Consider a target of particles of type $A$ with density $Q_A$ at rest (ideally dense, i.e., particles per volume = not mass density) and a beam of particles of type $B$ with density $Q_B$ coming at $A$ with velocity $v$. The vertices are $S_A$ and $S_B$.

The cross section should characterize a single scattering event. On the other hand, it is intuitively clear that the number of scattering events is proportional to $A_S A_B A_{S_B}$.

**Definition:**

$$
\sigma = \frac{\text{number of scattering events}}{\text{scattering cross section}}
$$

One can express the scattering cross section also in an alternative way:

Introduce the time $t$ which the beam needs to pass by:

$$
\tau = \frac{S_B}{v}
$$

the number of target particles is $Q_A A_S A$;

the scattering rate $R$ is the number of scattering events per target particle per time:

$$
R = \frac{\text{number of scattering events}}{Q_A A_S A \tau}
$$
\[ \text{flux} = \pi s_{y} = \frac{\pi}{2} s_{x} = \text{number of incoming particles} \]

\[ \pi s_{y} = \text{area of scattering surface} \]

\[ \pi s_{x} = \text{number of particles in scattering direction} \]

\[ s_{y} \]

by S matrix

\[ \text{probability of collision} \]

Initially the cross section is related to the probability that collision happens. We consider a reaction where two initial particles (of type A and B respectively) react into a final particle. Typically, the details concern the masses of the final states. We denote them by \( p_{f}, \delta \rightarrow \alpha \). The corresponding energy is \( E_{f} = \sqrt{p_{f}^{2} - m_{\alpha}^{2}} \). Suppose the mass of the \( i \) th final-state particle. We characterize the initial state by two wave packets for the particles of type A and B.

\[ |\psi_{A}\rangle = \int d\tau_{A} \frac{1}{\sqrt{2\pi} \Delta \tau_{A}} \varphi_{A}(\tau_{A}) e^{i p_{A} \tau_{A}} \]

\[ \text{wave packet} \]

\[ \text{correspondingly for } |\psi_{B}\rangle \]

The respective central momentum of the wave packet is \( p_{A} \) or \( p_{B} \).

\[ |\psi_{A}\rangle \sim \exp \left\{ -\left( \frac{p_{A}^{2} - p_{A'}^{2}}{8\Delta p_{A}^{2}} \right) \right\} \text{exp} \left\{ i p_{A} \tau_{A} \right\} \]

\[ \text{wave packet} \]

\[ \text{wave packet} \]

\[ \text{wave packet} \]
\[
\psi_\kappa(x_1, x_2) = \sum \frac{d^2 \rho_{x_1}^*}{(2\pi)^2} \frac{d^2 \rho_{x_2}^*}{(2\pi)^2} \frac{\langle \mathcal{E}(x_1) \rangle \langle \mathcal{E}(x_2) \rangle}{\langle \mathcal{E}(x_1) \rangle \langle \mathcal{E}(x_2) \rangle} \psi(a_{x_1}^* \psi_{x_2}^* \psi_{x_2})
\]

\[
= \left( \frac{d^2 \rho_{x_1}^*}{(2\pi)^2} \frac{d^2 \rho_{x_2}^*}{(2\pi)^2} \right) \left| \psi_{x_1}(x) \right|^2, \quad \text{same for } B
\]

Note: We use wave packets for the initial state and not plane waves to ensure a proper normalization.

The wave packets are prepared long before the reaction happens. The final moments are measured much later than the reaction has happened. It is reasonable that the initial particles scatter into a particle with arbitrary momenta \( p_i \), \( i = 1, \ldots, n \), as given by (in the Heisenberg picture)

\[
P(A, B \rightarrow 1, 2, \ldots, n) = \lim_{t \rightarrow \infty} \left( \frac{\langle d^2 \rho_{x_1}^* \rangle}{(2\pi)^2} \frac{\langle d^2 \rho_{x_2}^* \rangle}{(2\pi)^2} \right) \left| \psi_{x_1}(x_1) \right|^2 \left| \psi_{x_2}(x_2) \right|^2 \left| \psi_{x_3}(x_3) \right|^2 \left| \psi_{x_4}(x_4) \right|^2 \left| \psi_{x_5}(x_5) \right|^2 \left| \psi_{x_6}(x_6) \right|^2 \left| \psi_{x_7}(x_7) \right|^2 \left| \psi_{x_8}(x_8) \right|^2
\]

If we fix the momenta to small volumes \( d^2 \rho_{x_1} \) around \( \mathbf{p}_f \), the differential probability is

\[
\frac{dP}{d^2 \rho_{x_1}} = \left( \frac{\langle d^2 \rho_{x_1}^* \rangle}{(2\pi)^2} \frac{\langle d^2 \rho_{x_2}^* \rangle}{(2\pi)^2} \right) \left| \psi_{x_1}(x_1) \right|^2 \left| \psi_{x_2}(x_2) \right|^2 \left| \psi_{x_3}(x_3) \right|^2 \left| \psi_{x_4}(x_4) \right|^2 \left| \psi_{x_5}(x_5) \right|^2 \left| \psi_{x_6}(x_6) \right|^2 \left| \psi_{x_7}(x_7) \right|^2 \left| \psi_{x_8}(x_8) \right|^2
\]

with

\[
S_{x_1} = \left( \frac{\langle d^2 \rho_{x_1}^* \rangle}{(2\pi)^2} \frac{\langle d^2 \rho_{x_2}^* \rangle}{(2\pi)^2} \right) \left| \psi_{x_1}(x_1) \right|^2 \left| \psi_{x_2}(x_2) \right|^2 \left| \psi_{x_3}(x_3) \right|^2 \left| \psi_{x_4}(x_4) \right|^2 \left| \psi_{x_5}(x_5) \right|^2 \left| \psi_{x_6}(x_6) \right|^2 \left| \psi_{x_7}(x_7) \right|^2 \left| \psi_{x_8}(x_8) \right|^2
\]

and

\[
S_{x_2} \text{ defined from } S_{x_1}, \quad \mathbf{k} \rightarrow \mathbf{k}'}
We know already that the expectation value is governed by a unitary operator

$$\hat{S} = \hat{S}_1 \hat{S}_2 \ldots \hat{S}_n$$

with a unitary operator $S$ in each space, i.e., $S^2 = 1$. $S$ is called "$S$ matrix".

If no interaction happens, $S$ is just unity (then all matrix elements vanish for $n+2$). To include time interaction we introduce the "$T$ matrix"

$$S = T + i\Gamma$$

From $S^* S = 1$ one obtain

$$-i\Gamma T^* + iT^* \Gamma = 0$$
$$\Rightarrow -2\omega^2 T = i\Gamma T^*$$

unitarity relation

With $\omega = \frac{1}{2i}$

In principle $S$ and $T$ can be calculated if the Hamiltonian and in particular its interaction part is known.

In practice one often uses perturbation theory to determine $T$ approximately (see, e.g., the Section on quantum field theory). In the following we arrange that $T$ in known and relate it to the scattering cross section.
In an elementary reaction (in the absence of external fields or additional incident particles), total energy and total momentum are conserved. It is useful to define the incident matrix element or "Born matrix element" $M$ via

$$
\langle p_1, \ldots, p_n | T | k_a, k_2 \rangle = \left(2\pi\right)^n \delta(k_a^* \cdot k_2 - \frac{E_{p_1}}{2} p_1) M(k_a, k_2 \rightarrow p_1, \ldots, p_n)
$$

Now, the cross section we want that reaction take place, therefore it is sufficient to keep $i T$ and drop $M$ (or $S = M + i T$) in the calculation of the reaction probability

$$
\sigma = \left| \frac{1}{\sqrt{2\pi}} \int \frac{d^3 p_1}{(2\pi)^3 E p_1} \frac{d^3 p_2}{(2\pi)^3 E p_2} \frac{d^3 p_3}{(2\pi)^3 E p_3} \ldots \frac{d^3 p_n}{(2\pi)^3 E p_n} \right|^2 \left| S(k_a^* \cdot k_2 - \frac{E_{p_1}}{2} p_1) M(k_a, k_2 \rightarrow p_1, \ldots, p_n) \right|^2
$$

Now we have to specify our wave packets and make contact with the cross section. Usually we are satisfied with initial particles with fixed momenta, i.e. in plane waves, but we have to use wave packets to obtain normalized/formalizable data.

We use narrow wave packets such that one can replace $k_a, k_2$

$$\begin{align*}
\text{by} \quad p_a, \quad p_2 \quad \text{whenever possible,}
\end{align*}$$

\[\Rightarrow \quad \delta(\vec{k}_a \cdot \vec{p}_2 - \frac{E_{p_1}}{2} \vec{p}_1) \delta(E_{p_1} - \frac{E_{p_2}}{2} \vec{p}_2) M(p_a, k_2 \rightarrow p_1, \ldots, p_n) M^*(p_a, \vec{p}_2 \rightarrow \vec{p}_1, \ldots, \vec{p}_n)
\]

\[\Rightarrow \quad \delta(\vec{k}_a^* \cdot \vec{p}_3 - \frac{E_{p_1}}{2} \vec{p}_1) \delta(\vec{k}_2^* \cdot \vec{p}_4 - \frac{E_{p_2}}{2} \vec{p}_2) \ldots \left| M(p_a, p_3 \rightarrow \vec{p}_1, \ldots, \vec{p}_n) \right|^2
\]

Besides, the width of the wave packets is the hope to that the cross section does not depend on details of the shape of the wave packets.
\[ d\mathcal{P} = \left( \frac{1}{2\pi} \frac{\delta\mathbf{k}_3}{(2\pi)^2 2E_3} \right) \mathcal{A}_A(\mathbf{k}_A) \mathcal{A}_3^*(\mathbf{k}_3) \mathcal{A}_3(\mathbf{k}_3) \mathcal{A}_B^*(\mathbf{k}_B) \delta(\mathbf{k}_A + \mathbf{k}_3 - \mathbf{k}_B - \mathbf{E}_3) \, d\omega \]

where

\[ d\omega = \left( \frac{1}{2\pi} \frac{d\omega}{(2\pi)^2 2\omega} \right) (2\pi)^3 \delta(\mathbf{p}_4 - \mathbf{p}_5 - \mathbf{p}_6 - \mathbf{p}_7) \left| M(\mathbf{p}_4 \mathbf{p}_5 \mathbf{p}_6 \mathbf{p}_7) \right|^2 \]

does not depend on the details of the wave packets.

Introducing

\[ \psi_A(x) = \int \frac{d^3k}{(2\pi)^3 2E_A} \mathcal{A}_A(k) e^{i\mathbf{k}\cdot\mathbf{x}} \]

\[ (\mathbf{x} = \mathbf{x}_A + \mathbf{x}_3) \quad \text{and} \quad \text{same for } B \]

and rewriting

\[ (2\pi)^3 \delta(\mathbf{k}_A + \mathbf{k}_3 - \mathbf{k}_B - \mathbf{E}_3) = \int d^3k e^{i\mathbf{k}\cdot(\mathbf{x}_A + \mathbf{x}_3 - \mathbf{x}_B - \mathbf{E}_3)} \]

one gets

\[ d\mathcal{P} = \int d^4x \left| \psi_A(x) \right|^2 \left| \psi_B(x) \right|^2 d\omega \]

Next, we have to get closer to the situation for which the cross section is defined.

The wave packets \( \psi_A \) and \( \psi_B \) are strongly peaked around \( \mathbf{x}_A \) and \( \mathbf{x}_B \), respectively. Thus, the function \( \psi_A \) and \( \psi_B \) have large extensions in space (and time). It is typically the extension of target and beam, respectively. The density, e.g. in the target,

is given by

\[ g_A = \frac{\left| \psi_A(x) \right|^2}{\int d^4x \left| \psi_A(x) \right|^2} \]

name for B.
Note that the density is normalized, i.e., once we consider one target (and one projectile) particle.

We assume now that the densities \( f_a \) and \( f_b \) are (more or less) constant within the wave packets except for the edges.

\[
\begin{align*}
&\psi_a^2(x) \approx 0 \quad \text{outside of wave packet} \\
&\psi_b \approx \psi_b(\text{inside}) \quad \text{inside of wave packet}
\end{align*}
\]

and some for \( |\psi_b|^2 \).

In addition, we assume that during the scattering time the projectile wave function completely covers the target. Otherwise, one would not fully account for the target properties.

A classical example from two-body scattering might illustrate this point.

**Situation 1:**

![Situation 1 diagram]

**Situation 2:**

![Situation 2 diagram]

Only in situation 2 the whole target is occupied. Finally, we assume that \( \beta_a \) and \( \beta_b \) are anti-parallel (classical)

With these assumptions, one can calculate

\[
\mathcal{D}P = \int d\lambda \frac{|\psi_a(x)|^2}{|\psi_b(x)|^2} \frac{|\psi_b(x)|^2}{|\psi_a(x)|^2} d\lambda = \int d\lambda |\psi_b(\text{inside})|^2 \frac{|\psi_b(\text{inside})|^2}{|\psi_a(x)|^2} d\lambda
\]

noting that only the wave packet overlap.
3) Scattering rate $\frac{d\sigma}{d\omega} = \frac{1}{\pi} \left| \psi_2^*(\text{in蔵}) \right|^2 \int d^3k \left| \psi_1(k) \right|^2 \Delta n^2$.

$$\Delta n = \frac{1}{12} \left[ \frac{1}{|V_1 - V_8|} \right] \int d^3x \left| \psi_1^*(x) \right|^2 \int d^3y \left| \psi_2 (y) \right|^2 d\omega$$

3) Have to determine normalization integral

$$\int d^3k \left| \psi_1^*(k) \right|^2 = \int d^3k \frac{d^3k'}{2E_1} \frac{d^3k''}{2E_2} \frac{d^3k'''}{2E_3} f_k(z) f_{k'}(z') f_{k''}(z'') f_{k'''}(z''')$$

\[= \frac{1}{2E_1} \int d^3k \frac{d^3k'}{2E_2} \left| f_k(z) \right|^2 \]

\[= \frac{1}{2E_2} \int d^3k \frac{d^3k'}{2E_3} \left| f_k(z) \right|^2 \]

\[= \frac{2E_3}{2E_2} \int d^3k \left| f_k(z) \right|^2 \]

Finally

$$\Delta n = \frac{1}{4E_1 E_2} \frac{1}{|V_1 - V_8|} \left( \frac{1}{2E_3 E_4} \right)^2 \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \left| f_k(z) \right|^2$$

$\Delta n$ vanishes
The cross section is defined originally for the target at rest. It has the dimension of an area. The intuitive picture is an area perpendicular to the beam direction. Scattering happens, if the distance between the scattering partners is small, then \( \sqrt{S} \).

With respect to Lorentz transformation, it does not change if one boosts to frame, which moves along the beam axis. \( \mathbf{v} \) is the same for a target at rest, by beam drift, 1) center of mass frames (\( \mathbf{p}_a = \mathbf{p}_b = 0 \))

Note that target and beam appear in completely symmetric form in cross section formula.

In the frames where \( \mathbf{v} \) does not change, i.e., for \( \mathbf{p}_a = (\mathbf{p}_a) \) \( \mathbf{p}_a \parallel \mathbf{p}_b \), one can write down a Lorentz invariant expression for \( \sigma \):

\[
\sigma = \left( m_4^2 + m_5^2 + 2 \mathbf{p}_4 \cdot \mathbf{p}_b \right)
\]

\[
= m_4^2 + m_5^2 + 2 E_4 E_b - 2 E_4 V_a E_b v_8
\]

\[
\equiv \left( s - m_4^2 - m_5^2 \right) \left( s - 2 m_4 m_5 \right)
\]

\[
= \left( 2 E_4 E_b \right) \left[ (1 - V_a v_4) - 2 m_4 m_5 \right] \left[ (1 - V_a v_4) + 2 m_4 m_5 \right]
\]

\[
+ 4 E_4^2 E_b \left[ 1 - V_a v_4 \right] + 4 m_4^2 m_5^2 - 4 m_4 m_5 \left( 1 - V_a v_4 \right) \left( 1 - V_a v_4 \right)
\]

\[
\equiv \frac{4 E_4 E_b \left[ 1 - V_a v_4 \right]}{\sqrt{s - (m_4^2 + m_5^2) \left[ s - (m_4^2 + m_5^2) \right]}}
\]

\[
= 4 E_4 E_b \left[ v_4 + v_5 \right] \left( V_a v_4 \right) - 4 E_4 E_b \left[ v_4 + v_5 \right] \left( V_a v_4 \right)
\]

\[
\equiv 2 \sqrt{(s - (m_4^2 + m_5^2) \left[ s - (m_4^2 + m_5^2) \right])} = 4 E_4 E_b \left[ v_4 + v_5 \right]
\]
Consider two-body scattering:

\[ P_1, P_2 \rightarrow P_3, P_4 \]

1. \textit{Resultant kinematic case of scattering:}

It is most economic to work in the center-of-mass frame (since center-of-mass frame fixes motion in the absence of external forces):

\[ \vec{P}_3 + \vec{P}_4 = \vec{P}_1 + \vec{P}_2 = 0 \]

scattering is specified by momenta \(|\vec{P}_3| = |\vec{P}_4|\)

and scattering angle \(\theta\):

2. \textit{Five relevant variables:}

3. \textit{General consideration:}

the quantity which we want to calculate is \textit{in excess moment} 

\[ P_1, P_2 \rightarrow P_3, P_4 \]

\[ P_3 = P_1 + P_2 + P_3 \]

\[ P_4 = P_3 \]

\[ P_2 = P_1 + P_4 \]

\[ \text{Hence, } P_1, P_2 \text{ and } P_3 \text{ are energy-momentum conservation.} \]

\[ P_4^2 = m_3^2 \quad \Rightarrow \quad (P_1 + P_2 + P_3)^2 = m_4^2 \quad \{ \text{five relevant variables} \} \]
\[ s^2 = (p_1 + p_2)^2 = (p_3 + p_4)^2 \]

\[ \lambda^2 = (p_3 - p_4)^2 = (p_2 - p_1)^2 \]

\[ m_1^2 = (p_1 - p_2)^2 = (p_3 - p_4)^2 \]

They are related by

\[ s^2 + \lambda^2 = (p_1 + p_2)^2 + (p_3 - p_4)^2 = 2m_1^2 + 2m_2^2 + 2m_3^2 + 2m_4^2 \]

\[ = (m_1^2 + m_2^2 + m_3^2 + m_4^2)(p_1 - p_2 + p_3 - p_4) \]

\[ = m_1^2 + m_2^2 + m_3^2 + m_4^2 \]

\[ \lambda^2 = m_1^2 + m_2^2 + m_3^2 + m_4^2 \]

\[ \Rightarrow \lambda = m_1 \]

We can use, e.g., \( s \) and \( \lambda \) as independent variables.

In the center of mass frame we have

\[ s = (p_1 + p_2)^2 = (E_1 + E_2)^2 = \left( \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2} \right)^2 \]

\[ \Rightarrow p_1 = 0 \]

\[ \Rightarrow E_1 = m_1 \]

\[ \Rightarrow \sqrt{s} \text{ is total energy and } |p_1| \text{ can be expressed in terms of } s \text{ and vice versa.} \]

\[ \lambda = (p_3 - p_4)^2 = m_3^2 + m_4^2 - 2E_3E_4 + 2|p_3||p_4| \cos \theta \]

\[ \text{Since } s = (p_3 + p_4)^2 = \left( \sqrt{m_3^2 + p_3^2} + \sqrt{m_4^2 + p_4^2} \right)^2 \]

\[ \Rightarrow \frac{p_3}{p_4} = \frac{p_3}{p_4} \]

\[ \Rightarrow |p_3| = |p_4| \]

\[ \Rightarrow \text{we can express } |p_3| \text{ and } E_3 \text{ in terms of } s \]

\[ \Rightarrow \text{we can express } \theta \text{ in terms of } s \text{ and } \lambda. \]
Suppose $M$ is just a constant

\[ dS = \frac{1}{2E_{a}E_{b}} \frac{d^3p_a}{(2\pi)^3} \frac{d^3p_b}{(2\pi)^3} \delta(p_a + p_b - p_c - p_d) \ \frac{M^2}{M^2} \]

\(\text{for calculation of total cross section one has to determine the two-body plane space} \)

\[ T_2 = \frac{\int \frac{d^3p_a}{(2\pi)^3} \frac{d^3p_b}{(2\pi)^3}}{2E_a E_b} \delta(p_a + p_b - p_c - p_d) \]

This quantity is very important, i.e. it has the same value in any frame.

\(\text{Center of mass frame:} \)

\[ E_1 + E_2 = \sqrt{s}, \quad \mathbf{p}_1 = \mathbf{p}_2 = 0 \]

\[ M_2 = \frac{\gamma^3}{(2\pi)^3} \int \frac{d^3p_3}{2E_3} \frac{d^3p_4}{2E_4} \delta(E_3 + E_4 - \sqrt{s}) \delta(p_3 + p_4 - p_5 - p_6) \]

\[ = \frac{\gamma^3}{(2\pi)^3} \int \frac{d^3p_3}{2E_3} \frac{1}{2E_4} \delta(E_3 + E_4 - \sqrt{s}) \]

\[ \delta(p_3 + p_4 - p_5 - p_6) \]

\[ (E_3 - E_3 = E_3 - E_4) \]

The integral only depends on $|\mathbf{p}_3|$

To use spherical coordinates

\[ M_2 = \frac{\gamma^3}{(2\pi)^3} \int \cos \theta \ d\theta \sin \theta \ d\phi \ d^3p_3 \ d^3p_4 = \frac{E_3 E_4}{E_3 E_4} \delta(E_3 + E_4 - \sqrt{s}) \]

Instead of integrating over $p_3$ one might integrate over $E_3$

\[ E_3 = m_3^2 + p_3^2 \Rightarrow E_3 = E_3 - p_3 \ d^3p_3 \]

\[ E_{4,3} = \sqrt{m_4^2 + p_4^2} = \sqrt{m_4^2 + E_4^2 - m_3^2} \]

\[ T_2 = \frac{1}{(2\pi)^3} \int d^4p_3 \ d^4p_4 = \frac{1}{E_{4,3}} \delta(E_3 + E_4 - \sqrt{s}) \]
The $S$-function depends on the following values:

1. $E_3 - E_1 - E_2 = 0$
2. $E_1 + E_2 = E_4$
3. $(E_1 - E_2)^2 = m_1^2 - m_2^2 = E_3^2$
4. $E_1 - 2E_2 + E_3 = m_1^2 + m_2^2$
5. $E_1 - E_3 = \frac{5 + m_1^2 + m_2^2}{2\sqrt{2}}$
6. $E_{4,3} = \sqrt{s} = E_3 = \frac{5 + m_1^2 + m_2^2 + m_3^2}{2\sqrt{2}}$

The $E_3$ integration results at $m_3$, the $S$-function is an invariant, i.e., $E_3 \geq m_3$

1. $\leq 5 + m_1^2 + m_2^2 = 2\sqrt{2} m_3$
2. $(E_1 - m_3)^2 \geq m_3^3$

$S \geq m_3 + m_4$

This makes sense: The scattering can only happen if one has enough energy. At least one needs the rest energy of the final-state particles.

The momenta of particles 3 and 4 in the center-of-mass frame is

$$P_{cm4} = \sqrt{E_3^2 - m_3^2} = \sqrt{(s - \frac{m_3^2}{2}) - \frac{4E_1 E_2}{m_3^2}} = \frac{\sqrt{(s - m_3^2)(s - (m_3 + m_4)^2)}}{2\sqrt{2}}$$

Subject to $m_3 < m_4 + m_5$

Evaluating the $S$-function:

$$\frac{1}{8E_3} \left( (s - E_3 - E_{3,4}) \right) = \frac{1}{8E_3} \frac{E_3}{E_3} \left( E_3 - E_{3,4} \right) = \frac{E_3}{8E_3} \left( s(4s - E_3) + \frac{E_1^2}{E_1} s(4s - E_3) \right)$$
\[ T_x = \frac{1}{4\pi} \int d^2z \frac{1}{|z^2 - x^2|^{\frac{3}{2}}} S(z^2 - x^2) \]

\[ \frac{1}{2} \frac{P_2}{v_s} \left( \frac{T_x}{T_y} \right) \]

3. \[ \mathcal{O} = \frac{1}{4T_x} \{ v_x \cdot v_y \} \quad \text{if } \mathcal{O} \text{ constant} \]

we recall (page 34)

\[ 4 E_x E_y |v_x \cdot v_y| = 2 \sqrt{(1 - (v_x + v_y)^2)} \left( v_x + (v_x \cdot v_y)^2 \right) \]

expanding this expression with \( P_{e.x} \), we find

\[ 4 E_x E_y |v_x \cdot v_y| = 4 \sqrt{5} P_{e.x}^2 \]

with the momentum of particles 1 and 2 in the center of mass.

Assume:

\[ P_{e.m.} = \sqrt{\frac{(s - m_3 - m_4)^2}{2}} \]

\[ \mathcal{O} = \frac{1}{4T_x} \left( \frac{P_{e.m.}^2}{v_s^2} \right) \left( \frac{2}{3} \sqrt{5} \right) \]

\[ \mathcal{O} = \frac{1}{16 \pi s} \left( \frac{P_{e.m.}^2}{v_s^2} \right) \left( \frac{2}{3} \sqrt{5} \right) \]

for elastic scattering: \( P_{e.m.} = P_{e.m.} \)

\[ \mathcal{O} = \frac{1}{16 \pi s} \left( \frac{P_{e.m.}^2}{v_s^2} \right) \left( \frac{2}{3} \sqrt{5} \right) \]

If they are indistinguishable one counts all events twice

by integrating \( P_{e.m.} \) and \( P_{e.m.} \) over all momenta.

in this case

\[ \mathcal{O} = \frac{1}{32 \pi s} \left( \frac{P_{e.m.}^2}{v_s^2} \right) \left( \frac{2}{3} \sqrt{5} \right) \]