Fluid Flow

Flow of a fluid with particle density, $n$ and velocity field $\vec{v}$ through a volume, $V$. 
In time interval $dt$ all particles inside the volume element $dV = \vec{v}(\vec{r}) \, dS \, dt$ flow through surface.

- number of those particles: $dN = n(\vec{r}) \, dV$
- total number of particles per time, flowing out of $V$:

$$\frac{dN}{dt} = \text{flux} = \int_{\partial V} dS \, n(\vec{r}) \vec{v}(\vec{r}), \quad \partial V: \text{boundary of } V$$

Gauss’s Law (Proof for point particle outside $V$ I)

- split $\partial V$ in rectangular surface elements
- draw cone through surface elements $dS_1$ and $dS_2$

Gauss’s Law (Proof for point particle outside $V$ II)

- Surface element from $\partial V$ (blue) with its cone (black) compared to intersection of cone with sphere (red)

$$|d\vec{S}_2'| = \vec{i}_r d\vec{S}_2' = |d\vec{S}_2| \cos \theta = \vec{i}_r d\vec{S}_2$$