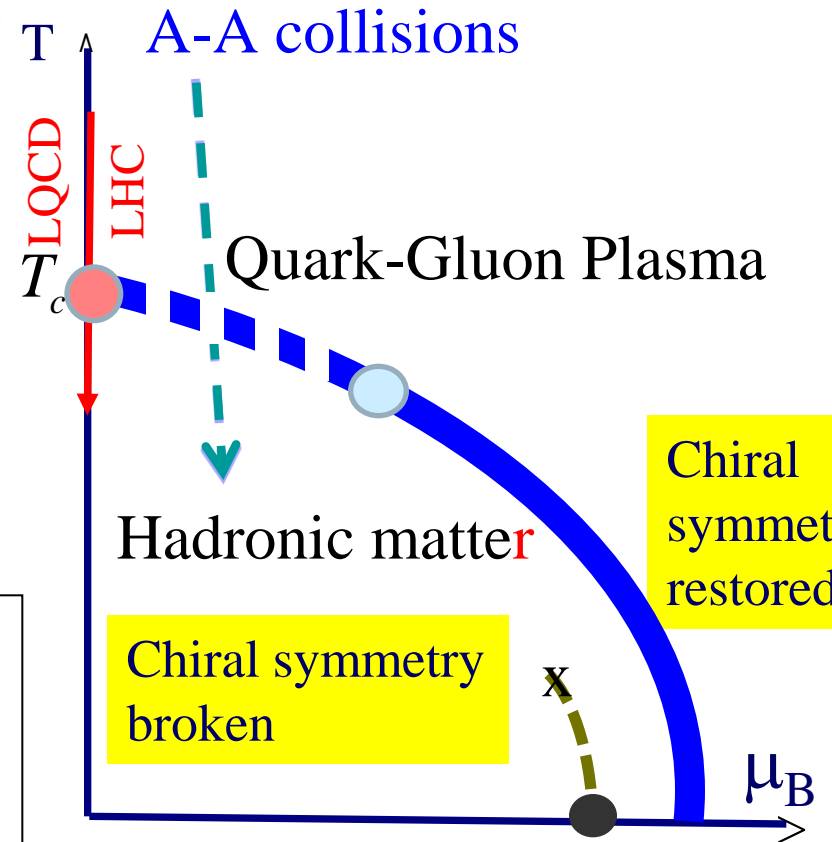
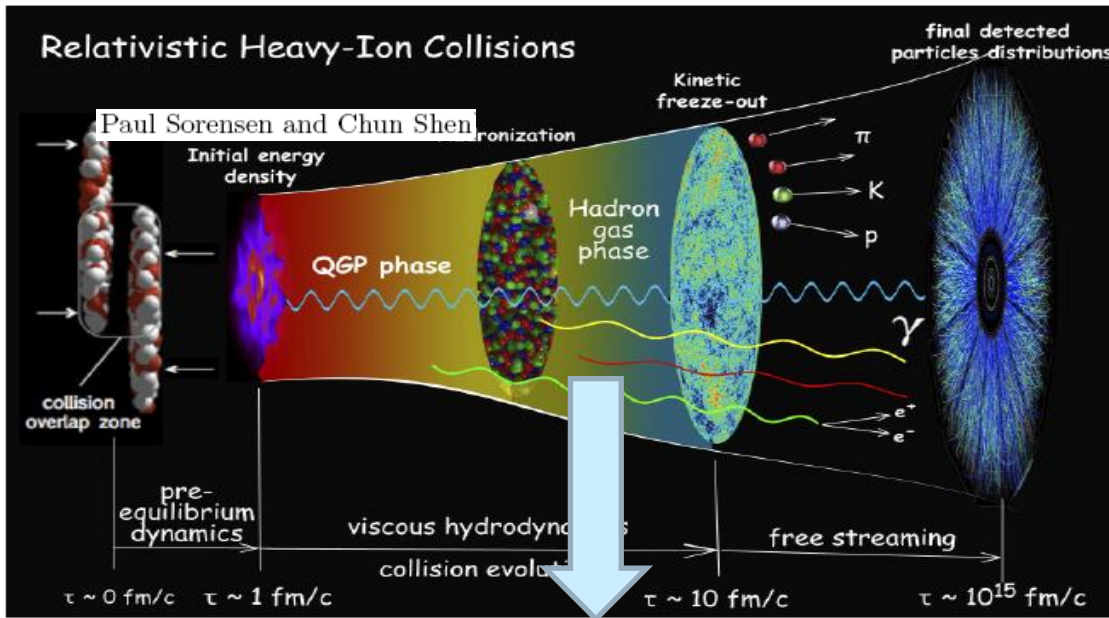


Resonance widths : fluctuations and particle momentum distribution near the QCD phase boundary

Krzysztof Redlich, Univ. of Wroclaw



Modeling hadronic resonances within S-matrix approach and its application in the analysis of pion spectra and strangeness fluctuations

Work done with: B. Friman, P. Huovinen, P.M. Lo, M. Marczenko and C. Sasaki
Phys.Rev. C92(2015) , Phys.Rev. D92 (2015), Eur.Phys.J. A52 (2016) , [arXiv:1608.06817](https://arxiv.org/abs/1608.06817)

Thermal particle production in HIC

Strongly interacting hadronic matter considered as **thermal medium in chemical equilibrium**

- resonance production dominates the interactions in hadronic reactions

clustering of hadrons and particle
antiparticle pair creations is included

- all information about interactions is hidden in the mass spectrum

$\tau(m^2) d(m^2)$



$$\tau(m^2) \sim m^a e^{m/T_H}$$

describes the number of hadrons and resonances in the mass interval $d(m^2)$

Statistical operator in HIC

$$\ln Z^{GC}(T, \vec{\mu}) = \int d^4 p \frac{2V_{\mu} p^{\mu}}{(2\pi)^3} \tau(p^2) e^{-\beta_{\mu} p^{\mu}}$$

- The statistical sum with the PDG discrete mass spectrum

$$\ln Z(T, \vec{\mu}) \approx \frac{VT}{2\pi^2} \sum_{i \in \text{hadrons}} d_i e^{\frac{\vec{Q}_i \cdot \vec{\mu}}{T}} \int ds s K_2\left(\frac{\sqrt{s}}{T}\right) F^{B-W}(m_i, s)$$

- and its particle composition

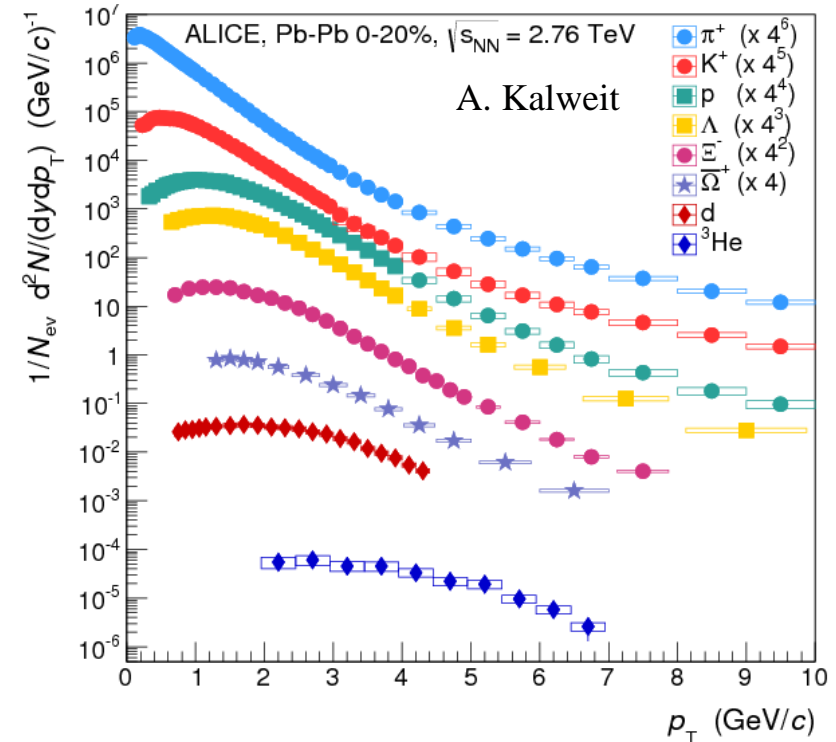
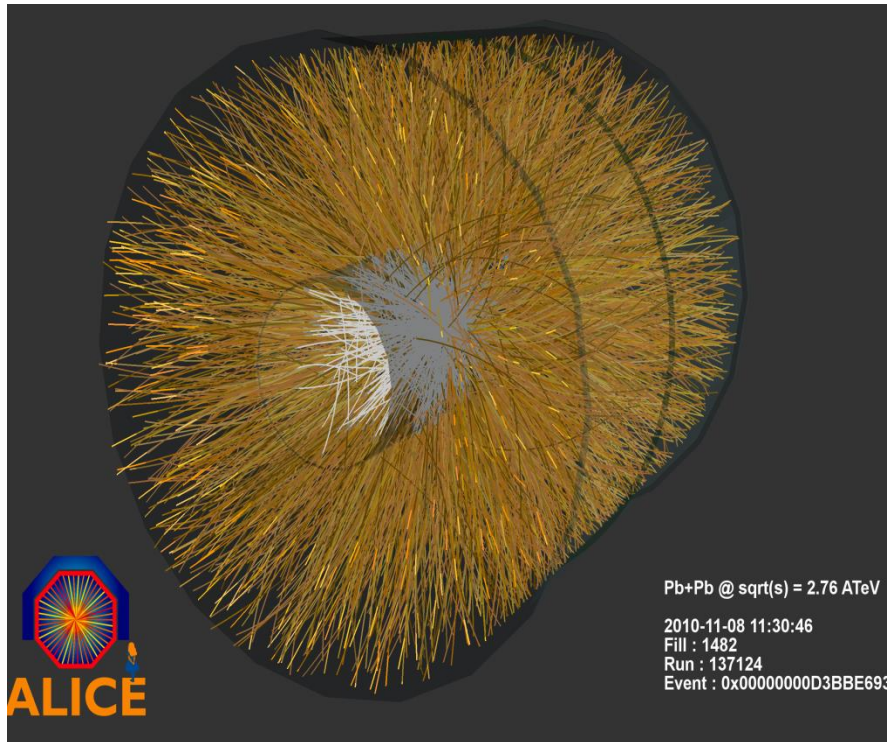
particle yield thermal density BR thermal density of resonances

$$\langle N_i \rangle = V \left[n_i^{th}(T, \vec{\mu}_B) + \sum_K \Gamma_{K \rightarrow i} n_i^{th-Res.}(T, \vec{\mu}_B) \right]$$

- Only 2-parameters needed to fix all particle yield ratios

Excellent data of ALICE Collaboration for particle yields

ALICE Collaboration



ALICE Time Projection Chamber (TPC), Time of Flight Detector (TOF), High Momentum Particle Identification Detector (HMPID) together with the Transition Radiation Detector (TRD) and the Inner Tracking System (ITS) provide information on the flavour composition of the collision fireball, vector meson resonances, as well as charm and beauty production through the measurement of leptonic observables.

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonance Gas (HRG):

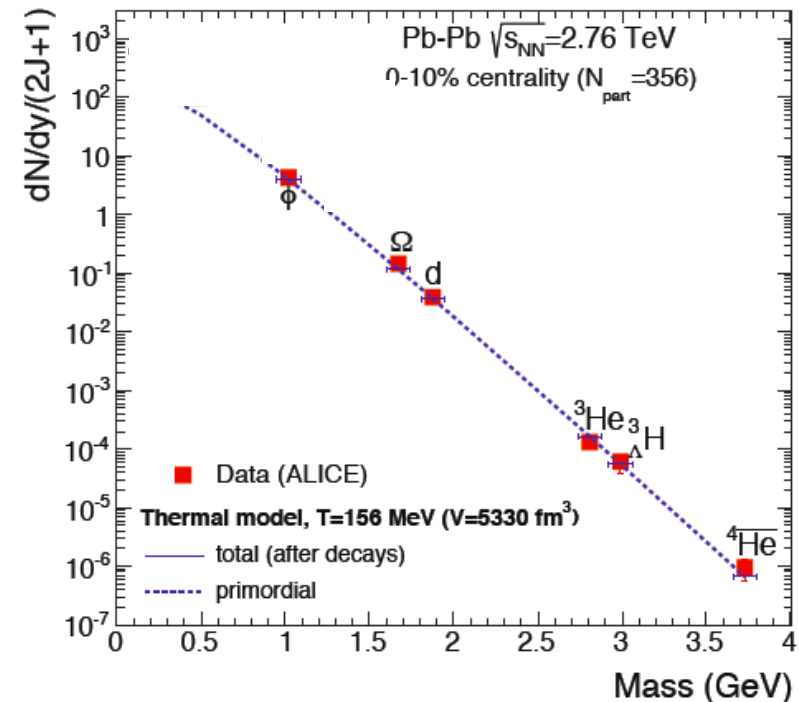
“uncorrelated” gas of hadrons and resonances

$$\langle N_i \rangle = V [n_i^{th}(T, \vec{\mu}) + \sum_K \Gamma_{K \rightarrow i} n_i^{th-Res.}(T, \vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.

Particle yields with no resonance decay contributions at the LHC:

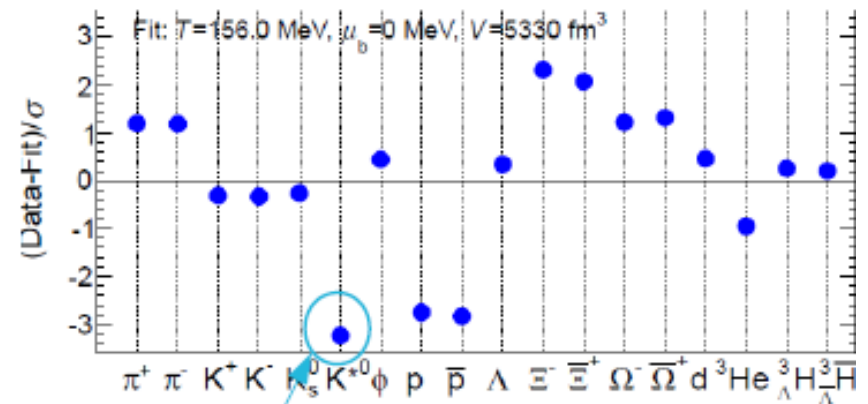
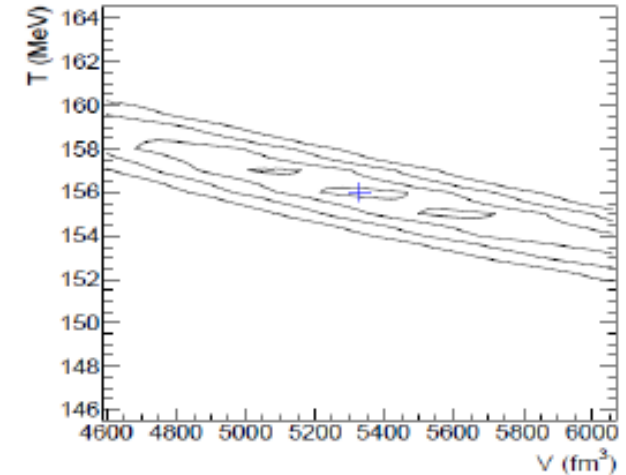
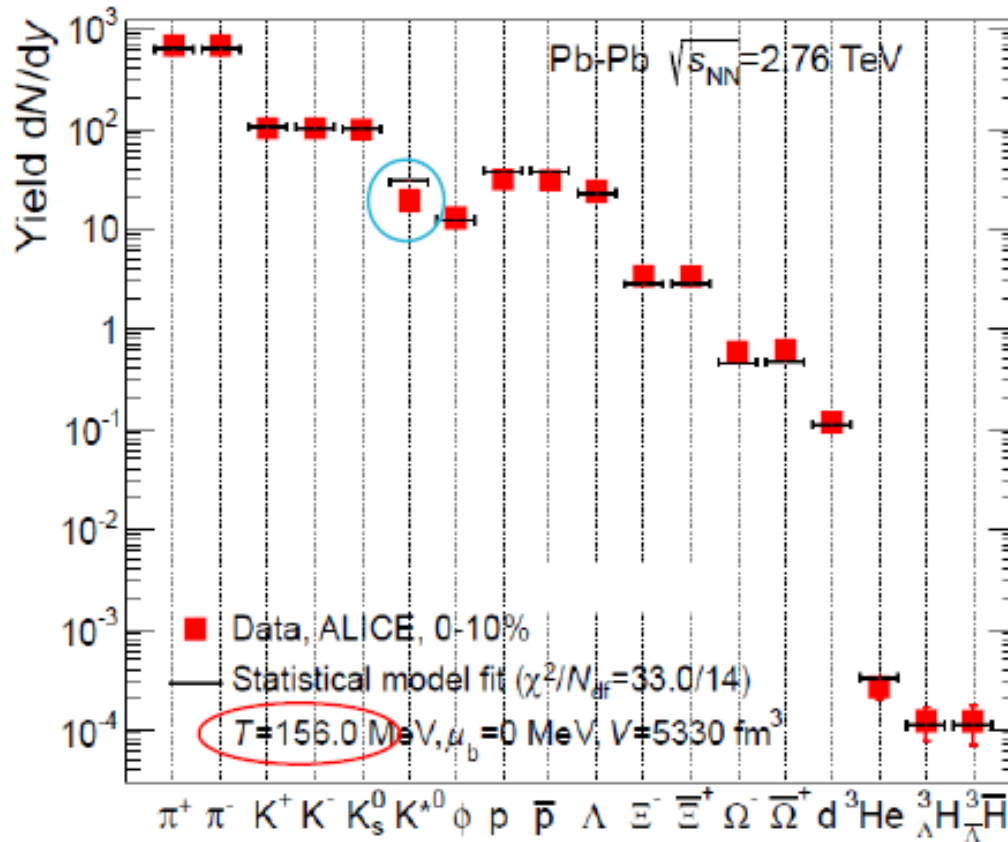
$$\frac{1}{2j+1} \frac{dN}{dy} = V (m/T)^2 K_2(m/T)$$



- Measured yields are reproduced with HRG at $T \approx 156$ MeV

Thermal equilibrium at the LHC with respect to Hagedorn's thermodynamic potential

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.



protons low by 2.8σ

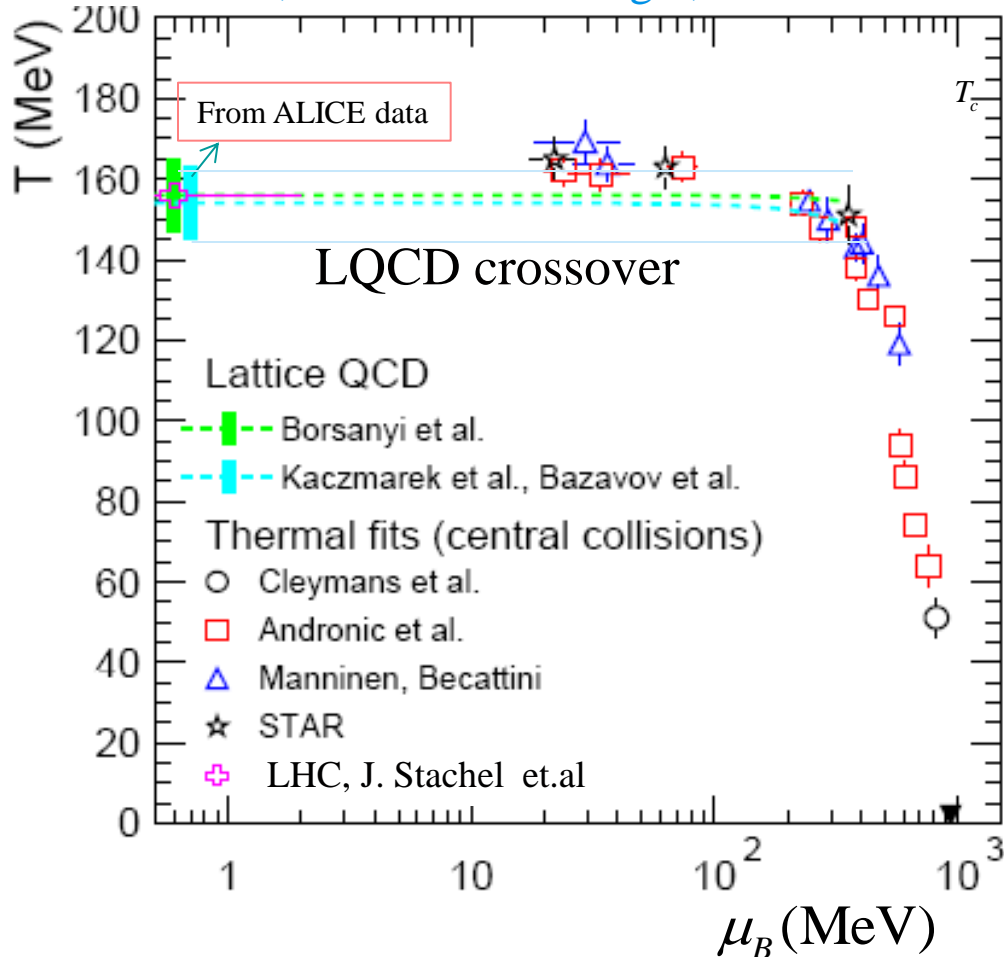
strongly decaying
resonance

$T = 156$ MeV
red. $\chi^2 = 2.33$

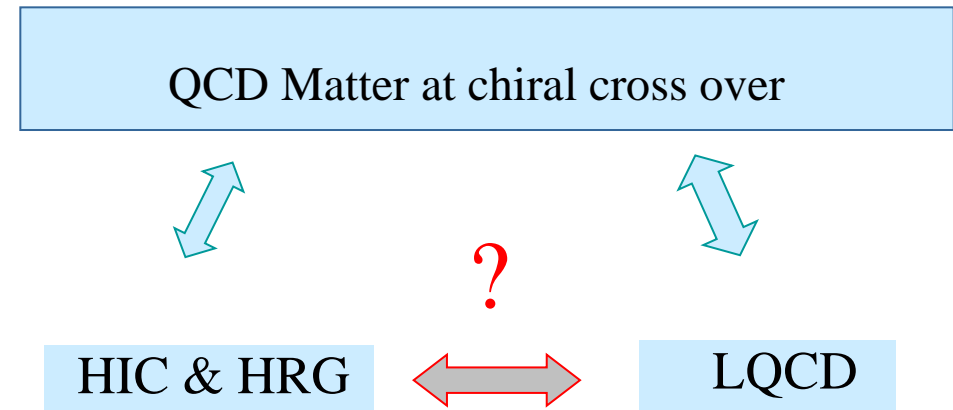
Chemical Freeze out and QCD Phase Boundary

Chemical freeze out defines a lower bound for the QCD phase boundary

A. Andronic, P. Braun-Munzinger, K.R. & J. Stachel



- The QCD phase boundary coincides with chemical freeze out conditions obtained from HIC data analyzed with the HRG model

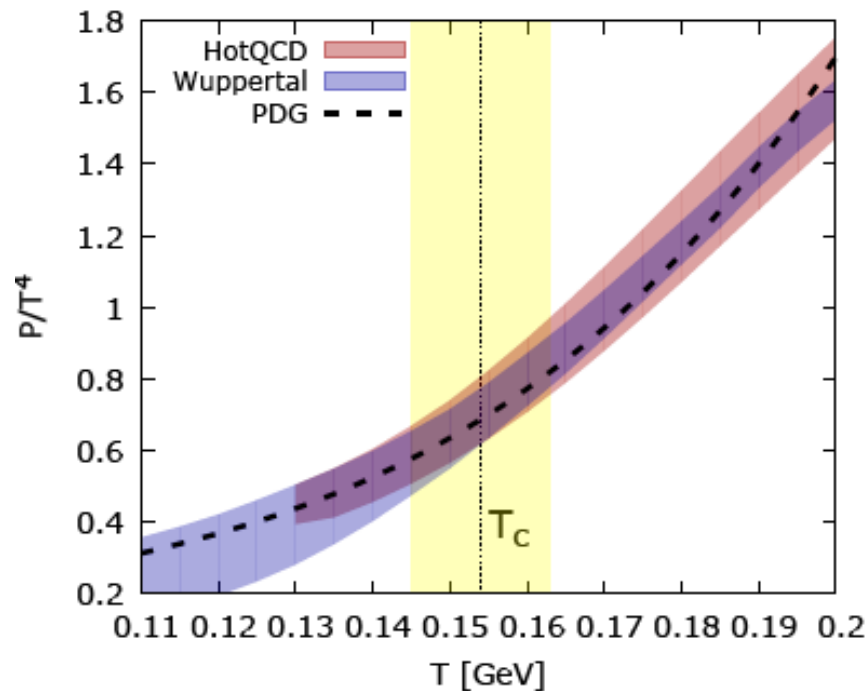


- The HRG should describe the QCD thermodynamics in the hadronic phase

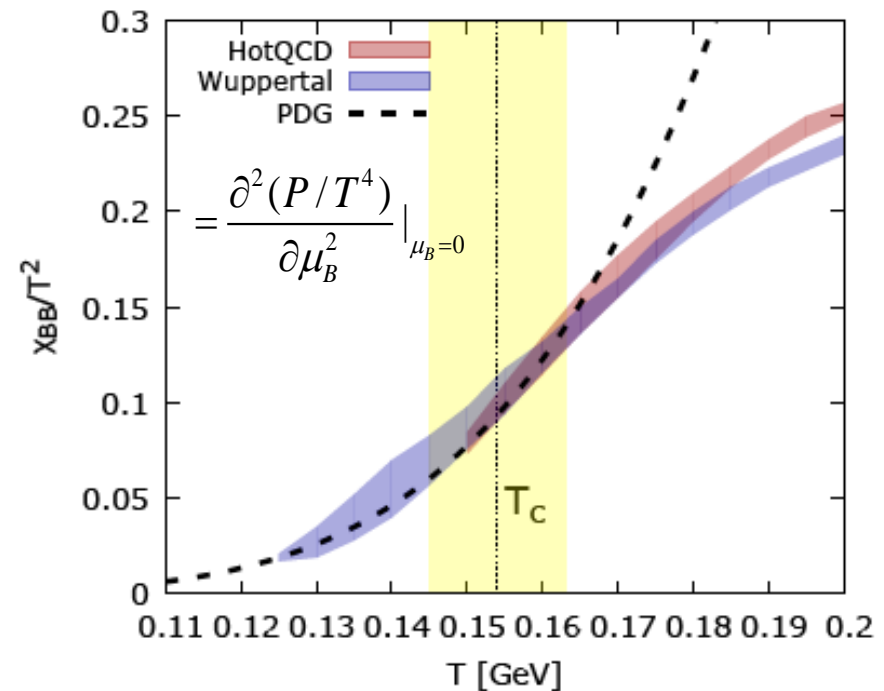
Combine data of HotQCD and Budapest-Wuppertal Coll.

P. M. Lo arXiv:1507.06398

Total thermodynamic pressure

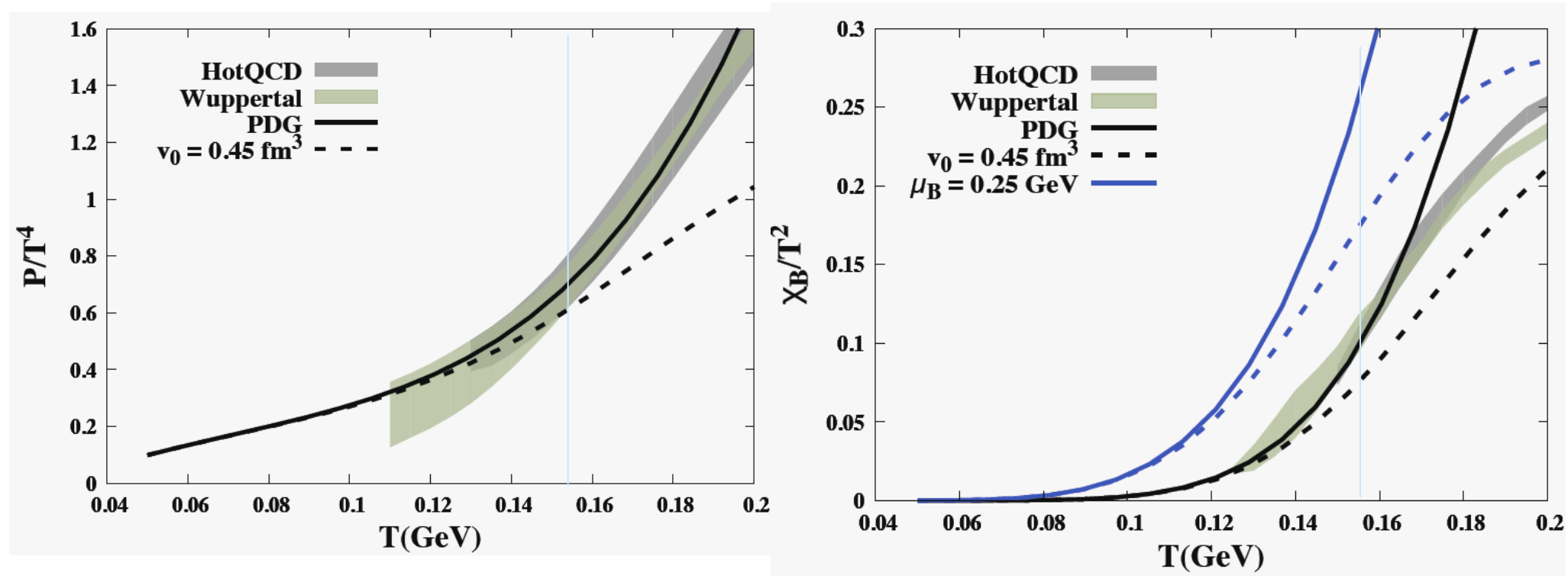


Baryon number fluctuations



- Consistent description of the equation of state up to the chiral crossover by the HRG

HRG with repulsive interactions - hard core



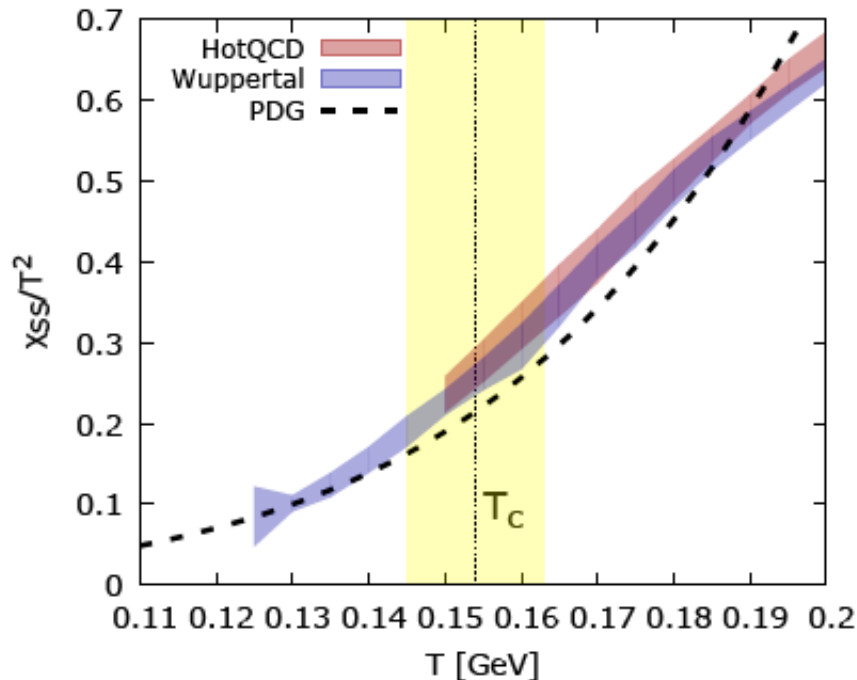
- LQCD excludes hard-core repulsive interactions between hadrons with $r_{hc} > 2 \text{ fm}$

Missing resonances in the strangeness sector

A. Bazavov, et al. Phys. Rev. Lett. 113 (2014)

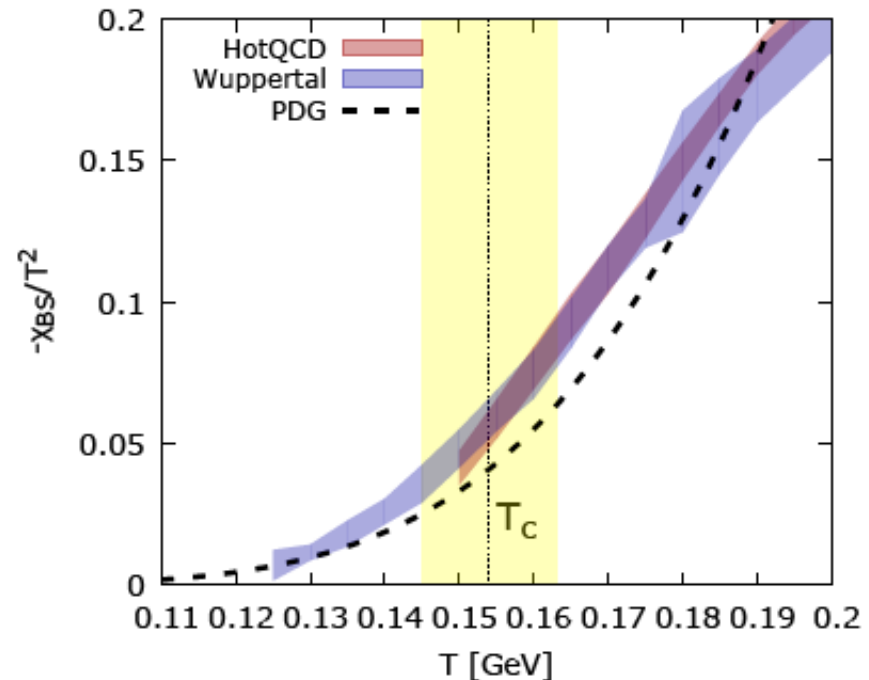
$$\chi_{SS} = \partial^2 P / \partial \mu_s^2$$

Strange mesonic sector



$$\chi_{BS} = \partial^2 P / \partial \mu_B \partial \mu_S$$

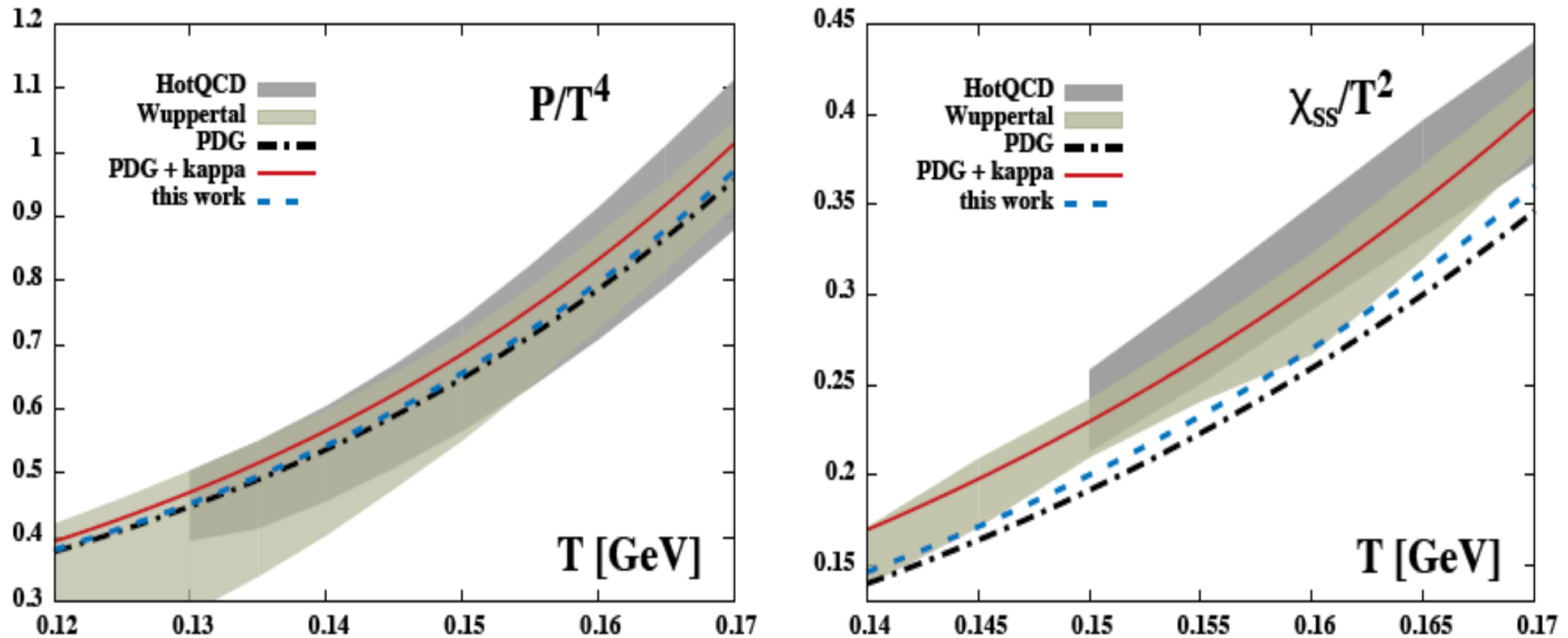
Strangeness fluctuations



- Go beyond PDG and include resonances in the Hagedorn's continuum mass spectrum: A. Majumder & B. Muller, Phys. Rev. Lett. (2010)

$$\rho(m) \rightarrow m^a e^{m/T_H}$$

Leading missing resonance contribution to strangeness fluctuations

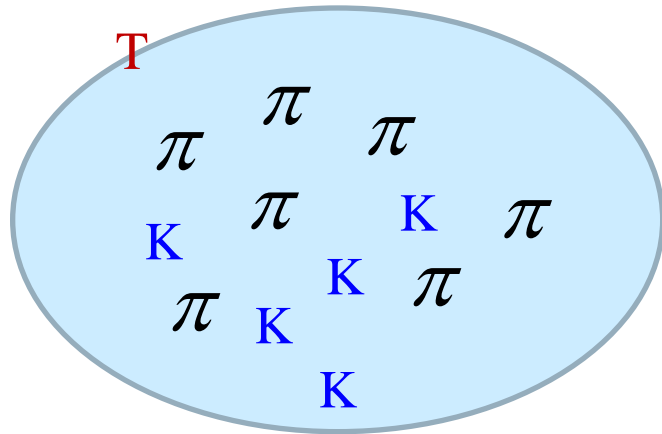


The strange scalar meson channel, with the unconfirmed kappa, $K^0(800)$ resonance is a prime candidate. $m = 0.682 \text{ GeV}$ $I(J^P) = \frac{1}{2}(0^+)$,

S-MATRIX APPROACH

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969)

W. Weinhold, & B. Friman
Phys. Lett. B 433, 236 (1998).



- Consider interacting pions and kaons gas in thermal equilibrium at temperature T
- Due to $K\pi$ scattering resonances are formed
 $l=1/2$, s -wave : $\kappa(800)$, $K_0^*(1430)$ [$JP = 0+$]
 $l=1/2$, p -wave : $K^*(892)$, $K^*(1410)$, $K^*(1680)$ [$JP = 1-$]
- In the S-matrix approach the thermodynamic pressure in the low density approximation

$$P(T) \approx P_{\pi}^{id} + P_K^{id} + P_{\pi K}^{int}$$

Thermodynamic pressure of an ideal gas:

$$P = P^{id} / T^4 = - \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left[1 - e^{-\sqrt{p^2 + M^2} - \mu} \right] + \ln \left[1 - e^{-\sqrt{p^2 + M^2} + \mu} \right] \right\}$$

S-MATRIX APPROACH: INTERACTING PART

The leading order corrections , determined by the two-body **scattering phase shift**, which is equivalent to the second virial coefficient

$$P_{\text{int}} = \int_{m_{\text{th}}}^{\infty} \frac{dM}{2\pi} B(M) P_T(M)$$

$$B(M) = 2 \frac{d}{dM} \delta(M)$$

$$\int_{m_{\text{th}}}^{\infty} \frac{dM}{2\pi} B(M) = 1$$

Effective weight function

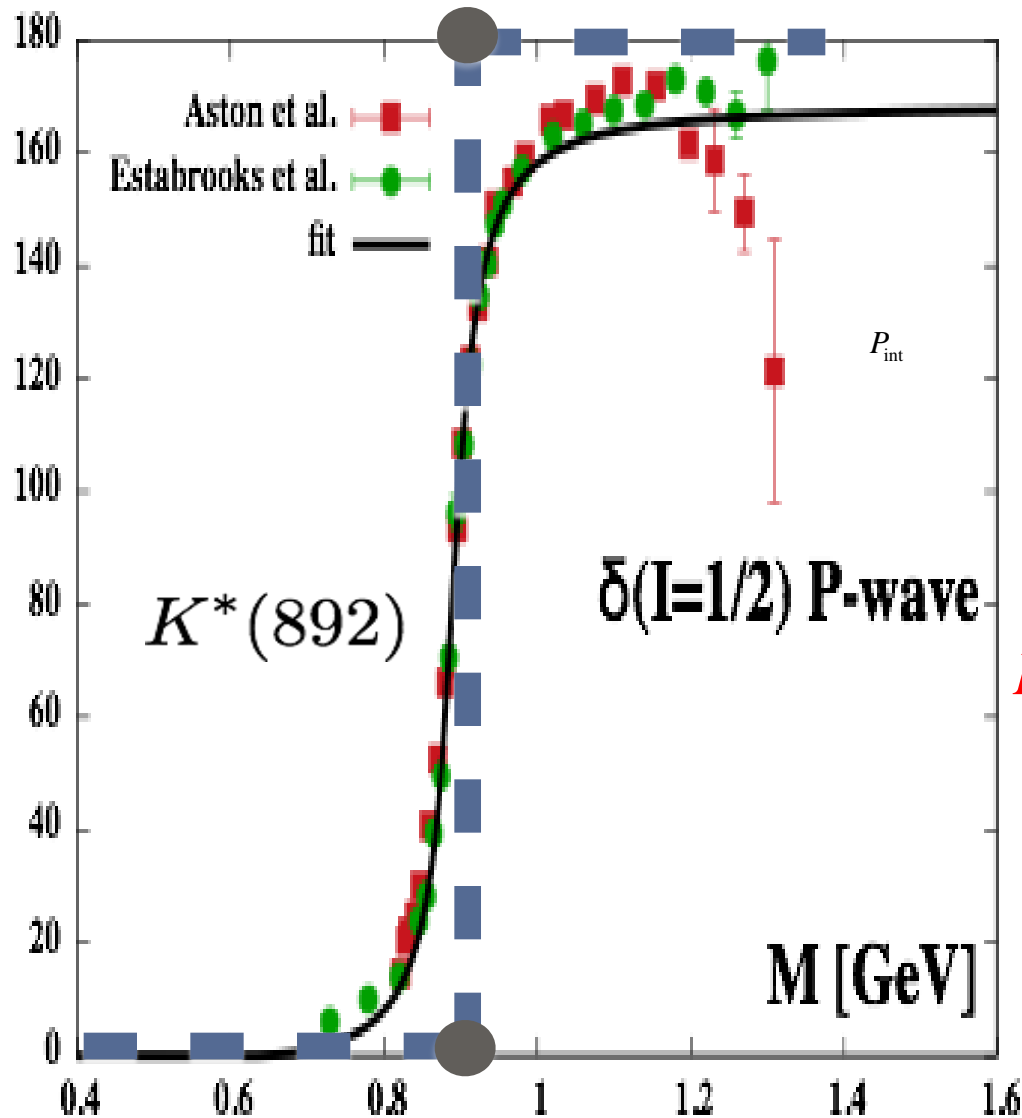
Scattering phase shift

Normalization

Pressure of an ideal gas of resonances with an invariant mass M

$$P_T(M) = -2 \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left[1 - e^{-\sqrt{p^2 + M^2} - \mu} \right] + \ln \left[1 - e^{-\sqrt{p^2 + M^2} + \mu} \right] \right\}$$

Experimental phase shift in P-wave channel



B. Friman et al, arXiv:1507.04183

For narrow resonance

$$B(M) = 2 \frac{d}{dM} \delta(M)$$

very well described by
the Breit-Wigner form

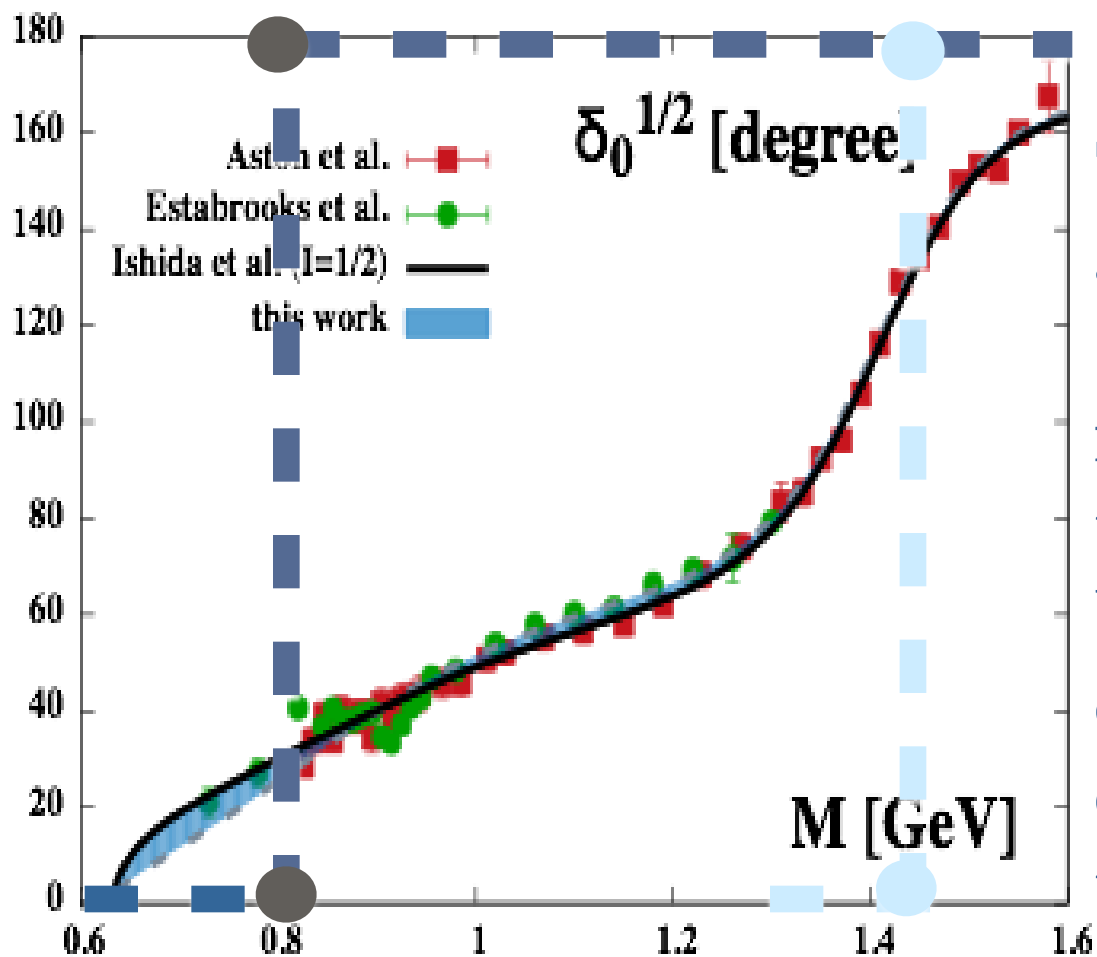
$$B(M) \approx M \frac{2M\gamma_{BW}}{(M^2 - M_0^2)^2 + M^2\gamma_{BW}^2}$$

for $\gamma_{BW} \rightarrow 0$

$$B(M) = \delta(M^2 - M_0^2) \quad \text{and}$$

$$P_{\pi K}^{int}(T) \approx P_{K^*}^{id}(T)$$

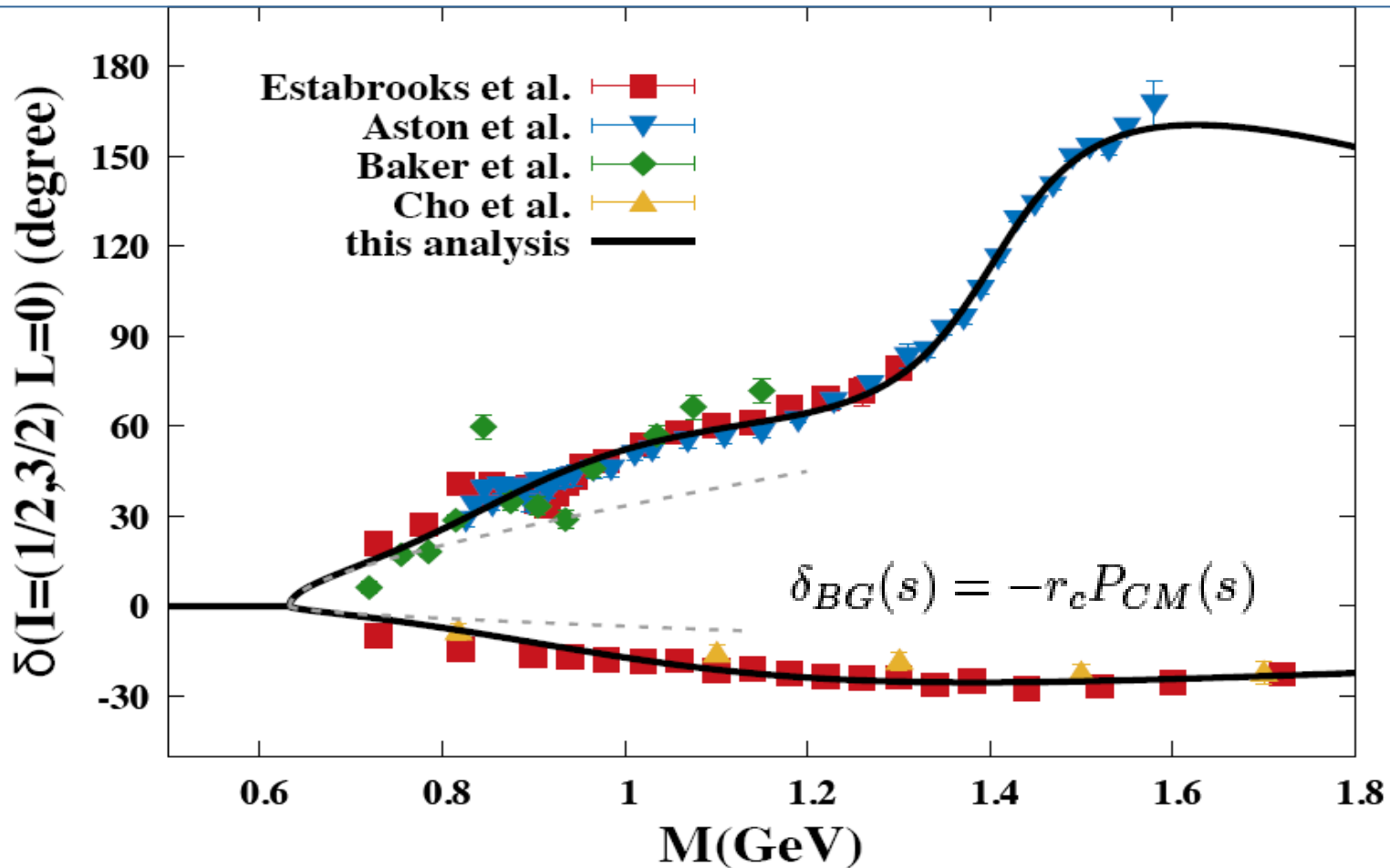
Experimental phase shift in S channel



The kappa resonance is a special example that:

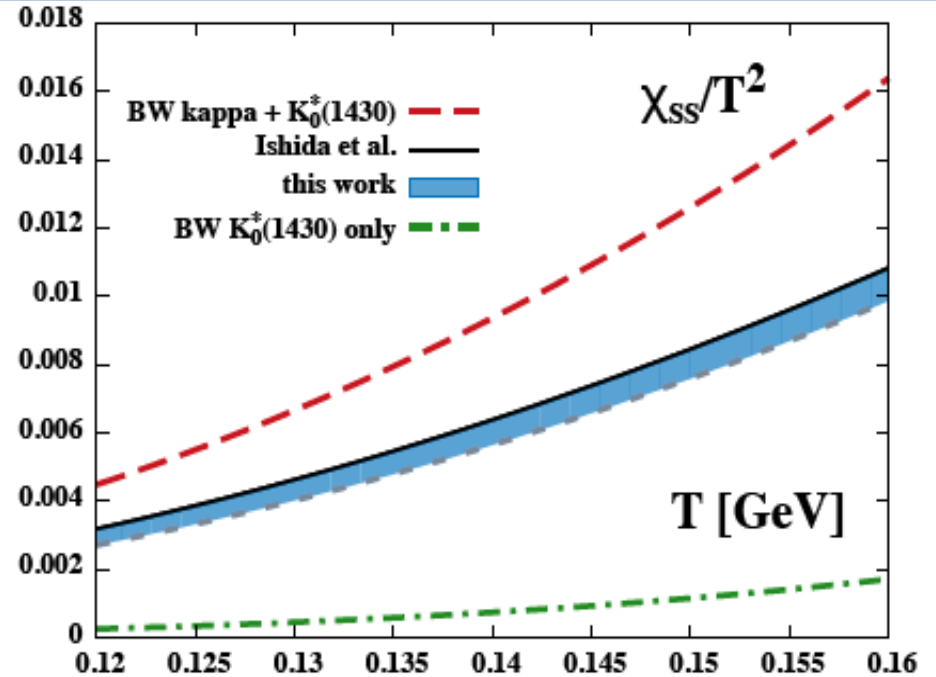
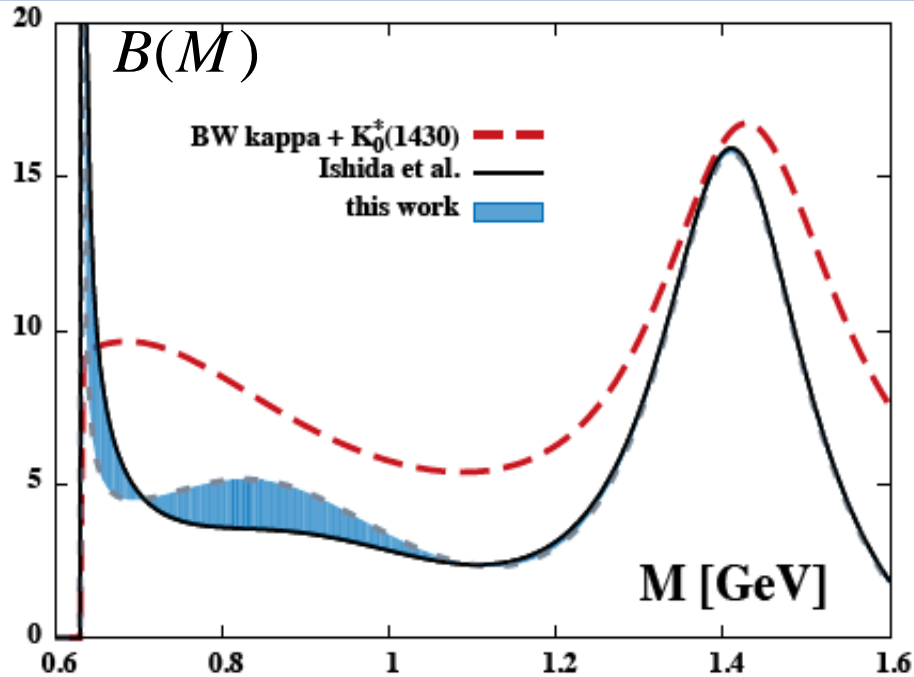
for broad resonances their contribution has to be taken with a special care, by considering the experimental or theoretically calculated phase shift

Non-resonance contribution- negative phase shift in S-wave channel



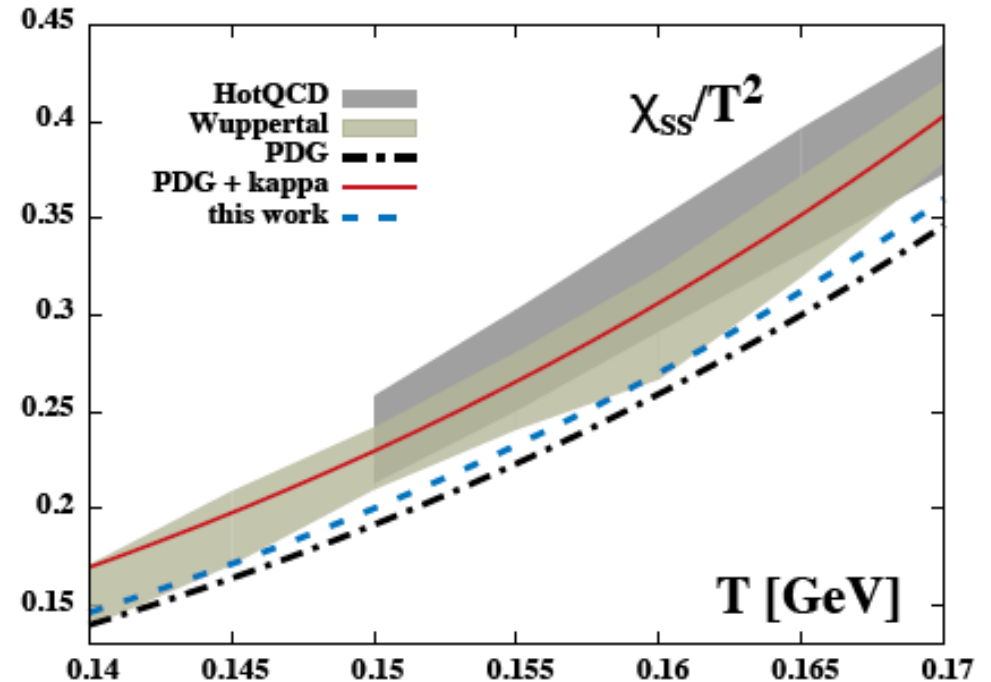
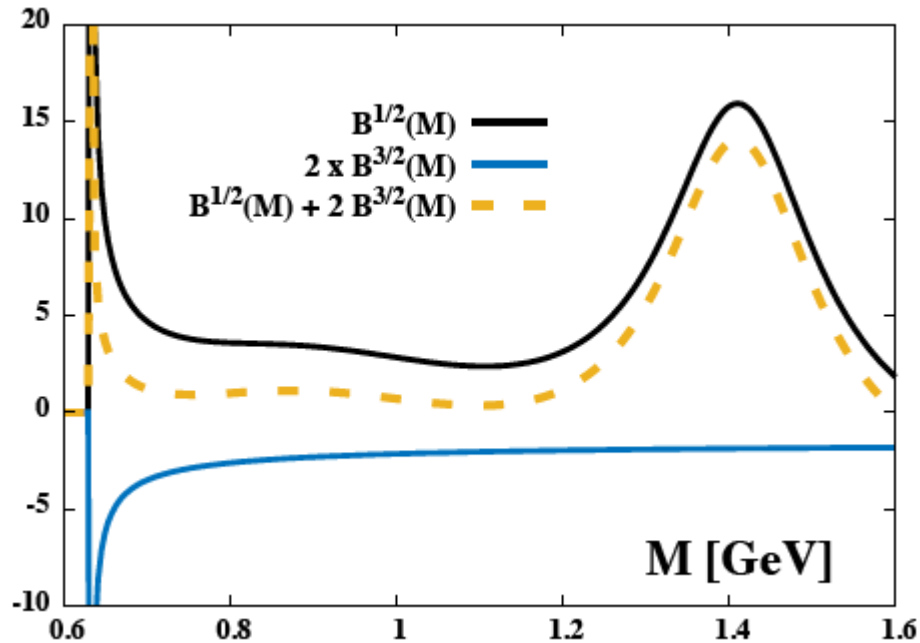
$$\delta_0^{1/2} = \delta_\kappa + \delta_{K_0^*} + \delta_{BG}. \quad \longrightarrow \quad B(M) = 2 \frac{d}{dM} \delta(M) \quad \longrightarrow \quad \chi_{SS}(T)$$

S-matrix approach to strangeness fluctuations



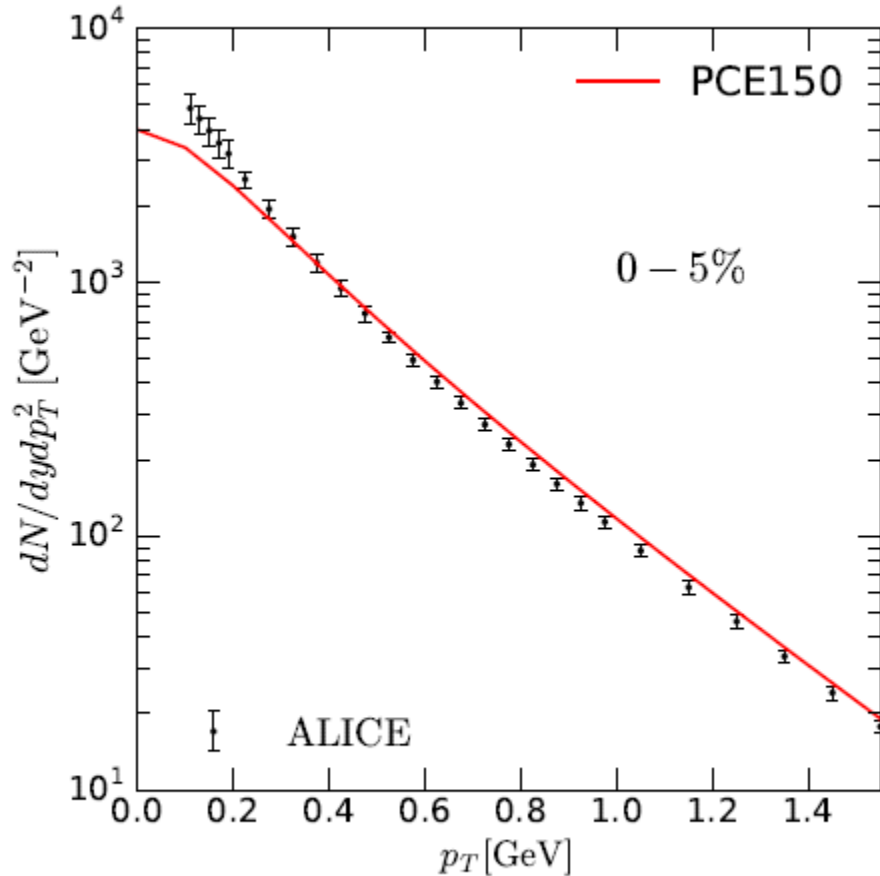
In the S-matrix approach essential reduction of the contribution of S-wave kappa relative to naive BW approach

S-matrix approach to strangeness fluctuations



In the S-matrix approach the contribution of S-wave kappa resonances to strangeness susceptibilities is small

Pion spectra in hydro calculations



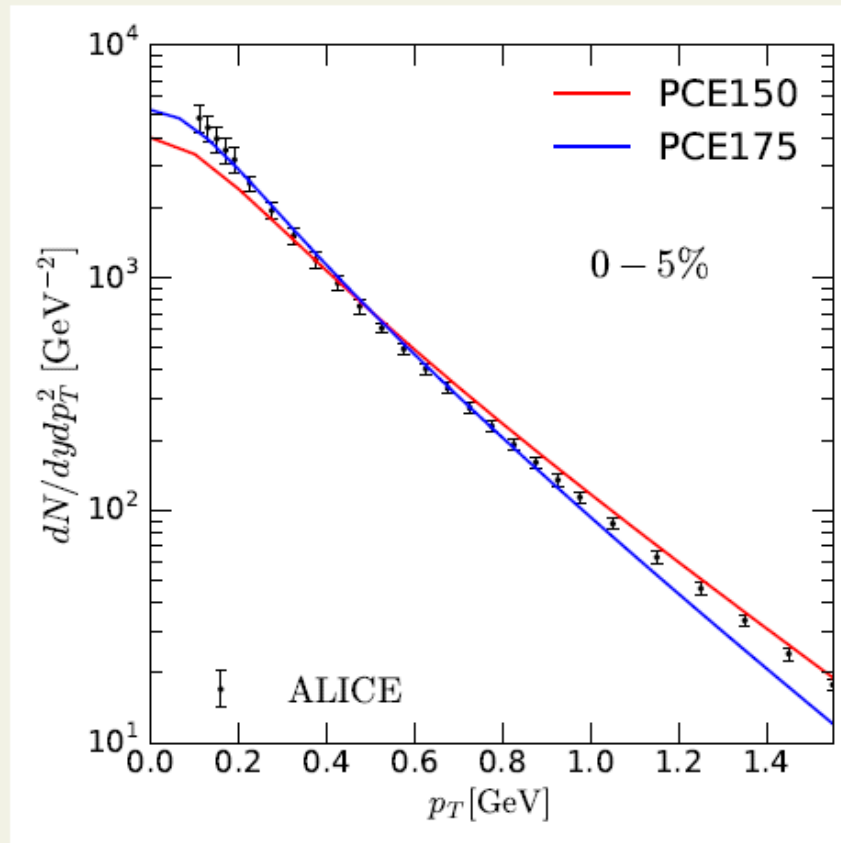
- viscous hydro
- initial state:
pQCD+saturation
- $\tau_0 \approx 0.2\text{fm}/c$

PCE150:
fit to π , K , p yields
no fit to spectrum

Continuous problem with
low p_t pion spectra in
hydro - calculations

Pion spectra in hydro calculations

Pion p_T spectrum at LHC



- viscous hydro
- initial state:
pQCD+saturation
- $\tau_0 \approx 0.2\text{fm}/c$

PCE150:
fit to π , K , p yields
no fit to spectrum

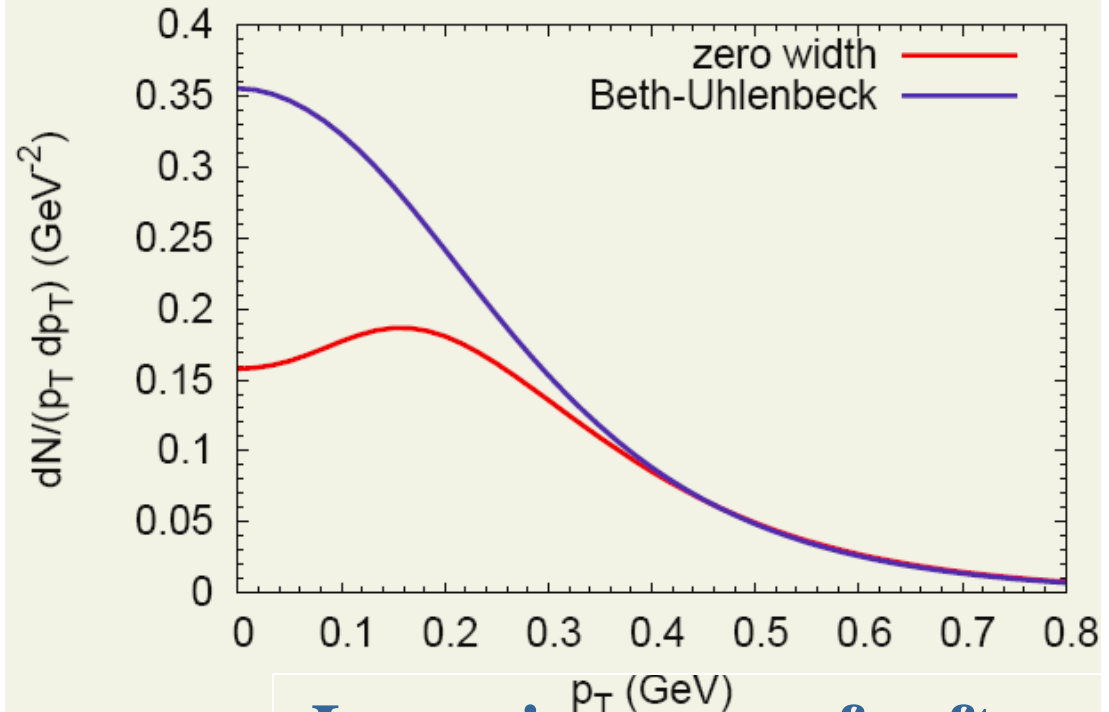
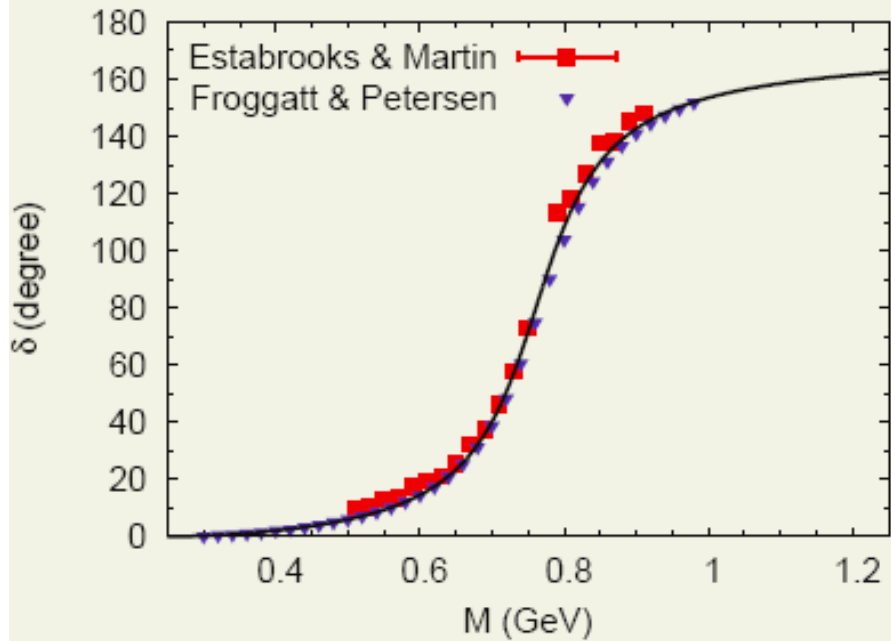
PCE175:
no fit to yields
fits the spectrum

©H. Niemi

Resonances are treated as point-like objects !!!!

S-matrix approach: Pion spectra

• $\pi\pi$ scattering, P-wave, i.e. ρ res



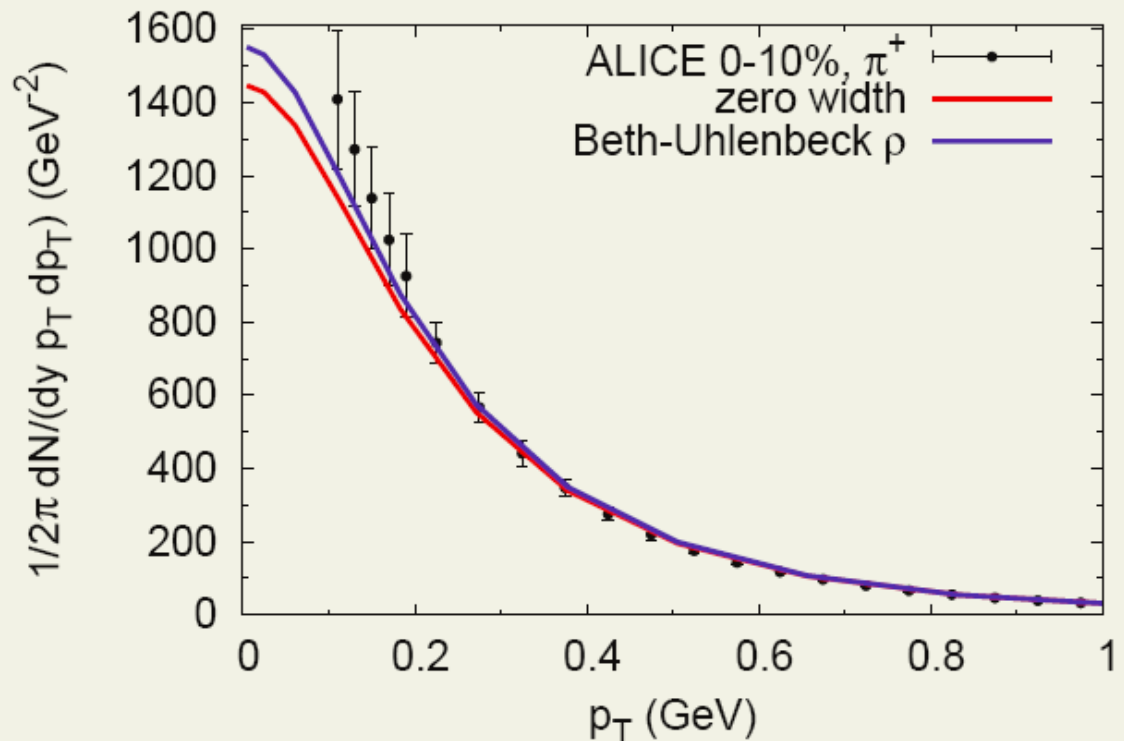
Large increase of soft pions obtained in the S-matrix approach

$$\frac{dN_{\pi}^{de}}{dy p_T dp_T d\phi} = V E_{\pi} \int dM_{\rho} \frac{1}{2\pi} \mathcal{B}(M_{\rho}) \times \frac{M_{\rho}}{2 p_{\pi} p_{CM}} \int_{E_{\rho}^{-}}^{E_{\rho}^{+}} dE_{\rho} E_{\rho} \frac{d\rho}{(2\pi)^3} f_{\rho}(E(M_{\rho}), T),$$

i) Pion production from an expanding fireball

Enhancement of soft pions from rho-decay with a correct treatment of resonance dynamics within S-matrix approach

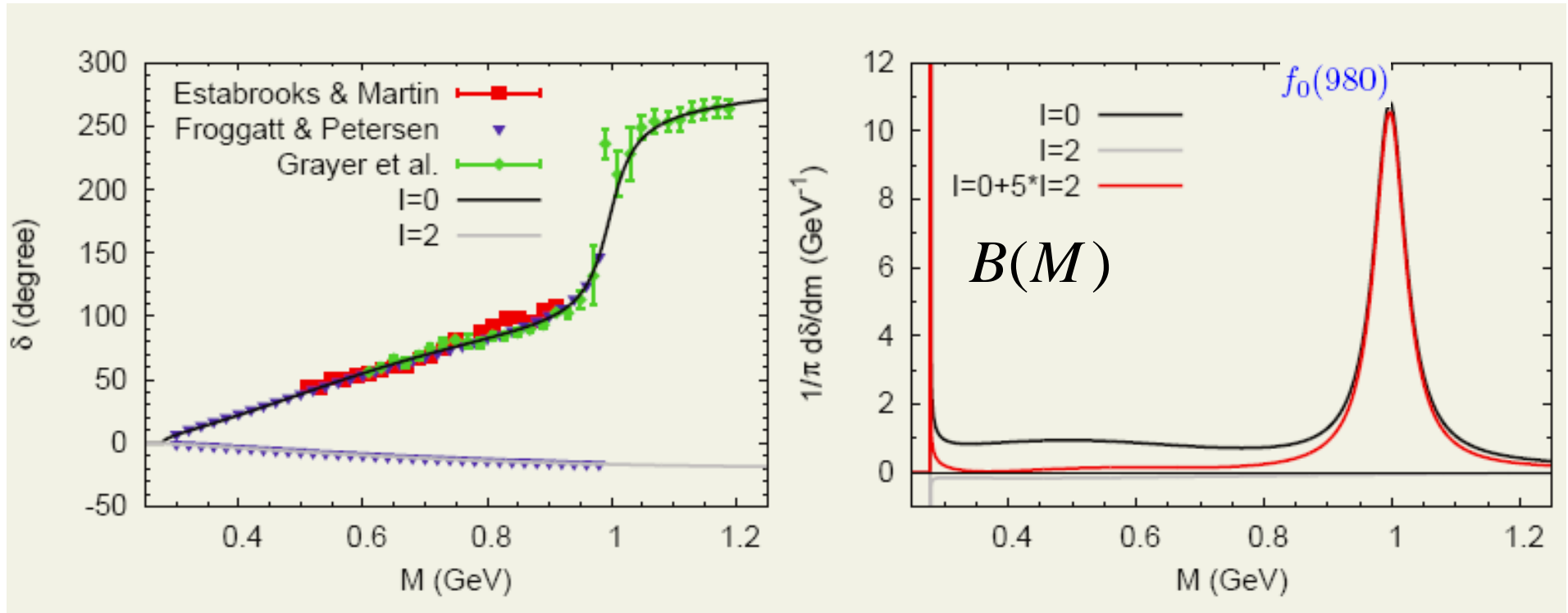
Pions from blast wave



- $\tau = 13.7$ fm
- $R = 10$ fm
- $v_{max} = 0.78$

- all resonances up to 2 GeV
- Beth-Uhlenbeck for rhos
- zero width for everything else

Pions from $\sigma(600)$ decay

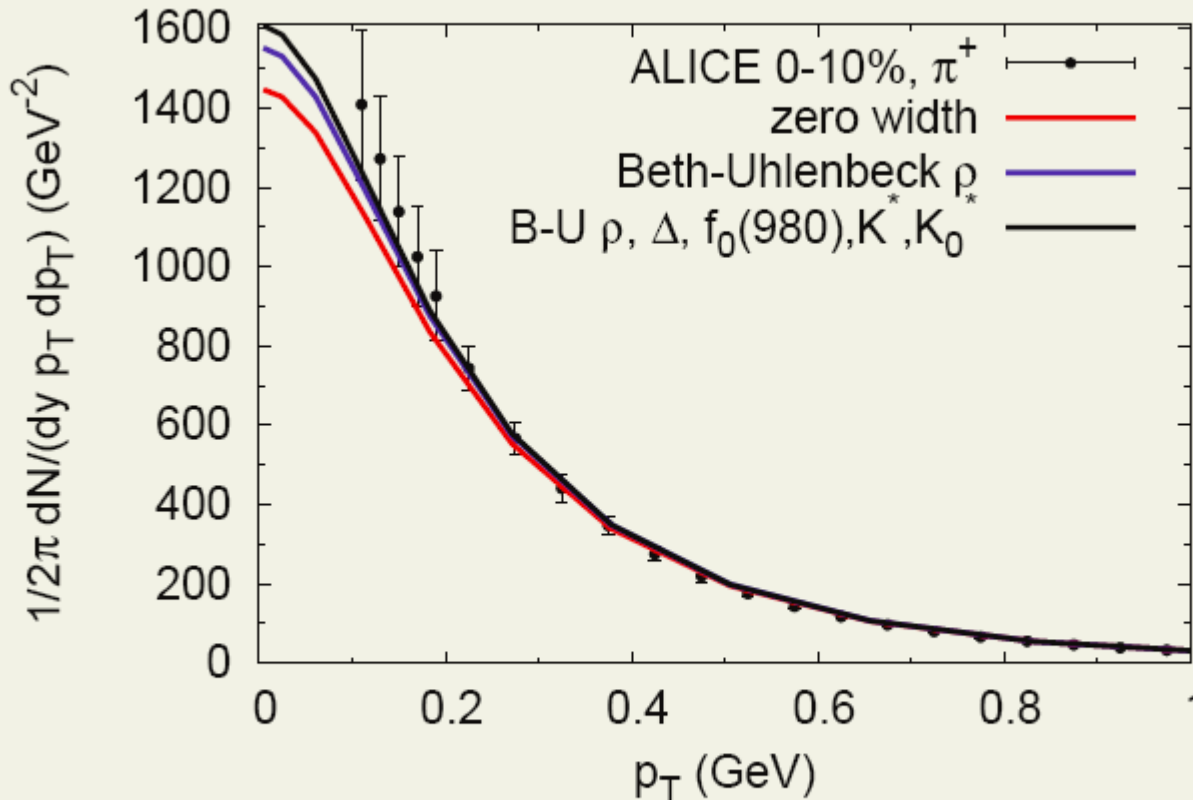


Large contribution from point-like sigma to soft pion spectra: **Negligible in the S-matrix approach**

Broniowski, Giacosa & Begun, PRC92, 034905 (2015)

ii) Pion production from an expanding fireball

Pions from blast wave



Clear enhancement of soft pions with only a few scattering channels treated within the S-Matrix approach, which accounts for resonance, and for non-resonance, repulsive interactions

- $\tau = 13.7 \text{ fm}$
- $R = 10 \text{ fm}$
- $v_{max} = 0.78$
- all resonances up to 2 GeV
- Beth-Uhlenbeck for $\rho, \Delta, f_0(980), K^*(892), K_0^*(1430)$
- zero width for everything else

Hagedorn's spectrum: parameters from the PDG data

P. M. Lo arXiv:1507.06398

- discrete mass spectrum

$$\rho(m) = \sum_i d_i \delta(m - m_i)$$

- The same information can be stored in the cumulant

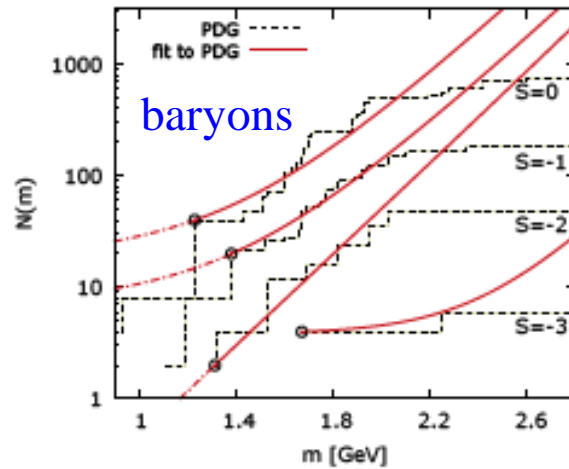
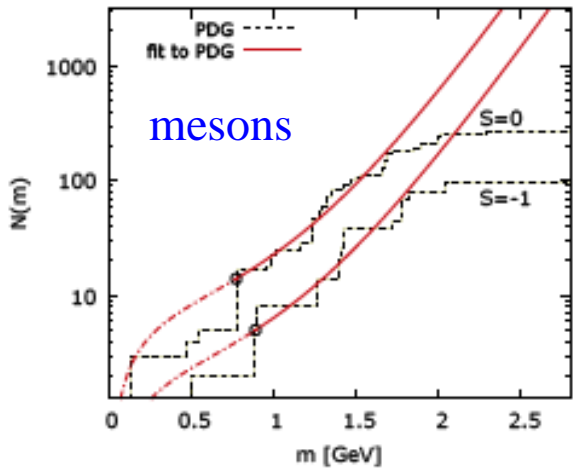
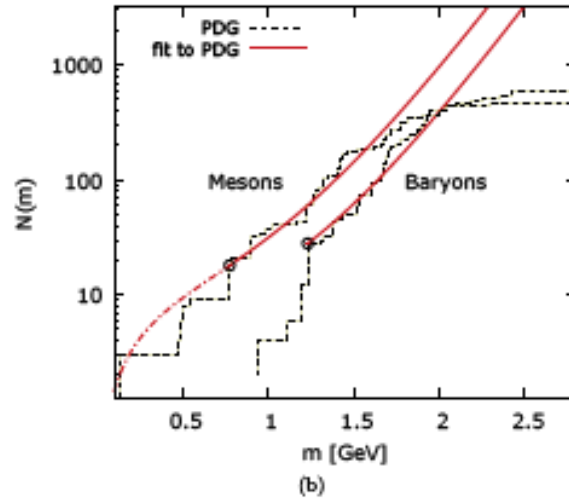
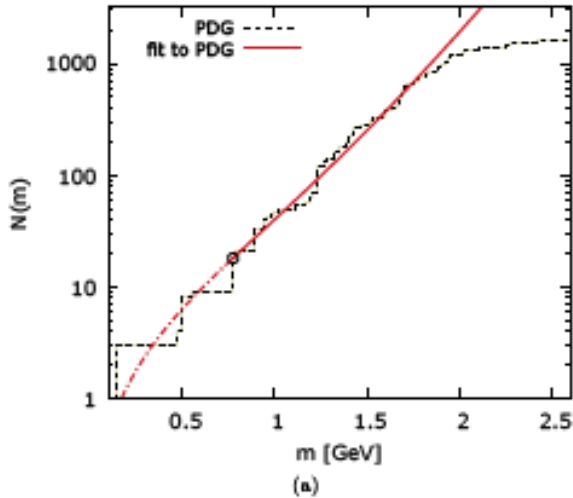
$$N(m) = \sum_i d_i \theta(m - m_i)$$

such that $\rho = \partial N / \partial m$

We use the following form for the mass spectrum and fit parameters to PDG

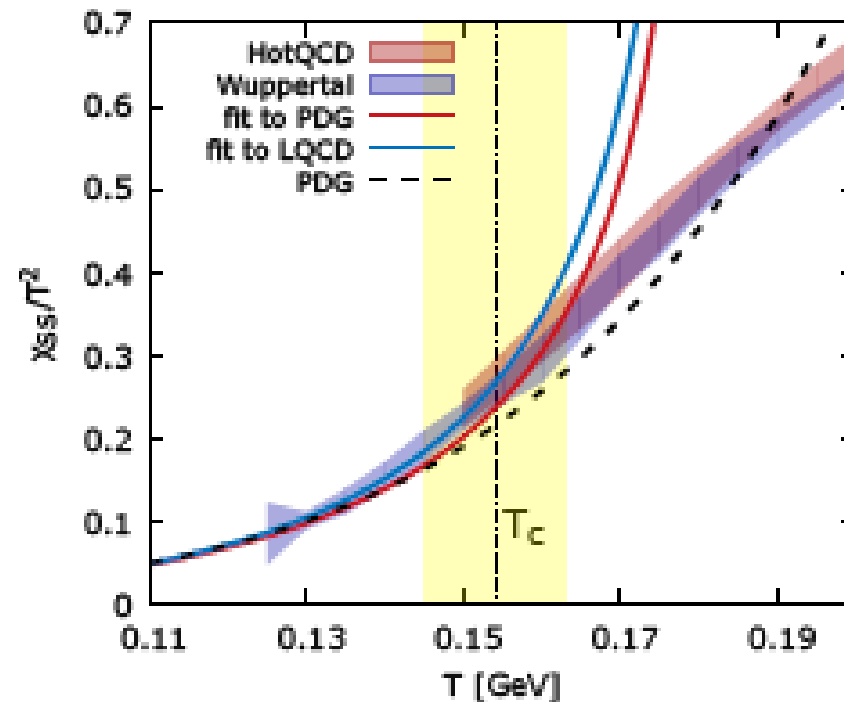
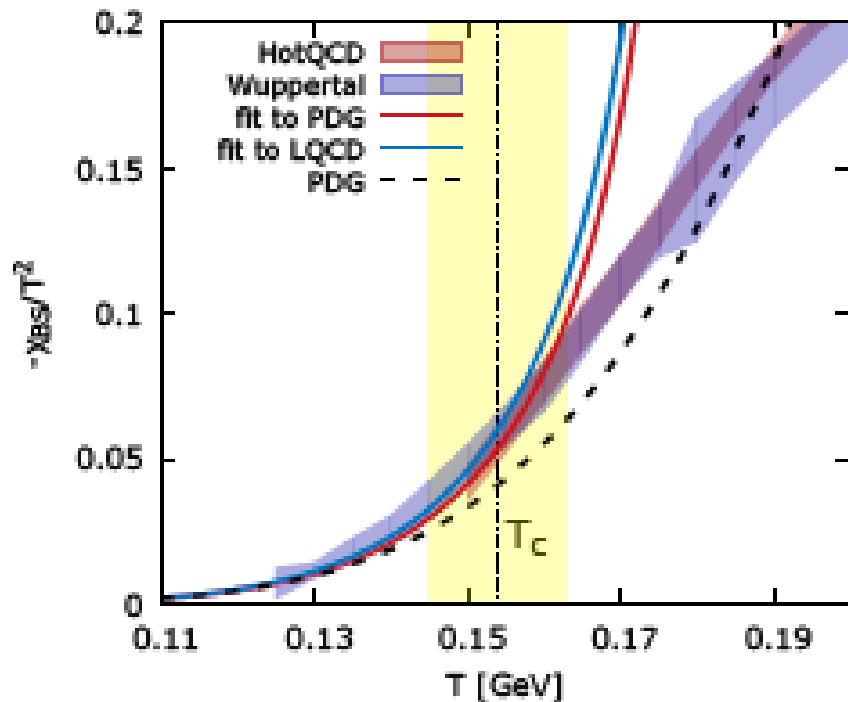
$$\rho^H(m) = \frac{A e^{m/T_H}}{(m^2 + m_0^2)^{5/4}},$$

$T_H = 0.18$ GeV common for all mesons and baryons in different sectors of quantum numbers



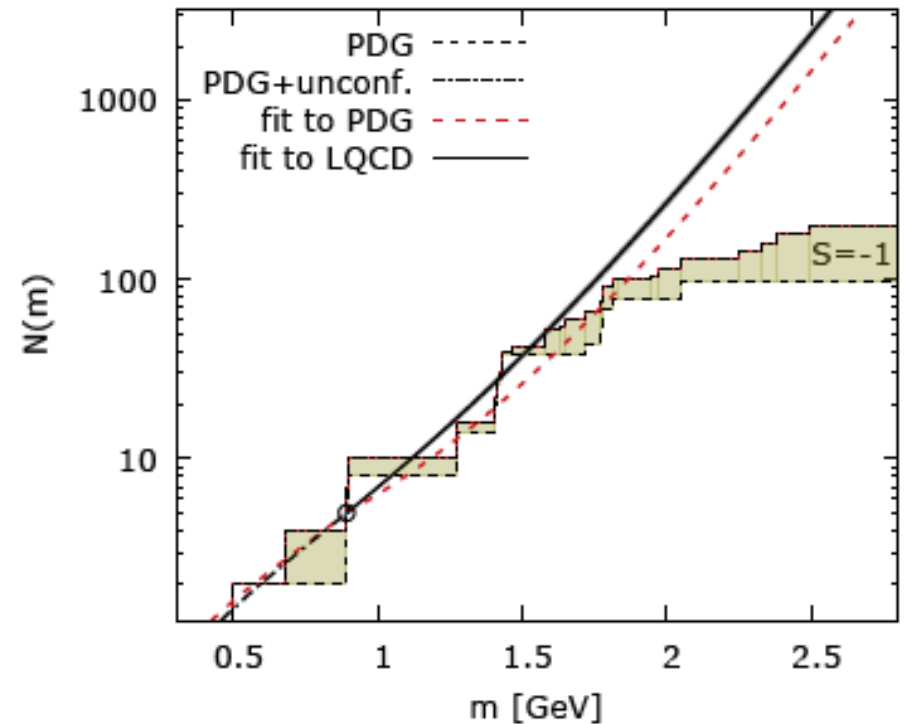
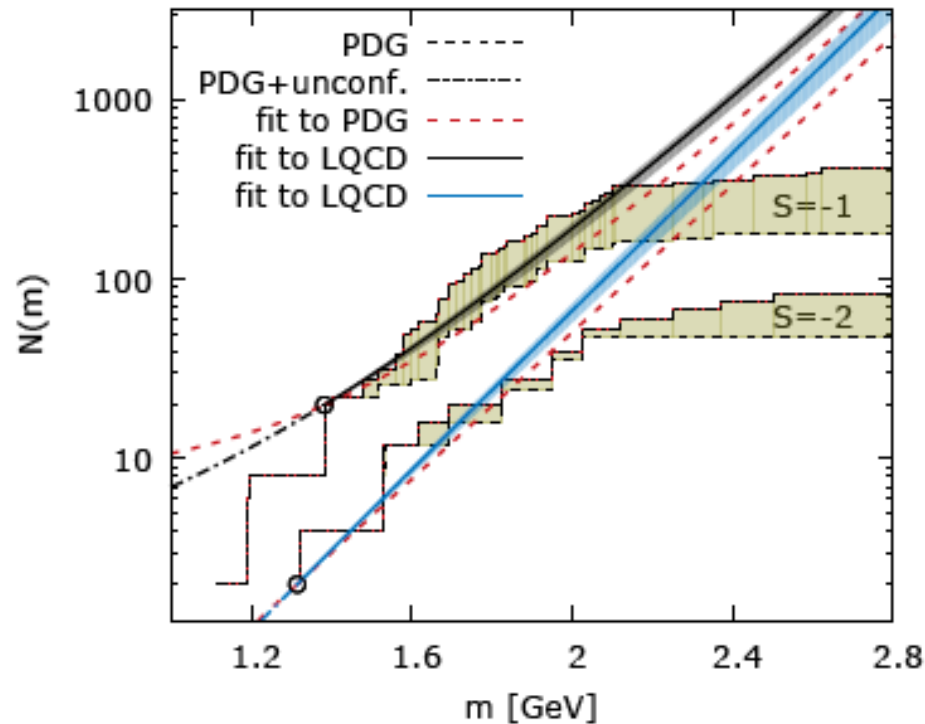
$$\rho(m) = \sum_{G.S.} d_i \delta(m - m_i) + \theta(m - m_x) \rho^H(m)$$

Hagedorn's continuum mass spectrum contribution to strangeness fluctuations



- Satisfactory description of LGT with asymptotic states from Hagedorn's spectrum fitted to PDG
- To find optimal results: extract $\rho^H(M)$ from LGT and compare with PDG that includes expected new states₂₆

Missing resonances in the PDG:



- In the strange baryon sector the optimal mass spectrum extracted from LGT is consistent with that expected and unconfirmed states in the PDG
- In the strange meson sector one expects new resonances with the mass $M < 2$ GeV

Conclusions

- The Hadron Resonance Gas (HRG) provide a very fair description of particle production yields in HIC from SIS up to LHC
- The HRG is also a good approximation of the QCD partition function in the hadronic phase
 - However, a more accurate description of the interaction contributions in HRG is needed, and can be done by using empirical scattering phase shifts within S-matrix approach, and accounting for missing resonances in the Hagedorn mass spectrum