Problem 1 (30 points)

Physics 208 Quiz 1

Solution

January 23, 2008

(a) In the figure above, add the vectors $\vec{r}_1$ and $\vec{r}_2$ geometrically.

See Figure.

(b) What are the components of these vectors, $(x_1, y_1)$ and $(x_2, y_2)$, and the sum $\vec{r}_1 + \vec{r}_2$.

$(x_1, y_1) = (2, 1)$ m, $(x_1, y_1) = (-1/2, 2)$ m, $(x_1 + x_2, y_1 + y_2) = (3/2, 3)$ m.

Do not forget to write the units!

(c) If at the end points of the vectors are particles with masses $m_1$ and $m_2$, what is the gravitational force, $\vec{F}_{12}$, exerted by particle 1 on particle 2? [You do not need to give numbers, just the expression in terms of $\vec{r}_1$ and $\vec{r}_2$, the masses etc.]

$$\vec{F}_{12} = \gamma \frac{m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

Always remember that a force is a vector. You must have a difference of “position vectors” of the points and then take the magnitude of this vector to calculate the distance of the two points. In general $|\vec{r}_1 - \vec{r}_2| \neq r_1 - r_2$! The gravitational force is always attractive, $\gamma$ is Newton's Gravitation Constant. Do not mix it up with $1/(4\pi\epsilon)$ in Coulombs Law, which applies to the electric force between to charges at rest!
Problem 2 (70 points)

(a) A particle with mass, \(m\), moves along the trajectory

\[
\vec{r}(t) = R \cos(\omega t) \vec{i}_x + R \sin(\omega t) \vec{i}_y, \tag{1}
\]

where \(\vec{i}_x\) and \(\vec{i}_y\) are unit vectors, perpendicular to each other (giving a Cartesian coordinate system); \(t\) is time and \(\omega = \text{const}\). Show that the particle moves along a circle with the center in the origin of the coordinate system. What is the radius of this circle? **Hint:** Calculate the distance of the particle from the origin to show that it is constant with time!

\[
r(t) = |\vec{r}(t)| = \sqrt{[R \cos(\omega t)]^2 + [R \sin(\omega t)]^2} = R = \text{const}, \tag{2}
\]

because \((\cos \alpha)^2 + (\sin \alpha)^2 = 1\) for all \(\alpha\), i.e., the particle has always the same distance, \(R\), from the origin. This means it runs along a circle of radius, \(R\), with the center in the origin of the coordinate system.

(b) Calculate the velocity, \(\vec{v}(t)\), and the acceleration, \(\vec{a}(t)\), of the particle. Determine the magnitude of these quantities.

\[
\vec{v}(t) = -R \omega \sin(\omega t) \vec{i}_x + R \omega \cos(\omega t) \vec{i}_y, \quad v(t) = |\vec{v}(t)| = R \omega, \\
\vec{a}(t) = -R \omega^2 \cos(\omega t) \vec{i}_x - R \omega^2 \sin(\omega t) \vec{i}_y, \quad a(t) = |\vec{a}(t)| = R \omega^2. \tag{3}
\]

(c) What is the force \(\vec{F}(t)\) exerted on the particle?

According to Newton’s 2nd law

\[
\vec{F}(t) = m \vec{a}(t) = -m R \omega^2 \cos(\omega t) \vec{i}_x - m R \omega^2 \sin(\omega t) \vec{i}_y. \tag{4}
\]

(d) Can you express the force in terms of \(\vec{r}\)? What is its direction and magnitude?

\[
\vec{F} = -m \omega^2 [R \cos(\omega t) \vec{i}_x + R \sin(\omega t) \vec{i}_y] = -m \omega^2 \vec{r}. \tag{5}
\]

To make the particle run on a circle of radius, \(R\), one has to exert a force of magnitude

\[
F = |\vec{F}| = m \omega^2 |\vec{r}| = m R \omega^2 = \frac{m v^2}{R}, \tag{6}
\]

pointing towards the center of the circle. That force is known as the centripetal force.