EQUATION OF STATE AND FLUCTUATIONS FROM THE LATTICE

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NSF

BEST COLLABORATION

Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses

- Given enough computer power, uncertainties can be kept under control

- Results from different groups, adopting different discretizations, converge to consistent results

- Unprecedented level of accuracy in lattice data

Low temperature phase: HRG model

Dashen, Ma, Bernstein; Prakash, Venugopalan, Karsch, Tawfik, Redlich

- Interacting hadronic matter in the ground state can be well approximated by a non-interacting resonance gas
- The pressure can be written as:

$$p^{HRG}/T^4 = \frac{1}{VT^3} \sum_{i \in \; mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in \; baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a})$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

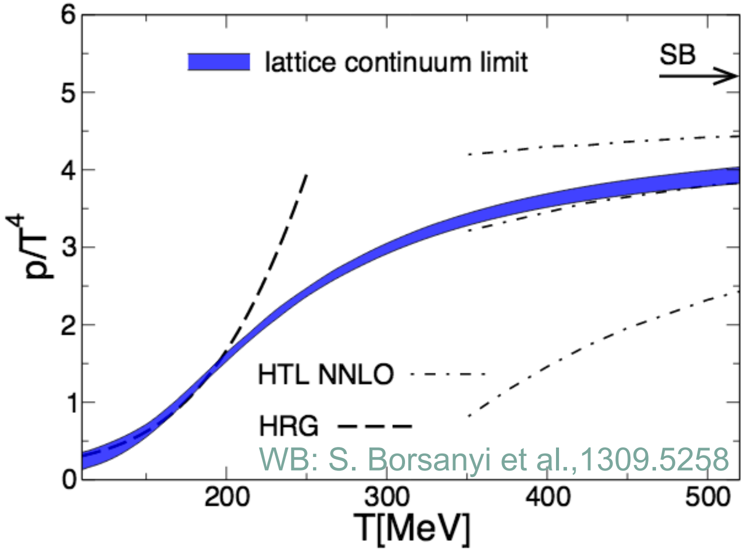
with energies $\varepsilon_i = \sqrt{k^2 + m_i^2}$, degeneracy factors d_i and fugacities

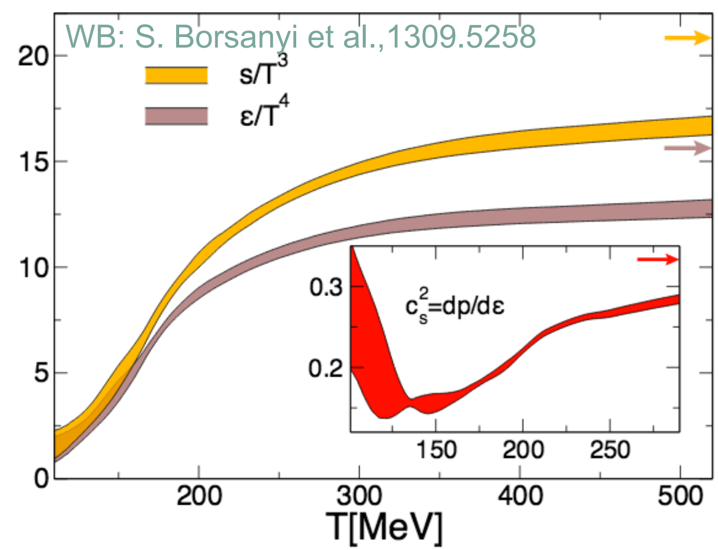
$$z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right) \quad .$$

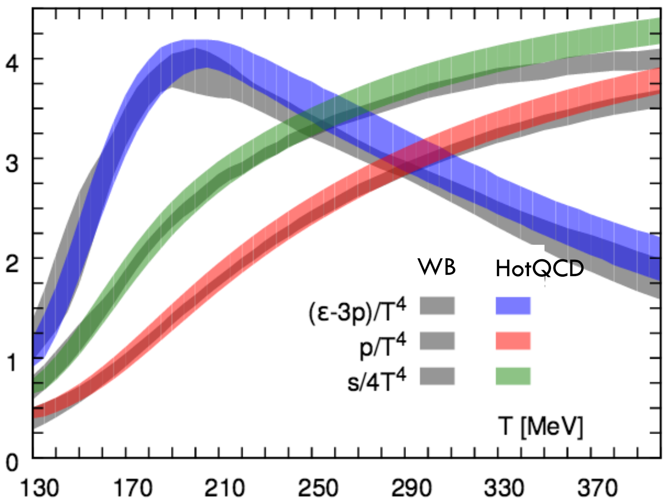
X^a: all possible conserved charges, including the baryon number B, electric charge Q, strangeness S.

- Needs knowledge of the hadronic spectrum

QCD Equation of state at μ_B=0







- EoS available in the continuum limit, with realistic quark masses
- Agreement between stout and HISQ action for all quantities

WB: S. Borsanyi et al., 1309.5258, PLB (2014)
HotQCD: A. Bazavov et al., 1407.6387, PRD (2014)

Sign problem

- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex → Monte Carlo simulations are not feasible

- We can rely on a few approximate methods, viable for small μ_B/T:
  - Taylor expansion of physical quantities around $\mu_B=0$ (Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003)
  - Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

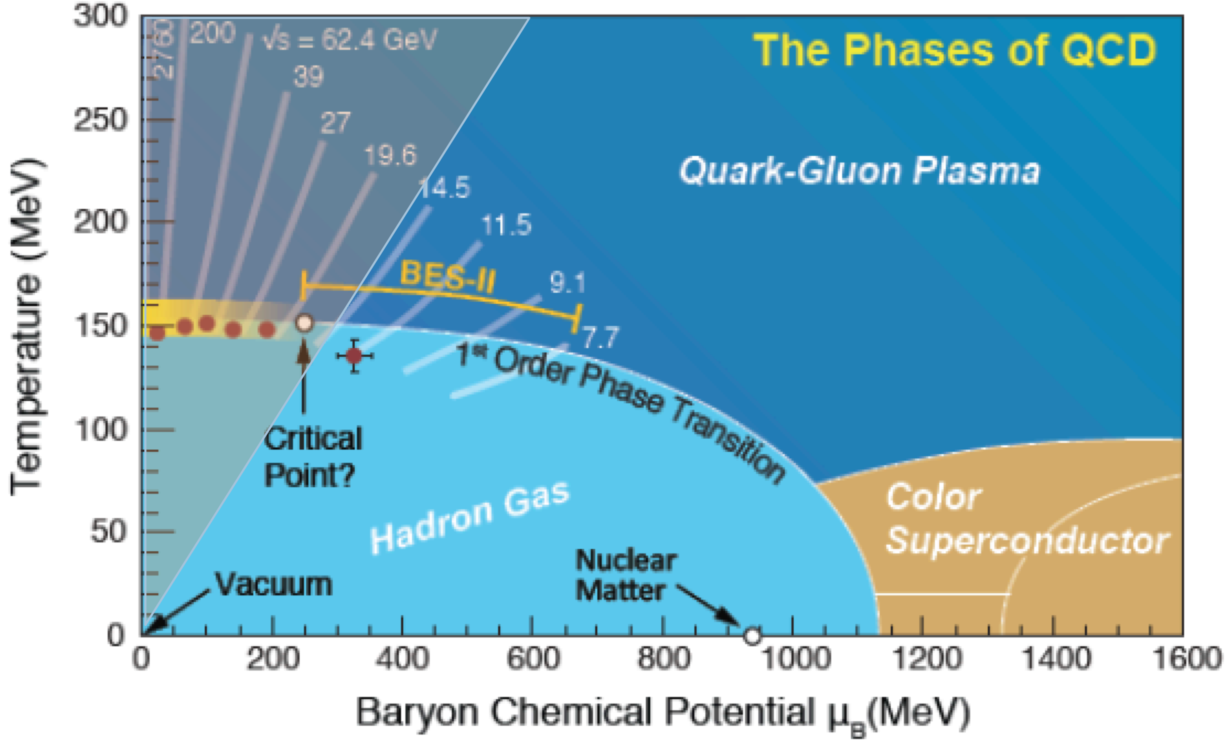
Equation of state as a Taylor expansion in μ_B

- Notation:

$$\hat{\mu}_B \equiv \mu_B/T \quad \hat{p} \equiv p/T^4 \quad \hat{n} \equiv n_B/T^3 \quad \hat{s} \equiv s/T^3$$

- Taylor expansion for the pressure:

$$\hat{p} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \ldots$$



Physics at imaginary μ

- At imaginary μ there is no sign problem
- The partition function is periodic in μ_I with period $2\pi T$

$$Z = \mathrm{Tr}\left(e^{-\beta\hat{H}+i\beta\mu_I\hat{N}}\right)$$

- For more chemical potentials: μ_B, μ_Q, μ_S, several trajectories are possible → useful for different physics
  - Here we use:

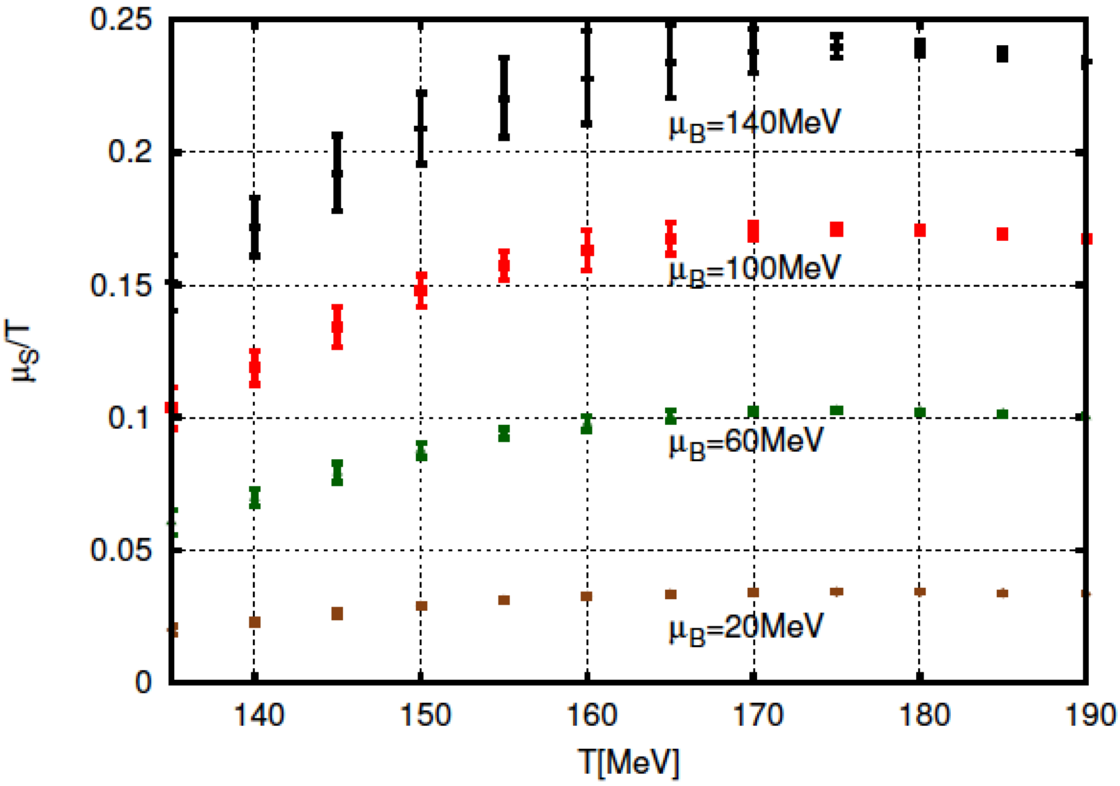
    $\langle n_S \rangle = 0$ $\qquad$ $\langle n_Q \rangle = 0.4 \langle n_B \rangle$

  - Other choices are possible, e.g.:

    $\mu_S = 0$ $\qquad$ $\mu_Q = 0$

Strangeness neutrality

- We simulate at μ_B, μ_S pairs such that $\langle n_S \rangle = 0$
- This requires a non-trivial fine tuning



Thermodynamic identities

- For the pressure we measure:

$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B} \frac{d\left(p/T^4\right)}{d\left(\mu_B/T\right)}\bigg|_{\langle n_S\rangle=0, \langle n_Q\rangle=0.4\langle n_B\rangle, T=\text{const.}}$$

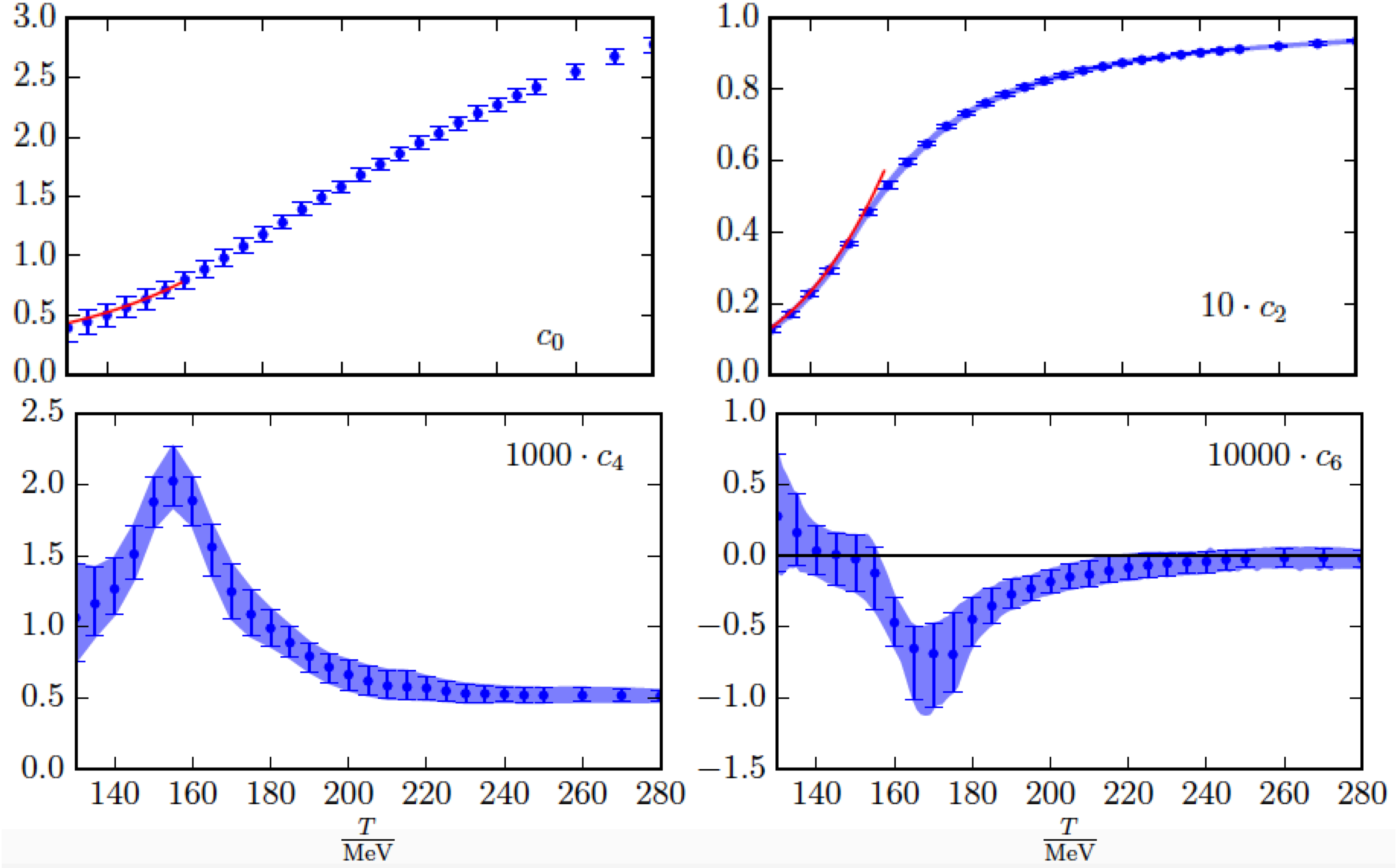
$$= n_B\left(1+0.4\frac{d\mu_Q}{d\mu_B}\right) = 2c_2 + 4c_4\left(\frac{\mu_B}{T}\right)^2 + 6c_6\left(\frac{\mu_B}{T}\right)^4 + \ldots$$

- For the entropy and energy:

$$s = [T^4\partial/\partial T + 4T^3](p/T^4)$$

$$\hat{\epsilon} = \hat{s} - \hat{p} + \hat{\mu}_Q\hat{n}_Q + \hat{\mu}_B\hat{n}_B$$

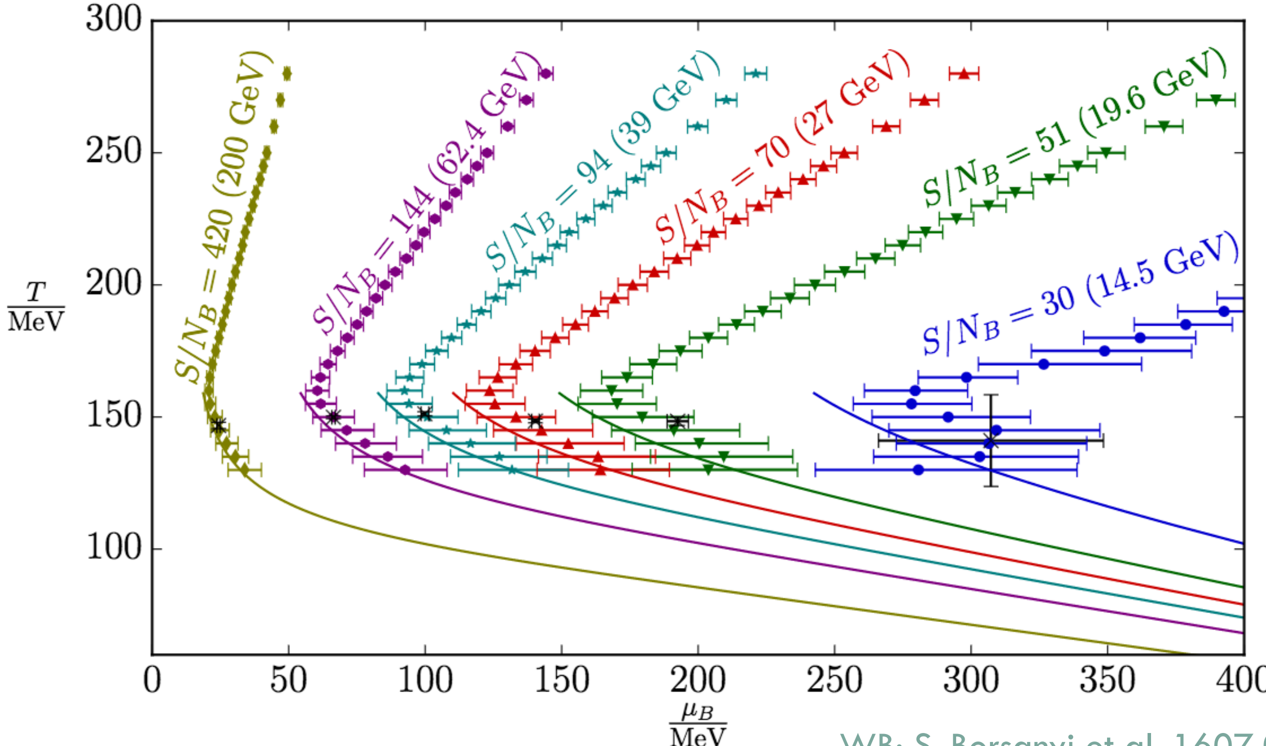
Taylor expansion of the pressure



WB: S. Borsanyi et al. 1607.02493 (2016)

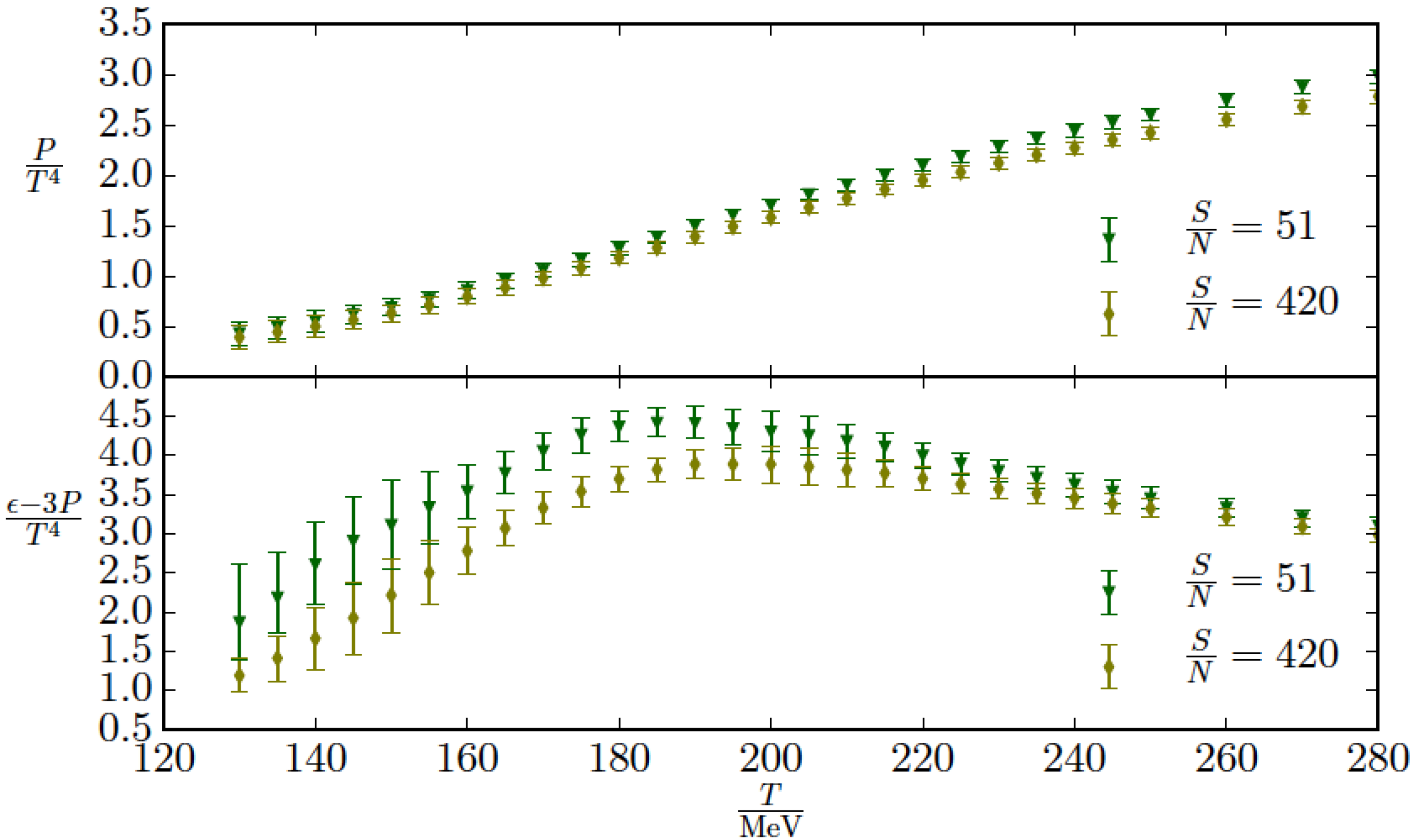
Equation of state at $\mu_B>0$

- Extract the isentropic trajectory that the system follows in the absence of dissipation
- The freeze-out point estimates are from Alba et al., Phys. Lett. B738 (2014)



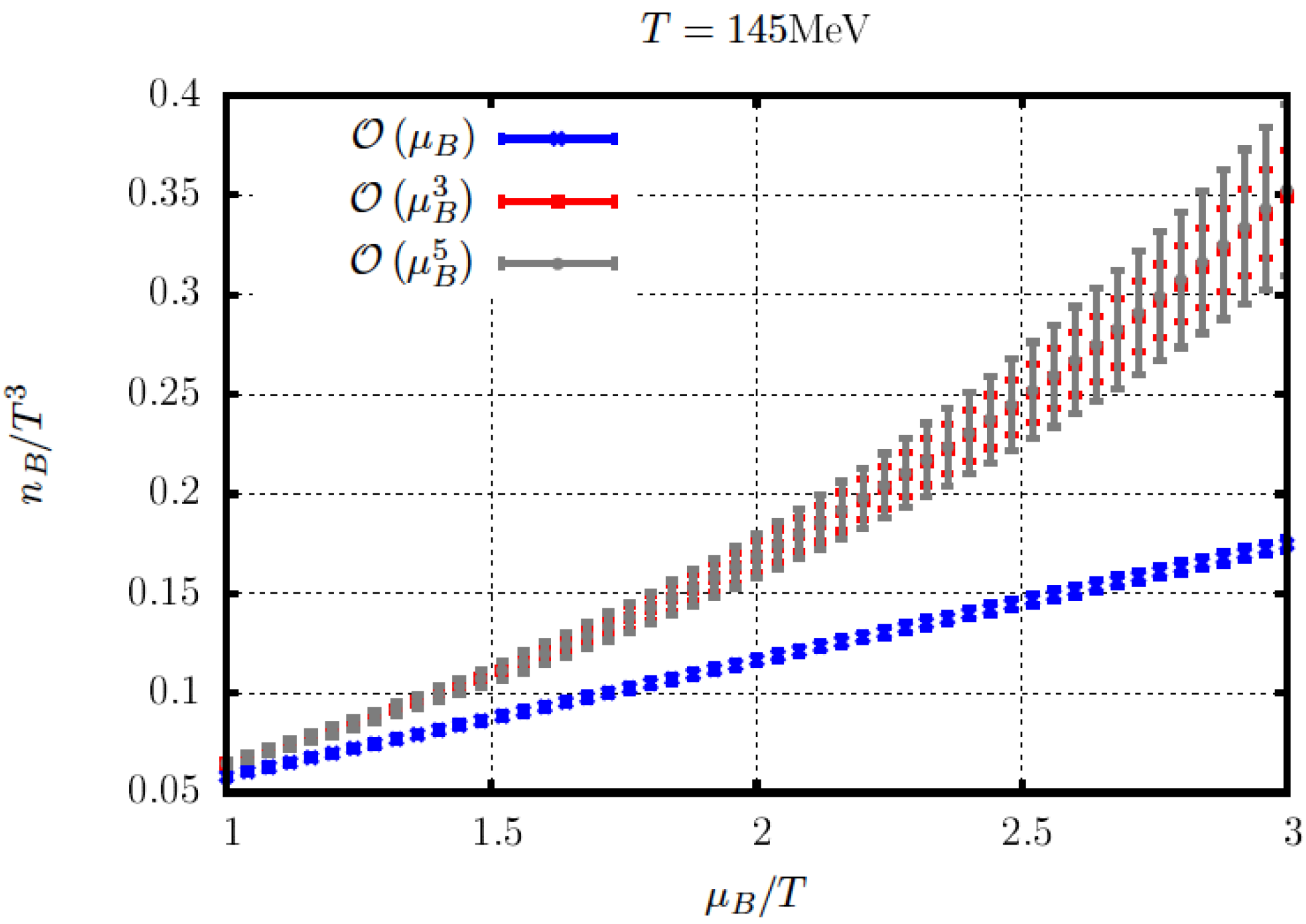
WB: S. Borsanyi et al. 1607.02493, (2016)

Equation of state along the trajectories



WB: S. Borsanyi et al. 1607.02493, (2016)

Different orders of μ_B expansion for n_B



Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned}
\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\
\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\
\mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.
\end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

- The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

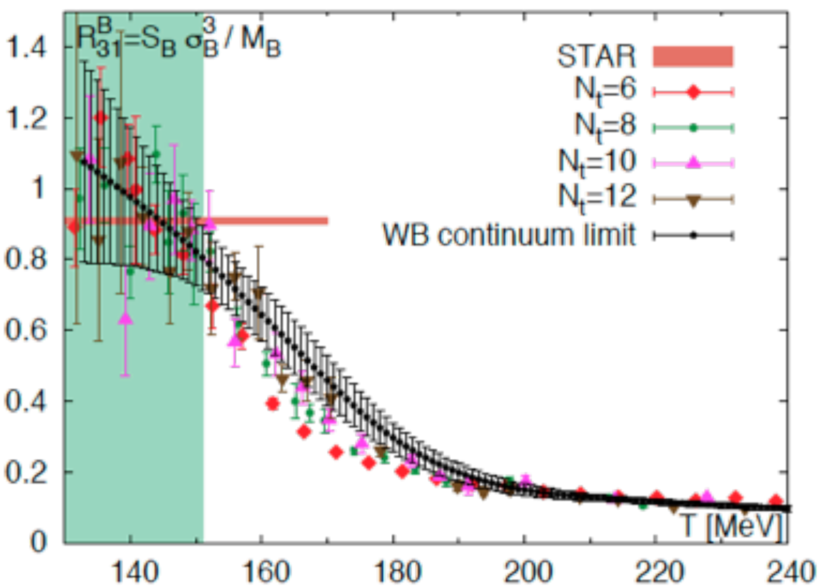
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

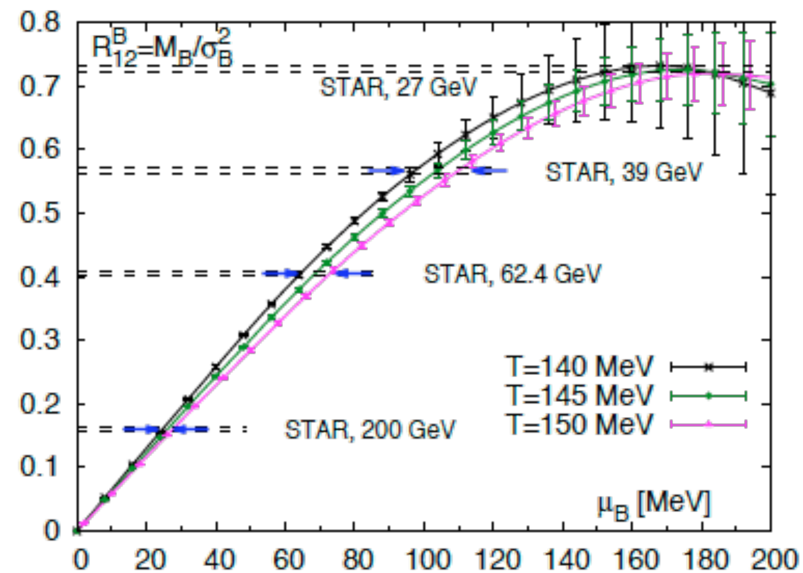
- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

J.Steinheimer et al., PRL (2013)

Freeze-out parameters from B fluctuations

- Thermometer: $\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = S_B \sigma_B^3/M_B$ Baryometer: $\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \sigma_B^2/M_B$



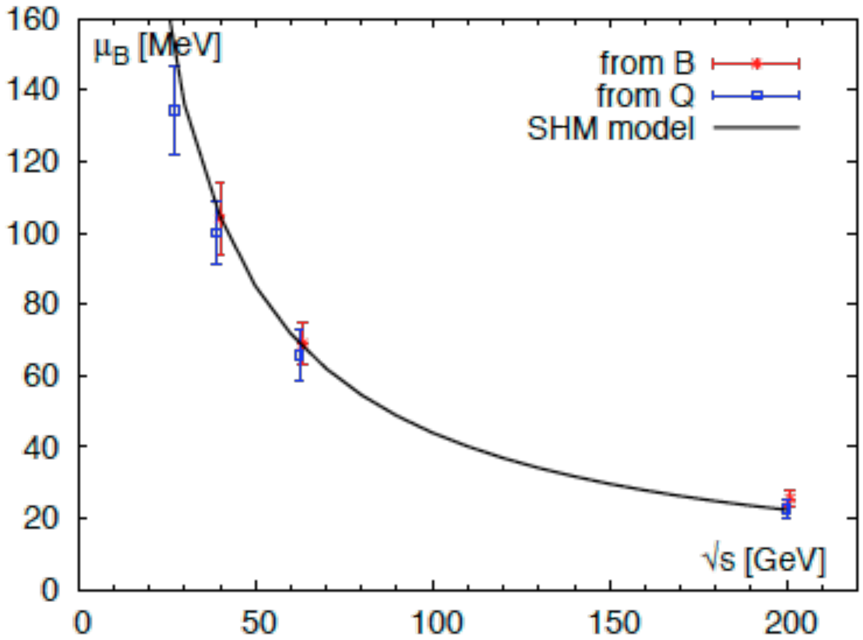


WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

- Upper limit: $T_f \leq 151 \pm 4$ MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

Freeze-out parameters from B fluctuations

- Thermometer: $\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$=$S_B\sigma_B^3/M_B$ Baryometer: $\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$=$\sigma_B^2/M_B$



$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

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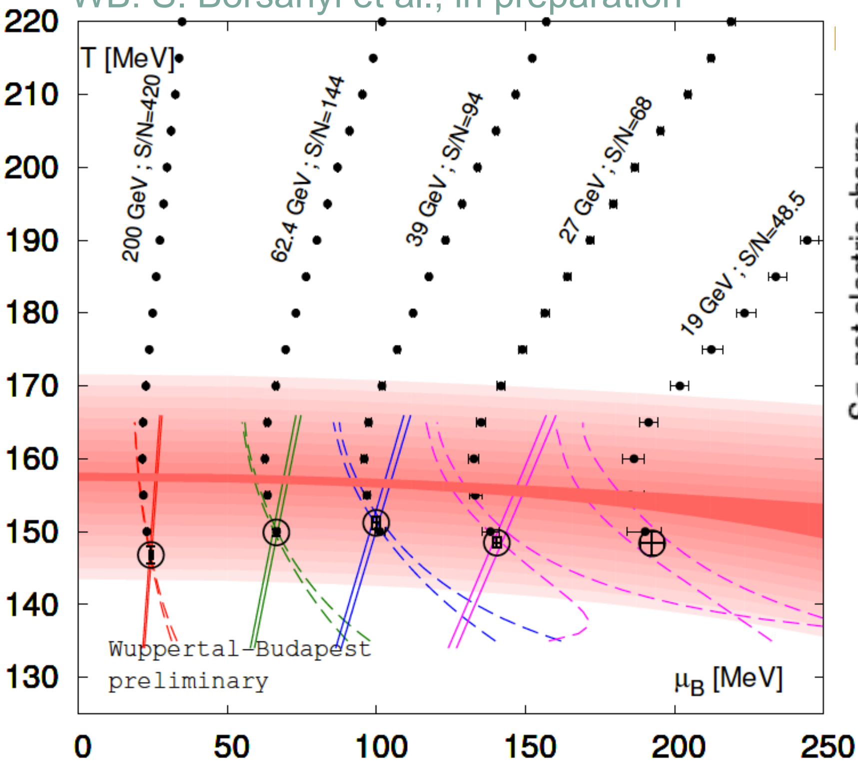
Freeze-out line from first principles

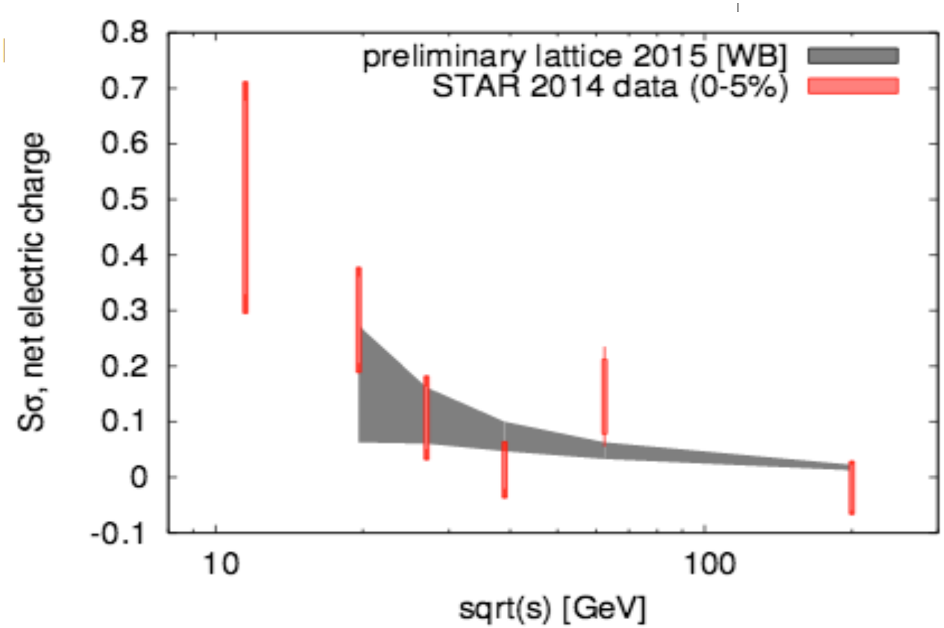
- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation

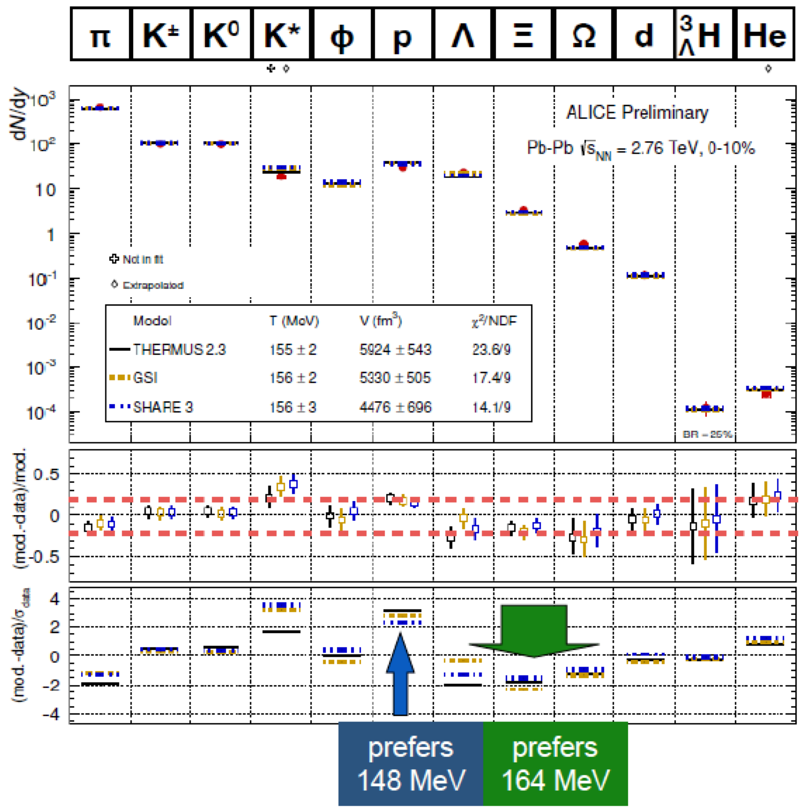




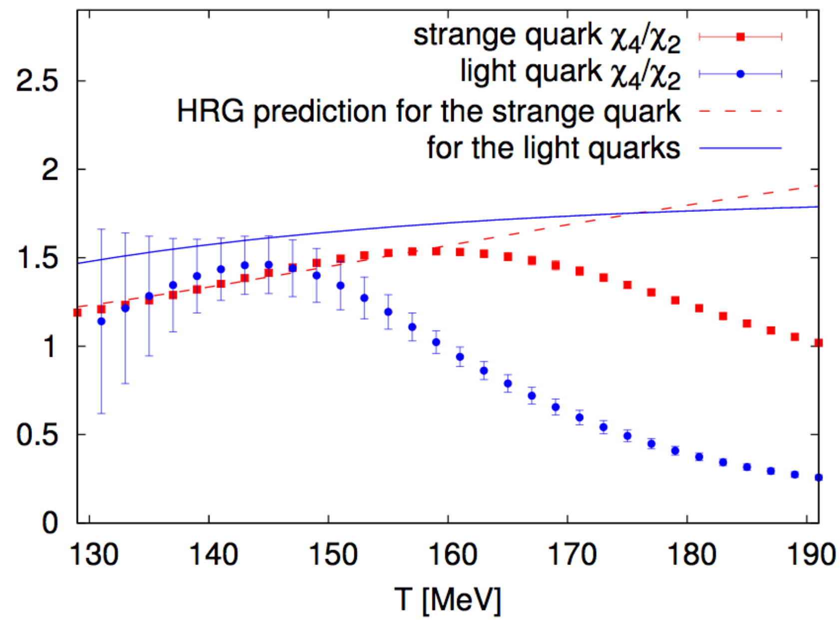
S/N=const from lattice EOS [WB 2015]
HRG analysis [Alba et al]
T_c from lattice [WB 1507.07510]

What about strangeness freeze-out?

- Yield fits seem to hint at a higher temperature for strange particles



M. Floris: QM 2014

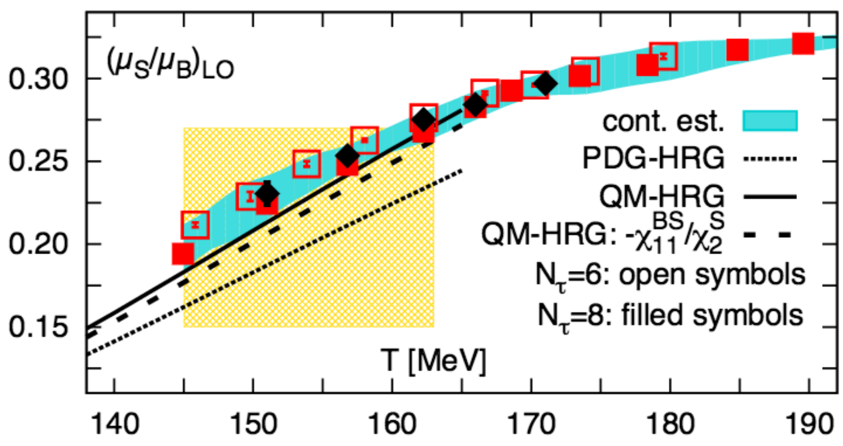


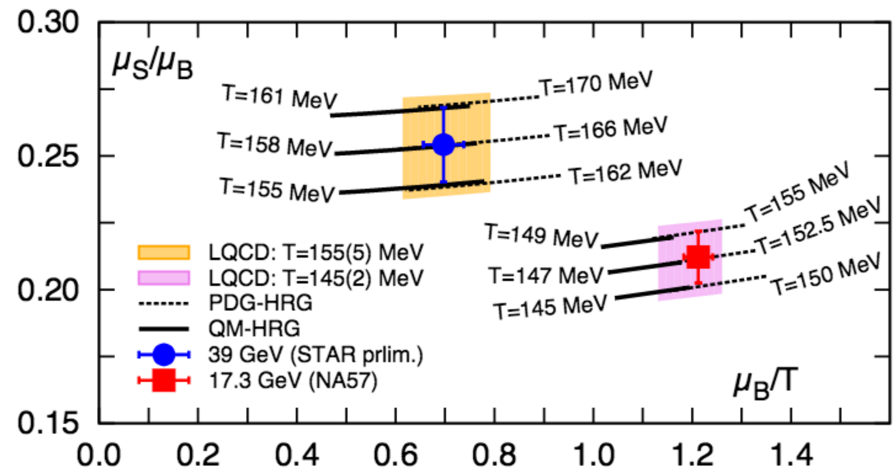
- Similar behavior found in lattice QCD results

R. Bellwied et al. (WB Collaboration): PRL2013

Missing strange states?

- Quark Model predicts not-yet-detected (multi-)strange hadrons





- QM-HRG improves the agreement with lattice results for the baryon-strangeness correlator:

$$(\mu_S/\mu_B)_{LO} = -\chi_{11}^{BS}/\chi_2^{S} + \chi_{11}^{QS}\mu_Q/\mu_B$$

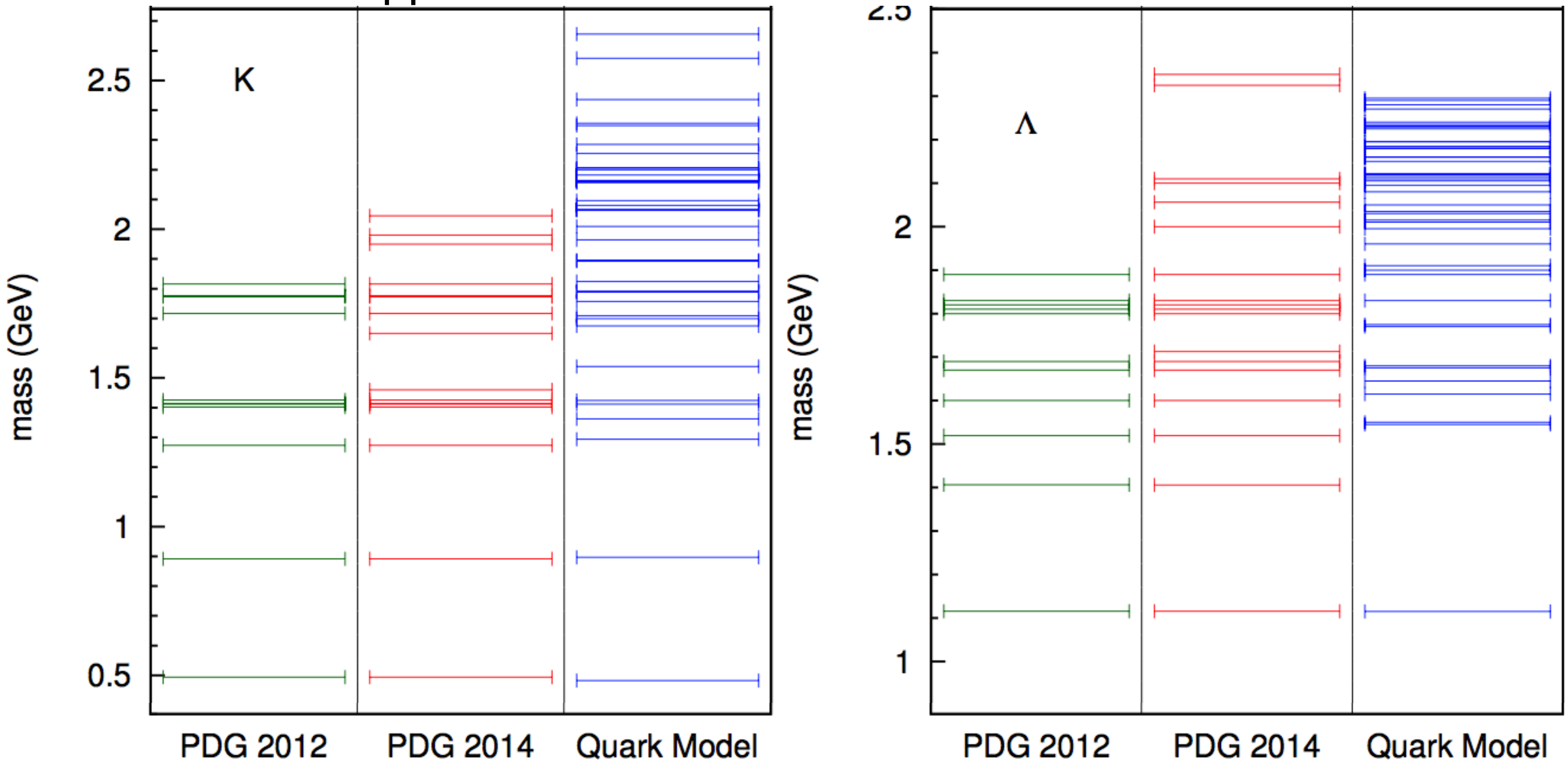
- The effect is only relevant at finite μ_B

- Feed-down from resonance decays not included

A. Bazavov et al., PRL (2014)

Missing strange states?

- New states appear in the 2014 version of the PDG



WB collaboration, in preparation

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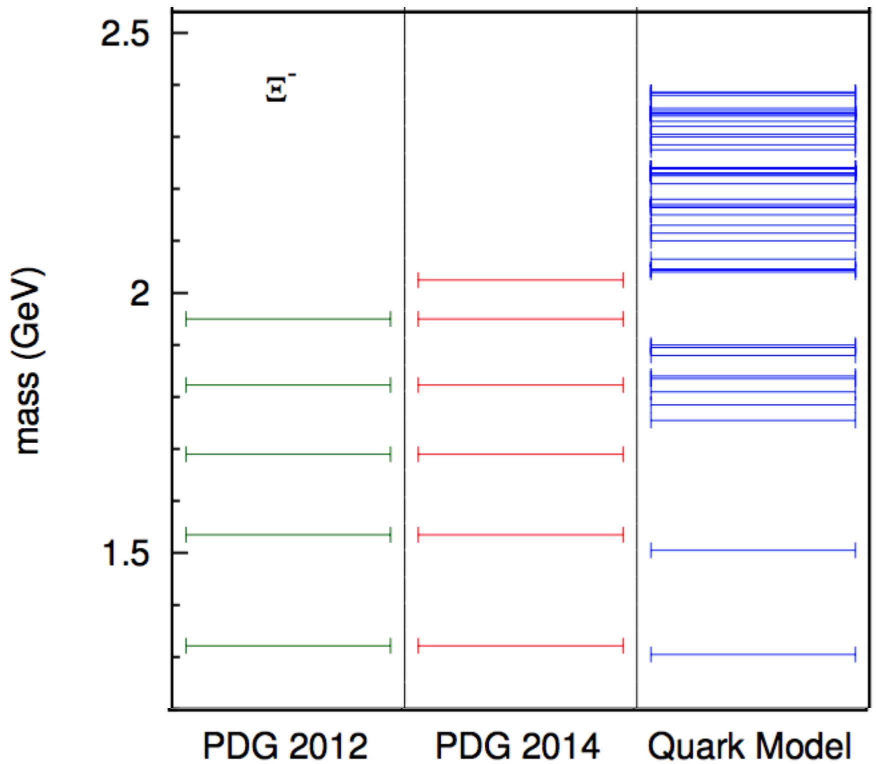
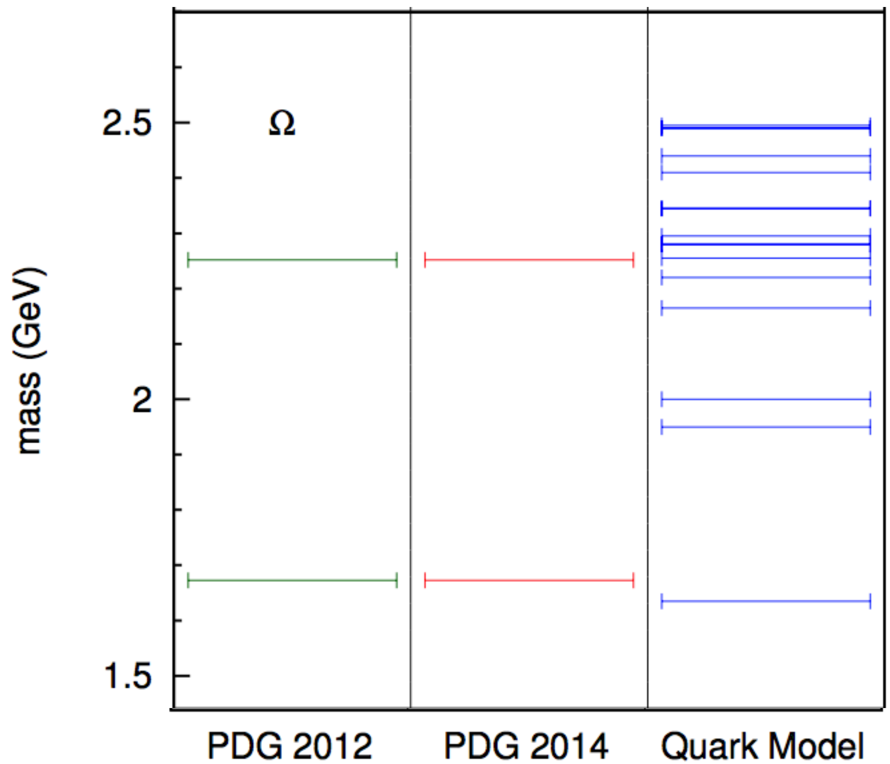
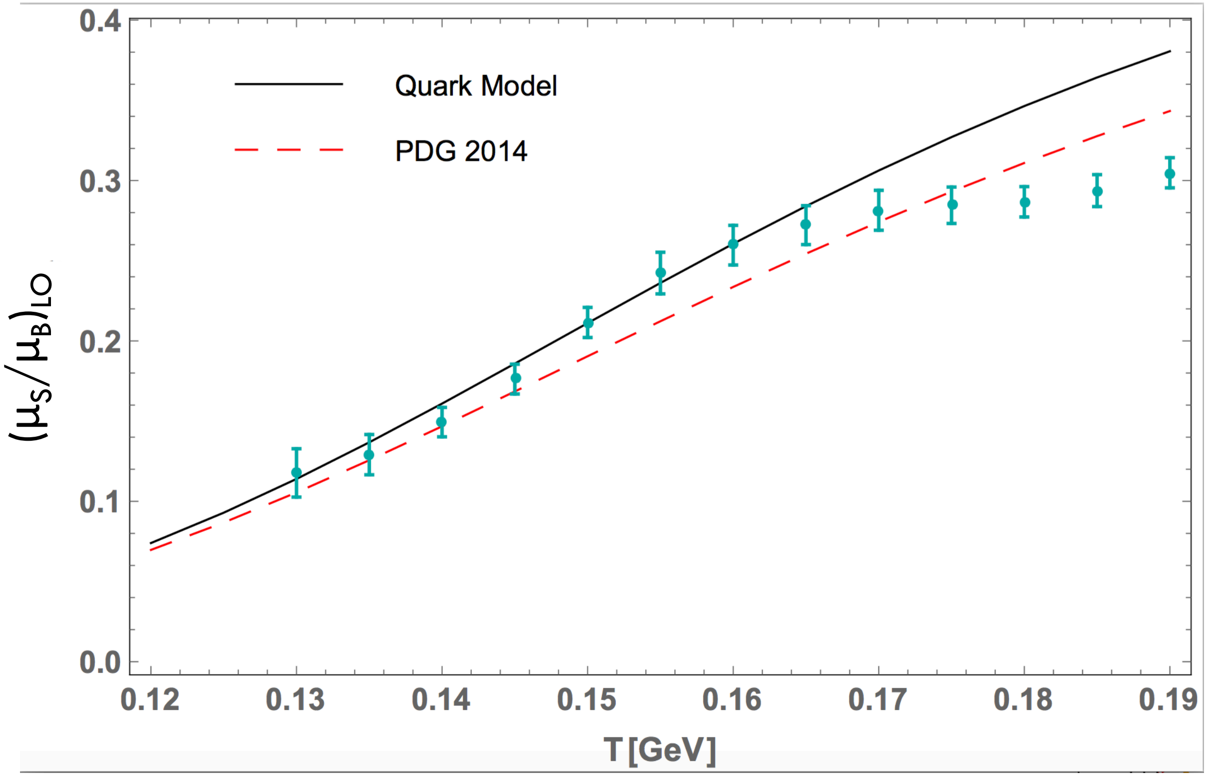




WB collaboration, in preparation

Missing strange states?

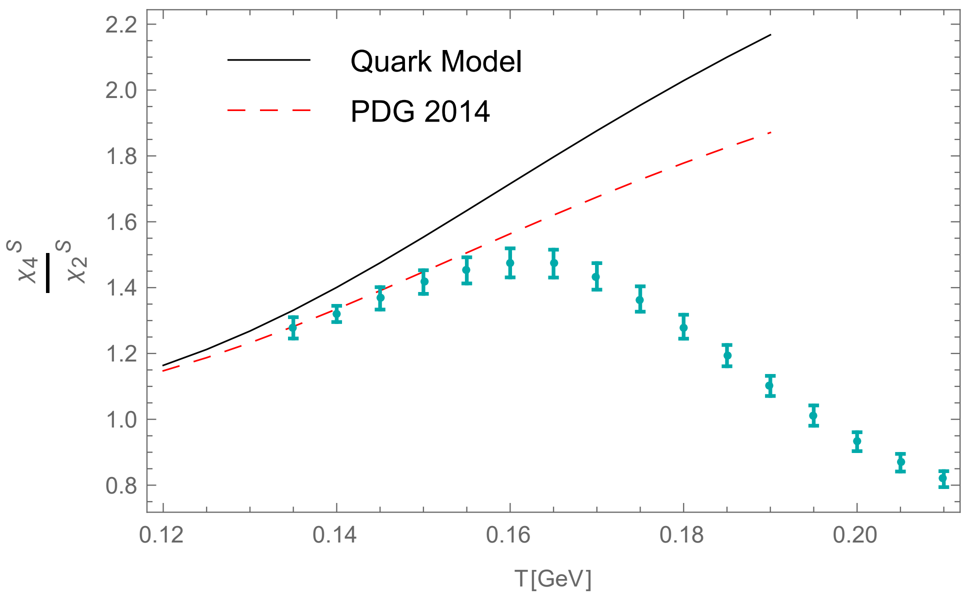
- The comparison with the lattice is improved for the baryon-strangeness correlator:

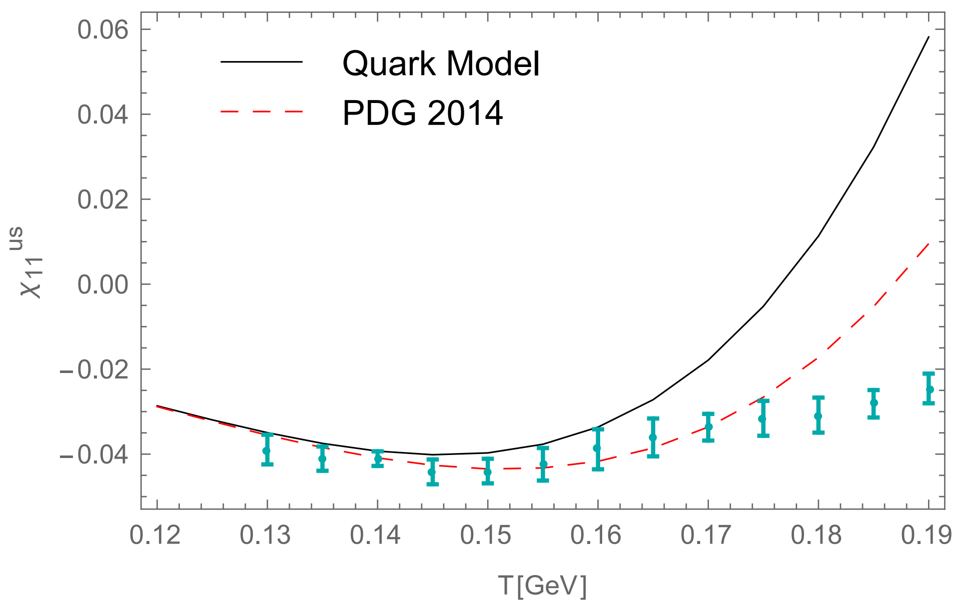


WB collaboration, in preparation

Missing strange states?

- Some observables are in agreement with the PDG 2014 but not with the Quark Model:





- χ_4^S/χ_2^S is proportional to <S^2> in the system
- It seems to indicate that the quark model predicts too many multi-strange states

WB collaboration, in preparation

Missing strange states?

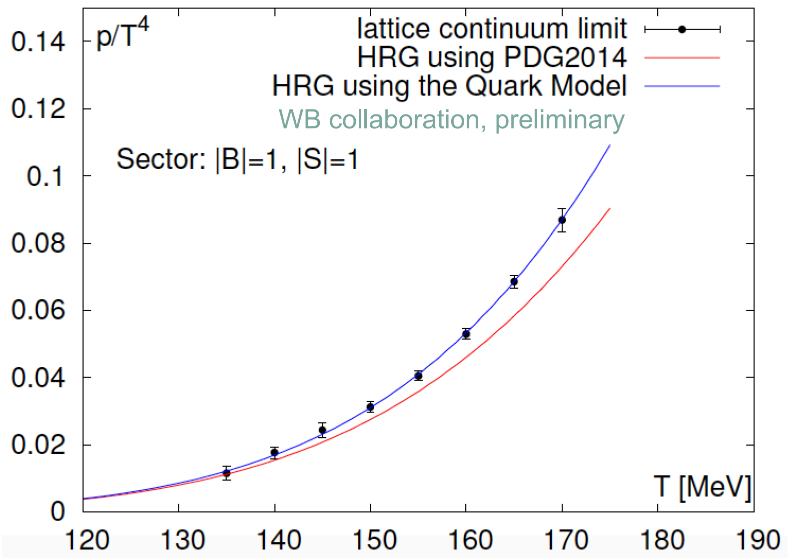
- Idea: define linear combinations of correlators which receive contributions only from particles with a given quantum number
- They allow to compare PDG and QM prediction for each sector separately

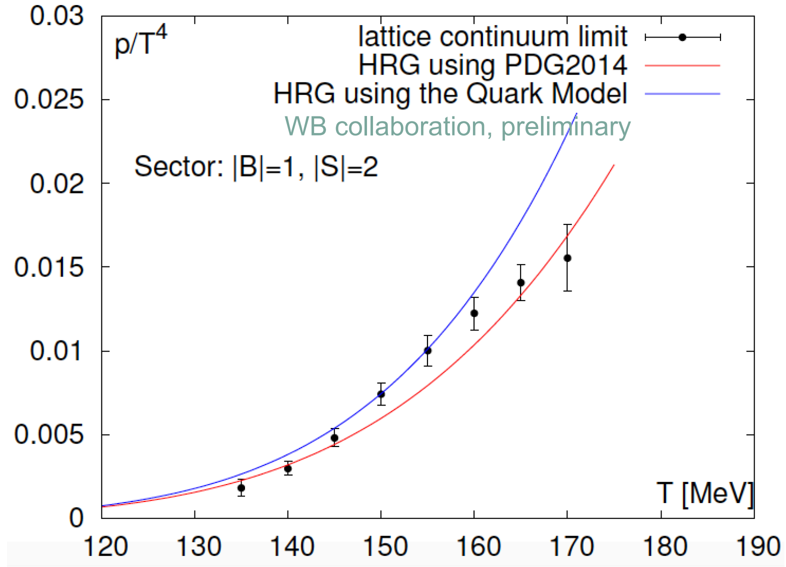
$$
\begin{aligned}
P_S(\hat{\mu}_B, \hat{\mu}_S) &= P_{0|1|} \cosh(\hat{\mu}_S) \\
&+ P_{1|1|} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
&+ P_{1|2|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
&+ P_{1|3|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)
\end{aligned}
$$

$$
\begin{aligned}
P_{0|1|} &= \chi_2^S - \chi_{22}^{BS} \\
P_{1|1|} &= \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right) \\
P_{1|2|} &= -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right) \\
P_{1|3|} &= \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right)
\end{aligned}
$$

A. Bazavov et al., PRL (2013)

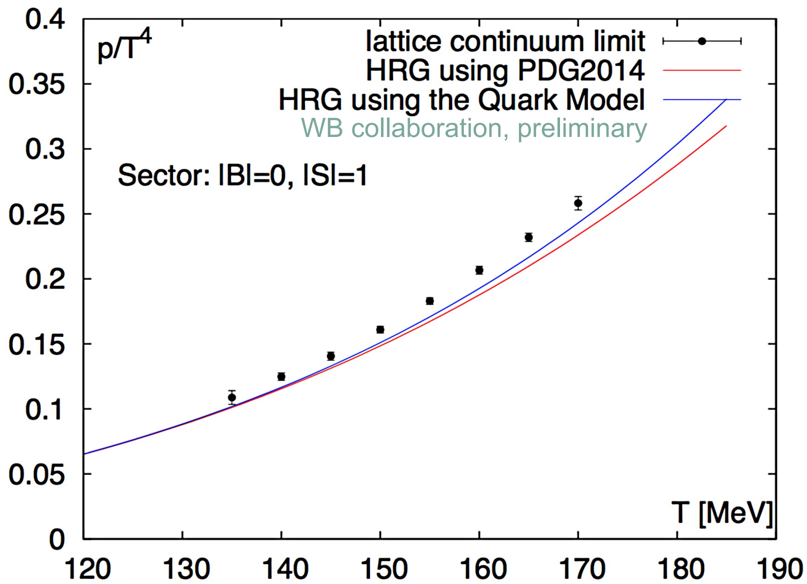
Missing strange states?

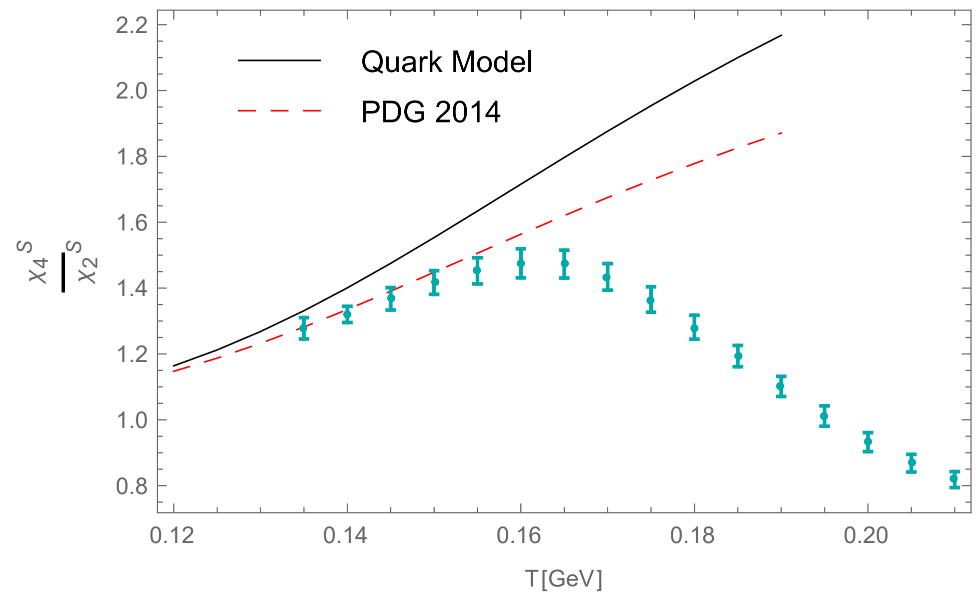




- The precision in the lattice results can allow to distinguish between the two scenarios
- Quark model pushes the agreement with the data for the strange baryons to higher temperatures

Not enough strange mesons





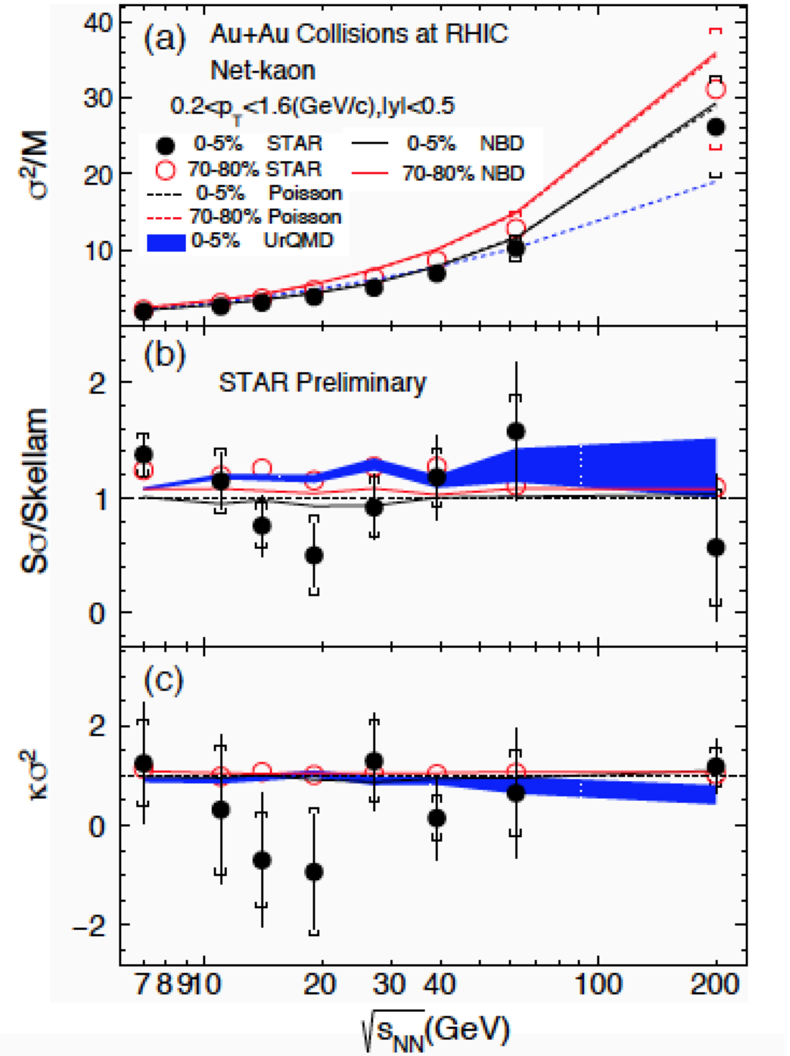
- Both Quark Model and PDG 2014 underestimate the partial pressure due to strange mesons
- This might explain why the QM overestimates χ_4^S/χ_2^S: more strange mesons would bring the curve down

P.M. Lo, K. Redlich, C. Sasaki, PRC (2015)

Kaon fluctuations

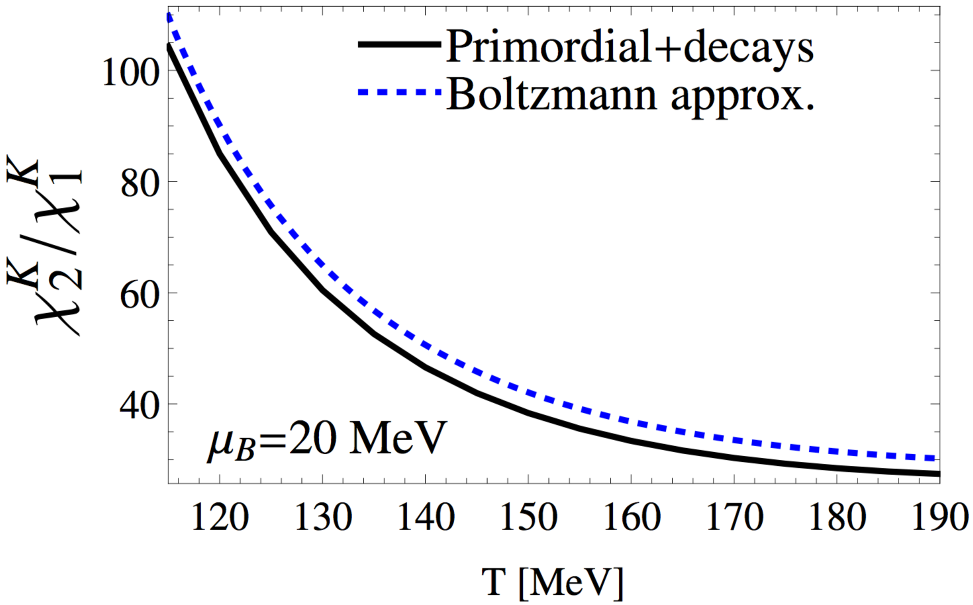
Talk by Ji XU at SQM 2016

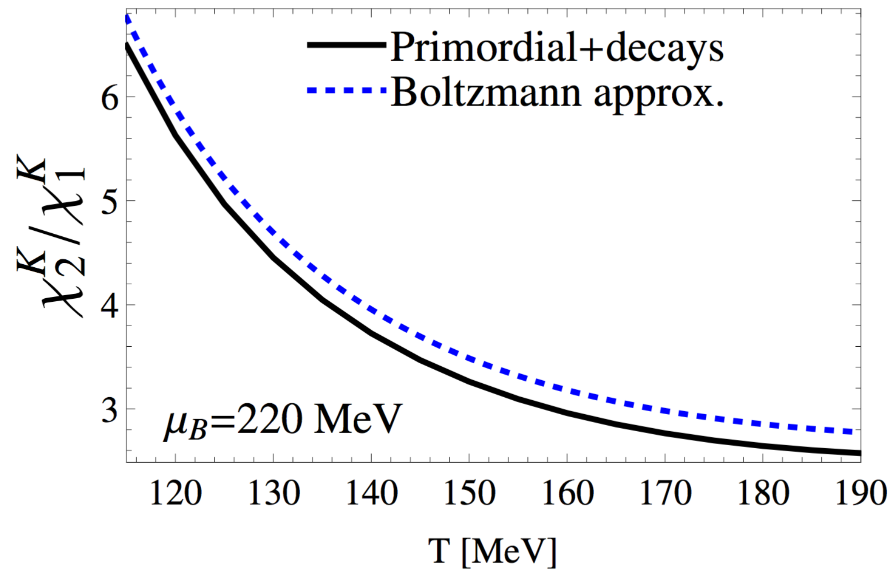
- Experimental data are becoming available.
- Exciting result but presently hampered by systematic errors
- BES-II will help
- Kaon fluctuations from HRG model will be affected by the hadronic spectrum and decays



Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527





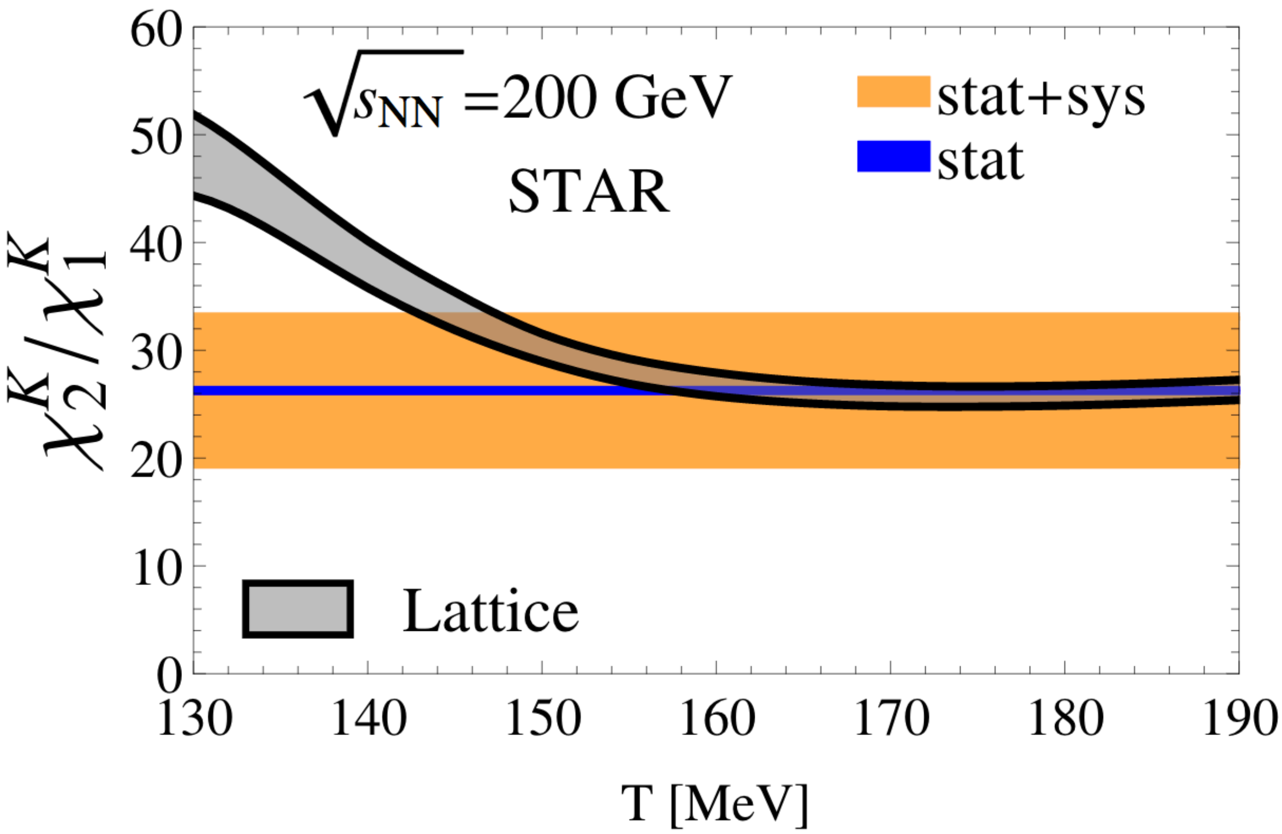
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527



- Experimental uncertainty does not allow a precise determination of T_f^K

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at μ_B=0 can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Comparison with experiment allows to determine properties of strongly interacting matter from first principles
- It is possible to identify kaon fluctuations in lattice QCD

Lattice details

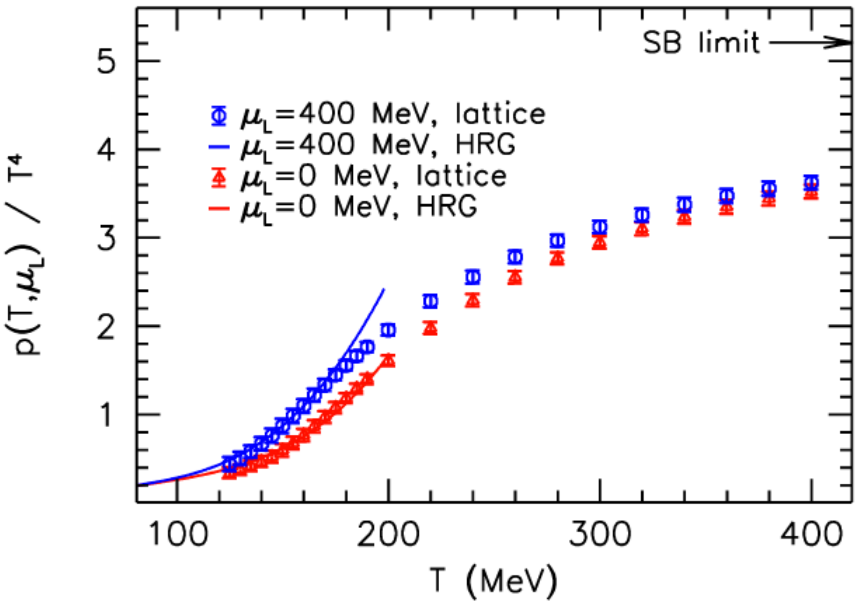
- **The 4stout staggered action**
  - 2+1+1 dynamical flavors
  - 4 levels of stout smearing in the fermionic action
  - The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping m_c/m_s=11.85
  - The scale is set in two ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the source of the systematic error in all of the plots

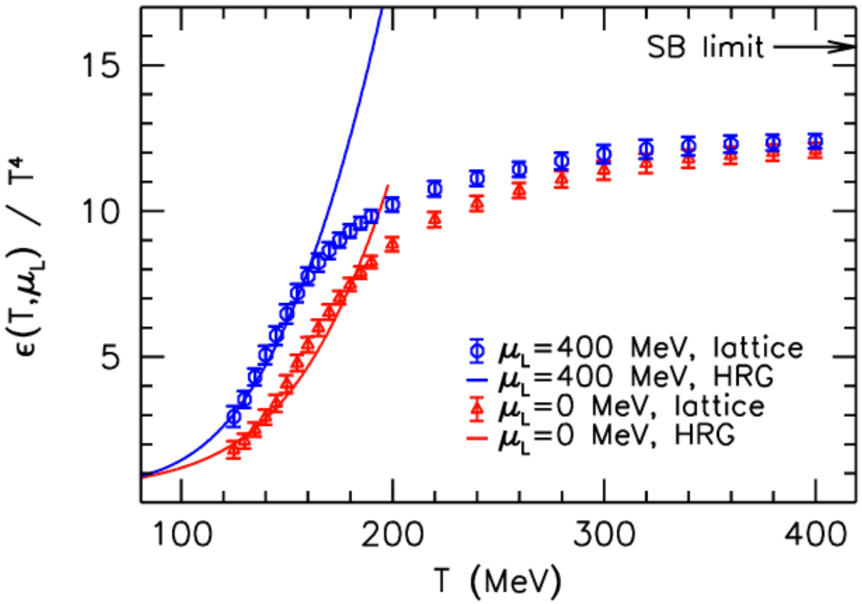
- **Ensembles**
  - Continuum limit from N_t=10, 12, 16
  - For imaginary μ we have $\mu_B = iT\pi j/8$, with j=3, 4, 5, 6, 6.5, 7

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4} = \frac{p(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \log \mathcal{Z}}{\partial\mu_i\partial\mu_j}\Big|_{\mu_i=\mu_j=0}$$





S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Analytical continuation – illustration of systematics