On spectral functions & transport coefficients in QCD

Jan M. Pawlowski

Universität Heidelberg & ExtreMe Matter Institute

Phuket, November 3rd 2016

GEFÖRDERT VOM
Bundesministerium
für Bildung
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Outline

- **Introduction**
- **Single particle spectral functions**
- **Spectral functions & transport coefficients**
- **Summary & outlook**

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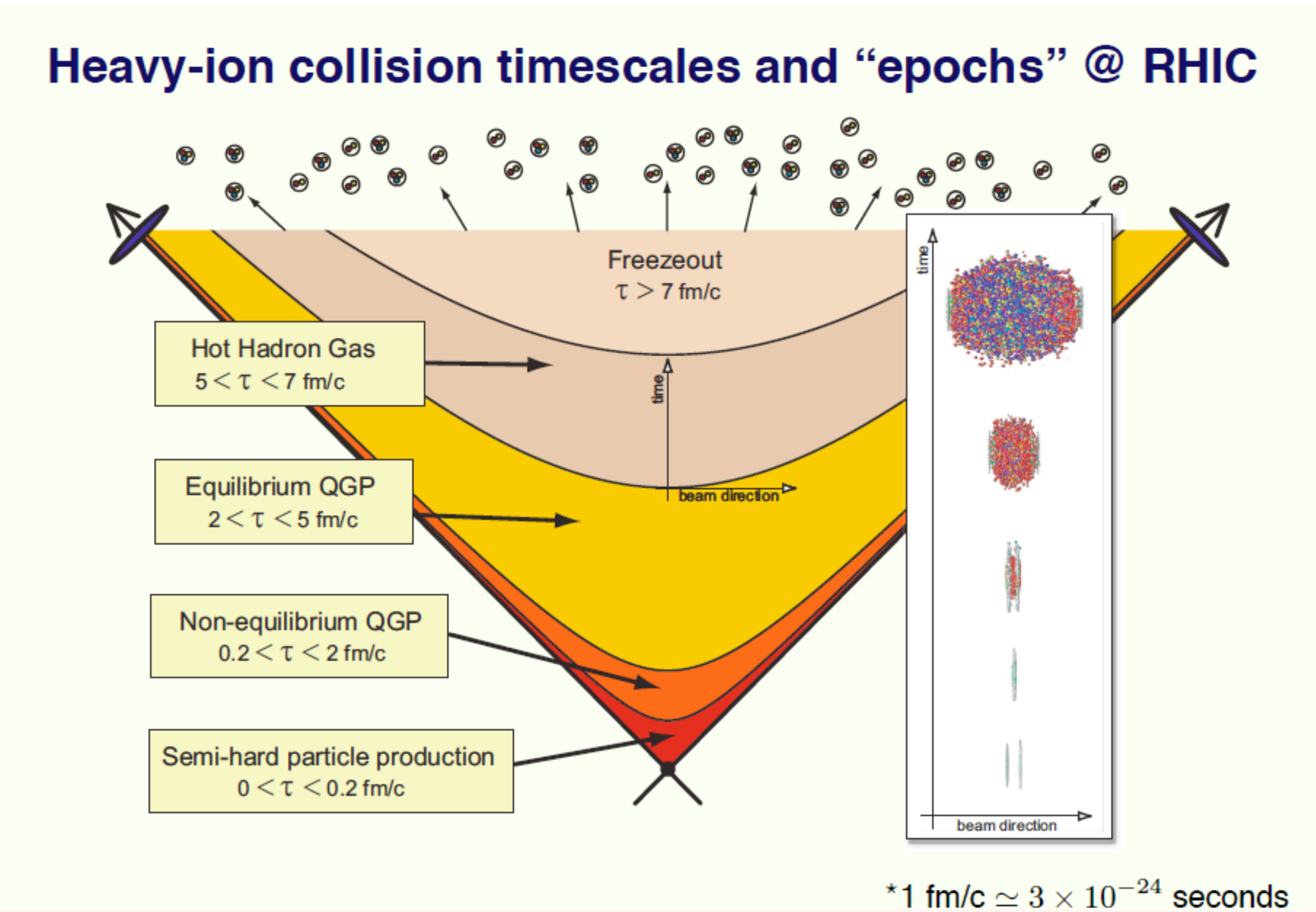
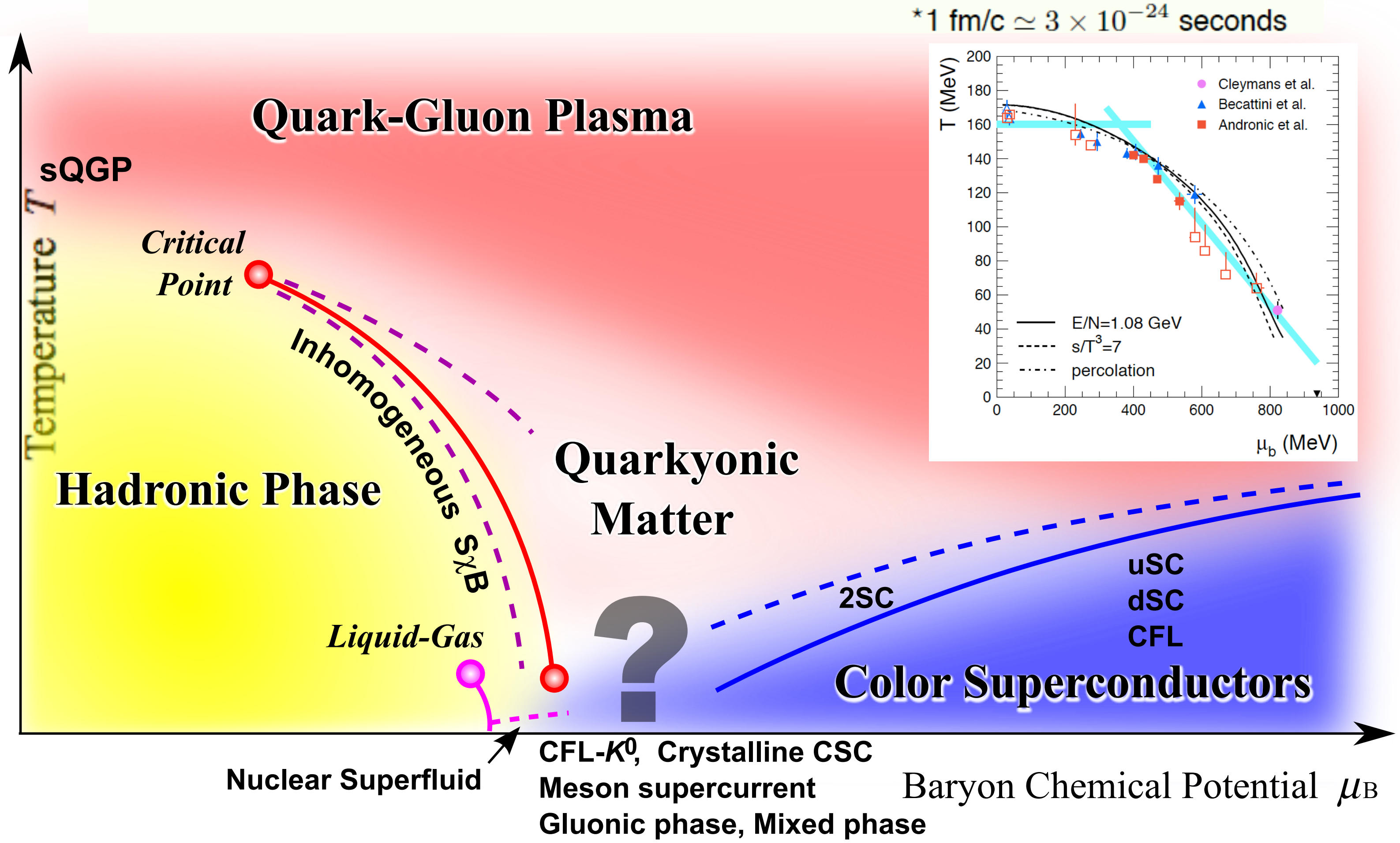
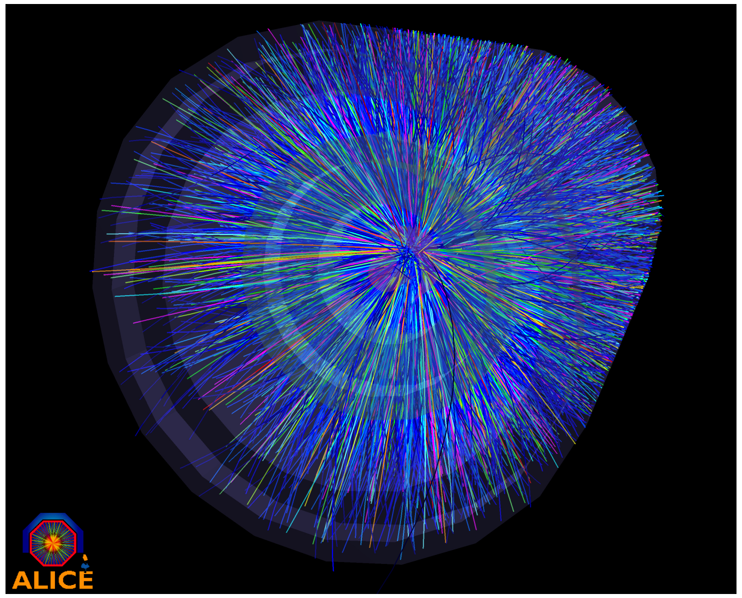
- Introduction
- Single particle spectral functions
- Spectral functions & transport coefficients
- Summary & outlook

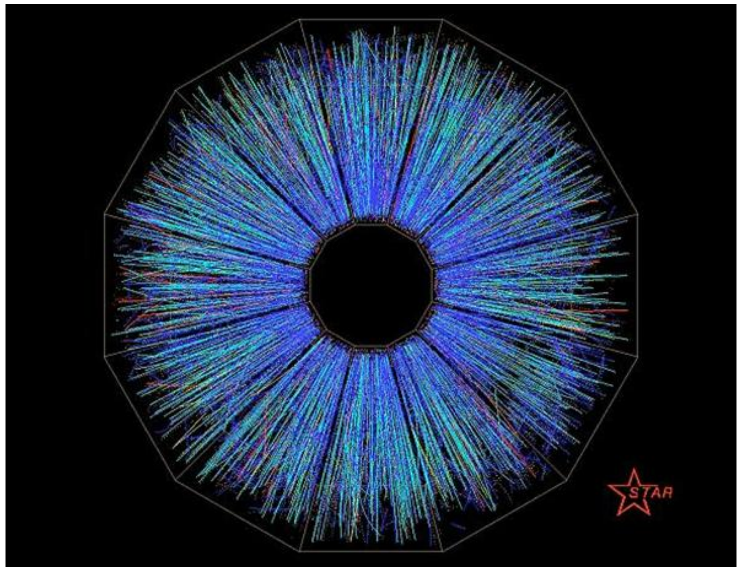
Heavy ion collisions



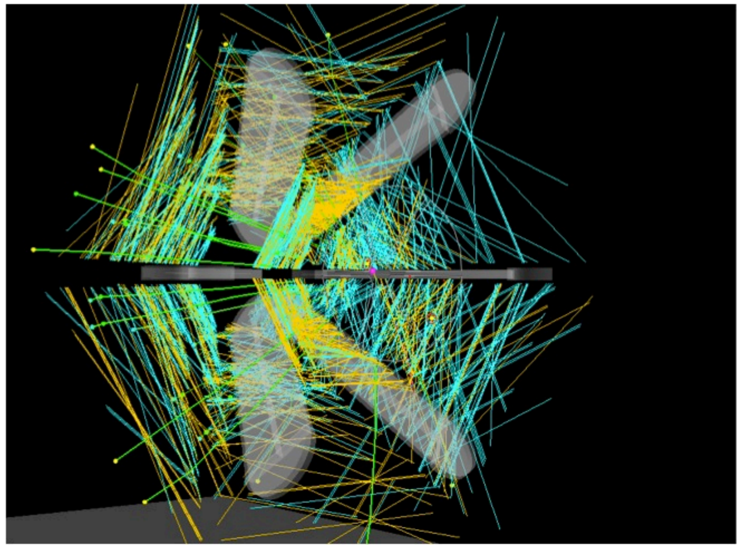




LHC

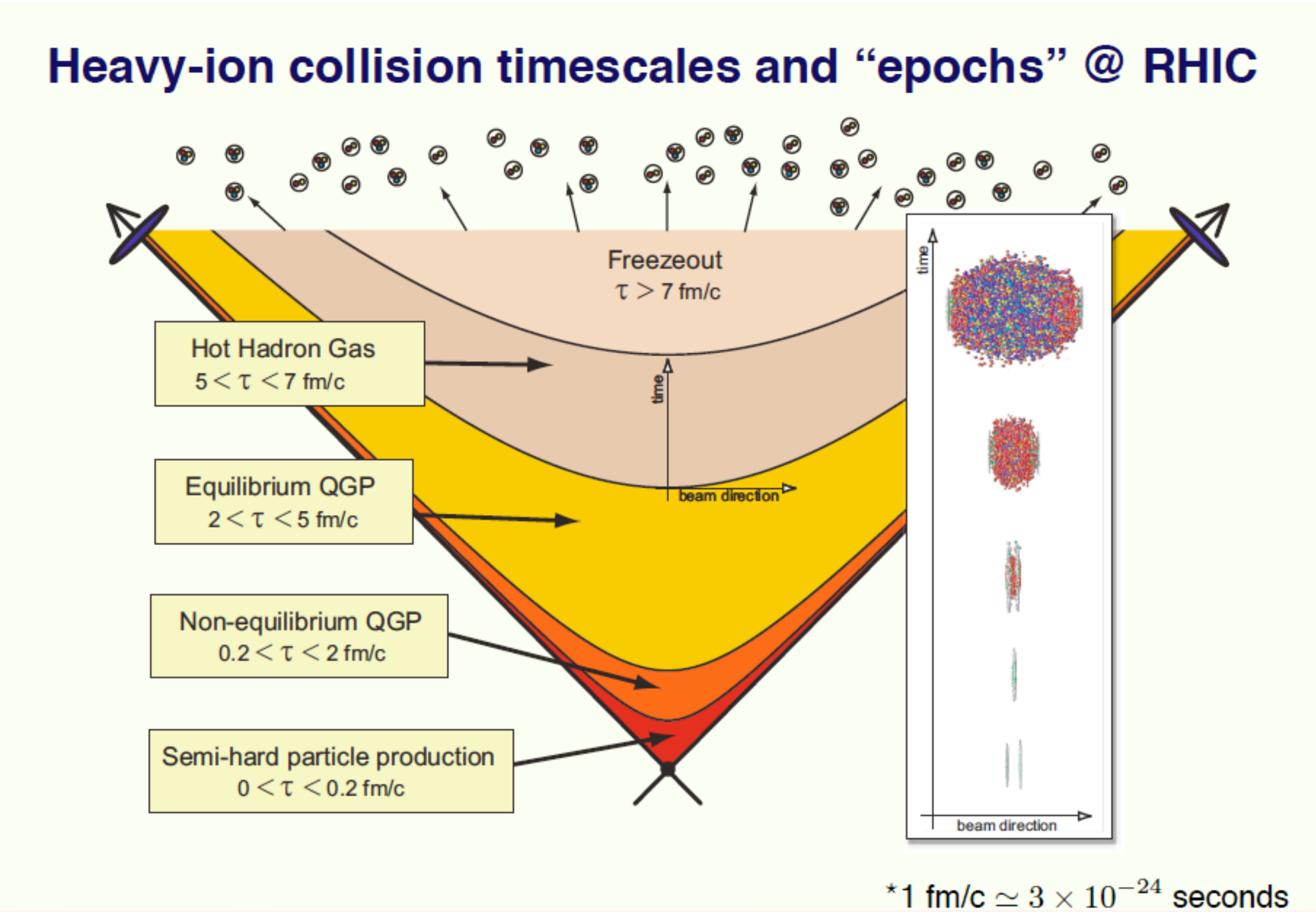


RHIC



GSI

Heavy ion collisions



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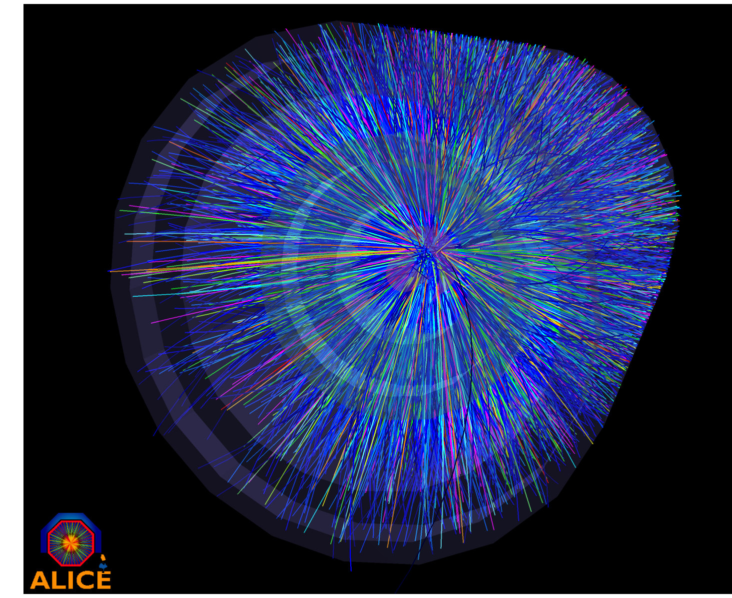

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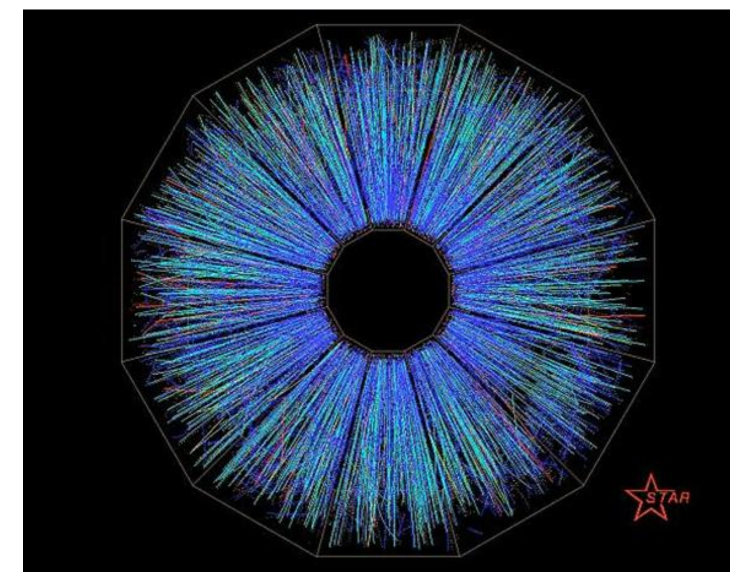

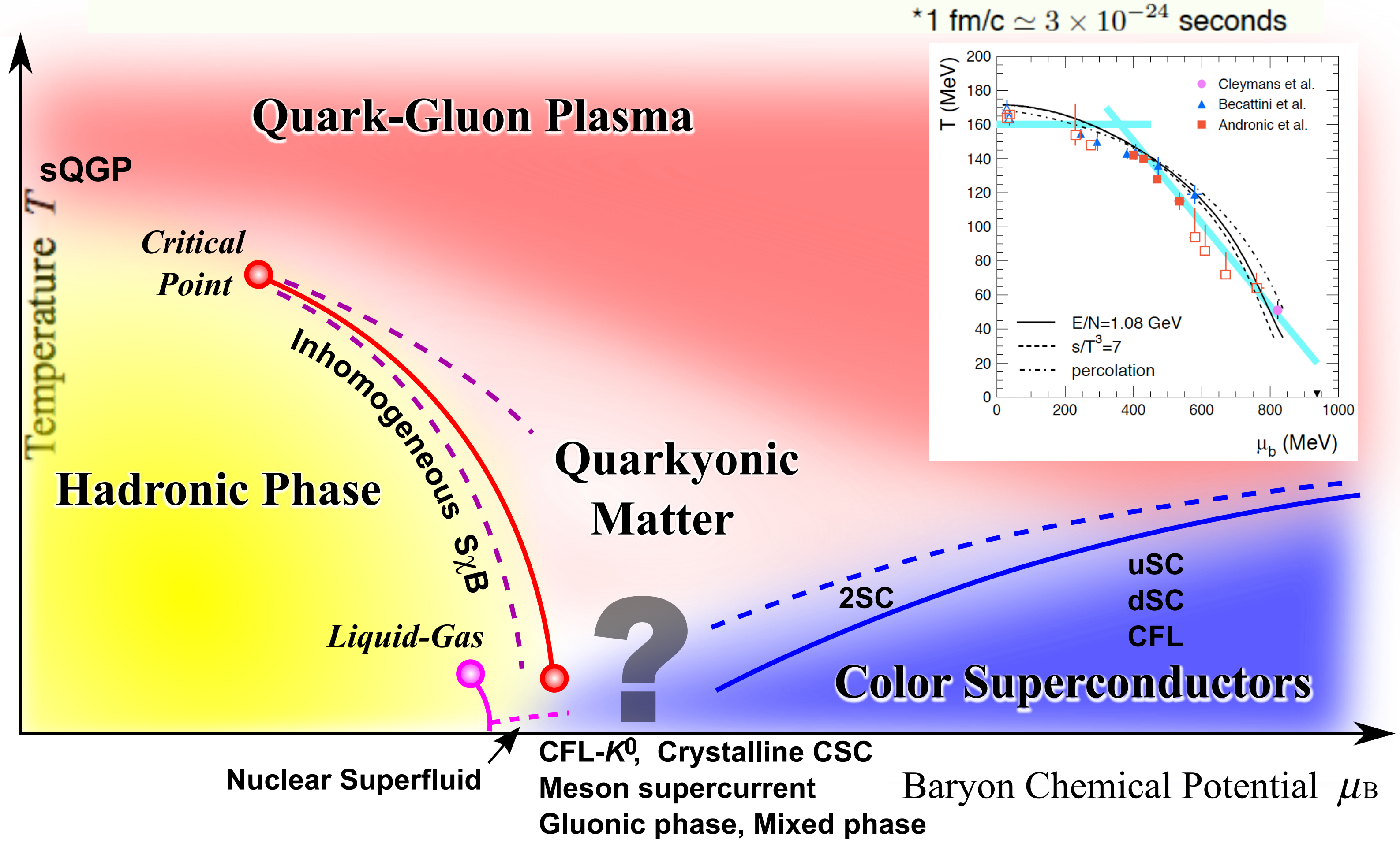

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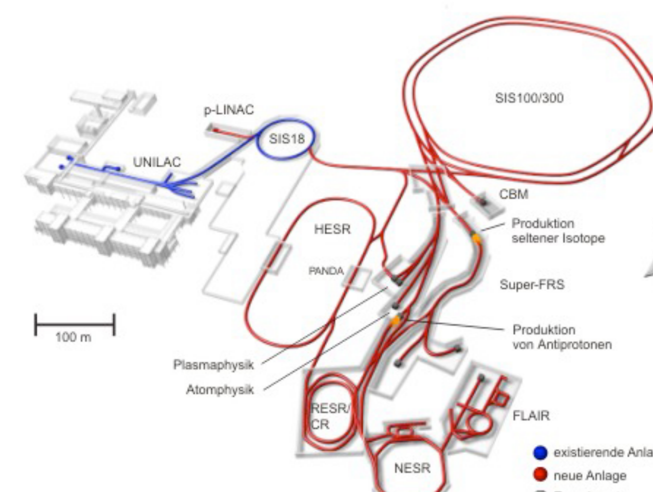

NICA

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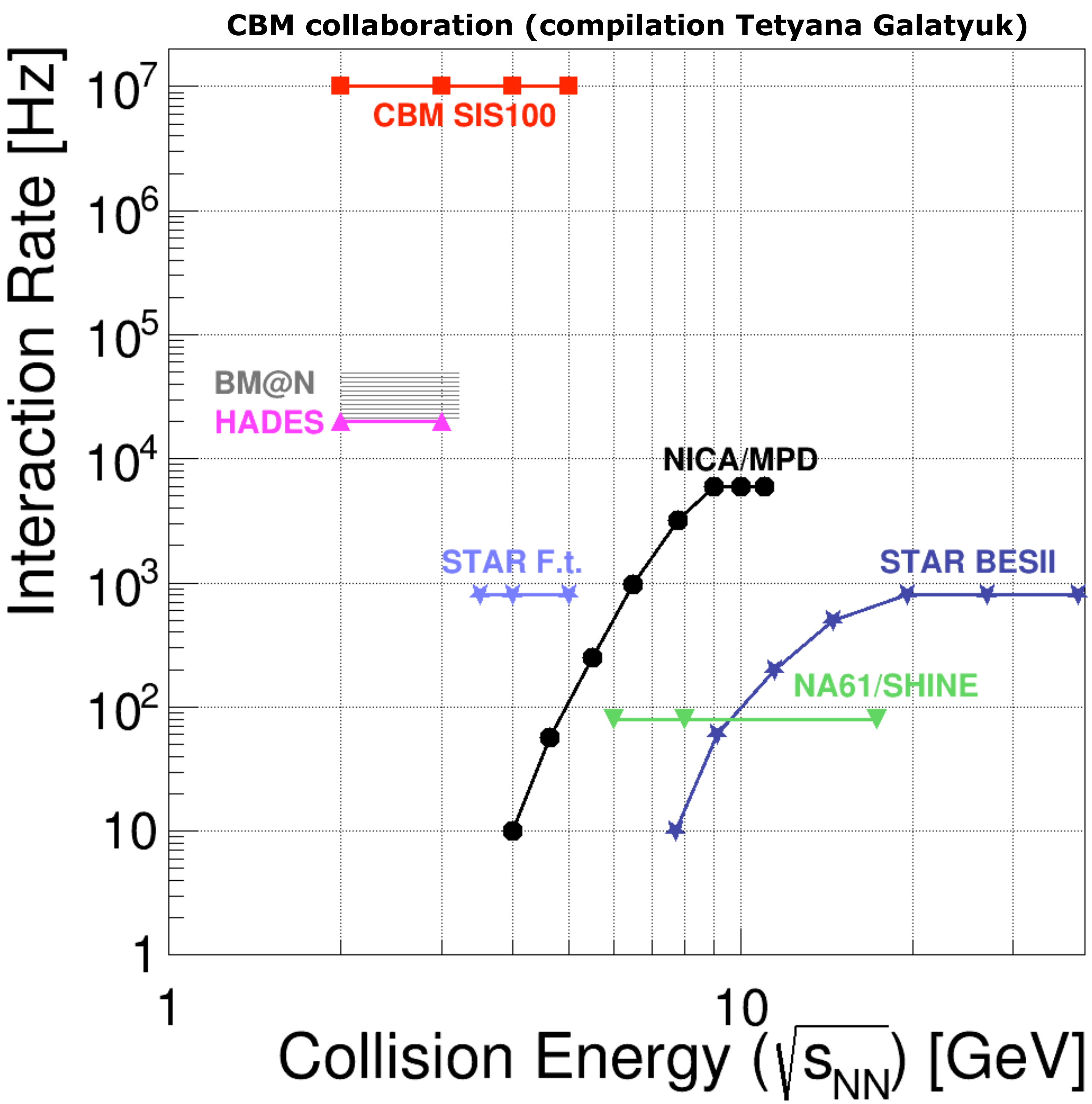

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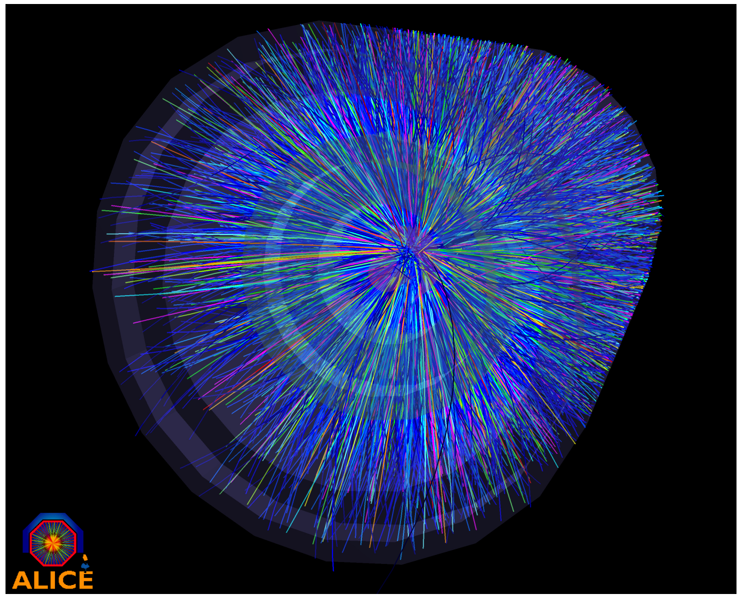

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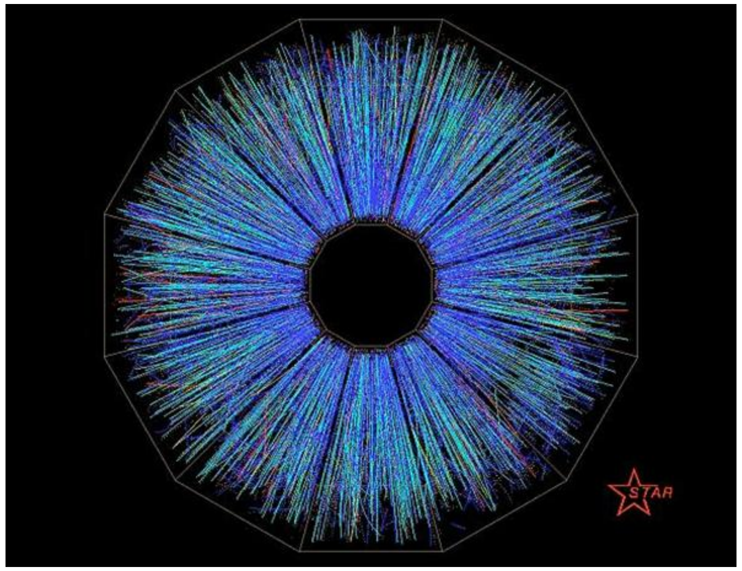

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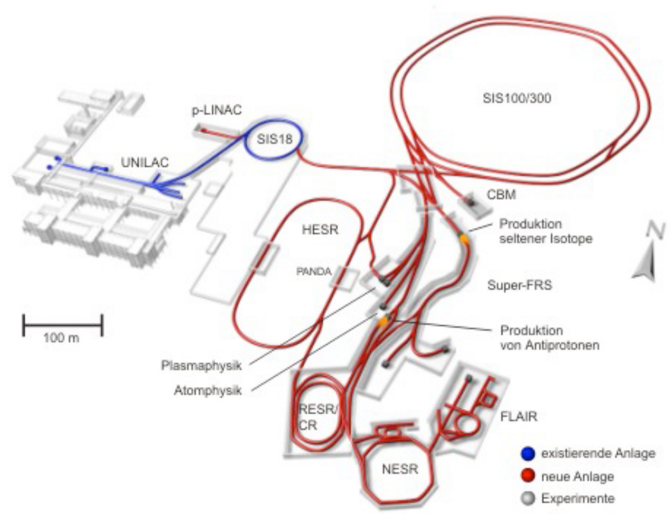
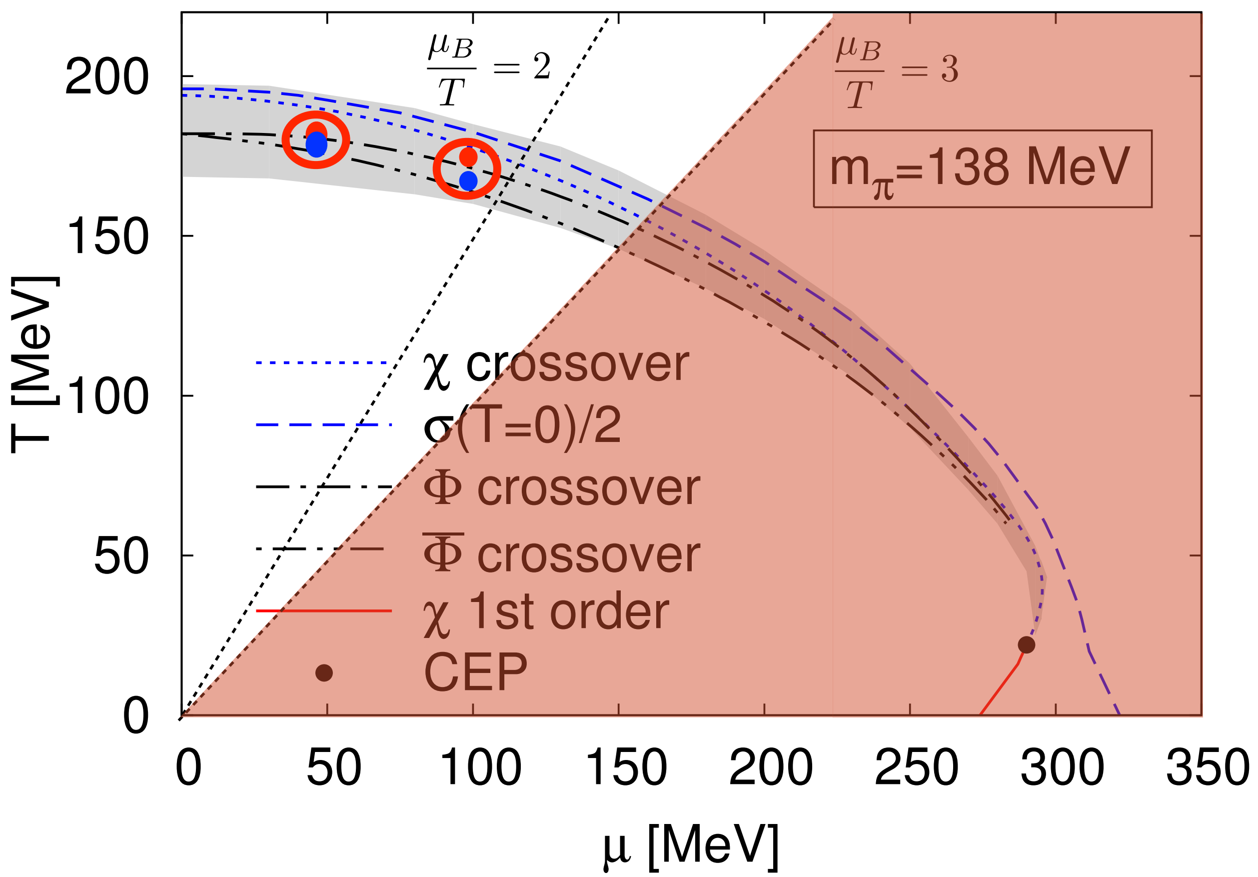


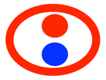
NICA

fQCD: motivation

Phase diagram of quantised 2-flavor PQM-model



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007



FRG QCD results at finite density

Haas, Braun, JMP '09, unpublished

Extension of FRG QCD results at imaginary chemical potential

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

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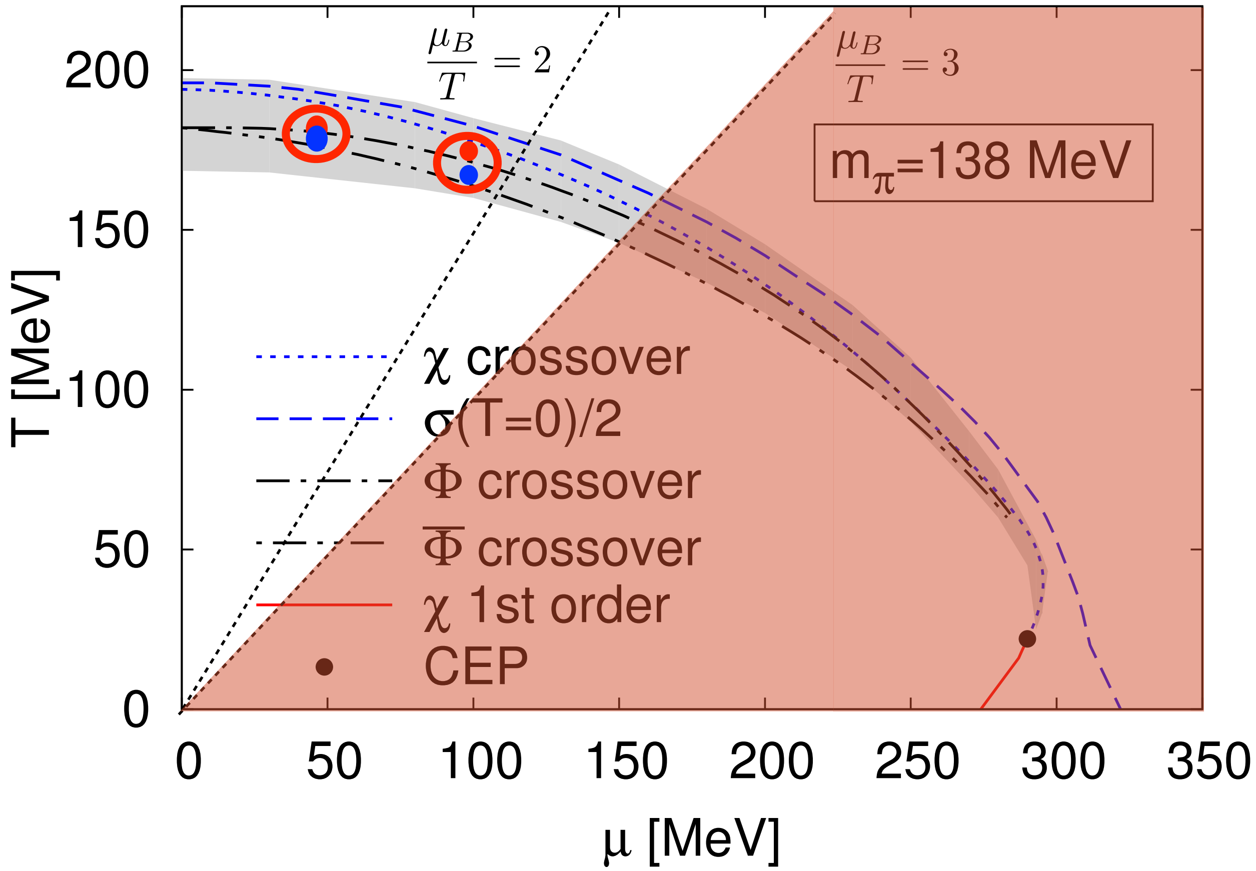

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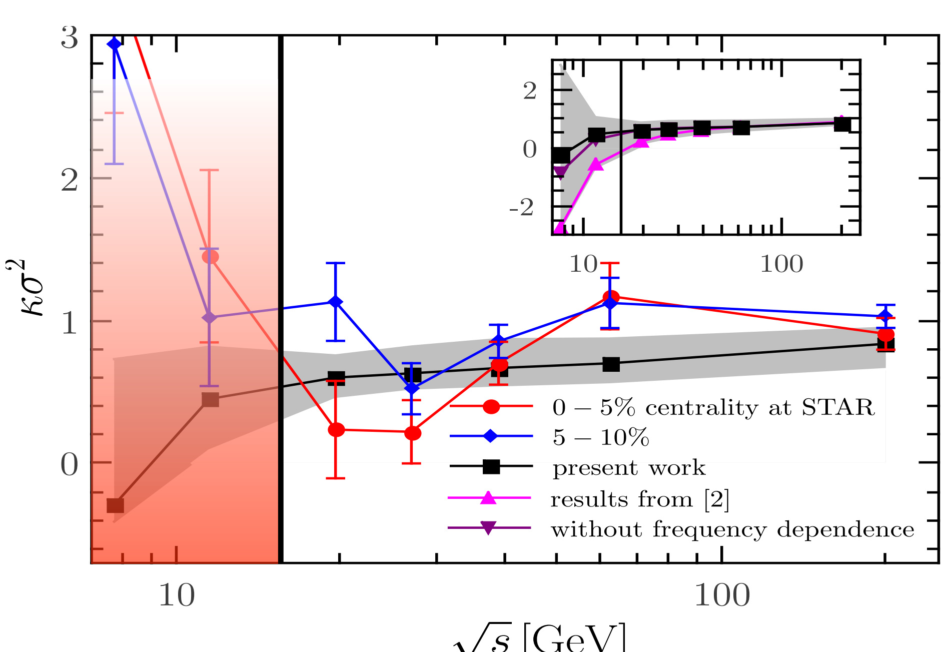
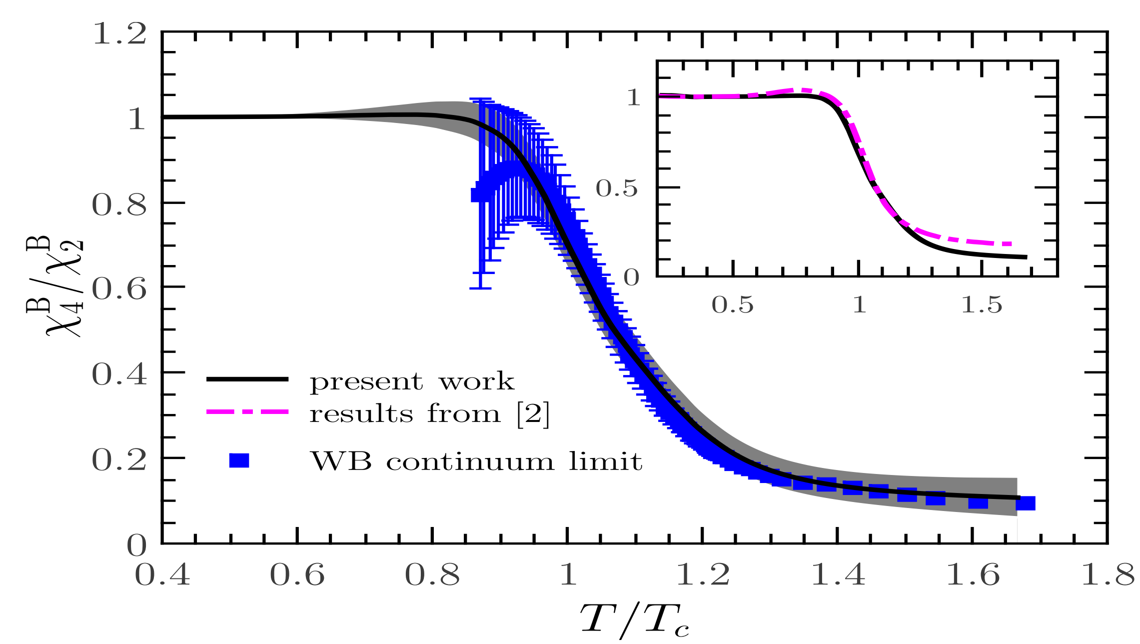
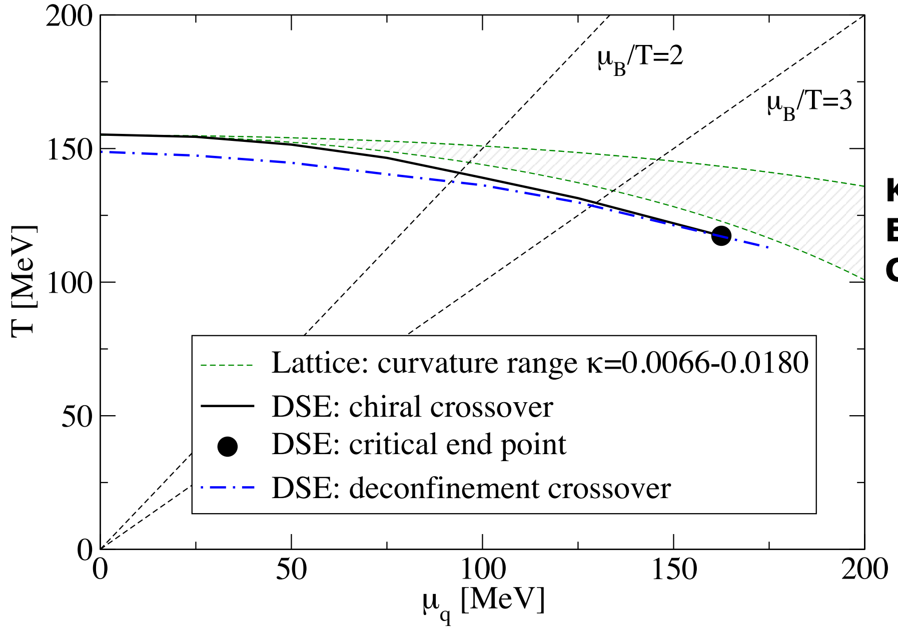




Fu, JMP, Schaefer, Rennecke, arXiv:1608.04302

Phase structure at finite density

Phase diagram of 2+1 flavor QCD



Kaczmarek at al. ’11
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Cea, Cosmai, Papa ’14

Fischer, Fister, Luecker, JMP, PLB732 (2014) 248

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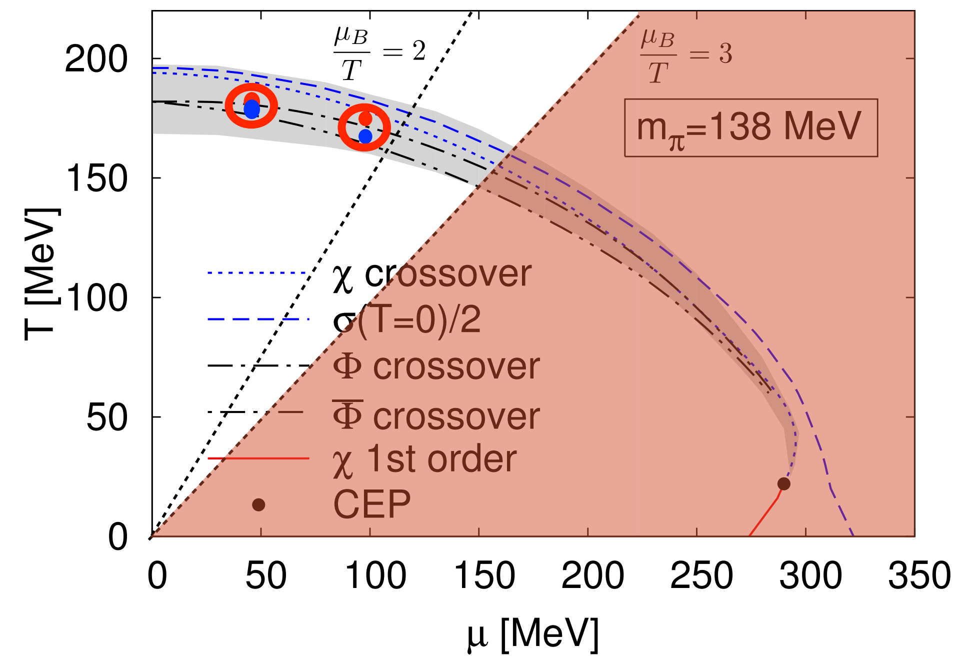
Chiral phase structure

Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301

Phase diagram of QCD-enhanced 2-flavor PQM-model



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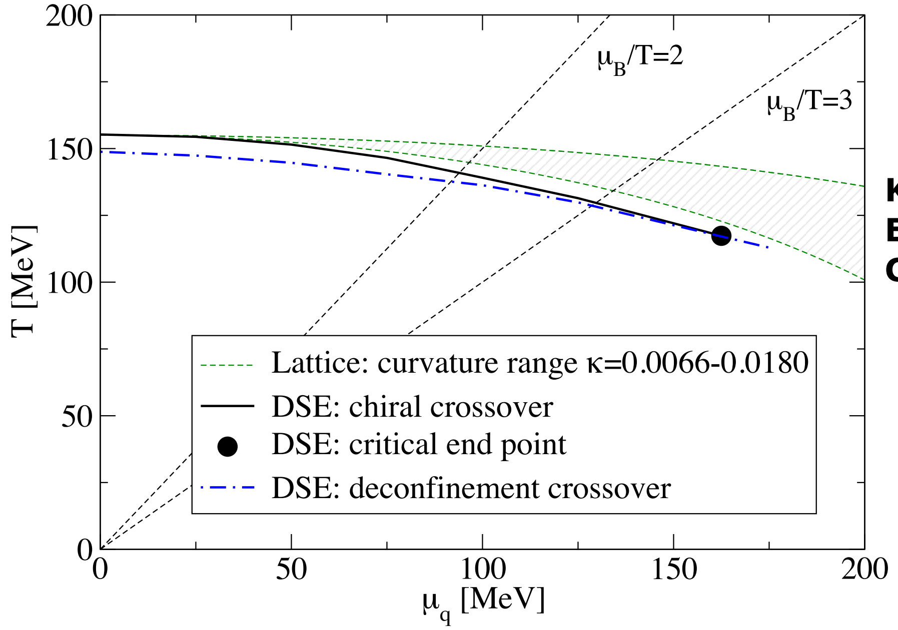

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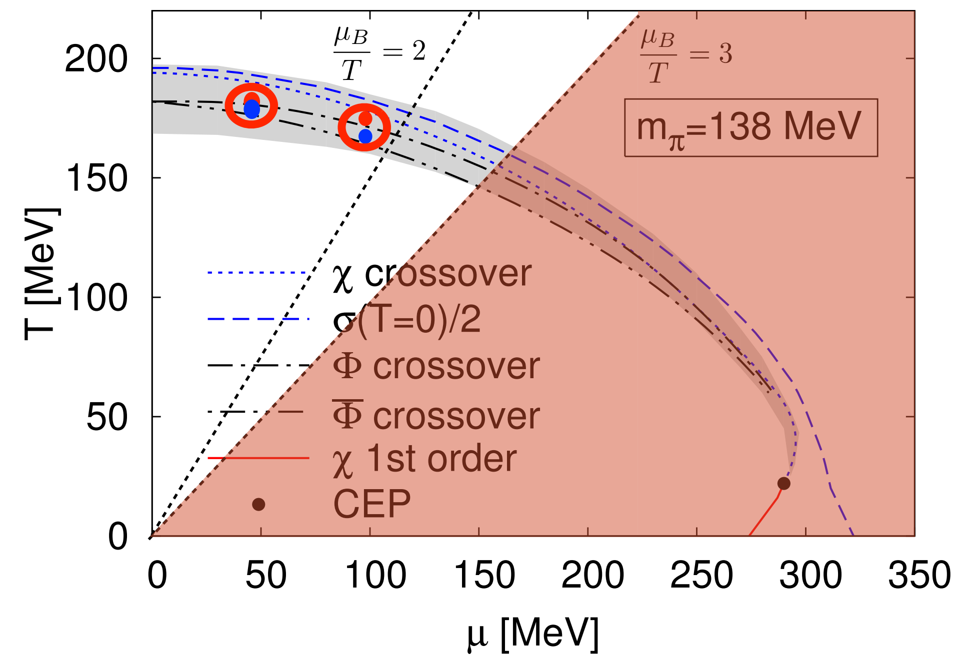

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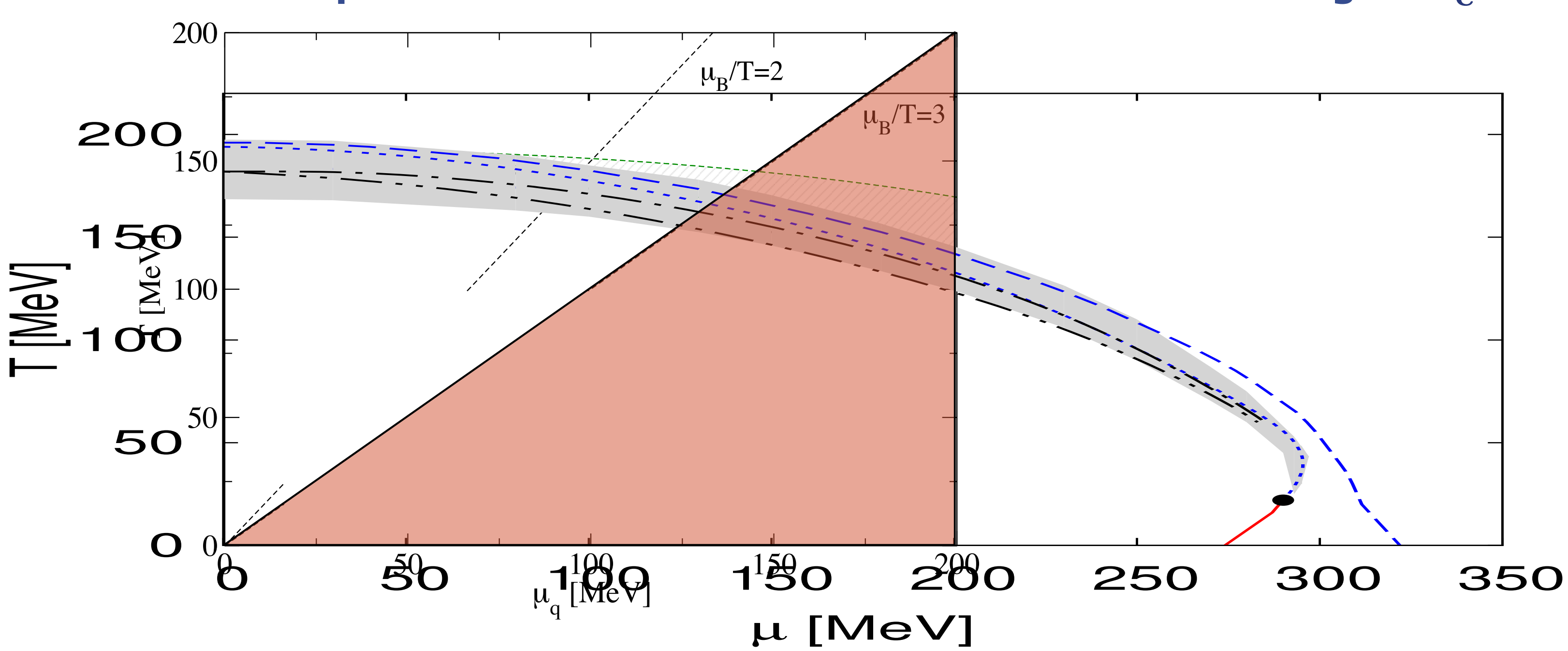

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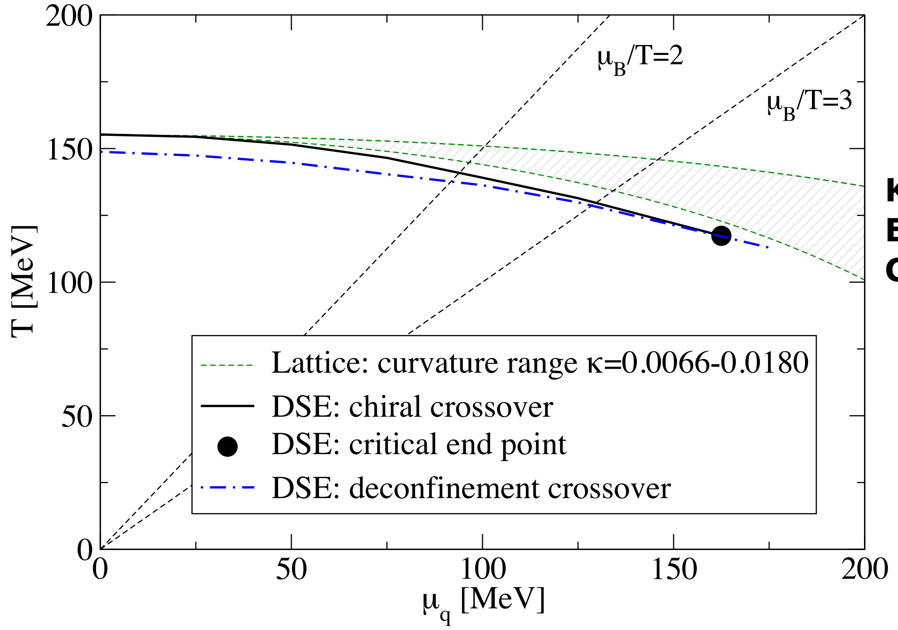

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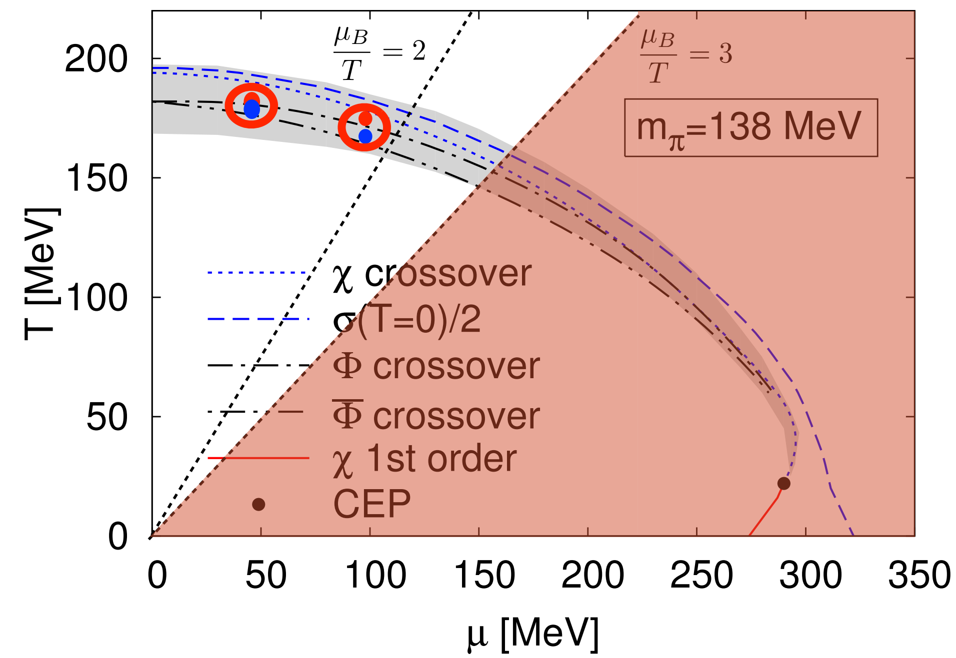

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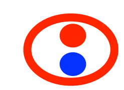

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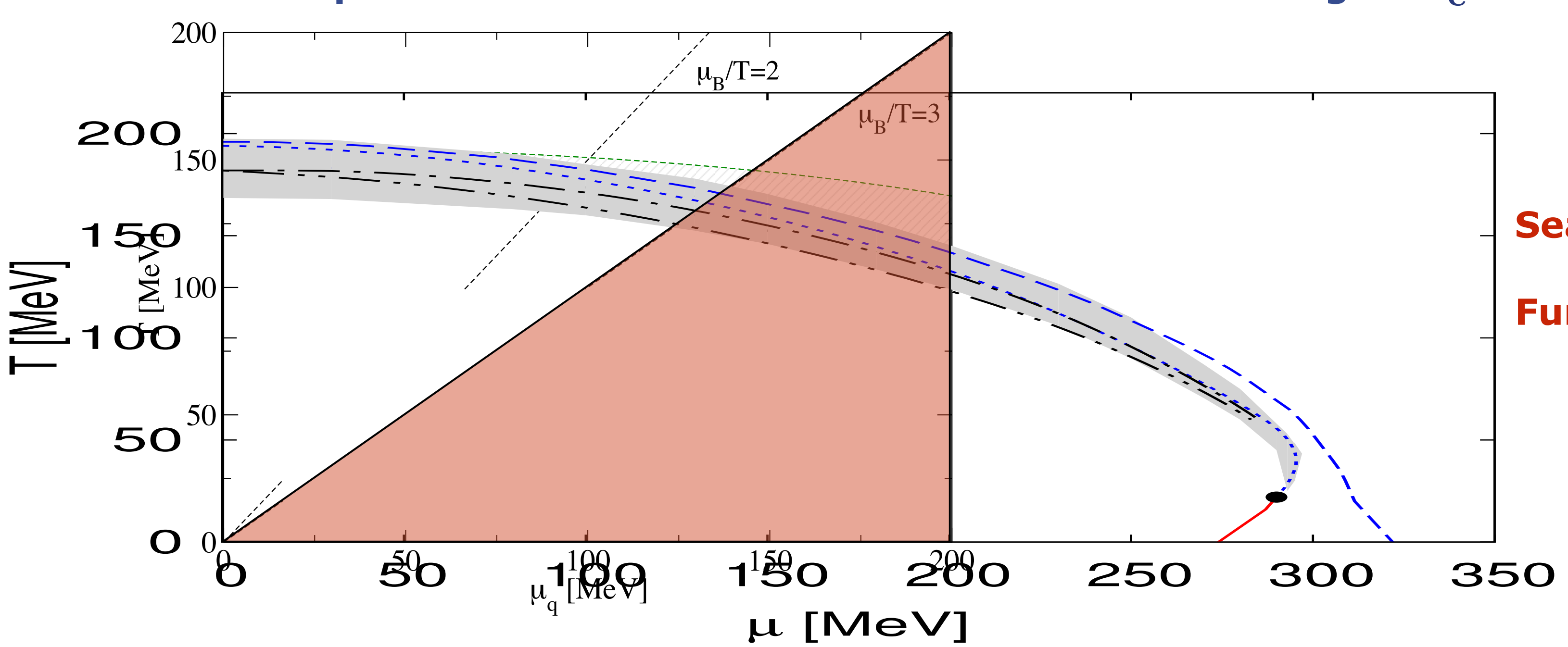


Search for the CEP at high density

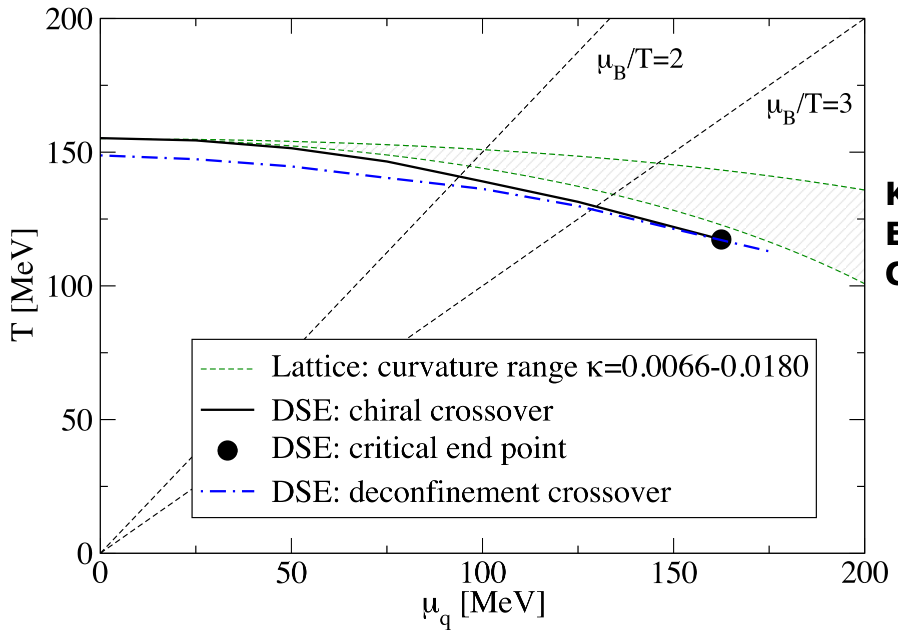
FunMethods:

best non-perturbative
1st principle methods

triggered by Claudia

Phase structure at finite density

Phase diagram of 2+1 flavor QCD



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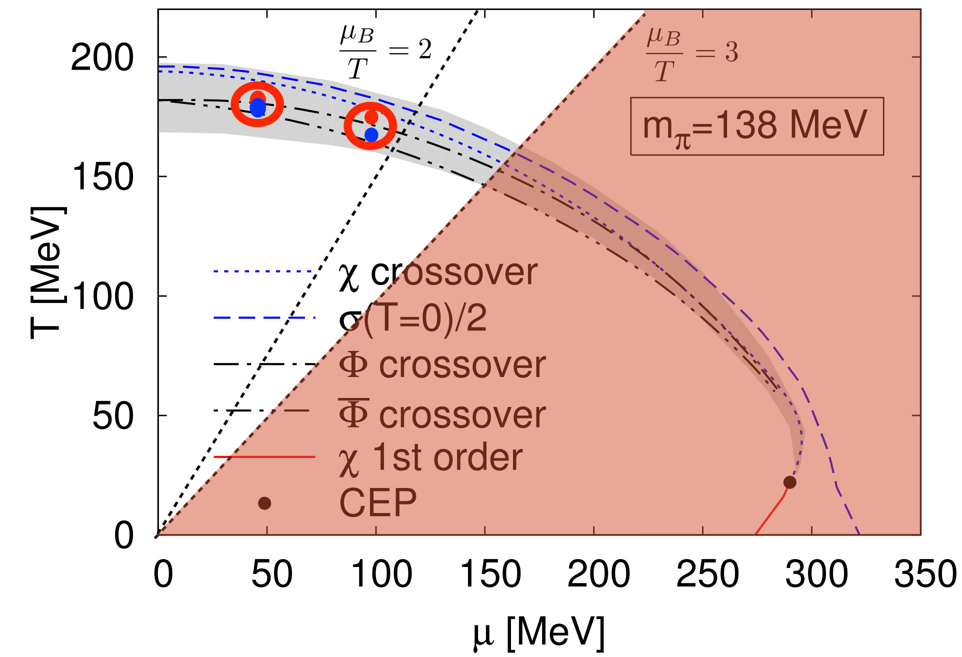

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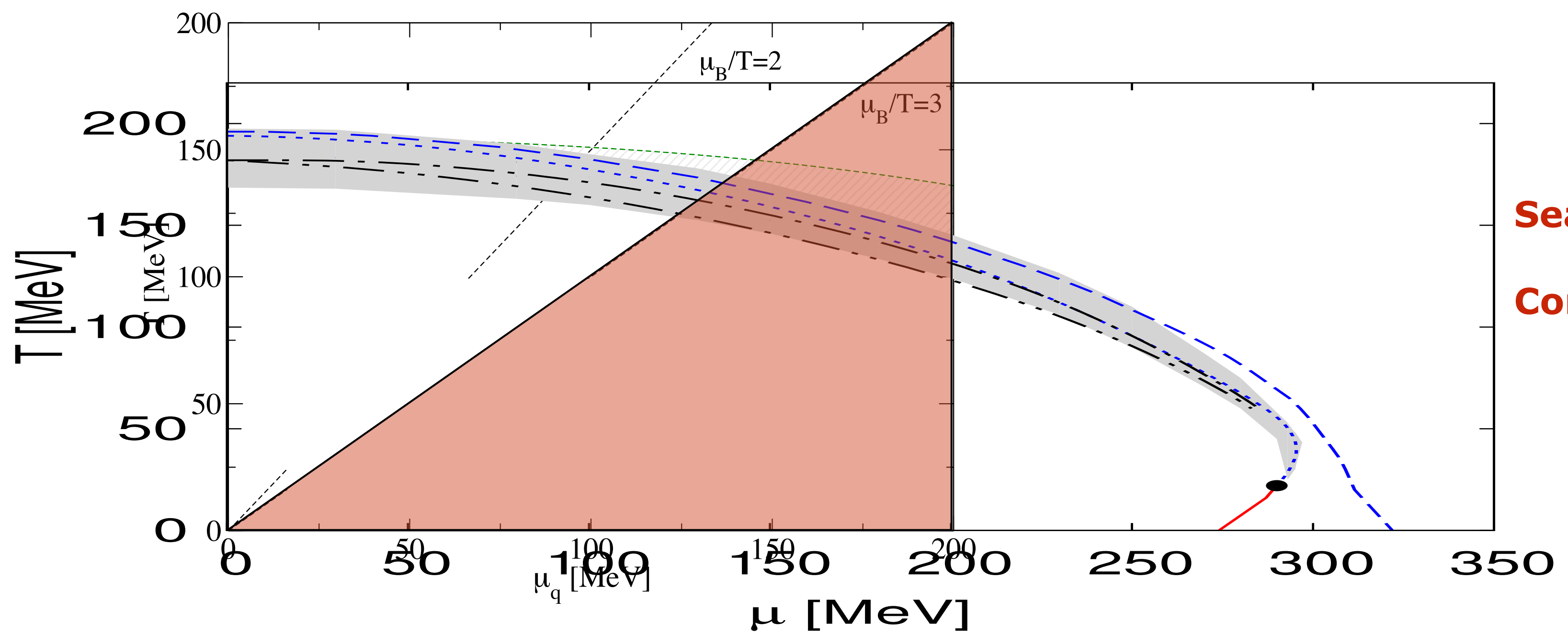

Search for the CEP at high density

Combination of FRG, DSE & lattice

best non-perturbative
1st principle method

Functional RG for QCD

fQCD collaboration: J. Braun, L. Corell, A. Cyrol, W.-j. Fu, M. Leonhardt, M. Mitter, JMP, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink

Darmstadt, Heidelberg

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Cyrol, Mitter, Strodthoff, arXiv:1610:09331

Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

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hardQCD: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

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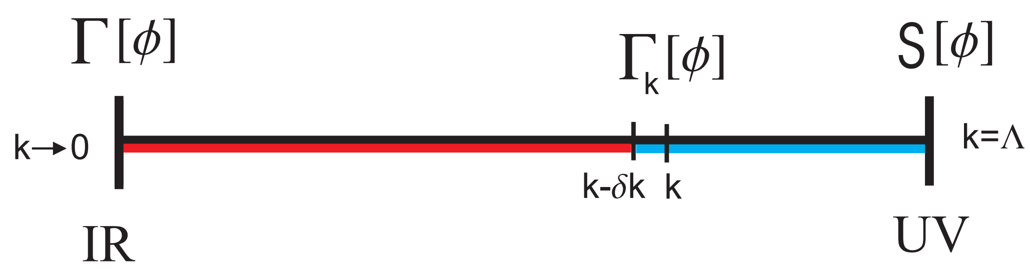
easyQCD: Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Rennecke, PRD 92 (2015) 076012

Functional RG for QCD

JMP, AIP Conf.Proc. 1343 (2011)
Nucl.Phys. A931 (2014) 113

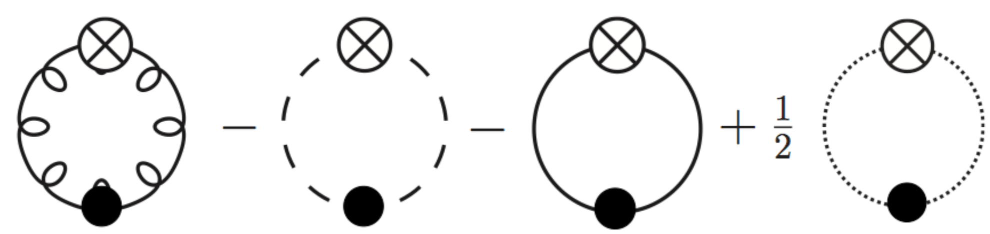
free energy at momentum scale k



ab initio

glue quantum fluctuations

hadronic quantum fluctuations

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ 

free energy/ grand potential

quark quantum fluctuations

RG-scale k: $t = \ln k$

closed form

Functional RG for QCD

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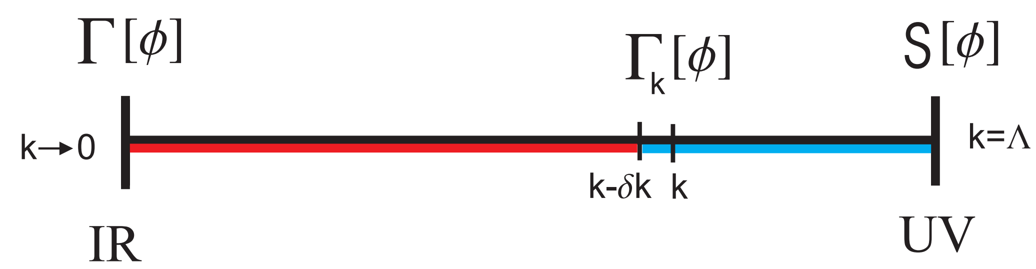

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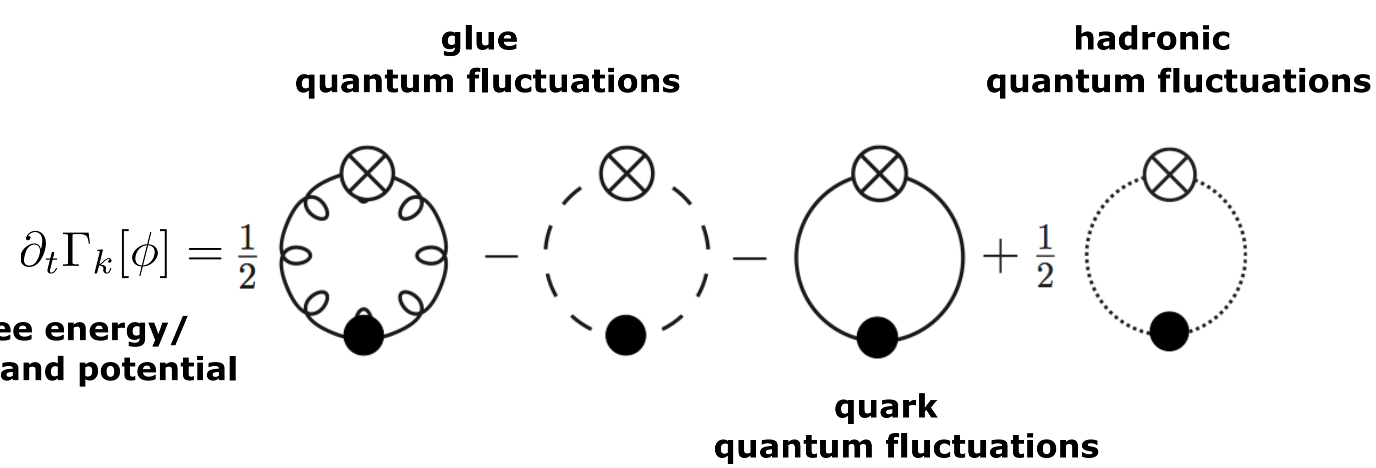

RG-scale k: $t = \ln k$

closed form

properties

- access to physics mechanisms
- numerically tractable
  no sign problem
  systematic error control via closed form
- low energy models naturally encorporated

Functional RG for QCD

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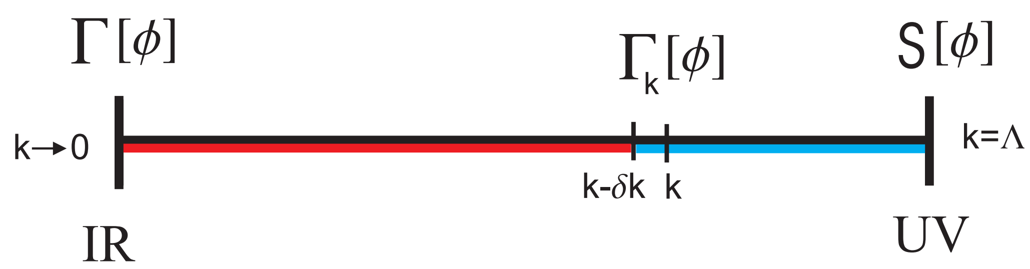

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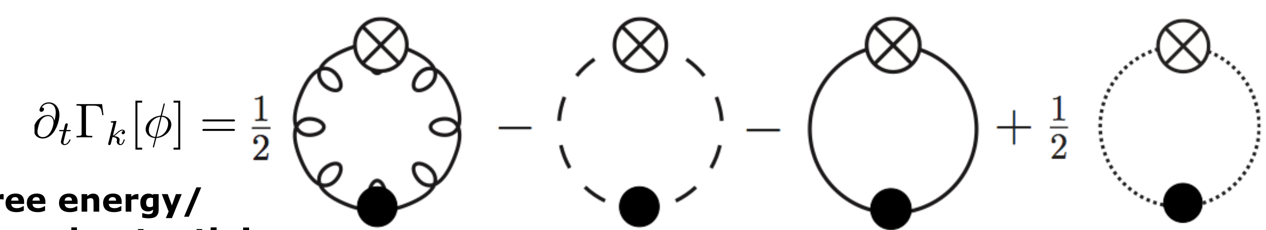

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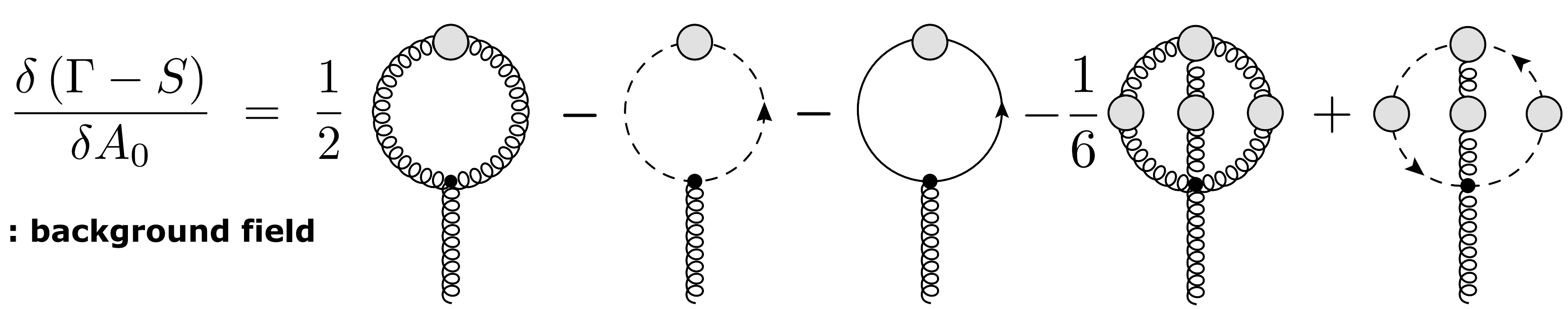
functional DSE :



A_0 : background field

fQCD: workflow

VertEXpand
Mathematica package for the derivation of vertices from a given action using FORM
(Denz,Held,Rodigast; unpub.)

DoFun
Mathematica package for the derivation of functional equations
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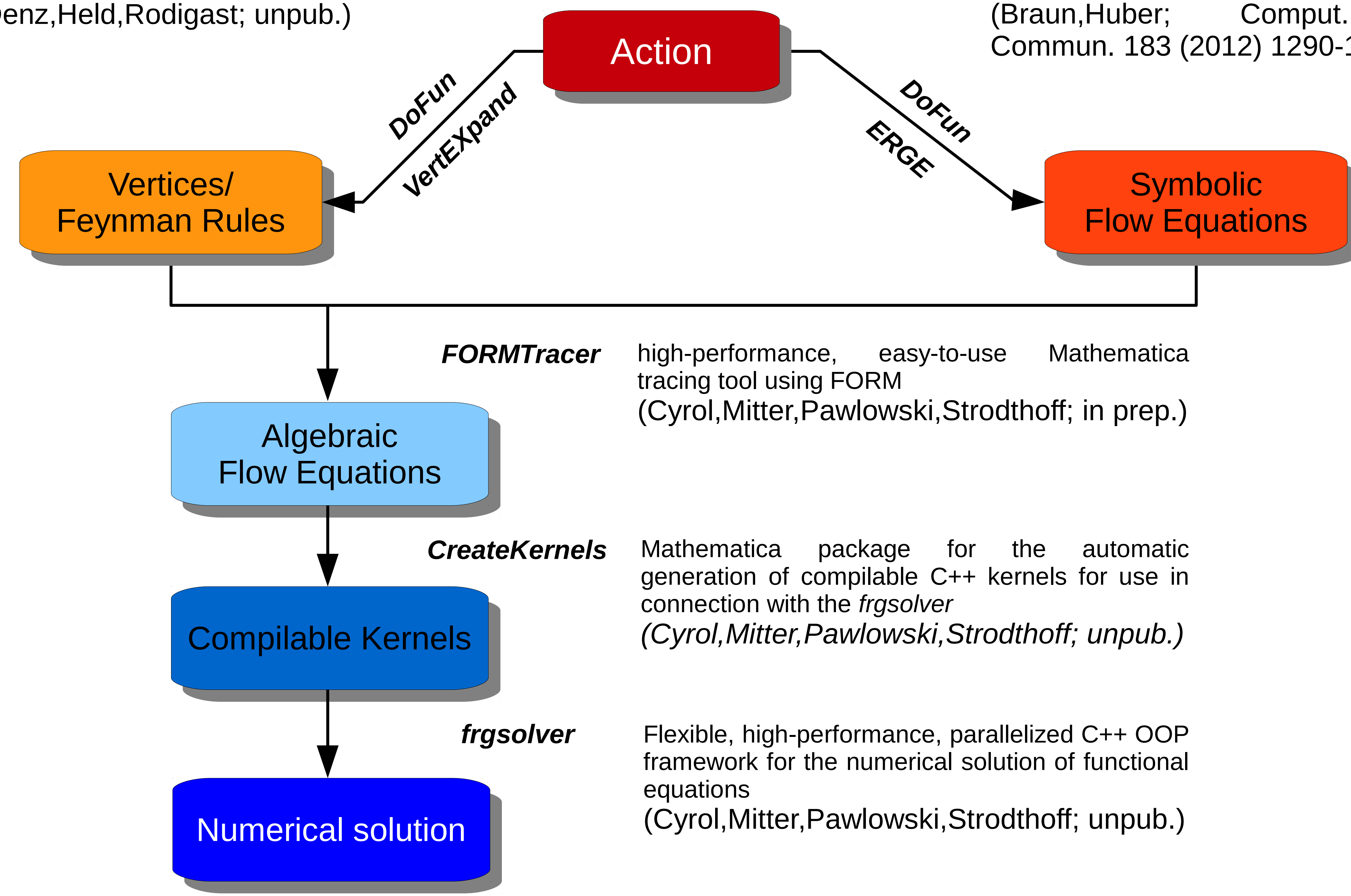

FORMTracer high-performance, easy-to-use Mathematica tracing tool using FORM
(Cyrol,Mitter,Pawlowski,Strodthoff; in prep.)

CreateKernels Mathematica package for the automatic generation of compilable C++ kernels for use in connection with the *frgsolver*
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frgsolver Flexible, high-performance, parallelized C++ OOP framework for the numerical solution of functional equations
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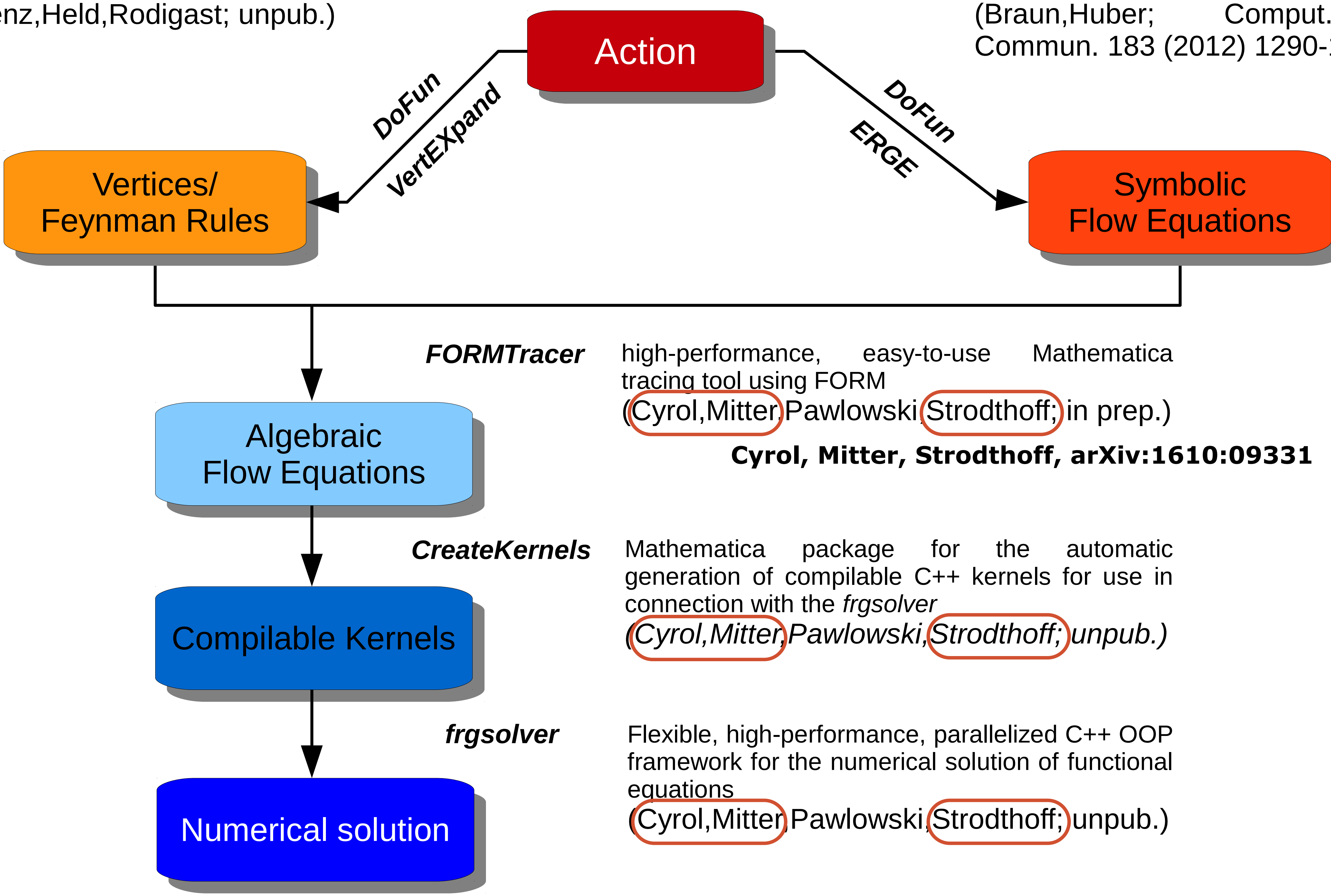

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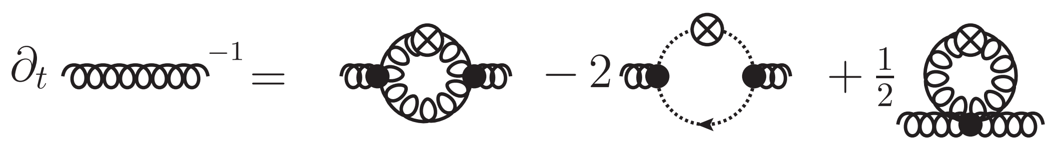
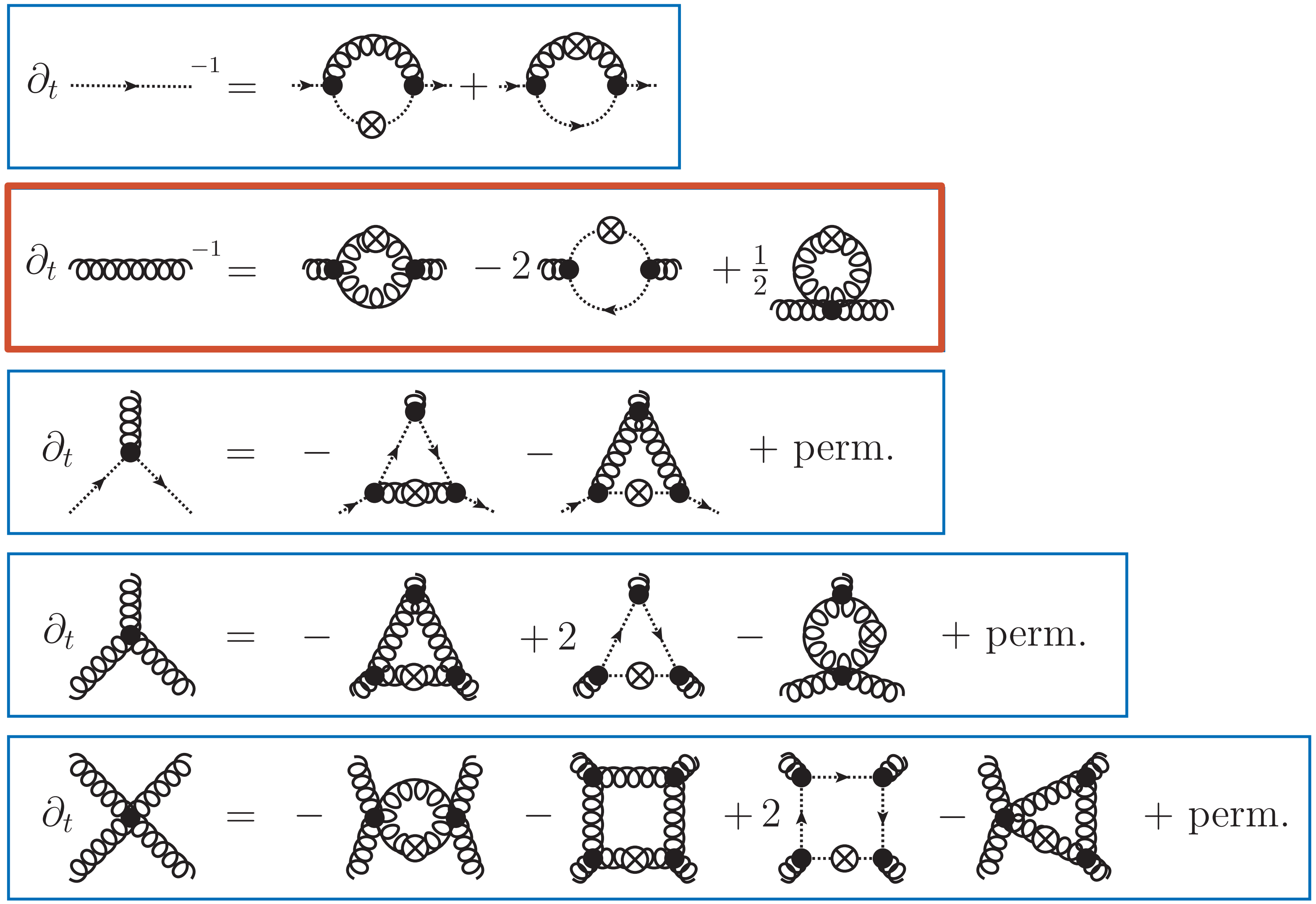
Gluonic correlation functions

$$\langle A\, A \rangle (p^2)$$



Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Gluonic correlation functions



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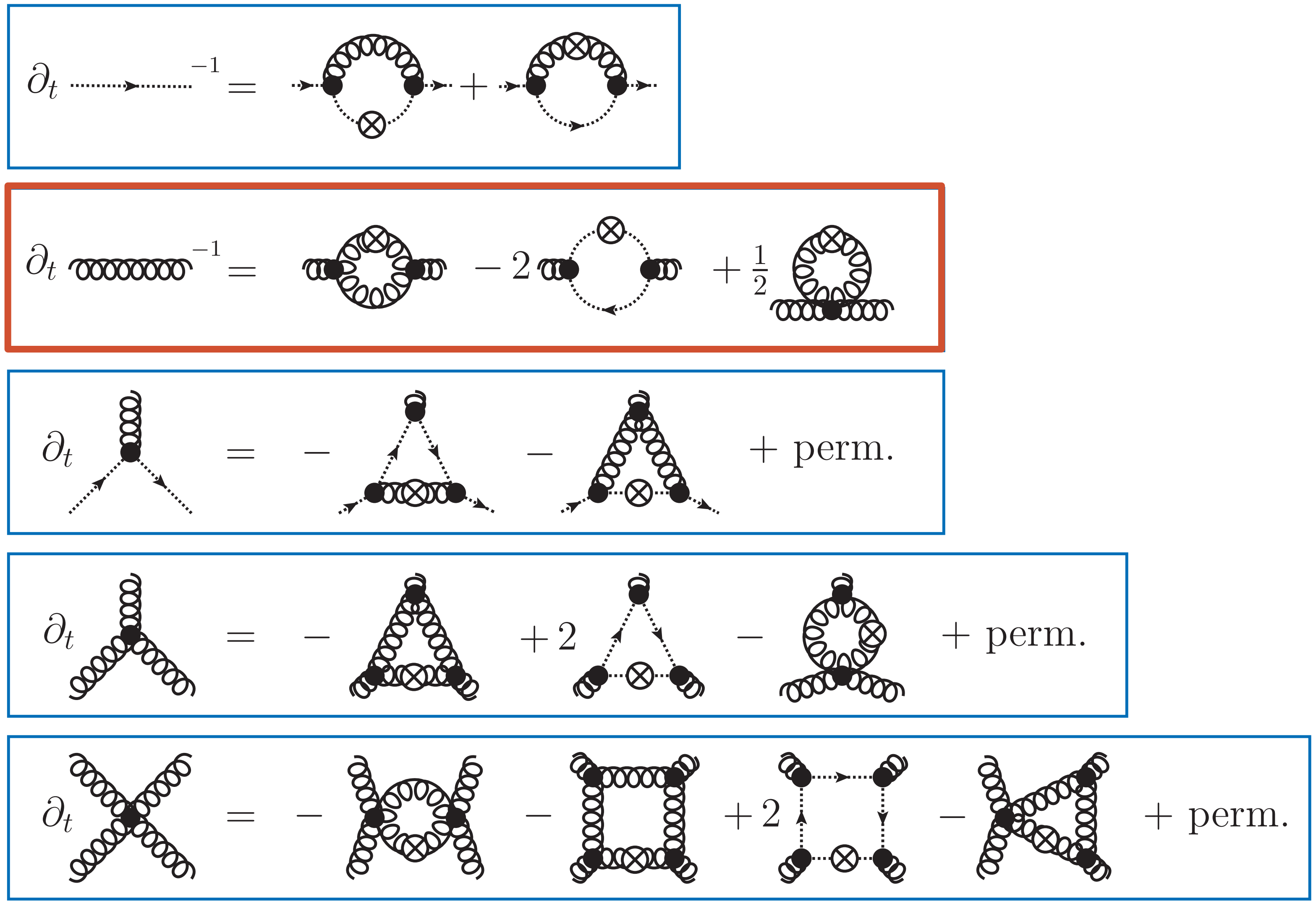

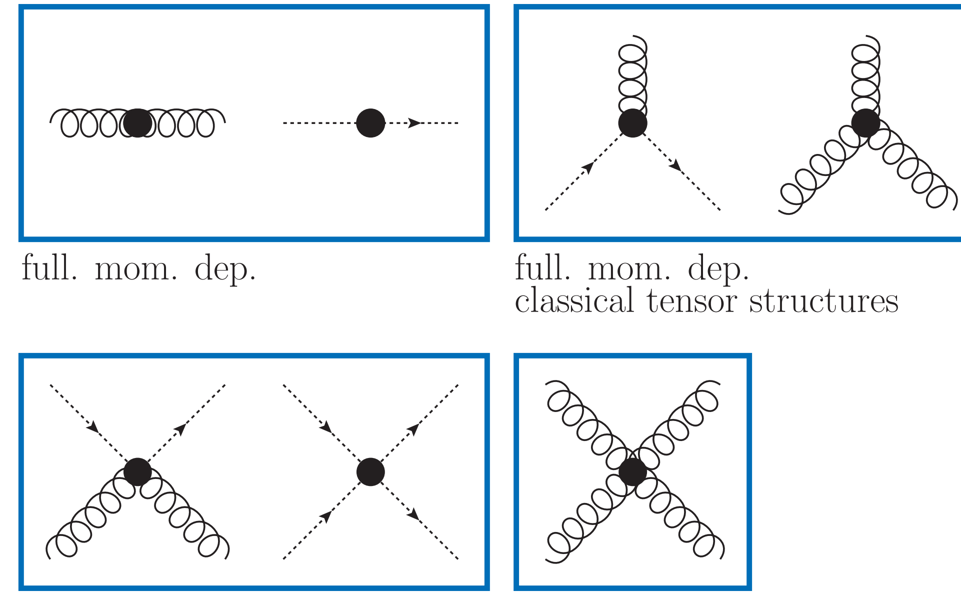

full. mom. dep.

full. mom. dep.
classical tensor structures

mom. dep. needed by tadpoles
full tensor basis

sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

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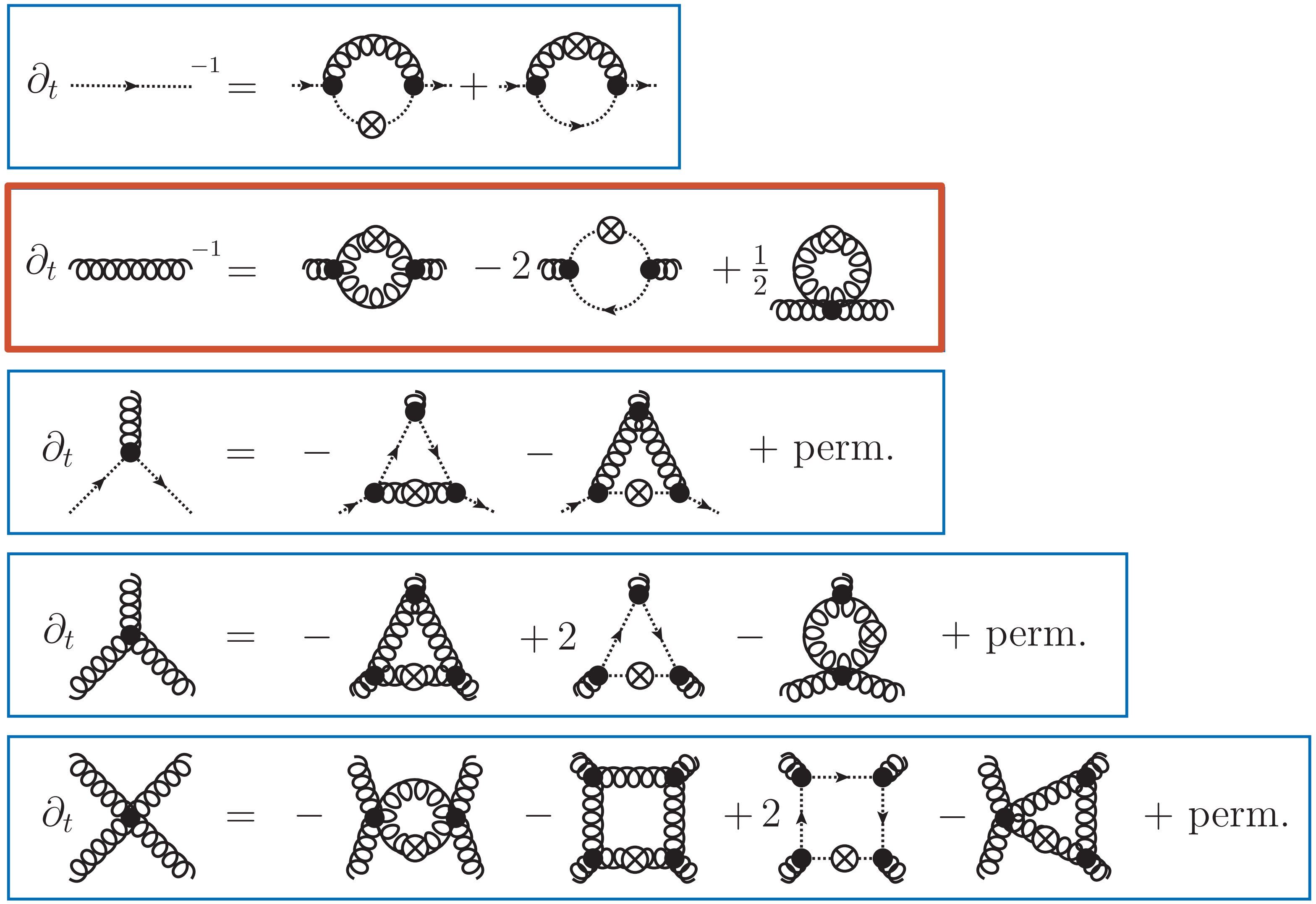
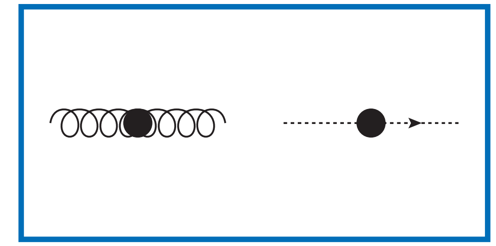
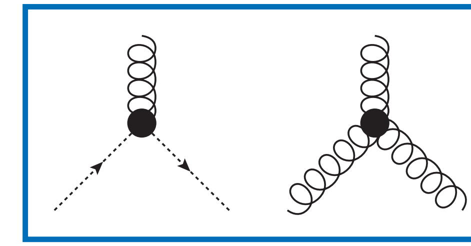
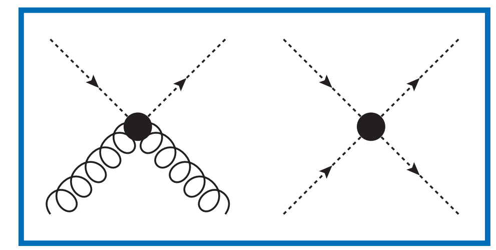




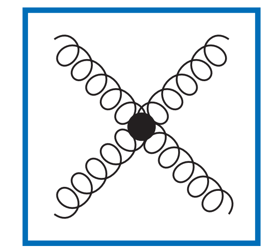
full. mom. dep.



full. mom. dep.
classical tensor structures



mom. dep. needed by tadpoles
full tensor basis



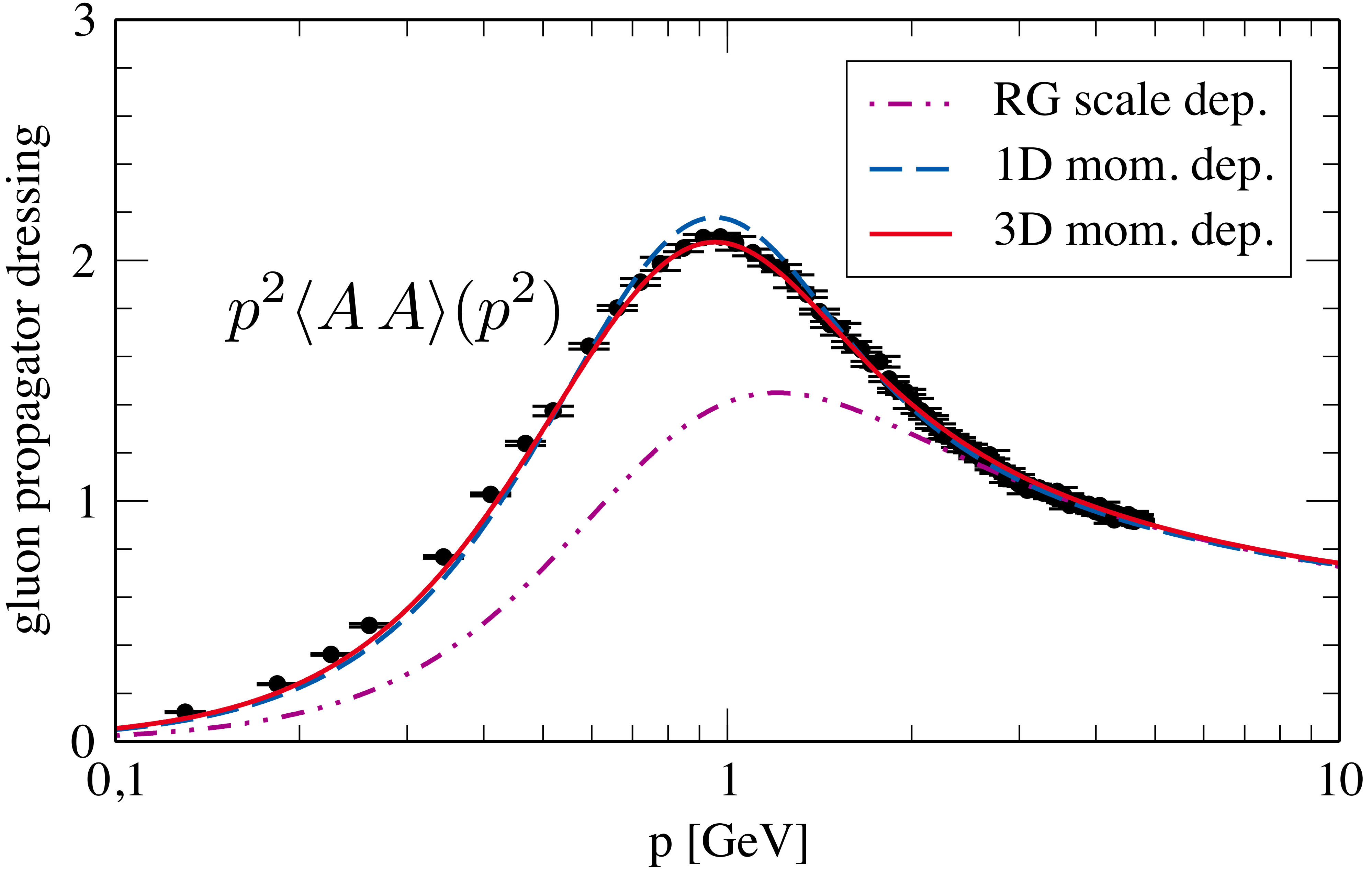
sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Euclidean gluon propagator

Functional Renormalisation Group



Lattice: Sternbeck, Ilgenfritz, Müller-Preussker, Schiller, Bogolubsky, PoS LAT2006, 076

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Euclidean gluon propagator

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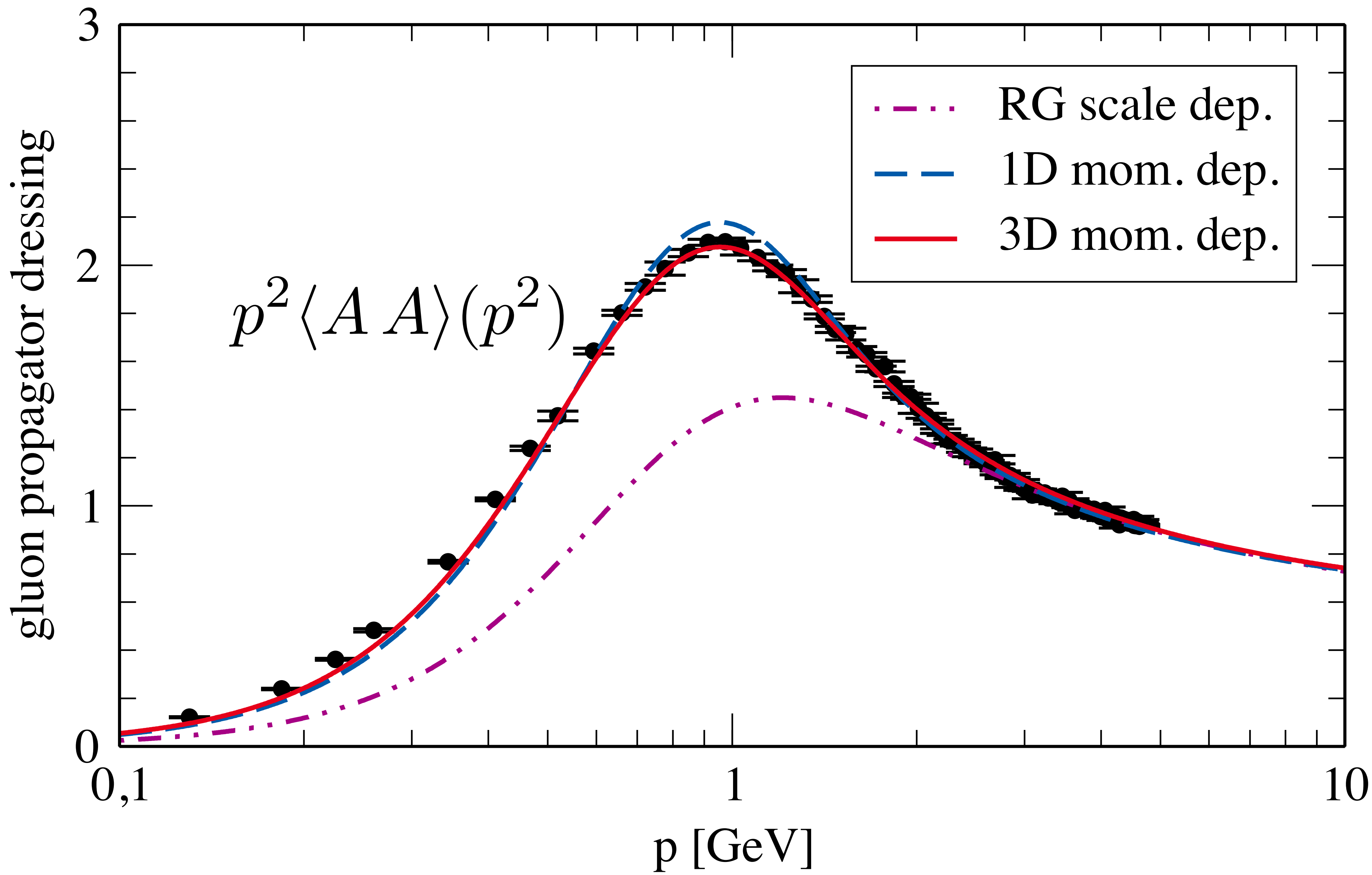


Lattice: Sternbeck, Ilgenfritz, Müller-Preussker, Schiller, Bogolubsky, PoS LAT2006, 076

Aiming at apparent convergence

up to date pinch technique:
Aguilar, Binosi, Papavassiliou, PRD 89 (2014) 085032

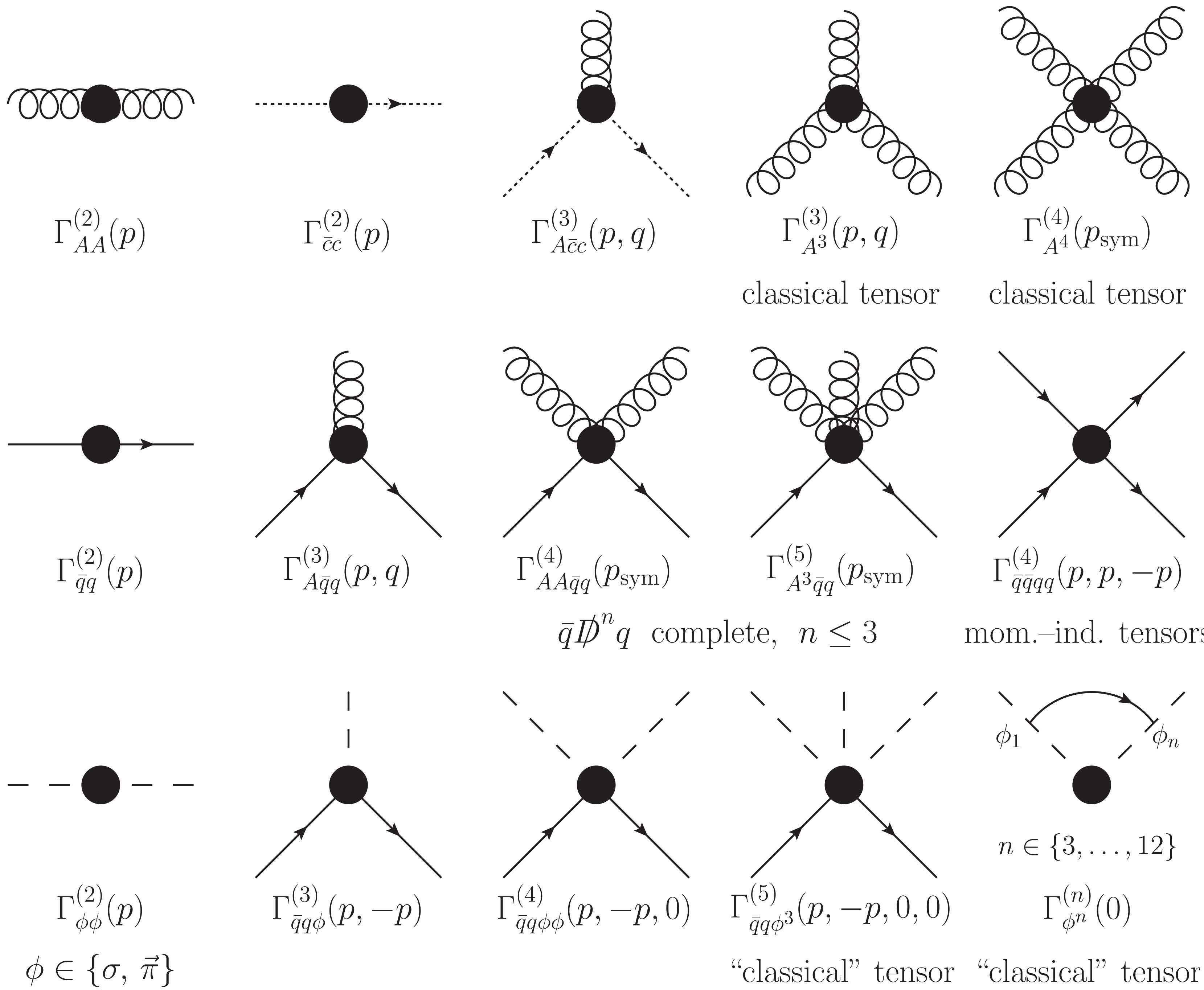
up to date DSE:
Cyrol, Huber, Smekal, EPJ C75 (2015) 102

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

... and now for something completely different ...

Aiming at apparent convergence

Approximation in full QCD



Aiming at apparent convergence

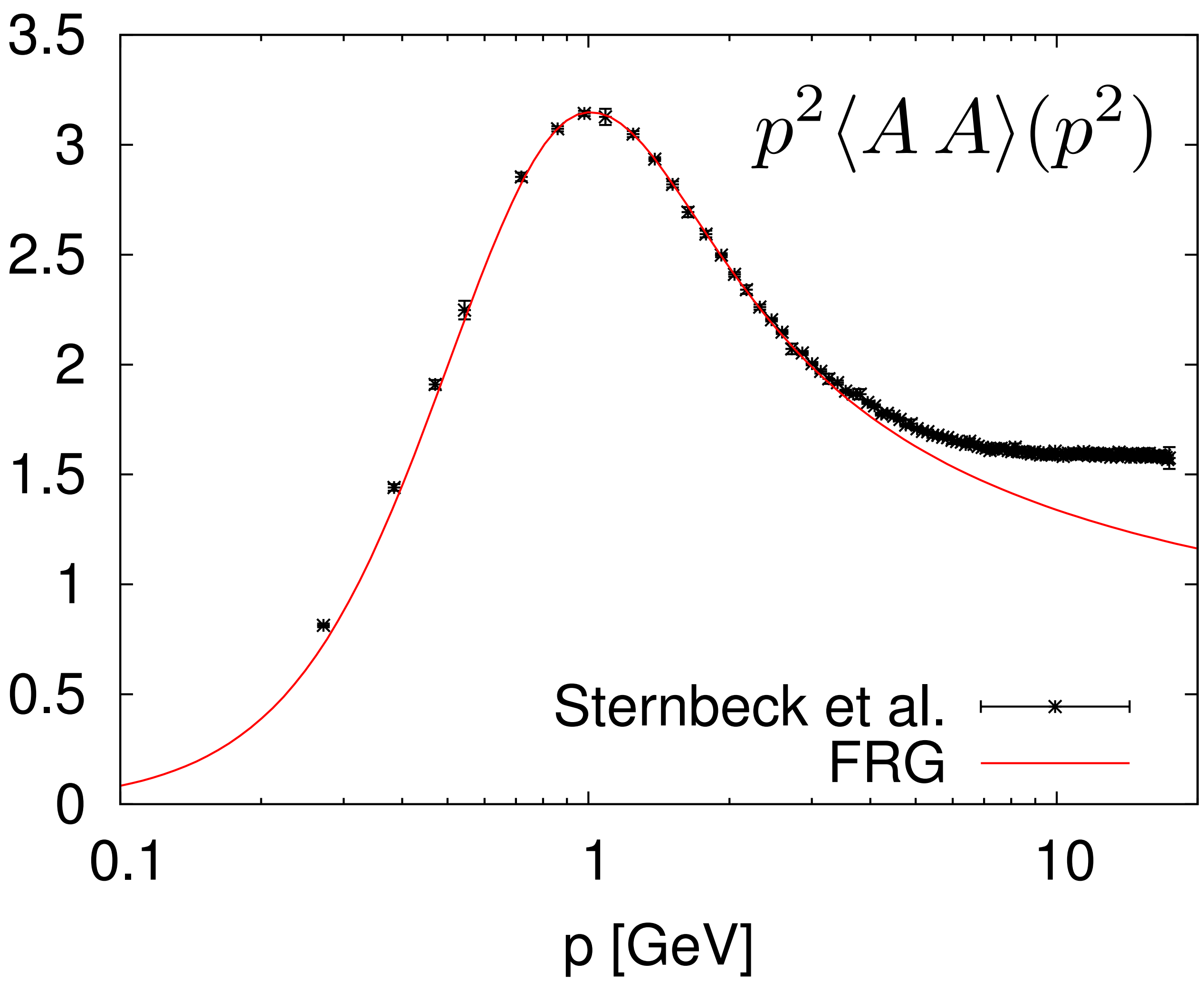
Cyrol, Mitter, JMP, Strodthoff, in prep.

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

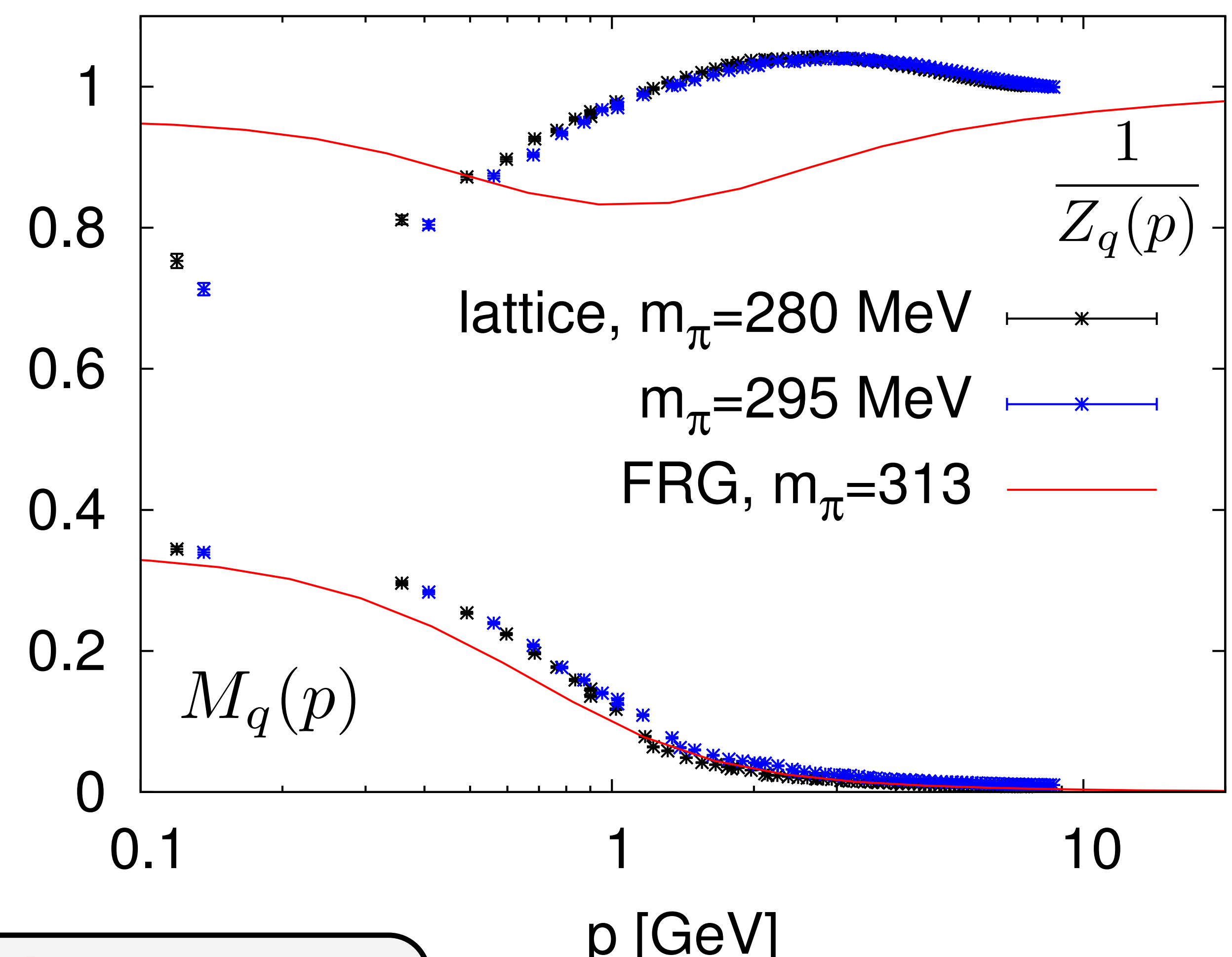
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Cyrol, Mitter, Strodthoff, arXiv:1610:09331

Euclidean propagators



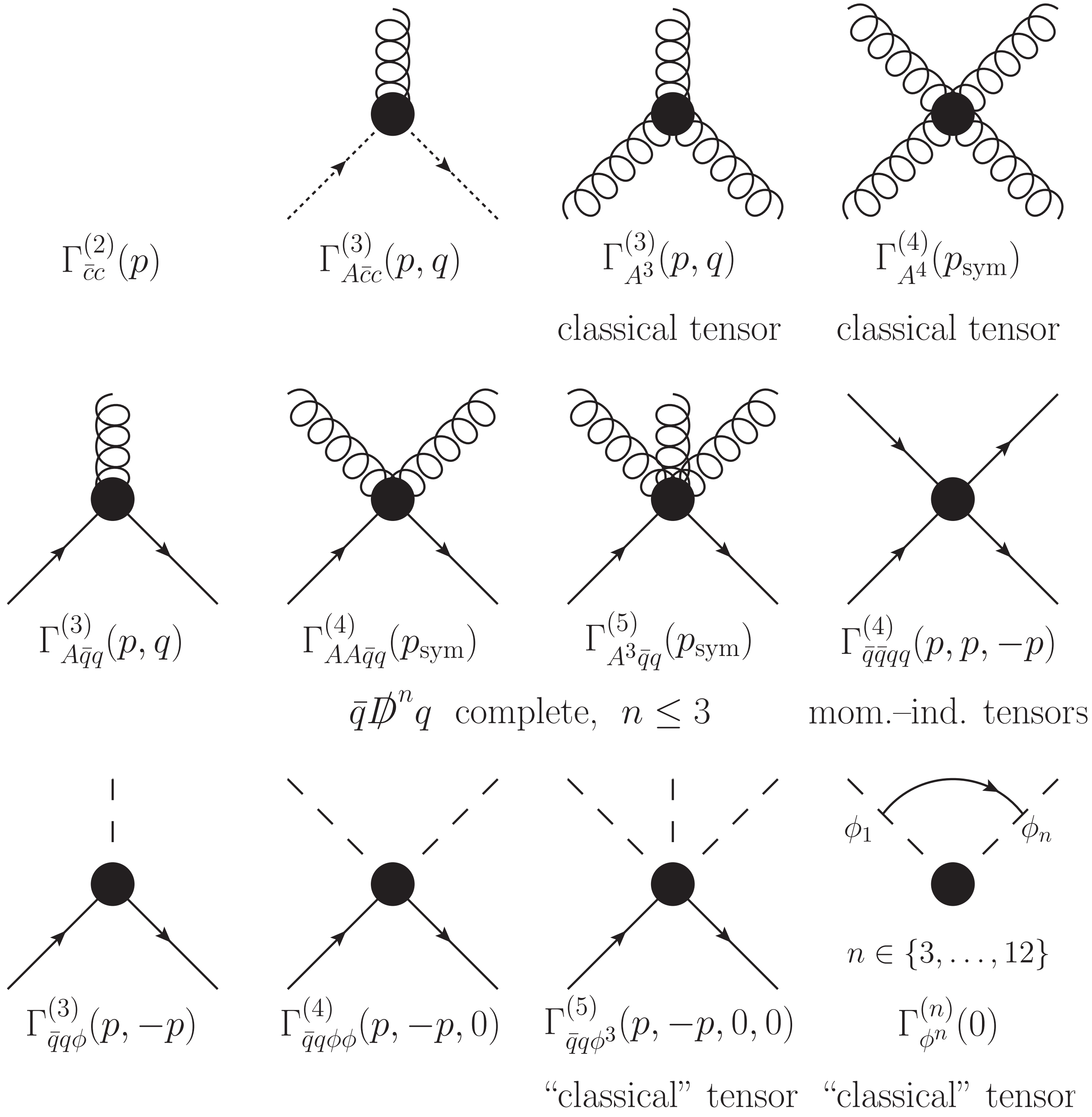
$$\frac{1}{Z_q(p)}\,\frac{1}{i\,\not{p} + M_q(p)}$$



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

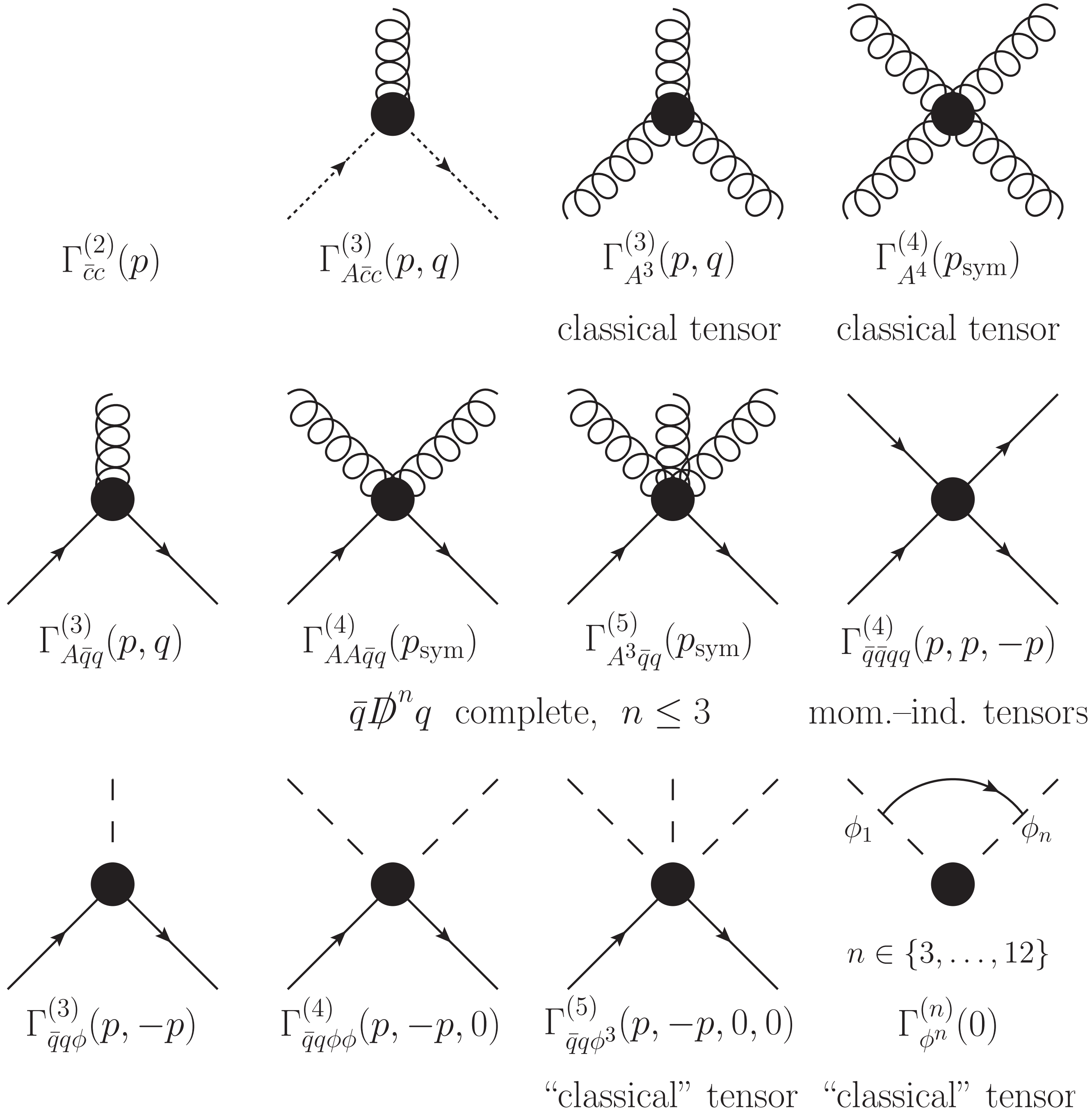
Vertices



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

Vertices

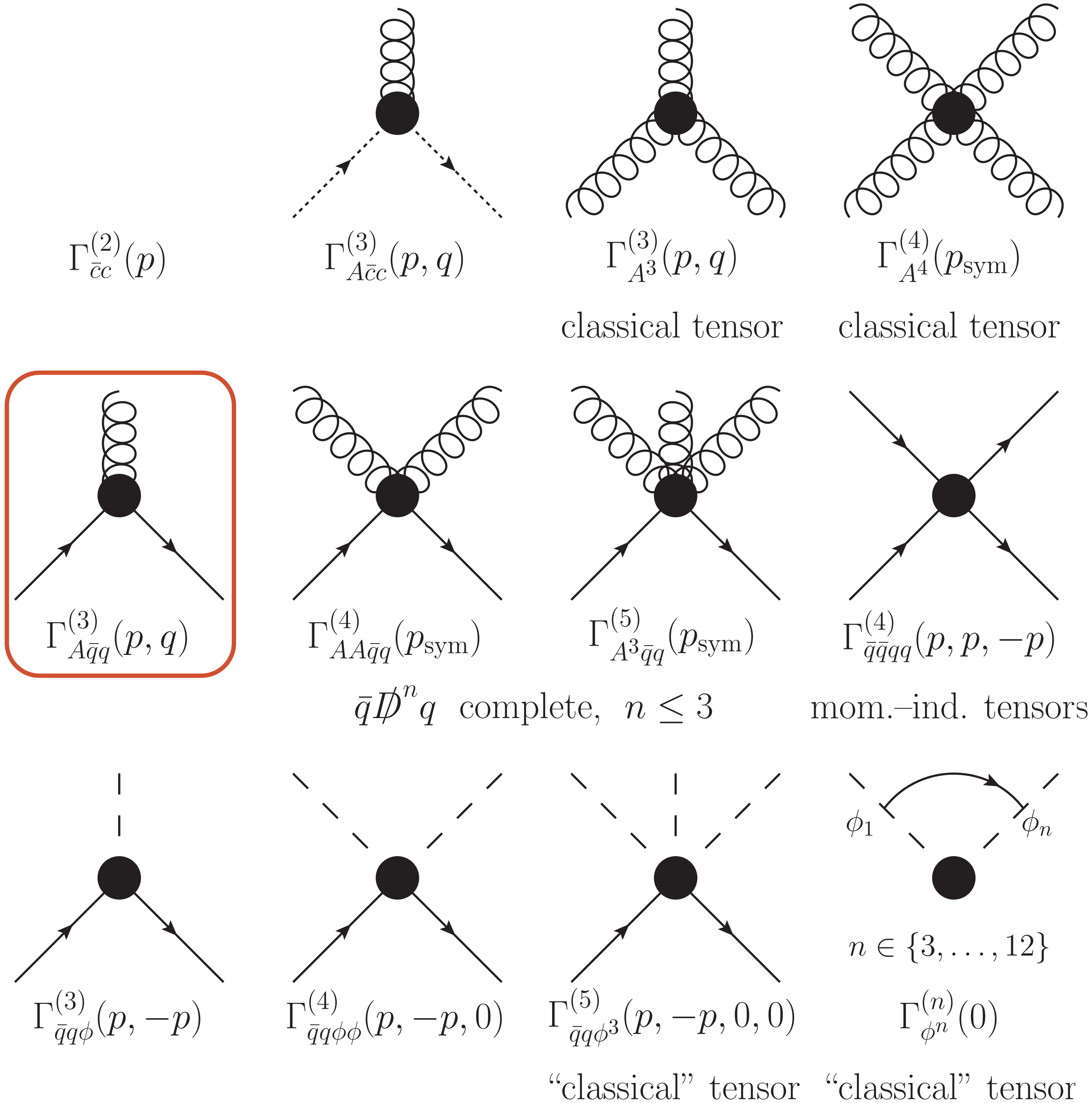


Welches Schweinderl hätten's denn gerne?

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

Vertices



Welches Schweinderl hätten's denn gerne?

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^a_\mu (p,q) = 1^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_\mu (p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q:$ $\quad \left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_\mu (p,q) = -i\,\gamma_\mu$

$\bar{q}\not{D}^2 q:$ $\quad \left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_\mu (p,q) = (p-q)_\mu\, 1_{4\times 4}$

$$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}-\not{q})\gamma_\mu$$

$$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_\mu (p,q) = (\not{p}+\not{q})\gamma_\mu$$

$\bar{q}\not{D}^3 q:$ $\quad \left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}+\not{q})(p-q)_\mu$

$$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_\mu (p,q) = i\,(\not{p}-\not{q})(p-q)_\mu$$

$$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_\mu (p,q) = \frac{i}{2}[\not{p},\not{q}]\gamma_\mu$$

Aiming at apparent convergence

quenched: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

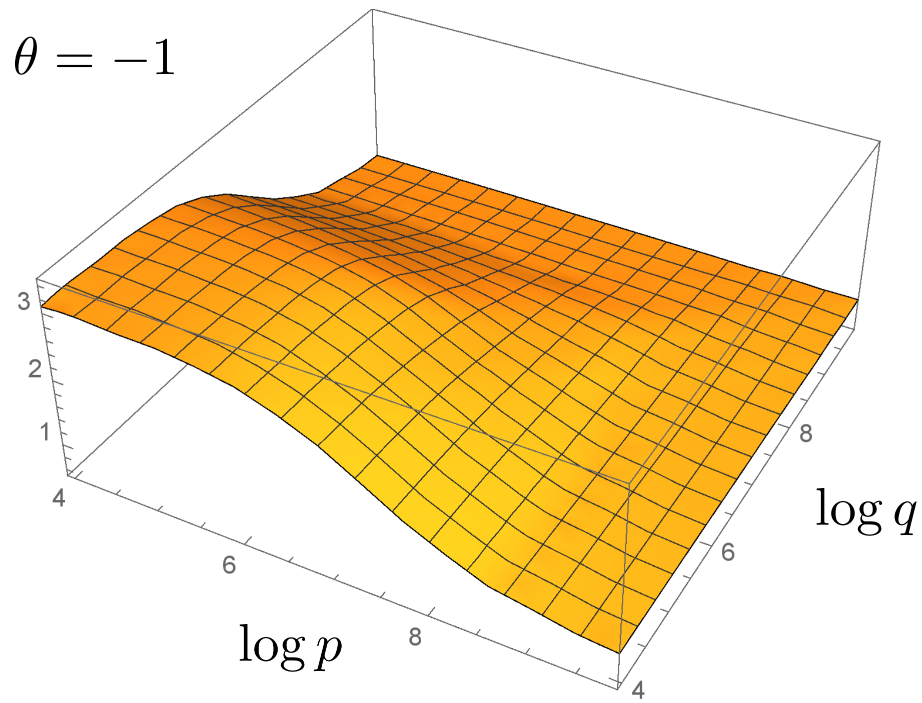
Cyrol, Mitter, JMP, Strodthoff, in prep.

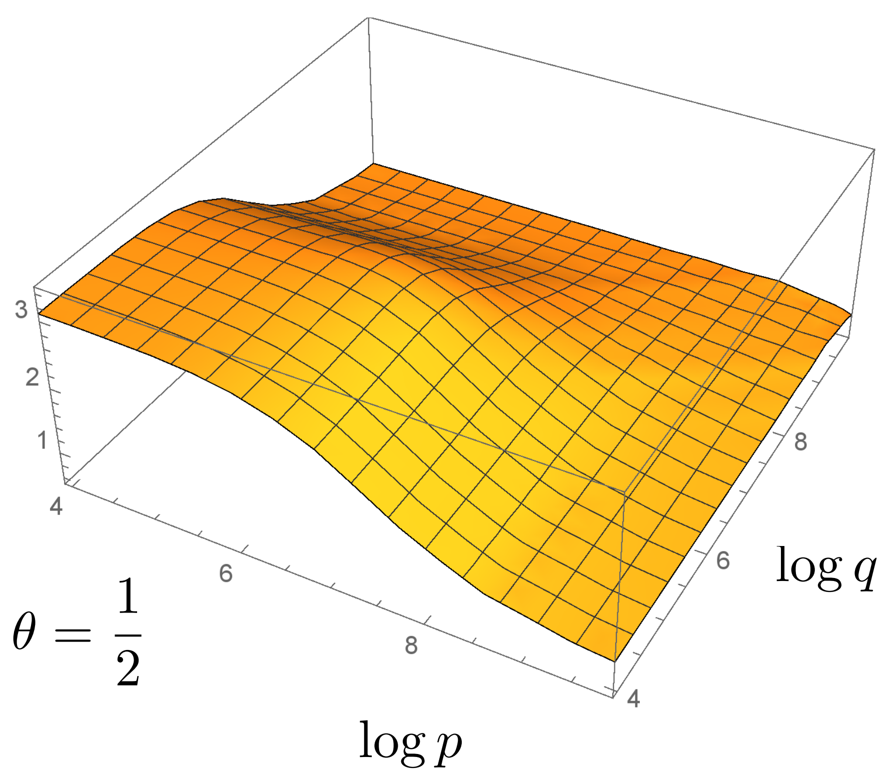
Quark-gluon vertex

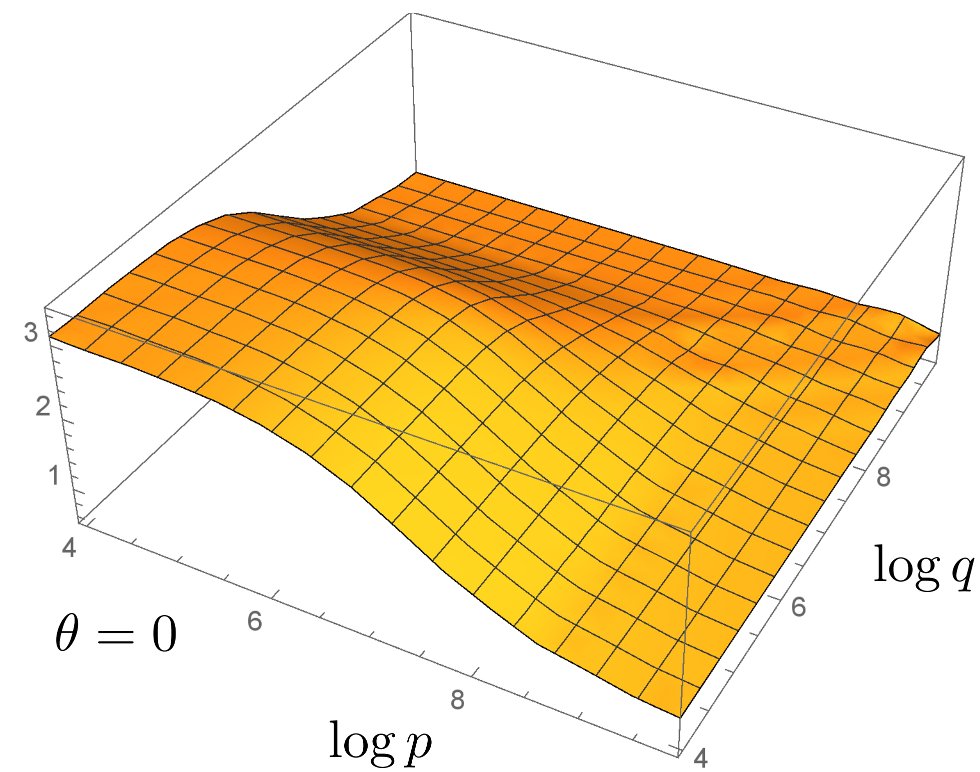
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV

$\lambda_1(p,q)$







Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

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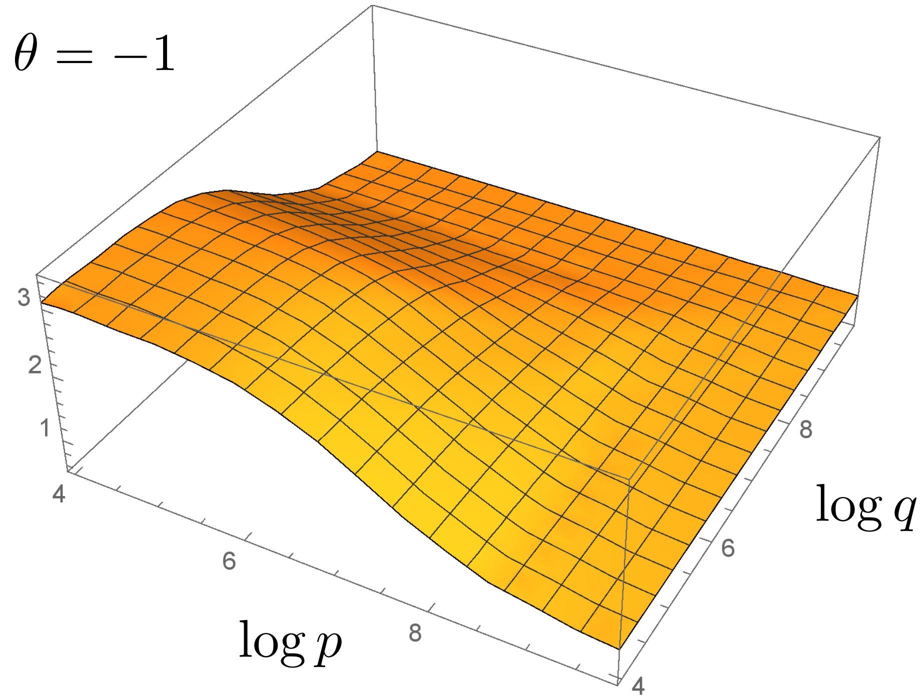
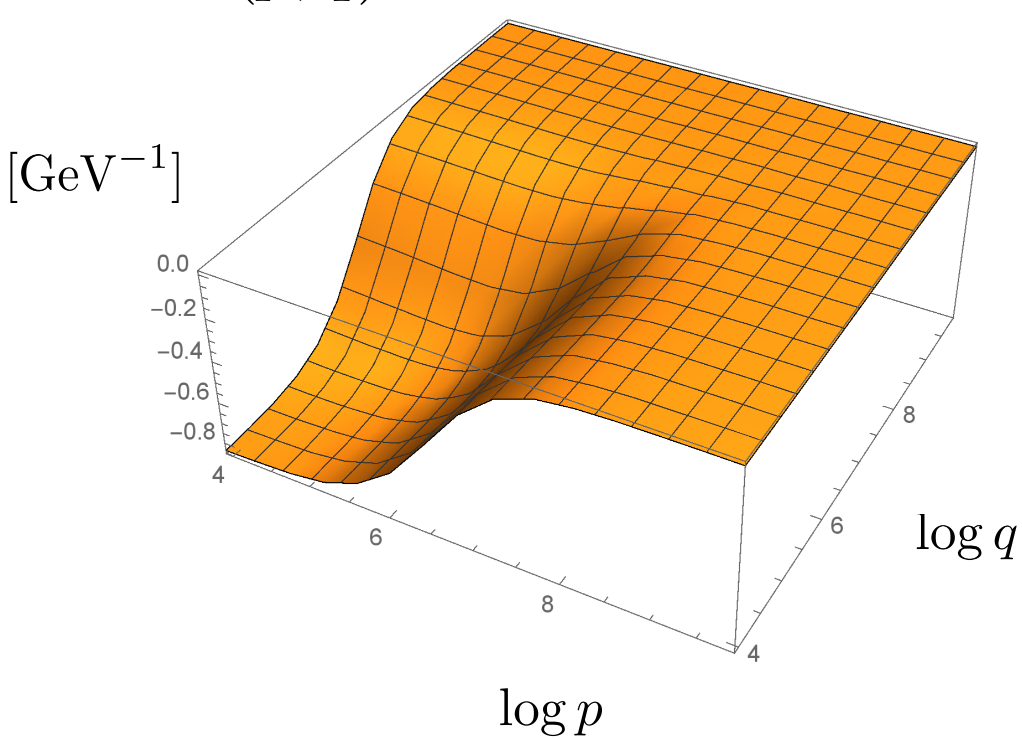
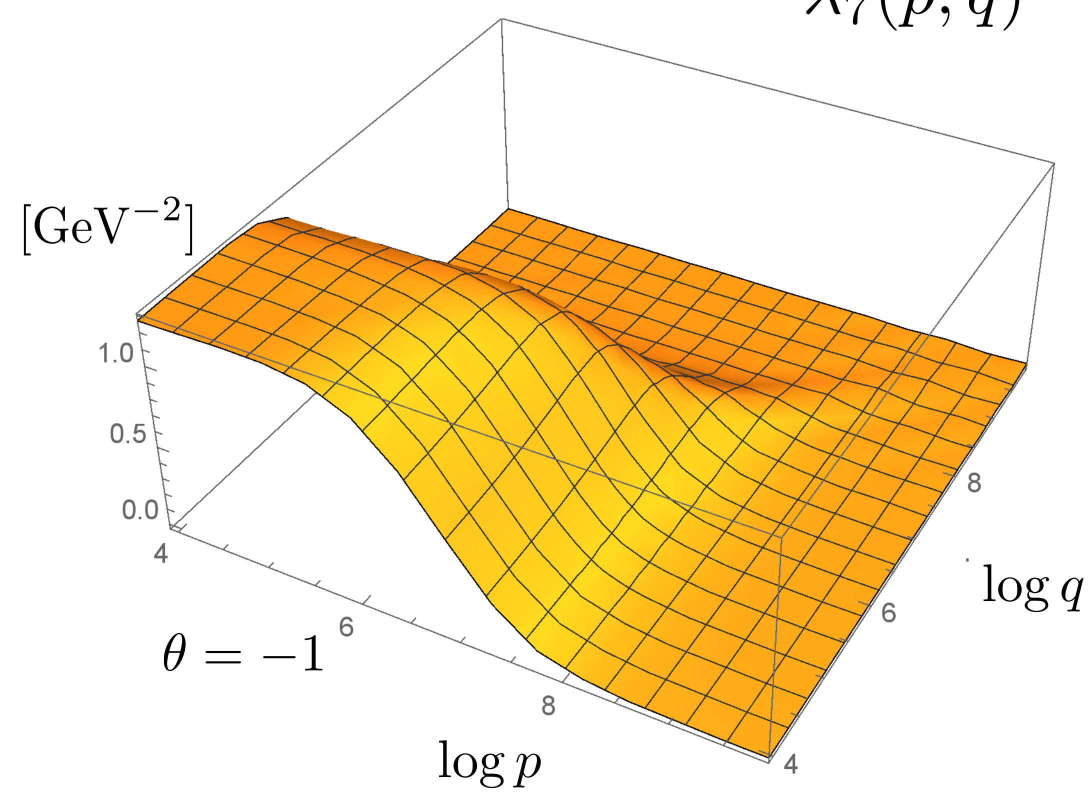


$\lambda_4(p, q)$



$\lambda_7(p, q)$



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

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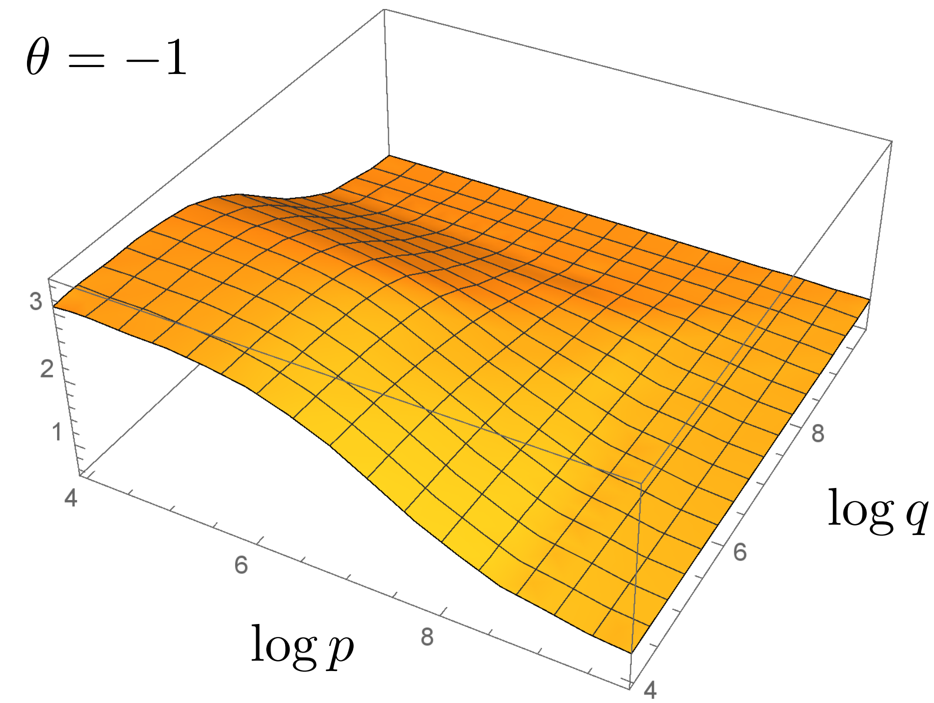


$\lambda_1(p, q)$

up-to-date 1st principle works:

FunMethods:
Williams, EPJ A51 (2015) 57
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026

Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, arXiv:1609.02568
Aguilar, Cardona, Ferreira, Papavassiliou, arXiv:1610.06158

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice: Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363

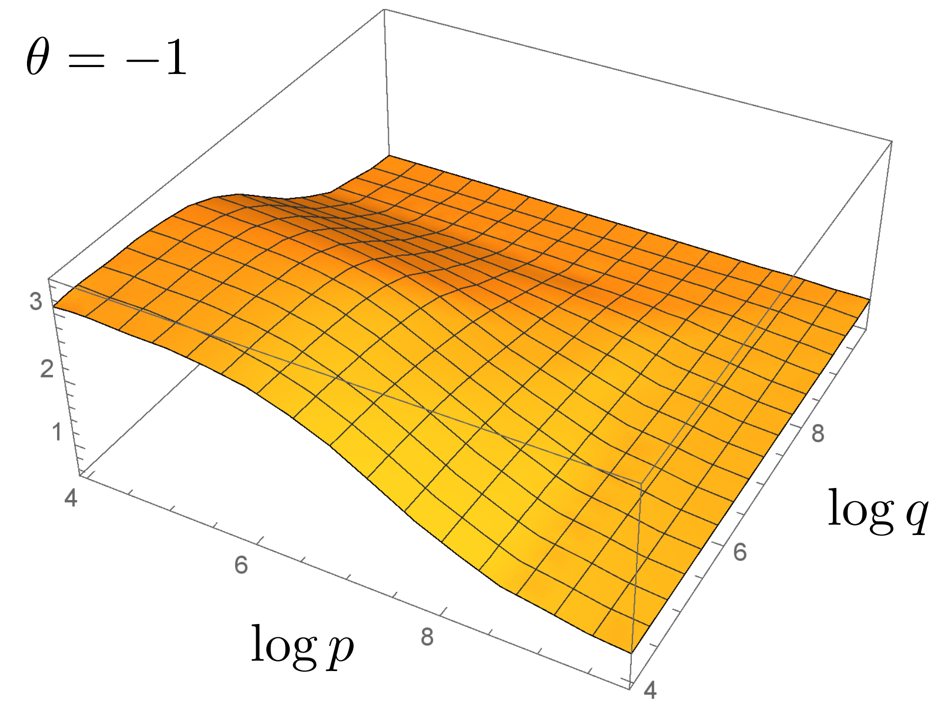
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

Quark-gluon vertex

$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



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Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

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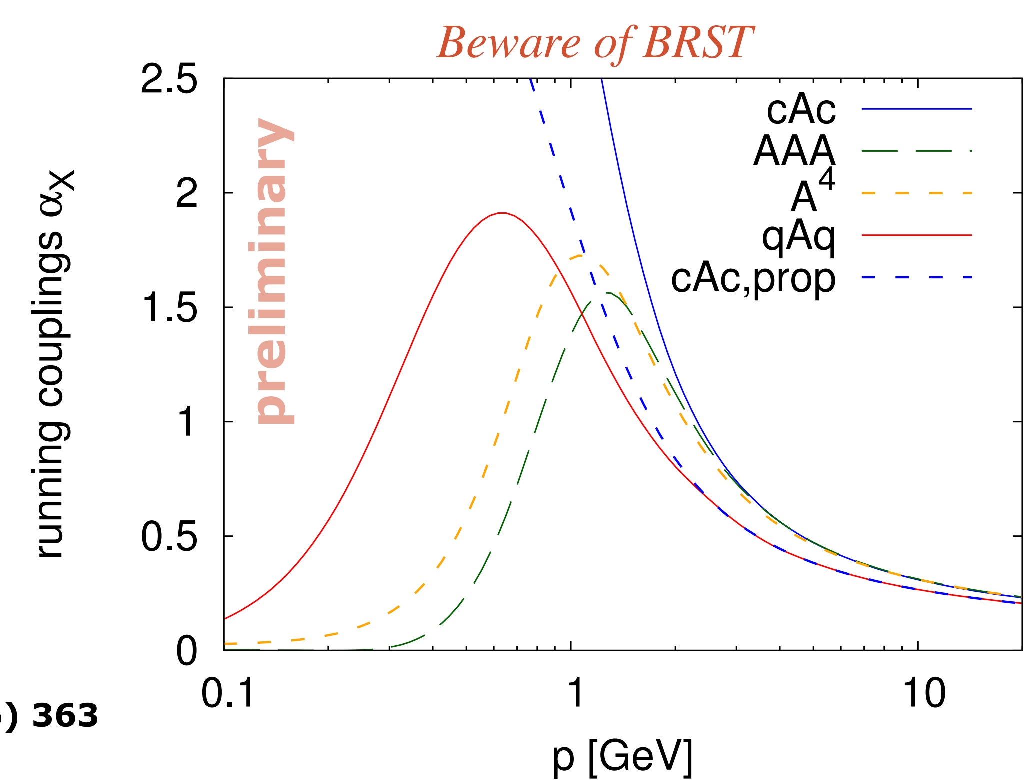
lattice: **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**

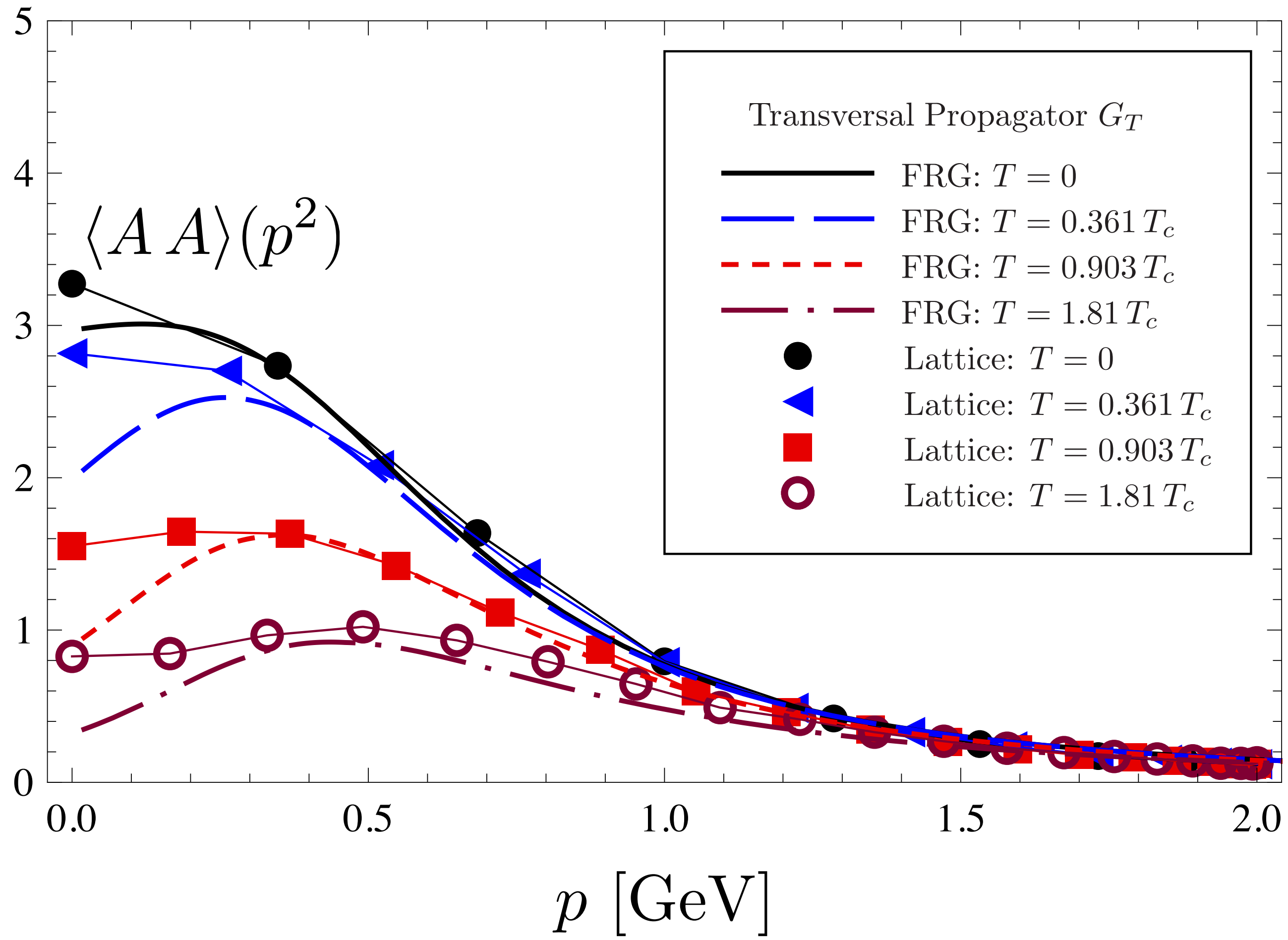


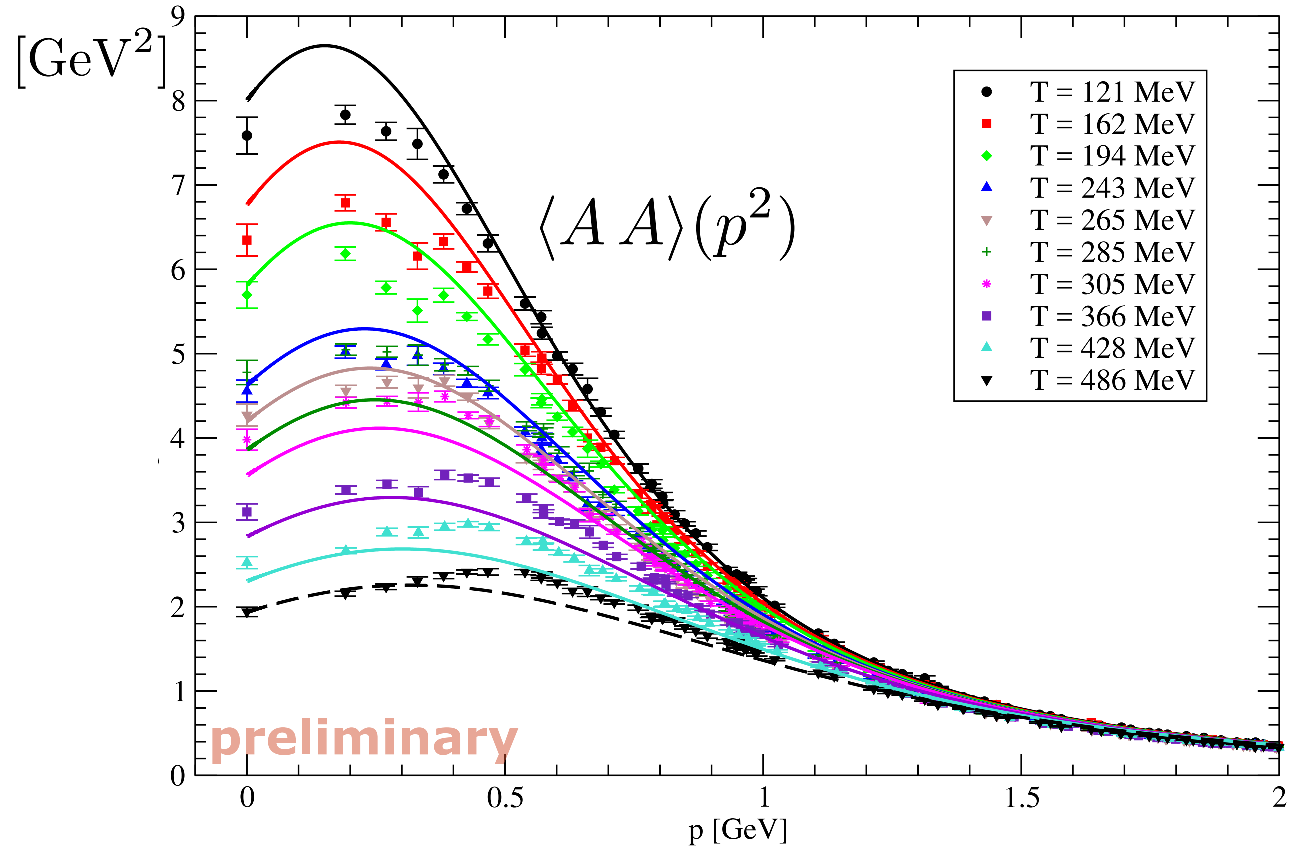
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, in prep.

Euclidean gluon propagator at finite T

Yang-Mills propagators, finite T





Fister, JMP, arXiv:1112.5440

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

CF model: Reinosa, Serreau, Tissier, Tresmontant, arXiv:1606.08012

Cyrol, Mitter, JMP, Strodthoff, in preparation

Lattice: Silva, Oliviera, Bicudo, Cardoso, PRD89 (2014) 7, 074503

Spectral functions & transport

Euclidean

Real time

Correlations of the energy-momentum tensor

$$\langle \pi(x_1) \cdots \pi(x_n) \rangle$$

MEM-type Methods

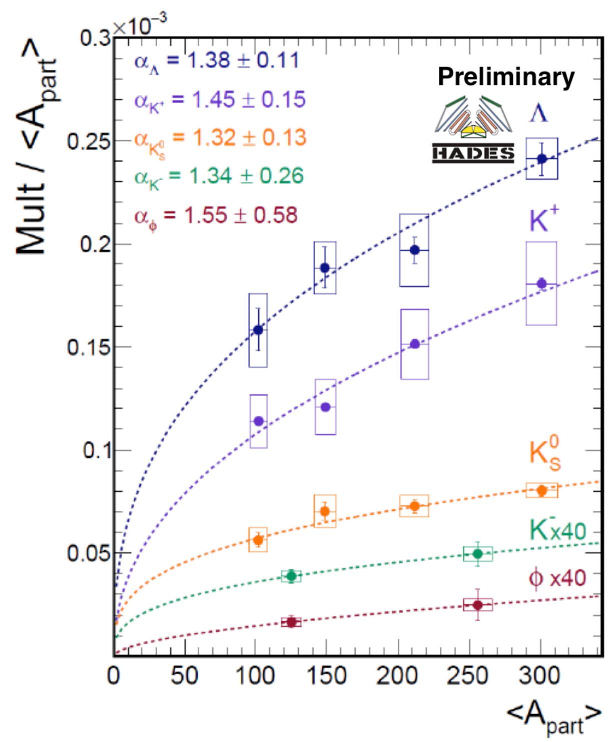
$$\langle \pi(x_1) \cdots \pi(x_n) \rangle$$

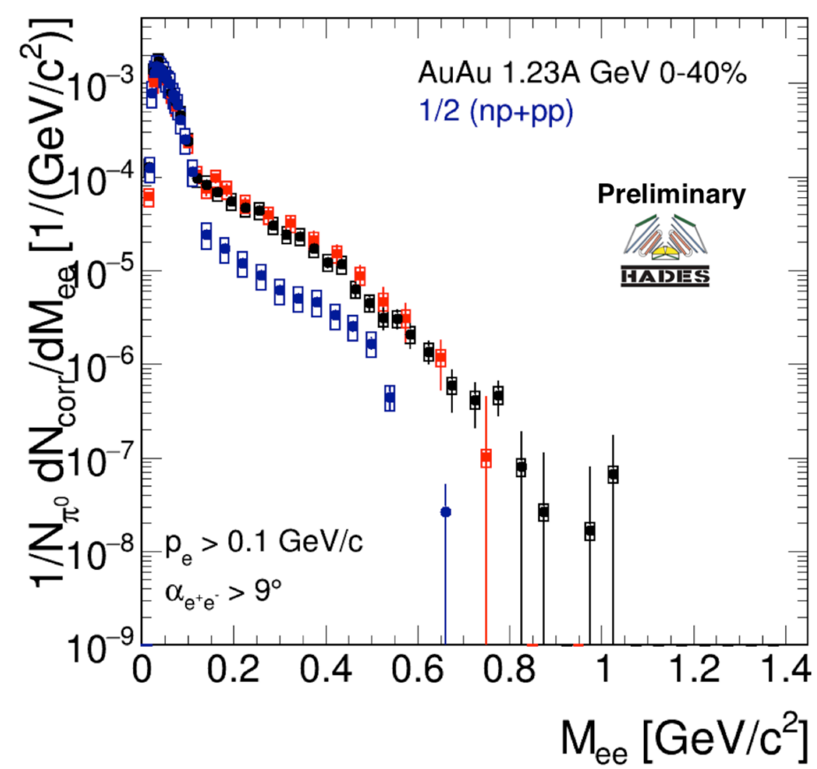
Transport coefficients, hadron resonances

Kubo formula

$$\frac{\eta}{s} \quad \dots$$

Spectral functions & transport





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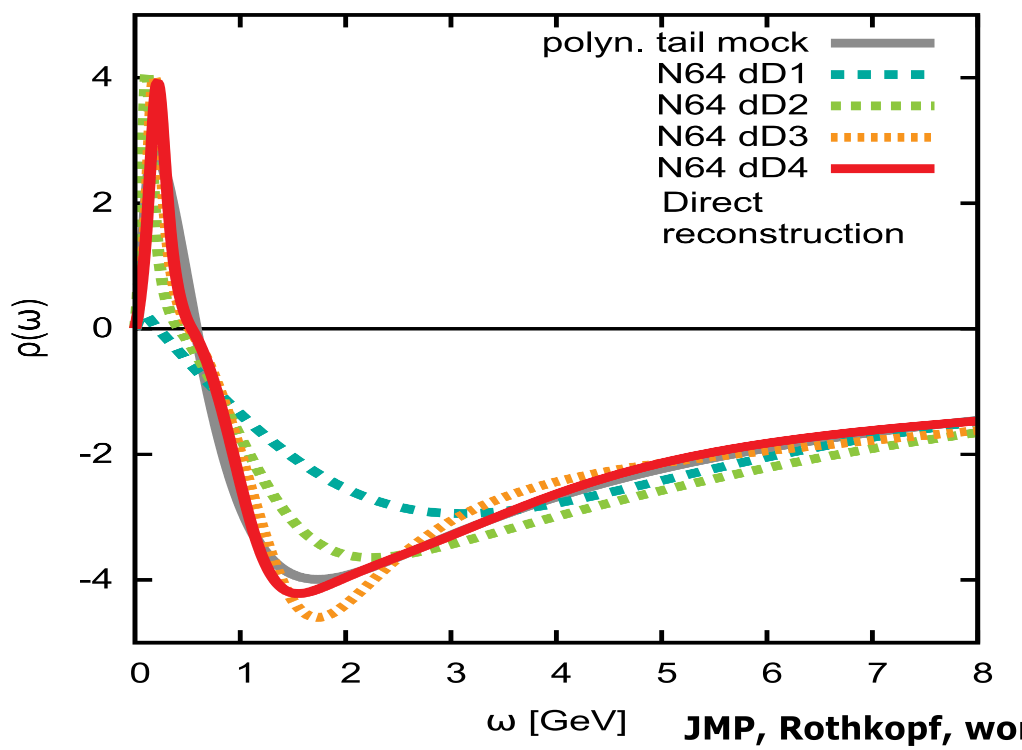
Spectral functions & transport

Euclidean



JMP, Rothkopf, work in progress

Real time

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Spectral functions & transport

Euclidean | **Real time**

Quark-gluon-hadron correlations

$$\langle A_\mu(x_1)\cdots q(x_{n+1})\cdots\rangle$$

direct computation

$$\langle A_\mu(x_1)\cdots q(x_{n+1})\cdots\rangle$$

Correlations of the energy-momentum tensor

$$\langle \pi(x_1)\cdots\pi(x_n)\rangle$$

direct computation

MEM-type Methods

$$\langle \pi(x_1)\cdots\pi(x_n)\rangle$$

Transport coefficients, hadron resonances

Kubo formula

$$\frac{\eta}{s} \quad \ldots$$

Spectral functions & transport

Euclidean | **Real time**

Quark-gluon-hadron correlations

$$\langle A_\mu(x_1)\cdots q(x_{n+1})\cdots\rangle$$

complex frequencies →

direct computation

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Correlations of the energy-momentum tensor

$$\langle \pi(x_1)\cdots\pi(x_n)\rangle$$

complex frequencies →

direct computation

MEM-type Methods

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Transport coefficients, hadron resonances

Kubo formula

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Spectral functions & transport

Euclidean

Real time

$$\langle A_\mu(x_1)\cdots q(x_{n+1})\cdots\rangle$$

direct computation

MEM-type Methods

$$\langle A_\mu(x_1)\cdots q(x_{n+1})\cdots\rangle$$

input in diagrams

Correlations of the energy-momentum tensor

direct computation

$$\langle \pi(x_1)\cdots\pi(x_n)\rangle$$

Transport coefficients, hadron resonances

Kubo formula

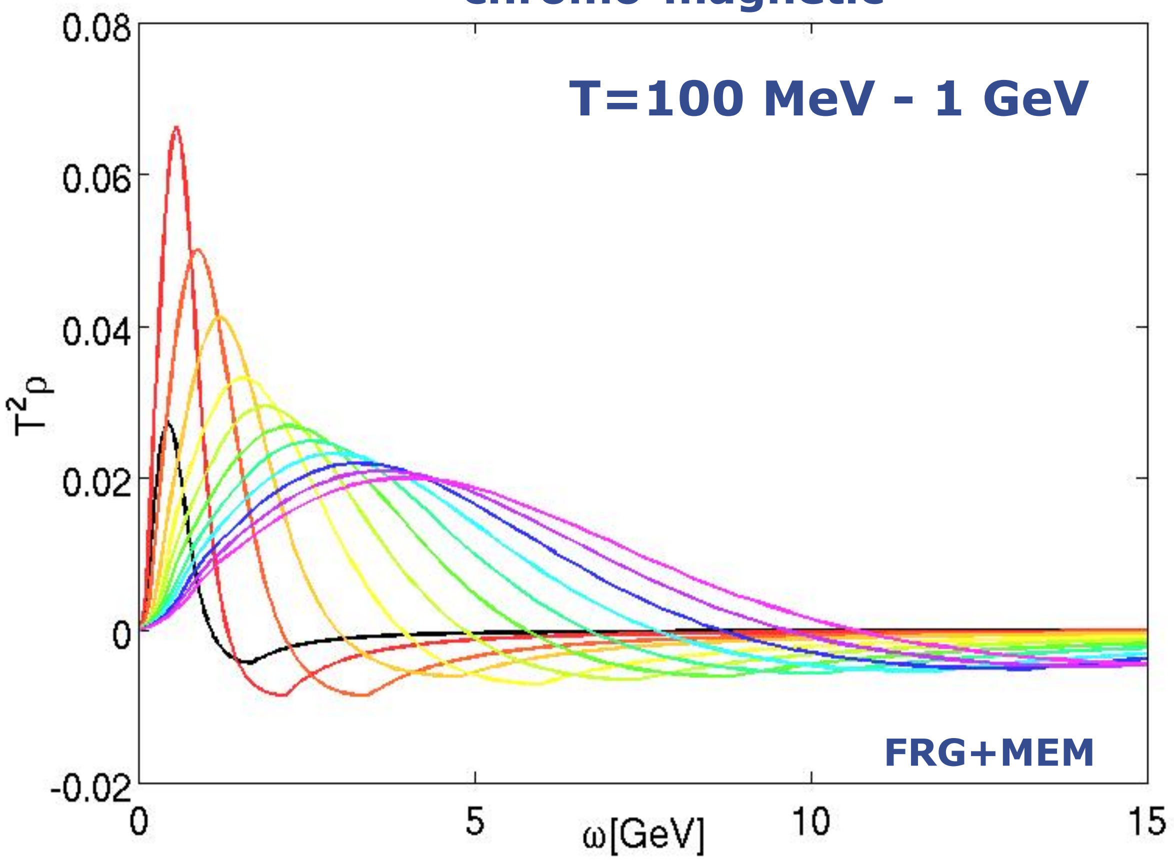
$$\frac{\eta}{s}\quad\ldots$$

Single particle spectral functions

$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

Single particle spectral functions

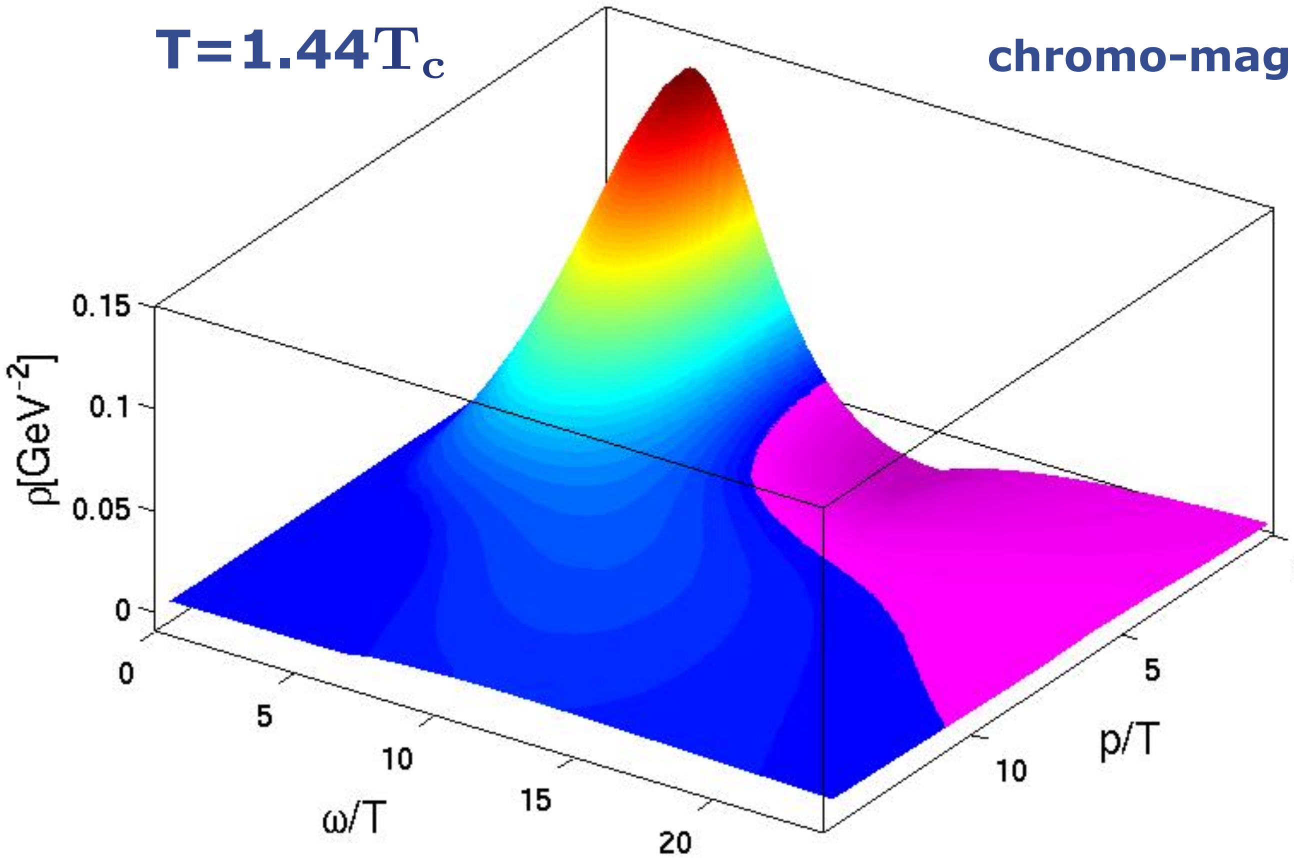
chromo-magnetic



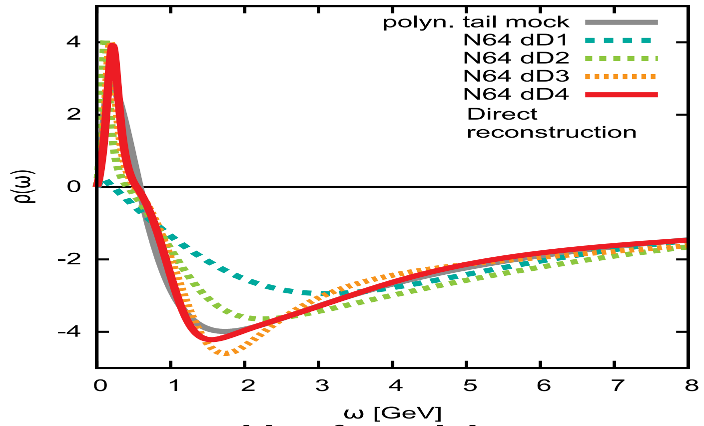
$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

T=1.44T_c

chromo-magnetic



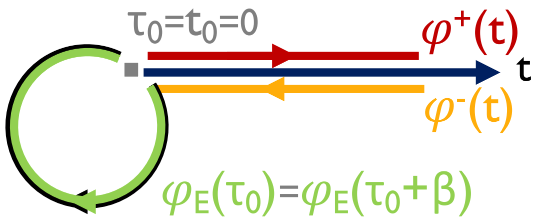
Maximum Entropy Method



JMP, Rothkopf, work in progress

Haas, Fister, JMP, PRD 90 (2014) 9, 091501

thermal spectral functions on the lattice



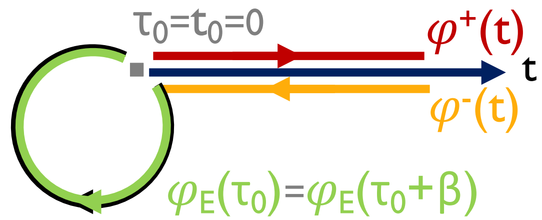
$$\underbrace{\int [d\varphi_0^+][d\varphi_0^-]\langle\varphi_0^+|e^{-\beta\hat{H}}|\varphi_0^-\rangle}_{\text{initial conditions}} \underbrace{\int_{\varphi_0^+}^{\varphi_0^-} \mathcal{D}\varphi e^{iS_M[\varphi^+]-iS_M[\varphi^-]}}_{\text{quantum dynamics}}$$

Stochastic quantisation

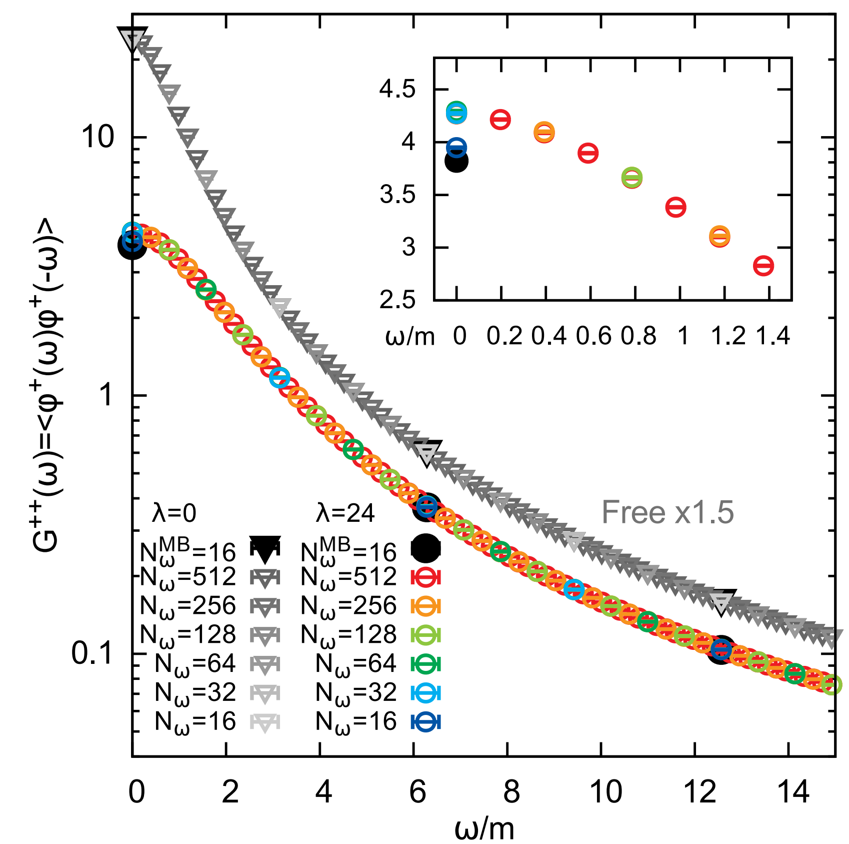
$$\partial_{t_5}\varphi^+(\omega_l) = -\frac{\delta S_0}{\delta\varphi^+(\omega_l)} - \frac{\delta S_E^{\text{int}}}{\delta\varphi^+(\tau_j)}\frac{\delta\varphi^+(\tau_j)}{\delta\varphi^+(\omega_l)} + \eta(\omega_l)$$

JMP, Rothkopf, arXiv:1610:09531

thermal spectral functions on the lattice



$$\underbrace{\int [d\varphi_0^+][d\varphi_0^-]\langle\varphi_0^+|e^{-\beta\hat{H}}|\varphi_0^-\rangle}_{\text{initial conditions}}\underbrace{\int_{\varphi_0^+}^{\varphi_0^-}\mathcal{D}\varphi e^{iS_M[\varphi^+]-iS_M[\varphi^-]}}_{\text{quantum dynamics}}$$



Stochastic quantisation

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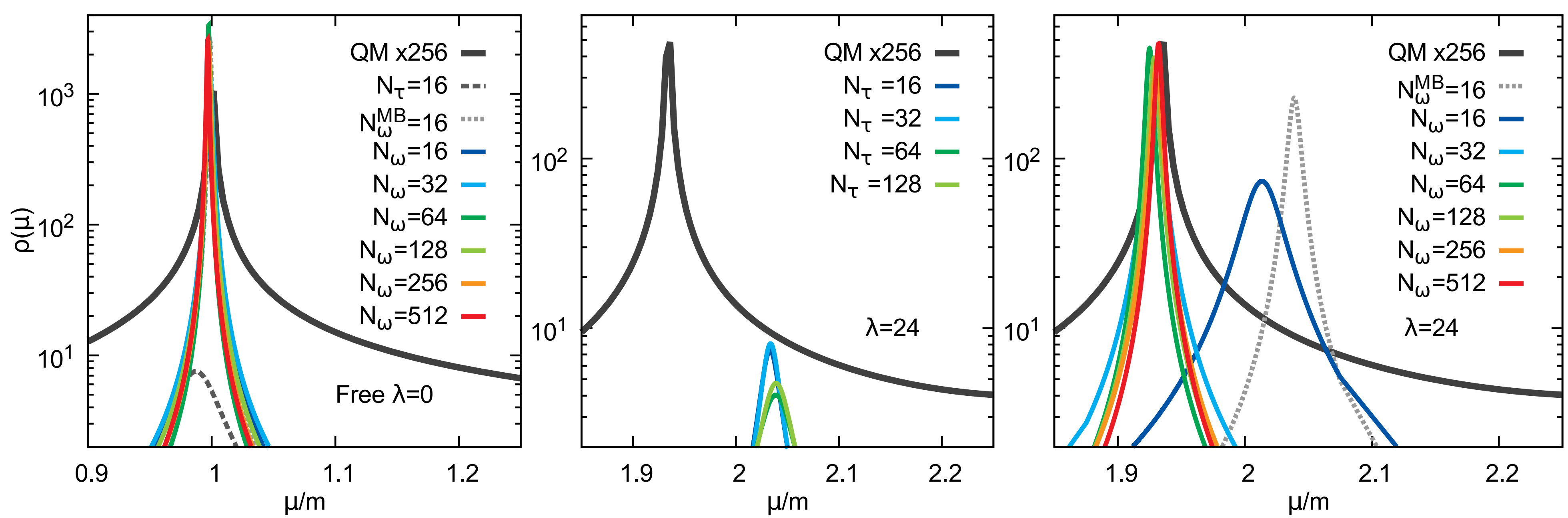
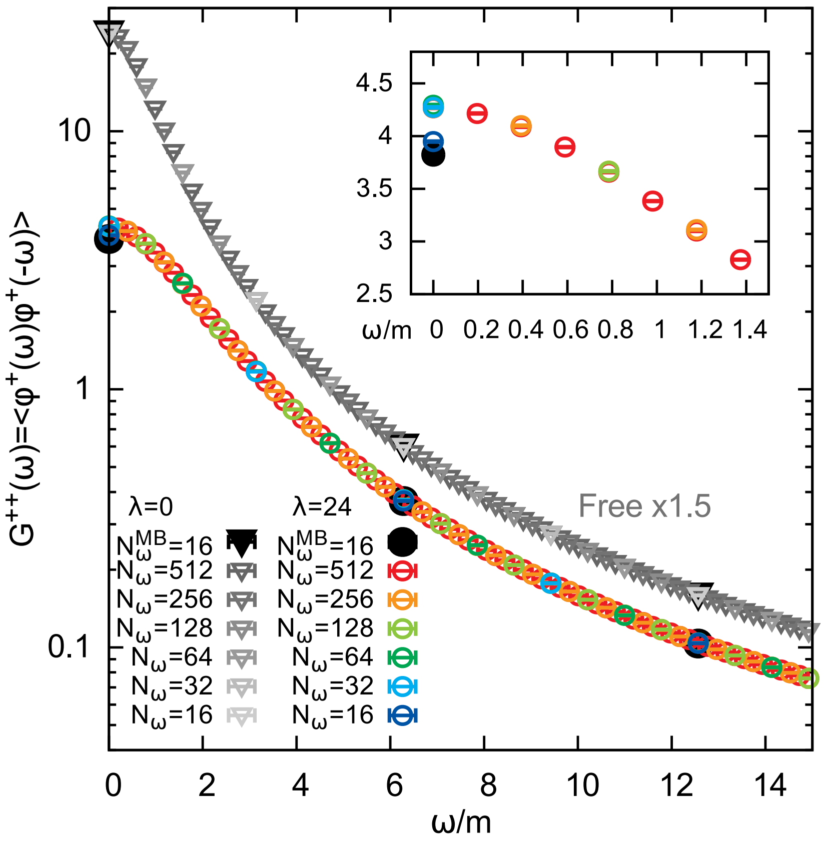
Test case: 1+0 dimensional scalar theory



JMP, Rothkopf, arXiv:1610:09531

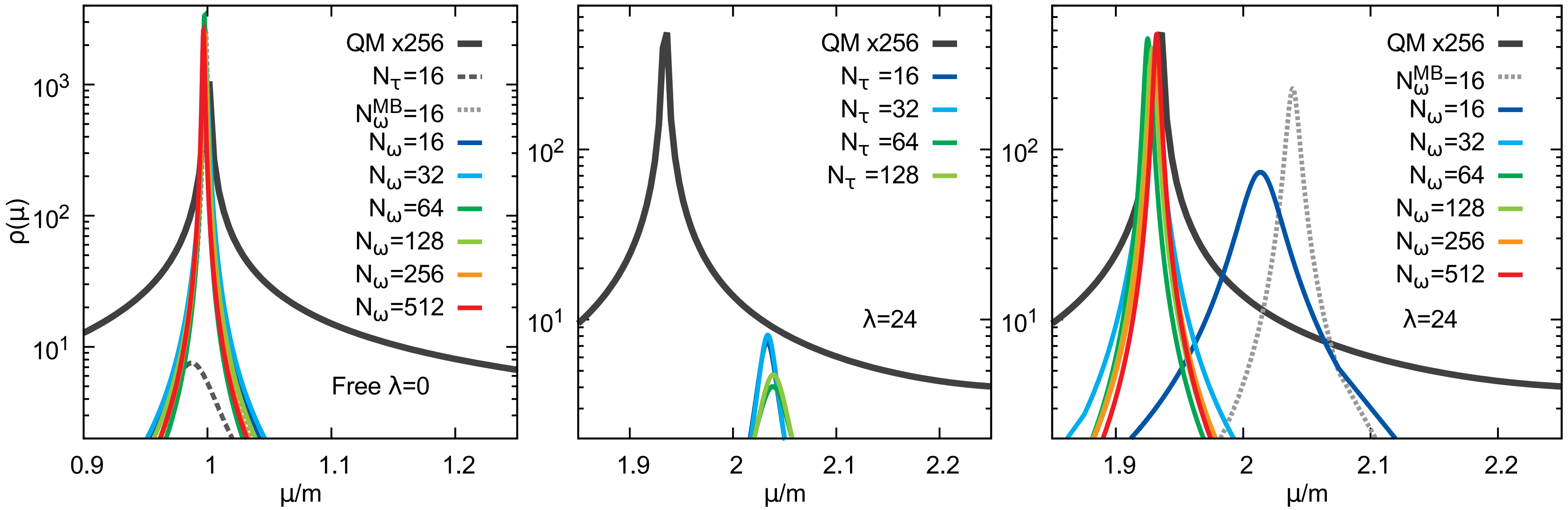
thermal spectral functions on the lattice



'Those are my methods (principles), and if you don't like them...well, I have others'

direct computation **Groucho Marx**

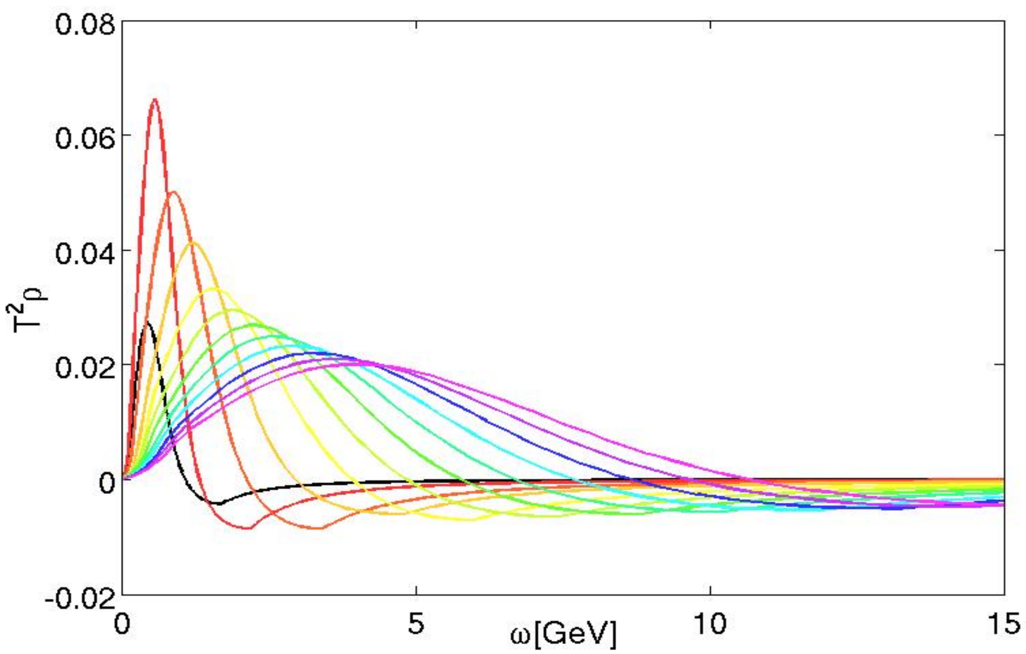
Test case: 1+0 dimensional scalar theory



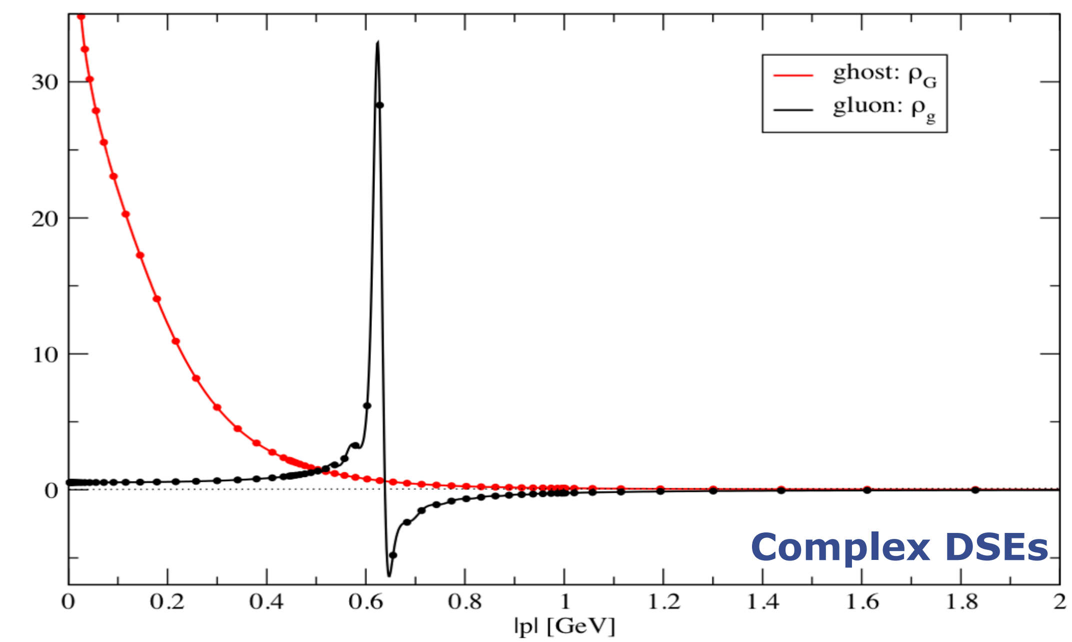
JMP, Rothkopf, arXiv:1610:09531

Transport

gluon spectral functions

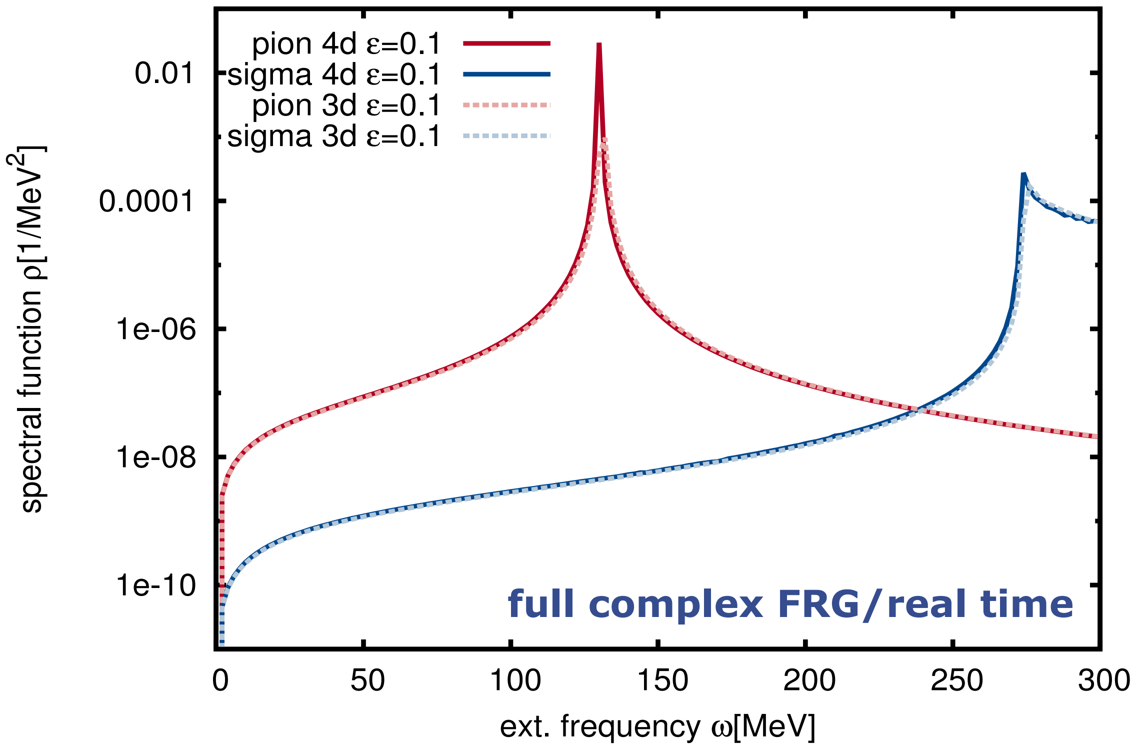


Haas, Fister, JMP, PRD 90 (2014) 9, 091501

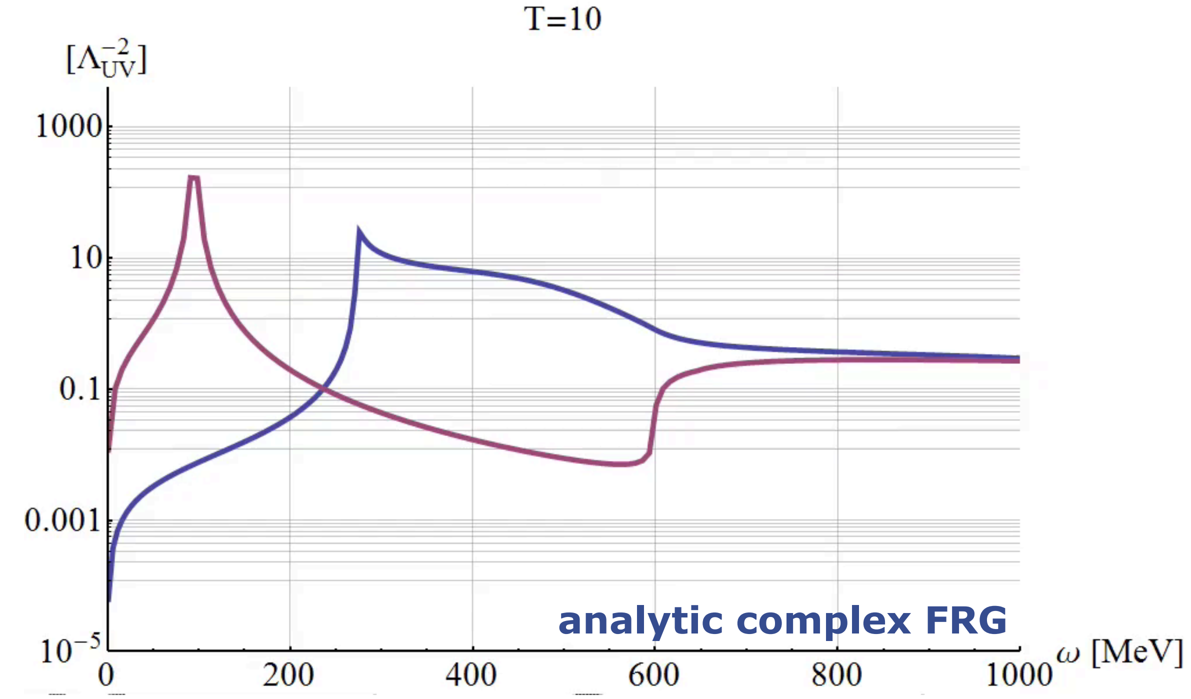


Strauss, Fischer, Kellermann, PRL 109 (2012) 252001

pion and sigma spectral functions



JMP, Strodthoff, PRD 92 (2015) 094009



Tripolt, Strodthoff, von Smekal, Wamach, PRD 89 (2014) 034010
Kamikado, Strodthoff, von Smekal, Wambach, EPJ C74 (2014) 2806

Outline

- Introduction
- Single particle spectral functions
- Spectral functions & transport coefficients
- Summary & outlook

Transport

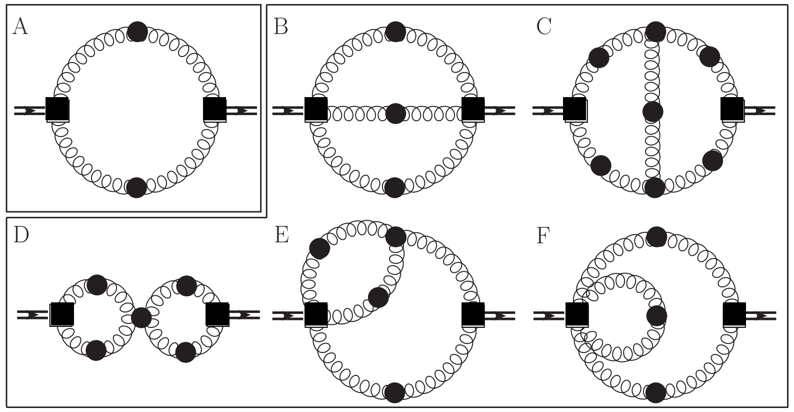
transport coefficients

Kubo relation

$$\eta = \frac{1}{20}\,\frac{d}{d\omega}\Big|_{\omega=0}\rho_{\pi\pi}(\omega,0)$$

‘3-loop’ exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



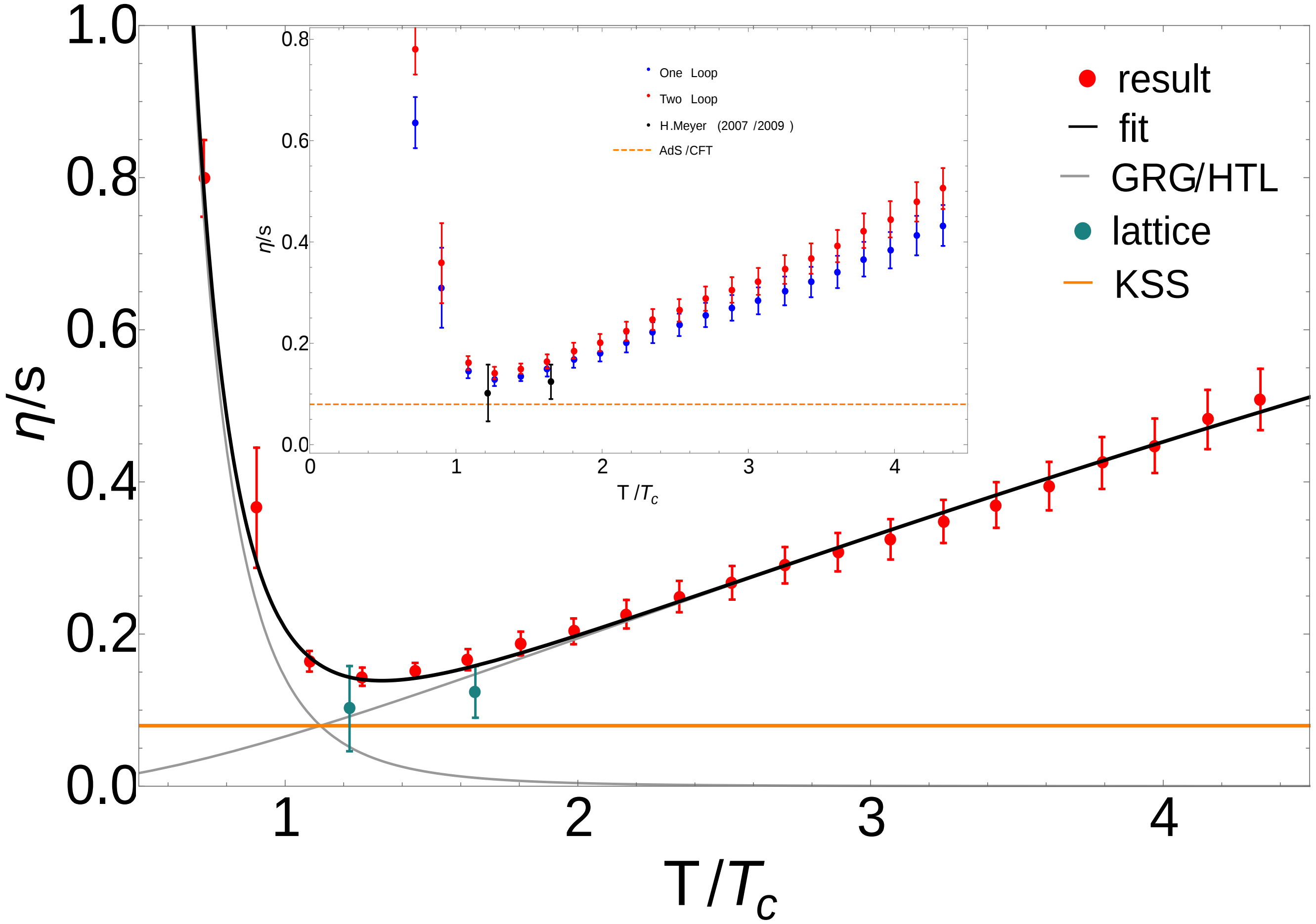
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

transport coefficients

Yang-Mills viscosity over entropy ratio



Aiming at apparent convergence

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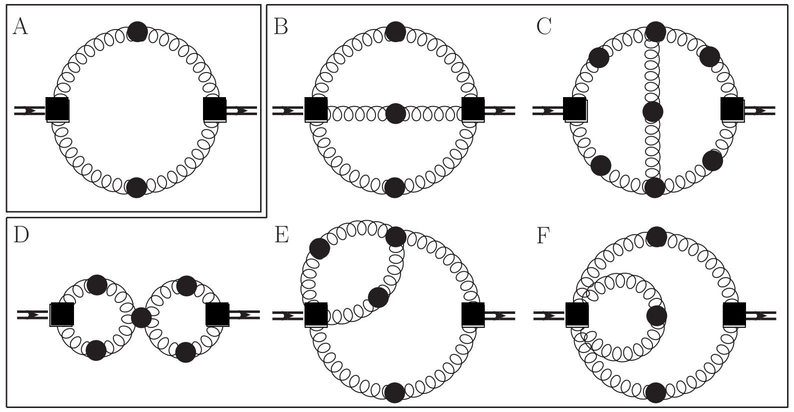


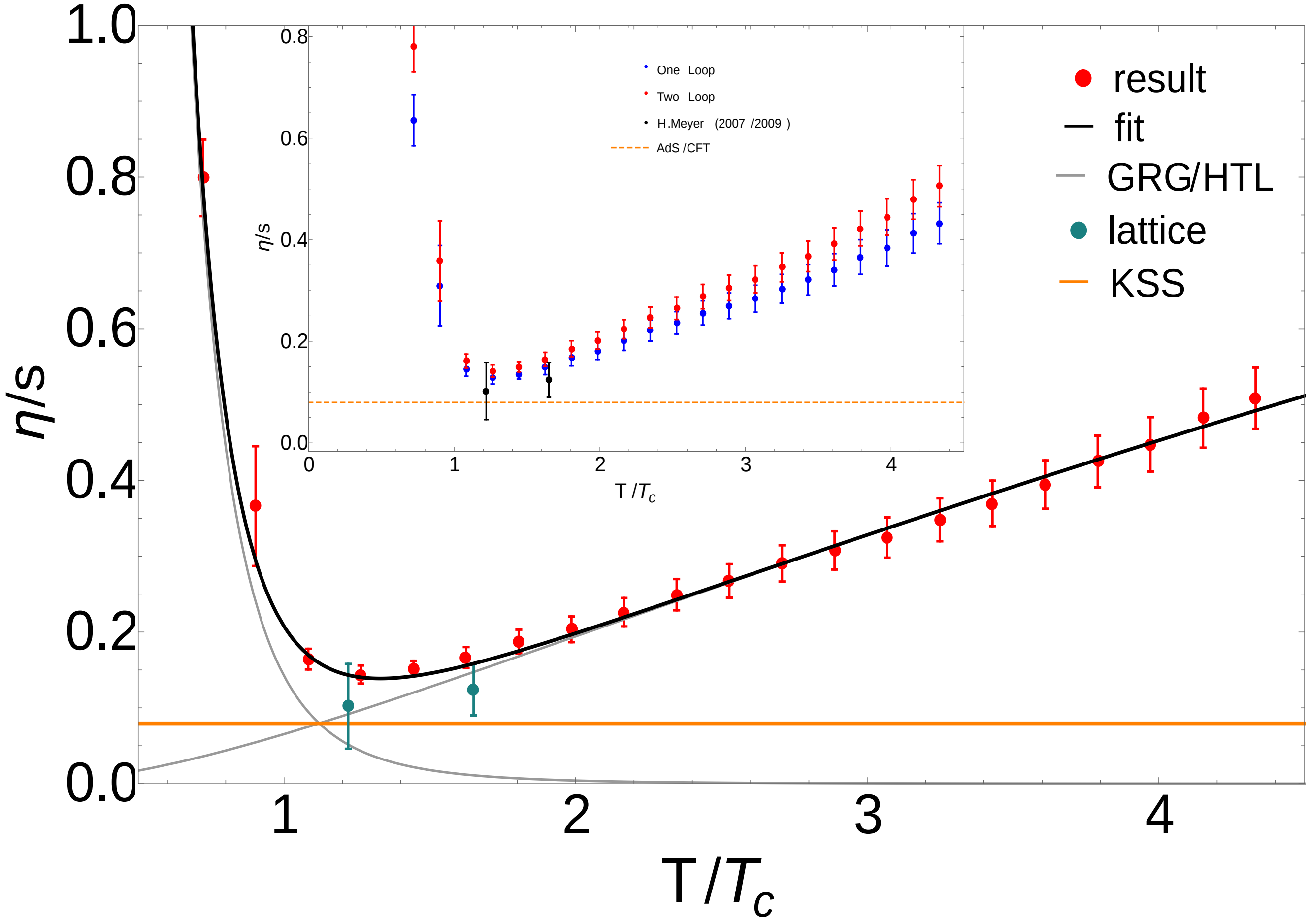
Haas, Fister, JMP, PRD 90 (2014) 9, 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$a_{\rm qgp} \approx 0.15$$

$$a_{\rm hrg} \approx 0.14$$

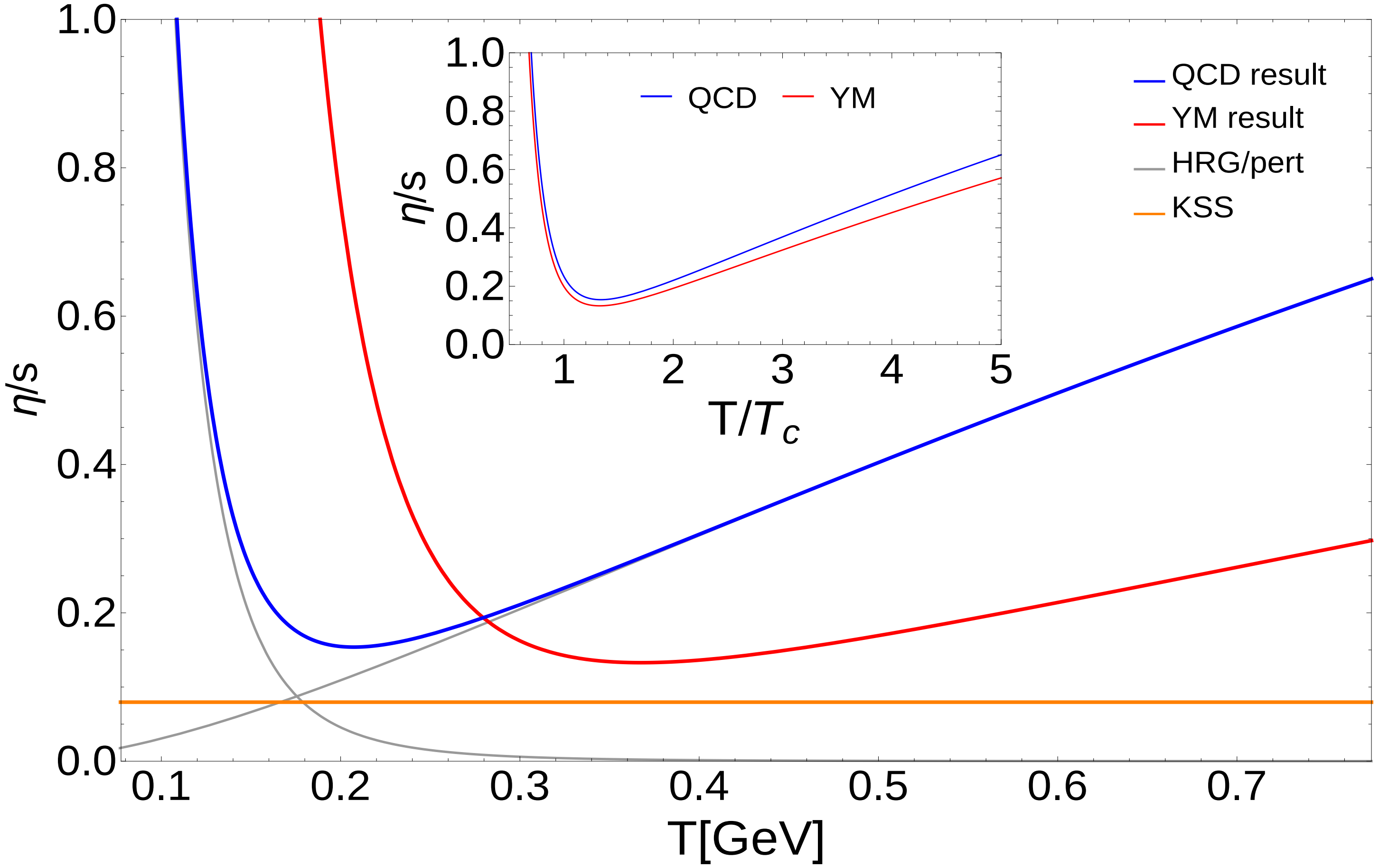
$$c \approx 0.66$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$a_{\rm qgp} \approx 0.2$$
$$a_{\rm hrg} \approx 0.16$$
$$c \approx 0.79$$

QCD

$$\gamma_{\rm grg} \approx 5$$
$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$
$$a_{\rm hrg} \approx 0.14$$
$$c \approx 0.66$$

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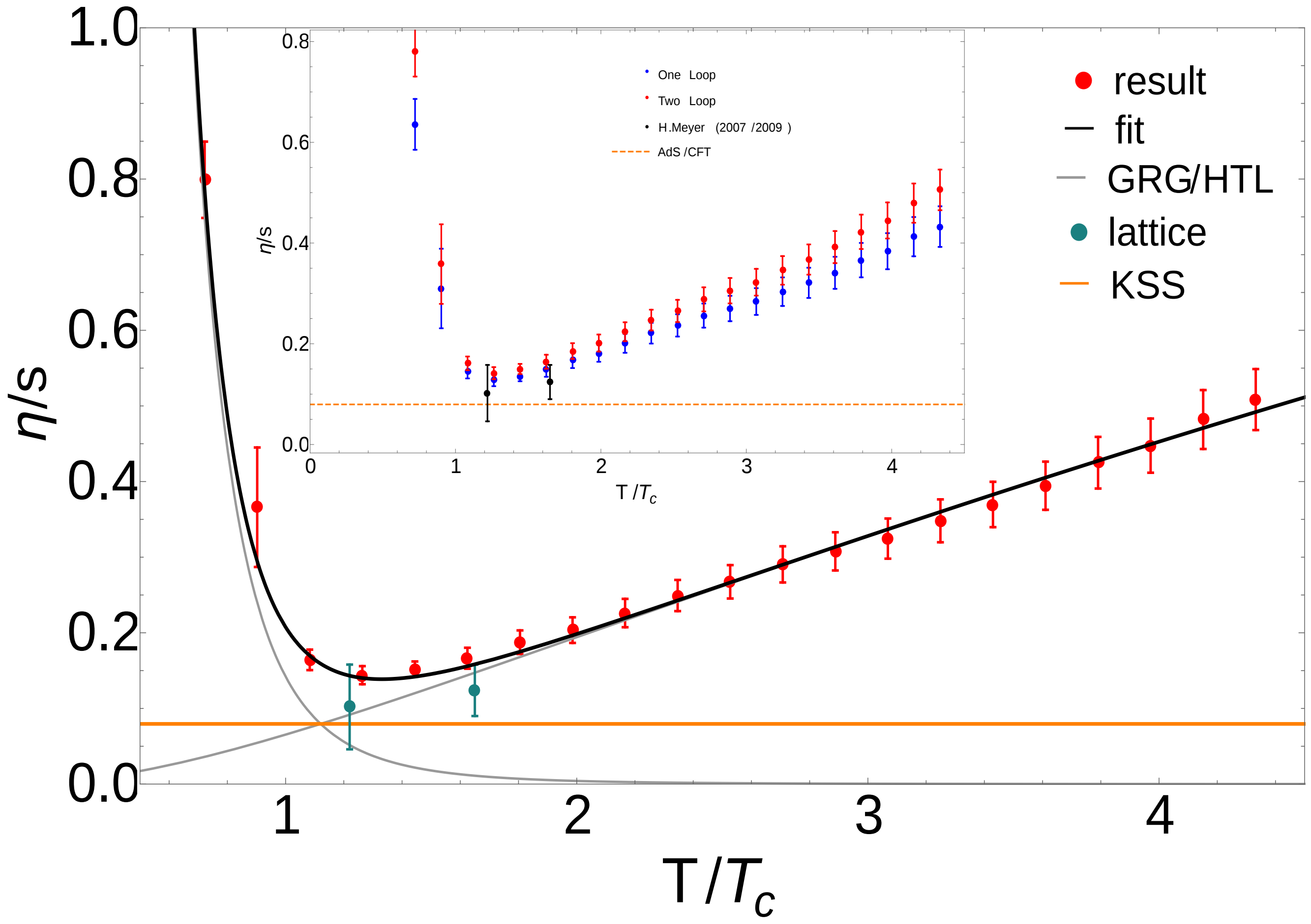
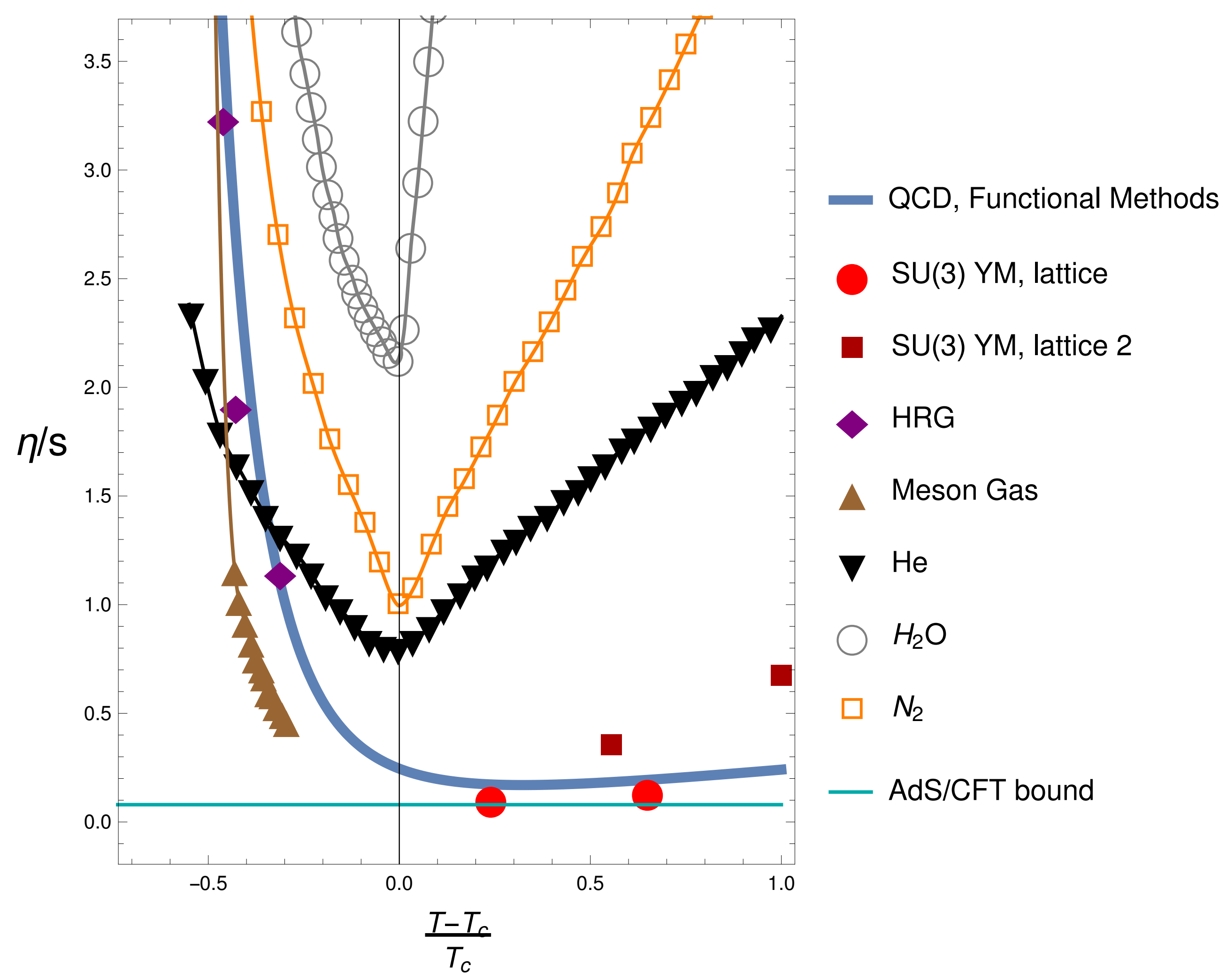
Transport

transport coefficients

Yang-Mills viscosity over entropy



QCD - estimate for viscosity over entropy ratio



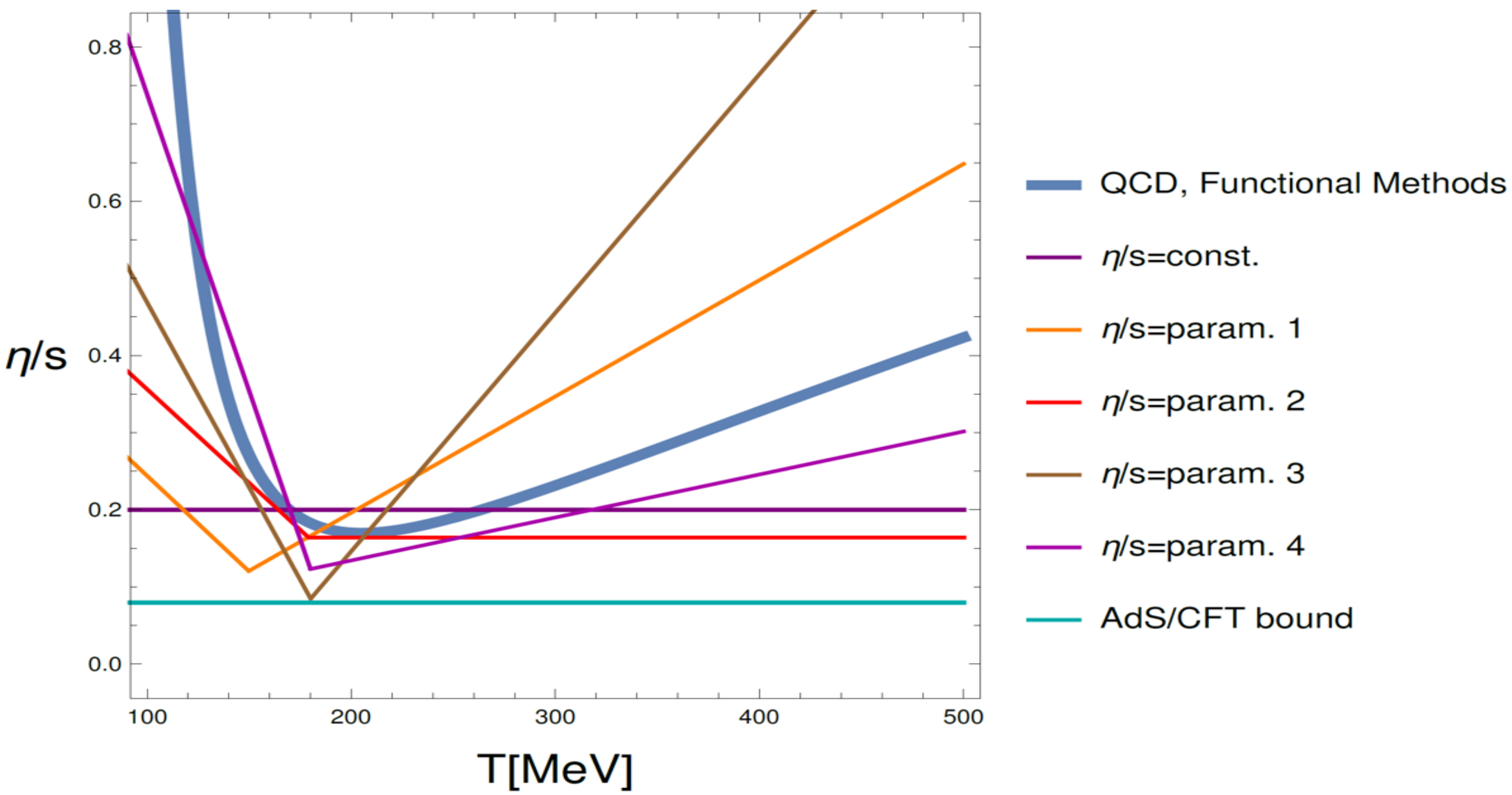
$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 11, 112002

Transport

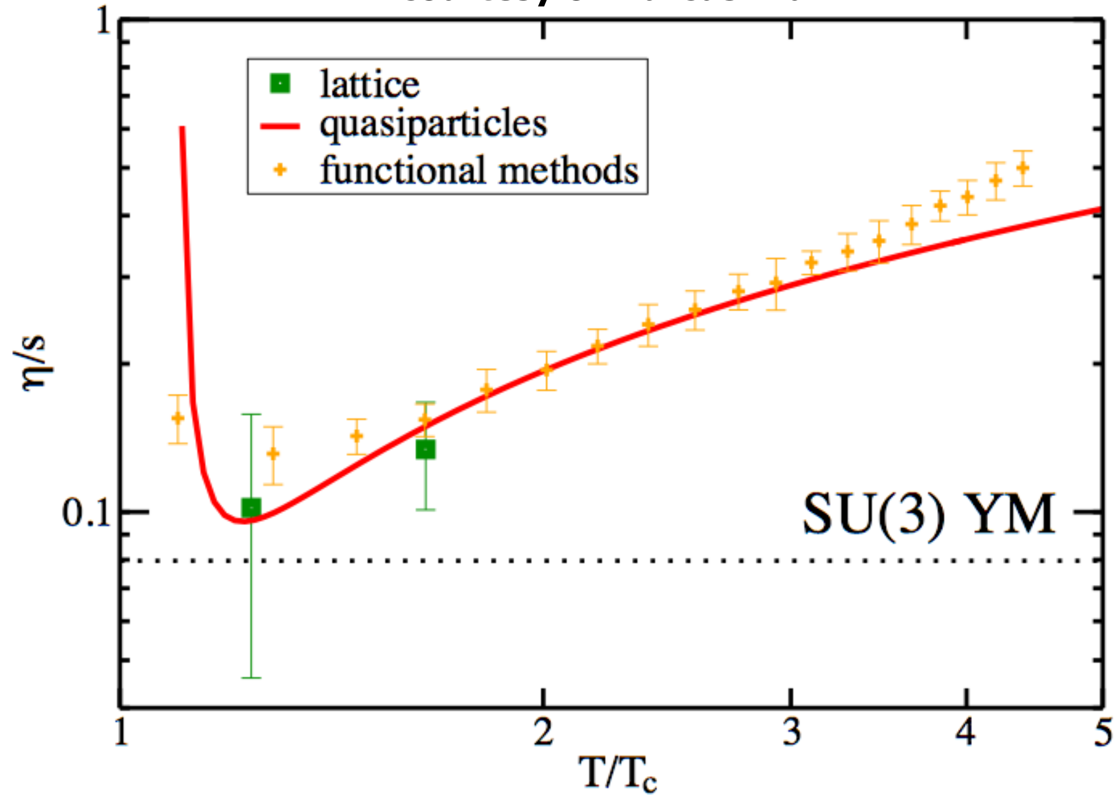
QCD transport & transport models

courtesy of Nicolai Christiansen



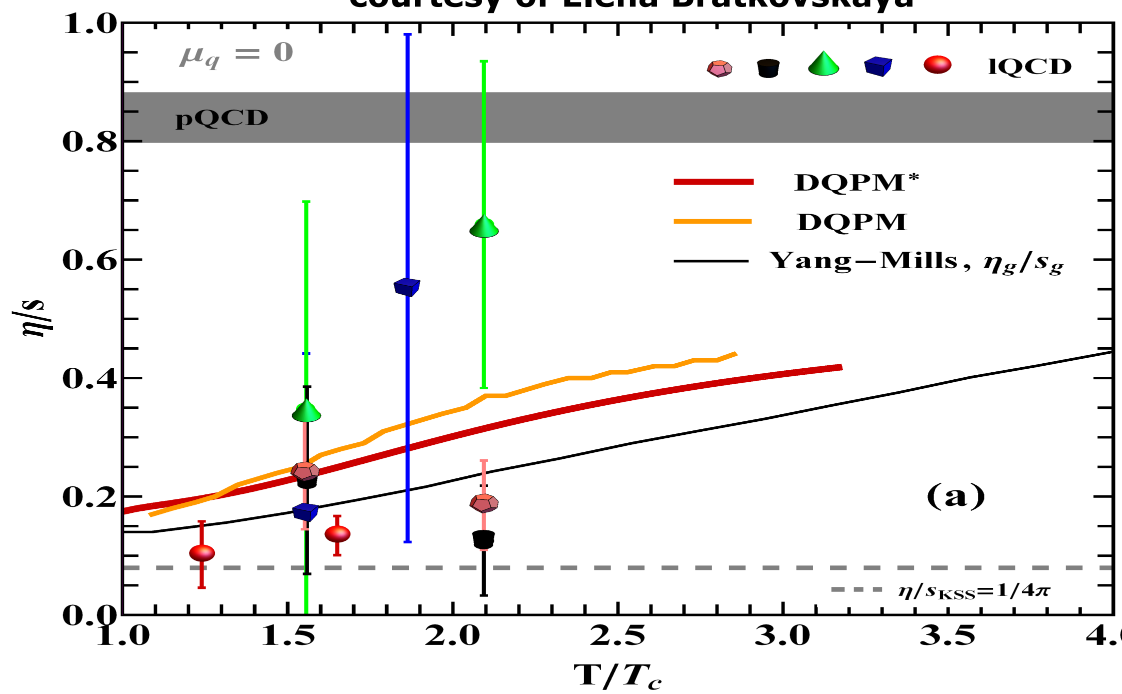
Niemi, Eskola, Paateleinen, PRC 93 (2016) 024907

courtesy of Marcus Bluhm



Bluhm, Kaempfer, Redlich, PRC 84 (2011) 025201

courtesy of Elena Bratkovskaya



Berrehrah, Cassing, Bratkovskaya, Steinert, PRC 93 (2016) 044914

$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

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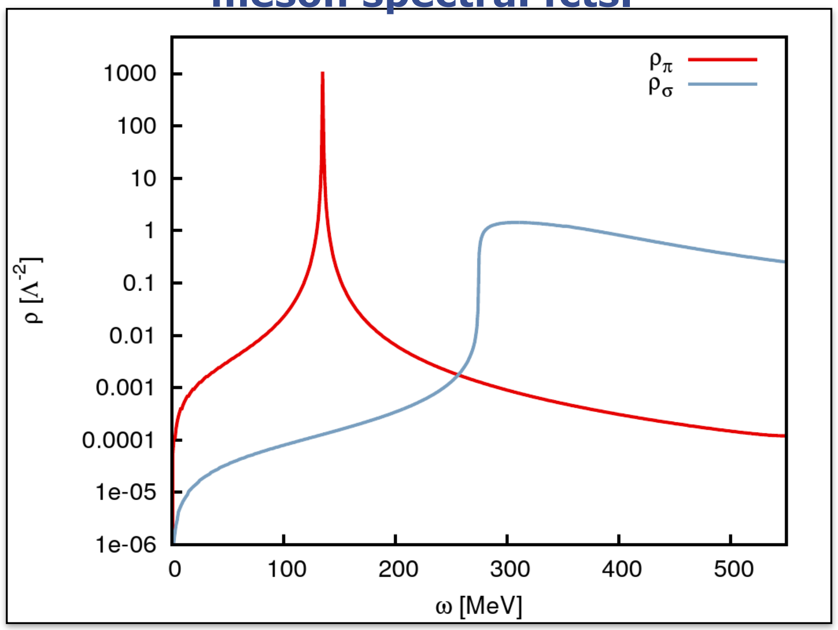
Outline

- **Introduction**
- **Confinement & transport**
- **Chiral symmetry breaking & the phase structure**
- **Summary & outlook**

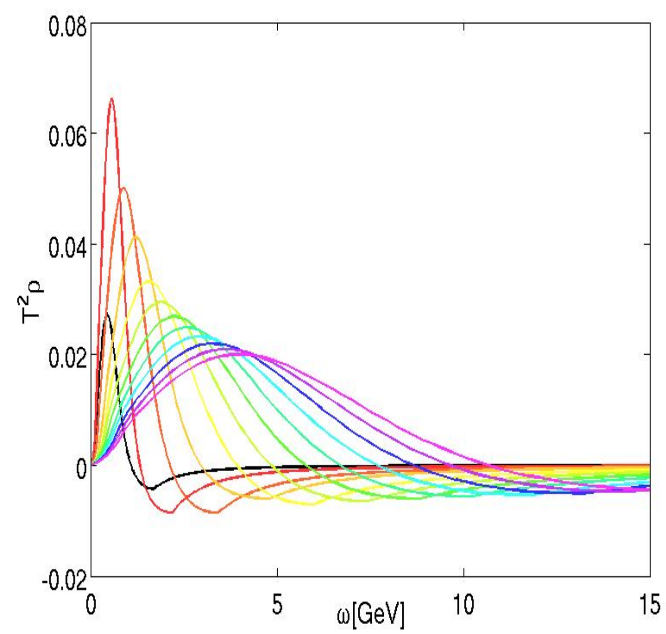
Summary & Outlook

▪ Spectral functions & transport coefficients

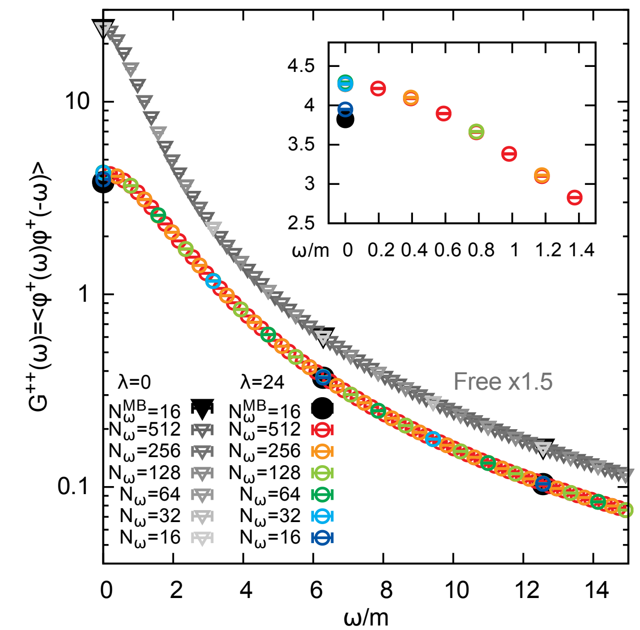
meson spectral fcts.



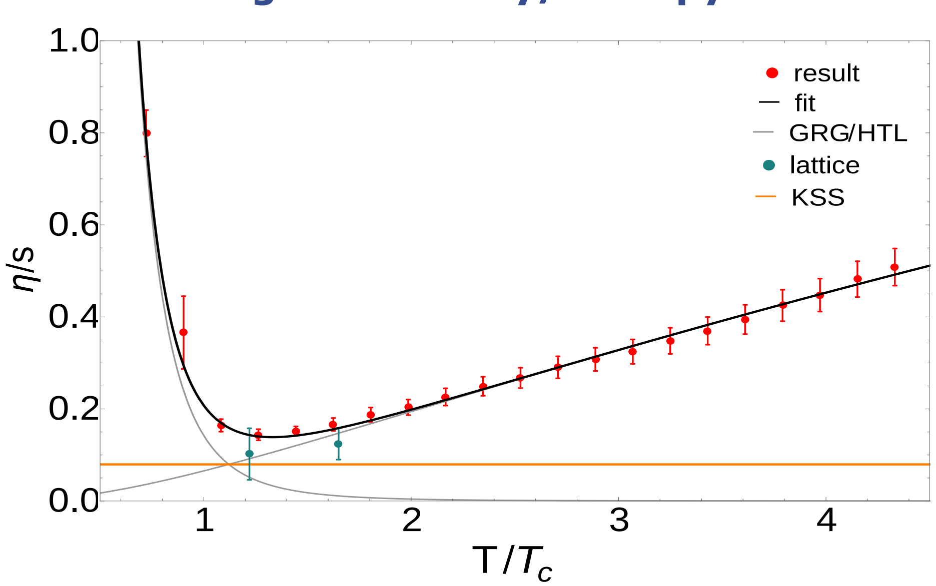
gluon spectral fcts.



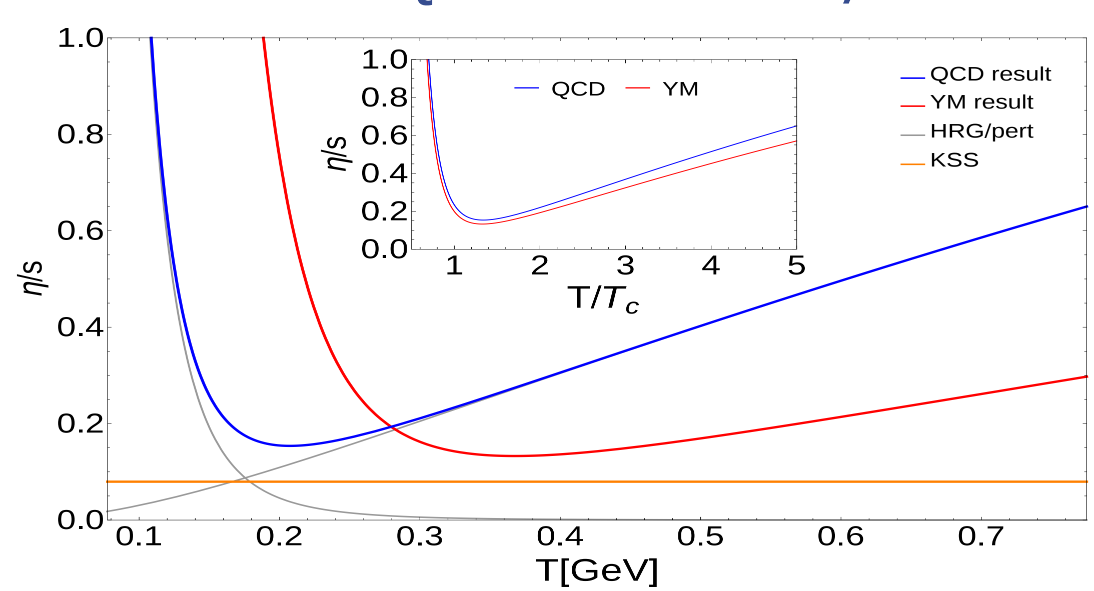
lattice reconstruction



glue viscosity/entropy



QCD estimate for eta/s



Summary & Outlook

- Single particle spectral functions

- transport coefficients

Summary & Outlook

- Single particle spectral functions

- transport coefficients

- direct computation of real time correlation functions in QCD

- bulk viscosity, relaxation time, ….

- Hadronic properties

  - glue balls, hadron spectrum & in medium modifications

  - low energy constants