Dilepton Production at SPS Energies

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   - $q\bar{q}$ annihilation in the QGP (thermal source)
   - Multi-pion processes (thermal source)
   - Meson t-channel exchange (thermal source)
   - $\rho$ decay after thermal freezeout
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Electromagnetic probes in heavy-ion collisions

- $\gamma, \ell^\pm$: no strong interactions
- reflect whole “history” of collision:
  - from pre-equilibrium phase
  - from thermalized medium
  - QGP and hot hadron gas
  - from VM decays after thermal freezeout

![Diagram showing electromagnetic probes in heavy-ion collisions](image)

Fig. by A. Drees
In-medium spectral functions and baryon effects

- **baryon effects** important $\leftrightarrow N_B + N_{\bar{B}}$ relevant (not $N_B - N_{\bar{B}}$)
  - large contribution to broadening of the peak
  - responsible for most of the strength at small $M$

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Medium Modifications of Hadrons
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Fireball and Thermodynamics

- **cylindrical fireball model**: \( V_{FB} = \pi (z_0 + v_{z0}t + \frac{az}{2}t^2) \left( \frac{a}{2}t^2 + r_0 \right)^2 \)

- **thermodynamics**:
  - isentropic expansion; \( S_{tot} \) fixed by \( N_{ch} \); \( T_c = T_{chem} = 175 \) MeV
  - \( T > T_c \): massless gas for QGP with \( N_f^{eff} = 2.3 \)
  - mixed phase: \( f_{HG}(t) = \left[ s_{c}^{QGP} - s(t) \right] / \left[ s_{c}^{QGP} - s_{c}^{HG} \right] \)
  - \( T < T_c \): hadron-resonance gas
  
  \[ \Rightarrow T(t), \mu_{baryon, meson}(t) \]

- **chemical freezeout**:
  - \( \mu_N^{chem} = 232 \) MeV
  - hadron ratios fixed
    \[ \Rightarrow \mu_N, \mu_\pi, \mu_K, \mu_\eta \] at fixed \( s/\rho_B = 27 \)

- **thermal freezeout**:
  - \( (T_{fo}, \mu_{\pi}^{fo}) \simeq (120, 80) \) MeV

Flow and particle/resonance distributions

- assume **local thermal equilibrium**: $T(t)$
- collective **radial flow**: $u(t, \vec{x}) = 1/\sqrt{1 - \vec{v}^2}(1, \vec{v})$
- $\vec{v}(t, \vec{x}) = a_{\perp} t \vec{x}_{\perp} / R(t)$
- phase-space distribution for hadrons

$$\frac{dN_i}{d^3\vec{p}d^3\vec{x}} = \frac{g_i}{(2\pi)^3} f_B/F \left( \frac{p \cdot u(t, \vec{x})}{T(t)} \right) \exp \left( \frac{\mu_i(t)}{T(t)} \right)$$

**NB:**
- covariant notation $d^3\vec{x}d^3\vec{p} = p_\mu d\sigma^\mu d^3\vec{p} / \sqrt{\vec{p}^2 + m^2}$
- $p u(t, \vec{x}) = \vec{p}_0$: energy of particle in rest frame of fluid cell
- phase-space distribution for **bosonic resonances**:

$$\frac{dN_i}{d^4p d^3\vec{x}} = \frac{g_i}{(2\pi)^4} f_B \left( \frac{p \cdot u(t, \vec{x})}{T(t)} \right) \exp \left( \frac{\mu_i(t)}{T(t)} \right) [ -2p_0 \text{Im} D_i(p) ]$$

$D_i(p)$: propagator of resonance, $A_i(p) = -2 \text{Im} D_i(p)$: spectral function
Radiation from thermal sources: $\rho$ decays

- model assumption: vector-meson dominance

$$\frac{dN^{(MT)}_{\rho \to l^+l^-}}{d^4x d^4q} = \frac{M}{q^0} \frac{\Gamma_{\rho \to l^+l^-}(M)}{d^3x} \frac{dN_{\rho}}{d^4q}$$

$$= -\frac{\alpha^2}{3\pi^3} \frac{L(M^2)}{M^2} \frac{m^4_\rho}{g^2_\rho} g_{\mu\nu} \text{Im} D_{\rho}^{\mu\nu}(M, \vec{q}) f_B \left( \frac{q \cdot u}{T(t)} \right) \exp \left( \frac{2\mu_\pi(t)}{T(t)} \right)$$

- special case of McLerran-Toimela (MT) formula
- $M^2 = q^2$: invariant mass, $M$, of dilepton pair
- $L(M^2) = (1 + 2m^2_l/M^2) \sqrt{1 - 4m^2_l/M^2}$: dilepton phase-space factor
- $D_{\rho}^{\mu\nu}(M, \vec{q})$: (four-transverse part of) in-medium $\rho$ propagator at given $T(t)$, $\mu_{\text{meson/baryon}}(t)$
- analogous for $\omega$ and $\phi$
Radiation from thermal sources: $q\bar{q}$ annihilation

- General: McLerran-Toimela formula

$$\frac{dN_{l^+l^-}^{(M^T)}}{d^4xd^4q} = -\frac{\alpha^2}{3\pi^3} \frac{L(M^2)}{M^2} g_{\mu\nu} \text{Im} \sum_i \Pi_{\text{em},i}(M, \vec{q}) f_B \left( \frac{q \cdot u}{T(t)} \right) \exp \left( \frac{\mu_i(t)}{T(t)} \right)$$

- $i$ enumerates partonic/hadronic sources of em. currents
- In-medium em. current-current correlation function

$$\Pi_{\text{em},i}^{\mu\nu} = i \int d^4x \exp(iqx) \Theta(x^0) \left\langle \left[ j_{\text{em},i}^\mu(x), j_{\text{em},i}^\nu(x) \right] \right\rangle$$

- In QGP phase: $q\bar{q}$ annihilation
- HTL improved electromagnetic current correlator

$$-i\Pi_{\text{em},\text{QGP}} = \gamma^*$$
Radiation from thermal sources: multi-\(\pi\) processes

- use vector/axial-vector correlators from \(\tau\)-decay data
- Dey-Eletsky-Ioffe mixing: \(\hat{\varepsilon} = 1/2 \varepsilon(T, \mu_\pi)/\varepsilon(T_c, 0)\)

\[
\Pi_V = (1 - \hat{\varepsilon}) \hat{\varepsilon} \Pi_{V,4\pi} + \frac{\hat{\varepsilon}}{2} \hat{\varepsilon} \Pi_{A,3\pi} + \frac{\hat{\varepsilon}}{2} (\hat{\varepsilon} \Pi_{V,4\pi} + \hat{\varepsilon} \Pi_{A,5\pi})
\]

- avoid double counting: leave out two-pion piece and \(a_1 \to \rho + \pi\) (already contained in \(\rho\) spectral function)
motivation: $q_T$ spectra too soft compared to NA60 data

thermal contributions not included in models so far

also for $\pi, a_1$
Radiation from thermal sources: Meson t-channel exchange

- t-channel exchange contributions become significant at high momenta
$\rho$ decay after thermal freezeout

- assume “sudden freezeout” at constant “lab time”: $t = t_{fo}$
- then Cooper-Frye formula with $d\sigma^\mu = (d^3\vec{x}, 0, 0, 0)$

$$\frac{dN_{\rho \to l^+l^-}^{(fo)}}{d^3\vec{x}d^4q} = \frac{\Gamma_{l^+l^-}}{\Gamma_{\rho}^{tot}} \frac{dN_i}{d^3\vec{x}d^4q}$$

$$= \frac{q_0}{M} \frac{1}{\Gamma_{\rho}^{tot}} \left[ \frac{dN_{\rho \to l^+l^-}^{(MT)}}{d^4xd^4q} \right]_{t=t_{fo}}$$

- use vacuum $\rho$ shape with in-medium width $\Gamma_{\rho}^{tot} \approx 260$ MeV
- NB: Momentum dependence for dilepton spectra from $\rho$ decays after thermal freezeout:

  like hadron spectra!

- $l^+l^-$ from thermal sources softer by Lorentz factor $M/q^0$
  compared to $l^+l^-$ from decay of freeze-out $\rho$'s
Decay of “primordial” $\rho$ mesons

- $\rho$ mesons, escaping from the fireball \textit{without thermalization}
- $pp$ data for initial $\rho$ spectra; Cronin effect via “Gaussian smearing”
- Schematic jet-quenching model

$$P_{\text{esc}} = \exp \left( - \int dt \sigma_{\text{abs}}^\rho(t) \varrho(t) \right),$$

$$\sigma_{\text{abs}}^\rho(t) = \begin{cases} 
\sigma_{\text{ph}} = 0.4 \text{ mb} & \text{for } t < q_0/m_\rho \tau_f \\
\sigma_{\text{had}} = 5 \text{ mb} & \text{for } t > q_0/m_\rho \tau_f 
\end{cases}$$

- check with pion $R_{AA}$ data
- “primordial $\rho$’s” + freezeout $\rho$’s
- hard $q_T$ spectra including jet quenching
Drell-Yan Annihilation and correlated charm decays

- invariant-mass spectrum for DY pairs

\[
\frac{dN_{DY}^{AA}}{dMdy} \bigg|_{b=0} = \frac{3}{4\pi R_0^2} A^{4/3} \frac{d\sigma_{NN}^{DY}}{dMdy}
\]

\[
\frac{d\sigma_{DY}^{NN}}{dMdy} = K \frac{8\pi\alpha}{9sM} \sum_{q=u,d,s} e_q^2 [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)]
\]

- parton distribution functions: GRV94LO
- higher-order effects
  - \(K\) factor
  - non-zero pair \(q_T\): for IMR and HMR fitted by Gaussian spectrum (NA50 procedure)
- extrapolation to LMR: constrained by photon point \(M \rightarrow 0\)
Drell-Yan Annihilation and correlated charm decays

- invariant-mass spectrum for DY pairs

\[
\left. \frac{dN_{AA}^{DY}}{dMdy} \right|_{b=0} = \frac{3}{4\pi R_0^2} A^{4/3} \frac{d\sigma_{NN}^{DY}}{dMdy}
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\frac{d\sigma_{NN}^{DY}}{dMdy} = K \frac{8\pi\alpha}{9sM} \sum_{q=u,d,s} e_q^2 \left[ q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2) \right]
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- parton distribution functions: GRV94LO

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- extrapolation to LMR: constrained by photon point \( M \rightarrow 0 \)

- Correlated decays of \( D \) and \( \bar{D} \) mesons
  - use data (provided by NA60 collaboration)
Invariant-mass spectra

- Fireball with “standard” EoS-A \( T_c = T_{\text{chem}} = 175 \text{ MeV} \)
- overall normalization ⇔ total fireball lifetime
- relative normalization of thermal radiation fixed by rates
- rates integrated over time, volume, \( \vec{q} \) including NA60 acceptance

- good description of data
Excess spectra: IMR and multi-pion contributions

- "$4\pi$ contributions" ($\pi + \omega, a_1 \rightarrow \mu^+ + \mu^-$)
- slightly enhanced by VA mixing
**Excess spectra: baryon effects**

- **in-medium VM spectral functions** without baryon effects
  - not enough broadening around $M = m_{\rho}^{\text{vac}}$
  - lack of strength at low mass $M \rightarrow m_{\text{thr}} = 2m_{\mu}$
Excess spectra: $q_T$ binning

Central In-In
$q_T < 0.5 \text{ GeV}$
$T_c = T_{ch} = 175 \text{ MeV}$

Central In-In
$q_T > 1.0 \text{ GeV}$
$T_c = T_{ch} = 175 \text{ MeV}$

Semicentral In-In
$q_T < 0.5 \text{ GeV}$
$T_c = T_{ch} = 175 \text{ MeV}$

Semicentral In-In
$q_T > 1.0 \text{ GeV}$
$T_c = T_{ch} = 175 \text{ MeV}$
$m_T$ spectra (central)

- fixed normalization in $0 \leq q_T \leq 0.5$ GeV bin
- satisfactory description of data
- high-mass bin slightly overestimated
- hard probes important for $q_T \geq 1$ GeV
$m_T$ spectra (semicentral)

- Theoretical spectra too soft in low-mass and $\rho$-region bin
- Room for more “primordial $\rho$’s”
Sensitivity to meson $t$-channel exchange contributions

- Use $2 \times$ of $\omega$-$t$ exchange to account for other mesons (e.g., $a_1$, $\pi$)
- Hardest among thermal sources
- Absolute strength not sufficient to resolve discrepancy with data
Sensitivity to $T_c$ and hadro-chemistry

- recent lattice QCD: $T_c \simeq 190\text{-}200$ MeV or $T_c \simeq 150\text{-}160$ MeV?
- thermal-model fits to hadron ratios: $T_{\text{chem}} \simeq 150\text{-}160$ MeV

**Graphs:****

- **EoS-A:** $T_c = T_{\text{chem}} = 175$ MeV
- **EoS-B:** $T_c = T_{\text{chem}} = 160$ MeV
- **EoS-C:** $T_c = 190$ MeV, $T_{\text{chem}} = 160$ MeV
  - $T_c \geq T \geq T_{\text{chem}}$: hadron gas in chemical equilibrium
  - keep fireball parameters the same (including life time)
Sensitivity to $T_c$ and hadro-chemistry

- recent lattice QCD: $T_c \approx 190\text{-}200 \text{ MeV}$ or $T_c \approx 150\text{-}160 \text{ MeV}$?
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- **EoS-A**: $T_c = T_{\text{chem}} = 175 \text{ MeV}$
- **EoS-B**: $T_c = T_{\text{chem}} = 160 \text{ MeV}$
- **EoS-C**: $T_c = 190 \text{ MeV}$, $T_{\text{chem}} = 160 \text{ MeV}$
  - $T_c \geq T \geq T_{\text{chem}}$: hadron gas in chemical equilibrium
- keep fireball parameters the same (including life time)
- mass spectra comparable to EoS-A ↔ slight enhancement of fireball lifetime
- in IMR QGP > multi-pion contribution
- higher hadronic temperatures ⇒ slightly harder $q_T$ spectra
- not enough to resolve discrepancy with data
- mass spectra comparable to EoS-A ↔ slight reduction of fireball lifetime
- in IMR multi-pion ≫ QGP contribution
- higher hadronic temperatures + high-density hadronic phase ⇒ harder $q_T$ spectra
- better agreement with data
EoS-B: QGP dominates over multi-pion radiation
- opposite in EoS-A and EoS-C
- multi-pion radiation dominantly from high-density hadronic phase
  
  reason: \( \frac{dN_{ll}}{dM dT} \propto \text{Im } \Pi_{em}(M, T) \exp(-M/T) T^{-5.5} \)
  
  radiation maximal for \( T = T_{\text{max}} = M/5.5 \)
  
  hadronic and partonic radiation “dual” for \( T \sim T_c \)
  
  compatible with chiral-symmetry restoration!
Inverse-slope analysis

- to extract $T_{\text{eff}}$ fit to
  \[ \frac{1}{q_T} \frac{dN}{dq_T} = \frac{1}{m_T} \frac{dN}{dm_T} = C \exp \left( -\frac{m_T}{T_{\text{eff}}} \right) \]

- fit of theoretical $q_T$ spectra: $1 \, \text{GeV} < q_T < 1.8 \, \text{GeV}$

- standard fireball acceleration: too soft $q_T$ spectra
- lower $T_c$ in EoS-B and EoS-C helps (higher hadronic temperatures)
- NB: here, Drell Yan contribution taken out
- enhance fireball acceleration to $a_\perp = 0.1c^2/fm$
- effective at all stages of fireball evolution
- agreement in IMR not spoiled $\Leftrightarrow$ dominated from earlier stages
- EoS-B harder $\Leftrightarrow$ relative contribution of harder freezeout $\rho$ decays vs. thermal $\rho$'s larger
sensitivity to contributions from meson $t$-channel exchange
- hardens low-mass region
- using vacuum $\rho$ in $t$-channel contribution: enhances slope in $\rho$ region

sensitivity to Drell-Yan contribution
- for IMR: describes effect seen in data (open vs. solid square data point)
- in LMR: too high around muon threshold $\Leftrightarrow$
  due to uncertainties in extrapolation to low $M$?!?
Slopes of thermal radiation

- use EoS-B with $a_\perp = 0.1c^2/fm$
- fit the dilepton slopes from thermal sources only

- freeze-out and primordial $\rho$'s crucial for $q_T > 1$ GeV
hadron slopes extracted by fit to freeze-out contribution

- $\eta$, $\omega$, and $\phi$ slopes described by EoS-A and smaller $a_\perp$
- $\rho$ slope by EoS-B with larger $a_\perp$
- possible resolution: “successive freezeout” $\Leftrightarrow$ vector mesons freeze out at different times, determined by interaction strength (i.e., width)
Conclusions and Outlook

- **dilepton spectra** ⇔ **in-medium em. current correlator**
- **model for dilepton sources**
  - radiation from **thermal sources**: QGP, $\rho$, $\omega$, $\phi$
  - $\rho$-decay after thermal freeze-out
  - decays of non-thermalized primordial $\rho$’s
  - Drell-Yan annihilation, correlated $D\bar{D}$ decays
Conclusions and Outlook

- **dilepton spectra** ⇔ **in-medium em. current correlator**
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- **invariant-mass spectra and medium effects**
  - excess yield dominated by radiation from **thermal sources**
  - baryons essential for **in-medium properties of vector mesons**
  - melting ρ with little mass shift robust signal! (independent of $T_c$)
  - IMR well described by scenarios with radiation dominated either by QGP or **multi-pion processes** (depending on EoS)
    - Reason: mostly from thermal radiation around
      - $160 \text{ MeV} \leq T \leq 190 \text{ MeV}$
      - “parton-hadron” duality of rates
      - compatible with chiral-symmetry restoration!
  - dimuons in In-In (NA60), Pb-Au (CERES/NA45), $\gamma$ in Pb-Pb (WA98)
Conclusions and Outlook

- fireball/freeze-out dynamics ⇔ $m_T$ spectra and effective slopes
  - “non-thermal sources” important for $q_T \gtrsim 1$ GeV
  - lower $T_c$ ⇒ higher hadronic temperatures ⇒ harder $q_T$ spectra
  - to describe measured effective slopes $a_\perp = 0.085 c^2/fm \rightarrow 0.1 c^2/fm$
  - off-equilibrium effects (viscous hydro)?

- Further developments
  - understand recent PHENIX results (large dilepton excess in LMR)
  - vector- should be complemented with axial-vector-spectral functions ($a_1$ as chiral partner of $\rho$)
  - constrained with IQCD via in-medium Weinberg chiral sum rules
  - direct connection to chiral phase transition!
Backup: parton-hadron duality of rates

- in-medium hadron gas matches with QGP
- similar results also for $\gamma$ rates
- “quark-hadron duality”!? 
- indirect evidence for chiral-symmetry restoration

![Graph showing dR_{ee}/dM^2 vs. M_{ee} for different temperatures with lines labeled as free HG, in-med HG, free QGP, and in-med QGP.]

Hendrik van Hees (Texas A&M University)
- good agreement also for dielectron spectra in 158 GeV Pb-Au
- allows further check of low-mass tail from baryon effects down to $M \to 2m_e$
Backup: Low-\(m_T\) rise

- observed low-\(q_T\) deviation from \(\frac{dN}{m_T dm_T} \propto \exp\left(-\frac{m_T}{T_{\text{eff}}}\right)\)
- asymptotic limit for \(M \gg T\) AND \(q_T \gg M\) for \(ll\) from fo-\(\rho\) decays
- for hadrons or radiation from freeze-out \(\rho\)'s:
  \[
  \frac{dN^{(\text{fo})}}{m_T dm_T} \propto \begin{cases} 
  \exp\left(-\frac{m_T}{T_{\text{eff}}}\right) & \text{for } q_T \gg M \gg T \\
  \sqrt{m_T} \exp\left(-\frac{m_T}{T_{\text{eff}}}\right) & \text{for } q_T \ll T \ll M.
  \end{cases}
  \]

- for radiation from thermal source:
  \[
  \frac{dN^{(\text{MT})}}{m_T dm_T} \propto \begin{cases} 
  \frac{1}{m_T} \exp\left(-\frac{m_T}{T_{\text{eff}}}\right) & \text{for } q_T \gg M \gg T \\
  \frac{1}{\sqrt{m_T}} \exp\left(-\frac{m_T}{T_{\text{eff}}}\right) & \text{for } q_T \ll T \ll M.
  \end{cases}
  \]

- with effective inverse slopes
  \[
  T_{\text{eff}} = T \sqrt{\frac{1 + \xi \beta^B}{1 - \xi \beta^B}} \quad (0 < \xi < 1), \quad T'_{\text{eff}} \simeq T + T - \frac{M}{2} \left\langle \beta^2(r)\right\rangle_r = T + \frac{M}{4} \beta^2_B.
  \]

- possibly also effect of Bose Enhancement due to \(\mu\)mesons
Backup: Low-$m_T$ rise

\[ \frac{dN}{d^3x \, dm_T} \text{ (a.u.)} \]

\[ m_T - M \text{ (GeV)} \]

M=0.4 GeV, T=0.15 GeV, $\beta_{\perp}^s=0.5$

M=2 GeV, T=0.15 GeV, $\beta_{\perp}^s=0.5$