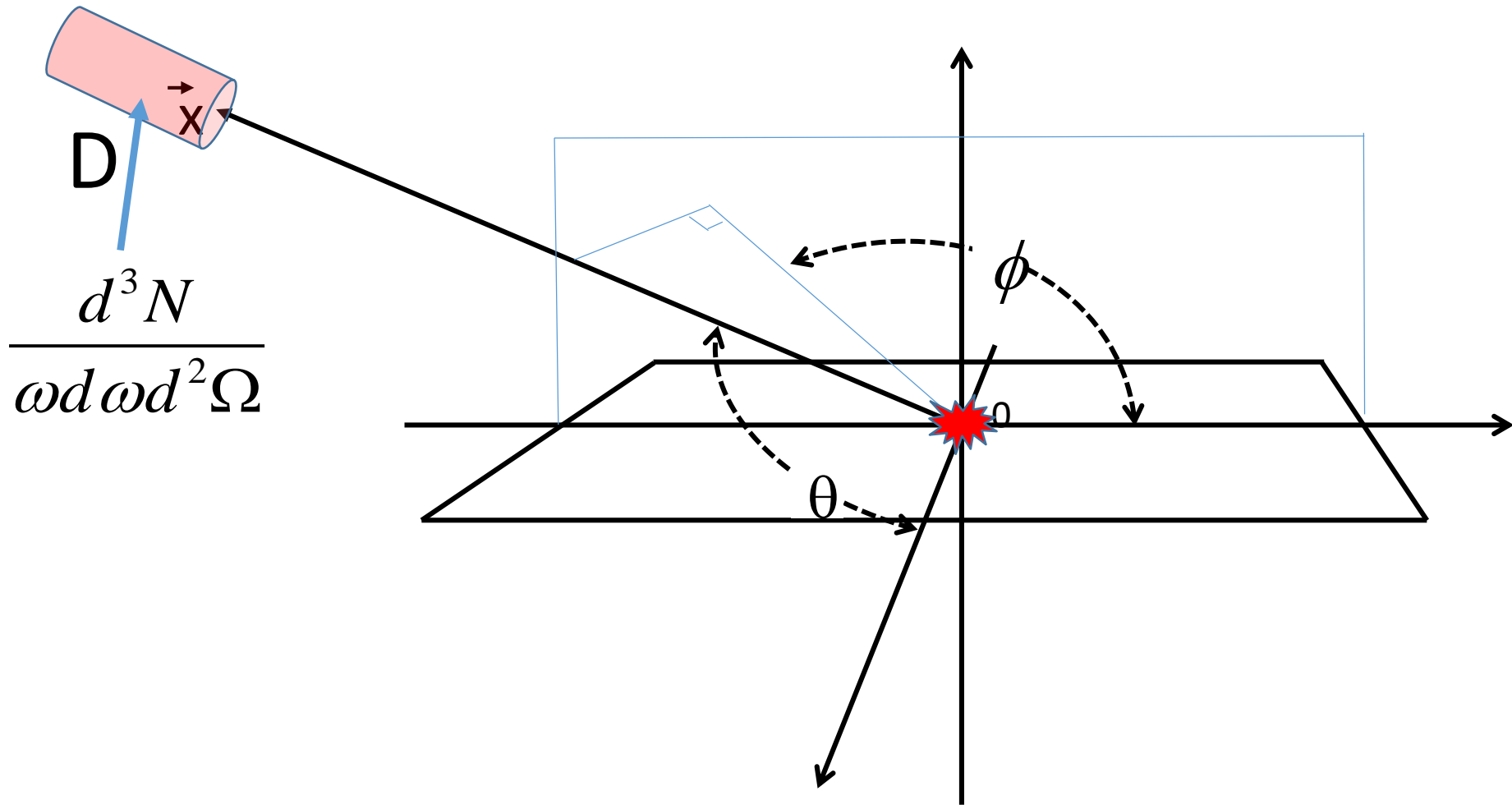
Charge current density







At large distance,

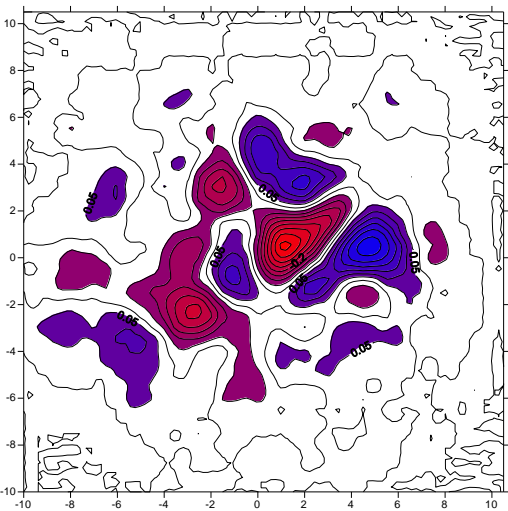


$$\frac{d^3 N}{d^2 \vec{p}_T dy} = \frac{d^3 N}{\omega d\omega d^2 \Omega} = \frac{1}{\pi} \left| \vec{A}(\omega, \vec{n}) \right|^2,$$

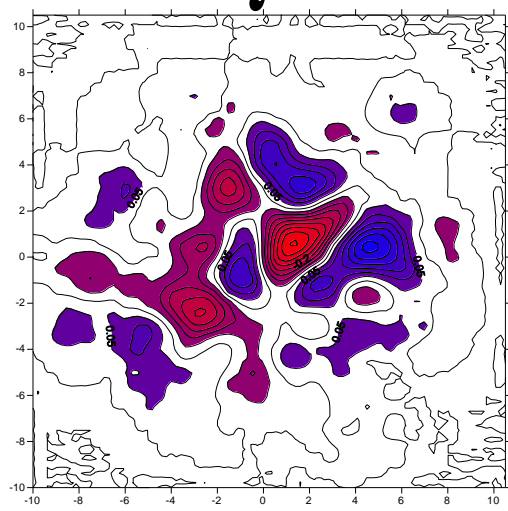
$$\vec{A}(\omega, \vec{n}) = -\frac{e}{4\pi} e^{i\omega r} \left(1 - \vec{n}\vec{n}^T \right) \int d^3 \vec{\xi} \int d\tau e^{i\omega(\tau - \vec{n} \cdot \vec{\xi})} \frac{\partial \vec{J}}{\partial \tau}(\vec{\xi}, \tau)$$

Mostly $J_z(\vec{\xi}, \tau)$ is dominant.

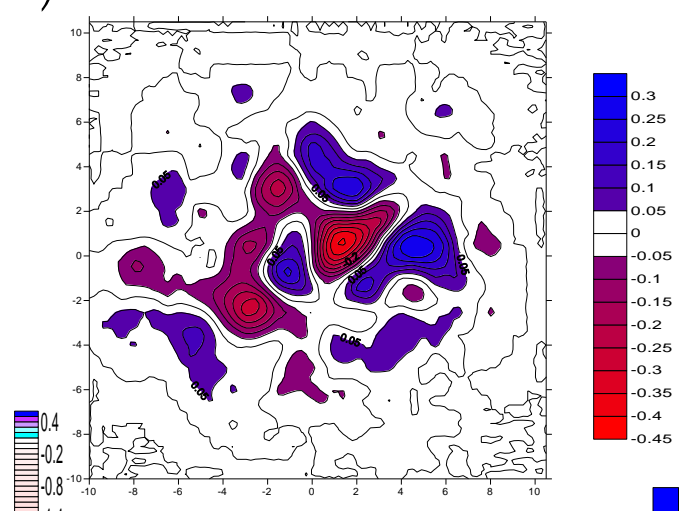
x-y profile of $\int d\xi_z dJ_z(\vec{\xi}, \tau) / dt$



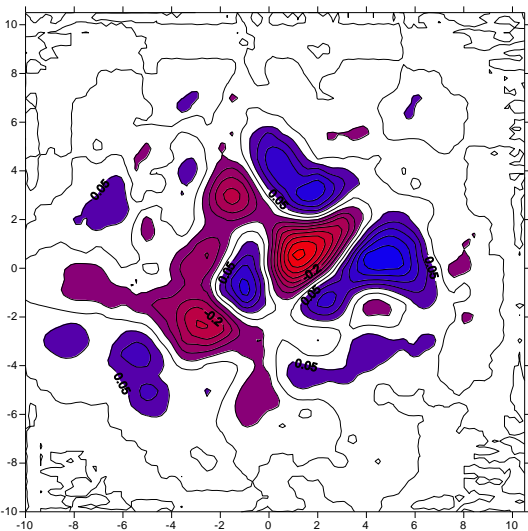
T=1.0 fm



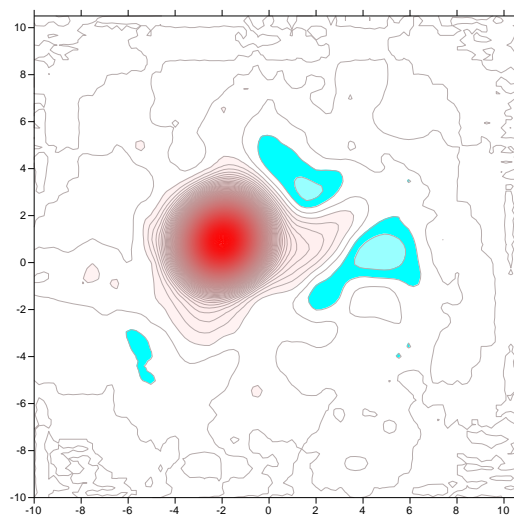
T=1.2 fm



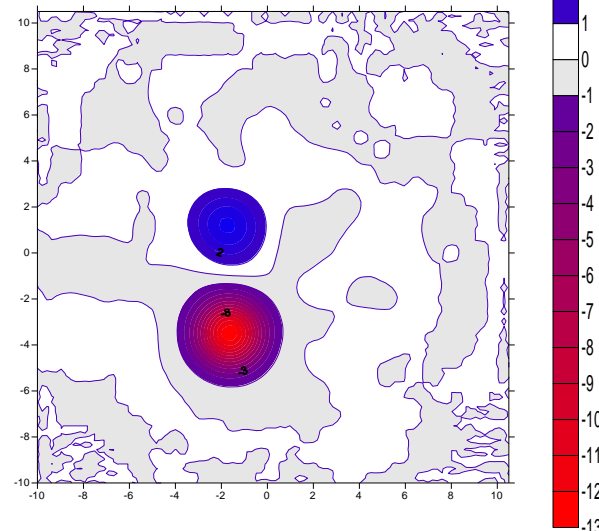
T=1.4 fm



T=1.6 fm

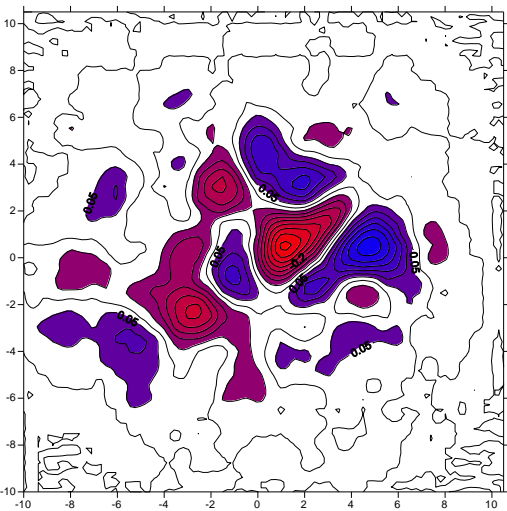


T=1.8 fm

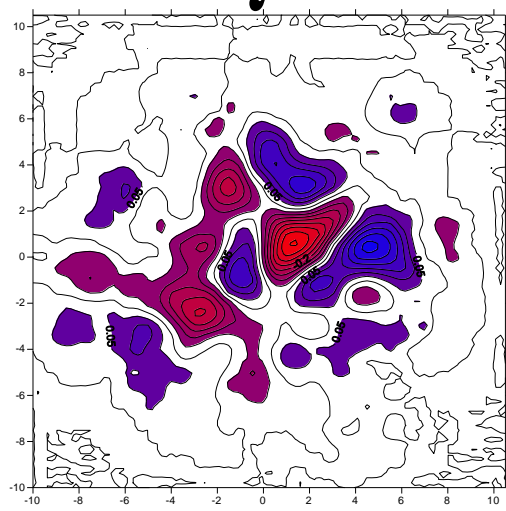


T=2.0 fm

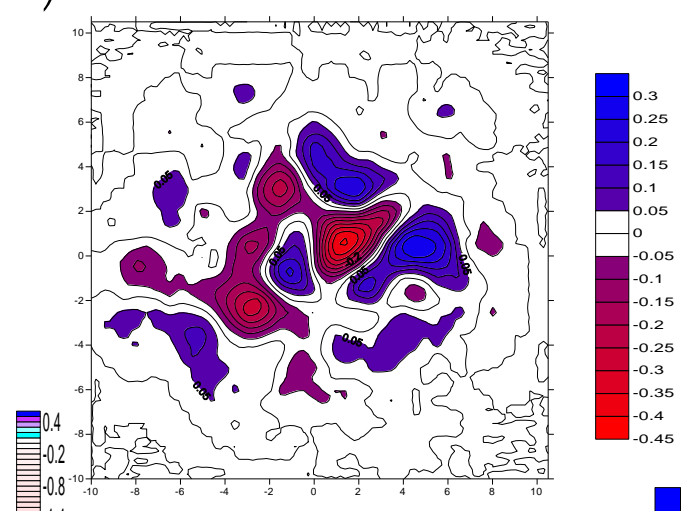
x-y profile of $\int d\xi_z dJ_z(\vec{\xi}, \tau) / dt$



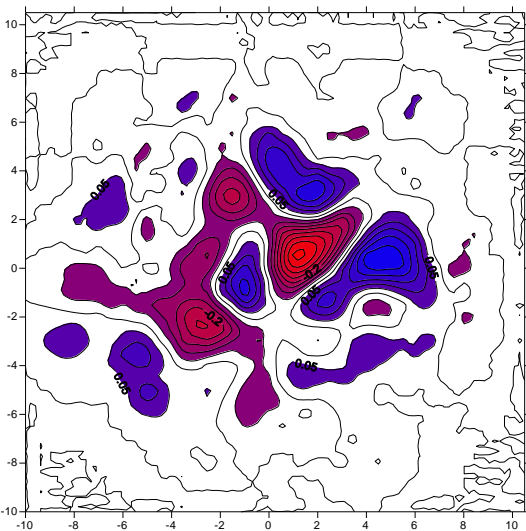
T=1.0 fm



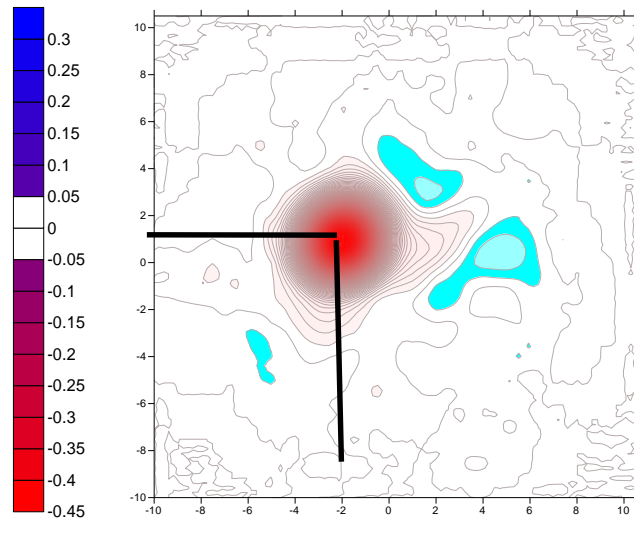
T=1.2 fm



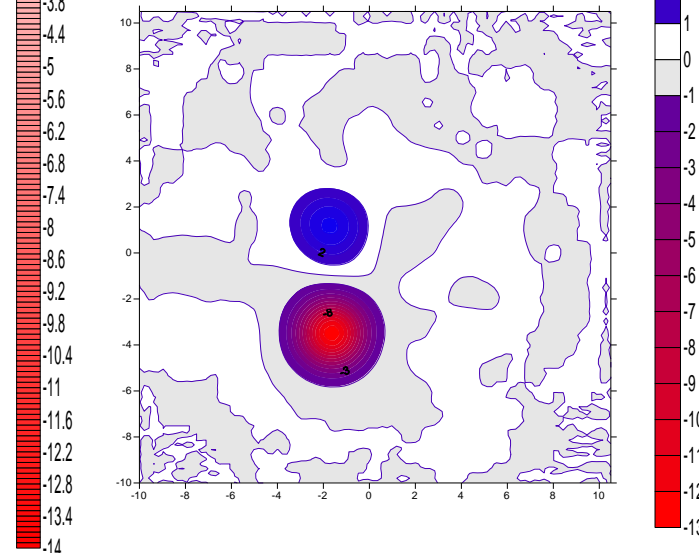
T=1.4 fm



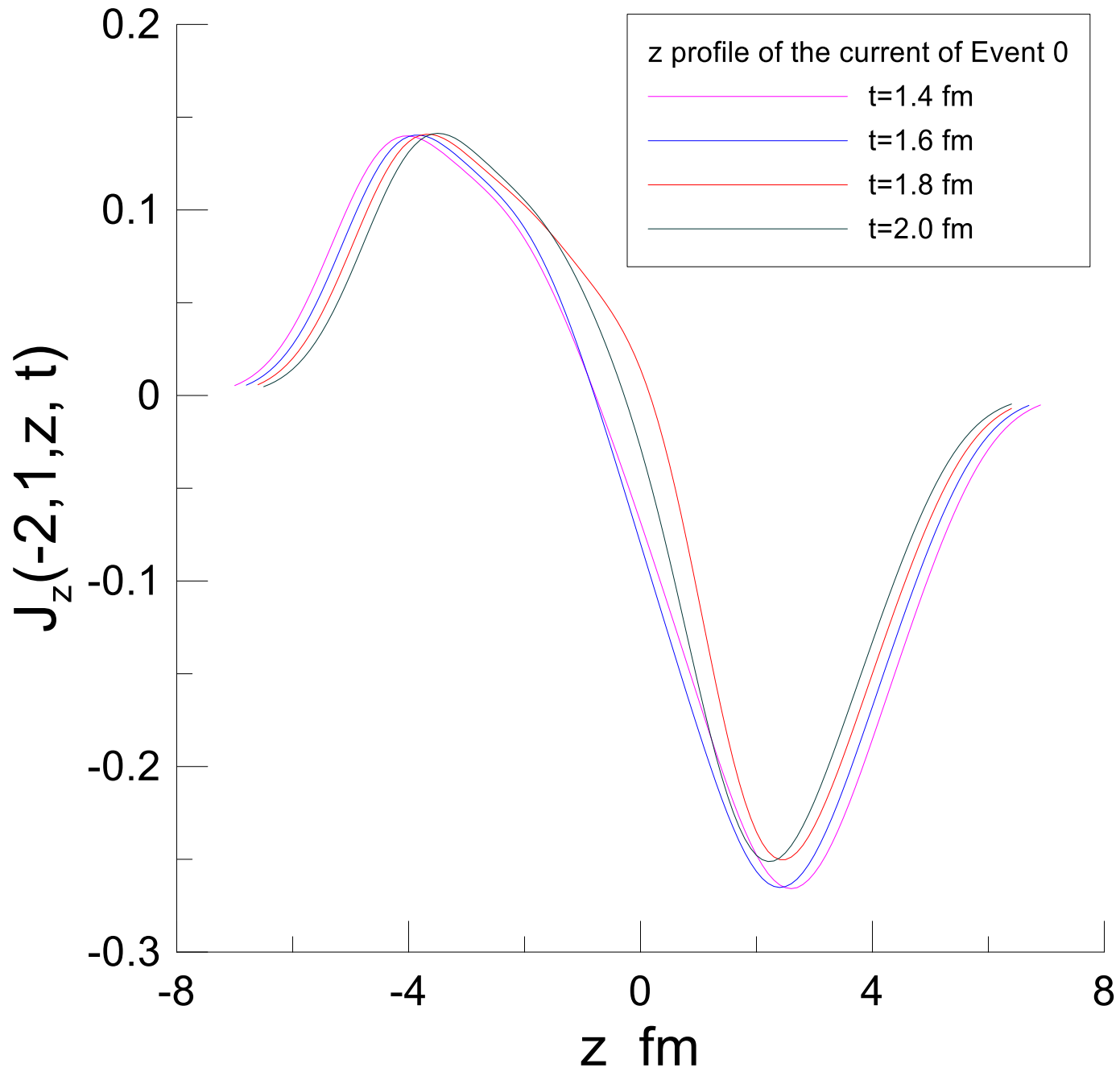
T=1.6 fm

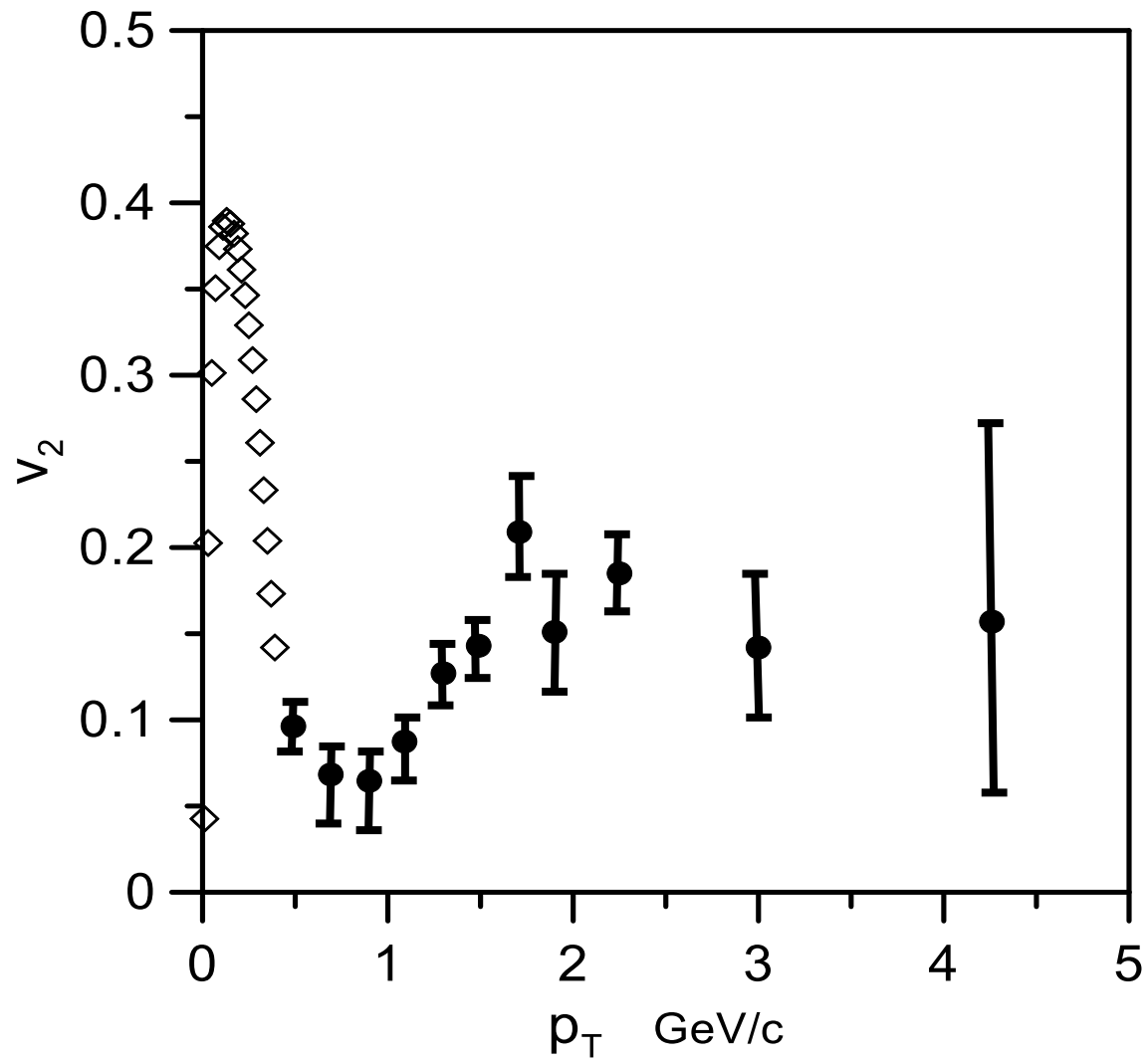


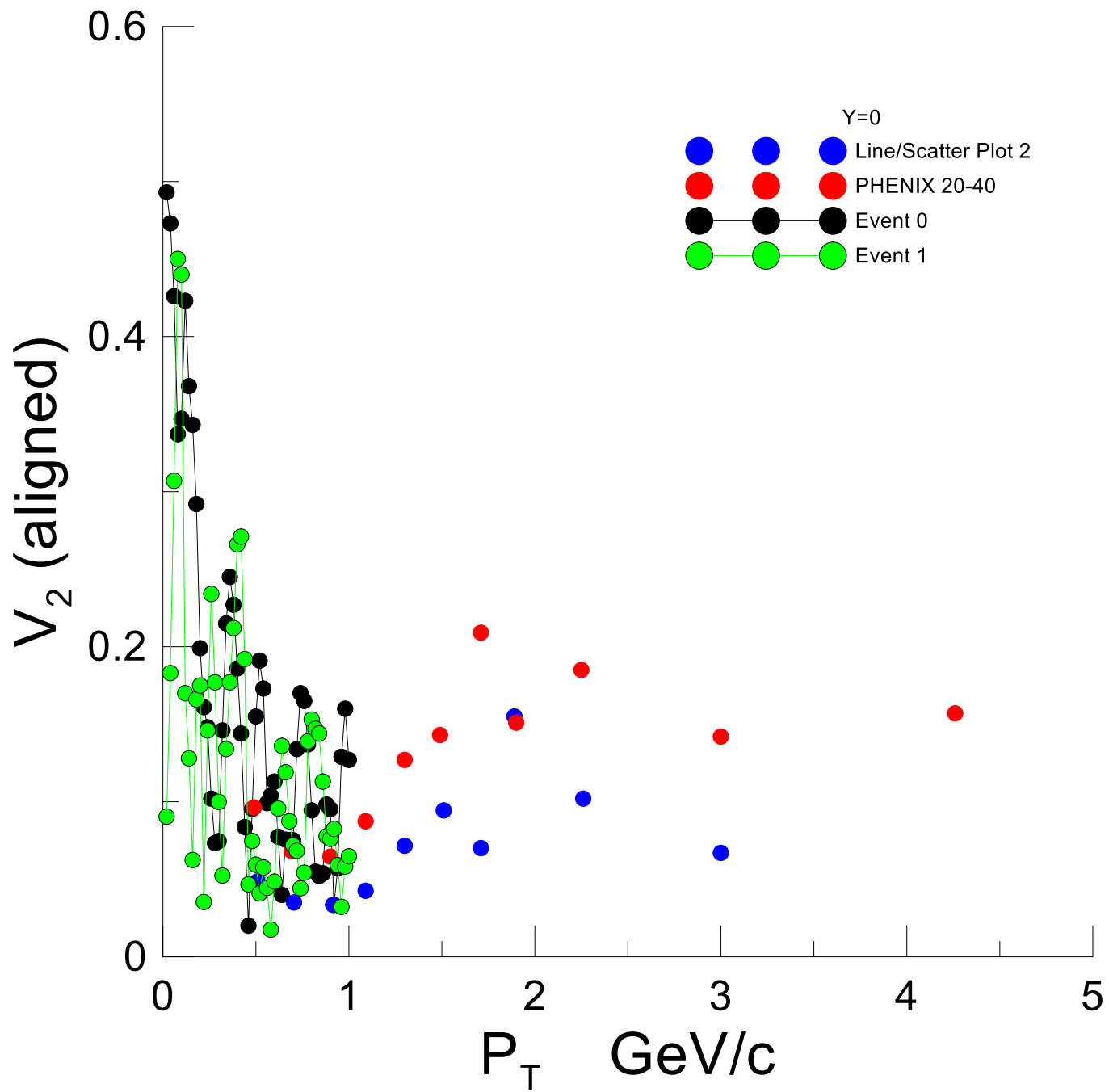
T=1.8 fm



T=2.0 fm







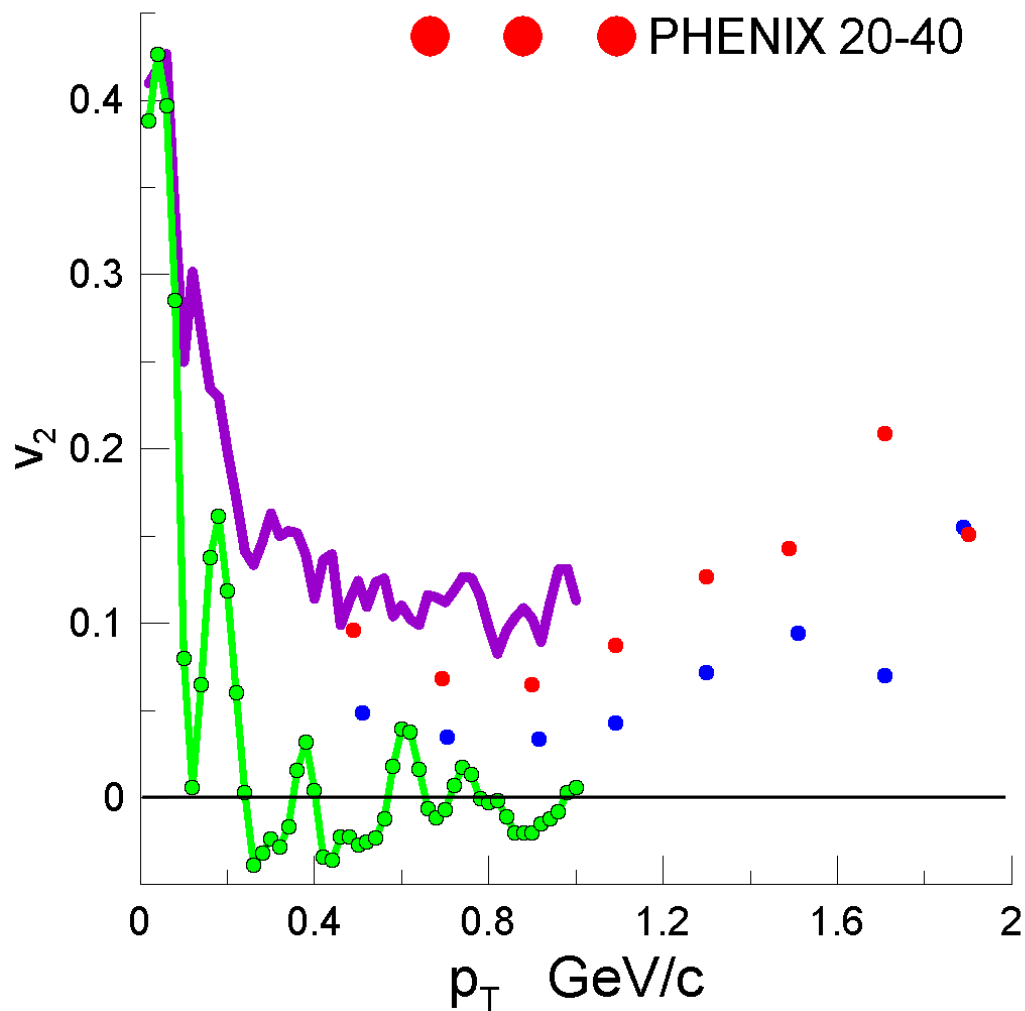
Event Averaged v_2 ($y=0$)

● ● ● Reaction Plane

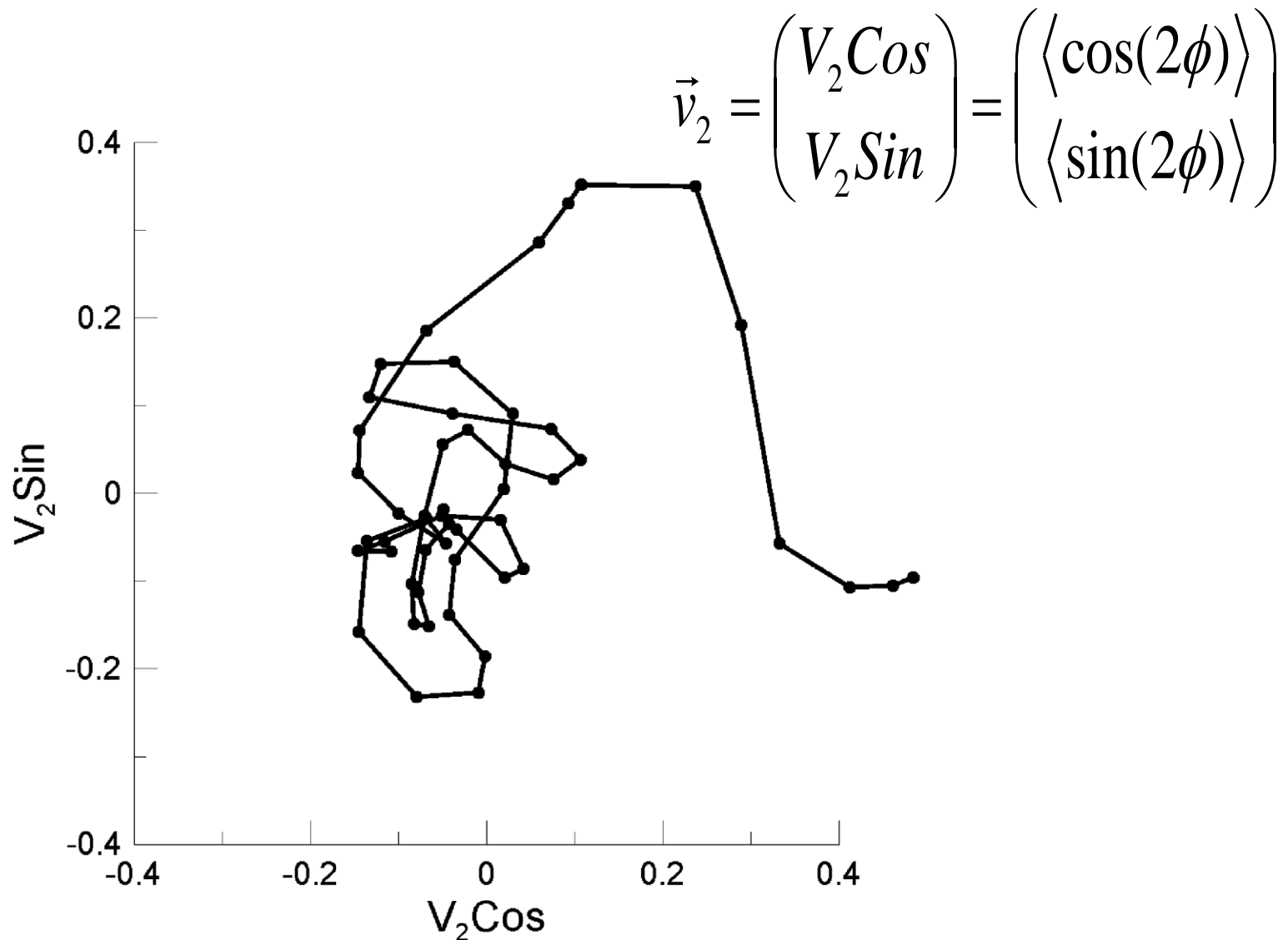
— Maximal Alignment

● ● ● PHENIX 0-20

● ● ● PHENIX 20-40

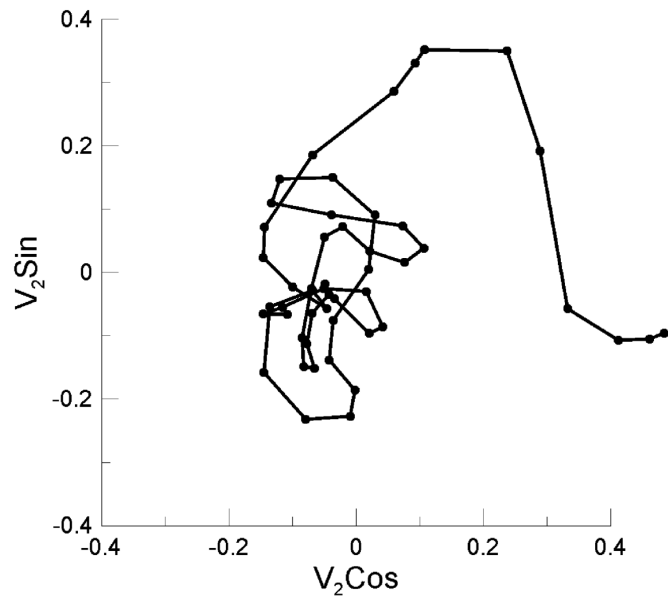


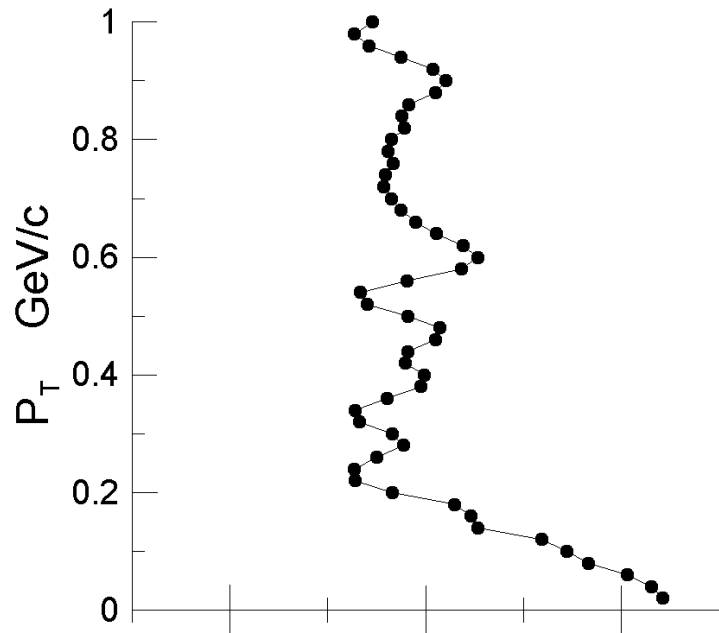
Change of \vec{v}_2 vector as function of energy



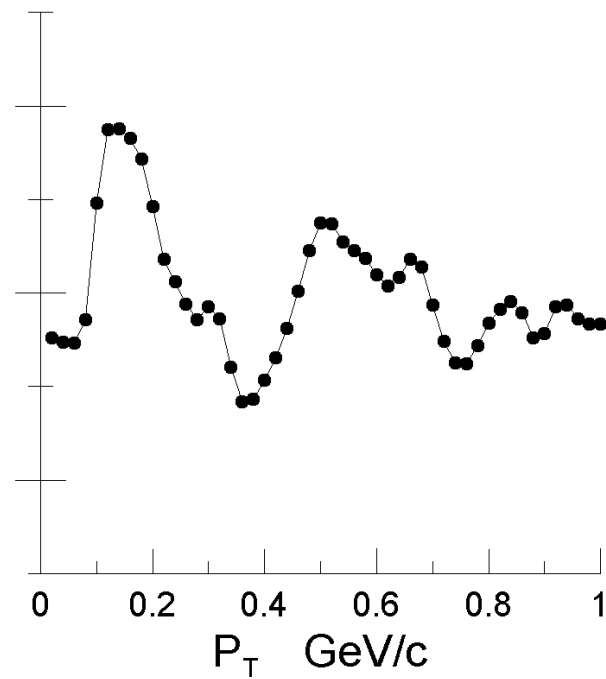
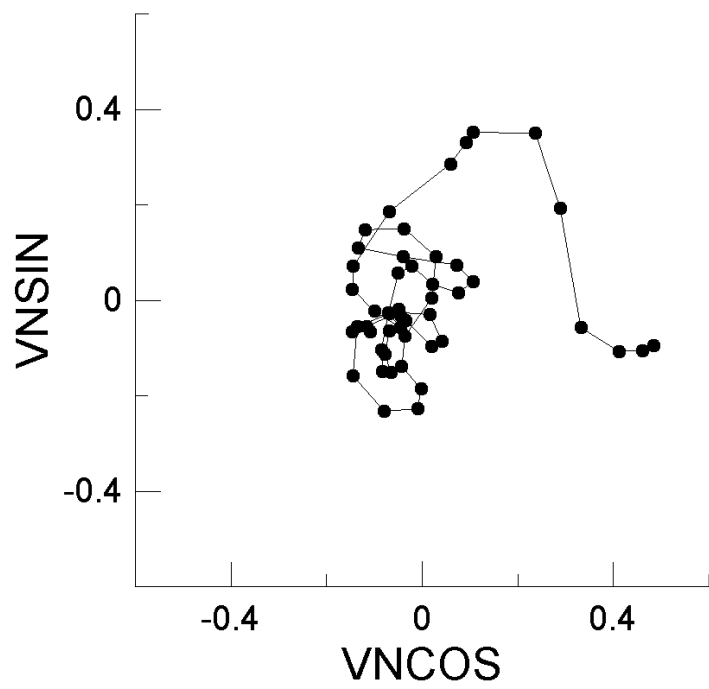
Change of \vec{v}_2 vector as function of energy

$$\vec{v}_2 = \begin{pmatrix} V_2 \cos \\ V_2 \sin \end{pmatrix} = \begin{pmatrix} \langle \cos(2\phi) \rangle \\ \langle \sin(2\phi) \rangle \end{pmatrix}$$

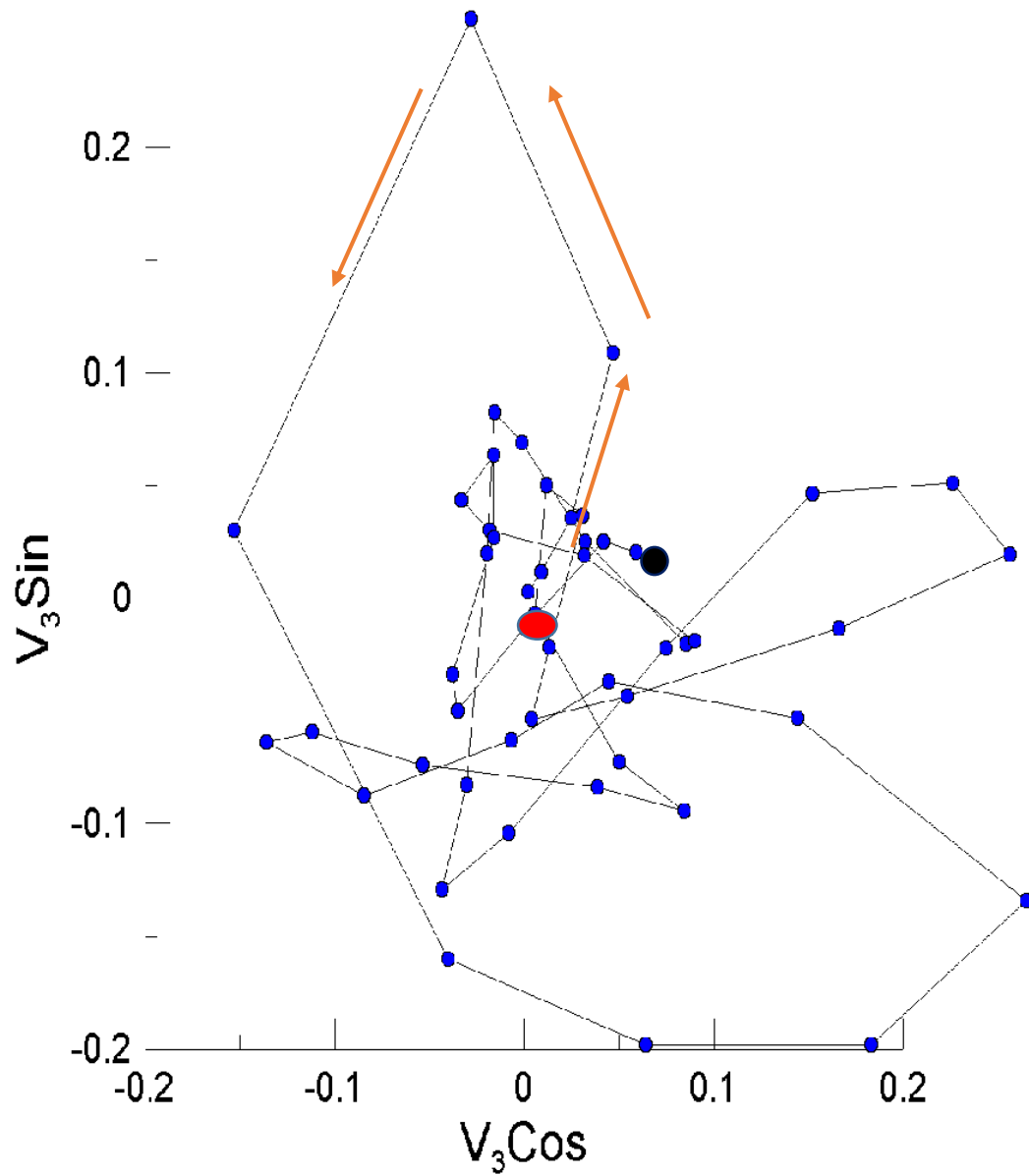


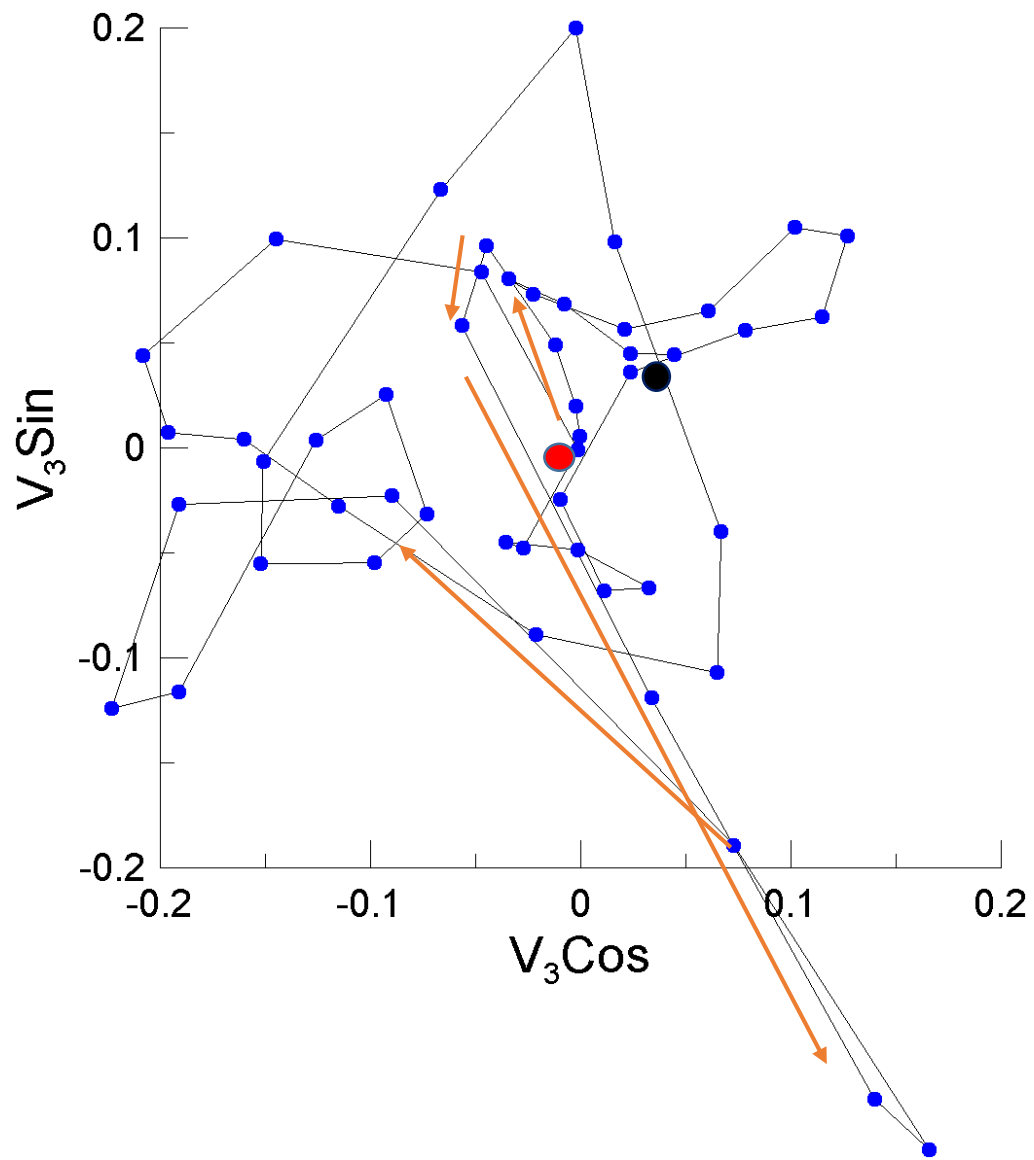


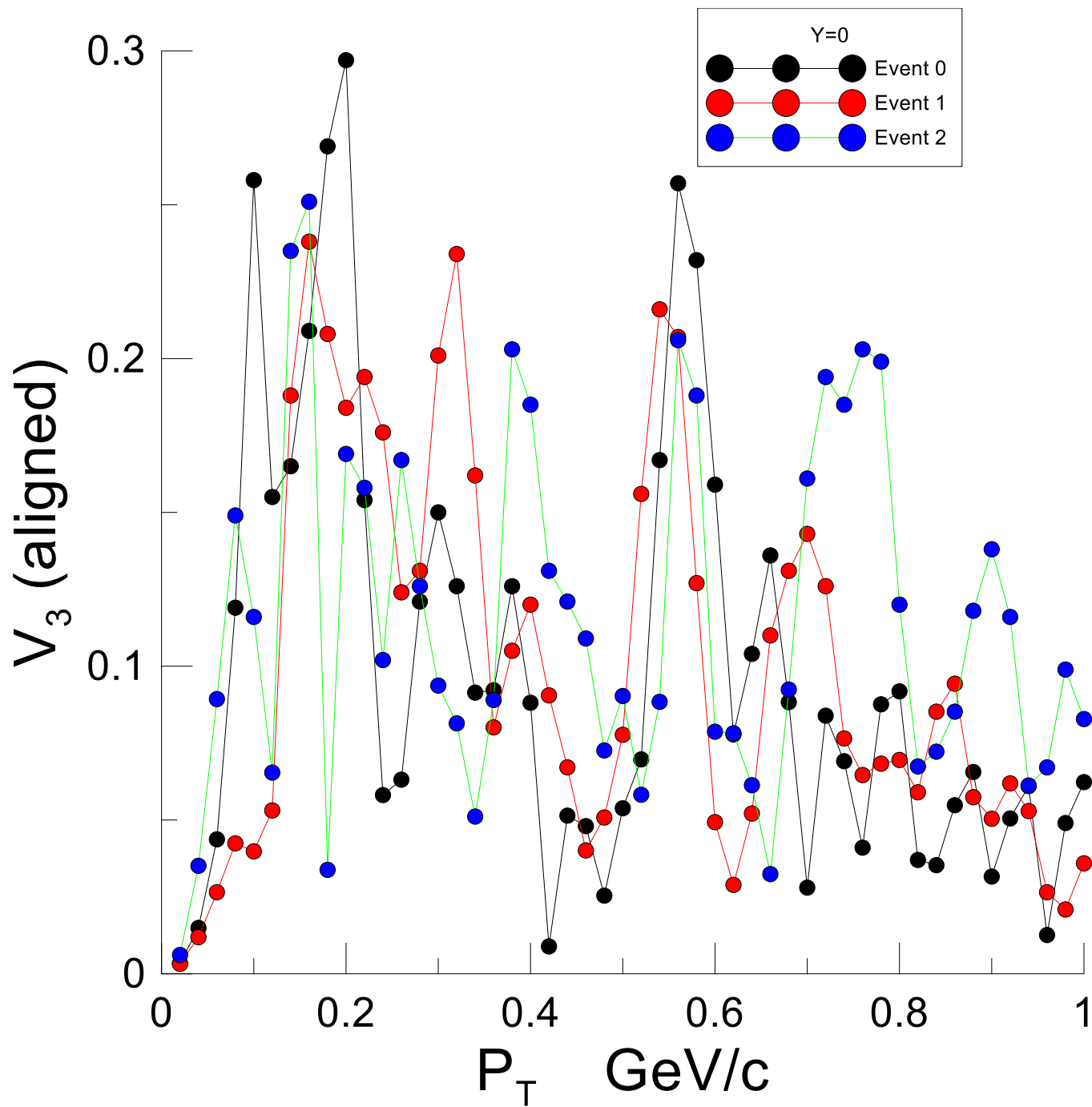
$$\vec{v}_2 = \begin{pmatrix} V_2 \cos \\ V_2 \sin \end{pmatrix} = \begin{pmatrix} \langle \cos(2\phi) \rangle \\ \langle \sin(2\phi) \rangle \end{pmatrix}$$



Similar behavior in $V_3 \dots$



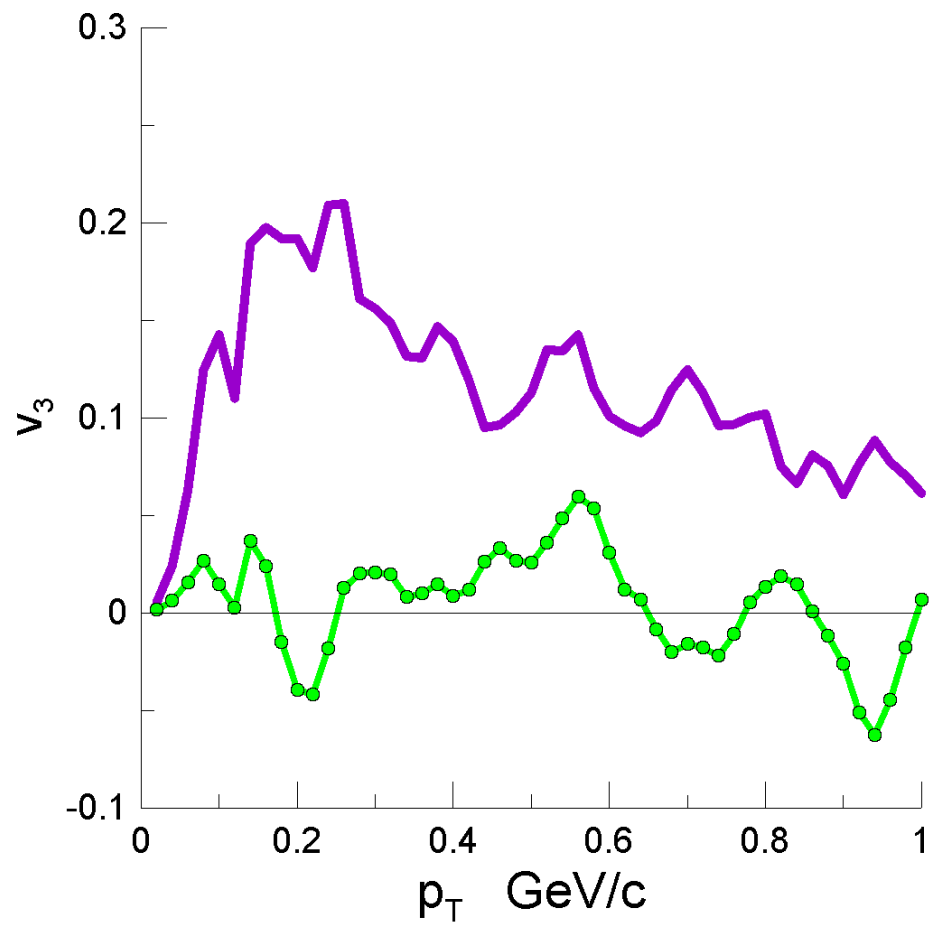




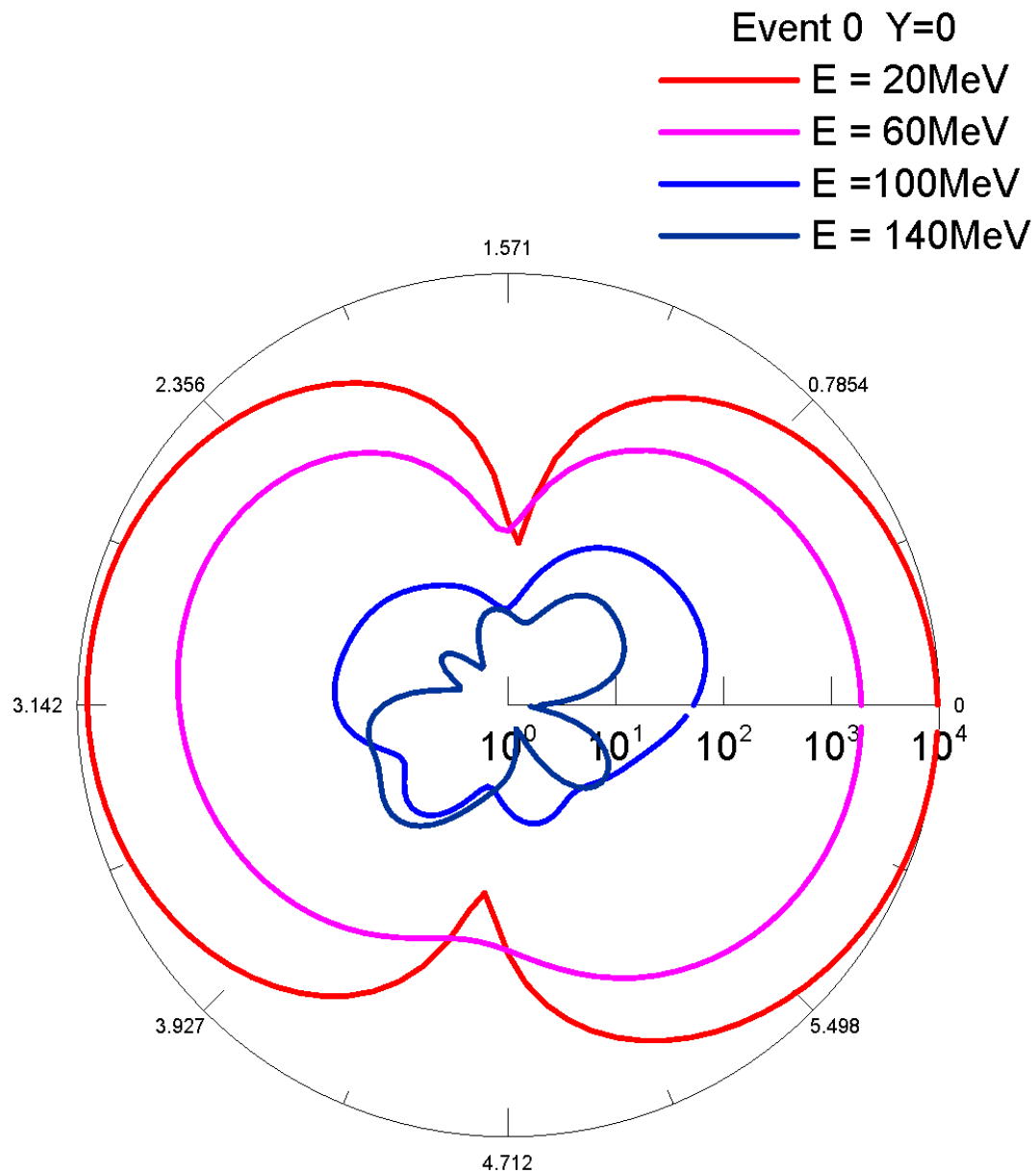
Event Averaged v_3

●—●—● Reaction Plane

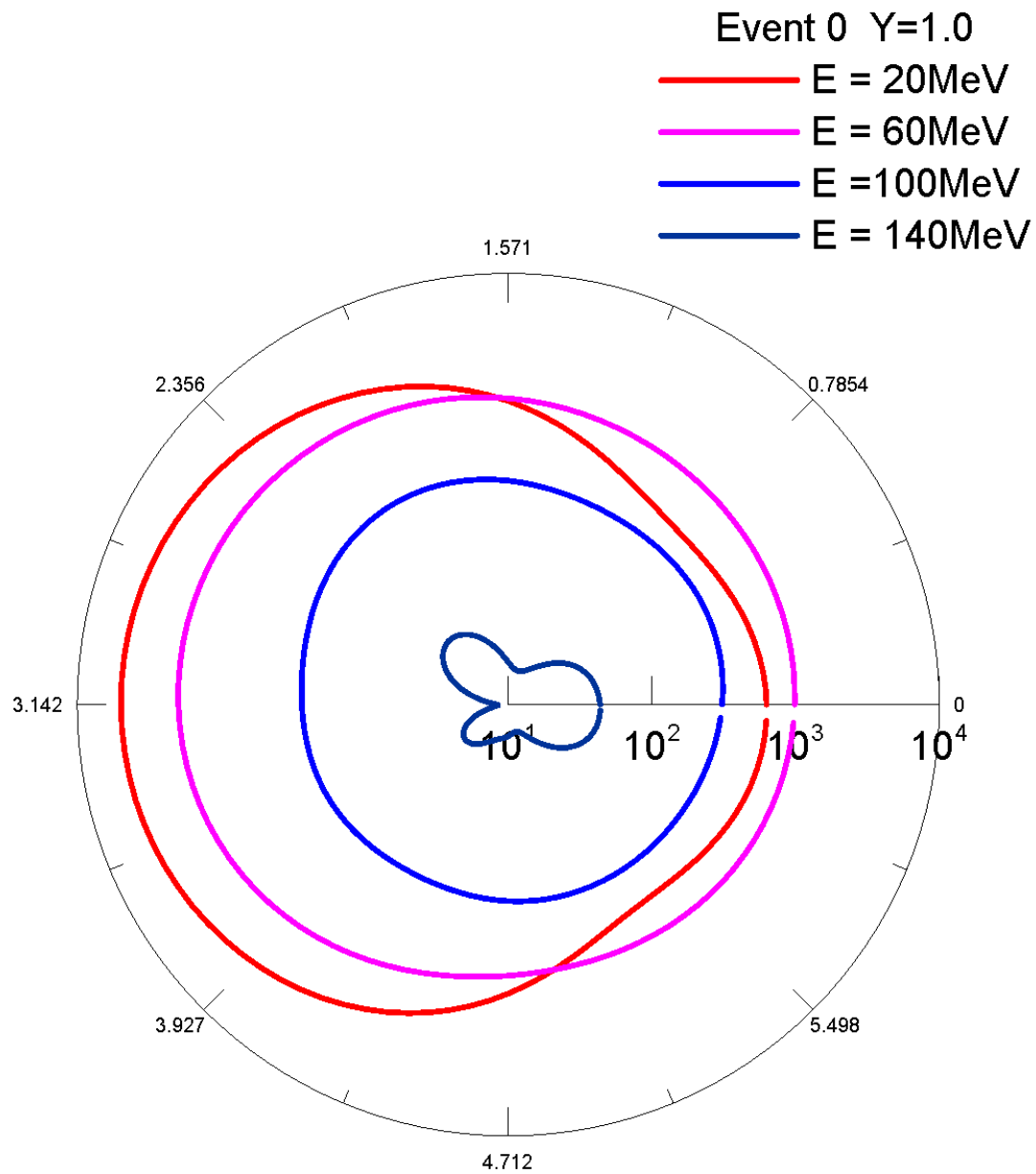
— Maximal Alignment

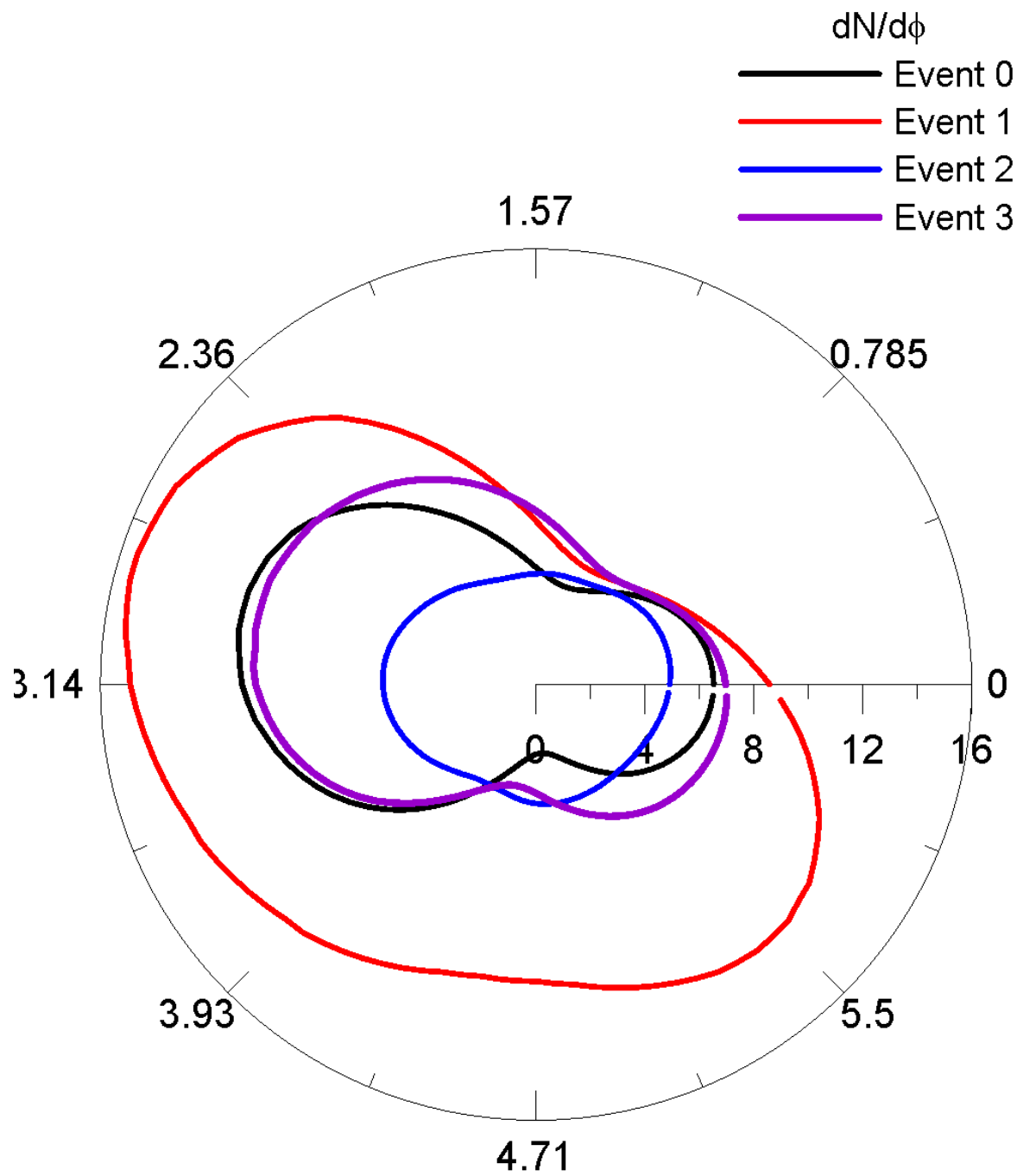


Azimuthal Angular distribution at $y=0$



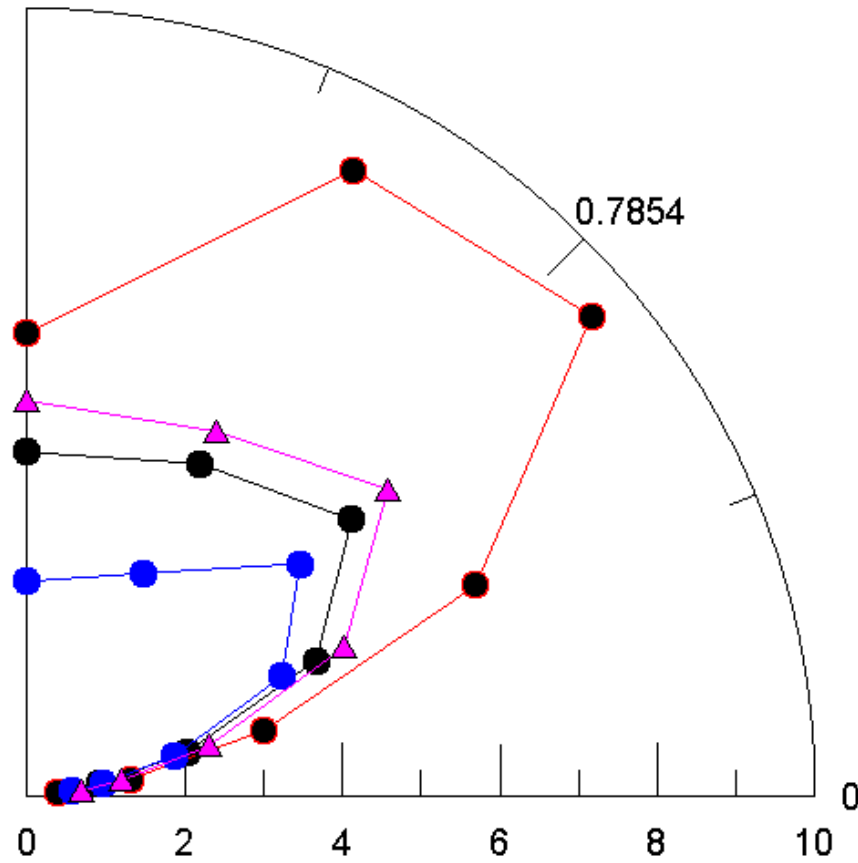
Azimuthal Angular distribution at $y=1.0$





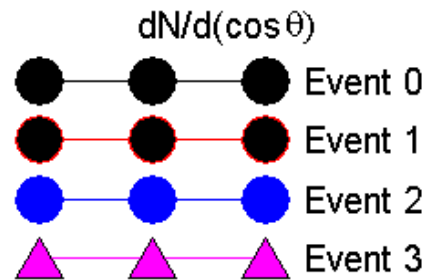
1.571

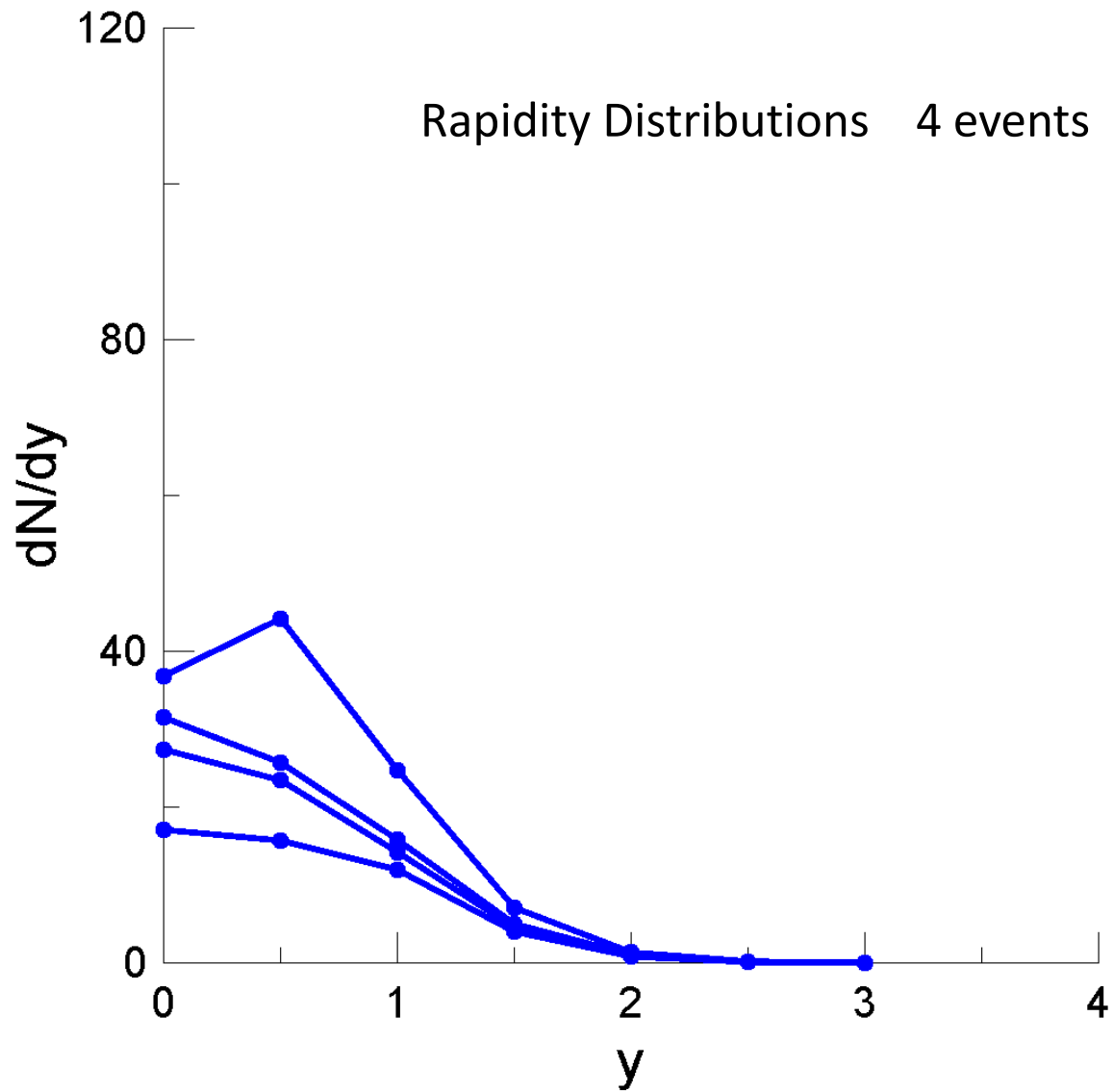
Zenith angle distribution (forward)



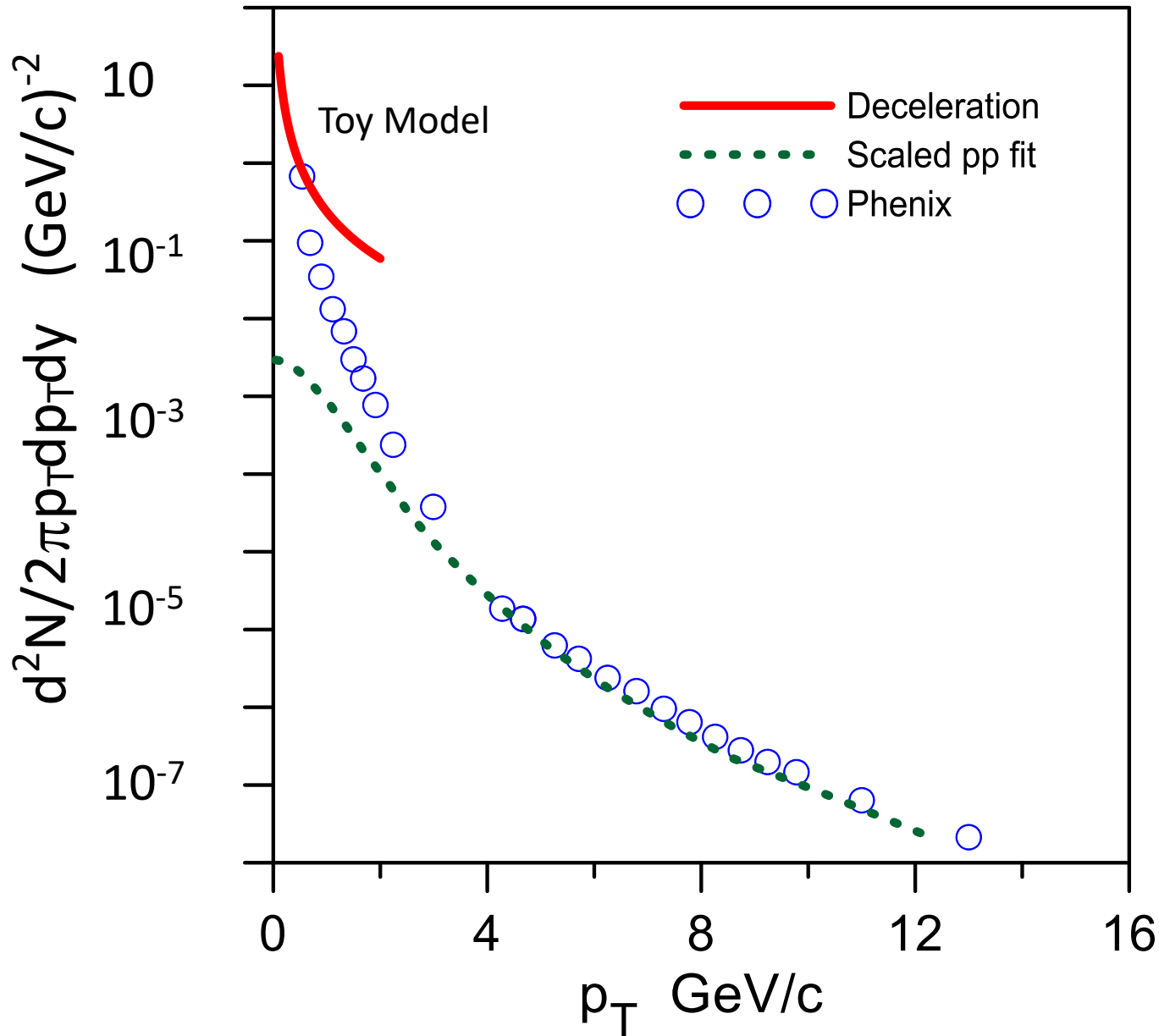
$$\sin \theta = \frac{1}{\cosh y},$$

$$\tan \theta = \sin y.$$



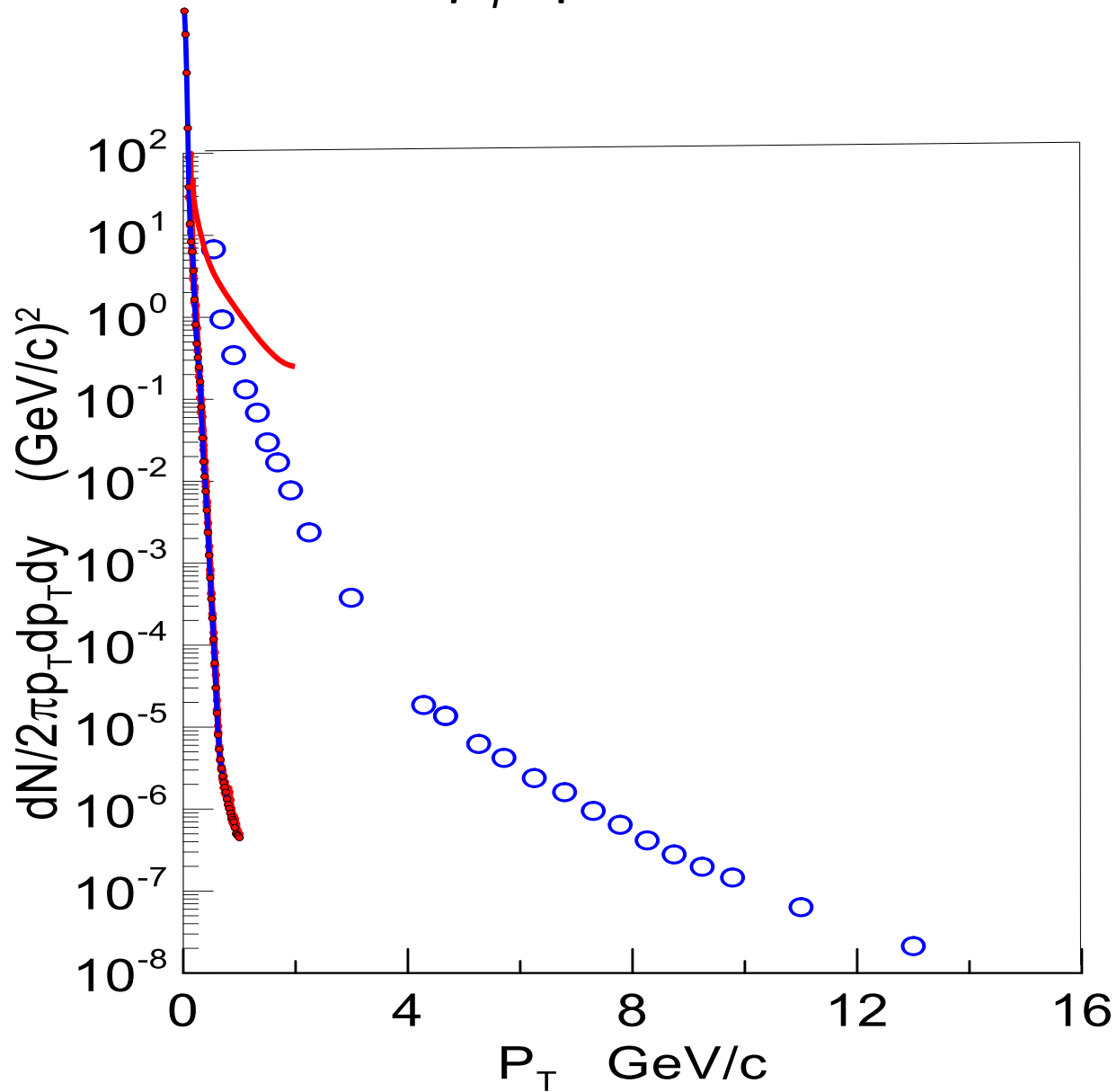


p_T Spectrum

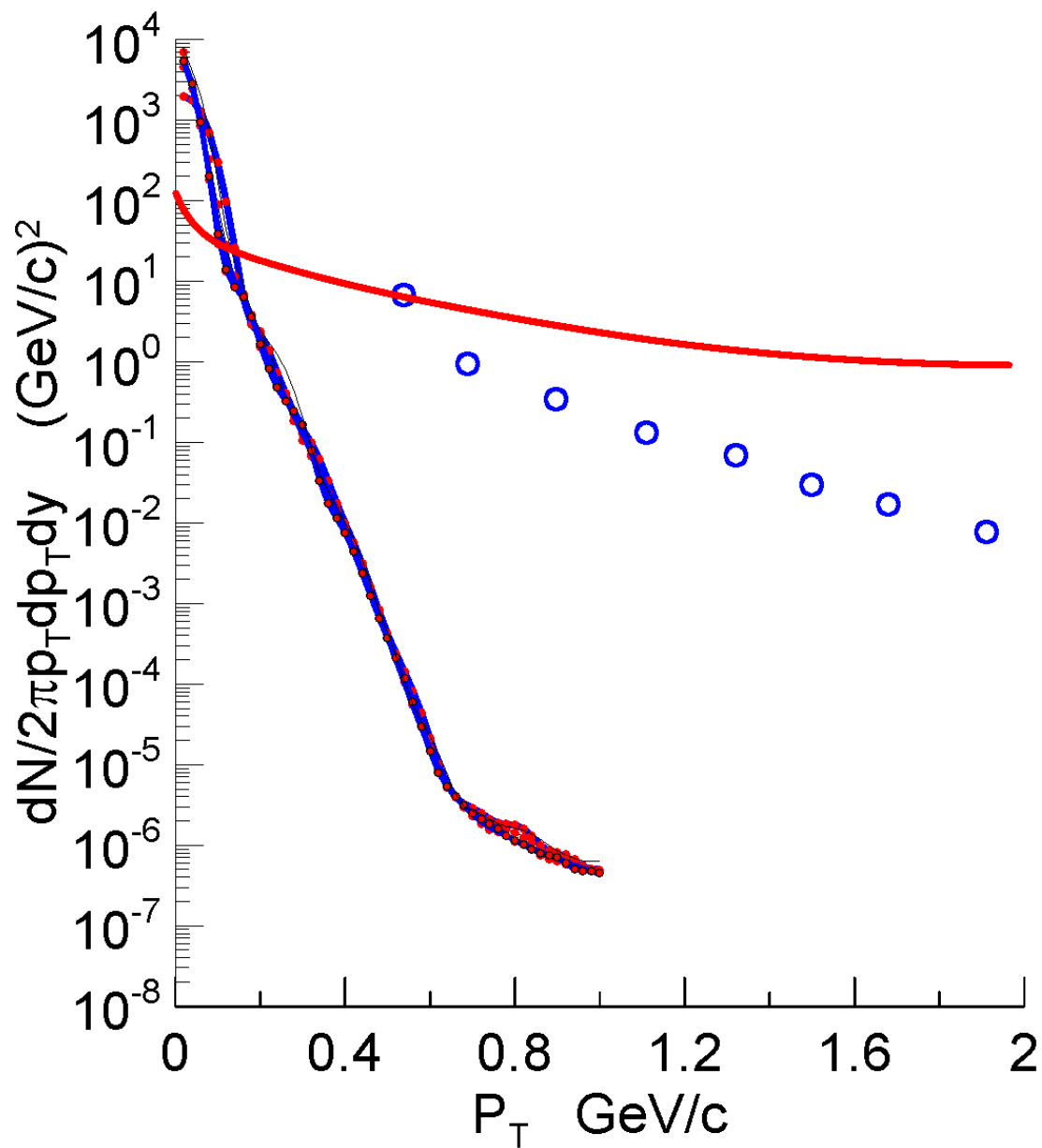


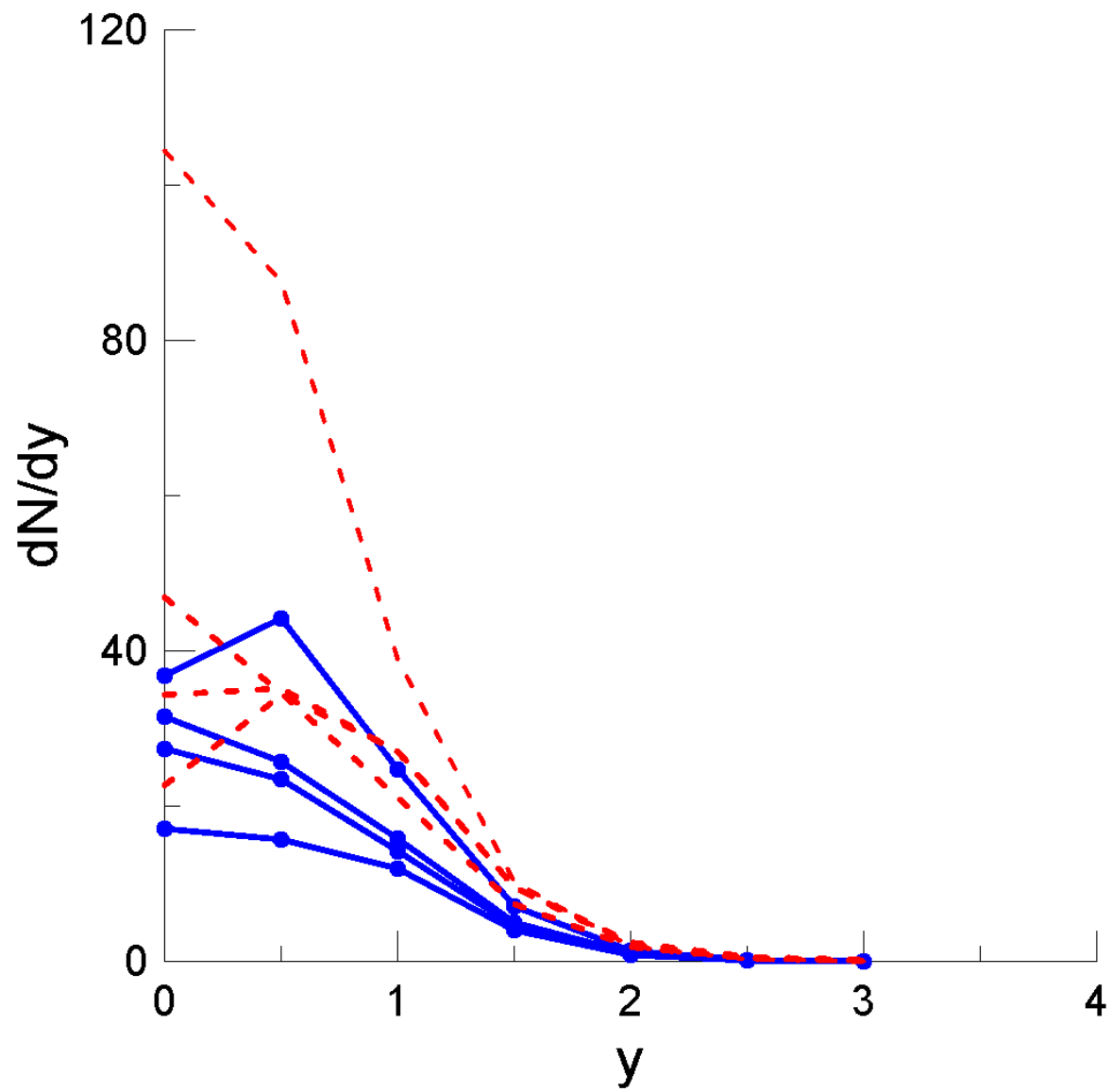
$$V_0 \approx 1,$$
$$Z_{eff} \approx 80,$$
$$d \approx 1 \text{ fm}$$

p_T Spectrum

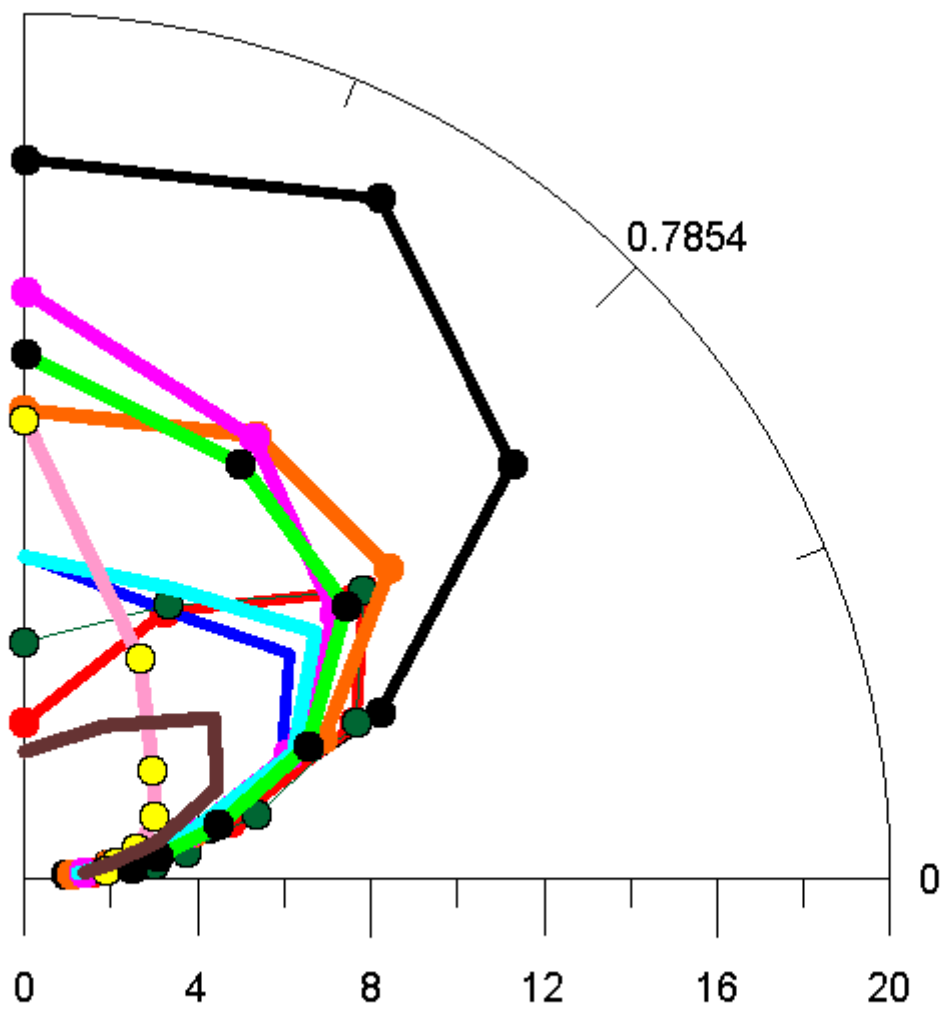


p_T Spectrum





1.571

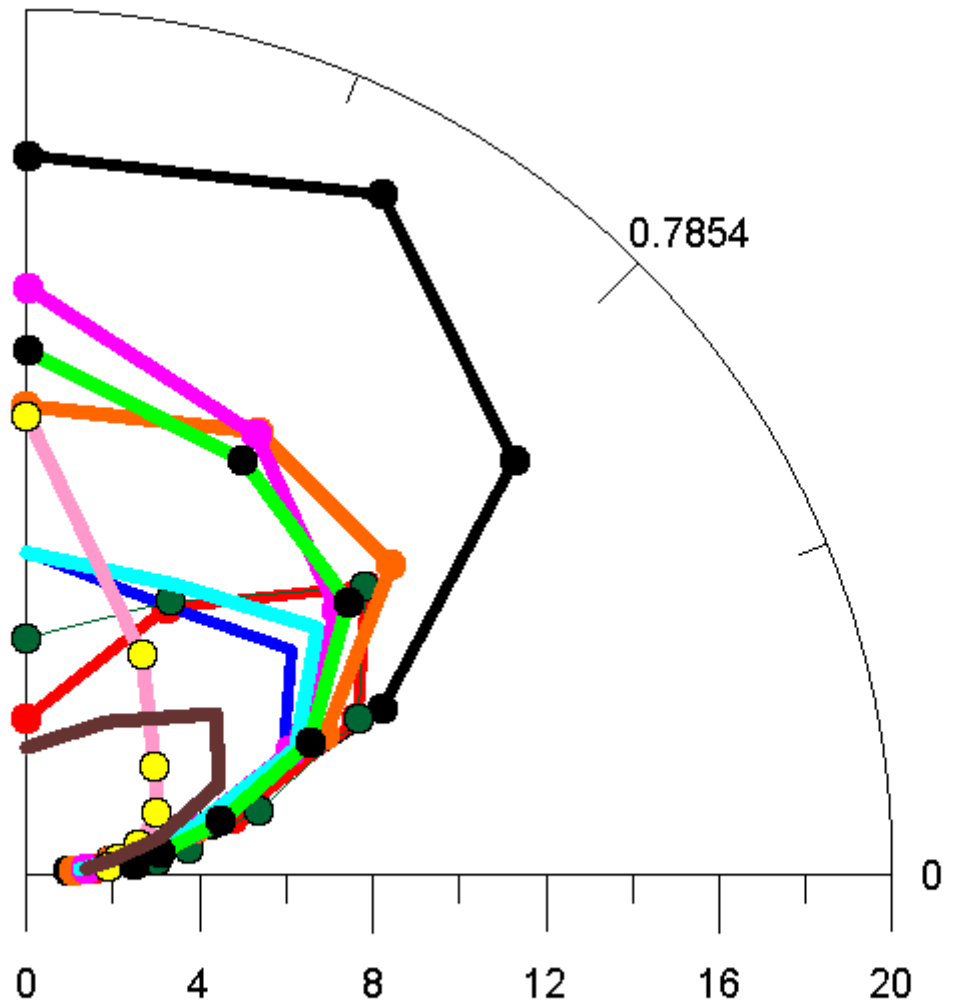
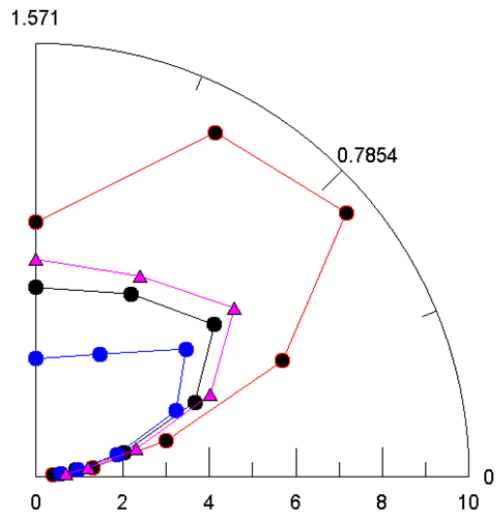


0.7854

0

0 4 8 12 16 20

1.571



Summary

- The classical electromagnetic fields radiated by the deceleration process of relativistic heavy ions were re-examined.
- A toy model and more sophisticated JAM simulations show a similar prediction for the v_2 anisotropy parameter, indicating that the photons from the charge deceleration (or stopping) process reflects very well the initial state in the EbyE basis.
- If the increase of v_2 in PHENIX data for lower p_T really tells something, we may think of some coherent (collective) deceleration mechanism of the incident charges.
- Indication of large EbyE fluctuations and formation of hot spots in JAM simulation.
- If this is the case, the classical EM radiation may offer a very interesting approach to determine the collision geometry, baryon stopping, etc..... But,
- Experimental difficulties due to critically low multiplicity ($\sim 10^2$ - 10^3). A further study is necessary to examine more in detail using different models of the initial condition.

Some Basic Questions

- How to deal with the string dynamics and charge current ?

$$\frac{\partial \vec{J}}{\partial \tau}(\vec{\xi}, \tau) = \frac{\partial}{\partial \tau} \langle \psi [Strings](\tau) | \hat{J}(\vec{\xi}) | \psi [Strings](\tau) \rangle$$

- How does the coarse-graining scale influence ?

ขอขอบคุณ !







